Bootcamp Asset Pricing

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Contents:

1 Introduction

These notes are an introduction to two extremes in the continuum of asset pricing models. The first model, the Lucas Tree model of Lucas (1978) looks at asset prices from a very high level, abstracting away (through the use of general equilibrium) the mechanism that generates prices, and thus allowing the modeler to focus on the behavioral (i.e. preference)

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components of market participants. It is a workhorse for modern finance because it can be used to understand the profit motive of investors, the importance of risk aversion and the (related) demand for diversification by investors, among other important phenomena. However, the Lucas model is quite difficult (and often intractable) when dealing with investors who have heterogeneous beliefs or preferences and, since it typically relies at least in part on consumption data, its use at frequencies higher than monthly has been less extensive.

The second model, elaborated by Kyle (1985), focuses on the specifics of the generation of market prices. By focusing on the market making process, it provides insight into what it means for the price of an asset to be the value P, and how that price changes in the short term if the information available to traders changes. It is also a commonly used model because it allows for heterogeneous information amongst traders, takes the price generation process seriously and can be applied to asset price data even at very high frequencies. However, it is fundamentally a static model so the only link between prices intertemporally is information. It is silent (at least in its most commonly used form) on the role of risk aversion and diversification and does not attempt to link investment strategy with a broader household objective function, beyond the assumption that investors maximize expected profit.

I discuss these two models by focusing on the question ‘‘What empirical facts can we learn about people by studying the prices of assets that we observe in the market.” Personally, I find this to be one of the most interesting questions in economics, and a prerequisite to understanding how to make good policy or make money in the marketplace.

In the marketplace, mutual funds, hedge funds and high-frequency traders must, beyond their study of the properties of the companies whose stocks they trade, also understand how individuals, households and institutions will react to new information, new regulatory environments and new market structures. Despite what is sometimes suggested in the popular press, the only incontrovertable truth about asset prices is that they are determined by supply and demand. A money manager hoping to understand how prices will move must understand how the properties of human participants affect the demand for those assets.

Similarly, a policy maker cannot hope to make effective policies without understanding the nature of the people who will be affected by them. Recent policy debates about (*inter alia*) market circuit breakers, incentivizing long-term saving through differing long- and short-term capital gains tax rates and the question of privatizing social security rely on the kinds of answers that can be had from an in-depth study of these two models.

2 The Lucas Tree Model of Asset Pricing

2.1 The probability space

We start with a state-space representation of the uncertainty in this model.

Suppose that there is a finite set Z of possible states that can occur each period. The state space is $\Omega = Z^{\infty}$, the set of infinite sequences of elements of Z. So a particular state $\omega \in \Omega$

is a path $\omega = (z_1, z_2, z_3, \ldots, z_t, \ldots)$.

Note This is not the most general formulation of this problem, but will give all of the results that we need for our goals. Specifically, the state-space representation we give can be generalized to an arbitrary probability space with a filtration that satisfies certain properties.

We will assume that many random variables are measurable with respect to the natural filtration generated by the space Ω at a particular time t.

Definition A random variable x is *measurable* with respect to the partition generated by Ω at time t if whenever $\omega_\tau = \omega'_\tau$ for all $\tau \leq t$, $x(\omega) = x(\omega')$. That is, x is measurable with respect to the partition generated by Ω at time t if whenever two paths are identical up to time t, the random variable x is the same across those two paths.

This property defines what can be known at any given time. If we require a random variable to be measurable with respect to the time t partition, we are saying that it can't depend on things that happen after time t.

Example Let $W = [1, 2, 3]$. Consider two paths $\omega = (1, 2, 3, \ldots)$ and $\omega' = (1, 2, 1, \ldots)$. Define the random variables $x = \omega_1 + \omega_2$ and $x' = \omega_1 + \omega_2 + \omega_3$. These random variables take on the following values

$$x(\omega) = 3$$
$$x(\omega') = 3$$

and

$$x'(\omega) = 6$$
$$x'(\omega') = 4$$

The random variable x is measurable with respect to the partition generated by Ω at times 1, 2 and 3. The random variable x' is not measurable with respect to the partition generated by Ω at times 1 and 2, but is measurable with respect to the partition generated at time 3. x' is not measurable with respect to the time 2 partition because ω and ω' are the same up to time 2 and yet x' assigned different values to these two paths. Note that x is measurable with respect to the time 3 partition because the previous definition does not require that a random variable take on different values across paths that differ.

Definition The sequence of partitions generated by Ω is called a *filtration*.

