Resetting the Innovation Clock: Endogenous Growth through Technological Turnover*

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Abstract

We propose a model of endogenous economic growth with "weak" scale effects and diminishing returns to innovation at the micro level. In our model, entrants introduce new technologies through *research* and incumbents incrementally improve them through *development*. Over time, further improvement becomes harder such that firms ultimately run out of ideas and exit, paving the way for entrants that discover new technologies with further room for improvement. This turnover gives rise to a continuous stream of (temporary) opportunities for technological improvements that sustain economic growth. In a stationary equilibrium, the growth rate is constant and *endogenous* to market incentives.

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1 Introduction

In this paper, we propose a model of innovation-driven growth that jointly replicates two empirical regularities that prior endogenous-growth frameworks have struggled to capture. First, economic growth has not accelerated despite sustained population growth (Jones, 1995). Second, within any specific technological field, discovering better ideas becomes increasingly difficult (Bloom, Jones, Van Reenen and Webb, 2020). However, even as new ideas become harder to find at both the macro and micro levels, our model delivers a constant rate of long-run economic growth that is endogenous to market incentives, particularly tax and R&D policy (Akcigit, Grigsby, Nicholas and Stantcheva, 2021; Dechezleprêtre, Einiö, Martin, Nguyen and Van Reenen, 2023).

Our model builds on the framework of Aghion and Howitt (1996) by distinguishing between two types of innovation: *research* and *development*. We define research as the discovery of new products, while development consists of incremental improvements to existing ones. The evolution of the camera industry illustrates this distinction clearly. Early film cameras were *developed* from simple box models into sophisticated single-lens reflex (SLR) systems with interchangeable lenses and precise electronic controls. However, as improvement opportunities in film-based imaging were exhausted, further progress required a fundamental shift. Ultimately, *research* delivered a new product: the digital camera, which opened up new avenues for subsequent development (e.g., sensor technology, image processing, and software-driven features).

The central mechanism of our model operates through the interplay between these two types of innovation. Development is conducted by incumbent firms seeking to improve their existing product lines. However, this process is subject to diminishing returns; as a product is refined, finding further improvements becomes increasingly difficult. Eventually, these development opportunities are exhausted, leading to the technological stagnation and eventual exit of incumbent firms. Their exit opens the door to potential entrants, which can introduce new products through research, effectively "resetting the innovation clock" by unlocking a new wave of development opportunities. This dynamic of creative destruction, driven by the lifecycle of technologies from research to development (and eventual obsolescence), generates a continuous stream of temporary innovation opportunities that sustain long-run economic growth.

The cycle of research and development is also what allows our model to reconcile *endogenous* economic growth with the two aforementioned empirical facts. Diminishing returns in development make ideas progressively harder to find along any product line and ultimately lead to their technological stagnation. However, the entry of new products through research continuously replenishes the pool of development opportunities. R&D policy can influence the pace of development for these new products, therefore affecting the growth rate of the economy in the long run. Finally, to rule out "strong scale effects", we allow for free entry into research as in second-generation endogenous growth models (Peretto, 1998; Dinopoulos and Thompson, 1998; Young, 1998; Segerstrom, 1998; Howitt, 1999). This ensures that the number of product lines scales with the population, keeping the ratio of developers per line constant.

Our model relies on two key assumptions that are empirically motivated: namely, that new entrants build on the shoulders of giants and that entry costs rise with technical progress. By "shoulders of giants" we mean that entrants inherit the prevailing state of technology, such that the initial quality of a new product scales with the contemporaneous average quality across products. This assumption is motivated by evidence that most measured aggregate U.S. productivity growth reflects improvements on existing products by incumbent firms rather than the net creation of new products (Garcia-Macia, Hsieh and Klenow, 2019). Without the "building on the shoulders of giants" effect, average quality growth would stall due to the inevitable exhaustion of development opportunities within a given product line, and our model would revert to a semi-endogenous model where growth is exclusively fueled by the creation of new products.

The second assumption, that the flow cost of creating a new product is tied to product-level profits (and thus to productivity), is crucial to neutralize strong scale effects. If entry costs rose faster than product-level profits, entry would fall, and the number of developers per line would rise over time due to population growth. This would lead to explosive growth as in first-generation endogenous growth models.[1] Denominating entry costs as constant in units of labor ensures that they scale with product-level profits, and that the number of product lines scales with the population. This is consistent with U.S. evidence that firm-level revenues increase with labor productivity (Klenow and Li, 2024), and that the number of firms scales with population while R&D employment per firm remains stable (Laincz and Peretto, 2006). We elaborate on these assumptions and the supporting evidence in Section 3.

