L9. HW

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1. a) As discussed in class,

$$\mathcal{L} = u(c_1) + \beta \sum_s u(c_2(s))\pi(s) + \lambda_1 (y_1 + a_1 - c_1 - q a_2) + \sum_s \lambda_2(s)[y_2(s) + a_2 - c_2(s)]$$

$$\frac{\partial L}{\partial c_1} = u'(c_1) - \lambda_1 = 0 \quad \Rightarrow \lambda_1 = \frac{1}{c_1}$$

$$\frac{\partial L}{\partial c_2} = \beta u'(c_2) - \lambda_2 = 0 \quad \Rightarrow \lambda_2 = \beta \frac{1}{c_2}$$

$$\frac{\partial L}{\partial a_2} = -\lambda_1 q + \lambda_2 = 0 \quad \Rightarrow \lambda_2 = q \cdot \lambda_1$$

$$\Rightarrow \frac{\beta}{c_2} = \frac{q}{c_1} \quad ①, \qquad \frac{c_1}{c_2} = \frac{q}{\beta}$$

$$\frac{\partial L}{\partial \lambda_1} = y_1 + a_1 - c_1 - q a_2 = 0 \quad \Rightarrow c_1 + q a_2 = y_1 + a_1 \quad ②$$

$$\frac{\partial L}{\partial \lambda_2} = y_2 + a_2 - c_2 = 0 \quad \Rightarrow a_2 = c_2 - y_2 \quad ③$$

let $c_1 = qk$, $c_2 = \beta k$,

$$qk + q(\beta k - y_2) = y_1 + a_1$$

$$k = \frac{y_1 + a_1 + q y_2}{(1+\beta)q} \quad \Rightarrow \quad \begin{cases} c_1^* = \dfrac{y_1 + a_1 + q y_2}{1+\beta} \\ c_2^* = \dfrac{\beta(y_1 + a_1 + q y_2)}{(1+\beta)q} \\ a_2^* = c_2 - y_2 = \dfrac{\beta(y_1 + a_1 + q y_2) - q y_2 - \beta q y_2}{(1+\beta)q} = \dfrac{\beta y_1 + \beta a_1 - q y_2}{(1+\beta)q} \end{cases}$$

b). As $a_1 \downarrow$, both c_1^* & $c_2^* \downarrow$ (with less wealth, people consume less)

c). when $q = \beta$, $c_1^* = c_2^* = \dfrac{y_1 + a_1 + q y_2}{1+\beta}$

when $q < \beta$, $c_2^* > c_1^*$

~~When the~~ When $q = \beta$, the extra profit (increase in utility) from bond is offset by the discounting of future values. When $q < \beta$, bond can bring more profit (added utility), than the discounted future values.

~~d). Likewise, from class notes,~~

~~$q_i u'(c_1) \geq \beta E[u'(c_2)]$~~

~~$\frac{q_i}{c_1} \neq \sum_s [\pi(s)]$~~

d). From class, Euler:

$$\frac{q_0}{c_1} = \beta\left[\pi(s_1)\cdot \cancel{\quad} u'(c_2(s_1)) + \pi(s_2)\cdot u'(c_2(s_2))\right] = \beta\left(\pi(s_1)\cdot\frac{1}{c_2(s_1)} + \pi(s_2)\frac{1}{c_2(s_2)}\right) \quad ①$$

$$\begin{cases} y_2(s_1)+a_2-c_2(s_1)=0 & ② \\ y_2(s_2)+a_2-c_2(s_2)=0 & ③ \\ y_1+a_1-c_1-qa_2=0 & ④ \end{cases}$$

substitute $a_2 = \frac{y_1+a_1-c_1}{q}$ to ② & ③

$$\begin{cases} c_2(s_1) = \bar{y}+a_2 = \bar{y}+\frac{y_1+a_1-c_1}{q} = \frac{\bar{y}q+y_1+a_1-c_1}{q} \\ c_2(s_2) = \underline{y}+a_2 = \underline{y}+\frac{y_1+a_1-c_1}{q} = \frac{\underline{y}q+y_1+a_1-c_1}{q} \end{cases}$$

$$\frac{q_0}{c_1} = \frac{\beta\pi(s_1)\cdot q}{\bar{y}q+y_1+a_1-c_1} + \frac{\beta\pi(s_2)\cdot q}{\underline{y}q+y_1+a_1-c_1}$$

