

Resetting the Innovation Clock: Endogenous Growth through Technological Turnover*

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Abstract

We propose a model of endogenous economic growth with “weak” scale effects and declining innovation efficiency at the micro level. In the spirit of [Aghion and Howitt \(1996\)](#), entrants introduce new technologies through *research* and incumbents incrementally improve those technologies through *development*. Over time, however, it becomes harder to improve upon an existing technology such that firms eventually run out of ideas and exit the market. Their departure paves the way for entrants that discover new technologies with further room for improvement, thereby “resetting the innovation clock”. This turnover through entry and exit gives rise to a continuous stream of temporary opportunities for technological improvements that sustain economic growth. Analysis of the model reveals that, in a stationary equilibrium, the rate of economic growth is constant and *endogenous* despite a growing population and declining innovation efficiency at the firm-level. We present macro- and micro-level evidence that supports the predictions of our model.

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1 Introduction

In this paper, we develop an endogenous growth model which captures three empirical facts that previous models have failed to reconcile. First, despite sustained population growth and an expanding supply of researchers, economic growth has not accelerated in recent decades. Second, innovation is responsive to changes in market incentives, in particular to changes in market size, tax policy, and R&D policy. Third, ideas eventually become harder to find on any particular product line.

That economic growth remained steady despite population growth was first noted by [Jones \(1995\)](#). Examining data from 1953 to 1995, [Jones \(1995\)](#) finds that the nearly ninefold increase in R&D personnel did not translate into faster economic growth.

On the effects of changes in market size, tax policy, and R&D policy on long-run innovation-led growth: [Acemoglu and Linn \(2004\)](#) document that, in the pharmaceutical industry, innovators tend to focus on drugs for wealthier consumers; similarly, recent work using French microdata by [Aghion, Bergeaud, Lequien and Melitz \(2024\)](#) finds that expanding a firm’s export markets increases its output of pioneering patents. [Akcigit, Grigsby and Nicholas \(2017\)](#) provides historical evidence showing a positive causal effect of public investments. And [Akcigit and Stantcheva \(2020\)](#) document a negative effect of corporate income tax on long-term innovation and productivity growth.

Finally, on ideas becoming harder in particular product lines: [Bloom, Jones, Van Reenen and Webb \(2020\)](#) document this phenomenon in sectors such as semiconductors, agriculture, and health. For instance, while Moore’s law suggests that the number of transistors on a chip should double roughly every two years, the number of researchers required to maintain this pace has increased dramatically—by a factor of eighteen since the early 1970s. A similar trend is observed in pharmaceuticals, where the number of new compounds approved by the FDA relative to research effort has fallen even as R&D employment has grown.

To account for the above empirical facts, in this paper we develop an endogenous growth model which features declining innovation efficiency at the product line level but yet sustained endogenous growth at the macro level. In the spirit of [Aghion and Howitt \(1996\)](#), what sustains economic growth in our model is the endogenous—research-driven—arrival of technological breakthroughs (i.e., new products, firms, or industries) that “reset” the research productivity clock continuously over time. Meanwhile, incumbent firms incrementally improve the quality of their products by investing in development. Over time, however, it becomes increasingly harder to improve product quality, to such an extent that incumbent firms eventually run out of ideas and exit the market. This in turn paves the way for new firms entering the market with new product lines

where development-led quality improvements subsequently occur. This “resetting of the innovation clock” and the resulting turnover between exiting incumbent firms and the arrival of new firms with new product lines, gives rise to a continuous stream of temporary opportunities for product quality improvements that sustain economic growth.

In a stationary equilibrium, this turnover ensures that the total number of firms grows in proportion to the population, such that any expansion in the latter is absorbed by the proliferation of new firms. At the same time, our model displays declining innovation efficiency at the micro level which takes the form of an idiosyncratic “obsolescence” shock. Whenever a firm experiences this shock, its capacity to further improve its product’s quality vanishes so that the firm’s innovation efficiency instantly falls to zero.

Overall, economic growth is sustained by the continuous replacement of obsolete firms by new entrants, each bringing new opportunities for quality improvement. Importantly, at any point in time, a fraction of firms has not yet experienced the obsolescence shock, and their incentives to improve their product’s quality remain responsive to policy. In fact, we show that, in stationary equilibrium, the growth rate of per capita consumption remains constant despite population growth, and yet it responds to changes in market size or innovation policy.

Literature Review

Our analysis in this paper first speaks to the long-standing economic history debate between those, who, following [Gordon \(2017\)](#), maintain that the era of great innovations is over and that innovation has entered a phase of sharply decreasing returns, and those who, following Joel Mokyr (e.g. see [Mokyr, 2014](#)), argue that the recent technological revolutions (IT and more recently AI), combined with the globalization of trade, have created the conditions for innovation and growth to prosper more than ever before. We contribute to this debate by developing a new growth model where ideas become harder to find within each particular line, but not in the economy as a whole as new lines keep being created.

Secondly, our paper can be seen as an attempt to settle the debate between endogenous growth and semi-endogenous growth advocates. On the one hand, a “first-generation” of innovation-based endogenous growth models (e.g. see [Romer \(1990\)](#); [Aghion and Howitt \(1992\)](#); and [Aghion, Akcigit and Howitt \(2015\)](#)) predict that market incentives—in particular property rights protection, market size, product market competition, or R&D subsidies—should affect the long-run growth rate of the economy. However, in these models, population size had to remain constant in order for growth not to

explode, a prediction which fails to withstand empirical scrutiny as many economies experienced constant economic growth despite a growing research workforce. We depart from this first wave of endogenous growth models by developing a framework which delivers similar effects of market incentives despite having declining innovation efficiency at the product line level and without having to assume population growth away.

On the other hand, [Jones \(1995\)](#)’s semi-endogenous growth model delivers constant long-run growth despite population growth. This in turn is achieved by allowing research productivity to decline with the accumulated stock of knowledge. A downside of that model, however, is that the long-run growth depends exclusively upon the rate of population growth, and in particular it is unaffected by market incentives and policy. We depart from the semi-endogenous growth paradigm by proposing a framework that generates constant long-run growth despite population growth, and yet where market incentives matter.

Most closely related to our analysis is the so-called “second generation” endogenous growth models (e.g. see [Dinopoulos and Thompson \(1998\)](#); [Peretto \(1998\)](#); [Young \(1998\)](#); and [Howitt \(1999\)](#)), which also deliver a constant long-run growth rate that can be affected by policy—although sometimes in a contrived way—despite population growth. This in turn was achieved by allowing the number of firms to grow in proportion to the population (through free entry), keeping the ratio of researchers per firm constant so that each firm keeps growing at a constant rate. However, the assumption that research productivity should remain constant at the firm level, is directly questioned by the evidence in [Bloom et al. \(2020\)](#) showing that research productivity is instead declining at the firm—or product line—level. We depart from this second wave of endogenous growth models by developing a model which produced the same basic policy predictions as those delivered by the first-generation endogenous growth models, even though we allow for both, population growth and declining research productivity at the product line level. What sustains economic growth in our model is the endogenous arrival of technological breakthroughs (i.e., new products, firms, or industries) that “reset” the research productivity clock continuously over time.

The remaining part of the paper is organized as follows. Section 2 lays out our growth model. Section 3 solves the model. It computes the expression for the balanced growth rate, then presents a simple calibration of the model. Section 4 discusses the predictions of the model and compares them to empirical evidence. Section 5 analyzes the constrained-optimal allocation of the model. Section 6 concludes.

