(4)

(a) $E(Y_t^{(i)}) = E(X^{(i)}) + E(\varepsilon_t^{(i)}) = \frac{9}{2} + \frac{9}{2} = 9.$

(b)

$Cov(Y_t^{(i)}, Y_s^{(i)}) = \cancel{Cov\ X_t^{(i)}}\ Cov(X^{(i)} + \varepsilon_t^{(i)}, X^{(i)} + \varepsilon_s^{(i)})$

$= Var(X^{(i)}) + Cov(X^{(i)}, \varepsilon_t^{(i)}) + Cov(X^{(i)}, \varepsilon_s^{(i)}) + Cov(\varepsilon_t^{(i)}, \varepsilon_s^{(i)})$

($X^{(i)}$ and $\varepsilon_t^{(i)}$ are independent)

$= Var(X^{(i)}) + Cov(\varepsilon_t^{(i)}, \varepsilon_s^{(i)})$

$Var(X^{(i)}) = \cancel{E(X^{(i)2})}\ E(X^{(i)2}) - E(X^{(i)})^2 = \frac{51}{2} - \frac{81}{4} = \frac{21}{4}$

$\cancel{Cov(\varepsilon_t^{(i)}, \varepsilon_s^{(i)}) = \begin{cases} E(\varepsilon_t^{(i)2}) - \frac{51}{2} & \text{if } t=s \\ 0 & \text{if } t \neq s \end{cases}}$ ~~$Cov(\varepsilon_t^{(i)}$~~

$\cancel{Cov(Y_t^{(i)}, Y_s^{(i)}) = \begin{cases} 4 \\ \frac{21}{4} \end{cases}}$

$Cov(\varepsilon_t^{(i)}, \varepsilon_s^{(i)}) = \begin{cases} E(\varepsilon_t^{(i)2}) - E(\varepsilon_t^{(i)})^2 = \frac{51}{2} - \frac{81}{4} = \frac{21}{4} & \text{if } t=s \\ 0 & \text{if } t \neq s \end{cases}$

$r(t,s) = Cov(Y_t^{(i)}, Y_s^{(i)}) = \begin{cases} \frac{21}{2} & \text{if } t=s \\ \frac{21}{4} & \text{if } t \neq s \end{cases}$

(c) $r(t,s)$ only depends on $t-s$ instead of t. so ~~$r_{t,s}$~~ $Y_t^{(i)}$ is weakly stationary.

(d) $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} Y_t^{(i)} = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} (X^{(i)} + \varepsilon_t^{(i)}) = \left(\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t^{(i)}\right) + X^{(i)}$

$E(\varepsilon_t^{(i)}) = \frac{9}{2}$ and $\{\varepsilon_t^{i}\}_{t=1}^{T}$ are iid.

∵ By weak Law of Large Number

$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t^{(1)} \xrightarrow{P} E(\varepsilon_t^{(1)}) = \frac{9}{2}$.

by sum rule of convergence in probability,

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} Y_t^{(1)} = X^{(1)} + \frac{9}{2} \neq 9 = E(Y_t^{(1)})$$

so $\frac{1}{T}\sum_{t=1}^{T} Y_t^{(1)}$ is not consistent for M_t.

(e)

$$\frac{1}{T}\sum_{t=1}^{T} Y_t^{(1)} \xrightarrow{P} X^{(1)} + E(\varepsilon_t^{(1)}) = X^{(1)} + \frac{9}{2} \neq 9 = E(Y_t^{(1)})$$

so $Y_t^{(1)}$ is not ergodic for mean.