With these assumptions, our model admits a tractable balanced growth path where the firm size distribution has an exact Pareto tail and the growth rate of consumption per capita is constant and, crucially, endogenous. In particular, economic growth is fueled by two forces: a semi-endogenous "love of variety" component tied to population growth and a product quality improvement component driven by development, as in Peters and Walsh (2021). Importantly, if population growth falls to zero, our model still

[1] Another way to sterilize this strong scale effect is to assume that vertical innovation faces "severely" diminishing returns to research labor as in Trammell (2025).

delivers growth in consumption per capita due to improvements in product quality. We show how different policies affect the latter. Subsidies for development directly increase the pace of quality improvements and raise long-run productivity growth. In contrast, subsidies for research, while encouraging entry, intensify competition and erode incumbents' development incentives, leading to a negative overall effect on growth (Peretto, 1998).

To assess the quantitative implications of our framework, we calibrate the model to match key features of the U.S. economy. We demonstrate that our deliberately parsimonious model can simultaneously replicate several establishment-level moments from the U.S. Business Dynamics Statistics, including the exit rate, the average establishment size, and the shape of the establishment size distribution. This simple calibration exercise allows us to quantify the impact of various policies. For instance, we find that a 10% subsidy to development expenditures raises the long-run growth rate of per-capita consumption by nearly 50 basis points.[2] However, the transition to this new steady state is gradual, with the growth rate initially slowing down due to the reallocation of labor away from production and research (entry).

The remainder of the paper is organized as follows. The following section reviews the relevant literature. Section 2 presents our model and its aggregate implications. Section 3 discusses its key assumptions. Section 4 presents a simple calibration of the model to illustrate the effects of different policies. Finally, Section 5 concludes. Throughout, we intentionally adopt a parsimonious specification to keep the mechanisms transparent and tractable; richer variants (stochastic innovation, alternative demand or exit, head-to-head competition, etc.) are discussed in Section 3.3.

Literature review

This paper makes two contributions. First, it speaks to the long-standing discussion on the future of technological progress, contested by techno-pessimists like Gordon (2017), who argue that transformative innovations are behind us, and optimists like Mokyr (2014), who see recent technological revolutions (e.g., IT and more recently AI) as sources of potentially unprecedented future growth. Our model offers a synthesis: it formally incorporates the view that ideas become harder to find within any single technological paradigm, yet it shows how the continual discovery of new paradigms can sustain aggregate economic growth indefinitely.

[2] While our model is intentionally parsimonious and the magnitude of this effect should be interpreted with caution, it is broadly consistent with growth effect estimates reported elsewhere in the literature, such as Acemoglu, Akcigit, Alp, Bloom and Kerr (2018) and Aghion, Bergeaud, Boppart, Klenow and Li (2025).

Second, our work contributes to the debate between *endogenous* and *semi-endogenous* growth paradigms. This debate has persisted since the mid-1990s, largely because of a seemingly insurmountable empirical challenge; convincingly discriminating whether market incentives (e.g., market size or R&D policies) have a causal effect on *long-run* productivity growth or level when transition dynamics can be slow. Consequently, the discussion has advanced by developing theoretical models from both sides and testing them against other, more measurable empirical moments.

The first generation of endogenous growth models (e.g., Romer (1990) and Aghion and Howitt (1992)) developed the framework through which market incentives could shape long-run economic growth. However, in these models, the population had to remain constant to prevent explosive growth, which ran counter to empirical evidence. In response, the semi-endogenous growth model of Jones (1995) eliminated this "strong scale effect" by assuming that innovation efficiency declines with the aggregate stock of knowledge. This solution, however, came at the cost of rendering long-run productivity growth unresponsive to market incentives.

A "second generation" of models subsequently sought to break the strong scale effect while preserving a role for market incentives (e.g., Peretto (1998), Dinopoulos and Thompson (1998), Young (1998), Segerstrom (1998), and Howitt (1999)). This was achieved by allowing the number of firms to grow in proportion to the population (through free entry), keeping the ratio of researchers per firm constant and delivering constant growth at the firm level. However, recent evidence from Bloom et al. (2020) directly challenges the assumption that a constant number of researchers per firm can deliver constant firm-level growth. They show across multiple contexts that innovation efficiency is instead consistently declining at the firm level.

