



# [Designing with a complete simulation test bench for op amps, Part 1: Output impedance](#)

[Ian Williams](#) - March 16, 2018

In a world of tight timelines and ever-increasing performance requirements, it's critical to create circuit designs right the first time; thus, engineers in the analog and mixed-signal industry often turn to simulation to improve their chances of success. A circuit simulation is only as good as the models that it contains, however. For crucial designs, it's important to verify that your models match the specs promised by their data sheets.

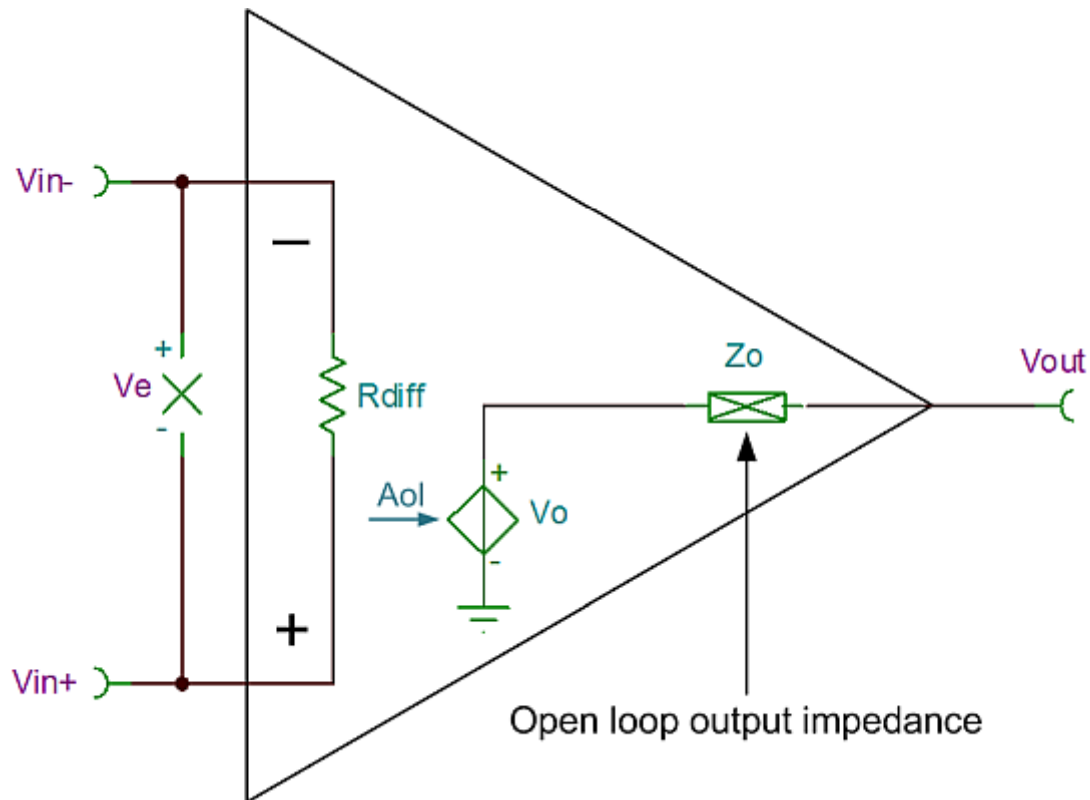
In this series, I'll provide a complete simulation test bench for [operational amplifiers](#) (op amps), covering every key op amp specification, how they impact application performance and the approach behind the test circuit designs.

## **Open loop output impedance - $Z_o$**

One of the most critical (and yet often overlooked) characteristics of an op amp – especially when performing small-signal stability analysis and dealing with small-signal output load transients such as driving an analog-to-digital converter (ADC) – is the open loop small-signal AC output impedance. Before getting into the details of output impedance, let's first define some terms.

Throughout this series, I'll use the term  $Z_o$  to indicate the *open loop* small-signal AC output impedance of an op amp and the term  $Z_{out}$  to indicate the *closed loop* small-signal AC output impedance of an op amp. It's important to distinguish between the two, for reasons that will become clear later. Unfortunately, there does not seem to be a standard for these terms in the analog semiconductor industry, with data sheets from different manufacturers using  $Z_o$ ,  $Z_{out}$ ,  $R_o$ , and  $R_{out}$  somewhat inconsistently.

$Z_o$  is an impedance in the op amp's small-signal path that occurs between the open loop gain stage ( $A_{ol}$ ) and the output pin ( $V_{out}$ ). This impedance interacts with  $A_{ol}$  across frequency to create the op amp's overall AC response. **Figure 1** shows a simplified representation of  $Z_o$  in an op amp small-signal model.

