Fiona

Pset 7.

1.a). when ρ is unchanged & σ_ε ~~changes~~ increases. the shape of z_t changes because there's more variance in the random part of the shock. When ρ is increased & σ is unchanged, z_t becomes bigger on average, while maintaining the same shape. because z_t is less random and more reliant on previous z_{t-1}.

$$\begin{array}{ccc} & l & h \\ l & \left[\begin{array}{cc} \pi_{11} & 1-\pi_{11} \\ 1-\pi_{22} & \pi_{22} \end{array}\right. & \\ h & & \end{array}$$

b). As shown in class, for AR.

$$\mathbb{E}[z_t^2] = \frac{\sigma_\varepsilon^2}{1-\rho^2}, \quad \mathbb{E}[z_t, z_{t+1}] = \rho \cdot \frac{\sigma_\varepsilon^2}{1-\rho^2}$$

For. MC.

$$\mathbb{E}[z_i^2] = \sum z^2 \cdot p(z) = (-\Delta)^2 \cdot \pi_{11} + \Delta^2 \cdot (1-\pi_{11})$$
$$= \Delta^2$$

$$E[z_t^2]_{Ar} = E[z^2]_{MC} = \frac{\sigma_\varepsilon^2}{1-\rho^2} = \Delta^2 \Rightarrow \Delta$$

MC: $\mathbb{E}[z'z] = \sum z' z_{l\,h} p(z' | z_{l\,h})$

$$= (-\Delta)(-\Delta) \cdot \pi_{11} + (-\Delta)(+\Delta)(1-\pi_{11})$$
$$= (2\pi_{11}-1)\Delta^2 = \rho \cdot \frac{\sigma_\varepsilon^2}{1-\rho^2}$$
$$= (+\Delta)(-\Delta)\pi_{22} + (+\Delta)(+\Delta)(1-\pi_{22})$$
$$= (1-2\pi_{22})\Delta^2 = \rho \frac{\sigma_\varepsilon^2}{1-\rho^2}$$

$$\Rightarrow \begin{cases} \pi_{11} = \frac{\rho+1}{2} = 0.975 \\ \pi_{22} = \frac{1-\rho}{2} = 0.025 \end{cases}$$

$$\Rightarrow \quad \Delta = \sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}} = 0.0224$$

It's a bad approximation. after all z can only take 2 values. The transition matrix says that if z is in high state, then it has a low probability of staying in high state, while if it's in low state, it has ~~very~~ a very high probability of staying there. I guess that explains why I'm getting a line in the low ; no matter what the initial condition is.

$$\begin{bmatrix} 1-\pi_{22} & \pi_{22} \end{bmatrix} \times \begin{bmatrix} \cdot \\ -\Delta \end{bmatrix} = \begin{bmatrix} \pi_{11}\Delta - \Delta(1-\pi_{11}) \\ (1-\pi_{22})\Delta - \Delta\pi_{22} \end{bmatrix}$$

$$\begin{cases} \rho\Delta + \varepsilon_{t+1} = \cdots & (1) \\ -\rho\Delta + \varepsilon_{t+1} = \cdots & (2) \end{cases}$$

$(1)-(2)$ $\quad 2\rho = \pi_{11} - 1 + \pi_{11} - 1 + \pi_{22} + \pi_{22} \quad = 2(\pi_{22} + \pi_{11} - 1)$.

$$\pi_{11} + \pi_{22} = \rho + 1$$

$(1)+(2)$ $\quad 2\varepsilon_{t+1} = \Delta[\pi_{11} + 1 - \pi_{22} - 1 + \pi_{11} - \pi_{22}]$

$$\varepsilon_{t+1} = \Delta(\pi_{11} + \pi_{22}) = \Delta(\rho+1).$$

$$\Delta = \varepsilon_{t+1}/\rho+1$$

代入 (1) $\quad \frac{\rho \cdot \varepsilon_{t+1}}{\rho+1} + \varepsilon_{t+1} = \pi_{11} \cdot \frac{\varepsilon_{t+1}}{\rho+1} - \frac{\varepsilon_{t+1}}{\rho+1}(1-\pi_{11})$

$$\rho\varepsilon_{t+1} + \varepsilon_{t+1}(\rho+1) = 2\pi_{11} \cdot \varepsilon_{t+1} - \varepsilon_{t+1} \cancel{+ \varepsilon_{t+1} \cdot \pi_{11}}$$

$$2\rho\varepsilon_{t+1} + \varepsilon_{t+1} = \pi_{11} \cdot \varepsilon_{t+1}.$$

$$\pi = \frac{1}{2}\rho +$$

please see other sheet.

