Calculating Core Losses in Transformers for Arbitrary Magnetizing Currents
A Comparison of Different Approaches

M. Albach   Th. Dürbaum
Philips Research Laboratory, Postfach 1980
D-52021 Aachen
E-mail albach@pfa.research.philips.com

A. Brockmeyer
Institut for Power Electronics and
Electrical Drives
Aachen University of Technology

Abstract - This paper presents a practical method for predicting the core losses in magnetic components for an arbitrary shape of the magnetizing current. This theory is based on a weighted time derivative of the magnetic flux density and offers the possibility to use the suppliers data, normally derived from measurements with sinewave currents, also for the real current shapes occuring in switch mode power supplies. By means of a special full-bridge converter a test setup is developed that allows the verification of the equations derived in this paper.

I. INTRODUCTION

Magnetic components are probably the most important parts in switch mode power supplies. They may determine cost, size, weight and efficiency in the design. The demand for miniaturisation leads to higher switching frequencies. This results in different design constraints. While the saturation of the magnetic components determines their design at low frequencies thermal constraints represent the limitations at higher switching frequencies. Two major loss mechanisms can be identified in magnetic devices, the winding losses, consisting of rms, skin and proximity losses, and the core losses.

At higher switching frequencies the core losses are no longer negligible. On the other hand the suppliers of magnetic materials provide information about the core losses only for sinewave excitation, as e.g. by Mulder [1]. However, the current waveshapes used in practice strongly deviate from an ideal sinewave. As an example Fig. 1 shows the flyback topology together with the magnetizing current of the transformer. Two major problems may arise during the calculation of core losses in case of arbitrary magnetizing current waveshapes:

- The shape of the magnetizing current is non-sinusoidal. In pwm converters it may have a triangular shape (with dead times, if operated in the discontinuous mode).

- In many situations the time derivative of the magnetizing current changes its sign several times within one switching cycle and as a consequence not only the surrounding of one major B-H loop contributes to the core losses but also the surroundings of several minor loops.
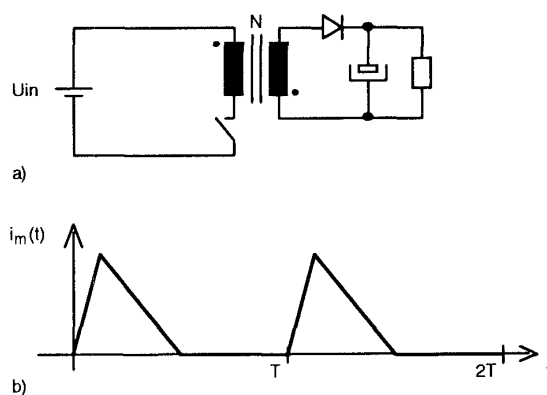

Figure 1: a) Circuit diagram of the flyback topology
   b) Corresponding magnetizing current of the transformer

This paper presents a simplified method for an accurate description of the core losses in case of arbitrary current waveshapes through magnetic devices. This method allows the calculation of core losses based on reliable data published by the supplier of the magnetic material. The influence of a dc-premagnetization is not investigated in this paper. Some information can be found by Brockmeyer [2].

II. COMPARISON WITH EXISTING THEORIES

From the literature two different approaches are well known for calculating the core losses. In the first approach a mathematical representation of the hysteresis loop is used to determine the energy lost in the core during one switching cycle. Roshen [3] models the hysteresis loop by means of two hyperbolas. Furthermore he assumes the loop to be independent of frequency and temperature. The frequency dependence of the total core losses is then described only by the bulk eddy currents in the core.

Unfortunately, the calculated eddy current losses are considerably lower than the observed losses (factor 2-3). This problem is then overcome by an adjustment of the core resistivity by a 'comparable' factor.

Hodgdon's model [4] provides a differential equation that is able to describe the essential features of ferromagnetic hysteresis loops including the existence of a major loop that contains all possible states of (H, B), stable minor loops and frequency dependence of the losses. As mentioned by Ossart et al. [5] the identification problem (that is determining the parameters of the model) is very sensitive to measurement errors.

