Problem Set 1

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Exercise 1

1. The state variables are $\{B_t\}_{t=1}^{\infty}$, where $B_0 = B$.

2. Let the amount of oil that she sells be S_t, The control variables are $\{S_t\}_{t=1}^{\infty}$, where $S_0 = 0$

3. The transition equation is $B_t = B_{t-1} - S_{t-1}$.

4.

Sequence Problem

$$V_{\infty}(B) = \max_{\{S_t\}_{t=1}^{\infty}} \quad \sum_{t=1}^{\infty} \frac{p_t S_t}{(1+r)^{(t-1)}} \quad s.t. \, B = \sum_{t=1}^{\infty} S_t$$

Bellman Equation

$$V(B_t) = \max_{S_t} \quad p_t S_t + \frac{V(B_{t+1})}{(1+r)} \quad s.t. \, S_t = B_t - B_{t+1}$$

5.

$$V(B_t) = \max_{B_{t+1}} \quad p_t(B_t - B_{t+1}) + \frac{p_{t+1}(B_{t+1} - B_{t+2}) + \frac{V(B_{t+2})}{1+r}}{1+r}$$

$$\frac{\partial V(B_t)}{\partial B_t} = p_t - \frac{p_{t+1}}{1+r}$$

$$\Rightarrow p_{t+1} = (1+r)p_t$$

6. If $p_t = p_{t+1}$, then the person would sell it all on the first period because the later she sells B, the less it will be worth. If $p_{t+1}(1+r) > p_t)$, the she would save it until infinity, because the longer she waits, the more it will be worth. The Euler equation is required to achieve an interior solution.

Exercise 2

1. The state variables are $\{k_t\}_{t=1}^{\infty}$, $\{z_t\}_{t=1}^{\infty}$, and consequently $\{y_t\}_{t=1}^{\infty}$, and $\{i_t\}_{t=1}^{\infty}$.

2. The control variables are $\{c_t\}_{t=1}^{\infty}$.

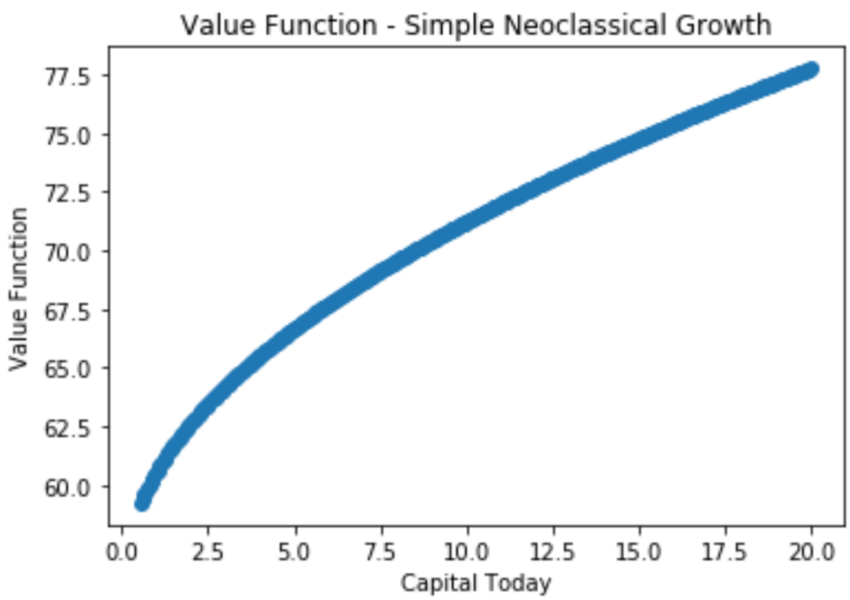
3.

$$V(k_t, z_t) = \max_{c_t} u(c_t) + \beta\,\mathbb{E}\,V(k_{t+1}, z_{t+1}) \quad s.t \quad c_t = z_t k_t^{\alpha} - k_{t+1} + (1-\delta)k_t$$

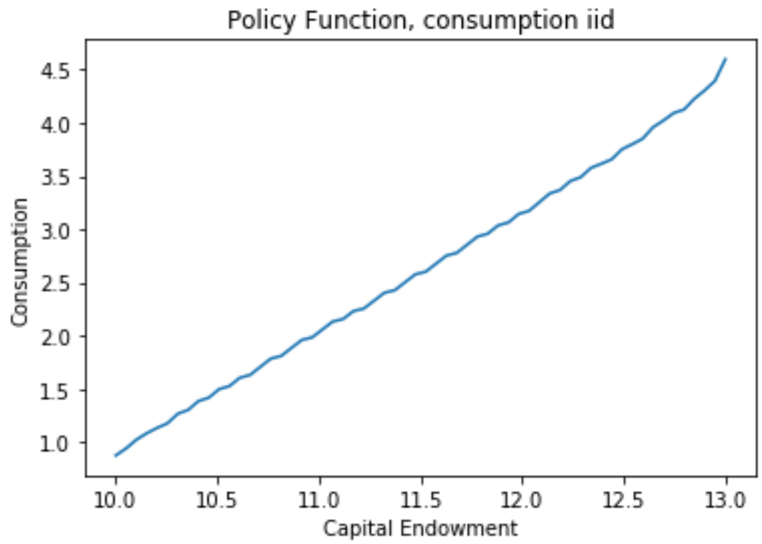
The Euler equations are:

$$u'(c_t) = \beta\,\mathbb{E}\left(\left(\alpha z_{t+1} k_{t+1}^{\alpha-1} + (1-\delta)\right) u'(c_{t+1})\right)$$

