DS Instability in an Annular (Corbino) Geometry

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June 19, 2020

1 Equations

We study the equations:

$$\begin{aligned} \frac{\partial J}{\partial t} + \frac{\partial}{\partial r}\left(\frac{J^2}{n} + n\right) - \frac{\eta}{m}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\frac{J}{n} = \gamma(J_0 - J) - \frac{J^2}{n}\frac{1}{r}, \\ \frac{\partial n}{\partial t} + \frac{\partial J}{\partial r} = -\frac{J}{r}. \end{aligned} \tag{1}$$

2 Asymmetric Boundary Conditions

Consider first the generalization of the “asymmetric" boundary conditions from [1]:

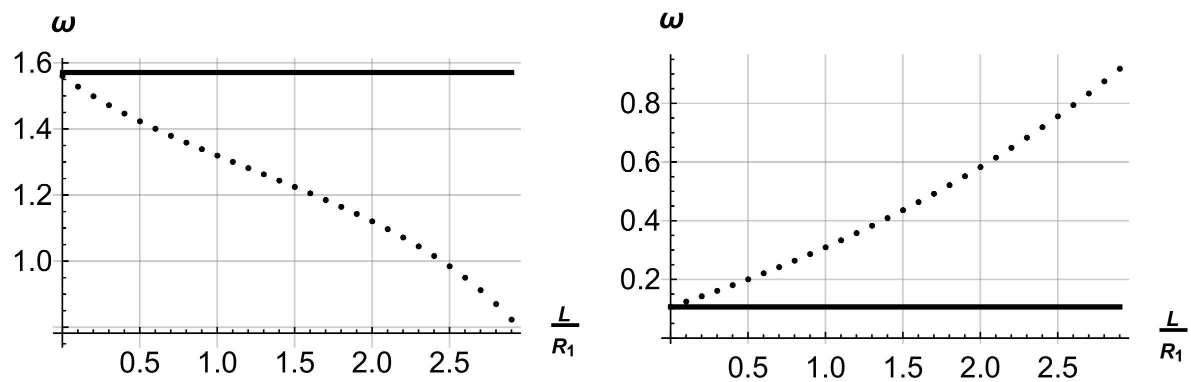
$$\begin{aligned} \left.\frac{\partial}{\partial r}(rJ)\right|_{r=R_1} &= 0, \\ J(r = R_2) &= v_0, \\ n(r = R_1) &= 1. \end{aligned} \tag{2}$$

2.1 Quasinormal Modes

We linearize Eqs. (1) with respect to the small perturbations $J = J_0+J_1 e^{-\mathrm{i}\omega t}$, $n = n_0+n_1 e^{-\mathrm{i}\omega t}$. The resulting, a Bessel Equation, can be solved numerically. At $v_0 = 0.14$, $\eta = 0.01$, and $\gamma = 0.04$, Fig. (1) shows the dependence of these values on the ratio $\mathcal{R} \equiv L/R_1$. In the limit of small $\mathcal{R}$, you get:

$$\omega \approx \left(\frac{\pi}{2} - \frac{\mathcal{R}}{\pi}\right) + \mathrm{i}\left(v_0(1 + 2\mathcal{R}) - \frac{\gamma}{2} - \frac{\eta\pi^2 n^2}{8}\right) \tag{3}$$

Increasing $\mathcal{R}$, making the problem “more annular" decreases the oscillation frequency. But at the same time, the growth rate becomes larger. That means the instability should persist at lower v_0 or higher η and γ, each of which could be desirable experimentally.



Real Part
(Frequency)

Imaginary Part
(Growth Rate)

Figure 1: Frequency and Growth Rate of the quasinormal modes under asymmetric boundary conditions. The solid black line gives the value for $\mathcal{R} = 0$, which corresponds to the rectangular geometry.

2.2 Simulations

Let's try first changing the ratio $\mathcal{R}$ and keeping the other parameters (v_0, η, γ) constant. Those results are given in Fig. (2).

As expected, we notice a decrease in the oscillation frequency as $\mathcal{R}$ increases. The instability also grows faster at higher $\mathcal{R}$. But the amplitude of the endpoint of the instability decreases with greater $\mathcal{R}$. I think this is a nonlinear effect— the linearized solutions would say the frequency keeps growing.

Let's check if we do find an instability in an area of parameter-space where we wouldn't have in the rectangular geometry. Eq.(2) of _ suggests that there should be *no* instability at the parameters $v_0 = 0.04$, $\eta = 0.01$, $\gamma = 0.1$. But running the simulation does give an instability as long as $\mathcal{R}$ is large enough. For instance, for $\mathcal{R} = 2.500$, the results are shown in Fig. (3).

