

Uniswap V3 Pricing Review for Lenders

Abstract

Stochastic Calculus Review

Ito's Lemma

Uniswap V3 Stochastic Analysis

Value of a Uniswap V3 Position

Uniswap State Equations

Pricing Derivations for Impermanent Loss

```
In[66]:= ethDailyVol = 0.0034;  
ethMeanYearly = 0.1;  
currentPrice = 1628;  
lowerBound = 1600;  
upperBound = 1700;  
initialValue = 10000;
```

```
In[109]:= currentLiquidityParams =  
  Liquidity[lowerBound, upperBound, currentPrice, initialValue] // N
```

```
Out[109]:= {{x → 4.37661, y → 2874.87, L → 8249.71}}
```

```

In[110]:= originalValue =
  x*p + y /. tokensGivenLiquidity /. {pa → lower, pb → higher, p → startPrice};
currentValue = x*p + y /. tokensGivenLiquidity /. {pa → lower, pb → higher};
IL =  $\frac{\text{currentValue} - \text{originalValue}}{\text{originalValue}}$ ;
humanReadable = {lower → pa, higher → pb, startPrice → p0};

In[77]:= IL /. humanReadable // Simplify;

```

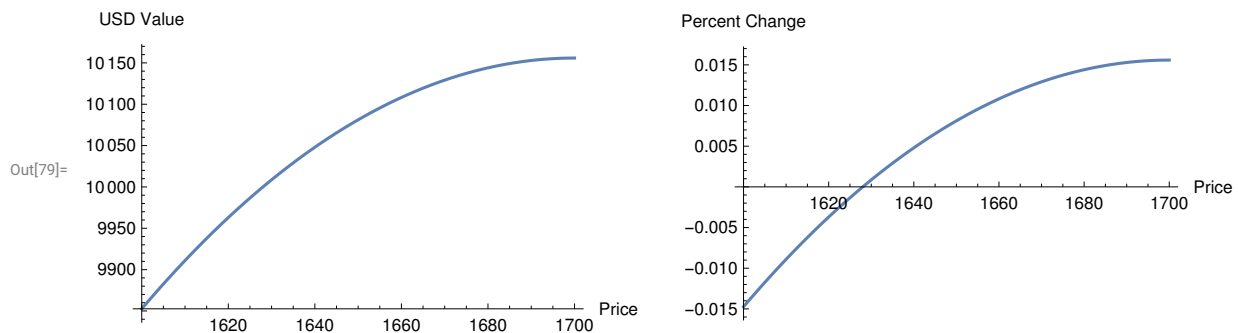
Plotting to Understand Value Curves

```

In[78]:= valueCurve = currentValue /. currentLiquidityParams;

In[79]:= GraphicsGrid[{{
  Plot[valueCurve /.
    {lower → lowerBound, higher → upperBound, startPrice → currentPrice},
    {p, lowerBound, upperBound}, AxesLabel → {"Price", "USD Value"}],
  Plot[
    IL /. {lower → lowerBound, higher → upperBound, startPrice → currentPrice},
    {p, lowerBound, upperBound}, AxesLabel → {"Price", "Percent Change"}]
}}]

```



Pricing With IL

```
In[80]:= resultIL = MVIto@@{IL, {p, t}, {{diffGBM /. {S -> p}}}} //
```

```
Expand //
```

```
(# /. {(dt)^2 -> 0, (dW)^2 -> dt, dt dW -> 0}) & //
```

```
Simplify;
```

```
resultIL /. humanReadable
```

$$\text{Out[81]} = \frac{dW \left(4 p \sigma - 4 \sqrt{p} \sigma \sqrt{p_b} \right) + dt \left(2 p \left(2 \mu + \sigma^2 \right) - \sqrt{p} \left(4 \mu + \sigma^2 \right) \sqrt{p_b} \right)}{4 \left(p_0 + \left(-2 \sqrt{p_0} + \sqrt{p_a} \right) \sqrt{p_b} \right)}$$

```
In[82]:= preprocessIL = resultIL /. currentLiquidityParams[[1]] /.
```

```
{p -> p[t], W -> W[t], lower -> lowerBound, higher -> upperBound, \mu -> \frac{\text{ethMeanYearly}}{365},
```

```
\sigma -> ethDailyVol, startPrice -> currentPrice} // N // Simplify
```

$$\text{Out[82]} = 0.000228404 dt \sqrt{p[t]} + 0.0028049 dW[t] \sqrt{p[t]} - 5.59742 \times 10^{-6} dt p[t] - 0.0000680288 dW[t] \times p[t]$$

```
In[83]:= procIL = ItoProcess[dV[t] == preprocessIL,
```

```
V[t], {V, 10 000}, {t, 0}, {W \approx WienerProcess[], p \approx
```