2 A New Growth Model

In this section, we present a simple model that captures the essence of our argument: firms invest in the *development* of existing lines to improve the quality of their products, but eventually run out of ideas and exit. Simultaneously, investments in *research* lead to the entry of new firms with entirely new product lines. As a consequence, opportunities for quality improvements are continuously replenished in the economy as a whole even though improvement opportunities are eventually exhausted on any existing lines.

2.1 The Economic Environment

Preferences

Consider a continuous-time economy where time is indexed by $t \in [0, \infty)$. This economy is populated by a representative household of measure N_t evolving according to:

$$\dot{N}_t = n \cdot N_t, \quad (1)$$

where $n > 0$. The household inelastically supplies one unit of a labor per period and has preferences over an individual consumption basket c_t such that lifetime utility is defined as:

$$U_0 = \int_0^\infty e^{-(\rho-n)t} \ln(c_t) dt \quad (2)$$

where $\rho > n$ denotes the rate of time preference. The consumption basket is a [Dixit and Stiglitz \(1977\)](#) aggregate of differentiated products indexed by $i \in \mathcal{I}_t$:

$$c_t = \left(\int_{i \in \mathcal{I}_t} (q_{it} \cdot c_{it})^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad (3)$$

where c_{it} is the quantity of product i consumed at time t , $q_{it} > 0$ is the quality of that product, $\theta > 1$ is the elasticity of substitution between products, \mathcal{I}_t is the set of available products, and its (endogenous) cardinality is denoted by $M_t \equiv |\mathcal{I}_t|$.

Here, a “product” is best understood as a new technology rather than an incremental improvement on an existing one. For instance, early film cameras included simple box cameras like the Kodak Brownie, which had fixed-focus lenses and limited control over exposure. Over time, manufacturers introduced more advanced 35 mm and single-lens reflex (SLR) film cameras—such as the Nikon F or the Canon AE-1—with features like interchangeable lenses, better metering systems, and more precise shutter controls. These advancements represented successive improvements of film camera technology.

Eventually, however, those improvement opportunities in film-based imaging were depleted, paving the way for a fundamentally new product: the digital camera, which opened up new dimensions of innovation (e.g., image sensors, on-board processing, and software-driven features).

Technology

Each product is produced by a single firm using production labor according to the linear technology:

$$y_{it} = l_{it}^P, \quad (4)$$

where y_{it} denotes the quantity of product i supplied at time t and l_{it}^P denotes the quantity of business labor used in production.

Over time, a firm can incrementally improve the quality of its product by directing business labor towards development. More precisely, a product's quality evolves according to the following controlled process:¹

$$\dot{q}_{it} = \gamma_{it} \cdot q_{it}, \quad (5)$$

where $\gamma_{it} < \bar{\gamma}$ is the proportional drift of the product's quality. The labor requirement l_{it}^D to achieve this quality drift is given by:

$$l_{it}^D = \frac{c_D (q_{it}/Q_t)^{\theta-1} \gamma_{it}^{1+\zeta}}{1+\zeta} \quad (6)$$

where $c_D > 0$ and $\zeta > 0$ measure the (reciprocal) efficiency and the elasticity of the development technology, and Q_t is an average quality index defined as:²

$$Q_t \equiv \left(M_t^{-1} \int_{i \in \mathcal{I}_t} q_{it}^{\theta-1} di \right)^{\frac{1}{\theta-1}}.$$

The development technology is such that there is a finite upper bound $\bar{\gamma} > 0$ on the drift of product quality to ensure that the optimal allocation does not involve any corner solutions as described in [Trammell \(2025\)](#).

However, at Poisson rate $\epsilon > 0$, a firm may receive an idiosyncratic “obsolescence”

¹Here, we specify a deterministic process for the sake of tractability, but the model could easily be extended to allow for idiosyncratic quality shocks.

²The development technology depends on a product's quality relative to the average index q_{it}/Q_t to ensure that product quality grows at the same rate for all firms in equilibrium (Gibrat's law), which delivers an analytical solution for the market allocation in this model.

shock after which it can no longer improve its product's quality. This is an extreme and stylized case of "ideas becoming harder to find" (Bloom et al., 2020) in which innovation efficiency literally falls to zero once the shock hits.³ After being hit by this shock, if a product's quality falls below a certain threshold $\underline{q}_t = \underline{q} \cdot Q_t$ where $\underline{q} \in (0, 1)$, the firm exogenously exits the market.⁴

In every point in time, a unit measure of potential entrants attempt to discover products that are entirely new to society through research. Specifically, these entrants can use $c_R(M_t/N_t)^\phi$ units of research labor to invent a unit flow of these new products. Here, the parameter $\phi > 0$ determines the degree of "congestion" in the entry cost technology. Once a product is discovered, its initial quality is drawn from a point mass at the lower bound \underline{q}_t of the product quality support.⁵

Resource Constraints

Labor supplied by the household can be allocated to production, development, or research, delivering the resource constraint:

$$L_t^P + L_t^D + L_t^R \leq N_t \quad (7)$$

where these aggregate labor allocations are defined as:

$$L_t^P \equiv \int_{i \in \mathcal{I}_t} l_{it}^P di, \quad L_t^D \equiv \int_{i \in \mathcal{I}_t} l_{it}^D di, \quad \text{and} \quad L_t^R = c_R(M_t/N_t)^\phi R_t$$

and where R_t is the flow of new products discovered at time t . Finally, the resource constraints for each product are given by:

$$c_{it} \leq y_{it} \quad \forall i \in \mathcal{I}_t. \quad (8)$$

³One could instead consider a more general formulation by which the efficiency of the development technology gradually decreases as a product's quality is improved.

⁴This simple exit rule is chosen for tractability as it delivers an analytic solution to the growth rate of consumption per capita, but it could easily be extended to an optimal stopping time problem in the presence of overhead costs.

⁵One could specify a more general learning process between entrants and incumbents (see Yao (2024) for a very general and still tractable specification), but we choose to keep the model as simple as possible to illustrate the driving mechanisms.

2.2 The Decision Problems

In this section, we define the decision problems of each economic agent. In terms of market structure, we assume that all agents partake in perfect competition in all markets besides firms who engage in monopolistic competition.

The Household's Problem

Taking prices as given, the household's problem is to choose its consumption of each product to maximize lifetime utility:

$$\max_{\{\{c_{it}\}_{i \in \mathcal{I}_t}\}_{t \geq 0}} \int_0^\infty e^{-(\rho-n)t} \ln(c_t) dt$$

subject to the flow budget constraint:

$$\dot{a}_t + \int_{i \in \mathcal{I}_t} p_{it} c_{it} di \leq (r_t - n)a_t + w_t - T_t.$$

Here, w_t is the wage rate, T_t are lump-sum per-capita taxes, a_t is the value of corporate assets per capita, and r_t is the rate of return on those assets:

$$a_t N_t = \int_{i \in \mathcal{I}_t} V_{it} di \quad \text{where} \quad \lim_{t \rightarrow \infty} e^{-\int_0^t (r_{t'} - n) dt'} a_t = 0,$$

and where V_{it} denotes the value of product i at time t . This problem thus delivers the usual intertemporal Euler equation and demand functions:

$$\dot{c}_t = (r_t - \rho)c_t \quad \text{and} \quad c_{it} = (P_t / p_{it})^\theta q_{it}^{\theta-1} c_t.$$

Since aggregate consumption is chosen as the numéraire, the price index P_t is normalized to one for all t :

$$P_t \equiv \left(\int_{i \in \mathcal{I}_t} (p_{it} / q_{it})^{1-\theta} di \right)^{\frac{1}{1-\theta}} = 1.$$

The Firms' Problem

Firms engage in monopolistic competition in the product market but perfect competition in the labor market, taking the wage and the demand for their product as given. A firm thus chooses the price at which to sell its product and its labor demands to maximize the expected present discounted value of its profits.