3 Symmetric Boundary Conditions

In the rectangular geometry, it's the asymmetric boundary conditions that allow the instability. But a paper I found from 2010, [2], guessed that since the annulus has a geometric asymmetry between the electrodes already, one might be able to get away with symmetric boundary conditions. By considering linearized equations, they found that the instability should exist and has *double the frequency* of the rectangular case. But they don't have any viscosity or

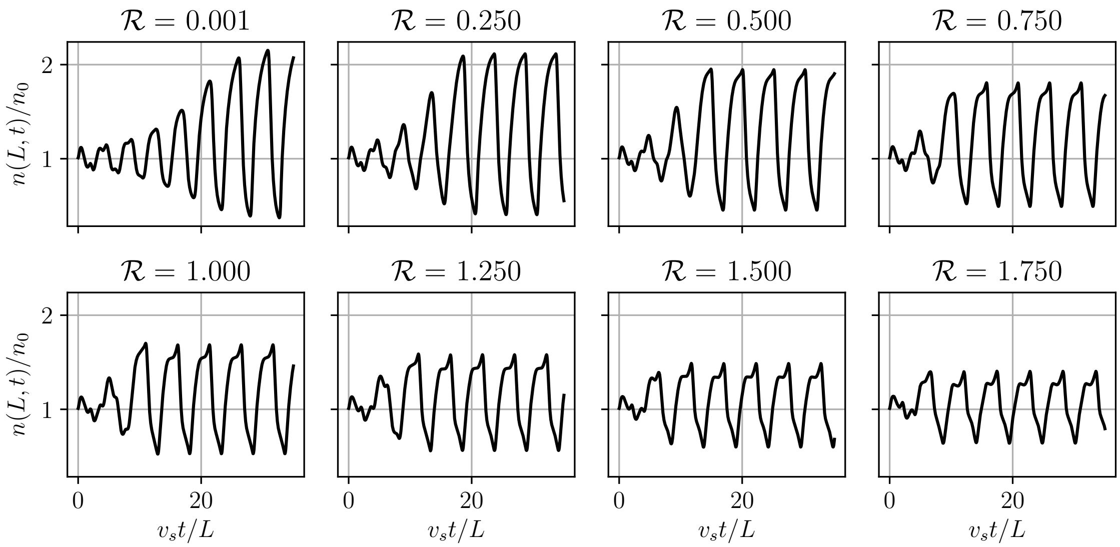


Figure 2: Keeping other parameters constant, change the ratio $\mathcal{R}$.

relaxation. The boundary conditions are:

$$n(r = R_1) = n(r = R_2) = 0. \tag{4}$$

I'll note that through the continuity equation, this puts a Neumann-like condition on the momentum J as well.

3.1 Quasinormal Modes

Anyway, I solved the linearized equations numerically with these new boundary conditions, and I found that we do get a higher frequency and an instability that is favoured at higher $\mathcal{R}$, as summarized in Fig. (4).

The parts with negative imaginary ω have no instability— but, with increasing $\mathcal{R}$, it quickly goes positive. So the instability should exist— but I haven't been able to observe the endpoint numerically yet. I suspect I'm overconstraining it, because I'm trying right now to apply four boundary conditions: two Dirichlet conditions on the density, and two Neumann-like conditions on the momentum. The best I can do right now is plot it in a regime where the instability is not supposed to exist, and at least show that the frequency is higher. That's given in Fig. (5).

References

[1] Christian B. Mendl, Marco Polini, and Andrew Lucas. Coherent Terahertz Radiation from a Nonlinear Oscillator of Viscous Electrons. sep 2019.