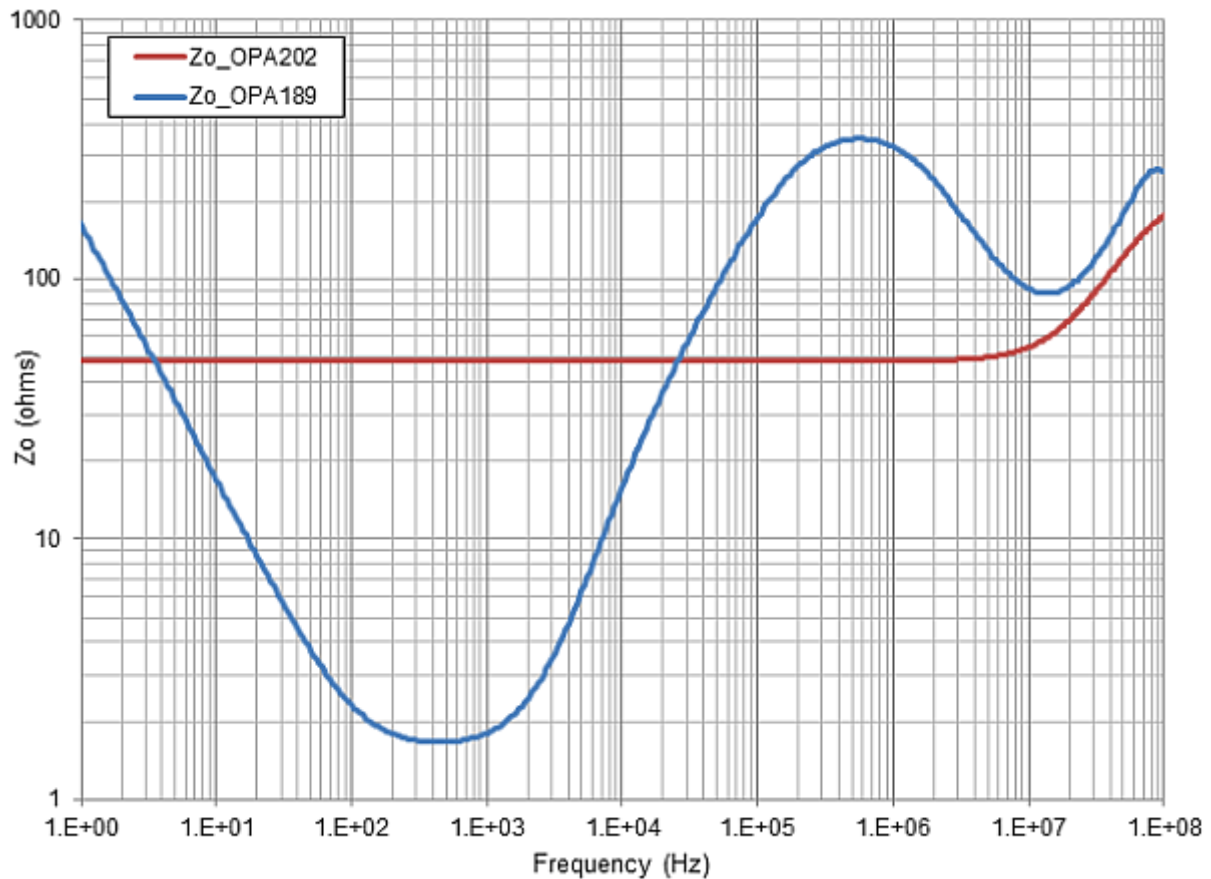


**Figure 1** Simplified op amp small-signal model (open loop)

In this model, a differential input signal ( $V_e$ ) develops across the op amp's input resistance ( $R_{diff}$ ).  $V_e$  is amplified by  $A_{ol}$  to generate the ideal output voltage ( $V_o$ ), which flows through  $Z_o$  before appearing at the output pin ( $V_{out}$ ).

