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① . As shown in class,

$\bar{w}(\bar{h}-1)\tilde{w} + \bar{w}\bar{h}\tilde{h}_t + A\bar{c}\tilde{c}_t = 0$

② . $c_t + i_t = r_t k_{t-1} + w_t h_t$

$\bar{c}e^{\tilde{c}_t} + \bar{i}e^{\tilde{i}_t} = \bar{r}\cdot e^{\tilde{r}_t}\bar{k}\cdot e^{\tilde{k}_{t-1}} + \bar{w}\cdot\bar{h}\cdot e^{\tilde{w}_t+\tilde{h}_t}$

$\bar{c}(1+\tilde{c}_t) + \bar{i}(1+\tilde{i}_t) = \bar{k}\bar{r}(1+\tilde{r}_t+\tilde{k}_{t-1}) + \bar{w}\cdot\bar{h}\cdot(1+\tilde{w}_t+\tilde{h}_t)$

$-\bar{c}\tilde{c}_t - \bar{i}\tilde{i}_t + \bar{k}\bar{r}\tilde{r}_t + \bar{k}\bar{r}\tilde{k}_{t-1} + \bar{w}\bar{h}\tilde{w}_t + \bar{w}\bar{h}\cdot\tilde{h}_t = \bar{c}+\bar{i}-\bar{k}\bar{r}-\bar{w}\bar{h} = 0.$

$\because \bar{c}+\bar{i} = \bar{r}\bar{k}+\bar{w}\bar{h} \Rightarrow 0$

③ . $i_t = k_t - k_{t-1}(1-\delta)$

$\bar{i}(1+\tilde{i}_t) = \bar{k}(1+\tilde{k}_t) - \bar{k}(1-\delta)(1+\tilde{k}_{t-1})$

$-\bar{i}\tilde{i}_t + \bar{k}\tilde{k}_t - \bar{k}(1-\delta)\tilde{k}_{t-1} = \bar{i} - \bar{k} - \bar{k}(1-\delta) = 0$

$\because \bar{i} = \bar{k} - \bar{k}(1-\delta)$

④ $y_t = e^{z_t}\cdot k_{t-1}^{\alpha} h_t^{1-\alpha}$

$\bar{y}e^{\tilde{y}_t} = e^{z_t}\cdot(\bar{k}e^{\tilde{k}_{t-1}})^{\alpha}\cdot(\bar{h}\cdot e^{\tilde{h}_t})^{1-\alpha}$

$\bar{y} = \bar{k}^{\alpha}\bar{h}^{1-\alpha}e^{(z_t+\alpha\tilde{k}_{t-1}+(1-\alpha)\tilde{h}_t-\tilde{y}_t)}$

$\bar{y} = \bar{k}^{\alpha}\bar{h}^{1-\alpha}(1+z_t+\alpha\tilde{k}_{t-1}+(1-\alpha)\tilde{h}_t-\tilde{y}_t).$

$\because \bar{y} = \bar{k}^{\alpha}\bar{h}^{1-\alpha} \Rightarrow$

"z_t" $+\alpha\tilde{k}_{t-1}+(1-\alpha)\tilde{h}_t-\tilde{y}_t = 0$"

⑤ . $r_t = \alpha\cdot e^{z_t}k_{t-1}^{\alpha-1}h_t^{1-\alpha}$

$\bar{r}e^{\tilde{r}_t} = \alpha\cdot e^{z_t}\cdot(\bar{k}e^{\tilde{k}_{t-1}})^{\alpha-1}(\bar{h}e^{\tilde{h}_t})^{1-\alpha}$

$\bar{r} = \alpha\cdot\bar{k}^{\alpha-1}\cdot\bar{h}^{1-\alpha}\cdot e^{z_t+(\alpha-1)\tilde{k}_{t-1}+(1-\alpha)\tilde{h}_t-\tilde{r}_t}$