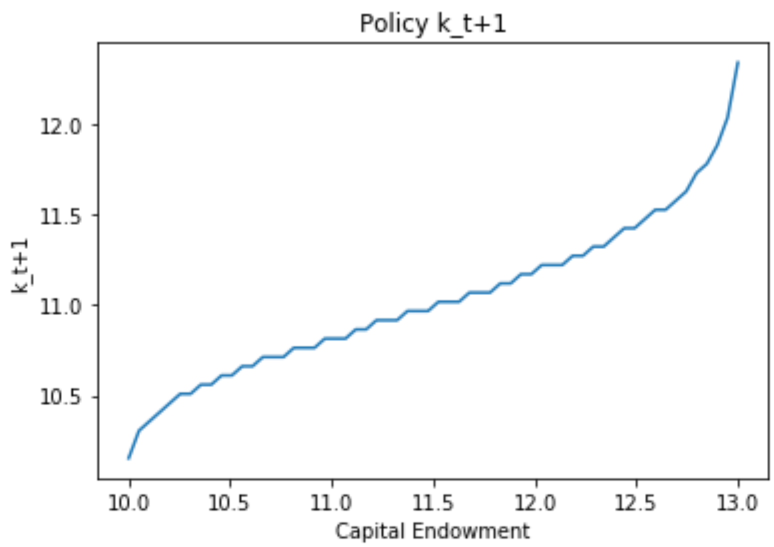
4.



i)



ii)



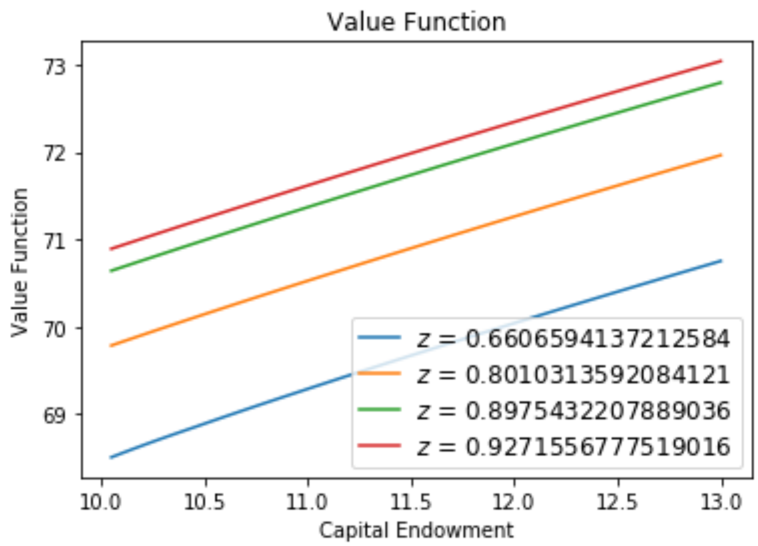
iii)

Exercise 3

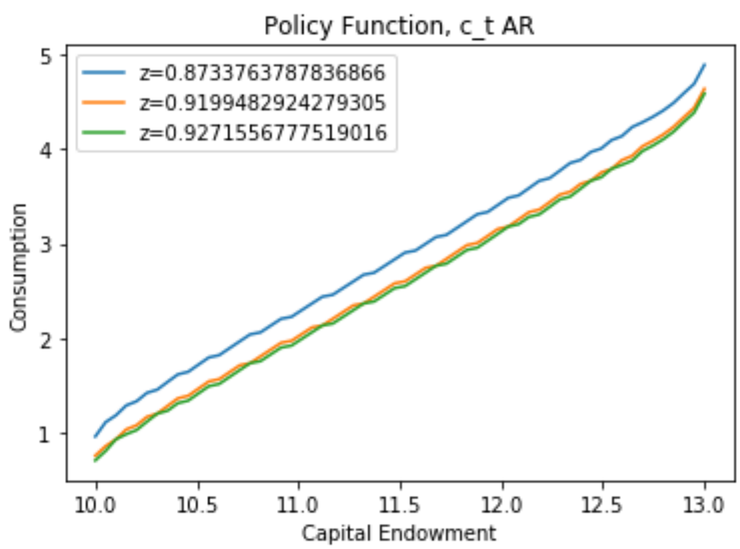
1.

$$V(k_t, z_t) = \max_{c_t} u(c_t) + \beta\,\mathbb{E}_{z_{t+1}|z_t} V(k_{t+1}, z_{t+1}) \quad s.t \quad c_t = z_t k_t^\alpha - k_{t+1} + (1-\delta)k_t$$

