Productivity Growth in Canada*

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Abstract

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1 Introduction

2 Theoretical Framework

Input-Output Definitions. Consider an economy with N industries indexed by $i \in \{1, \ldots, N\}$ and two factors of production: capital and labor. Let $\mathbf{b}_t$ be the $(N+2) \times 1$ vector whose i-th element is equal to the share of industry i in aggregate nominal value-added:

$$b_{it} \equiv \frac{P_{it} Y_{it}}{\sum_{j=1}^{N} P_{jt} Y_{jt}}, \quad \forall i \{1, \ldots, N\}.$$

The first N elements of $\mathbf{b}_t$ correspond to industries, while the last two correspond to capital and labor. Since factors do not enter in final demand, the last two elements of $\mathbf{b}_t$ are equal to zero. Let us further define the cost-based input-output matrix $\mathbf{\Omega}_t$ of dimension $(N+2) \times (N+2)$ whose (i, j)-th element is equal to industry i's expenditures on inputs from j as a share of its total expenditures:

$$\Omega_{ijt} \equiv \frac{P_{jt} X_{ijt}}{\sum_{k=1}^{N} P_{kt} X_{ikt}}.$$

The first N rows of $\mathbf{\Omega}_t$ correspond to industries, while the last two correspond to capital and labor. Since capital and labor require no intermediate inputs, the last two rows of $\mathbf{\Omega}_t$ are filled with zeros. The Leontief inverse of the cost-based input-output matrix is defined as:

$$\mathbf{\Psi}_t \equiv (\mathbf{I} - \mathbf{\Omega}_t)^{-1}.$$

Finally, we define the cost-based Domar weights as:

$$\boldsymbol{\lambda}_t' \equiv \mathbf{b}_t' \mathbf{\Psi}_t.$$

For expositional convenience, we denote by λ_t^K and λ_t^L the last two elements of $\boldsymbol{\lambda}_t'$, which measure their importance in final demand, indirectly through the production network of the economy.

Total-Factor Productivity. As shown by Baqaee and Farhi (2019), aggregate total-factor productivity (TFP) growth in inefficient economies can be calculated as:

$$\mathrm{d}\ln(A_t) = \mathrm{d}\ln(Y_t) - \lambda_t^K \mathrm{d}\ln(K_t) - \lambda_t^L \mathrm{d}\ln(L_t)$$

where A_t is aggregate TFP, Y_t is aggregate real value-added, K_t is aggregate capital, and L_t is aggregate labor. The growth rate of aggregate real value-added is given by:

$$\mathrm{d}\ln(Y_t) \equiv \sum_{i=1}^{N} b_{it}\mathrm{d}\ln(Y_{it}).$$

Here, the growth rate of real value-added in industry i is given by:[1]

$$\mathrm{d}\ln(Y_{it}) = \mathrm{d}\ln(A_{it}) + \alpha_{it}^{K}\mathrm{d}\ln(K_{it}) + \alpha_{it}^{L}\mathrm{d}\ln(L_{it})$$

where A_{it} is industry i's TFP, K_{it} its capital input, L_{it} its labor input, and α_{it}^{K} and α_{it}^{L} are the elasticities of its output with respect to capital and labor, respectively. Here, we make the assumption of constant returns to scale at the industry level such that $\alpha_{it}^{K} + \alpha_{it}^{L} = 1$. Under the additional assumption that the representative firm in industry i minimizes its costs and is a price-taker in input markets, the elasticity of output with respect to the capital input is given by:

$$\alpha_{it}^{K} = \frac{r_{it}K_{it}}{r_{it}K_{it} + w_{it}L_{it}}.$$

The growth rate of aggregate capital is given by:

$$\mathrm{d}\ln(K_t) \equiv \sum_{i=1}^{N} \omega_{it}^{K}\mathrm{d}\ln(K_{it}) \quad \text{where} \quad \omega_{it}^{K} \equiv \frac{r_{it}K_{it}}{\sum_{j=1}^{N} r_{jt}K_{jt}}$$

and the growth rate of aggregate labor is given by:

$$\mathrm{d}\ln(L_t) \equiv \sum_{i=1}^{N} \omega_{it}^{L}\mathrm{d}\ln(L_{it}) \quad \text{where} \quad \omega_{it}^{L} \equiv \frac{w_{it}L_{it}}{\sum_{j=1}^{N} w_{jt}L_{jt}}$$

and where r_{it} and w_{it} are the rental rate of capital and the wage rate in industry i at time t, respectively.

[1] Here, we make the assumption of constant returns to scale at the industry level.

Productivity Growth Accounting. Combining the above equations, we can express aggregate TFP growth as:

$$\begin{aligned}\mathrm{d}\ln(A_t) = &\sum_{i=1}^{N} b_{it}\mathrm{d}\ln(A_{it}) \\ &+ \sum_{i=1}^{N}(b_{it}\alpha_{it}^{K} - \omega_{it}^{K}\lambda_t^{K})\mathrm{d}\ln(K_{it}) \\ &+ \sum_{i=1}^{N}(b_{it}\alpha_{it}^{L} - \omega_{it}^{L}\lambda_t^{L})\mathrm{d}\ln(L_{it}).\end{aligned}$$