$$(\bar{y}q+y_1+a_1-c_1)(\underline{y}q+y_1+a_1-c_1) = \beta\pi(s_1)\cdot c_1(\underline{y}q+y_1+a_1-c_1)+\beta\pi(s_2)c_1(\bar{y}q+y_1+a_1-c_1)$$

$$\bar{y}q^2\underline{y}+\bar{y}q(y_1+a_1-c_1)+y_1(\underline{y}q+y_1+a_1-c_1)+a_1(\underline{y}q+y_1+a_1-c_1)+\cancel{c_1^2}$$
$$= \cancel{-\beta\pi(s_1)c_1^2} + \beta\pi(s_1)c_1(\underline{y}q+y_1+a_1) \cancel{-\beta\pi(s_2)c_1^2} + \beta\pi(s_2)\cdot c_1(\bar{y}q+y_1+a_1).$$

$$\cancel{\bar{y}\underline{y}q^2}\ \cancel{(1+\beta)c_1^2 - [(1+\beta\pi(s_1))\underline{y}q+(2+\beta)(y_1+a_1)+(1+\beta\pi(s_2))\bar{y}q]c_1}$$
$$\cancel{+(\bar{y}q+y_1+a_1)(\underline{y}+y_1+a_1)=0}$$

$$(1+\beta)c_1^2 - [\underline{y}q+\bar{y}q+2(y_1+a_1)+\beta\pi(s_1)\underline{y}q+\beta\pi(s_2)\bar{y}q+\beta(y_1+a_1)]$$
$$+(\bar{y}q+y_1+a_1)(\underline{y}q+y_1+a_1)=0$$

$$(1+\beta)c_1^2 - [\underline{y}q(1+\beta\pi(s_1))+\bar{y}q(1+\beta\pi(s_2))+(2+\beta)(y_1+a_1)]c_1$$
$$+(\bar{y}q+y_1+a_1)(\underline{y}q+y_1+a_1)=0.$$

For answer pls see next page.

$$c_2^*(s_1) = \bar{y}+\frac{y_1+a_1-c_1}{q}$$

$$c_2^*(s_2) = \underline{y}+\frac{y_1+a_1-c_1}{q}$$

ii). As $\pi(s_1)\downarrow$, c_1^*: can't say, depending on how small a is.

By income effect, $\pi(s_1)\downarrow$, she will have lower income, and thus consume less.

By substitution effect, $\pi(s_1)\downarrow$, she will consume more

y_up=u
y_down = v
pi_s1=a
pi_s2=b
a_1=c
y_1=y

$$x = \frac{1}{2(\beta+1)}\left(-\sqrt{\left((-a\,\beta q\,u - b\,\beta q\,v - \beta c - 2\,c - q\,u - q\,v - \beta\,y - 2\,y)^2 - 4\,(\beta+1)\left(c^2 + 2\,c\,q\,v + 2\,c\,y + q^2\,v^2 + 2\,q\,v\,y + y^2\right)\right)} + a\,\beta q\,u + b\,\beta q\,v + \beta c + 2\,c + q\,u + q\,v + \beta\,y + 2\,y\right) \text{ and } \beta + 1 \neq 0$$

2. $$\mathcal{L}_{a_1,a_2,c_2,x_2} = \log(c_1) + \beta\{s[\log c_2 + \log(a_2+x_2-c_2)] + (1-s)\log(a_2)\}$$
$$+\lambda[a_1 - c_1 - qa_2 - sq(1+\varphi)x_2] = 0$$

① $$\frac{\partial L}{\partial c_1} = \frac{1}{c_1} - \lambda = 0 \quad \Rightarrow \quad \lambda = \frac{1}{c_1}$$

② $$\frac{\partial L}{\partial c_2} = \frac{\beta s}{c_2} - \frac{\beta s}{a_2+x_2-c_2} = 0 \quad \Rightarrow \quad c_2^* = \frac{a_2+x_2}{2}$$

③ $$\frac{\partial L}{\partial a_2} = \frac{\beta s}{a_2+x_2-c_2} + \frac{\beta(1-s)}{a_2} - q\lambda = 0 \quad \Rightarrow \quad \frac{2\beta s}{a_2+x_2} + \frac{\beta(1-s)}{a_2} = \frac{q}{c_1} \quad ⑥$$

