Valuation Dynamics

in Models with Financial Frictions

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Research Objective

Compare/contrast implications of DSGE models with financial frictions through the study of their nonlinear transition mechanisms

- Environment
  - Continuous time with Brownian shocks
  - Financial intermediaries
  - Heterogeneous productivity or market access

- Comparison Targets
  - Macroeconomic quantity implications
  - Asset pricing implications
  - Macro- and micro-prudential policies

Challenges

Model features:

- ▷ Nonlinear transition mechanism
- ▷ Some shock configurations have big consequences
- ▷ Endogenous transitions across two regimes: (one where sector wide constraints are binding and another when they are not)

Model assessments:

- ▷ Alter or extend linear methods of analysis
- ▷ Perform cross model comparisons

Approaches

- Opening the black box: structural approach hold fixed some aspects of the economic environment (including parameters) while changing others and exploring implications
- Imposing observational constraints: holding fixed some implications and changing parameters accordingly to match these while altering the economic environment

"Nesting" Model

- Technology

  Technology

  - A-K production function with $a_e \geq a_h$ and adjustment costs
  - (total factor) productivity shocks
  - growth rate and stochastic vol shocks (long-run risk)
  - idiosyncratic shocks

- Markets
  - capital traded with shorting constraint at a price Q_t
  - experts face a skin-in-the-game constraint where the fraction of held capital, χ_t, is restricted ($\chi_t \geq \underline{\chi}$)

- Preferences
  - recursive utility, discount rate δ, IES $\frac{1}{\rho}$, and risk aversion γ
  - different preferences with $\gamma_h \geq \gamma_e$
  - OLG for technical reasons

Models Nested

- Complete markets with long run risk

  Bansal & Yaron (2004)
  Hansen, Heaton & Li (2008)
  Eberly & Wang (2011)
- Complete markets with heterogeneous preferences

  Longstaff & Wang (2012)
  Garleanu & Panageas (2015)
- Incomplete market/limited participation

  Basak & Cuoco (1998)
  Kogan & Makarov & Uppal (2007)
  He & Krishnamurthy (2012)
- Incomplete market/capital misallocation

  Brunnermeier & Sannikov (2014)
- Complete markets for aggregate risk and stochastic volatility

  Di Tella (2017)

Diagnostic Tools I

- Quantities
  - consumption/wealth ratio
  - investment rate
  - output growth
- Prices
  - risk-free rate
  - risk-price vectors (one per agent)
  - capital price
- State dynamics
  - drift and diffusion of the aggregate state vector
  - ergodic density of state vector

Models of Asset Valuation

Two channels:

- Stochastic growth modeled as a process $G = \{G_t\}$ where G_t captures growth between dates zero and t.
- Stochastic discounting modeled as a process $S = \{S_t\}$ where S_t assigns risk-adjusted prices to cash flows at date t.

Date zero prices of a payoff G_t are

$$\pi = \mathbb{E}\left(S_t G_t | \mathcal{F}_0\right)$$

where $\mathcal{F}_0$ captures current period information.

Stochastic discounting reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors.

Impulse Problem

Ragnar Frisch (1933):

> *There are several alternative ways in which one may approach the impulse problem .... One way which I believe is particularly fruitful and promising is to study what would become of the solution of a determinate dynamic system if it were exposed to a stream of erratic shocks that constantly upsets the continuous evolution, and by so doing introduces into the system the energy necessary to maintain the swings.*

Irving Fisher (1930):

> *The manner in which risk operates upon time preference will differ, among other things, according to the particular periods in the future to which the risk applies.*

Impulse Response Functions for Linear Models

- Model:

$$Y_{t+1} - Y_t = \mathbb{D} \cdot Z_t + \mathbb{F} \cdot W_{t+1}$$
$$Z_{t+1} = \mathbb{A} Z_t + \mathbb{B} W_{t+1}$$

  where W_{t+1} is a multivariate standard normal independent of date t information.

