Sandwich Panels

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1 Descriptive Parameters

t : face thickness

c : core thickness

l : beam length

b : beam width

W : beam Weigth

ρ_c^*: core foam density

ρ_s: core solid density

$\frac{\rho_c^*}{\rho_s}$: core relative density (typically 0.02 to 0.3)

$\frac{t}{l}$: typically 1/2000 to 1/200 = 0.0005 to 0.005

2 loading configuration constants

config independent

$C_2 = 3/8 = 0.375$

$C_{11} = 0.15$ (also named C4) constant of proportionality for the core shear strength

3 Point Bending

$B_1 = 48$

$B_2 = 4$

$B_3 = 4$

$B_4 = 2$

4 Point Loading

$B_1 = 28.2$

$B_2 = 3$

Cantilever

$B_3 = 1$

3 Deflection δ

Effective Rigidity $$(EI)_{eq} = \frac{E_f \cdot b \cdot t^3}{6} + \frac{E_c \cdot b \cdot c^3}{12} + \frac{E_f \cdot b \cdot t}{2} \cdot (c+t)^2$$

given that: $(E - f >> E_c^* \ \& \ c >> t)$

$(EI)_{eq}$ can be approxiated to: $$(EI)_{eq} = \frac{E_f \cdot b \cdot t \cdot c^2}{2}$$

Deflection (bending): $$\delta_b = \frac{P \cdot l^3}{EI_{eq}}$$

(shear): $$\delta_s = \frac{P \cdot l}{B_2 \cdot b \cdot c \cdot G_c^*}$$

$$\delta = \delta_b + \delta_s$$

precise $$\delta = \frac{P \cdot l^3}{B_1 \cdot (EI)_{eq}} + \frac{P \cdot l}{B_2 \cdot b \cdot c \cdot G_c^*}$$

simplified $$\delta \approx \frac{2 \cdot P \cdot l^3}{B_1 \cdot E_f \cdot b \cdot t \cdot c^2} + \frac{P \cdot l}{B_2 \cdot b \cdot c \cdot G_c^*}$$

with $$G_c^* \approx C_2 \cdot E_s \cdot (\rho^* / \rho_s)^2$$ (open-cell foam model)

$$C_2 = 3/8 = 0.375$$

4 Minimum weight (W) for a given stiffness

$$(\frac{c}{l})_{opt} = 4.3 \cdot \{\frac{C_2 \cdot B_2}{B_1^2} \cdot (\frac{\rho_f}{\rho_s})^2 \cdot \frac{E_s}{E_f^2} \cdot \frac{P}{\delta \cdot b}\}^{1/5}$$

$$(\frac{t}{l})_{opt} = 0.32 \cdot \{\frac{1}{B_1 \cdot B_2^2 \cdot C_2^2} \cdot (\frac{\rho_s}{\rho_f})^4 \frac{1}{E_f \cdot E_s^2} \cdot (\frac{P}{\delta \cdot b})^3\}^{1/5}$$

$$(\frac{\rho_c^*}{\rho_s})_{opt} = 0.59 \cdot \{\frac{B_1}{B_2^3 \cdot C_2^3} \cdot (\frac{\rho_s}{\rho_f}) \frac{E_f}{E_s^3} \cdot (\frac{P}{\delta \cdot b})^2\}^{1/5}$$

Note: $\frac{W_{faces}}{W_{cores}} = \frac{1}{4}$ $\quad \frac{\delta_b}{\delta} = \frac{1}{3}$ $\quad \frac{\delta_s}{\delta} = \frac{2}{3}$

5 Failure mode

face:	can yield
	compressible face can buckle locally - "wrinkling"
core:	can fail in shear
also:	can have debonding and indentation
	we will assume perfect bond and load distributed sufficiently to avoid indentation.

Face yielding: (the faces carry all the normal stress since $E_f >> E_c$):

$$\sigma_f = \frac{M \cdot y}{(EI)_{eq}} \cdot E_f \approx M \cdot \frac{c}{2} \cdot \frac{2}{E_f \cdot b \cdot t \cdot c^2} \cdot E_f = \frac{M}{b \cdot t \cdot c}$$

for a beam with a concentrated load P, the yielding occurs when

$$\sigma_f \approx \frac{P \cdot l}{B_3 \cdot b \cdot t \cdot c} = \sigma_{yf}$$

Face wrinkling: when normal stress in the face = local buckling stress

$$\sigma_f = 0.57 \cdot E_f^{1/3} \cdot E_s^{2/3} \cdot (\frac{\rho_c^*}{\rho_s})^{4/3} \approx \frac{P \cdot l}{B_3 \cdot b \cdot t \cdot c}$$

Core shear failure occurs when:

$$\tau_{c\ max} = \frac{P}{B_4 \cdot b \cdot c} = C_{11} \cdot (\frac{\rho_c^*}{\rho_s})^{3/2} \cdot \sigma_{ys}$$

$$C_{11} \approx 0.15$$

5.1 failure transitions

5.2 Face yielding and face wrinkling

$$(\rho_c^* / \rho_S) = (\frac{\sigma_{yf}}{0.57 \cdot E_f^{1/3} \cdot E_s^{2/3}})^{(3/4)}$$

i.e. for given face and core materials, at constant (ρ_c^* / ρ_s)

5.3 Face yield and core shear

$$(\frac{t}{l}) = \frac{C_{11} \cdot B_4}{B_3} \cdot (\frac{\rho_c^*}{\rho_s})^{3/2} \cdot (\frac{\sigma_{ys}}{\sigma_{yf}})$$

5.4 Face wrinking and core shear

$(\frac{t}{l}) = (\frac{C_{11} \cdot B_4}{0.57 \cdot B_3}) \cdot (\frac{\sigma_{ys}}{E_f^{1/3} \cdot E_s^{2/3}}) \cdot (\frac{\rho_c^*}{\rho_s})^{1/6}$

6 Minimum weight design for stifness and strength

$$W = 3.18 \cdot b \cdot l^2 [\frac{1}{B_1 \cdot B_2^2 \cdot C_2^2} \cdot \frac{\rho_f \cdot \rho_s^4}{E_f \cdot E_s^2} \cdot (\frac{P}{\delta \cdot b})^3]^{1/5}$$