The Jiles-Atherton model [6] is based on a physical model. Their differential equation describes the static behaviour of ferro- or ferrimagnetic material using four parameters. Jiles et al. [7] give an iterative procedure to find the parameters of the static model where the initial parameters have to be determined from the graphic derivation of parts of a reference hysteresis loop. Brockmeyer et al. [8] extend this model to dynamic magnetization. This leads to a more complicated differential equation with two additional parameters.

The Preisach model as described e.g. by Hui et al. [9] represents a basic physical model. It assumes a collection of hysteretic cells with two magnetization states. An additional weight function describes the material characteristics. The classical Preisach model exhibits two essential drawbacks - congruency of minor loops and being only a static model. Mayergoyz [10] reports on an extension of the classical model that circumvents these drawbacks. The identification problem connected with these Preisach models results in a tremendous experimental effort that is not justified by a comparable increase in accuracy.

All of the above mentioned models have the following problems with respect to the design of inductors and transformers for switched mode power supplies:

- The models calculate the hysteresis loop. This means that an additional step is needed to integrate the area of the B-H loop to obtain the desired value of the core losses.

- The parameters of the models are not provided by the suppliers of the magnetic materials.

- The temperature dependence of the hysteresis loop is not taken into account.

The second and completely different approach is to use the empirical equation of Steinmetz for a direct calculation of the core losses without using the intermediate step of a hysteresis loop description. As on the other hand the suppliers of magnetic materials publish core loss data as a function of frequency, excitation level and temperature obtained from measurements with sinusoidal excitation of the core, it seems to be the most practical way to use these data and to adapt the Steinmetz equation in such a way that it can also be used to calculate the losses in case of nonsinusoidal magnetizing currents.

In [11] an attempt is made to overcome these problems by using a Fourier expansion of the arbitrary waveforms. Due to the nonlinear dependence of the losses with both the flux density and frequency according to eq. (1) this method can only be applied to very specific situations, e.g. strictly limited frequency range and also low peak flux densities. For a comparison of different circuit topologies and different operation modes (continuous or discontinuous current shapes, various switching frequencies) for a given application this method is therefore not very helpful.

A study of dynamic hysteresis models shows that the physical origin of dynamic losses in magnetic materials is the average remagnetization velocity rather than the remagnetization frequency. This is the reason why the most common calculation rule for magnetic core losses the Steinmetz-equation has to be modified [12].

III. CORE LOSS DESCRIPTION BY MEANS OF THE STEINMETZ EQUATION

Before we describe the specific situations occurring in switch mode power supplies the results for sinusoidal excitation given by Mulder[1] are shortly reviewed.

The specific power density P_v in W/m^3 is calculated from the basic Steinmetz equation

$$P_v(\tau) = C_m * f^x * B^y * \left(ct2*\tau^2 - ct1*\tau + ct\right), \qquad (1)$$

where τ is the operating core temperature in °C, B is the amplitude of the flux density in T and f is the frequency of

Material grade	Frequency in kHz	Cm	x	y	ct2 $*10^4$	ct1 $*10^2$	ct
3 C 80	10 - 100	16.7	1.3	2.5	1.17	2.0	1.83
3 C 85	20 - 100	11	1.3	2.5	0.91	1.88	1.97
	100 - 200	1.5	1.5	2.6	0.91	1.88	1.97
3 F 3	20 - 300	0.25	1.6	2.5	0.79	1.05	1.26
	300 - 500	$2 * 10^{-2}$	1.8	2.5	0.77	1.05	1.28
	500 -1000	$36 * 10^{-7}$	2.4	2.25	0.67	0.81	1.14
3 F 4	500 -1000	$12 * 10^{-2}$	1.75	2.9	0.95	1.10	1.15
	1000-3000	$11 * 10^{-9}$	2.8	2.4	0.34	0.01	0.67

TABLE 1: Curve fit results for typical specific power losses

the sinewave in Hz. The specific power losses in W are then obtained by multiplication of P_v with the effective volume V_e of the selected core type.

Table 1 shows the coefficients to be used in this equation in case of different materials. For some material grades the frequency range has been split up into several sections to obtain a more reliable description of the material behaviour. As can be seen in Table 1 the coefficients x and y themselves are frequency dependent.

The data set in Table 1 is obtained from measurements with sinewave currents and this is common to all the values published in data books from different suppliers. To predict the core losses in order to optimize the design of magnetic components the mentioned 'real life' situations have to be taken into account. In the following chapters the mathematical background will be described.