- What is the impact of W_1 on the future of Y.

$$\mathbb{R}^y_{j+1} = \mathbb{R}^y_j + \mathbb{D}' \mathbb{R}^z_{j-1} + \mathbb{F}'$$
$$\mathbb{R}^z_{j+1} = \mathbb{A} \mathbb{R}^z_j$$

  with initializations $\mathbb{R}^y_0 = \mathbb{F}', \mathbb{R}^z_0 = \mathbb{B}$ and j denotes the time gap.

Diagnostic Tools II

Transition dynamics and valuation through altering cash flow exposure to shocks.

- Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
- Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
- Construct pricing counterpart to impulse response functions.

Unpack the Term Structure of Risk Premia!

Counterparts to impulse response functions pertinent to valuation:

- shock-exposure elasticities
- shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Hansen-Scheinkman (*Finance and Stochastics*), Borovička and Hansen ((*Journal of Econometrics*), Borovička-Hansen-Scheinkman (*Mathematical and Financial Economics*)

Construct Elasticities

- Construct shock elasticities as counterparts to impulse response functions

- Use (exponential) martingale perturbation $D_{(\tau,\tau+s)}$ to an underlying positive (multiplicative) process M where:

$$d \ln M_t = \mu_m(X_t)dt + \sigma_m(X_t) \cdot dW_t$$

$$\epsilon_m(x,t,\tau) := \lim_{s \downarrow 0} \frac{d}{ds} \log \mathbb{E}\left[\frac{M_t}{M_0} D_{(\tau,\tau+s)} | X_0 = x\right]$$

  where dW_t is a vector of Brownian increments.

- Apply to a cash-flow G_t and stochastic discount factor S_t
  - shock exposure elasticity $\epsilon_g(x,t,\tau)$;
  - shock cost elasticity $\epsilon_{sg}(x,t,\tau)$;
  - shock price elasticity $\epsilon_g(x,t,\tau) - \epsilon_{sg}(x,t,\tau)$

In what follows $\tau = 0$ or $\tau = t$.

Interpret Elasticities

Recall

$$\epsilon_m(x,t,\tau) = \lim_{s \downarrow 0} \frac{d}{ds} \log \mathbb{E}\left[\frac{M_t}{M_0} D_{(\tau,\tau+s)} | X_0 = x\right]$$

where $D_{(\tau,\tau+s)}$ is an exponential martingale perturbation.

Two interpretations:

- Change in probability measure - local impulse response
- Change in cash flow exposure - local risk risk return

Depend on current state, horizon, and date when the perturbation occurs.

What do These Elasticities Contribute?

- What shocks investors do care about as measured by expected return compensation?
- How do these compensations vary across states and over horizons?
- How do the shadow compensation differ across agent type?

Overview of Solution Method

- Markov equilibrium – aggregate state vector X_t:
  exogenous states: Z_t (growth), V_t (agg. stochastic vol.), and ς_t (idio. stochastic vol.)
  endogenous state: $W_t := \frac{N_{e,t}}{N_{e,t}+N_{h,t}}$ (wealth share)
- "Value function" approach: $U_i\left(N_{i,t}, X_t\right) = N_{i,t}^{1-\gamma_i}\xi_i(X_t)$ and $N_{i,t}$ is the individual net wealth
- (ξ_e, ξ_h) solutions to second order non-linear PDEs – implicit FD scheme with artificial time derivative
- Each time-step: compute aggregate state dynamics and prices using the value functions from the previous time-step
- Endogenous state partition due to occasionally-binding constraints
- Implementation in C++ allowing for HPC

Computation: Value Functions

Scaled value functions ξ_i solve PDEs like

$$0 = K_i + A_i \xi_i + B_i \cdot \partial_x \xi_i + \text{trace}[C_i C_i' \partial_{xx'} \xi_i], \quad x = (w, z, v, \varsigma),$$

where the coefficients are:

$$\begin{aligned} K_i &= K_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ A_i &= A_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ B_i &= B_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ C_i &= C_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \end{aligned}$$

The dependence of A, B, C on (ξ_e, ξ_h) arises due to general equilibrium.