$\bar{r} = \alpha\cdot\bar{k}^{\alpha-1}\cdot\bar{h}^{1-\alpha}\cdot(1+z_t+(\alpha-1)\tilde{k}_{t-1}+(1-\alpha)\tilde{h}_t-\tilde{r}_t)$

$\because \bar{r} = \alpha\cdot e^{\bar{z}}\bar{k}^{\alpha-1}\bar{h}^{1-\alpha} = \alpha\bar{k}^{\alpha-1}\bar{h}^{1-\alpha}$

$\Rightarrow z_t + (\alpha-1)\tilde{k}_{t-1} + (1-\alpha)\tilde{h}_t - \tilde{r}_t = 0.$

⑥ . $w_t = (1-\alpha)\cdot e^{z_t}k_{t-1}^{\alpha}h_t^{-\alpha}$

$\bar{w}e^{\tilde{w}_t} = (1-\alpha)\cdot e^{z_t}\cdot(\bar{k}\cdot e^{\tilde{k}_{t-1}})^{\alpha}\cdot(\bar{h}\cdot e^{\tilde{h}_t})^{-\alpha}$

$\bar{w} = (1-\alpha)\cdot\bar{k}^{\alpha}\cdot\bar{h}^{-\alpha}\cdot e^{(z_t+\alpha\cdot\tilde{k}_{t-1}-\alpha\tilde{h}_t-\tilde{w}_t)}$

$\because \bar{w} = (1-\alpha)e^{\bar{z}}\cdot\bar{k}^{\alpha}\cdot\bar{h}^{-\alpha} = 1$ $(1+z_t+\alpha\tilde{k}_{t-1}-\alpha\tilde{h}_t-\tilde{w}_t)$

$\Rightarrow z_t + \alpha\cdot\tilde{k}_{t-1} - \alpha\tilde{h}_t - \tilde{w}_t = 0.$

Euler:

$$1 = \beta \, E \left\{ \frac{C_t}{C_{t+1}} [r_{t+1} + (1-\delta)] \right\}$$

$$1 = \beta \, E \left\{ \frac{\bar{c} e^{\tilde{c}_t}}{\bar{c} e^{\tilde{c}_{t+1}}} [\bar{r} e^{\tilde{r}_{t+1}} + (1-\delta)] \right\}$$

$$1 = \beta \, E \left\{ e^{\tilde{c}_t - \tilde{c}_{t+1} + \tilde{r}_{t+1}} \cdot \bar{r} + (1-\delta) \cdot e^{\tilde{c}_t - \tilde{c}_{t+1}} \right\}$$

$$1 = \beta \, E \left\{ (1 + \tilde{c}_t - \tilde{c}_{t+1} + \tilde{r}_{t+1}) \cdot \bar{r} + (1-\delta) \cdot (1 + \tilde{c}_t - \tilde{c}_{t+1}) \right\}$$

$$1 = \beta \, E \{ (\bar{r} + (1-\delta)) \} + \beta \, E \{ (\bar{r} + 1 - \delta) \tilde{c}_t - (\bar{r} + 1 - \delta) \tilde{c}_{t+1} + \bar{r} \tilde{r}_{t+1} \}$$

$$\because \quad \bar{r} + (1-\delta) = \beta^{-1}$$

$$0 = \beta \, E \{ (\bar{r} + 1 - \delta) \tilde{c}_t - (\bar{r} + 1 - \delta) \tilde{c}_{t+1} + \bar{r} \tilde{r}_{t+1} \}$$

