(a)

$$\Phi_1 = \begin{pmatrix} 0.98 & 0 & -0.04 \\ -0.04 & 0.71 & -0.03 \\ 0.66 & -0.5 & 0.66 \end{pmatrix}$$

check eigenvalues of Φ_1

$$\begin{vmatrix} 0.98-\lambda & 0 & -0.04 \\ -0.04 & 0.71-\lambda & -0.03 \\ 0.66 & -0.5 & 0.66-\lambda \end{vmatrix} = 0 \Rightarrow \begin{cases} \lambda_1 = 0.064 \\ \lambda_2 = 0.944 \\ \lambda_3 = 0.742 \end{cases}$$

so the process is covariance stationary

(b)

$$\begin{pmatrix} 1 & 0 & 0 \\ -a_{21} & 1 & 0 \\ -a_{31} & -a_{32} & 1 \end{pmatrix}\begin{pmatrix} d_t \\ \eta_t \\ r_t \end{pmatrix} = B_1 \begin{pmatrix} d_{t-1} \\ \eta_{t-1} \\ r_{t-1} \end{pmatrix} + u_t$$

by LDL decomposition.

$$\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0.7 & 1 & 0 \\ -1.8 & 0.5 & 1 \end{pmatrix}\begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 2.45 & 0 \\ 0 & 0 & 1.56 \end{pmatrix}\begin{pmatrix} 1 & 0.7 & -1.8 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}$$

Let $L = \begin{pmatrix} 1 & 0 & 0 \\ 0.7 & 1 & 0 \\ -1.8 & 0.5 & 1 \end{pmatrix}$

$$L^{-1}\begin{pmatrix} d_t \\ \eta_t \\ r_t \end{pmatrix} = L^{-1}\Phi_1 \begin{pmatrix} d_{t-1} \\ \eta_{t-1} \\ r_{t-1} \end{pmatrix} + L^{-1}\varepsilon_t.$$

$u_t = L^{-1}\varepsilon_t$, $\quad Var(u_t) = L^{-1}\varepsilon_t(L^{-1})' = L^{-1}\varepsilon_t(L')^{-1} = \underbrace{\begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 2.45 & 0 \\ 0 & 0 & 1.56 \end{pmatrix}}_{=D}$

$E(u_t) = E(L^{-1}\varepsilon_t) = 0$

$\forall t \neq s \quad E(u_t u_s') = E(L^{-1}\varepsilon_t \varepsilon_s'(L^{-1})') = L^{-1}E(\varepsilon_t\varepsilon_s')(L^{-1})' = 0. \Rightarrow u_t \sim WN(0, D)$

$$\begin{pmatrix} 1 & 0 & 0 \\ -0.7 & 1 & 0 \\ 2.15 & -0.5 & 1 \end{pmatrix} \begin{pmatrix} d_t \\ y_t \\ r_t \end{pmatrix} = \begin{pmatrix} 0.98 & 0 & -0.04 \\ -0.73 & 0.71 & 0 \\ 2.79 & -0.86 & -0.01 \end{pmatrix} \begin{pmatrix} d_{t-1} \\ y_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{pmatrix}$$

$$u_t = \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{pmatrix} \sim WN\left(0, \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.45 & 0 \\ 0 & 0 & 0.06 \end{pmatrix}\right)$$

equation by equation.

$$d_t = 0.98 d_{t-1} - 0.04 r_{t-1} + u_{1t}$$

$$y_t = 0.7 d_t - 0.73 d_{t-1} + 0.71 y_{t-1} + u_{2t}$$

$$r_t = -2.15 d_t + 0.5 y_t + 2.79 d_{t-1} - 0.86 y_{t-1} - 0.01 r_{t-1} + u_{3t}$$

(c)

$$\begin{pmatrix} 1 & 0 & 0 \\ -a_{21} & 1 & -a_{23} \\ -a_{31} & -a_{32} & 1 \end{pmatrix} \begin{pmatrix} d_t \\ y_t \\ r_t \end{pmatrix} = B_1 \begin{pmatrix} d_{t-1} \\ y_{t-1} \\ r_{t-1} \end{pmatrix} + u_t$$

We cannot have a structural VAR from reduced form estimation. The reason is that there are multiple Σ that satisfies.

$$\Omega = \Sigma D \Sigma'$$ where D is diagonal matrix and Σ^{-1} has the form $\begin{pmatrix} 1 & 0 & 0 \\ -a_{21} & 1 & -a_{23} \\ -a_{31} & -a_{32} & 1 \end{pmatrix}$

For example

$$\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0.56 & 1 & -0.6 \\ -3.7 & 2.5 & 1 \end{pmatrix} \begin{pmatrix} 0.15 & 0 & 0 \\ 0 & 0.31 & 0 \\ 0 & 0 & 1.22 \end{pmatrix} \begin{pmatrix} 1 & 0.56 & -3.7 \\ 0 & 1 & 2.5 \\ 0 & -0.6 & 1 \end{pmatrix}$$

then $B_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0.56 & 1 & -0.6 \\ -3.7 & 2.5 & 1 \end{pmatrix}^{-1}$, also we have

$$\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0.79 & 1 & -1.8 \\ -2.2 & 3.9 & 1 \end{pmatrix} \begin{pmatrix} 0.23 & 0 & 0 \\ 0 & 0.91 & 0 \\ 0 & 0 & 2.4 \end{pmatrix} \begin{pmatrix} 1 & 0.79 & -2.2 \\ 0 & 1 & 3.9 \\ 0 & -1.8 & 1 \end{pmatrix} \quad B_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0.79 & 1 & -1.8 \\ -2.2 & 3.9 & 1 \end{pmatrix}^{-1}$$