We solve this PDE system with an iterative approach with two steps:

- ▷ given coefficients, we solve the linear PDE and obtain $\{\xi_i\}_{i=e,h}$
- ▷ given PDE solution $\{\xi_i\}_{i=e,h}$, we update coefficients using equilibrium constraints

Computation: Constraints

Capital distribution $\kappa \in [0, 1]$ and expert equity issuance $\chi \in [\underline{\chi}, 1]$ determine the occasionally-binding constraints of the models:

$$0 = \min(1 - \kappa, -\alpha_h)$$
$$0 = \min(\chi - \underline{\chi}, \alpha_e),$$

where α_i is agent *i*'s endogenous premium on capital.

Economic intuition.

- Experts hold all capital ($\kappa = 1$) if and only if households obtain no premium for holding it ($\alpha_h < 0$)
- Experts issue as much equity as possible ($\chi = \underline{\chi}$) if and only if their inside equity compensation exceeds the outside equity compensation ($\alpha_e > 0$)

Computation: Constraints

Variational inequalities. Algebraic equations on part of the state space (when constraints bind) and first-order non-linear elliptic PDEs on the complement (when constraints are slack).

$$
\begin{aligned}
0 &= \min(1-\kappa, -\alpha_h) \\
0 &= \min(\chi - \underline{\chi}, \alpha_e),
\end{aligned}
$$

where

$$
\begin{aligned}
\alpha_h &= F_h(x, \kappa, \partial_x \kappa, \chi, \partial_x \chi) \\
\alpha_e &= F_e(x, \kappa, \partial_x \kappa, \chi, \partial_x \chi).
\end{aligned}
$$

Solution method. Use "explicit" first difference scheme with false transient. See Oberman (2006)

$$
\frac{\kappa^{t+\Delta} - \kappa^t}{\Delta} = \min\left[1 - \kappa^t, F_h(x, \kappa^t, \partial_x \kappa^t, \chi^t, \partial_x \chi^t)\right]
$$

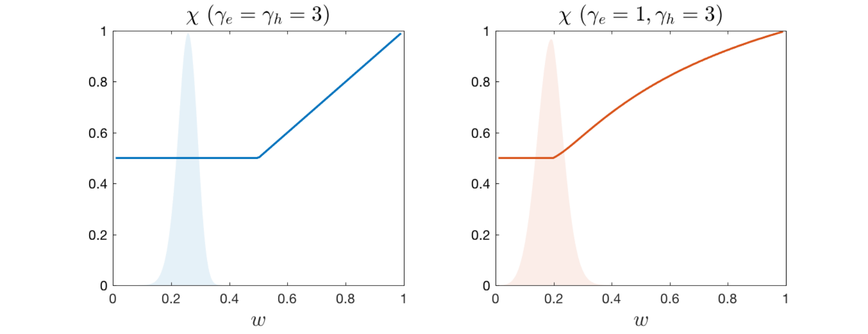
Binding Constraints

- When is does the constraint always bind?

- Economic setting
  - experts are the only producers
  - skin-in-the-game constraint $\chi \geq \underline{\chi} = .5$
  - TFP shocks only
  - EIS = 1