From this point on, we abandon the i -index notation since a firm is entirely described by its product's quality q and its obsolescence status. We denote the latter by $S \in \{O, N\}$ where O represents an "old" firm that has received the obsolescence shock, and N represents a "new" firm that has not. The new firm's value function satisfies a standard Hamilton-Jacobi-Bellman (HJB) equation:

$$r_t V_t^N(q) = \max_{\mathbf{u}_t^N(q) \geq \mathbf{0}} \{ (1 - \tau^C)[p_t(q)y_t(q) - w_t l_t^P(q)] + (1 - \tau^D)w_t l_t^D(q) \\ + \gamma_t(q)q\partial_q V_t^N(q) \} + \epsilon[V_t^O(q) - V_t^N(q)] + \dot{V}_t^N(q)$$

where $\mathbf{u}_t^N(q) \equiv \{p_t(q), l_t^P(q), l_t^D(q)\}$ is the vector of control variables, $\tau^C > 0$ is the corporate income tax rate, and $\tau^D > 0$ is a subsidy on the firm's development expenditures. Similarly, the "old" firm's value function satisfies the HJB equation:

$$r_t V_t^O(q) = \max_{\mathbf{u}_t^O(q) \geq \mathbf{0}} \{ (1 - \tau^C)[p_t(q)y_t(q) - w_t l_t^P(q)] \} + \dot{V}_t^O(q)$$

where $\mathbf{u}_t^O(q) \equiv \{p_t(q), l_t^P(q)\}$. The profit-maximization problem implies that a firm sets its price to a constant markup above marginal cost irrespective of its obsolescence status:

$$p_t(q) = \mu \cdot w_t, \quad \forall q \quad \text{where} \quad \mu \equiv \frac{\theta}{\theta - 1}.$$

Hence, a firm's flow profits can be expressed as:

$$\pi_t^S(q) = (1 - \tau^C)(q/Q_t)^{\theta-1} c_t N_t / (\theta M_t) - \mathbb{1}_{\{S=N\}}(1 - \tau^D)w_t l_t^D(q)$$

where the optimal labor allocation to development is given by:

$$l_t^D(q) = \frac{c_D(q/Q_t)^{\theta-1} \gamma_t(q)^{1+\zeta}}{1 + \zeta} \quad \text{where} \quad \gamma_t(q) = \left[\frac{q\partial_q V_t^N(q)}{(1 - \tau^D)w_t c_D(q/Q_t)^{\theta-1}} \right]^{1/\zeta}.$$

The Entrant's Problem

Entrants engage in perfect competition on the final goods market and, thus, choose their allocation L_t^R of labor to research to maximize future expected profits while taking the wage rate as given:

$$V_t^E = \max_{L_t^R} \left\{ \frac{V_t^N(q_t) L_t^R}{c_R(M_t/N_t)^\phi} - (1 - \tau^R)w_t L_t^R \right\}$$

where $\tau^R > 0$ is a research subsidy. The first-order condition of the entrant's problem delivers what will be referred to as the free-entry condition:

$$V_t^N(q_t) = (1 - \tau^R)w_t c_R(M_t/N_t)^\phi.$$

2.3 The Equilibrium Market Allocation

Having defined the decision problems of each economic agent, we can now define the concept of an equilibrium market allocation, and lay out the equations that determine the long-run equilibrium growth rate of the aggregate economy.

Definition 1. *Given the initial conditions $\{N_0, Q_0, \{m_0^N(q), m_0^O(q)_{q=q_t}^\infty\}\}$, where $m_0^S(q)$ is the initial measure of type-S firms with product quality q , a market allocation consists of time paths for quantities, prices, and policy functions such that the following conditions hold:*

1. $\{\{c_t(q)\}_{q=q_t}^\infty\}_{t \geq 0}$ solve the household's problem.
2. $\{\{p_t(q), l_t^P(q), l_t^D(q)\}_{q=q_t}^\infty\}_{t \geq 0}$ solve the firm's problem.
3. $\{L_t^R\}_{t \geq 0}$ solve the entrant's problem.
4. $\{\{p_t(q)\}_{q=q_t}^\infty\}_{t \geq 0}$ clear the product markets.
5. $\{w_t\}_{t \geq 0}$ clear the labor market.
6. $\{r_t\}_{t \geq 0}$ clear the asset market.
7. The government's budget is balanced.

The market allocation in this model admits a remarkably simple aggregation such that output per capita is given by:

$$c_t = M_t^{\frac{1}{\theta-1}} Q_t.$$

That is, consumption per capita is increasing in the measure of products (owing to a taste for variety) and the average quality across those products.

3 Solving the Model

3.1 Equilibrium Balanced Growth Path

In Appendix A.1 we show that on a balanced growth path (BGP), the measure of products grows at the same rate as the population, the average quality index grows at a constant rate, and, thus, the growth rate of consumption per capita is constant and given by:

$$g = \frac{n}{\theta - 1} + g^Q. \quad (9)$$

We can derive an expression for the growth rate of the average quality index:

$$g^Q = \gamma \cdot \frac{n + d}{n + \epsilon} - \frac{n(1 - \underline{q}^{\theta-1})}{\theta - 1} \quad \text{where} \quad \gamma = \left[\frac{(\theta - 1)(1 - \tau^R)c_R \mathcal{M}^{\frac{1}{\theta-1}}}{(1 - \tau^D)c_D \underline{q}^{\theta-1}} \right]^{1/\zeta} \quad (10)$$

which is composed of two additive terms. The first term reflects the positive growth contribution of incremental product quality improvements by incumbent firms. This term is the product of the common product quality drift γ among firms that haven't yet received the obsolescence shock and the fraction of such firms in the economy given by $\frac{n+d}{n+\epsilon}$. The second term reflects the negative growth contribution of net product entry occurring at rate n . Indeed, entering firms draw their initial product quality at the lower bound $\underline{q} \in (0, 1)$ of the quality support, which drags down the average quality index.

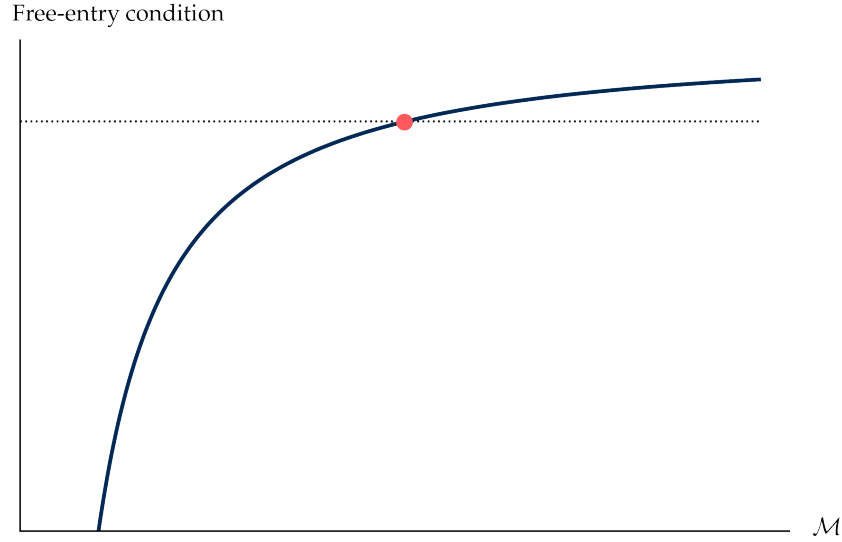
Notice that the quality drift depends on the measure of products per capita (denoted by $\mathcal{M}_t \equiv M_t/N_t$), which is determined in equilibrium through the free-entry condition illustrated in Figure 1 using the parameter values detailed in Section 3.2. In Appendix A, we show that this equilibrium is unique.