Definition A sequence of random variables $(x^1, x^2, \ldots)$ where each x^τ is measurable with respect to the partition generated by Ω at time τ is said to be *adapted to the filtration generated by* Ω.

A sequence of random variables adapted to the filtration generated by Ω has the property that its values at time t depend only on things that have happened up to time t. This is a property that we will require of most of the random sequences that we study. So much so, that we use the notation $x = (x_1, x_2, x_3, \ldots)$ to denote random variables that are adapted to the filtration generated by our space Ω.

A probability distribution μ is a function from the measurable sets into the real line. For the kinds of models studied here, one can think of probabilities as mapping from *cylinders* of paths, which are sets of the form $\{\omega : (\omega_1, \omega_2, \omega_3, \ldots, \omega_t) = (z_1, z_2, \ldots, z_t)\}$. These probabilities will correspond to the probability of reaching a node on the event tree (in the definition above, the node whose first t realizations are $(z_1, z_2, \ldots, z_t)$. Expectation of a random variable x, given the partition Ω_t is then taken in the usual way,

$$Ex = \sum_{w \in \Omega_t} x(w)\mu(w)$$

If x is adapted to the partition Ω_t, then $x(\omega) = x(\omega')$ for any $\omega, \omega' \in w \in \Omega_t$, so $x(w)$ in the above expression is defined to be the value of x along any path in the cylinder defined by w.

2.2 The Investor's Problem

Consider an investor who has preferences over her lifetime consumption that take the form

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t(\omega))\right]. \tag{1}$$

This expectation is taken over the set of paths Ω and (as noted above) the random sequence $c = (c_0, c_1, \ldots)$ is assumed to be adapted to the filtration generated by Ω. The sequences $\{c_t\}_{t=0}^{\infty}$ are stochastic and depend on the realization of uncertainty and the investment choices of the individual in a way that will become clear.

Note We will assume that $u(\cdot)$ is strictly concave so that $E\left[\sum_{t=0}^{\infty} \beta^t u(c_t(\omega))\right]$ is also strictly concave. This allows us to dispense with second order conditions.

The individual has access to assets labeled $1, \ldots, N$. Each asset n pays a dividend d_{nt} at time t that is also assumed to be stochastic and is paid in units of consumption at time t. The investor starts at time 0 with shares of the assets given by $\overline{\boldsymbol{\theta}_0} = \left(\overline{\theta_1}, \overline{\theta_2}, \ldots, \overline{\theta_n}\right)'$. Asset holdings at the end of time t and the beginning of time $t+1$ are $\boldsymbol{\theta}_t$.

The prices of the assets at time t are given by $\boldsymbol{p}_t = (p_{1t}, p_{2t}, \ldots, p_{nt})'$ (all vectors in these notes are assumed to be column vectors unless otherwise stated). We assume that these prices are expressed in terms of units of the consumption good available at time t.

The budget constraint of the investor at time 0 is

$$c_0 + \boldsymbol{p}_1' \overline{\boldsymbol{\theta}_0} = \boldsymbol{d_0'} \overline{\boldsymbol{\theta}} + \boldsymbol{p_0'} \overline{\boldsymbol{\theta}}.$$

The budget constraint at time t is given by

$$c_t + \boldsymbol{p}_t' \boldsymbol{\theta}_t = (\boldsymbol{d_t'} + \boldsymbol{p_t'}) \boldsymbol{\theta}_{t-1}.$$

Rewriting this we obtain the equation

$$c_t = \boldsymbol{d}_t'\boldsymbol{\theta}_t + \boldsymbol{p}_t'\left(\boldsymbol{\theta}_{t-1} - \boldsymbol{\theta}_t\right). \tag{2}$$

Notice that all of the random variables (and random vectors) in the problem above depend implicitly on the realization ω and are assumed to be adapted to the filtration generated by Ω.