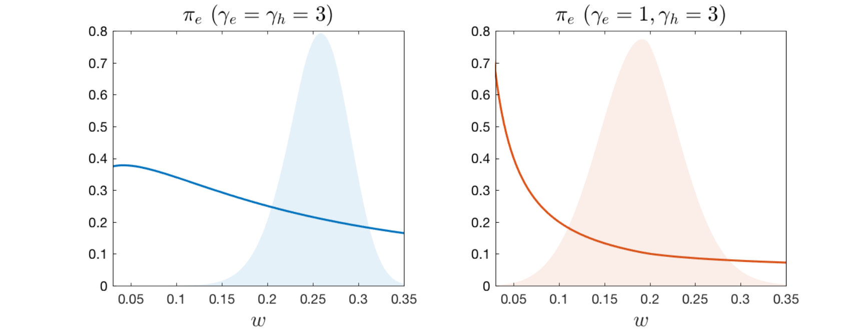
- Compare homogeneous RRA ($\gamma_e = \gamma_h$) to heterogeneous RRA ($\gamma_e < \gamma_h$)

Binding Constraints



Expert's skin-in-the-game χ in the two models.

Binding Constraints



Expert's shadow risk prices π_e in the two models.

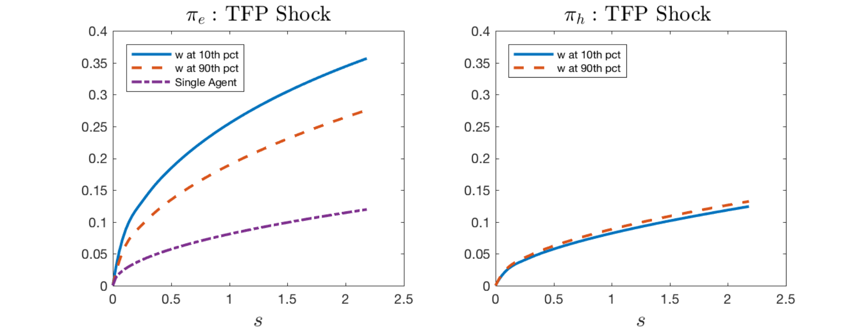
Shocks and Financial Frictions

- How do financial frictions affect agents' attitudes about shocks?

- Economic setting of focus
  - experts are the only producers
  - shocks to producitivity, growth rate, and volatility
  - RRA = 3, EIS = 1

- Compare model with a skin in the game constraint ($\chi \geq \underline{\chi} = .5$) vs. model without frictions ($\underline{\chi} = 0$)

Shocks and Financial Frictions



$\underline{\chi} = .5$ $\qquad$ $\underline{\chi} = 0$

Expert's and household's productivity risk prices π_e, π_h.

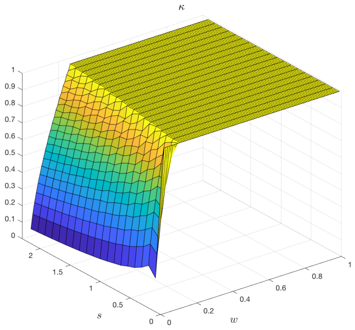
Heterogeneity: Productivity vs. Risk Aversion

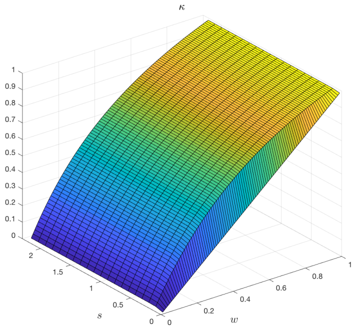
- “Expert” agents in the economy are either more productive or less risk averse?

- Economic setting of focus
  - experts and households can both produce
  - no equity-issuance $\chi \equiv \underline{\chi} = 1$
  - shocks to TFP level, growth rate, and volatility
  - EIS = 1

- Compare an economy with differences in productivity ($a_e > a_h$ but $\gamma_e = \gamma_h = 3$) to one in with differences in risk aversion ($\gamma_e = 2, \gamma_h = 8$ but $a_e = a_h$)