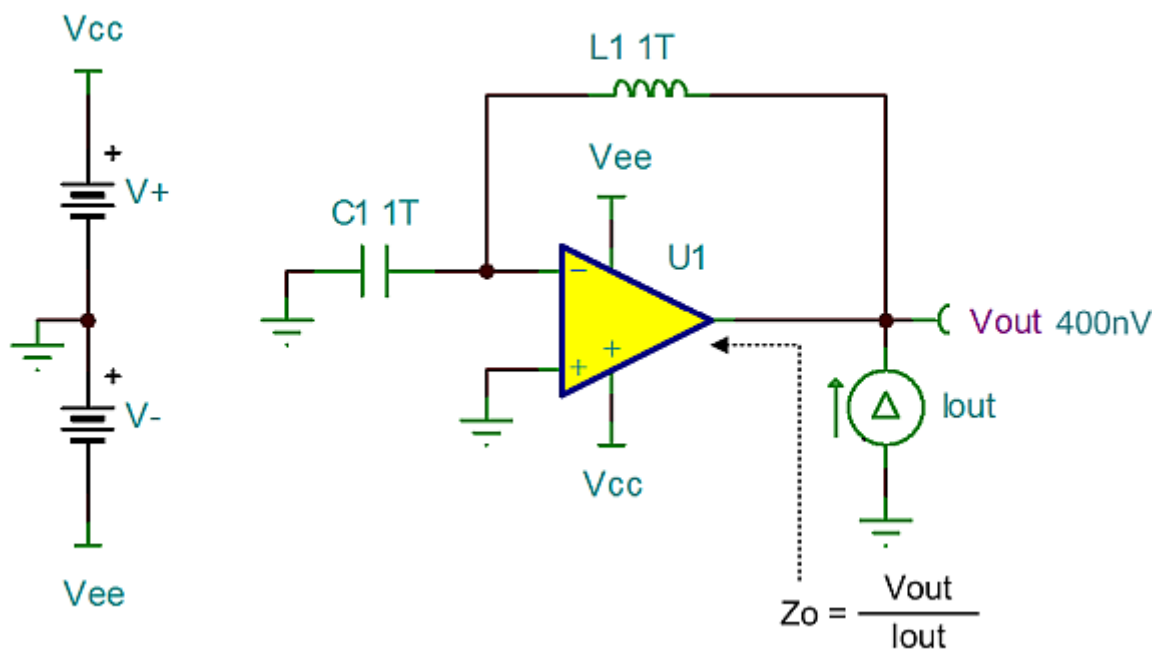
$Z_o$  is a characteristic of the output stage of the op amp. In the past, when bipolar amplifiers with simpler designs dominated the industry, the open loop output impedance of most devices was resistive, or constant, over frequency. Now,  $Z_o$  can be a highly complex characteristic with capacitive, inductive, and resistive regions that roll off sharply over frequency. This is a consequence of the various techniques that analog integrated circuit (IC) designers must use to meet customer needs today, such as rail-to-rail input and output, high open loop gain, high common-mode rejection ratio, low noise, and low power.

**Figure 2** compares the open loop output impedance of the Texas Instruments [OPA202](#), a modern bipolar amplifier with a classic output stage design, to the [OPA189](#), a complementary metal-oxide semiconductor (CMOS) amplifier with ultra-high DC precision and rail-to-rail output. Take note of the resistive nature of the  $Z_o$  curve for the OPA202, while the  $Z_o$  curve for the OPA189 is alternately capacitive and inductive throughout different frequency regions.



**Figure 2** Open loop output impedance: OPA202 vs. OPA189

If op amp manufacturers do not model  $Z_o$  accurately, then the overall small-signal AC behavior of an op amp simulation model is incorrect and not reliable for stability analysis. Thankfully, it's easy to verify that a model's  $Z_o$  matches the data sheet. **Figure 3** shows the recommended test circuit.



**Figure 3** Open loop output impedance test circuit

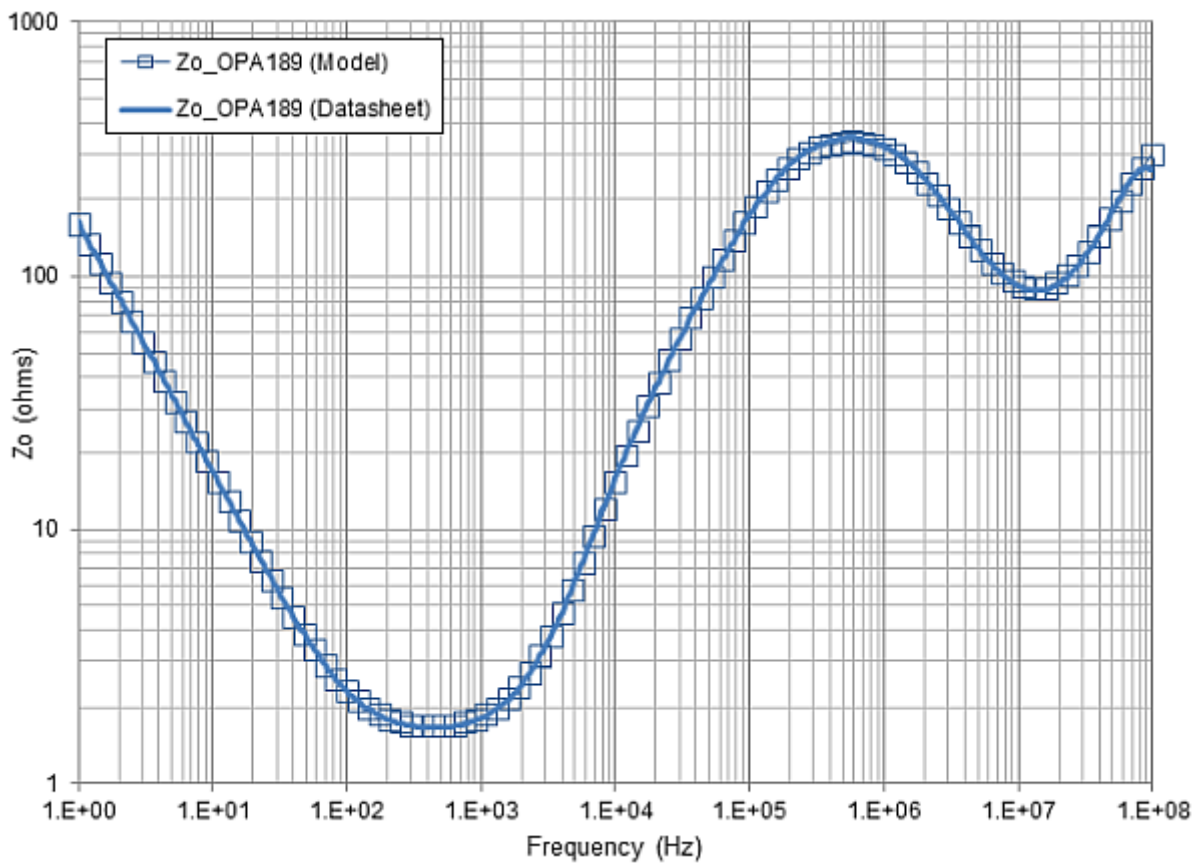
In this test circuit, inductor L1 creates closed loop feedback at DC while allowing for open loop AC analysis, and capacitor C1 shorts the inverting input to ground at AC to prevent the node from