3.2 A Simple Calibration

In this section, we present a straightforward calibration of the parameters of our model to illustrate its mechanisms. Although this exercise is not meant as a rigorous empirical quantification for direct comparison with economic data, it provides insight into the potential magnitude of different forces.

We set the pure rate of time preference, ρ , to 0.04 and assume an annual population growth rate, n , of 1%. Consistent with Garcia-Macia, Hsieh and Klenow (2019), the elasticity of substitution across products, θ , is set to 4. We fix ζ , which captures the degree of decreasing returns to development labor, at 1—aligning with microeconomic evidence on the tax elasticity of R&D summarized in Acemoglu, Akcigit, Alp, Bloom

Figure 1: The Market Allocation



Note: The root of this equation determines the unique value of M that satisfies the free-entry condition.

and Kerr (2018).

Four additional parameters require calibration: the development cost parameter c_D , the research cost parameter c_R , the initial quality of new entrants \underline{q} , and the obsolescence shock ϵ . Since these parameters are less conventional in the literature or not directly observable, we calibrate them by jointly matching the following four empirical moments:

1. Per capita consumption grows at 2% per year.
2. The firm-level entry rate is 10%.
3. The average firm has 20 employees.
4. New entrants are 95% as large as incumbents.

The first three moments are representative of the U.S. economy, while the fourth reflects a stylized fact from Einav, Klenow, Murciano-Goroff and Levin (2022), who document that the average entrant achieves roughly the same sales per customer as the average incumbent. The calibrated parameter values are reported in Table 1.

While the parameters $\{c_D, c_R, \underline{q}, \epsilon\}$ are jointly calibrated to match the aforementioned moments, we provide some intuition for their identification. The development cost parameter c_D is primarily identified by the growth rate of per capita consumption, as it governs the pace at which firms improve their product's quality and, in turn, economic

Table 1: Calibration

Parameter	Value	Source
ρ	0.04	Standard
n	0.01	Population growth
θ	4	Garcia-Macia et al. (2019)
ζ	1	Acemoglu et al. (2018)
c_D	$\exp(8.48)$	Consumption per capita growth
c_R	$\exp(4.34)$	Average firm size
\underline{q}	0.1	Relative size of entrants
ϵ	0.1	Entry rate

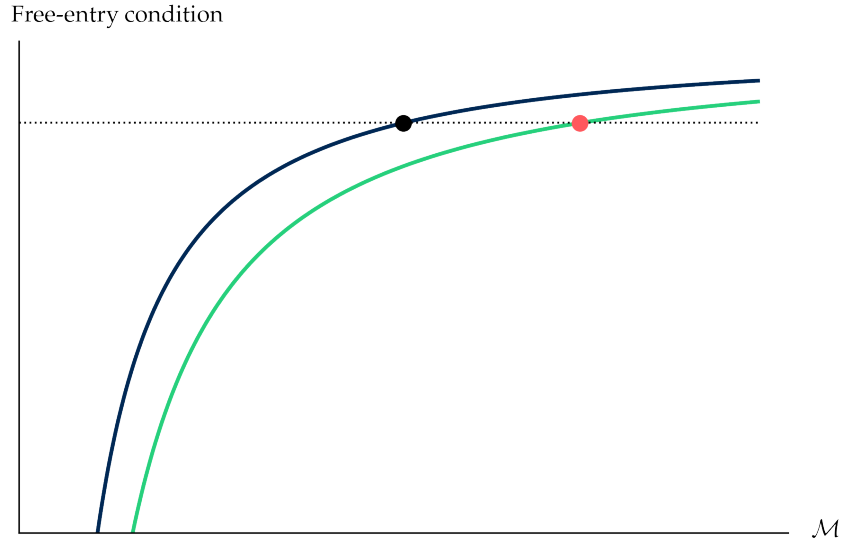
growth. The research cost parameter c_R is determined by the average firm size, since it influences the cost of entry and thereby the equilibrium number of firms. The initial quality of new entrants \underline{q} is directly linked to the relative size of entrants compared to incumbents. Finally, the obsolescence shock, ϵ , is identified from the firm-level entry rate because it governs how quickly firms become obsolete and ultimately exit the market. Under a balanced growth path, the entry rate equals the sum of the exit rate and the population growth rate.

Under this calibration, Figure 2 illustrates the equilibrium effect of introducing a 100% corporate income subsidy—equivalent to doubling the size of the market. This subsidy shifts the free-entry condition to the right, resulting in a higher equilibrium measure of products per capita and, therefore, increasing the growth rate of consumption per capita by 45 basis points.

Figure 3 illustrates the equilibrium effects of R&D policy. In Panel 3(a), a 30% subsidy on incumbent firms' product development expenditures is implemented. Although this subsidy raises the per capita consumption growth rate by 71 basis points, it has little to no effect on the equilibrium value of \mathcal{M} . This indicates that the subsidy primarily affects economic growth through its direct effect on the product quality drift γ , rather than through the measure of products per capita.

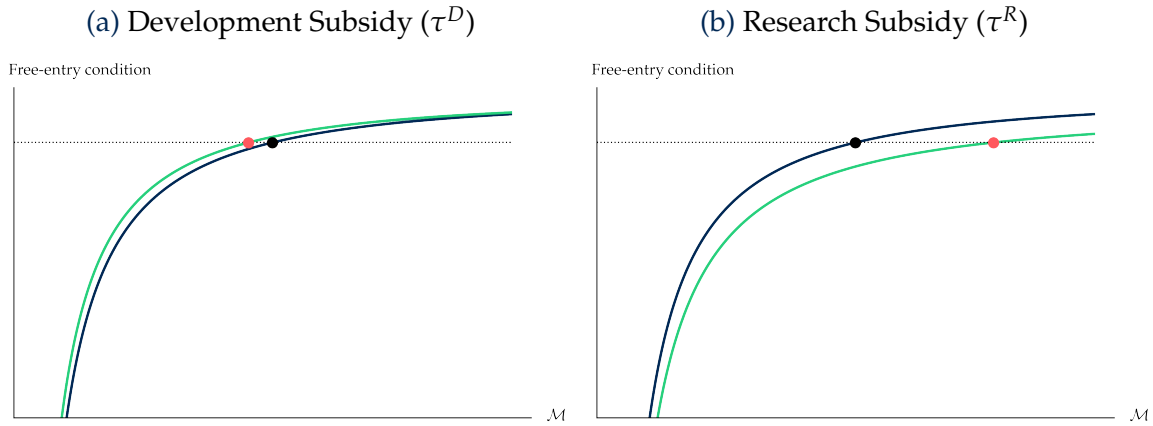
In contrast, Panel 3(b) illustrates that a 30% subsidy on research expenditures for new entrants substantially increases the equilibrium value of \mathcal{M} . This policy has two opposing effects on economic growth. On one hand, the lower entry cost reduces the firm's value through the free-entry condition, thereby dampening development incentives. On the other hand, the higher measure of products per capita increases the cost of entry relative to development. Ultimately, the first effect dominates, resulting in a 36 basis point decline in the growth rate of consumption per capita.

Figure 2: Corporate Income Subsidy (τ^C)



Note: The green line represents the free-entry condition when introducing a corporate income subsidy of 100%.

Figure 3: The Equilibrium Effects of R&D Policy



Note: The green lines represent the free-entry condition when introducing subsidies of 30% to development and research, respectively.

4 Discussion

While partly building on it and sharing an important empirical motivation with it, our model departs from the canonical semi-endogenous growth framework in ways which allow us to match additional empirical facts.