④ $$\frac{\partial L}{\partial x_2} = \frac{\beta s}{a_2+x_2-c_2} - sq(1+\varphi)\lambda = 0 \quad \Rightarrow \quad \frac{2\beta s}{a_2+x_2} = \frac{sq(1+\varphi)}{c_1} \quad ⑦$$

⑤ $$\frac{\partial L}{\partial \lambda} = a_1 - c_1 - qa_2 - sq(1+\varphi)x_2 = 0$$

② & ④ : $\frac{\beta}{C_2} = \frac{q(1+\varphi)}{C_1}$ $\quad C_2 = \frac{\beta}{q(1+\varphi)} C_1$

⑥ $\frac{sq(1+\varphi)}{C_1} + \frac{\beta(1-s)}{a_2} = \frac{q}{C_1}$

$\frac{1}{C_1}[sq(1+\varphi) - q] = \frac{1}{a_2}\beta(s-1)$

$a_2 = \frac{\beta(s-1)}{sq(1+\varphi)-q} C_1$

~~into~~ ⑨. $X_2 = 2C_2 - a_2 = [\frac{2\beta}{q(1+\varphi)} - \frac{\beta(s-1)}{sq(1+\varphi)-q}] C_1$

into ⑤ $a_1 - [1 + \frac{\beta(s-1)}{s(1+\varphi)-1} + 2s\beta - \frac{\beta(s-1) \cdot sq(1+\varphi)}{sq(1+\varphi)-q}] C_1 = 0$

$[1 + 2s\beta + \beta(1-s)] C_1 = a_1$

$C_1^* = \frac{a_1}{1+\beta+s\beta}$

$a_2^* = \frac{\beta(s-1)}{sq(1+\varphi)-q} \cdot \frac{a_1}{1+\beta+s\beta} = \frac{\beta(s-1)\, a_1}{q[s(1+\varphi)-1](1+\beta+s\beta)}$

$C_2^* = \frac{\beta}{q(1+\varphi)} \cdot \frac{a_1}{1+\beta+s\beta} = \frac{\beta a_1}{q(1+\varphi)(1+\beta+s\beta)}$

$X_2^* = [\frac{2\beta}{q(1+\varphi)} - \frac{\beta(s-1)}{sq(1+\varphi)-q}] \frac{a_1}{1+\beta+s\beta}$

$= \frac{2\beta a_1}{q(1+\varphi)(1+\beta+s\beta)} - \frac{\beta a_1(s-1)}{q[s(1+\varphi)-1](1+\beta+s\beta)}$

$b_1^* = a_2^*$

$b_2^* = a_2^* + X_2^* - C_2 = a_2 + 2C_2 - a_2 - C_2 = C_2^*$

~~when $\varphi = 0$~~,

$$\frac{q}{(1+\varphi)^2} = \frac{2\beta s}{a_2 \cancel{+ x_2}} \qquad 1 + \frac{\beta(1-s)}{a_2} \cdot \frac{a_2 + x_2}{2\beta s} = \frac{a_2 \cdot 2\beta s + \beta(1-s)(a_2 + x_2)}{a_2 \cdot 2\beta s}$$

$$a_2 \cdot 2\beta s \cdot q = (1+\varphi)^2 [a_2 \cdot 2\beta s + \beta(1-s)(a_2 + x_2)]$$

$$= (1+\varphi)^2 \cdot \beta(1-s) \cdot x_2$$

$$a_2 [2\beta s \cdot q - (1+\varphi)^2 \cdot 2\beta s - (1+\varphi)^2 \cdot \beta(1-s)] = (1+\varphi)^2 \beta(1-s) x_2.$$

$$a [2\beta s \cdot q - (1+\varphi)^2 (\beta s + \beta)] = (1+\varphi)^2 (1-s)\beta x_2$$

$$a [2sq - (1+\varphi)^2 (s+1)$$

when $\varphi = 0$ $\quad b_1^* = a_2^* = \frac{\beta a_1}{q(1+\beta+s\beta)}$

$$b_2^* = c_2^* = \frac{\beta a_1}{q(1+\beta+s\beta)}$$

$\Rightarrow b_1^* = b_2^*$, $a_2^* = c_2^*$

When $\varphi \uparrow$, can't tell how x_2^* will change; depends on a_1, β, s & q.

$b^* = c_2^*$, when $\varphi \uparrow$, c_2^* & $b^* \downarrow$

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