IV. EXTENSION TO ARBITRARY MAGNETIZING CURRENTS

The proposed method starts from the specific energy density given in Ws/m³

$$e_v = P_v(\tau)/f = C_m * f^{x(f)-1} * B^{y(f)} * \left(ct2*\tau^2 - ct1*\tau + ct\right). \quad (2)$$

This equation describes the energy density, which is lost during one complete sinewave cycle. To obtain the losses this value has to be multiplied by both the frequency and the effective magnetic volume V_e. With increasing switching frequency the area covered by the B-H loop is increased. This effect is represented in equation (2) by the exponent $x(f) > 1$. This means that the energy e_v depends on the velocity of the surrounding of the B-H loop or in other words on the time derivative of the magnetic flux dB/dt. In the measurements with sinewave signals the varying dB/dt is represented in the results only as the mean value of the complete sinewave. An extension of equation (2) to an arbitrary shape of the magnetizing current is possible by means of a weighted value for dB/dt.

If $\dot{B}_w = dB_w/dt$ denotes the weighted time derivative of B and if we assume a piecewise linear description of B, as for example provided by simulation programs in form of the magnetizing current through magnetic components, we can calculate the following sum

$$\dot{B}_w = \sum_{k=2}^{K} \dot{B}_k * \frac{B_k - B_{k-1}}{B_{max} - B_{min}}, \quad (3)$$

where $\dfrac{B_k - B_{k-1}}{B_{max} - B_{min}}$ is the weighting factor and $\dot{B}_k = \dfrac{B_k - B_{k-1}}{t_k - t_{k-1}}$ is the time derivative. Equation (3) may also be written as an integral representation

$$\dot{B}_w = \int_0^T \frac{\dot{B}^2}{B_{max} - B_{min}} \, dt. \quad (4)$$

For the test signal $B = \hat{B} \sin wt$, which has been used to derive the coefficients in Table 1, the integral (4) can be solved and leads to

$$\dot{B}_{w\sin} = (B_{max} - B_{min}) * \frac{\pi^2}{2} * f_{\sin} = \hat{B} * \pi^2 * f_{\sin}. \quad (5)$$

The next step is to calculate $\dot{B}_w$ by means of equation (3) or (4) for the given current shape. This result is then compared with a sinewave signal of the same flux density swing $B_{max} - B_{min}$. The frequency of this sinewave signal, referred to as equivalent sinewave frequency $f_{\sin eq}$, can now be determined in such a way that this sinewave signal would give the same results.

$$\int_0^T \frac{\dot{B}^2}{B_{max} - B_{min}} \, dt \overset{!}{=} (B_{max} - B_{min}) * \frac{\pi^2}{2} * f_{\sin eq} \quad (6)$$

For a piecewise linear current shape the sum (3) can be used instead of the integral (4) and equation (6) yields

$$f_{\sin eq} = \frac{2}{\pi^2} \sum_{k=2}^{K} \left(\frac{B_k - B_{k-1}}{B_{max} - B_{min}} \right)^2 * \frac{1}{t_k - t_{k-1}} \quad (7)$$

By inserting $f_{\sin eq}$ in equation (2) and after division by the switching period T the specific power density is obtained.

$$P_v(\tau) = \frac{1}{T} * C_m * f_{\sin eq}^{x-1} * B^y * \left(ct2*\tau^2 - ct1*\tau + ct\right) \quad (8)$$

Of course the coefficients taken from Table 1 have to be selected according to $f_{\sin eq}$.

V. SUBLOOPS

As already mentioned above in some converters the current may have several maxima and minima within one switching cycle. In these cases it is not sufficient to calculate the difference $B_{max} - B_{min}$ between the peak flux values and to use the equations given above. A far better procedure is to separate the various B-H loops on the time axis. This will be explained by means of the next two figures. Let's first have a look on the current waveshape of an LLC converter shown in Fig. 2.

As a first step the dc value of the subloop is determined.