floating. The op amp must be in its linear operating region (as shown in **Figure 3**), where  $V_{out}$  is equal to a small offset voltage. Always check your supply voltage and common-mode voltage to make sure that no limits are being violated.

The AC current source  $I_{out}$  back-drives the op amp output, and by measuring the resulting voltage at  $V_{out}$ , you can calculate  $Z_o$  using Ohm's law, as shown in Equation 1:

$$Z_o = \frac{V_{out}}{I_{out}} = \frac{V_{out}}{1A} = V_{out} \quad (1)$$

Since you're using a 1A AC current source for  $I_{out}$ ,  $Z_o$  is simply equal to  $V_{out}$ . To plot  $Z_o$ , run an AC transfer function over the desired frequency range and plot the voltage at  $V_{out}$ . Note that many simulators default to showing the results in decibels. If you plot the measurement on a logarithmic scale,  $V_{out}$  is equivalent to ohms. Let's use this circuit to test the  $Z_o$  of the [OPA189 SPICE model](#).



**Figure 4** OPA189  $Z_o$  results

In this case, the op amp's  $Z_o$  is modeled very closely to the data sheet curve and can be used for small-signal analysis to achieve results that will match the real world.

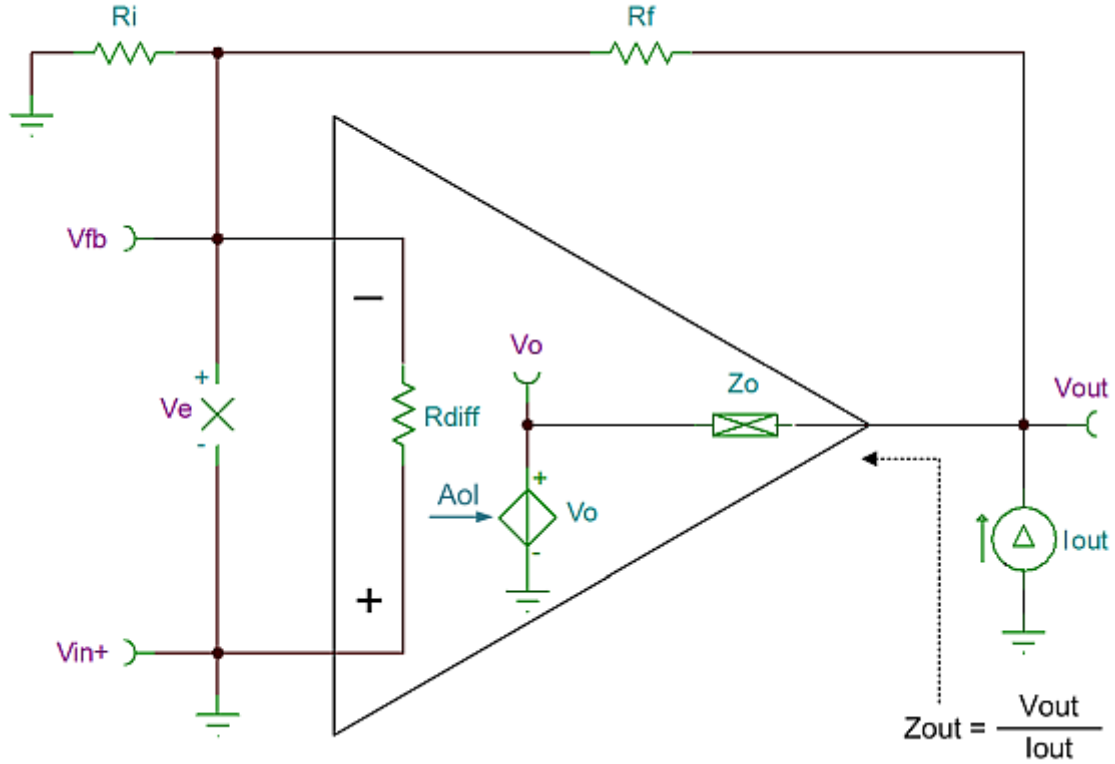
### Closed loop output impedance - $Z_{out}$