First, our model predicts that market size matters for long-run innovation and growth. In the growth formula (10), the market size effect is captured by the term $\mathcal{M}^{\frac{1}{\theta-1}}$. In words, as the market expands - and therefore profits increase - more firms are incentivized to enter, thereby raising the number of firms per capita. A higher firm density in turn reduces the relative cost of development compared to research, ultimately accelerating TFP growth. This market size prediction is borne by several empirical studies show that larger market size e.g. driven by an increased market demand, stimulates innovation and growth in the long run. Thus, at the sectoral level, [Acemoglu and Linn \(2004\)](#) find that a 1% expansion in a U.S. pharmaceutical market leads to a 4 to 6% rise in new drugs. [Dubois, De Mouzon, Scott-Morton and Seabright \(2015\)](#), using updated data, report a smaller elasticity of 0.23, implying that a 10% market increase induces a 2.3% rise in drug development, with \$2.5 billion in additional revenue needed per new molecule. [Blume-Kohout and Sood \(2013\)](#) show that Medicare Part D in 2006 significantly boosted pharmaceutical revenues and led to a 50% increase in new clinical trials. In vaccines, [Finkelstein \(2004\)](#) documents that policy-driven market expansions increased vaccine trials by 2.5 times, with \$0.06 of R&D investment per additional dollar of anticipated revenue. At the firm level, [Aghion et al. \(2024\)](#) study French manufacturers and find that the most productive firms increase patenting in response to export demand shocks. They interpret this fact using the lens of the Schumpeterian growth model, where enhanced competition discourages less efficient firms from innovating with a counteracting effect driven by the positive effect of competition on market size and therefore on innovation. Similarly, [Bustos \(2011\)](#) finds that Argentinian firms exposed to MERCOSUR tariff reductions in the 1990s adopted advanced technologies and intensified R&D and [Lileeva and Trefler \(2010\)](#) show that Canadian manufacturers entering export markets after the Canada - US Free Trade Agreement experienced productivity gains, product innovation, and technological upgrades.

Second, our model predicts that tax policy impacts long term innovation and growth. For example, the growth formula in equation (10) implies that an increase in τ^D has a negative effect on long-run innovation and growth. Empirically we know that R&D tax credits impact long-run innovation (e.g. see ([Dechezleprêtre, Einiö, Martin, Nguyen and Van Reenen, 2023](#))) even when these are not necessarily the most efficient instruments to stimulate certain kinds of innovation. Moreover, the same growth formula shows that an increase in corporate taxation τ^R leads to a decline in growth and innovation, in line with [Kennedy, Dobridge, Landefeld and Mortenson \(2022\)](#); [Mukherjee, Singh and Žaldokas \(2017\)](#).

A third departure from the semi-endogenous growth model, is that in our model innovation by incumbent firms is an important driver of growth. In fact, [Garcia-Macia](#)

et al. (2019) document that over half of U.S. TFP growth between 1983 and 2013 is attributable to existing product improvements by incumbent firms.

We can modify our model in two ways to recover the canonical semi-endogenous growth framework. First, we can rule out quality improvements of existing products (i.e., take $c_D \rightarrow \infty$). Under this assumption, all products retain a constant quality level, so growth is driven solely by the entry of new products. In this case, the per capita consumption growth rate becomes:

$$g = \frac{n}{\theta - 1}$$

which is the standard result in the semi-endogenous growth literature. However, this modification fails to capture the empirical observation that improvements in existing products are the primary driver of growth in the U.S. economy.

Second, we could assume that new entrants draw their initial product quality from a fixed distribution that does not scale with the existing average quality index. With this assumption, new products begin at a given quality level, improve until the obsolescence shock occurs, and then stagnate, modification of the model, however, also contradicts the empirical findings of Garcia-Macia et al. (2019), since it implies that product improvements play no role in driving overall economic growth.

5 The Constrained-Optimal Allocation

In Appendix A.2, we pose the problem of a planner that maximizes social welfare while taking the incumbent-to-incumbent and incumbent-to-entrant technology spillovers as given. More precisely, the planner takes Q_t and M_t in the research and development technologies as given. Because the planner does not internalize such spillovers, we refer to the solution to this problem as *constrained-optimal*.

To decentralize this constrained-optimal allocation, we show that a government could implement subsidies to both research and development of $\tau^R = \tau^D = 1/\theta$. That is, the market allocation features too little research (entry) and development, disregarding technology spillovers. As in Dhingra and Morrow (2019), there is too little entry because new entrants do not internalize that they raise demand for existing firms due to the “taste for variety.” Moreover, there is too little development because firms exert market power. Indeed, firms produce at levels below what would prevail under perfect competition, which reduces the “market size” incentive for development. Note, however, that if our model featured head-to-head creative destruction, the market allocation could feature

too much development.

6 Conclusion

In this paper we have developed a growth model which combines insights from both, the endogenous and semi-endogenous growth literatures. Thus, on the one hand the model predicts constant long-run growth despite population growth, and that the productivity of innovation should decline on any particular line. On the other hand the model predicts that market size or tax policy should affect long run growth and innovation. The analysis in the paper can be extended in several interesting directions. A first extension would be to conduct a welfare analysis....A second extension would....

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A Theoretical Appendix

This section of the Appendix provides derivations and proofs for the results presented in the paper.

A.1 The Market Allocation

Hamilton-Jacobi-Bellman Equations. The old firm's value function satisfies the HJB equation:

$$r_t V_t^O(q) = (1 - \tau^C)(q/Q_t)^{\theta-1} c_t N_t / (\theta M_t) + \dot{V}_t^O(q).$$

Defining $x_t \equiv \ln(q_t/Q_t)$, we can rewrite:

$$r_t V_t^O(x) = (1 - \tau^C) \exp[(\theta - 1)x] c_t N_t / (\theta M_t) - g_t^Q \partial_x V_t^O(x) + \dot{V}_t^O(x)$$

where $g_t^Q \equiv \dot{Q}_t/Q_t$ denotes the growth rate of the average quality index. Let us guess that this value function takes the following form:

$$V_t^O(x) = V_t^O \exp[(\theta - 1)x].$$

Substituting this guess in the HJB equation, we obtain the following ordinary differential equation (ODE):

$$\dot{V}_t^O = [r_t + (\theta - 1)g_t^Q] V_t^O - (1 - \tau^C) c_t N_t / (\theta M_t),$$

which verifies our guess. The new firm's value function satisfies the HJB equation:

$$\begin{aligned} (r_t + \epsilon) V_t^N(q) &= (1 - \tau^C)(q/Q_t)^{\theta-1} c_t N_t / (\theta M_t) - (1 - \tau^D) w_t l_t^D(q) \\ &\quad + \gamma_t(q) q \partial_q V_t^N(q) + \epsilon V_t^O(q) + \dot{V}_t^N(q). \end{aligned}$$

Using the change of variable defined above, we can rewrite:

$$\begin{aligned} (r_t + \epsilon) V_t^N(x) &= (1 - \tau^C) \exp[(\theta - 1)x] c_t N_t / (\theta M_t) \\ &\quad - (1 - \tau^D) w_t c_D \exp[(\theta - 1)x] \gamma_t(x)^{1+\zeta} / (1 + \zeta) \\ &\quad + [\gamma_t(x) - g_t^Q] \partial_x V_t^N(x) + \epsilon V_t^O(x) + \dot{V}_t^N(x). \end{aligned}$$