Following Halperin and Mazlish (2024), adding and subtracting the term $\sum_{i=1}^{N} b_{it_0}\mathrm{d}\ln(A_{it})$ and summing from time t_0 to t_1, we can further decompose the cumulative aggregate TFP growth between these two dates into the following components:

$$\sum_{t=t_0}^{t_1}\mathrm{d}\ln(A_t) = \sum_{t=t_0}^{t_1}\sum_{i=1}^{N} b_{it_0}\mathrm{d}\ln(A_{it}) \qquad \text{Productivity} \tag{1}$$

$$+ \sum_{t=t_0}^{t_1}\sum_{i=1}^{N}(b_{it} - b_{it_0})\mathrm{d}\ln(A_{it}) \qquad \text{Baumol} \tag{2}$$

$$+ \sum_{t=t_0}^{t_1}\sum_{i=1}^{N}(b_{it}\alpha_{it}^{K} - \omega_{it}^{K}\lambda_t^{K})\mathrm{d}\ln(K_{it}) \qquad \text{Capital} \tag{3}$$

$$+ \sum_{t=t_0}^{t_1}\sum_{i=1}^{N}(b_{it}\alpha_{it}^{L} - \omega_{it}^{L}\lambda_t^{L})\mathrm{d}\ln(L_{it}) \qquad \text{Labor.} \tag{4}$$

In this equation, term (1) weighs industry-level TFP growth by each industry's initial share of value-added. That is, if we freeze each industry's share of nominal value-added, we capture the contribution of within-industry productivity improvements alone–how much aggregate TFP has grown purely because each industry became more productive over time.

Term (2) instead weighs industry-level TFP growth by the change in each industry's share of value-added between dates t_0 and t_1. Therefore, it measures how the changing industrial composition of the economy contributes to aggregate TFP growth. This term is negative when industries with low productivity growth become more important in the economy, or vice-versa.

Terms (3) and (4) capture the contribution of reallocating factors across sectors. It weighs changes in industry-level capital and labor inputs by the difference between two terms. The first term is the product of the industry's share of total value-added and the elasticity of its output with respect to inputs. The second term is the product of the industry's share of total factor expenditures and the elasticity of aggregate output with respect to inputs. Intuitively, moving inputs into industries where they are more productive than the economy-wide average ($\alpha_{it}^K > \lambda_t^K$ or $\alpha_{it}^L > \lambda_t^L$) and where the receiving industries are more important in output than costs ($b_{it} > \omega_{it}^K$ or $b_{it} > \omega_{it}^L$) (i.e., high markup industries) raises aggregate productivity growth.

Baqaee and Farhi (2019) also show that aggregate TFP growth in inefficient economies can be approximated as:

$$\mathrm{d}\ln(A_t) = \underbrace{\sum_{i=1}^{N} \lambda_{it}\mathrm{d}\ln(A_{it})}_{\text{Technology}} - \underbrace{\sum_{i=1}^{N} \lambda_{it}\mathrm{d}\ln(\mu_{it}) - \lambda_t^K\mathrm{d}\ln(\Gamma_t^K) - \lambda_t^L\mathrm{d}\ln(\Gamma_t^L)}_{\text{Allocative efficiency}}$$

where μ_{it} is the wedge between the price of industry i's output and the cost of its inputs, and Γ_t^K and Γ_t^L are the aggregate capital and labor shares of value-added, respectively. They decompose aggregate TFP growth into a technology component and an allocative efficiency component. However, since wedges are not directly observable, we treat them as residuals using the definition of aggregate TFP growth above.

Following the same logic as above, we can further decompose cumulative aggregate TFP growth between two dates t_0 and t_1 into the following components:

$$\sum_{t=t_0}^{t_1} \mathrm{d}\ln(A_t) = \sum_{t=t_0}^{t_1}\sum_{i=1}^{N} \lambda_{it_0}\mathrm{d}\ln(A_{it}) \qquad \text{Productivity} \tag{5}$$

$$+ \sum_{t=t_0}^{t_1}\sum_{i=1}^{N} (\lambda_{it} - \lambda_{it_0})\mathrm{d}\ln(A_{it}) \qquad \text{Baumol} \tag{6}$$

$$- \sum_{t=t_0}^{t_1}\sum_{i=1}^{N} \lambda_{it}\mathrm{d}\ln(\mu_{it}) - \lambda_t^K\mathrm{d}\ln(\Gamma_t^K) - \lambda_t^L\mathrm{d}\ln(\Gamma_t^L) \qquad \text{Allocative efficiency.} \tag{7}$$

3 Data

To calculate and decompose aggregate TFP growth in Canada, we need data on the following variables:

1. Industry-level nominal value-added ($\{P_{it}Y_{it}\}_{i=1}^N$): We use data from 1961 to 2019 on gross domestic product at the 3-digit NAICS level from Table 36-10-0217-01 of Statistics Canada.

2. Industry-level total-factor productivity ($\{A_{it}\}_{i=1}^N$): We use data from 1961 to 2019

on multifactor productivity based on value-added at the 3-digit NAICS level from Table 36-10-0217-01 of Statistics Canada.

3. Industry-level capital ($\{K_{it}\}_{i=1}^{N}$): We use data from 1961 to 2019 on capital inputs at the 3-digit NAICS level from Table 36-10-0217-01 of Statistics Canada.

4. Industry-level labor ($\{L_{it}\}_{i=1}^{N}$): We use data from 1961 to 2019 on labor inputs at the 3-digit NAICS level from Table 36-10-0217-01 of Statistics Canada.

5. Industry-level capital expenditures ($\{r_{it}K_{it}\}_{i=1}^{N}$): We use data from 1961 to 2019 on capital cost at the 3-digit NAICS level from Table 36-10-0217-01 of Statistics Canada.