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Closed loop output impedance,  $Z_{out}$ , is the impedance looking in to the output of the op amp when the op amp is in a closed loop configuration with negative feedback. Unlike  $Z_o$ , which is an inherent

property of the op amp and does not change (for the most part) with variations in load or feedback,  $Z_{out}$  is a function of  $Z_o$ ,  $A_{ol}$  and  $\beta$ , the feedback factor set by the feedback network.

Let's return to the op amp small-signal model, now expanded to represent a closed loop circuit, shown in **Figure 5**.



**Figure 5** Simplified op amp small-signal model (closed loop)

As shown in Equation 2, the output impedance is still equal to  $V_{out}$  divided by  $I_{out}$ , even though the op amp loop has closed. You can derive  $Z_{out}$  as a function of the other circuit elements in order to gain a better understanding of its behavior.

$$Z_{out} = \frac{V_{out}}{I_{out}} \quad (2)$$

Feedback factor  $\beta$  is the ratio of the voltage that appears at feedback node  $V_{fb}$  vs. the output voltage  $V_{out}$ . The standard resistor divider equation given in Equation 3 computes the voltage at  $V_{fb}$ :

$$\beta = \frac{V_{fb}}{V_{out}} = \frac{V_{out} * \frac{R_i}{(R_f + R_i)}}{V_{out}} = \frac{R_i}{R_f + R_i} \quad (3)$$

Since the noninverting input of the op amp is grounded,  $V_e$ , or the error voltage between the op amp input pins, is equal to  $V_{fb}$ . By rearranging the terms of Equation 3, you can determine that  $V_{fb}$  is equal to  $V_{out}$  multiplied by  $\beta$ , as shown in Equation 4:

$$V_e = V_{fb} = V_{out} * \beta \quad (4)$$

$V_e$  is amplified by the op amp's  $A_{ol}$  to generate  $V_o$ . Since  $V_e$  is a positive voltage at the op amp's inverting input while the noninverting input is grounded, adding a negative sign preserves the

correct polarity, as shown in Equation 5:

$$V_o = -V_e * A_{ol} \quad (5)$$

You can now calculate the output voltage in Equation 6 by adding  $V_o$  to the voltage drop across  $Z_o$  induced by output current  $I_{out}$ . To simplify the calculations, in this step assume that the output impedance is much lower than the impedance of the feedback network so that all of  $I_{out}$  flows through  $Z_o$ .

$$V_{out} = V_o + I_{out} * Z_o \quad (6)$$

Substituting Equation 5 into Equation 6 for  $V_o$  results in Equation 7:

$$V_{out} = -V_e * A_{ol} + I_{out} * Z_o \quad (7)$$

Substituting Equation 4 into Equation 7 for  $V_e$  results in Equation 8:

$$V_{out} = -V_{out} * \beta * A_{ol} + I_{out} * Z_o \quad (8)$$

Rearranging the equation to bring all  $V_{out}$  terms to the left side results in Equation 9:

$$V_{out} + V_{out} * \beta * A_{ol} = I_{out} * Z_o \quad (9)$$

Factoring the left side of the equation for  $V_{out}$  results in Equation 10:

$$V_{out} * (1 + \beta * A_{ol}) = I_{out} * Z_o \quad (10)$$

Dividing both sides of the equation by the term  $(1 + \beta * A_{ol})$  results in Equation 11, the new definition of  $V_{out}$ :

$$V_{out} = \frac{I_{out} * Z_o}{1 + \beta * A_{ol}} \quad (11)$$

Substituting Equation 11 into Equation 2 results in Equation 12, the definition of  $Z_{out}$ :