Let us guess that this value function takes the following form:

$$V_t^N(x) = V_t^N \exp[(\theta - 1)x].$$

Substituting this guess in the optimal product quality drift, we obtain:

$$\gamma_t = \left[\frac{(\theta - 1)V_t^N}{(1 - \tau^D)w_t c_D} \right]^{1/\zeta}$$

which is independent of a product's quality. Substituting this result in the HJB equation of the new firm, we obtain the following ODE, which verifies our guess:

$$\dot{V}_t^N = [r_t + \epsilon - (\theta - 1)(\gamma_t - g_t^Q)]V_t^N - \epsilon V_t^O - \frac{(1 - \tau^C)c_t N_t}{\theta M_t} - \frac{(1 - \tau^D)w_t c_D \gamma_t^{1+\zeta}}{1 + \zeta}.$$

Fokker-Planck Equations. The Fokker-Planck (FP) equations describing the evolution of the density of log relative quality among new and old firms are given by:

$$\begin{aligned} \dot{m}_t^N(x) &= -(\gamma_t - g_t^Q)\partial_x m_t^N(x) - \epsilon m_t^N(x) + \delta(x - \underline{x}_t)R_t, \\ \dot{m}_t^O(x) &= g_t^Q\partial_x m_t^O(x) + \epsilon m_t^N(x), \end{aligned}$$

where $\delta(\cdot)$ denotes the Dirac delta function and $\underline{x}_t \equiv \ln(\underline{q}_t/Q_t)$. Therefore, the law of motion for the measure of new and old products are given by:

$$\dot{M}_t^N = R_t - \epsilon M_t^N \quad \text{and} \quad \dot{M}_t^O = \epsilon M_t^N - d_t M_t$$

where d_t denotes the exit rate at the lower bound of the quality support:

$$d_t \equiv g_t^Q \lim_{x \rightarrow \underline{x}_t} m_t^O(x) / M_t.$$

Hence, the total measure of products evolves according to:

$$\dot{M}_t = (e_t - d_t)M_t$$

where $e_t \equiv R_t/M_t$ denotes the entry rate.

Equilibrium Conditions. Using our previous results, the free-entry condition can be rewritten as:

$$V_t^N \underline{q}^{\theta-1} = (1 - \tau^R)w_t c_R (M_t/N_t)^\phi.$$

The labor market clearing condition can be rewritten as:

$$N_t = \frac{c_t N_t}{\mu w_t} + \frac{c_D \gamma_t^{1+\zeta}}{1 + \zeta} \cdot \int_{\underline{x}_t}^{\infty} \exp[(\theta - 1)x] m_t^N(x) dx + c_R (M_t/N_t)^\phi e_t M_t.$$

The only endogenous variable for which a corresponding equation is missing is the growth rate of the average quality index. Using the change of variable defined above, the expression for this index implies:

$$M_t^{-1} \int_{\underline{x}_t}^{\infty} \exp[(\theta - 1)x] m_t(x) dx = 1.$$

Balanced Growth Path. On a BGP, the growth rate of the average quality index and the drift of product quality for new firms are both constant. Moreover, the measure of new and old firms both grow at the same rate as the population. This implies that the entry and exit rates are constant, and the former is equal to $e = n + d$. The share of new and old products are also constant and equal to:

$$\frac{M_t^N}{M_t} = \frac{n + d}{n + \epsilon} \quad \text{and} \quad \frac{M_t^O}{M_t} = \frac{\epsilon - d}{n + \epsilon}.$$

Defining $f_t^N(x) \equiv m_t^N(x) / M_t^N$, the stationary FP equation for this distribution of log relative quality among new firms is:

$$-(\gamma - g^Q) \partial_x f^N(x) - (n + \epsilon) f^N(x) = -(n + \epsilon) \delta(x - \underline{x}).$$

With parameter values such that $\gamma > g^Q$, the solution to this ODE is:

$$f^N(x) = \lambda_N \exp[-\lambda_N(x - \underline{x})] \quad \text{where} \quad \lambda_N \equiv \frac{n + \epsilon}{\gamma - g^Q}.$$

Similarly, defining $f_t^O(x) \equiv m_t^O(x) / M_t^O$, the stationary FP equation for this distribution of log relative quality among old firms is:

$$g^Q \partial_x f^O(x) - n f^O(x) = -\frac{\epsilon(n + d)}{\epsilon - d} \cdot f^N(x).$$

Dividing through by g^Q , and multiplying by the integration factor $\exp(-\lambda_O x)$ where $\lambda_O \equiv n/g^Q$, we can rewrite:

$$\partial_x [\exp(-\lambda_O x) f^O(x)] = -\frac{\epsilon(n + d)}{(\epsilon - d)g^Q} \cdot \exp(-\lambda_O x) f^N(x).$$

Integrating this equation and solving for $f^O(x)$, we obtain:

$$f^O(x) = \left[C - \frac{\epsilon(n + d)}{(\epsilon - d)g^Q} \cdot \int_{\underline{x}}^x f^N(x) \exp(-\lambda_O x) dx \right] \exp(\lambda_O x)$$

where C is an integration constant. For $f^O(x)$ to be integrable, we must have:

$$C = \frac{\epsilon(n+d)}{(\epsilon-d)g^Q} \cdot \int_{\underline{x}}^{\infty} f^N(x) \exp(-\lambda_O x) dx.$$

Substituting this expression back in the solution for $f^O(x)$, we obtain:

$$f^O(x) = \frac{\epsilon(n+d)f^N(x)}{(\epsilon-d)(\lambda_N g^Q + n)}.$$

For $f^O(x)$ to be a probability distribution, we must verify that:

$$\epsilon(n+d) = (\epsilon-d)(\lambda_N g^Q + n).$$

Using the definition of the exit rate, which delivers $d = \epsilon g^Q / \gamma$, it is straightforward to verify that this condition is satisfied. Therefore, we have that the stationary distribution of quality among old firms is identical to that of new firms. Let us define normalized variables, which are constant on a BGP:

$$\mathcal{V}^O \equiv \frac{V_t^O}{c_t}, \quad \mathcal{V}^N \equiv \frac{V_t^N}{c_t}, \quad \mathcal{M} \equiv \frac{M_t}{N_t}, \quad \mathcal{W} \equiv \frac{w_t}{c_t}.$$

With these definitions, we can express the free-entry condition as:

$$\mathcal{V}^N \underline{q}^{\theta-1} = (1 - \tau^R) \mathcal{W} \mathcal{M}^\phi c_R.$$

Substituting this expression in the product quality drift, we obtain:

$$\gamma = \left[\frac{(\theta-1)(1-\tau^R) \mathcal{M}^\phi c_R}{(1-\tau^D) c_D \underline{q}^{\theta-1}} \right]^{1/\zeta}.$$

To obtain that last expression for the growth rate of the average quality index, we use the definition of the index itself:

$$\frac{\lambda_N \cdot \underline{q}^{\theta-1}}{\lambda_N + 1 - \theta} = 1 \quad \Leftrightarrow \quad g^Q = \gamma - \frac{(n+\epsilon)(1-\underline{q}^{\theta-1})}{\theta-1}.$$

The labor market clearing condition can be rewritten as:

$$1 = \frac{1}{\mu \mathcal{W}} + \frac{(n+d) \mathcal{M} c_D \gamma^{1+\zeta}}{(n+\epsilon)(1+\zeta)} + c_R \mathcal{M}^{1+\phi} (n+d).$$

The HJB equation for the value of a new firm can be rewritten as:

$$\mathcal{V}^N = \frac{[\rho + \epsilon + (\theta - 1)g^Q](1 - \tau^C)}{\theta \mathcal{M}[\rho + (\theta - 1)g^Q][\rho + \epsilon - (n + \epsilon)(1 - \underline{q}^{\theta-1})]} + \frac{(1 - \tau^D)\mathcal{W}c_D\gamma^{1+\zeta}}{(1 + \zeta)[\rho + \epsilon - (n + \epsilon)(1 - \underline{q}^{\theta-1})]}.$$

A.2 The Constrained-Optimal Allocation

Notation. We introduce the following notation to define the inner product between two square-integrable functions $f(x), g(x) : \Omega \rightarrow \mathbb{R}$ over their common domain:

$$\langle f(x), g(x) \rangle_{x \in \Omega} \equiv \int_{\Omega} f(x)g(x)dx.$$

Second, let us denote the (partial) Gateaux derivative of a functional F with respect to the function $f(x)$ in direction $\tau(x)$ as:

$$\delta F[f(x); \tau(x)] \equiv \left. \frac{\partial F[f(x) + \varepsilon \cdot \tau(x), \cdot]}{\partial \varepsilon} \right|_{\varepsilon=0}$$

where the functional F can take additional arguments through the “dot” notation and the “test” function $\tau(x)$ is assumed to vanish on the boundaries of the relevant integration domain.