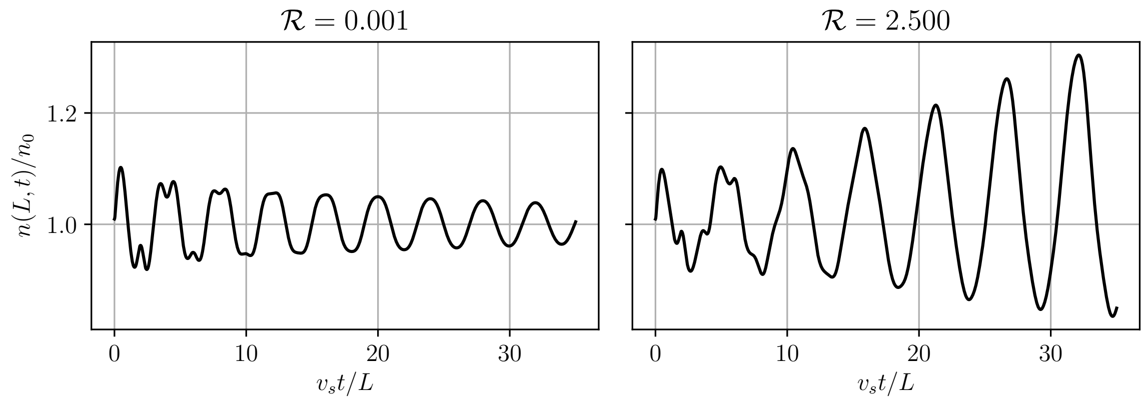
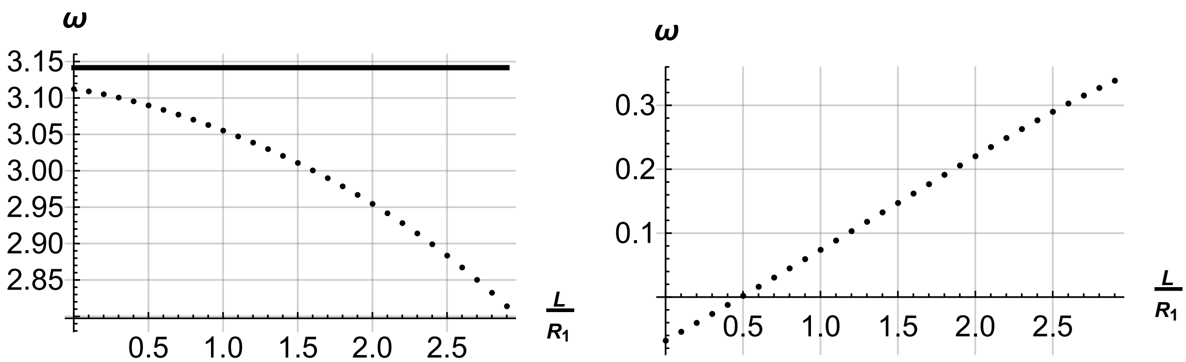


Figure 3: When $\mathcal{R} = 0.001$, which basically corresponds to the rectangular geometry, there is no instability. But the instability reappears at, for instance, $\mathcal{R} = 2.500$.

[2] O. Sydoruk, R. R.A. Syms, and L. Solymar. Plasma oscillations and terahertz instability in field-effect transistors with Corbino geometry. *Applied Physics Letters*, 2010.



Real Part
(Frequency)

Imaginary Part
(Growth Rate)

Figure 4: Using the symmetric boundary conditions (4).

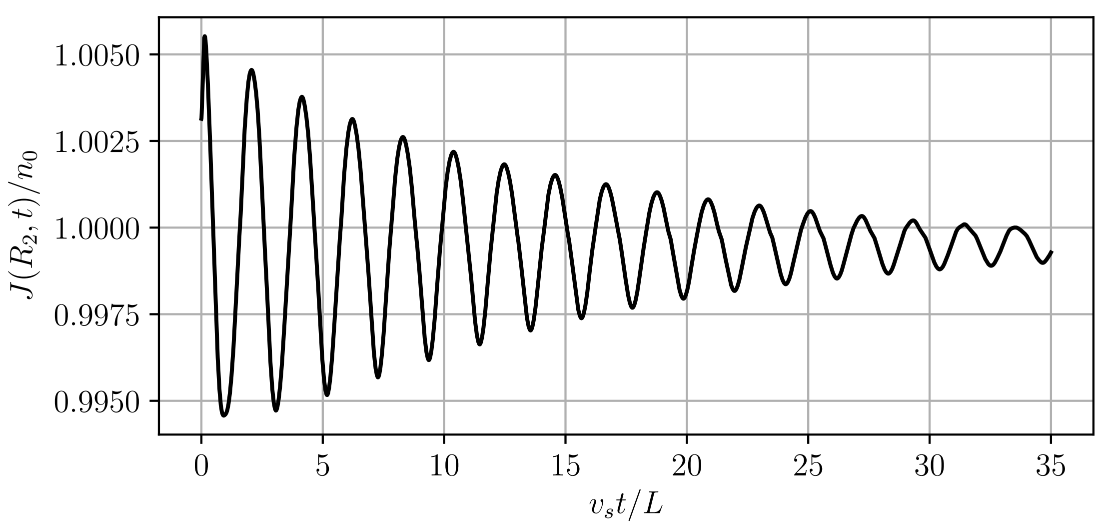


Figure 5: Solution with symmetric boundary conditions. While the oscillations have double the frequency, so far, I have only seen decay.