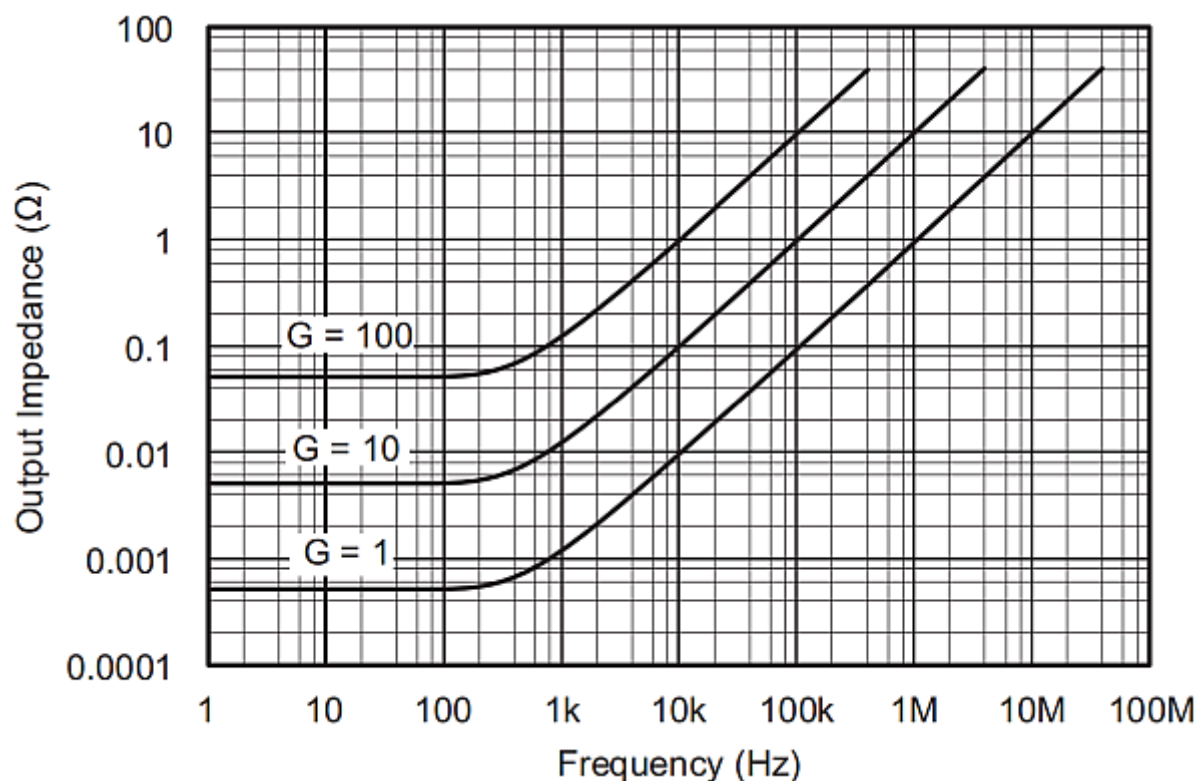
$$Z_{out} = \frac{V_{out}}{I_{out}} = \frac{\frac{I_{out} * Z_o}{1 + \beta * A_{ol}}}{I_{out}} = \frac{Z_o}{1 + \beta * A_{ol}} \quad (12)$$

The definition of  $Z_{out}$  shows that in a closed loop configuration,  $Z_o$  is reduced by  $A_{ol} * \beta$ , the loop gain of the op amp. Since  $A_{ol}$  is typically very large (especially at low frequencies),  $Z_{out}$  will therefore be very small. However, once the bandwidth of the amplifier is exceeded and no loop gain remains,  $Z_{out}$  approaches  $Z_o$ .

### Specifying $Z_o$ rather than $Z_{out}$

Most op amp manufacturers specify  $Z_o$  rather than  $Z_{out}$  in their data sheets, since  $Z_o$  is a better representation of the op amp's inherent output impedance without the influence of other circuit elements. However, the data sheets for many older devices provide a plot for  $Z_{out}$ . Also remember that the terms  $Z_o$  and  $Z_{out}$  are not standard in the industry, so take care when reading a data sheet to make sure that you know which property is being characterized.