6. Industry-level labor expenditures ($\{w_{it}L_{it}\}_{i=1}^{N}$): We use data from 1961 to 2019 on labor compensation at the 3-digit NAICS level from Table 36-10-0217-01 of Statistics Canada.

7. Industry-level intermediate input expendiures ($\{\{P_{jt}X_{ijt}\}_{j=1}^{N}\}_{i=1}^{N}$): We use data from 1961 to 2019 on the symmetric input-output tables at the 3-digit NAICS level from Table 36-10-0001-01 of Statistics Canada and its previous versions.

4 Empirical Results

Table 1: TFP Growth Decomposition #1

	1961-2019	1961-1980	1980-2000	2000-2019
Productivity	0.78%	1.11%	0.76%	0.12%
Baumol	-0.27%	-0.08%	-0.16%	-0.20%
Capital	-0.11%	-0.19%	-0.08%	-0.08%
Labor	0.10%	0.16%	0.08%	0.07%
Total	0.50%	0.99%	0.60%	-0.09%

Note:

Figure 1: Price and TFP Growth

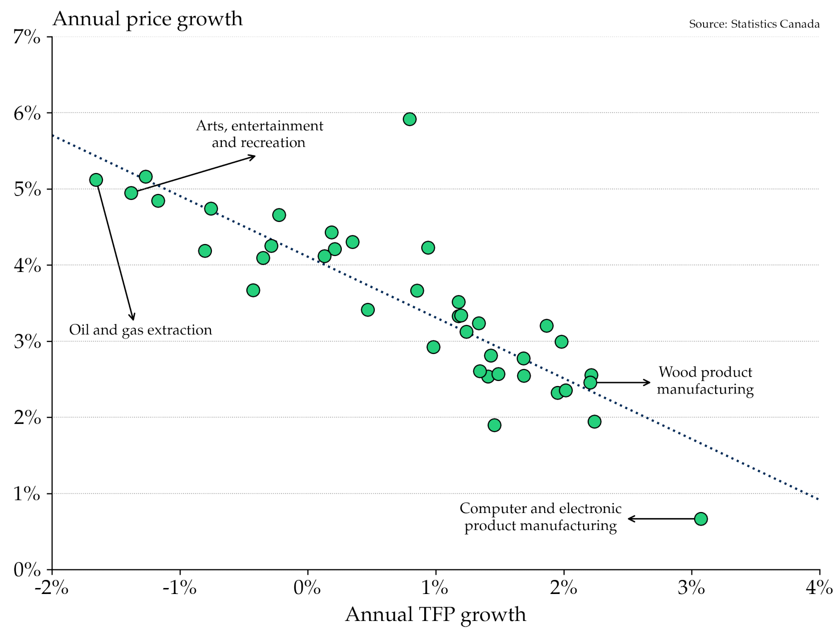


Figure 2: Wage and TFP Growth

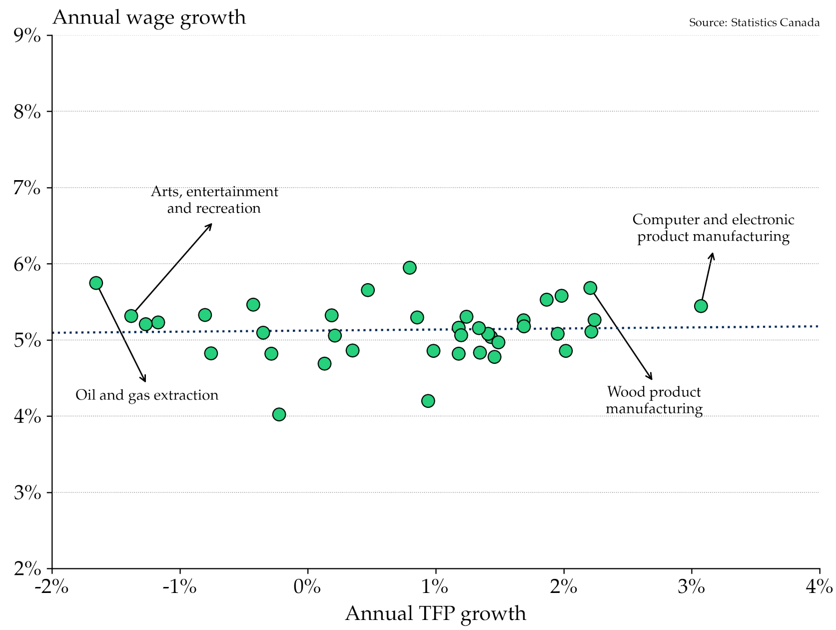


Figure 3: Nominal GDP and TFP Growth

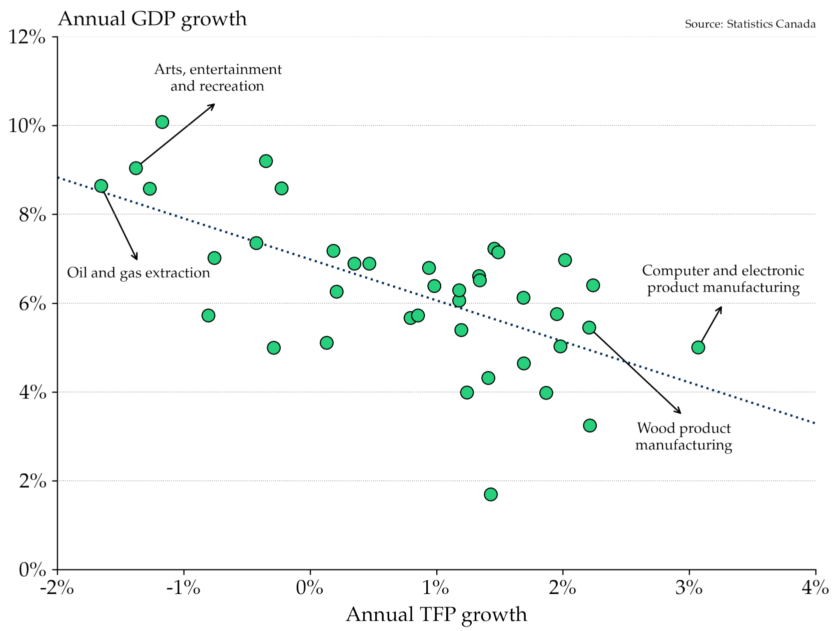


Figure 4: Real GDP and TFP Growth

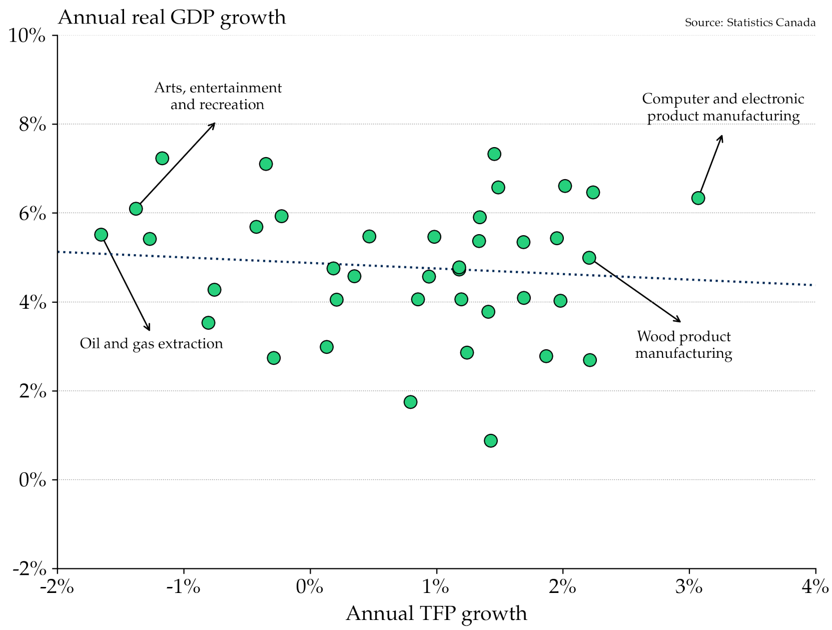


Figure 5: Input Cost Shares and TFP Growth

(a) Capital (b) Labor

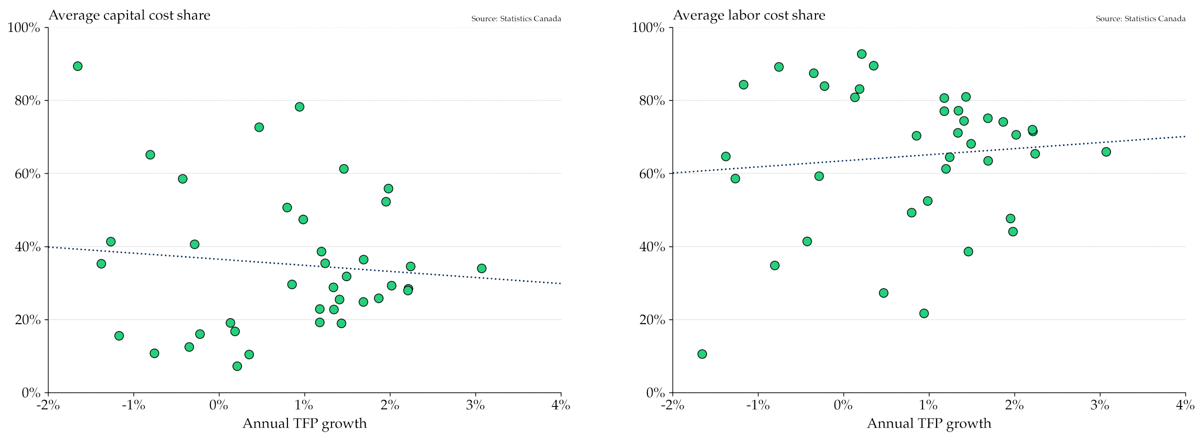


Table 2: TFP Growth Decomposition #2

	1961-2019	1961-1980	1980-2000	2000-2019
Productivity	1.46%	2.00%	1.92%	0.24%
Baumol	-0.35%	-0.14%	-0.47%	-0.25%
Allocative efficiency	-0.60%	-0.87%	-0.85%	-0.09%
Total	0.50%	0.99%	0.60%	-0.09%

Note:

Figure 6: TFP Growth and the Baumol Effect

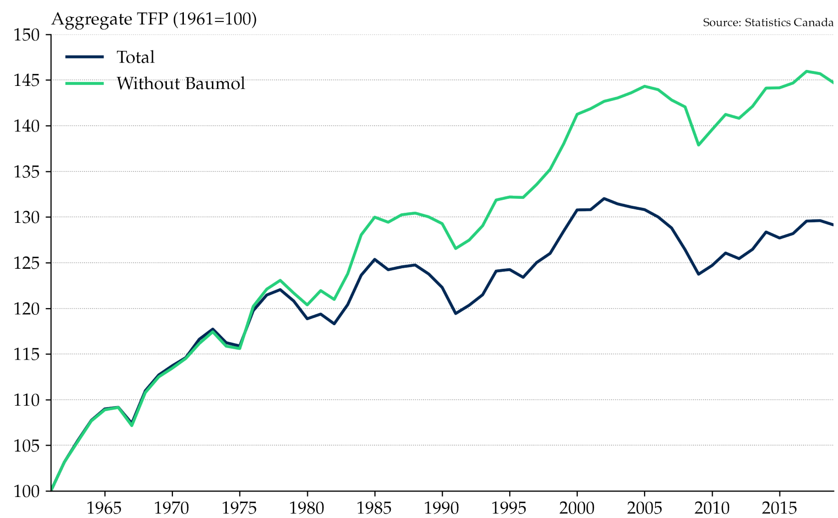


Figure 7: TFP Growth and Misallocation

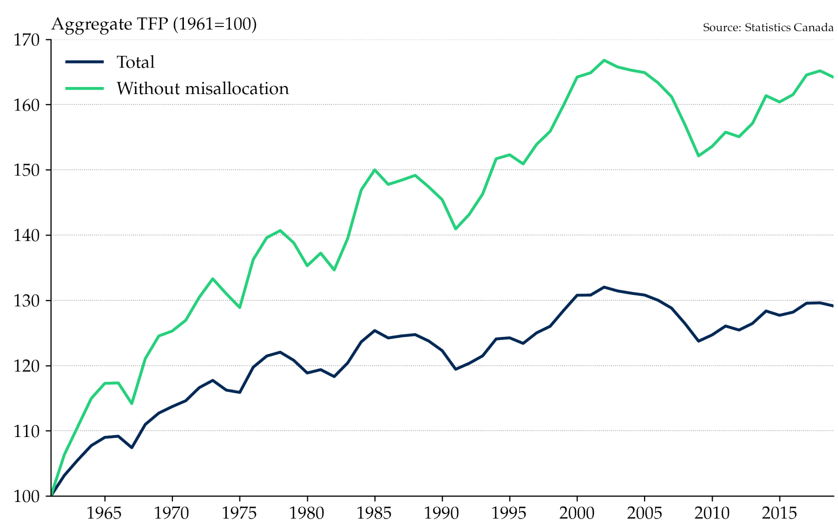


Figure 8: TFP Growth Decomposition #1

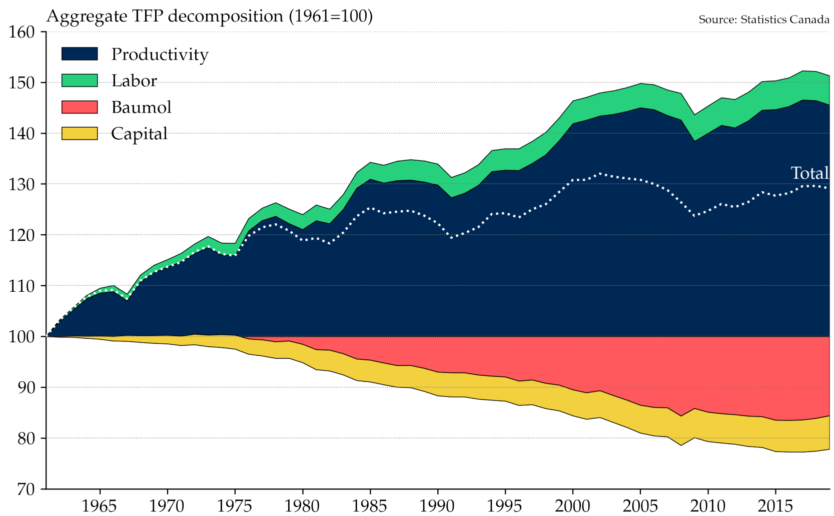