Heterogeneity: Productivity vs. Risk Aversion

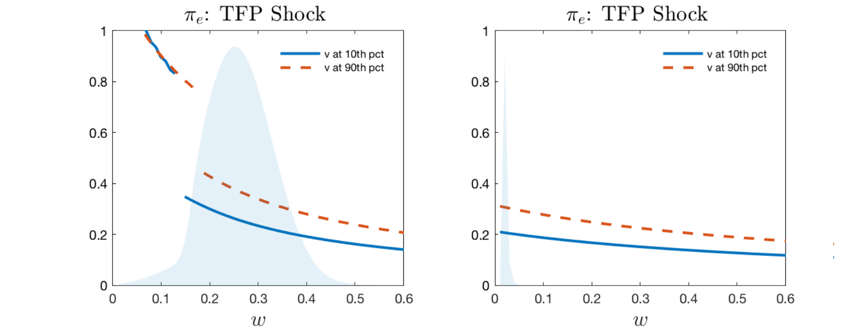




$a_e > a_h, \gamma_e = \gamma_h$ $\qquad$ $\gamma_e < \gamma_h, a_e = a_h$

Relative capital distribution κ.

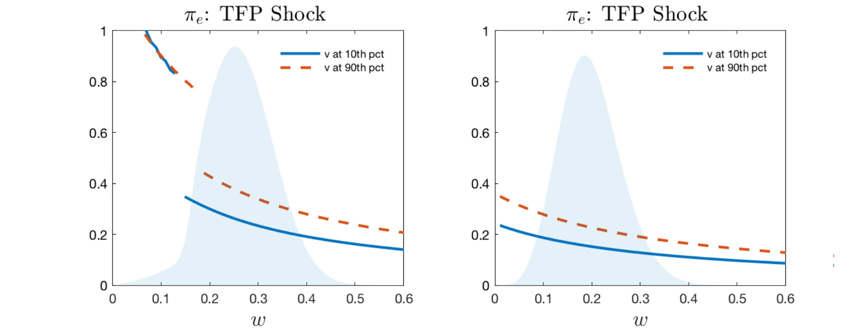
Heterogeneity: Productivity vs. Risk Aversion



$a_e > a_h, \gamma_e = \gamma_h$ $\quad$ $\gamma_e < \gamma_h, a_e = a_h$

Expert's TFP risk price π_e in the two models.

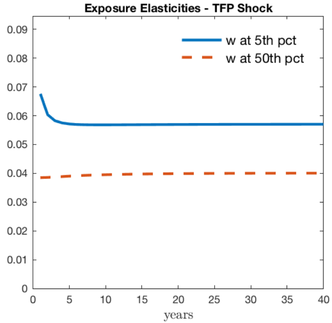
Heterogeneity: Productivity vs. Risk Aversion

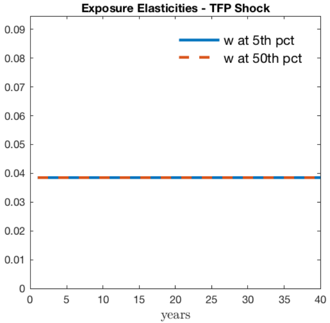


$a_e > a_h, \gamma_e = \gamma_h$ $\quad\quad$ $\gamma_e < \gamma_h, a_e = a_h$

Expert's productivity risk price π_e in the two models.