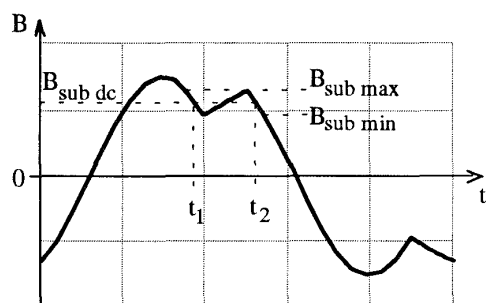$$B_{sub\,dc} = \frac{B_{sub\,max} + B_{sub\,min}}{2} \quad (9)$$

Figure 2: Current waveshape with subloop

From this result the times t_1 and t_2 can be obtained which correspond to the begin and end of the subloop. In the next step the waveform is split up into the separate waveforms shown in Fig. 3.
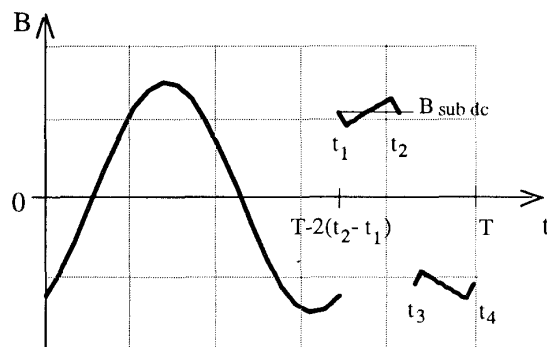

Figure 3: Separation of the different loops

Each waveshape in Fig. 3 is now treated separately. Depending on its equivalent sinewave frequency and its flux density swing $B_{max} - B_{min}$ the specific power losses are calculated by means of the equations given above. The summation of all these loss contributions finally leads to the total specific power losses in the core.

Using the equations derived so far the specific core losses may be calculated for any realistic magnetizing current waveform. In the following chapter the simulation results are compared to own and published measurements for some characteristic waveshapes of the magnetizing current. Additionally a comparison is made with the Fourier expansion approach as described in [11].

VI. EXPERIMENTAL VERIFICATION

As a first example the specific core losses for the triangular current shape given in Fig. 4 have been calculated with the

duty cycle δ as parameter. If these values are referred to the specific core losses of a sinewave current with both the same amplitude and same switching period T the result in Fig. 5 is obtained. Of course this curve depends on the selected material and also on the chosen frequency $1/T$, which is due to the fact that the exponent x in eq. (8) itself depends on these parameters. However, the basic characteristic of this curve remains the same.

The triangular current with equal rise and fall times ($\delta = .5$) has less specific core losses compared with the sinewave current. If, however, the duty cycle is decreased and high dB/dt values occur the losses become much higher. These results are in good agreement with published measurements. Chen [13] demonstrated that the core losses under squarewave voltage excitation are lower if compared to those under sinusoidal excitation. In case of ferrite material he has measured a decrease of about 6% to 12 % which is comparable with the decrease predicted by our method. The strong dependence on the duty cycle has been verified by Triner [14].
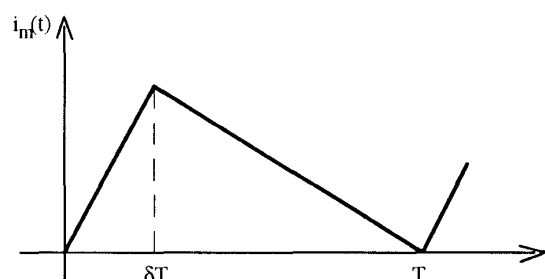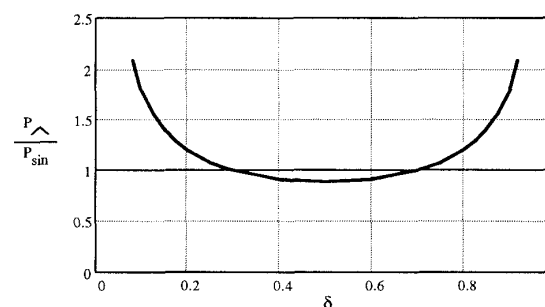

Figure 4: Triangular magnetizing current shape

Figure 5: Specific core losses of the current shown in Fig. 4 with a sinewave current as reference

To verify the results from the outlined equations by means of own measurements a special test setup is developed. It consists of an IGBT full bridge inverter that drives an inductor containing the ferrite core under test. By adjustment of the switching times and the voltage of the dc-link it is possible to generate different magnetizing

currents through the test-core. The total core losses are measured using the oscilloscope method according to the European Standard CECC 25300 and CECC 25000 [15].