Our contribution to this debate is to propose a model that reconciles this recent empirical evidence with the desirable predictions of the second wave of endogenous growth models: long-run economic growth is responsive to market incentives, yet constant even as the population expands. In our model, this reconciliation is achieved by the endogenous arrival of technological breakthroughs (i.e., new products, firms, or industries) that continuously "reset the innovation clock."

2 A new growth model

In this section, we present an intentionally simple model that captures the essence of our argument: firms invest in the *development* of existing products to improve their quality, but eventually run out of ideas and exit. Simultaneously, investments in *research* lead to the entry of new firms with entirely new product lines. As a consequence, quality-upgrading opportunities are continuously replenished in aggregate even though they are ultimately exhausted on any existing lines. We keep the environment intentionally lean to isolate the core mechanism; richer variants are discussed in Section 3.3.

2.1 The economic environment

Preferences

Consider a continuous-time economy where time is indexed by $t \in [0, \infty)$. This economy is populated by a representative household of measure N_t evolving according to:

$$\dot{N}_t = n \cdot N_t, \tag{1}$$

where $n \geq 0$. The household inelastically supplies one unit of labor at every point in time, and has logarithmic preferences over per-capita consumption c_t such that lifetime utility is defined as:

$$U_0 = \int_0^{\infty} e^{-(\rho - n)t} \ln(c_t) \mathrm{d}t. \tag{2}$$

Here, $\rho > n$ denotes the rate of time preference. The individual consumption basket is a Dixit and Stiglitz (1977) aggregator of differentiated products indexed by $i \in \mathcal{I}_t$:

$$c_t = \left(\int_{i \in \mathcal{I}_t} (q_{it} \cdot c_{it})^{\frac{\theta - 1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta - 1}}, \tag{3}$$

where c_{it} is the consumed quantity of product i (per capita), $q_{it} > 0$ is the quality of that product, $\theta > 1$ is the elasticity of substitution between products, $\mathcal{I}_t$ is the set of available products, and its (endogenous) cardinality is denoted by $M_t \equiv |\mathcal{I}_t|$.

Each of those products is produced by a single firm using production labor according to the linear technology:

$$y_{it} = l_{it}^{P}, \tag{4}$$

where y_{it} denotes the quantity of product i supplied at time t by the firm and l_{it}^{P} denotes the quantity of labor used in production.

Over time, a firm can incrementally improve the quality of its product by directing labor towards development. More precisely, a product's quality evolves according to the following controlled process:

$$\dot{q}_{it} = \gamma_{it} \cdot q_{it}, \tag{5}$$

where γ_{it} is the proportional drift of the product's quality. The labor requirement l_{it}^D to achieve this quality drift is given by:

$$l_{it}^D = \frac{c_D (q_{it}/Q_t)^{\theta-1} \gamma_{it}^{1+\zeta}}{1+\zeta} \tag{6}$$

where $c_D > 0$ determines the scale of the development cost function, $\zeta > 0$ measures its elasticity, and Q_t is an average quality index defined as:

$$Q_t \equiv \left(M_t^{-1} \int_{i \in \mathcal{I}_t} q_{it}^{\theta-1} \mathrm{d}i \right)^{\frac{1}{\theta-1}}.$$

The development technology is such that there is a finite upper bound $\overline{\gamma} > 0$ on the drift of product quality to ensure that the optimal allocation does not involve any corner solutions as described in Trammell (2025).[3,4]

However, at Poisson rate $\epsilon > 0$, a firm may receive an idiosyncratic "obsolescence" shock after which it can no longer improve its product's quality. This is an extreme and stylized case of "ideas becoming harder to find" (Bloom et al., 2020) in which innovation efficiency literally falls to zero once the shock hits. After being hit by this shock, the product quality eventually falls below a certain threshold $\underline{q}_t = \underline{q} \cdot Q_t$ where $\underline{q} \in (0, 1)$, and the firm exogenously exits the market.

In every point in time, a unit measure of potential entrants attempt to discover through research products that are entirely new to society. Specifically, these entrants can direct c_R units of labor to research in order to invent a unit flow of these new products. Once a product is discovered, its initial quality is drawn from a point mass at the lower bound $\underline{q}_t$ of the product quality support.

Labor supplied by the household can be allocated to either production, development, or research, delivering the labor market clearing condition:

$$L_t^P + L_t^D + L_t^R \leq N_t, \tag{7}$$

where these aggregate labor allocations are defined as:

$$L_t^P \equiv \int_{i \in \mathcal{I}_t} l_{it}^P \mathrm{d}i, \quad L_t^D \equiv \int_{i \in \mathcal{I}_t} l_{it}^D \mathrm{d}i, \quad \text{and} \quad L_t^R \equiv c_R R_t$$

and where R_t is the flow of new products discovered at time t. Finally, the resource

[3] We thank Phillip Trammell and Chad Jones for pointing this out to us.