Figure 9: TFP Growth Decomposition #2

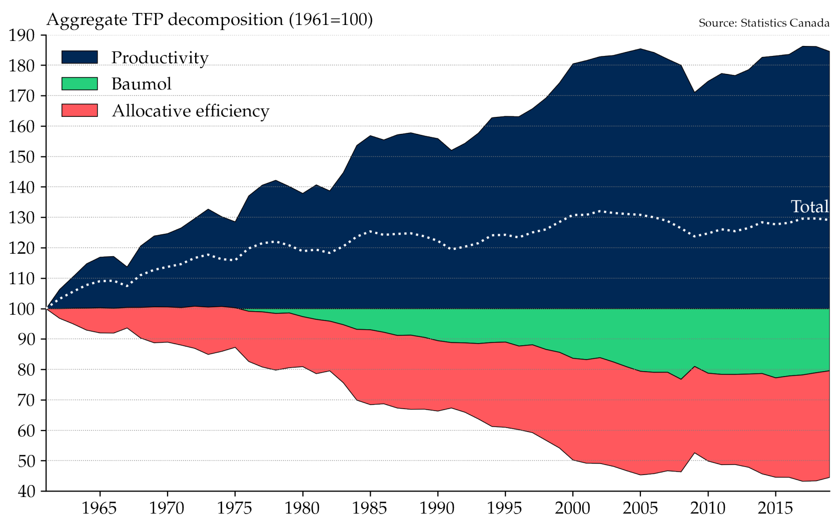


Figure 10: Industrial Contributions to TFP Growth

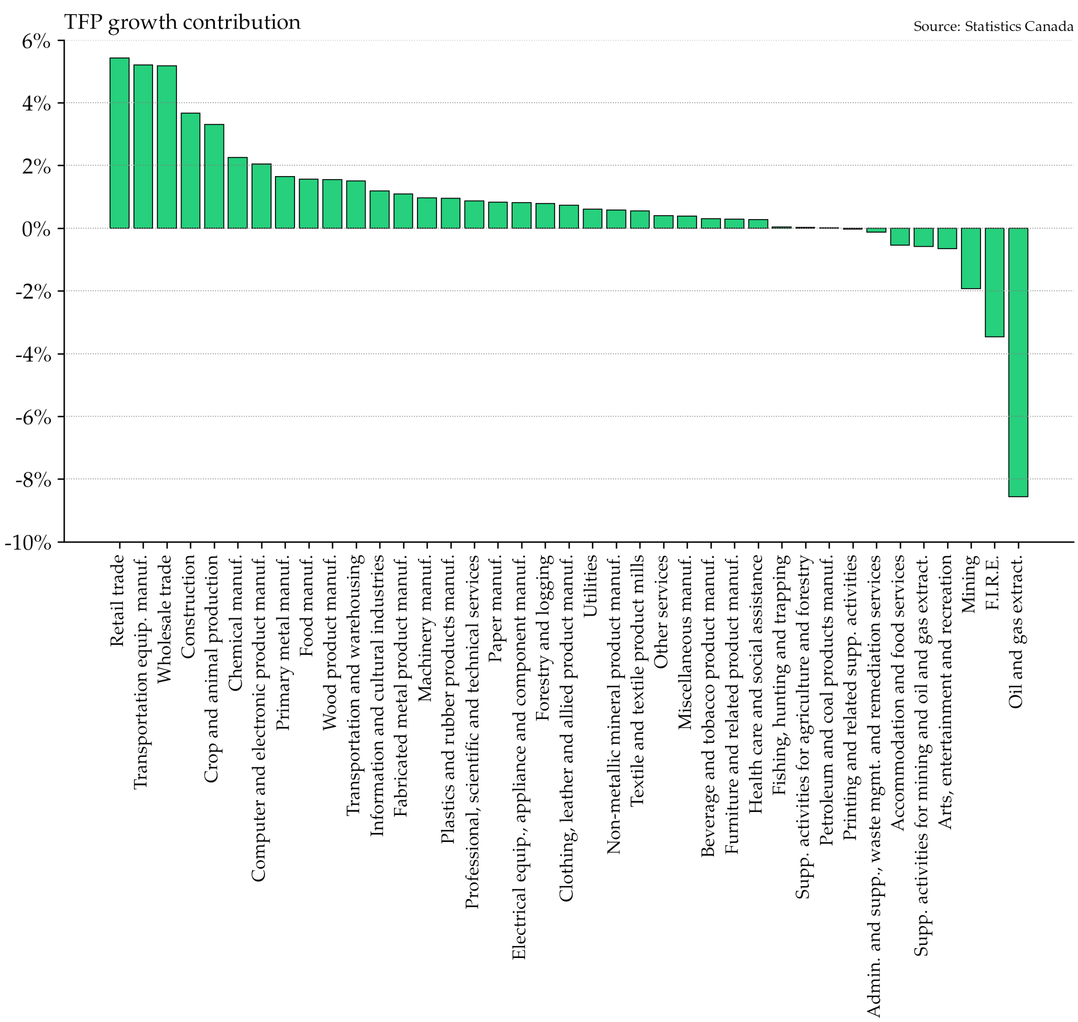


Figure 11: TFP Growth by Industry