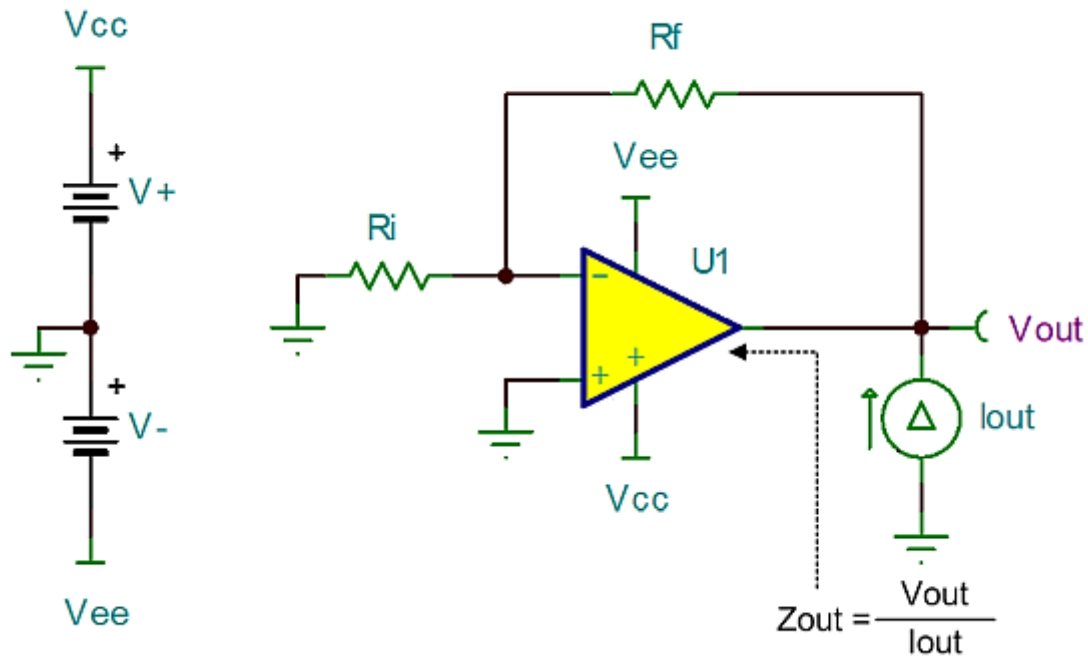
**Figure 6** shows the closed loop output impedance vs. the frequency curve for the [OPA350](#), an op amp released in 2000.



**Figure 6** Closed loop output impedance for the OPA350

Notice how low the closed loop output impedance becomes after being reduced by the op amp's loop gain. In this case, the low-frequency  $Z_{out}$  ranges from about 1m $\Omega$  to 100m $\Omega$ . Also note the shape of the curves, which look like inverted versions of a classic open loop gain plot. This shouldn't be a surprise given what you now know about  $Z_{out}$ . Finally, see that curves are provided for closed loop gains equal to 1, 10 and 100V/V. Since  $\beta$  sets the closed loop gain, this means that the plot defines  $Z_{out}$  for three common values of  $\beta$ .

If your circuit design uses op amps whose data sheets specify  $Z_{out}$ , then you will need to verify a model's  $Z_{out}$  rather than its  $Z_o$ . **Figure 7** shows the recommended test circuit.



**Figure 7** Closed loop output impedance test circuit

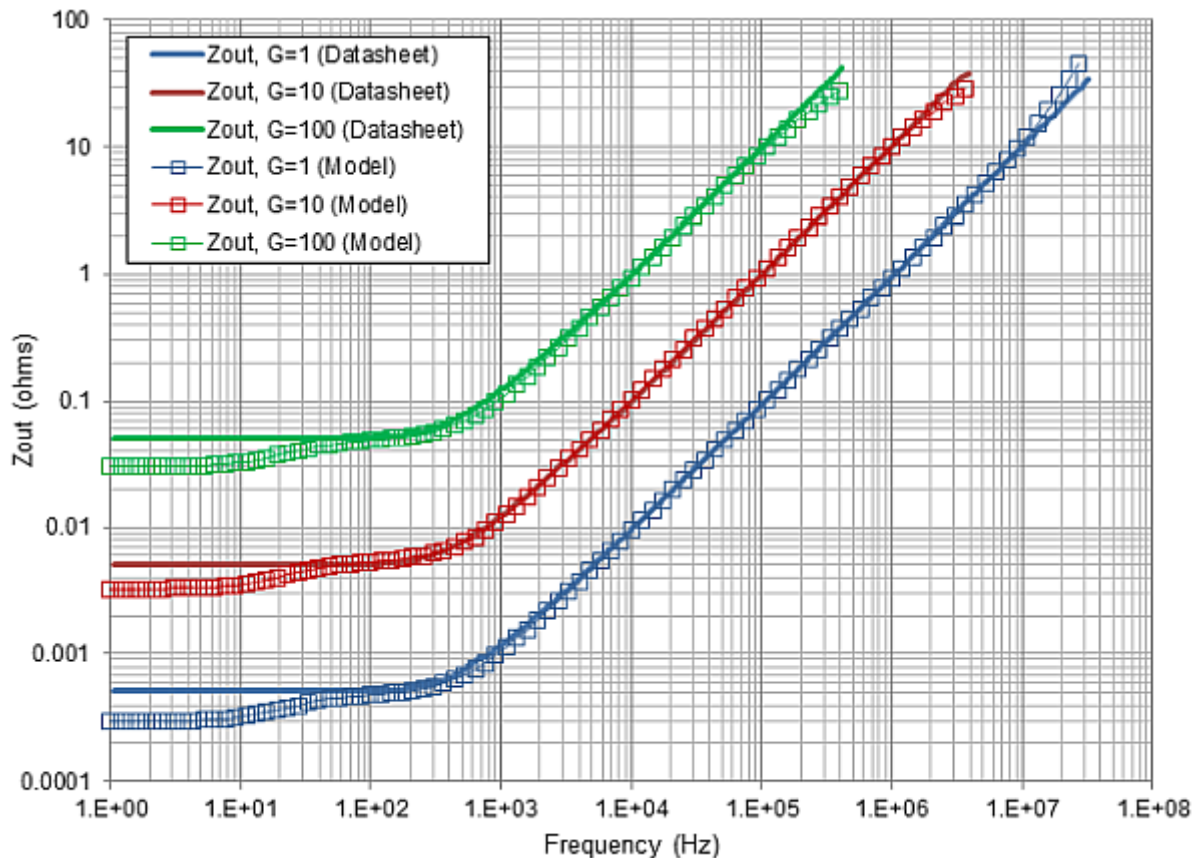
In this test circuit, feedback resistors  $R_f$  and  $R_i$  close the loop around the op amp to create a test condition that matches the model used in the derivation of  $Z_{out}$ . As before, AC current source  $I_{out}$  back-drives the op amp output. By measuring the resulting voltage at  $V_{out}$ , you can determine  $Z_{out}$  using Ohm's law, as shown in Equation 13:

$$Z_{out} = \frac{V_{out}}{I_{out}} = \frac{V_{out}}{1A} = V_{out} \quad (13)$$

To plot  $Z_{out}$ , run an AC transfer function over the desired frequency range and plot the voltage at  $V_{out}$ . If your simulation software supports it, you can step through values of  $R_f$  and  $R_i$  to create closed loop gain settings that match the data sheet curves. Let's use this circuit to test the  $Z_{out}$  of the [OPA350 SPICE model](#) at  $G = 1, 10$  and  $100V/V$ .

For  $G = 1V/V$ , set  $R_f$  equal to  $1m\Omega$  (or a short circuit) and  $R_i$  equal to  $1T$  (or an open circuit). This places the op amp in a standard unity-gain configuration. For  $G = 10V/V$ , set  $R_f = 10 * R_i$ . For  $G = 100V/V$ , set  $R_f = 100 * R_i$ .





**Figure 8** OPA350 Zout results

Again, the output impedance of the model matches the data sheet curves quite closely, with the exception of some small deviations at low frequency. Thankfully, these low-frequency deviations are negligible for small-signal stability analysis, which is usually dominated by the high-frequency characteristics of Zout. Based on this verification, you can be confident that any stability compensation applied to this simulation model will give you a good indication of the behavior of real silicon. For an introduction to common stability issues and stability compensation techniques, watch the [TI Precision Labs - Op Amps video series on stability](#).

In summary:

- $Z_o$  is the open loop small-signal AC output impedance of an op amp. It is an inherent characteristic of the design of the op amp output stage and does not change based on feedback or load.
- $Z_{out}$  is the closed loop small-signal AC output impedance of an op amp. It is the effect of  $Z_o$  being reduced by the open loop gain of the op amp as well as the feedback factor  $\beta$ .
- Output impedance is a key contributor to an op amp's small-signal behavior. If performing analysis on critical circuits with simulation, always verify the accuracy of the op amp model's output impedance.

Thank you for reading the first installment in this series. In [part 2](#), I'll continue the discussion on op amp small-signal characteristics and show how to verify open loop gain ( $A_{ol}$ ), closed loop gain ( $A_{cl}$ ) and small-signal step response.

### Additional resources

- Learn more about stability simulation in Bruce Trump's "[SPICEing Op Amp Stability](#)" blog post.
- Download the [Solving Op Amp Stability Issues](#) presentations by Tim Green and Collin Wells.
- Read these Analog Applications Journal articles:

- [“Modeling the output impedance of an op amp for stability analysis.”](#)
- [“Op amp stability and input capacitance.”](#)

## References

1. Frederiksen, Thomas M. [“Intuitive Operational Amplifiers, From Basics to Useful Applications,](#) Revised Edition.” McGraw-Hill Book Company: New York, New York, 1988.

## Acknowledgements

The author acknowledges Tim Green as “the wizard of Zo” and thanks him for his continued mentorship on the vast topic of output impedance.

*[Ian Williams](#) is an applications engineer and SPICE model developer for the Precision Amplifiers group at Texas Instruments.*

## Related articles:

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