The Planner’s Problem. Consider the problem of a planner seeking to maximize the following objective:

$$U_0 = \int_0^\infty e^{-(\rho-n)t} \ln(c_t) dt$$

subject to the constraints:⁶

$$\begin{aligned} Y_t &= [\sum_{T \in \{N, O\}} \langle (q y_t^T(q))^{\frac{\theta-1}{\theta}}, m_t^T(q) \rangle_{q \in [\underline{q}_t, \infty)}]^{\frac{\theta}{\theta-1}}, \\ N_t &\geq \sum_{T \in \{N, O\}} \langle y_t^T(q), m_t^T(q) \rangle_{q \in [\underline{q}_t, \infty)}, \\ Y_t &\geq C_t + R_t + \langle d_t(q), m_t^N(q) \rangle_{q \in [\underline{q}_t, \infty)}, \\ \dot{m}_t^N(q) &= -\partial_q[\gamma_t(q)q m_t^N(q)] - \epsilon m_t^N(q) + \delta(q - \underline{q}_t)R_t / (c_R Q_t M_t^{\frac{1}{\theta-1}}), \\ \dot{m}_t^O(q) &= \epsilon m_t^N(q) \end{aligned}$$

by choosing $\{\{y_t^T(q)\}_{T \in \{N, O\}}, d_t(q)\}_{q=\underline{q}_t}^\infty, R_t\}_{t=0}^\infty$. The solution to the planner’s problem is “constrained-optimal” in the sense that the planner takes externalities across firms

⁶Here, we substituted the product resource constraints in the labor resource constraint.

(technology spillovers) as given. More precisely, the planner takes M_t and Q_t as given in the research and development technologies. Reformulating this problem using the current-value Hamiltonian, we obtain:

$$\begin{aligned}\mathcal{H}_t = & \ln(c_t) + \nu_t^L [N_t - \sum_{T \in \{N, O\}} \langle y_t^T(q), m_t^T(q) \rangle_{q \in [\underline{q}_t, \infty)}] \\ & + \nu_t^Y \{ [\sum_{T \in \{N, O\}} \langle (q y_t^T(q))^{\frac{\theta-1}{\theta}}, m_t^T(q) \rangle_{q \in [\underline{q}_t, \infty)}]^{\frac{\theta}{\theta-1}} - c_t N_t - R_t - \langle d_t(q), m_t^N(q) \rangle_{q \in [\underline{q}_t, \infty)} \} \\ & - \langle \nu_t^N(q), \partial_q [\gamma_t(q) q m_t^N(q)] \rangle_{q \in [\underline{q}_t, \infty)} + \epsilon \langle \nu_t^O(q) - \nu_t^N(q), m_t^N(q) \rangle_{q \in [\underline{q}_t, \infty)} \\ & + \nu_t^N(\underline{q}_t) R_t / (c_R Q_t M_t^{\frac{1}{\theta-1}})\end{aligned}$$

where $\{\nu_t^L, \nu_t^Y, \{\nu_t^N(q), \nu_t^O(q)\}_{q=\underline{q}_t}^\infty\}_{t=0}^\infty$ are the costate functions. Using integration by parts, we can rewrite:

$$\begin{aligned}\mathcal{H}_t = & \ln(c_t) + \nu_t^L [N_t - \sum_{T \in \{N, O\}} \langle y_t^T(q), m_t^T(q) \rangle_{q \in [\underline{q}_t, \infty)}] \\ & + \nu_t^Y \{ [\sum_{T \in \{N, O\}} \langle (q y_t^T(q))^{\frac{\theta-1}{\theta}}, m_t^T(q) \rangle_{q \in [\underline{q}_t, \infty)}]^{\frac{\theta}{\theta-1}} - c_t N_t - R_t - \langle d_t(q), m_t^N(q) \rangle_{q \in [\underline{q}_t, \infty)} \} \\ & + \langle \gamma_t(q) q \partial_q \nu_t^N(q), m_t^N(q) \rangle_{q \in [\underline{q}_t, \infty)} + \epsilon \langle \nu_t^O(q) - \nu_t^N(q), m_t^N(q) \rangle_{q \in [\underline{q}_t, \infty)} \\ & + \nu_t^N(\underline{q}_t) R_t / (c_R Q_t M_t^{\frac{1}{\theta-1}}).\end{aligned}$$

The first-order condition with respect to $y_t^T(q)$ implies:

$$\nu_t^Y \delta Y_t[y_t^T(q); \varrho(q)] = \nu_t^L \langle m_t^T(q), \varrho(q) \rangle_{q \in [\underline{q}_t, \infty)}$$

where $\delta Y_t[y_t^T(q); \varrho(q)]$ is the Gateaux derivative of Y_t with respect to $y_t^T(q)$ in direction $\varrho(q)$, which is an arbitrary function that vanishes on the boundaries of $[\underline{q}_t, \infty)$:

$$\delta Y_t[y_t^T(q); \varrho(q)] = Y_t^{1/\theta} \langle q^{\frac{\theta-1}{\theta}} y_t^T(q)^{-1/\theta} m_t^T(q), \varrho(q) \rangle_{q \in [\underline{q}_t, \infty)}.$$

Since that first-order condition must hold for any function $\varrho(q)$, we obtain the relative demand functions:

$$y_t^T(q) / Y_t = q^{\theta-1} (\nu_t^Y / \nu_t^L)^\theta, \quad \forall q \in [\underline{q}_t, \infty).$$

Integrating this expression, we find:

$$\nu_t^L = \nu_t^Y M_t^{\frac{1}{\theta-1}} Q_t$$

such that we can rewrite:

$$y_t^T(q) = \frac{v_t^Y Y_t(q/Q_t)^{\theta-1}}{v_t^L M_t}, \quad \forall q \in [\underline{q}_t, \infty).$$

The first-order condition with respect to $d_t(q)$ implies:

$$\langle \gamma_t(q) q \partial_q v_t^N(q) m_t^N(q) / d_t(q), \varrho(q) \rangle_{q \in [\underline{q}_t, \infty)} / (1 + \zeta) = v_t^Y \langle m_t^N(q), \varrho(q) \rangle_{q \in [\underline{q}_t, \infty)}.$$

Since that first-order condition must hold for any function $\varrho(q)$, we obtain:

$$\gamma_t(q) = \left[\frac{q \partial_q v_t^N(q)}{v_t^Y c_D Q_t N_t^{\frac{1}{\theta-1}} (q/Q_t)^{\theta-1}} \right]^{1/\zeta}, \quad \forall q \in [\underline{q}_t, \infty).$$