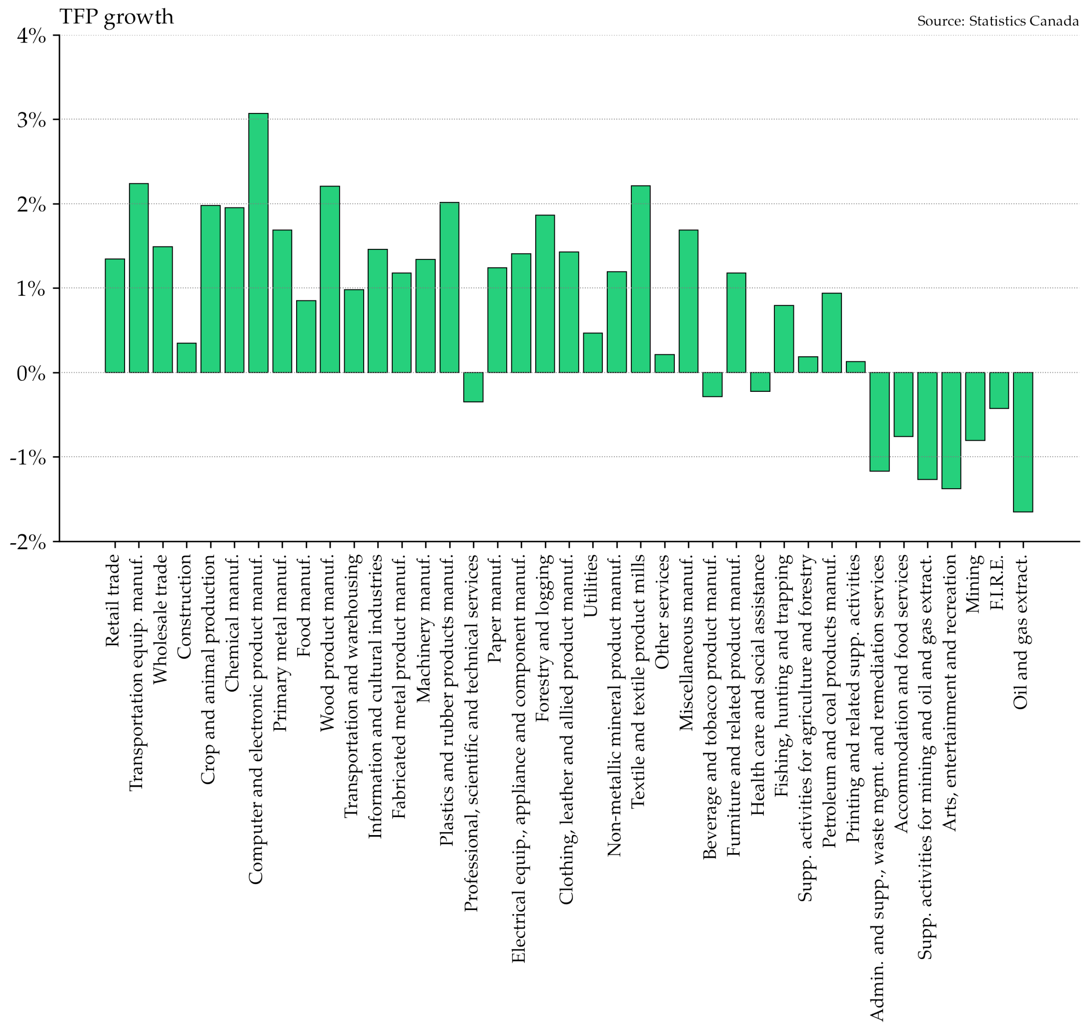
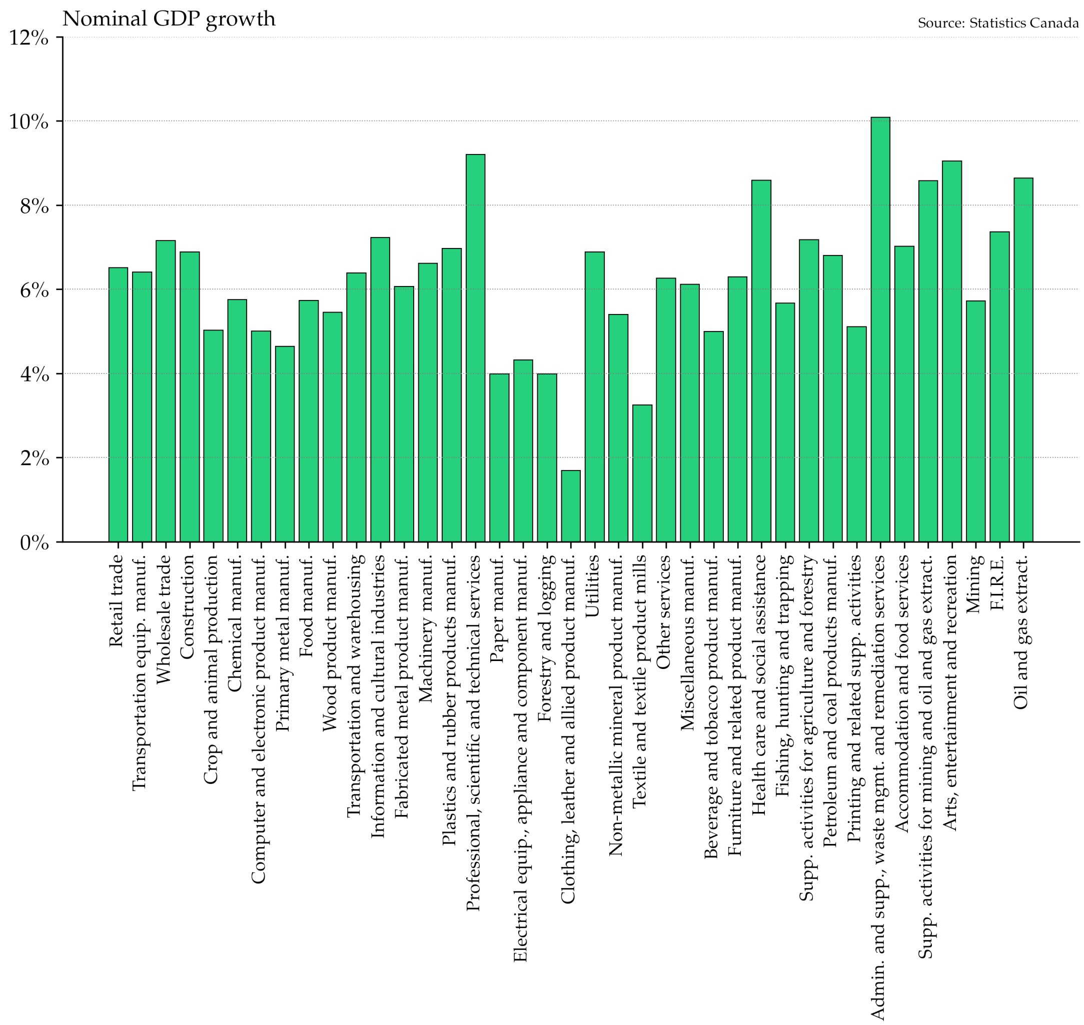


Figure 12: GDP Growth by Industry



References

Baqaee, David Rezza and Emmanuel Farhi, "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 09 2019, *135* (1), 105–163.