After plugging the budget constraint in equation (2) into the problem in equation (1) we are left with the problem

$$\max_{\boldsymbol{\theta}_0,\boldsymbol{\theta}_1,\ldots} E\left[\sum_{t=0}^{\infty}\beta^t u\left(\boldsymbol{d}_t'\boldsymbol{\theta}_t + \boldsymbol{p}_t'\left(\boldsymbol{\theta}_{t-1} - \boldsymbol{\theta}_t\right)\right)\right].$$

Consider the first order conditions for this problem with respect to $\boldsymbol{\theta}_t$. These are

$$E\left[\beta^t u'(c_t)\left(\boldsymbol{d}_t - \boldsymbol{p}_t\right) + \beta^{t+1}u'(c_{t+1})\boldsymbol{p}_{t+1}\right] = 0$$

which can be simplified to

$$u'(c_t)\left(\boldsymbol{d}_t - \boldsymbol{p}_t\right) + \beta E\left[u'(c_{t+1})\boldsymbol{p}_{t+1}|\Omega_t\right] = 0.$$

Consider the row corresponding to asset i of these vector Euler conditions. It is

$$u'(c_t)\left(d_{i,t} - p_{i,t}\right) + \beta E\left[u'(c_{t+1})p_{i,t+1}|\Omega_t\right] = 0.$$

Solving this equation for the price of asset i at time t we obtain the relationship

$$p_{i,t} = \beta E\left[\frac{u'(c_t+1)}{u'(c_t)}(d_{i,t+1} + p_{i,t+1})|\Omega_t\right]. \tag{3}$$

This fundamental relationship allows us to answer lots of questions about how the value of assets is determined. We will look at this relationship in several different ways.

2.3 The stochastic discount factor (SDF)

Let the adapted random variable m_{t+1} be defined as

$$m_{t+1} = \beta\frac{u'(c_{t+1})}{u'(c_t)}.$$

This sequence of random variables $m = (m_1, m_2, \ldots)$ is called the *stochastic discount factor* or SDF. Notice that with this definition, equation (3) can be written as

$$p_{i,t} = E[m_{t+1}(p_{i,t+1} + d_{i,t+1})|\Omega_t]. \tag{4}$$

The SDF has intuitive meaning. To understand it, lets first divide both sides of the previous equation by $p_{i,t}$ to get

$$1 = E\left[m_{t+1}\left(\frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}}\right)|\Omega_t\right].$$

Define the expression

$$R_{i,t+1} = \frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}}$$

covariance of X and Y is $cov(X, Y) = E(XY) - E(X)(Y)$. Thus, we can rewrite equation (4) as

$$1 = cov(m_{t+1}, R_{i,t+1}) + E[m_{t+1}|\Omega_t]E[R_{i,t+1}|\Omega_t] \tag{5}$$

Since the SDF is the ratio of marginal utilities, it is large in states next period when marginal utility is high compared to today. Under standard assumptions of decreasing marginal utility, that means that the SDF is big in states where consumption is small. Likewise, in states where consumption is large, the SDF is small. Therefore, the term $cov(m_{t+1}, R_{i,t+1})$ will be large if *on average* the return to asset i is large when marginal utility next period is large. That is, $cov(m_{t+1}, R_{i,t+1})$ will be large if on average the return to asset i is high when consumption next period is low. This result leads to the common practice in the literature of calling $-cov(m_{t+1}, R_{i,t+1})$ the *risk* of asset i.

Now consider the previous equation for asset j. Combining it with the equation for asset i gives the relationship

$$cov(m_{t+1}, R_{i,t+1}) + E[m_{t+1}|\Omega_t]E[R_{i,t+1}|\Omega_t] = cov(m_{t+1}, R_{j,t+1}) + E[m_{t+1}|\Omega_t]E[R_{j,t+1}|\Omega_t] \tag{6}$$

This demonstrates a fundamental relationship between the expected return to an asset and that asset's covariance with the investor's marginal utility. Since the term $E[u'(c_{t+1})|\Omega_t]$ is common across assets i and j, if asset i has a higher expected return than asset j then the investor will select a portfolio such that the covariance of asset i with marginal utility (the SDF) is low. That is, assets with high risk must also have high returns. In other words, the individual will choose a portfolio where the expected return of each asset exactly compensates the trader for the risk born in holding the asset.

2.4 The risk-free rate of return

Using the SDF, we can calculate several things that are of interest. Consider first the price of an asset that pays one unit of consumption next period in every state of the world. Following that, the asset ceases to exist. The price of that asset today will allow us to calculate the risk-free rate of return in this economy from period t to period $t+1$. From equation (4), the price today of such an asset would be

$$p_{i,t} = E[m_{t+1}|\Omega_t].$$

Thus, one unit of consumption tomorrow costs $E[m_{t+1}|\Omega_t]$ today. Therefore, the (gross) risk-free rate of return today is

$$1 + r_f = \frac{1}{E[m_{t+1}|\Omega_t]}. \tag{7}$$

This relationship demonstrates the reason behind the name SDF. The present value of one unit consumption tomorrow is

$$E[m_{t+1}|\Omega_t],$$

while in general the present value of any random payoff x tomorrow is

$$p_t = E[m_{t+1}x|\Omega_t].$$

Armed with this definition, we can now reinterpret equation (6). Rewriting, this equation becomes

$$cov(m_{t+1}, R_{i,t+1}) + \frac{E[R_{i,t+1}|\Omega_t]}{1+r_f} = cov(m_{t+1}, R_{j,t+1}) + \frac{E[R_{j,t+1}|\Omega_t]}{1+r_f}.$$