2. Firm $\max_{k_t^d(s_t),\, h_t^d(s_t)} e^{z_t(s_t)} [k_t^d(s_t)]^{\alpha} [h_t^d(s_t)]^{1-\alpha} - r_t(s_t) k_t^d(s_t) - w_t(s_t) h_t^d(s_t)$

$$\frac{\partial \pi(k_t, h_t)}{\partial k_t^d} = \alpha e^{z_t(s_t)} \cdot k_t^d(s_t)^{\alpha-1} h_t(s_t)^{1-\alpha} - r_t(s_t) = 0$$

$$\frac{\partial \pi(k_t, h_t)}{\partial h_t} = (1-\alpha) e^{z_t(s_t)} k_t(s_t)^{\alpha} \cdot h_t^{-\alpha} - w_t(s_t) = 0$$

$$\Rightarrow \begin{cases} r_t(s_t) = \alpha \cdot e^{z_t(s_t)} \cdot k_t^d(s_t)^{\alpha-1} \cdot h_t(s_t)^{1-\alpha} \\ w_t(s_t) = (1-\alpha) e^{z_t(s_t)} k_t^d(s_t)^{\alpha} \cdot h_t^{-\alpha} \end{cases}$$

Scanned by CamScanner

Consumer:

$$\max_{\{c_t(s_t), l_t(s_t), k_{t+1}(s_t)\}} \sum_{t=0}^{\infty}\sum_{s_t} \beta^t u(c_t(s_t), l_t(s_t))\pi_t(s_t) \quad \text{s.t.} \quad h_t(s_t)+l_t(s_t)=1 \ \&$$

$$c_t(s_t)+k_{t+1}(s_t) = k_t(s_{t-1})(1-\delta)+r_t(s_t)k_t(s_{t-1})+w_t(s_t)h_t(s_t)$$

$$\mathcal{L} = \sum_{t=0}^{\infty}\sum_{s_t} \beta^t u(c_t(s_t), l_t(s_t))\pi_t(s_t) + \sum_{t=0}^{\infty}\sum_{s_t} \lambda_t(s_t)\{-c_t(s_t)-k_{t+1}(s_t)+k_t(s_{t-1})(1-\delta)+r_t(s_t)\cdot k_t(s_{t-1})+w_t(s_t)(1-l_t(s_t))\}$$

$$\frac{\partial L}{\partial c_t(s_t)} = \beta^t u_c(c_t(s_t), l_t(s_t))\pi_t(s_t) - \lambda_t(s_t) = 0$$

$$\frac{\partial L}{\partial l_t(s_t)} = \beta^t u_l(c_t(s_t), l_t(s_t))\pi_t(s_t) - w_t(s_t)\lambda_t(s_t) = 0$$

$$\frac{\partial L}{\partial k_{t+1}(s_t)} = -\lambda_t(s_t) + \sum_{s_{t+1}} \lambda_{t+1}(s_{t+1})[(1-\delta)+r_t] = 0.$$

$$\Rightarrow \left\{ w_t(s_t) = \frac{\cancel{\beta^t} u_c(c_t(s_t), l_t(s_t))\cancel{\pi_t(s_t)}}{\cancel{\beta^t} u_l(c_t(s_t), l_t(s_t))\cancel{\pi_t(s_t)}} = \frac{u_c}{u_l} \right\}$$

Euler:

$$\cancel{\beta^t} u_c(c_t(s_t), l_t(s_t)) = \beta^{\cancel{t+1}} \sum_{s_{t+1}} u_c(c_{t+1}(s_{t+1}), l_{t+1}(s_{t+1}))(1-\delta+r_t)\cdot\pi(s_{t+1}|s_t)$$

$$\boxed{u_c(c_t, l_t) = \beta\cdot E_t[u_c(c_{t+1}, l_{t+1})\cdot(1-\delta+r_t)]} \quad (3)\ 1^\circ$$

suppose $u(c_t, l_t) = \log c_t + A\log l_t$

$$u_c = \frac{1}{c_t} \quad (3)\ 2^\circ$$

$$\Rightarrow \text{Euler}: \boxed{\frac{1}{c_t} = \beta E_t\left\{\frac{1}{c_{t+1}}(r_{t+1}+(1-\delta))\right\}}$$

~~From Firm's problem~~

$$\cancel{w_t = \frac{u_c}{u_l} = \frac{1}{c_t}\cdot\frac{l_t}{A}}$$

$$\begin{cases} k_t^d = k_{t-1} \\ \Rightarrow \boxed{w_t = (1-\alpha)e^{z_t(s_t)} k_{t-1}^{\alpha}(s_t)\cdot h_t^{-\alpha}} \quad (1) \end{cases}$$

When market clears,

$$\Rightarrow w_t(s_t) = \frac{u_c}{u_l} = \frac{1}{c_t}\cdot\frac{l_t}{A} = (1-\alpha)\cdot e^{z_t} k_{t-1}^{\alpha} h_t^{-\alpha}$$

$$\Rightarrow \boxed{\frac{1}{c_t} = \frac{A/(1-h_t)}{(1-\alpha)e^{z_t}\cdot k_{t-1}^{\alpha}\cdot h_t^{-\alpha}}} \quad (4)$$

$$\boxed{\Rightarrow r_t = \alpha e^{z_t}\cdot k_{t-1}(s_t)^{\alpha-1} h_t(s_t)^{1-\alpha}} \quad (2)$$

As w_t ((1)), r_t ((2)) & Euler (3) $1^\circ \in CE$ are the same as the paretal efficient situation detaild in lecture note, the system is paretal efficient. (except in lecture nots $k_t^d = k_t$, but here it's $k_t^d = k_{t-1}$, the rest is the same). The welfare therem holds.