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Exercice : Copule produit

Démontrer que $C^{\perp}(u_1,u_2)=u_1 u_2$ avec $(u_1,u_2)\in[0,1]^2$

→ Savoir démontrer que C est une copule ou non

Etape 1 : Conditions aux bords

$$\left.\begin{aligned} C^{\perp}(u,0) &= u\cdot 0 = 0 \\ C^{\perp}(0,u) &= 0\cdot u = 0 \end{aligned}\right\} \checkmark$$

$$\left.\begin{aligned} C^{\perp}(1,u) &= 1\cdot u = u \\ C^{\perp}(u,1) &= u\cdot 1 = u \end{aligned}\right\} \checkmark$$

Donc $C^{\perp}(u_1,u_2)=u_1\cdot u_2$

Etape 2 : Vérifier la propriété 2-increasing

On doit avoir :

$$\Delta = C^{\perp}(v_1,v_2) - C^{\perp}(v_1,u_2) - C^{\perp}(u_1,v_2) + C^{\perp}(u_1,u_2) \geq 0$$

On remplace $C^{\perp}(u_1,u_2)=u_1\,u_2$

$$\Delta = v_1 v_2 - v_1 u_2 - u_1 v_1 + u_1 u_2$$

On factorise : $\left.\left(\underbrace{(v_1-u_1)}_{>0}\underbrace{(v_2-u_2)}_{>0}\right) > 0 \right\} \checkmark$

Exercice (perso) : Fonction Copule

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- $C(u_1, u_2) = \min(u_1, u_2)$

Étape 1 : Marge : $C(u,0) = C(0,u) = \min(u,0) = \min(0,u) = 0$

$C(u,1) = C(1,u) = \min(u,1) = \min(1,u) = u$

Étape 2 : 2-increasing : $u_1 = 0{,}2$, $v_1 = 0{,}6$, $u_2 = 0{,}3$, $v_2 = 0{,}7$

$c(v_1, v_2) = \min(0{,}6, 0{,}7) = 0{,}6$

...

$\Delta = 0{,}6 - 0{,}3 - 0{,}2 + 0{,}2 = 0{,}3 \geq 0$

$\rightarrow$ Copule Fréchet supérieure

On est dans $[0,1]$ donc $u-1 \leq 0$

- $C(u_1, u_2) = \max(u_1 + u_2 - 1, 0)$

Étape 1 : Marge : $C(u,0) = \max(u + 0 - 1, 0) = \max(u-1, 0) = 0$

$C(0,u) = \max(0 + u - 1, 0) = \max(u-1, 0) = 0$

$C(u,1) = \max(u + 1 - 1, 0) = \max(u, 0) = u$

$C(1,u) = \max(1 + u - 1, 0) = u$

Étape 2 : 2-increasing : $u_1 = 0{,}1$, $v_1 = 0{,}4$, $u_2 = 0{,}2$, $v_2 = 0{,}5$

$C(v_1, v_2) = \max(0{,}4 + 0{,}5 - 1, 0) = \max(-0{,}1, 0) = 0$

$C(v_1, u_2) = \max(0{,}4 + 0{,}2 - 1, 0) = \max(-0{,}4, 0) = 0$

...

$\Delta = 0 - 0 - 0 + 0 = 0 \geq 0$ $\rightarrow$ Copule Fréchet inférieure

- $C(u_1, u_2) = u_1^2 u_2$

Étape 1 :

$C(u,0) = u_1^2 \cdot 0 = 0$

$C(u,1) = u_1^2 \cdot 1 = u_1^2$ ✗

$\rightarrow$ Marge non respectée