[4] We choose the value of $\overline{\gamma}$ such that this constraint does not bind in equilibrium. This delivers a restriction on parameters such that $\overline{\gamma}$ must be greater than the corresponding expression in equation (11).

constraints for each product are given by:

$$c_{it}N_t \leq y_{it}, \ \ \forall i \in \mathcal{I}_t. \tag{8}$$

2.2 The decision problems

In this section, we define the decision problems of each economic agent. In terms of market structure, we assume that all agents partake in perfect competition in all markets besides firms who engage in monopolistic competition.

The household's problem

Taking prices as given, the household's problem is to choose its consumption of each product to maximize lifetime utility:

$$\max_{\{\{c_{it}\}_{i\in\mathcal{I}_t}\}_{t\geq 0}} \int_0^\infty e^{-(\rho-n)t}\ln(c_t)\mathrm{d}t \quad \text{s.t.} \quad c_t = \left(\int_{i\in\mathcal{I}_t}(q_{it}\cdot c_{it})^{\frac{\theta-1}{\theta}}\mathrm{d}i\right)^{\frac{\theta}{\theta-1}}$$

subject to the flow budget constraint:

$$\dot{a}_t + \int_{i\in\mathcal{I}_t} p_{it}c_{it}\mathrm{d}i \leq (r_t - n)a_t + w_t - T_t.$$

Here, p_{it} is the price of product i, w_t is the wage rate, T_t are lump-sum per-capita taxes, a_t is the value of corporate assets per capita, and r_t is the rate of return on those assets:

$$a_tN_t = \int_{i\in\mathcal{I}_t} V_{it}\mathrm{d}i \quad \text{where} \quad \lim_{t\to\infty} e^{-\int_0^t(r_{t'}-n)\mathrm{d}t'}a_t = 0,$$

and where V_{it} denotes the value of product i at time t. This problem thus delivers the usual intertemporal Euler equation:

$$\dot{c}_t = (r_t - \rho)c_t$$

and the following demand functions:

$$c_{it} = (P_t/p_{it})^\theta q_{it}^{\theta-1}c_t. \tag{9}$$

Since aggregate consumption is chosen as the numéraire, the price index P_t is normalized to one for all t:

$$P_t \equiv \left(\int_{i \in \mathcal{I}_t} (p_{it}/q_{it})^{1-\theta} \mathrm{d}i \right)^{\frac{1}{1-\theta}} = 1.$$

The firm's problem

Firms engage in monopolistic competition in the product market but perfect competition in the labor market, taking the wage and the demand for their product as given. A firm thus chooses the price at which to sell its product and its labor demands to maximize the expected present discounted value of its profits.

From this point on, we abandon the i-index notation since a firm is entirely described by its product's quality q and its obsolescence status. We denote the latter by $S \in \{O, N\}$ where O represents an "old" firm that has received the obsolescence shock, and N represents a "new" firm that has not. The new firm's value function satisfies a standard Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} r_t V_t^N(q) = \max_{\mathbf{u}_t^N(q) \geq \mathbf{0}} \{ & (1-\tau^C)[p_t(q) y_t(q) - w_t l_t^P(q)] - (1-\tau^D) w_t l_t^D(q) \\ & + \gamma_t(q) q \partial_q V_t^N(q) \} + \epsilon [V_t^O(q) - V_t^N(q)] + \dot{V}_t^N(q) \end{aligned}$$

subject to the production technology in equation (4), the development technology in equation (6), the demand function in equation (9), and the product resource constraint in equation (8). Here, $\mathbf{u}_t^N(q) \equiv \{p_t(q), l_t^P(q), l_t^D(q)\}$ is the vector of control variables, $\tau^C \geq 0$ is the corporate income tax rate, and $\tau^D \geq 0$ is a subsidy on the firm's development expenditures. Similarly, the "old" firm's value function satisfies the HJB equation:

$$r_t V_t^O(q) = \max_{\mathbf{u}_t^O(q) \geq \mathbf{0}} \{ (1-\tau^C)[p_t(q) y_t(q) - w_t l_t^P(q)] \} + \dot{V}_t^O(q)$$

subject to the production technology in equation (4), the demand function in equation (9), the product resource constraint in equation (8), and where $\mathbf{u}_t^O(q) \equiv \{p_t(q), l_t^P(q)\}$. The profit-maximization problem implies that a firm sets its price at a constant markup above marginal cost irrespective of its obsolescence status:

$$p_t(q) = \mu \cdot w_t, \ \ \forall q, \quad \text{where} \quad \mu \equiv \frac{\theta}{\theta - 1}.$$