The first-order condition with respect to R_t implies:

$$v_t^Y c_R Q_t M_t^{\frac{1}{\theta-1}} = v_t^N(\underline{q}_t).$$

The first-order condition with respect to $m_t^N(q)$ implies:

$$\begin{aligned} \langle (\rho - n) v_t^N(q) - \dot{v}_t^N(q), \varrho(q) \rangle_{q \in [\underline{q}_t, \infty)} &= \langle (q/Q_t)^{\theta-1}, \varrho(q) \rangle_{q \in [\underline{q}_t, \infty)} v_t^Y Y_t / [(\theta - 1) M_t] \\ &+ \langle \gamma_t(q) q \partial_q v_t^N(q) - v_t^Y d_t(q) + \epsilon[v_t^O(q) - v_t^N(q)], \varrho(q) \rangle_{q \in [\underline{q}_t, \infty)}. \end{aligned}$$

Since this condition must hold for any function $\varrho(q)$, we have:

$$\begin{aligned} (\rho - n) v_t^N(q) - \dot{v}_t^N(q) &= (q/Q_t)^{\theta-1} v_t^Y Y_t / [(\theta - 1) M_t] - v_t^Y d_t(q) \\ &+ \gamma_t(q) q \partial_q v_t^N(q) + \epsilon[v_t^O(q) - v_t^N(q)], \quad \forall q \in [\underline{q}_t, \infty). \end{aligned}$$

Similarly, the first-order condition with respect to $m_t^O(q)$ implies:

$$(\rho - n) v_t^O(q) - \dot{v}_t^O(q) = (q/Q_t)^{\theta-1} v_t^Y Y_t / [(\theta - 1) M_t], \quad \forall q \in [\underline{q}_t, \infty).$$

Defining the following functions:

$$V_t^{N*}(q) \equiv v_t^N(q) / (\mu v_t^Y), \quad V_t^{O*}(q) \equiv v_t^O(q) / (\mu v_t^Y), \quad w_t^* \equiv v_t^L / (\mu v_t^Y), \quad r_t^* \equiv \dot{c}_t / c_t + \rho,$$

and substituting them in the new firm's social HJB equation, we obtain:

$$\begin{aligned} r_t^* V_t^{N*}(q) - \dot{V}_t^{N*}(q) &= (q/Q_t)^{\theta-1} Y_t / (\theta M_t) - d_t(q) / \mu \\ &+ \gamma_t(q) q \partial_q V_t^{N*}(q) + \epsilon [V_t^{O*}(q) - V_t^{N*}(q)] \end{aligned}$$

where $\gamma_t(q)$ is given by:

$$\gamma_t(q) = \left[\frac{\mu q \partial_q V_t^{N*}(q)}{c_D Q_t N_t^{\frac{1}{\theta-1}} (q/Q_t)^{\theta-1}} \right]^{1/\zeta}.$$

Doing so for the old firm's social HJB equation, we obtain:

$$r_t^* V_t^{O*}(q) - \dot{V}_t^{O*}(q) = (q/Q_t)^{\theta-1} Y_t / (\theta M_t).$$

Finally, substituting these functions in the social free-entry condition and the labor resource constraint, we obtain:

$$\frac{c_R Q_t M_t^{\frac{1}{\theta-1}}}{\mu} = V_t^{N*}(q_t) \quad \text{and} \quad N_t = \frac{Y_t}{\mu w_t^*}.$$

This demonstrates that the constrained-optimal allocation can be implemented with research and development subsidies of $\tau^R = \tau^E = 1/\theta$.

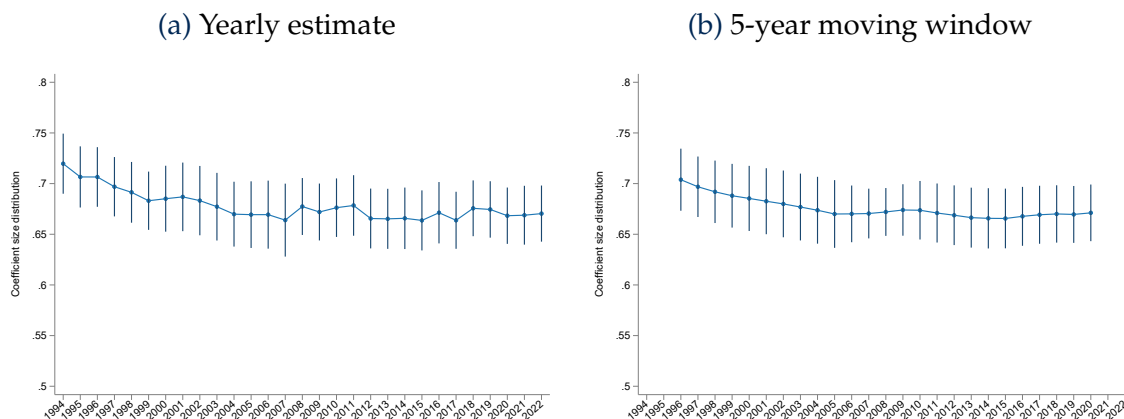
B Motivating Facts

Figure B1: Average size of firms



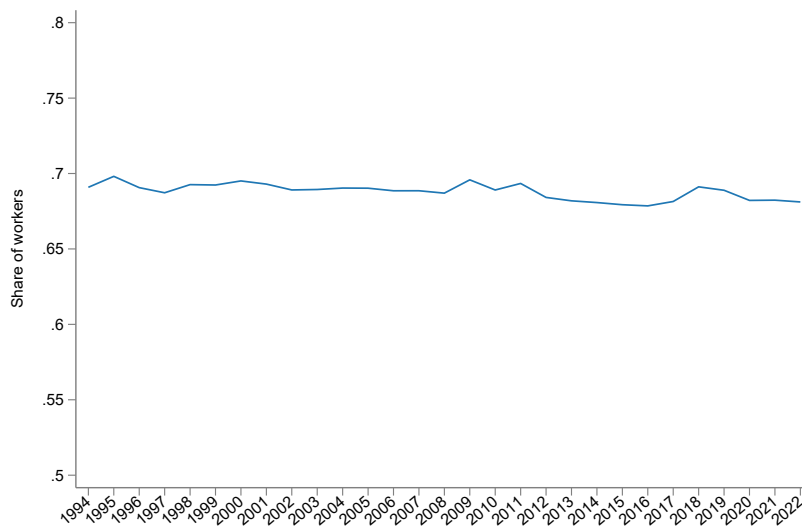
Note: Average size of firms is measured using a count of full time equivalent number of employees. Production workers are defined by their occupation category (*“Professions et Catégories Socioprofessionnelles (PCS)”* starting with 23, 31, 35, 37, 38, 47, 48, 55, 62, 63, 64, 65, 66, 67, 68 or 69). Private business firms (*“catégorie juridique”* starting with 5) based in mainland France. Source: BTS-Postes.

Figure B2: Slope of the size distribution



Note: Coefficients and 95% confident intervals from estimating the correlation between the logarithm of the rank and the logarithm of the employment shares of firms. Correlation are obtained from estimating a linear model with the OLS with 5-digit sector fixed effects. Standard errors are clustered by 2-digit sector. Rank and employment share are measured every year in a given 2-digit sector level. 5-year moving window estimate the model by including all observations from $t - 2$ to $t + 2$. The estimation implement the correction from Gabaix and Ibragimov (2011). Source: BTS-Postes.

Figure B3: Share of production workers



Note: Unweighted average share of production workers at the firm level. The number of production and other workers are measured in full time equivalent. Production workers are defined by their occupation category (*"Professions et Catégories Socioprofessionnelles (PCS)"* starting with 23, 31, 35, 37, 38, 47, 48, 55, 62, 63, 64, 65, 66, 67, 68 or 69). Private business firms (*"catégorie juridique"* starting with 5) based in mainland France. Source: BTS-Postes.