$$A: \; k_t \begin{bmatrix} 0 \\ 0 \\ \bar{k} \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad B: \; k_{t-1} \begin{bmatrix} 0 \\ -\bar{r}\bar{k} \\ -\bar{k}(1-\delta) \\ \alpha \\ \alpha - 1 \\ \alpha \end{bmatrix} \qquad C: \begin{array}{c} \begin{matrix} c_t & y_t & i_t & h_t & w_t & r_t \end{matrix} \\ \begin{bmatrix} \bar{A}\bar{c} & 0 & 0 & \bar{w}\bar{h} & \bar{w}(\bar{h}-1) & 0 \\ -\bar{c} & 0 & -\bar{i} & \bar{w}\bar{h} & \bar{w}\bar{h} & \bar{r}\bar{k} \\ 0 & 0 & -\bar{i} & 0 & 0 & 0 \\ 0 & -1 & 0 & 1-\alpha & 0 & 0 \\ 0 & 0 & 0 & 1-\alpha & 0 & -1 \\ 0 & 0 & 0 & -\alpha & -1 & 0 \end{bmatrix} \end{array}$$

$$D: \; z_t \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad F = 0 \quad G = 0 \quad J = 0$$

$$J: [\, -(\bar{r}+1-\delta) \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,]$$

$$K: [\, \bar{r}+1-\delta \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \bar{r} \,]$$

initial states:

$$\begin{cases} \bar{r} = \alpha \cdot \dfrac{\bar{y}}{\bar{k}} & ① \\ \bar{y} = \bar{r}\bar{k} + \bar{w}\bar{h} & \\ \bar{y} = \bar{k}^{\alpha} \bar{h}^{1-\alpha} & ③ \end{cases} \Rightarrow$$

combining ① & ③.

$$\bar{r} = \alpha \cdot \bar{k}^{\alpha-1} \cdot \bar{h}^{1-\alpha}$$

$$\bar{k} = \sqrt[\alpha-1]{\frac{\bar{r}}{\alpha}} \cdot \bar{h}$$

$$\bar{y} = \bar{k} \cdot \bar{r} / \alpha$$

$$\bar{c} = \bar{y} - \bar{i} = \bar{y} - \delta \cdot \bar{k}$$

$$\bar{w} = (1-\alpha) \, \bar{y} / \bar{h}$$

$$\bar{i} = \delta \cdot \bar{k}$$

$$\bar{z} = 0$$

Indivisable Labor Model

$\tilde{u}(C_t, H_t) = \log(C_t) - B H_t = \log(C_t) - B(1-l_t)$

$W_t = \frac{\tilde{u}_l}{\tilde{u}_c}$ (from class).

$\frac{\partial \tilde{u}}{\partial C_t} = \frac{1}{C_t}$

$\frac{\partial \tilde{u}}{\partial l_t} = B$

$W_t = \frac{u_l}{u_c} = B \cdot C_t$.

$\bar{w} \cdot e^{\tilde{w}_t} = B \cdot \bar{c} \cdot e^{\tilde{c}_t}$

$\bar{w} \cdot (1+\tilde{w}_t) = B \cdot \bar{c} \cdot (1+\tilde{c}_t)$

$\cancel{\bar{w} - B\bar{c}} \Rightarrow 0 = B\bar{c}\tilde{C}_t - \bar{w}\hat{w}_t$

$\bar{w} = B \cdot \bar{c}$

$B = \frac{\bar{w}}{\bar{c}} = 2.56$

	% SD output	σ_c/σ_y	σ_i/σ_y	σ_h/σ_y	σ_w/σ_y	σ_h/σ_w	corr(h,
standard	4.54	0.61	2.57	0.29	0.82	0.29	0.51
indivisable L	4.17	0.67	2.44	0.55	0.67	0.82	0.35
paper US	1.92	0.45	2.78	0.78	0.57	1.37	0.07

The pct standard deviation for output, & corr(h, w) of both models are higher than US data. and σ_h/σ_y much lower. The indivisable labor model significantly increases and σ_h/σ_w σ_h/σ_y, the volatility of hours worked versus ~~and~~ that of output. In addition, % SD output, σ_w/σ_y, σ_h/σ_w, corr(h, w)'s performances have all been improved with the introduction of indivisable labor. σ_c/σ_y, σ_i/σ_y's performance got worse slightly due to the new inclusion