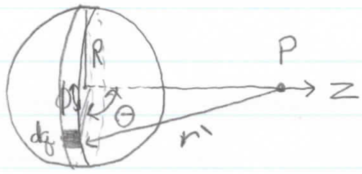
Problem Set 2

September 24th, 2021 — Jules Faucher

PHYS-512 — 260926201

1.



$$dq = \sigma dA = \sigma R^2 \sin\theta \, d\theta \, d\phi$$

$$r'^2 = R^2 + z^2 - 2Rz\cos\theta$$

$$dE = \frac{dq}{4\pi\varepsilon_0 r'^2} \cdot \frac{(z - R\cos\theta)}{r'}$$

$$= \frac{\sigma R^2 \sin\theta (z - R\cos\theta)}{4\pi\varepsilon_0 (R^2 + z^2 - 2Rz\cos\theta)^{3/2}} d\theta d\phi$$

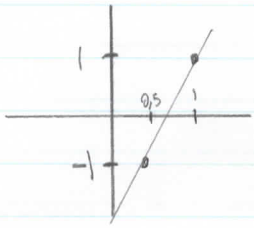
$$\therefore E = \int_0^{2\pi}\int_0^{\pi} \frac{\sigma R^2 \sin\theta (z - R\cos\theta)}{4\pi\varepsilon_0 (R^2 + z^2 - 2Rz\cos\theta)^{3/2}} d\theta d\phi$$

$$= \frac{\sigma R^2}{2\varepsilon_0} \int_0^{\pi} \frac{\sin\theta (z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}} d\theta$$

Hence, we only need to compute this integral at multiple values of z.

There is a singularity in the integrand at $R = z$. This makes the integral blow up to infinity. Using my integrator, an error message appears stating that there was an attempt at dividing by 0. However, quad doesn't care and simply skips this data point.

3. Since Chebyshev polynomials only exist on the interval $[-1, 1]$, we need to map the interval $[0{,}5, 1]$ to the former.



$$m = \frac{\Delta y}{\Delta x} = \frac{2}{0{,}5} = 4 \Rightarrow y = 4x - 3$$

We may also need the inverse: $x = \frac{y}{4} + \frac{3}{4}$

After we fit the data $(4x-3, \log_2 x)$, we obtain a way to evaluate $\log_2 x$ for any $x \in [0{,}5, 1]$. Using logarithm properties, we can put the $\log_2 x$ of any $x > 0$ in terms of the $\log_2 x$ $\forall x \in [0{,}5, 1]$.

Recall that $\log(a^b) = b \cdot \log(a)$

and $\log(ab) = \log(a) + \log(b)$

so if we have $\log(x)$ where x is any number > 0,

$$\log_2(x) = \log_2(m \cdot 2^e)$$
$$= \log_2(m) + \log_2(2^e)$$
$$= \log_2(m) + e$$

m is always $\in [0{,}5, 1]$ for positive numbers

Hence, by breaking down x into its mantissa and exponent, we can evaluate any logarithms despite the fact that we can only evaluate directly $x \in [0{,}5, 1]$.

Computers probably compute their logs in base 2 so they can focus on modeling $\log_2 x$ $\forall x \in [0{,}5, 1]$ with great accuracy, and not the whole function.