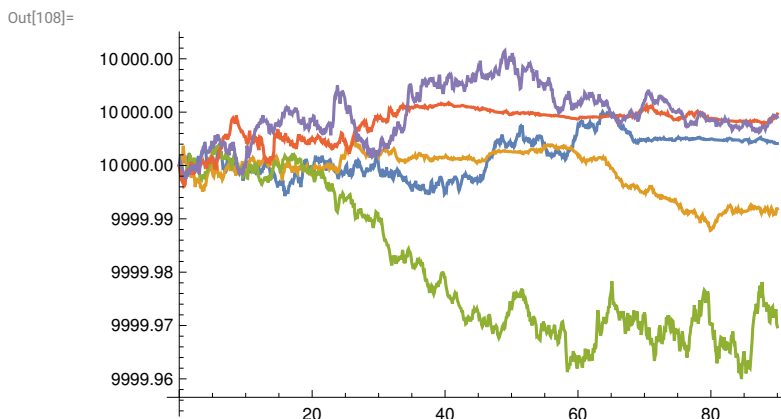
```
GeometricBrownianMotionProcess[\frac{\text{ethMeanYearly}}{365}, \text{ethDailyVol}, \text{currentPrice}]]];
```

```
fsIL = RandomFunction[procIL, {0, 90}, 5];
```

```
In[85]:= Mean[fsIL[90]]
```

```
Out[85]= 10 000.
```

```
In[108]:= Show[{
  ListLinePlot[fsIL, FillingStyle -> Axis]
}]
```



Pricing with Position Value

```
In[129]:= preprocessPV =
  currentValue /. {lower → lowerBound, higher → upperBound, p → p[t]} /.
  currentLiquidityParams;
```

```
In[130]:= preprocessPV
```

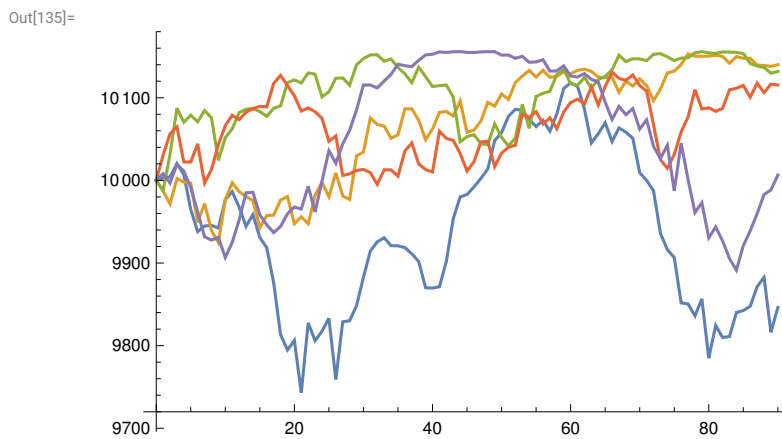
```
Out[130]:= {8249.71 (-40 + √p[t]) + 200.085 (10 √17 - √p[t]) √p[t]}
```

```
In[131]:= procPV = TransformedProcess[preprocessPV, {p ≈
  GeometricBrownianMotionProcess[ $\frac{\text{ethMeanYearly}}{365}$ , ethDailyVol, currentPrice]},
  t];
fsPV = RandomFunction[procPV, {0, 90, 1}, 5];
```

```
In[134]:= Mean[fsPV[90]]
```

```
Out[134]:= 10 048.
```

```
In[135]:= ListLinePlot[fsPV, FillingStyle → Axis]
```



Further Results

```
In[94]:= TableForm[{
  {"Impermanent Loss in Uniswap V3", Hyperlink[
    "https://lambert-guillaume.medium.com/an-analysis-of-the-expected-value-of-the-
    impermanent-loss-in-uniswap-bfbfebbefed2"]},
  {"Uniswap Liquidity V3 Math",
    Hyperlink["http://atiselsts.github.io/pdfs/uniswap-v3-liquidity-math.pdf"]}
}]
```

Out[94]//TableForm=

Impermanent Loss in Uniswap V3	https://lambert-guillaume.medium.com/an-analysis-of-impermanent-loss-in-uniswap-bfbfebbefed2
Uniswap Liquidity V3 Math	http://atiselsts.github.io/pdfs/uniswap-v3-liquidity-math.pdf

Perpetual Lending Stochastic Analysis

Mean-Reverting Additional Value Term

Above, we saw Uniswap V3