For comparison of the calculated results from the equations presented in this paper with the existing approaches and also with the test results the current waveshape as shown in Fig. 6 has been used. The triangular current shape with period T_O has been kept constant and only the dead time $n*T_O$ between two cycles has been varied. For the measurements a E42/42/15 core of Philips 3C85 ferrite has been used. The frequency $f_O = 1/T_O$ has been set to 20 kHz and the temperature was kept constant at 100°C. If we now assume that the losses occuring within the switching cycle $0 \leq t \leq T_O$ are independent of the following dead time $n*T_O$ and if we further assume that the losses during the dead
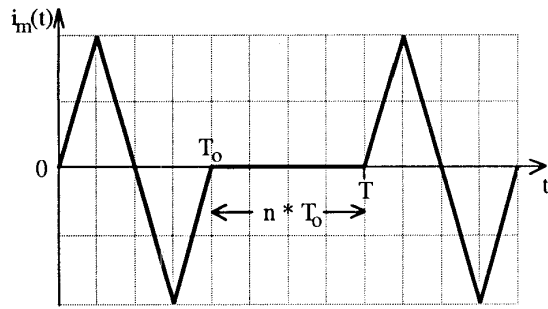

Figure 6: Discontinuous magnetizing current shape used within the experimental setup (T_O represents the switching period in case of n = 0)

time are zero the total losses should be proportional to the following expression

$$P \approx \frac{1}{1+n}.$$ (10)

This relation is exactly represented by the equations given above as can be easily verified. The current shape during the time interval $0 \leq t \leq T_O$ is transferred to an equivalent frequency $f_{sin\ eq}$ according to eq. (7)

$$f_{sineq} = \frac{2}{\pi^2} * \left[\frac{1}{4} * \frac{1}{T_o/4} + \frac{1}{T_o/2} + \frac{1}{4} * \frac{1}{T_o/4} \right] = \frac{8}{\pi^2 * T_o},$$ (11)

which is inserted in the modified Steinmetz equation (8). The losses are finally obtained by division with the total period T.

Fig. 7 shows the losses in W as a function of the total period $T = (1+n)*T_O$. For n = 0 (T = T_O = 50 µs) the losses from the conventional equation are slightly too high, which is in agreement with Fig 5. This would be tolerable, but with increasing dead times the losses from this

equation are far too low, whereas the results obtained from the modified equations can hardly be distinguished from the measured values. It is obvious that the new equations are in very good agreement with the material behaviour. The previously made assumptions that the two different time intervals are in terms of loss contributions more or less independent of each other are thus also justified. For this reason the separation of subloops on the time axes as proposed in this paper will also lead to sufficiently accurate results.

☐ measurement   + Steinmetz modified
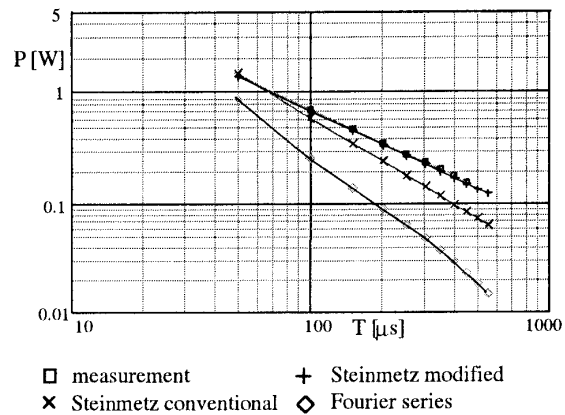✕ Steinmetz conventional ◇ Fourier series

Figure 7: Comparison of losses obtained from measurements with the losses calculated by different methods

To get rid of the idea that the losses can also be obtained by using a Fourier series for the waveform and by summing the losses due to each harmonic the results of this method are also shown in Fig. 7. Due to the highly nonlinear behaviour of the ferrite materials (compare the coefficients x and y from Table 1) this method can only lead to acceptable results for some special cases. In the example described here the magnetic flux density was about 0.2 T and the effective volume of the core is V_e = 17.3e-6 m^3. Using these data the reader may check himself the failure of the Fourier approach. For n = 0 the Fourier series gives only 64 % of the measured losses, for n = 3 (T = 200 µs), however, the Fourier series predicts only 25 %.