Heterogeneity: Productivity vs. Risk Aversion

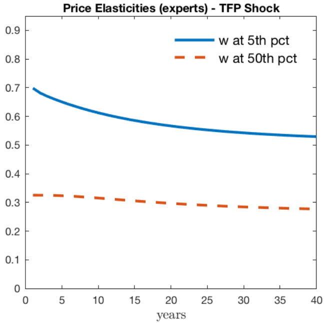


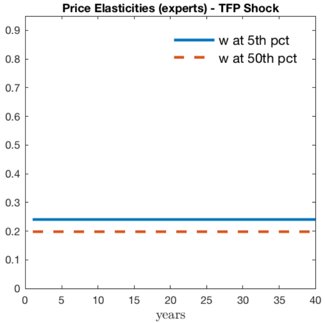


$a_e > a_h, \gamma_e = \gamma_h$ $\gamma_e < \gamma_h, a_e = a_h$

Productivity shock-exposure elasticities

Heterogeneity: Productivity vs. Risk Aversion

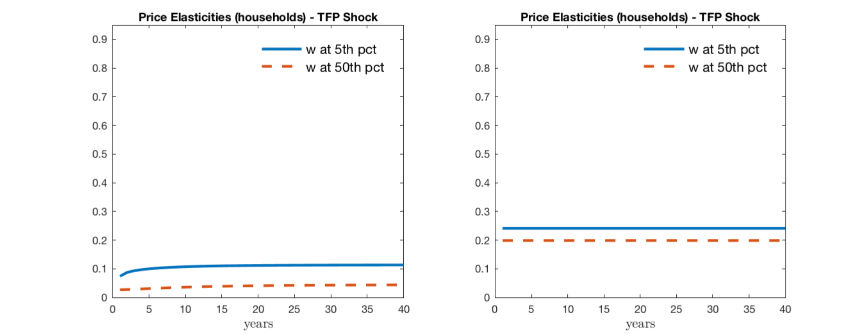




$a_e > a_h, \gamma_e = \gamma_h$ $\gamma_e < \gamma_h, a_e = a_h$

Productivity shock price elasticities for experts.

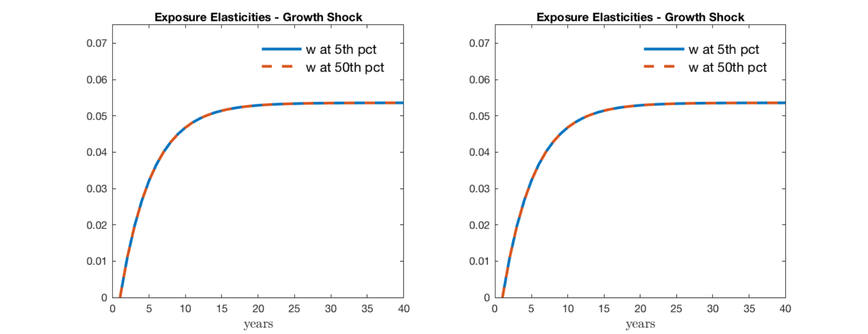
Heterogeneity: Productivity vs. Risk Aversion



$$a_e > a_h, \gamma_e = \gamma_h \qquad \gamma_e < \gamma_h, a_e = a_h$$

Productivity shock price elasticities for households.

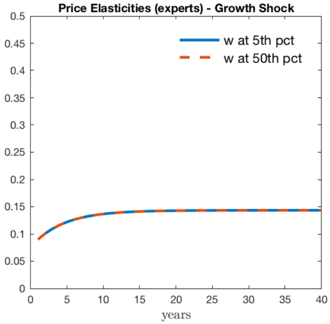
Heterogeneity: Productivity vs. Risk Aversion

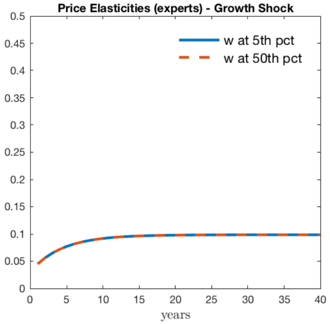


$a_e > a_h, \gamma_e = \gamma_h$ $\quad\quad$ $\gamma_e < \gamma_h, a_e = a_h$

Growth-rate shock exposure elasticities for aggregate consumption.