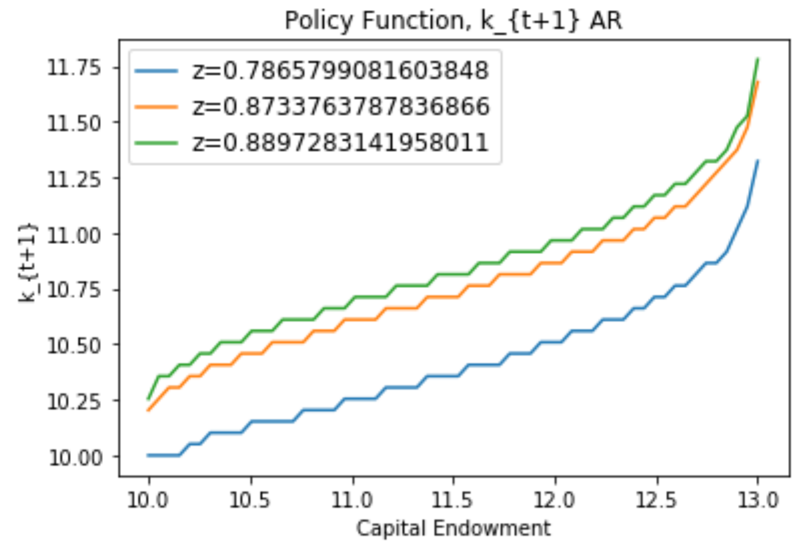
2.


Value Function
Value Function
Capital Endowment
z = 0.6606594137212584
z = 0.8010313592084121
z = 0.8975432207889036
z = 0.9271556777519016

i)


Policy Function, c_t AR
Consumption
Capital Endowment
z=0.8733763787836866
z=0.9199482924279305
z=0.9271556777519016

ii)



iii)

Exercise 4

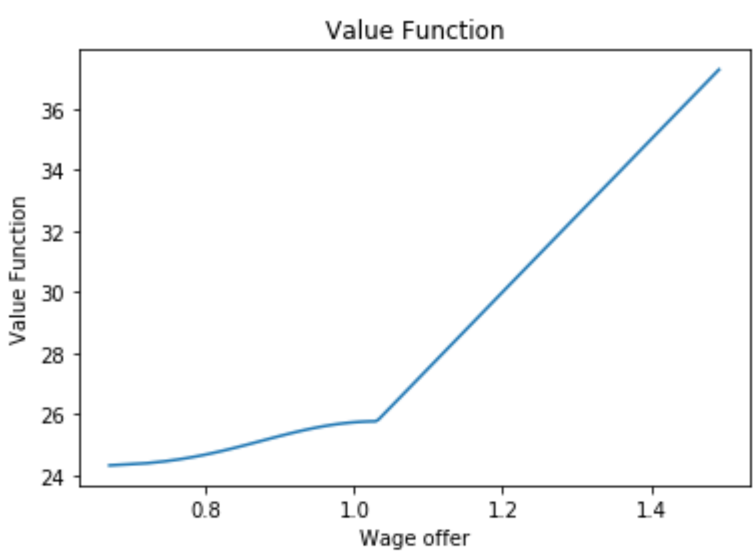
1.

$$V(w) = \max\{V^1(w), V^0(w)\}$$

$$V^1(w) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t w = \frac{w}{1-\beta}$$

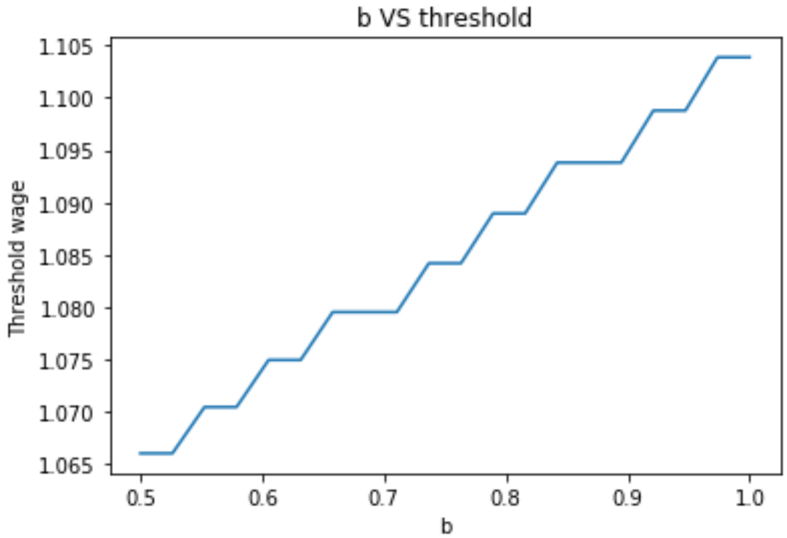
$$V^0(w) = b + \beta \mathbb{E} V(w')$$

2.



i)

ii) The optimal wage is 1.104



iii)