VII. SUMMARY

Suppliers of magnetic materials characterise their products by means of core loss data obtained from sinusoidal excitation. However, the operation within switch mode power supplies leads to magnetizing current waveshapes which strongly deviate from the sinewave. In order to use the published data from the suppliers a mathematical approach has been proposed that takes the real waveform

of the magnetizing current into account. By means of the derived equations an equivalent frequency is calculated such that the losses caused by a sinusoidal excitation with the same flux density swing will be the same as those produced by the arbitrary magnetizing current. Instead of the switching frequency this equivalent frequency is used in combination with the published data to obtain the core losses for a certain application.

Two examples have been simulated by means of the proposed theory. The first example demonstrates the dependence of the core losses on the duty cycle of a PWM converter operating at the border between the discontinuous and continuous mode. The theory reveals that at 50% duty cycle the core losses are lower than those of sinusoidal excitation. On the other hand the core losses increase dramatically for larger differences between the on- and off-time. These results are in agreement with published measurements. The second example investigates the influence of dead times on the core losses. The proposed theory as well as the described measurement demonstrate that the energy dissipated during one surrounding of the B-H curve depends only on the waveform of the magnetizing current rather than on the dead time between two succeeding loops.

As a conclusion the proposed equations provide a simple and accurate possibility to determine the core losses of magnetic components for arbitrary waveforms. The data published by the suppliers of the magnetic materials are sufficient and no extra parameter extraction is necessary. Furthermore the piecewise linear description of the currents through magnetic devices as obtained from many simulation tools, e.g. SPICE can directly be used in combination with the equations given above.

REFERENCES

[1] Mulder, S. A., "Fit Formulae for Power Loss in Ferrites and their Use in Transformer Design", 1993, PCIM ´93 Europe Power Conversion Conference Proceedings , 345-359

[2] Brockmeyer, A. "Experimental Evaluation of the Influence of DC-Premagnetization on the Properties of Power Electronic Ferrites", 1996, APEC Conference Proceedings

[3] Roshen, W., "Ferrite Core Loss for Power Magnetic Components Design", 1991, IEEE Transaction in Magnetics 27, 4407-4415

[4] Hodgdon, M. L., "Applications of a Theory of Ferromagnetic Hysteresis", 1988, IEEE Transaction in Magnetics 24, 218-221

[5] Ossart, F., Meunier, G., "Comparison between various hysteresis models and experimental data", 1990, IEEE Transaction in Magnetics 26, 2837-2839

[6] Jiles, D. C., Atherton, D. L., "Theory of Ferromagnetic Hysteresis", 1986, Journal of Magnetism and Magnetic Materials 61, 48-60

[7] Jiles, D. C., et al., "Numerical Determination of Hysteresis Parameters for the Modeling of Magnetic Properties Using the Theory of Ferromagnetic Hysteresis", 1992, IEEE Transaction in Magnetics 28, 27-35

[8] Brockmeyer, A., Schülting, L., "Modelling of Dynamic Losses in Magnetic Materia", 1993, EPE 112-117

[9] Hui, S. Y. R., Zhu, J., "Magnetic Hysteresis Modeling and Simulation using the Preisach Theory and TLM Technique", 1994, PESC´94, 837-842

[10] Mayergoyz, I. D., "Dynamic Preisach Models of Hysteresis", 1988, IEEE Transaction on Magnetics 24, 2925-2977

[11] R. Severns, "HF core losses for non-sinusoidal waveforms", HFPC 1991, 140-148

[12] Dürbaum, Th. and Albach M., Core Losses in Transformers with an Arbitrary Shape of the Magnetizing Current. EPE Proc., vol. 1, p. 1.171-1.176, Sevilla (Spain), 1995. European Power Electronics Association EPE.

[13] Chen, D. Y., "Comparison of the High Frequency Magnetic Core Losses under two different Driving Conditions: A Sinusoidal Voltage and a Squarewave Voltage", 1978, PESC´78, 237-241

[14] Triner, J. E., "Analyze Magnetic Loss Characteristics Easily Using a High Power Wideband Operational Amplifier", 1981, Power Conversion International , 55-63

[15] CECC 25 300. Magnetic oxide cores for power application. *Harmonized system of quality assessment for electronic components; sectional specification.*, 1981.