Expressions of the form $\frac{E[R_{i,t+1}|\Omega_t]}{1+r_f}$ can now be interpreted as the *risk-premium* associated with asset i. That is, the expected return above the return that one would obtain for investing in a risk-free asset. This equation then says that assets with high risk (assets for which $-cov(m_{t+1}, R_{i,t+1})$ is small) will have high expected returns when the trader is holding an optimal portfolio.

2.5 Exercises

These exercises will give some familiarity with the GMM estimation of asset pricing models. Assume that the joint distribution of (m_{t+1}, R_{t+1}) is constant for all t (this would occur, for example, if the stochastic process ω is Markov.) Define $m_{t+1} = \beta\frac{u'(c_{t+1})}{u'(c_t)}$ with $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Download data on real personal consumption expenditures (FRED series PCECC96) and returns to the aggregate U.S. stock market (e.g. the Wilshire 5000 given in FRED series

WILL5000INDFC) as well as the risk-free rate (FRED series GDPDEF, the GDP deflator, is one possible measure of inflation). One possibility for data on the risk-free rate is the 3-month T-Bill rate given in FRED series TB3MS. Note that you will need to transform the Wilshire 5000 series and the T-Bill series into quarterly data as well as calculate the real returns to the Wilshire 5000 by accounting for inflation.

Answer the following questions:

1. Assume that $\beta = 0.99$ (since this is quarterly data, this implies an annual risk-free rate of about 4%). Calculate empirical analogs of the expected values and covariance in (5). Using `scipy.optimize.broyden1`, solve for the value of γ that leads to equation (5) holding.
2. How would you determine if this parameter is reasonable?
3. Now, consider the moment restriction given in equation (7). Continue to assume that $\beta = 0.99$. Estimate the γ that generates the SDF using only this moment condition. How does this γ differ from the estimate of γ found previously?
4. Finally, estimate *beta* and *gamma* using both equation (5) and equation (7). You will need to specify a weighting matrix (the identity matrix is a great place to start). Plot the objective function for different (β, γ) pairs. How does your estimation differ from the previous two ways of estimating those parameters?

Note. The idea behind these problems comes from Hansen and Singleton (1982), although the implementation has some differences. For a survey of this literature, see Campbell (2003).

3 Orderbooks and Asset Pricing

3.1 The Kyle model of orderbook shape

This model of the orderbook is given in Kyle (1985). We have uninformed traders, informed traders (or insiders in Kyle's language) and market makers. Uninformed traders have random demand that has mean 0 and a known variance. Let the demand for uninformed traders be given by

$$u \sim N(0, \sigma_u^2).$$

Informed traders observe the future value of the asset v and select their demand x as a function of this observed v and the price set by the market makers. Assume that

$$v \sim N(p_0, \Sigma_0).$$

A competitive set of market makers establish an order book parameterized as a price that is a function of the number of orders that they observe $y = x + u$. Since these market makers are competitive, they will in expectation earn zero profit. It is assumed that market makers cannot distinguish between informed and uninformed liquidity requests.

The expected profit to a market maker given the demand for liquidity y is

$$E[\pi|y] = E[(v-p)(-y)|y] = -yE[v-p|y]$$

(recall that the market maker must take the opposite position of the liquidity demanders, who have position y). If expected profits are to be zero for every y, then in equilibrium $p = E[v|y]$. In order to find an equilibrium in this model, we proceed by making an assumption about the behavior of market makers, which at the end we verify to be true. Let us assume that market makers set the price according to a linear function of the demand that they observe. Specifically, assume that the equilibrium price is given by a function of the formula

$$P(y) = \mu + \lambda y$$

Now, we consider the problem of informed traders. The expected profit to informed traders is

$$E\pi = E[(v - p(y))x|v].$$

In order to maximize this profit, informed traders solve (given the price function of market makers)

$$\max_x E[\pi] = \max_x E[(v - \mu - \lambda(x+u))x|v]$$

We start by passing the expectation into this equation to get (we note that v, μ and λ are assumed to be known in equilibrium)

$$\max_x vx - \mu x - \lambda x^2$$

(Recall that $Eu = 0$). The first order condition for this problem (which can be verified to be strictly concave) is

$$v - \mu - 2\lambda x = 0$$

This has the solution $x = -\frac{\mu}{2\lambda} + \frac{1}{2\lambda}v$. Notice if the market makers use a linear pricing rule, then the informed traders will set their demand as a linear function of the signal that they receive.