Heterogeneity: Productivity vs. Risk Aversion

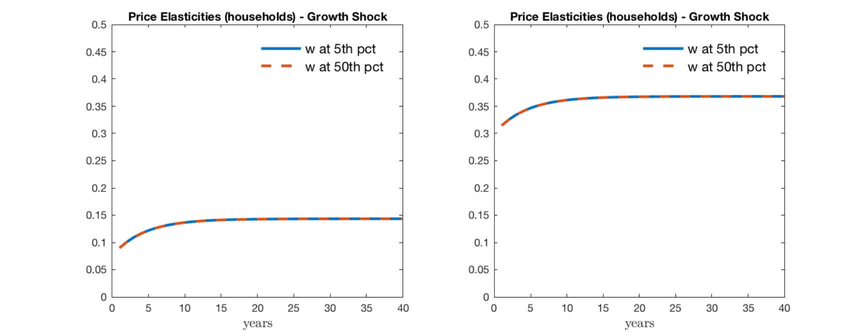




$a_e > a_h, \gamma_e = \gamma_h$ $\qquad$ $\gamma_e < \gamma_h, a_e = a_h$

Growth-rate shock price elasticities for experts.

Heterogeneity: Productivity vs. Risk Aversion



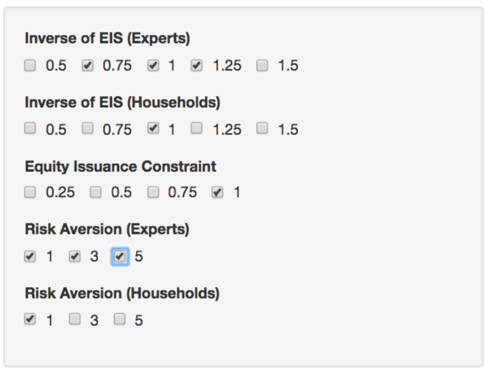
$a_e > a_h, \gamma_e = \gamma_h$ $\gamma_e < \gamma_h, a_e = a_h$

Household growth-rate shock price elasticities.

Conclusion / Next Steps

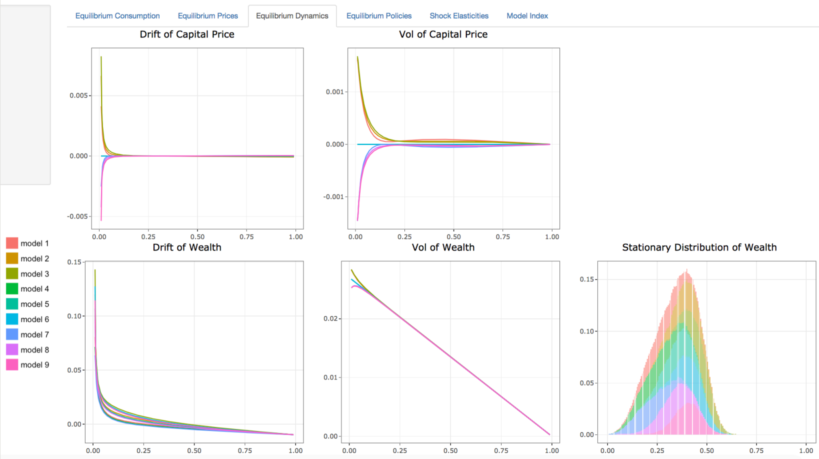
- Consider additional types of financial constraints
- Compare to smooth within regime models that target different time periods
- Analyze link between heterogenous preference models, heterogenous belief models, financial frictions' models
- Provide user-friendly web application to compare and contrast models...

Web Application



Select your desired constellation of models...

Web Application



Tabs separating outcomes for prices, dynamics, etc...

Technology

Efficiency units of capital K_t follow

$$dK_t = K_t \left[(Z_t + \iota_t - \delta)\, dt + \sqrt{V_t}\sigma \cdot dW_t \right]$$

Exogenous state variables (S_t, Z_t) follow

$$dZ_t = \lambda_z(\bar{z} - Z_t)dt + \sqrt{V_t}\sigma_v \cdot dW_t$$
$$dV_t = \lambda_v(\bar{v} - V_t)dt + \sqrt{V_t}\sigma_v \cdot dW_t$$

Adjustment costs: investment $\iota_t K_t dt$ costs $\Phi\left(\iota_t\right) k_t dt$ in output

back