Halperin, Basil and Zachary J. Mazlish, "Decomposing the Great Stagnation: Baumol's Cost Disease vs. "Ideas Are Getting Harder to Find"," Working Paper 2024.

Appendix

A Empirical Appendix

Table A.3: TFP Growth Decomposition #1 (without O&G)

	1961-2019	1961-1980	1980-2000	2000-2019
Productivity	0.81%	1.14%	0.82%	0.21%
Baumol	-0.16%	0.03%	-0.15%	-0.10%
Capital	-0.10%	-0.17%	-0.06%	-0.07%
Labor	0.10%	0.16%	0.08%	0.07%
Total	0.65%	1.16%	0.68%	0.11%

Note:

Table A.4: TFP Growth Decomposition #2 (without O&G)

	1961-2019	1961-1980	1980-2000	2000-2019
Productivity	1.52%	2.07%	2.02%	0.38%
Baumol	-0.21%	0.03%	-0.45%	-0.13%
Allocative efficiency	-0.66%	-0.94%	-0.88%	-0.14%
Total	0.65%	1.16%	0.68%	0.11%

Note:

Figure A.13: TFP Growth and the Baumol Effect (without O&G)

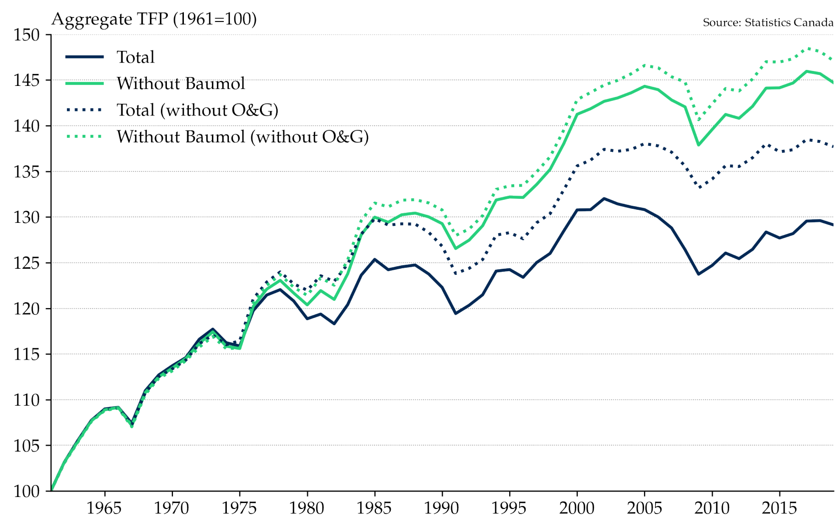


Figure A.14: TFP Growth and Misallocation (without O&G)

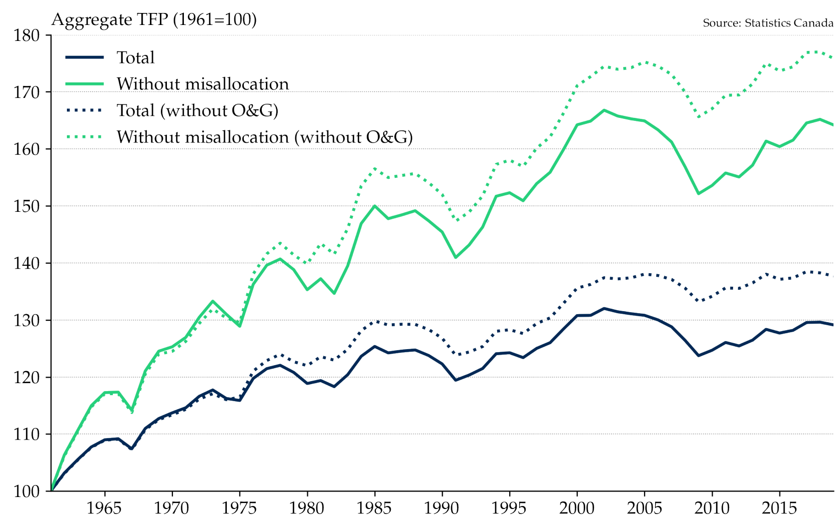


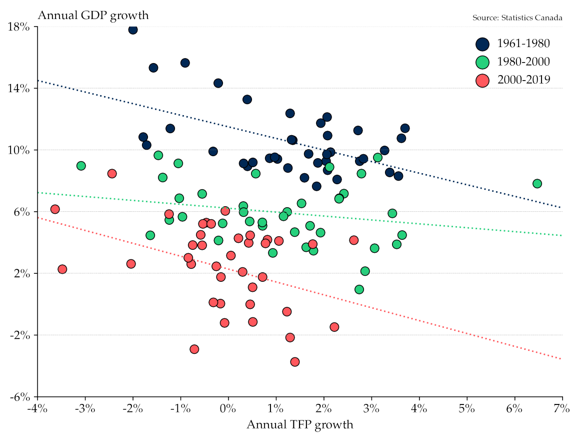
Figure A.15: Prices, Wages, and TFP Growth

(a) Prices

(b) Wages

Figure A.16: GDP and TFP Growth

(a) Nominal GDP



(b) Real GDP