Now we go back to the problem of market makers. Given that the demand that they observe is $y = x + u$, they need to calculate $E[v|x+u]$. Note that v and y are related since v determines x which is a part of y. In fact, since y is a linear function of v, y and v are jointly normally distributed. The demand y is $y \sim N(-\frac{\mu}{2\lambda} + \frac{1}{\lambda}p_0, \frac{1}{\lambda^2}\Sigma_0)$. The covariance between y and v is

$$
\begin{aligned}
E(y-Ey)(v-Ev) &= E(-\frac{\mu}{2\lambda}+\frac{1}{2\lambda}v+u+\frac{\mu}{2\lambda}+\frac{1}{2\lambda}p_0)(v-p_0) \\
&= E(\frac{1}{2\lambda}(v-p_0))(v-p_0) \\
&= \frac{1}{2\lambda}\Sigma_0
\end{aligned}
$$

For two jointly normal random variables (a,b), $E[a|b] = Ea + \frac{cov(a,b)}{\sigma_b^2}(b-Eb)$. Applying this formula to y and v gives

$$
E[v|y] = p_0 + \frac{\frac{1}{2\lambda}\Sigma_0}{\frac{1}{4\lambda^2}\Sigma_0+\sigma_u^2}(y+\frac{\mu}{2\lambda}-\frac{1}{2\lambda}p_0).
$$

Now, recall our original assumption that $P(y) = \mu + \lambda y$. Since $P(y) = E[v|y]$, we can compare coefficients and we notice that the coefficient on y implies that

$$
\lambda = \frac{\frac{1}{2\lambda}\Sigma_0}{\frac{1}{4\lambda^2}\Sigma_0+\sigma_u^2}
$$

Solving this, for λ produces the result that

$$
\lambda = \frac{\sqrt{\Sigma_0}}{2\sigma_u}
$$

Furthermore, it can be seen by inspection that $\mu = p_0$ is a solution to that equation.

Therefore, an equilibrium price function of this game is

$$
P(y) = p_0 + \frac{\sqrt{\Sigma_0}}{2\sigma_u}y.
$$

3.2 Interpreting the model

Traditionally, orderbooks are drawn on a two-dimensional graph with the price on the horizontal axis and the number of units available (to buy or sell) on the vertical axis.

To place the equilibrium above in a manner similar to the usual picture, we solve to y to get

$$
y(P) = \frac{2\sigma_u}{\sqrt{\Sigma_0}}(P-p_0) \tag{8}
$$

Notice, that for prices greater than the market makers' prior mean about the value of the asset (p_0), the market maker's position will be positive (i.e. when the market maker sells $y > 0$) and when the market maker buys $y < 0$. As such, the common picture of the orderbook is

$$OB(P) = \left| \frac{2\sigma_u}{\sqrt{\Sigma_0}}(P - p_0) \right|$$

However, we will work with the easier to deal with version given in (8). Let's make a few observations about this orderbook.

Properties of the Kyle Orderbook

1. The BBO (sometimes just called the spread, or price) will occur at $P = p_0$.
2. Relatively "flat" or "thin" orderbooks arise when the standard deviation of noise trader demand is small relative to the standard deviation of informed trader demand. This makes sense, since on average large swings in demand will be caused by informed traders in this scenario. Market makers are concerned about trading with informed traders, so they offer relatively small amounts of shares near the BBO (best bid and offer). Similarly, "steep" of "thick" orderbooks arise when there is a lot of noise and relatively little variance in informed trader demand.
3. In this model, all changes in orderbooks can be thought of as changes in beliefs in the fundamental value of the asset (changes in p_0) and changes in beliefs about the properties of traders (changes in $\frac{2\sigma_u}{\sqrt{\Sigma_0}}$.)

3.3 Estimating the parameters of the model

Equation (8), can be estimated by way of linear regression. If we transform equation (8) into slope intercept form we get

$$y = -p_0 \frac{2\sigma_u}{\sqrt{\Sigma_0}} + \frac{2\sigma_u}{\sqrt{\Sigma_0}} P \tag{9}$$

So, estimating the equation

$$y = \gamma + \beta P \tag{10}$$

using the price and quantity amounts from the orderbook, we can back out the parameters p_0 and $\frac{2\sigma_u}{\sqrt{\Sigma_0}}$ by solving

$$\hat{\gamma} = -p_0 \frac{2\sigma_u}{\sqrt{\Sigma_0}}$$
$$\hat{\beta} = \frac{2\sigma_u}{\sqrt{\Sigma_0}}$$

for p_0 and $\frac{2\sigma_u}{\sqrt{\Sigma_0}}$ which yields

$$p_0 = -\frac{\hat{\gamma}}{\hat{\beta}}$$
$$\frac{2\sigma_u}{\sqrt{\Sigma_0}} = \hat{\beta}$$

Note. For hypothesis testing, care must be taken here since *if* $\hat{\gamma}$ and $\hat{\beta}$ have a t-distribution then the distribution of the estimate of $\frac{2\sigma_u}{\sqrt{\Sigma_0}}$ is the ratio of two t-distributions which is non-trivial.[1]

3.4 Implications of the model

By looking at comparative statics on equation (9) we see that changes in the midpoint represent changes in the beliefs of market makers about p_0. Changes in the slope of the orderbook relate to changes in market maker beliefs about the ratio $\frac{2\sigma_u}{\sqrt{\Sigma_0}}$. Thus, changes in the BBO reflect (according to this model) changes in the value of the asset directly, whereas changes in the slope reflect changes in the relative volatility of trade between informed and uninformed traders.

Consider a simple model of market maker learning about the true value of the asset. Market makers have prior beliefs that the asset is distributed $v \sim N(p_0, \Sigma_0)$. Suppose that they receive a public signal about the asset of the form $s = v + \epsilon$ where ϵ is independent of v and distributed $\epsilon \sum N(0, 1/\rho_\epsilon)$. Let $\rho = 1/\Sigma_0$. Then,

$$v|s \sim N\left(\frac{\rho p_0 + \rho_\epsilon s}{\rho + \rho_\epsilon}, \rho + \rho_\epsilon\right) \tag{11}$$

If a sequence $s_1, s_2, \ldots, s_t$ have been received, each i.i.d., then the updated beliefs of the market makers are

$$v|s_1, \ldots, s_t \sim N\left(\frac{\rho p_0 + \rho_\epsilon \left(\sum_{i=1}^t s_i\right)}{\rho + t\rho_\epsilon}, \rho + \rho_\epsilon\right) \tag{12}$$

Here, as additional signals are received *for the same prior belief*, the precision of the beliefs goes up and the variance of the market makers' beliefs about the asset's worth goes down. Because of this, the slope of the orderbook and the liquidity of the market go up. Likewise, this model implies that if one observes the market become less liquid, that must mean that the signals now being received by market makers convey information about a different distribution that what was originally believed to be true about v.

[1] See Press (1969) (https://www.jstor.org/stable/2283732?seq=1#page_scan_tab_contents) for details.

3.5 Exercises

These exercises are intended to help you use the Kyle model to understand the orderbooks of 5 tickers on a particular day (July 11, 2017). The data have been compiled from the NASDAQ ITCH data files for those days. The data are Python pickle files where each file contains a list of the following form `[(timestamp1,{P1: Q1, P2: Q2, P3:Q3, ... }), (timestamp2, {P1: Q1, P2, Q2, ...})... ]`. That is, the data comes as a list of tuples, where each tuple contains a timestamp, and an orderbook, where the orderbook is a dictionary where the keys are prices and the values are the quantities available at each price. These data have been compiled in such a way that we have the orderbook every 60 seconds throughtout the trading day.

1. Write a Python class that represents a single orderbook, where the data for that single orderbook is an instance property. Write a method for that orderbook that will use the data to the parameters γ and β from equation (10). Plot the orderbook and the estimated equation. Where does the model fit well and where does it not? Might one want to estimate the model using only a subset of the orderbook? If so, where?
2. For each ticker, plot the estimated values of $\frac{2\sigma_u}{\sqrt{\Sigma_0}}$ throughout the day.
3. Compare the estimates of p_0 to the actual midpoint of the BBO over time. What do these differences say about the nature of the orderbook and the estimation procedure you're using?
4. Compare/contrast the results for each ticker. Are the noise/signal ratios that you calculate consistent with what you would have expected? What about the movements in p_0?