Hence, a firm's flow profits can be expressed as:

$$\pi_t^S(q) = (1-\tau^C)(q/Q_t)^{\theta-1} C_t/(\theta M_t) - \mathbb{1}_{\{S=N\}}(1-\tau^D) w_t l_t^D(q)$$

where the optimal quality drift is

$$\gamma_t(q) = \left[\frac{q \partial_q V_t^N(q)}{(1-\tau^D) w_t c_D (q/Q_t)^{\theta-1}} \right]^{1/\zeta}.$$

Substituting this into equation (5) gives the optimal labor allocation to development.

The entrant's problem

Entrants engage in perfect competition on the labor market and, thus, choose research labor L_t^R to maximize future expected profits while taking the wage rate as given:

$$V_t^E = \max_{L_t^R} \left\{ V_t^N(\underline{q}_t) L_t^R / c_R - (1-\tau^R) w_t L_t^R \right\}$$

where $\tau^R > 0$ is a research subsidy. The first-order condition of the entrant's problem delivers what will be referred to as the free-entry condition:

$$V_t^N(\underline{q}_t) = (1-\tau^R) w_t c_R.$$

2.3 The market equilibrium allocation

Having defined the decision problems of each economic agent, we can now define the concept of a market equilibrium allocation, and lay out the equations that determine the long-run equilibrium growth rate of the aggregate economy.

Definition 1. *Given the initial conditions $\{N_0, Q_0, \{m_0^N(q), m_0^O(q)_{q=\underline{q}_t}^{\infty}\}\}$, where $m_0^S(q)$ is the initial measure of type-S firms with product quality q, a market allocation consists of time paths for quantities, prices, and policy functions such that the following conditions hold:*

1. $\{\{c_t(q)\}_{q=\underline{q}_t}^{\infty}\}_{t\geq 0}$ *solve the household's problem.*
2. $\{\{p_t(q), l_t^P(q), l_t^D(q)\}_{q=\underline{q}_t}^{\infty}\}_{t\geq 0}$ *solve the firm's problem.*
3. $\{L_t^R\}_{t\geq 0}$ *solve the entrant's problem.*
4. $\{\{p_t(q)\}_{q=\underline{q}_t}^{\infty}\}_{t\geq 0}$ *clear the product markets.*
5. $\{w_t\}_{t\geq 0}$ *clear the labor market.*
6. $\{r_t\}_{t\geq 0}$ *clear the asset market.*

7. *The government's budget is balanced:* $T_t = \tau_R w_t c_R R_t + \tau^D w_t L_t^D - \tau^C C_t/\theta$.

The market allocation in this model admits a remarkably simple aggregation such that aggregate consumption is given by:

$$C_t = M_t^{\frac{1}{\theta-1}} Q_t L_t^P.$$

That is, aggregate consumption is increasing in the measure of products (owing to a taste for variety), the average quality across those products, and the labor input used in production. In Appendix A.1 we show that on a balanced growth path (BGP), the measure of products grows at the same rate as the population, the average quality index grows at a constant rate, and, thus, the growth rate of consumption per capita is constant and given by:

$$g = \frac{n}{\theta - 1} + g^Q. \tag{10}$$

We can derive an expression for the growth rate of the average quality index which is composed of two additive terms:

$$g^Q = \gamma \cdot \frac{n+d}{n+\epsilon} - \frac{n(1-\underline{q}^{\theta-1})}{\theta-1} \quad \text{where} \quad \gamma = \left[\frac{(\theta-1)(1-\tau^R)c_R}{(1-\tau^D)c_D \underline{q}^{\theta-1}}\right]^{1/\zeta} \tag{11}$$

and where $d = \epsilon g^Q/\gamma$ is the endogenous exit rate. The first is the product of the common product quality drift γ among firms that haven't yet received the obsolescence shock and the fraction of such firms in the economy given by $\frac{n+d}{n+\epsilon}$.[5] The second term reflects the negative growth contribution of net product entry occurring at rate n. Indeed, entering firms draw their initial product quality at the lower bound $\underline{q}_t$ of the quality support, which drags down the average quality index. This term is divided by $\theta - 1$ since with a higher substitution elasticity θ, varieties become closer substitutes and the Dixit and Stiglitz (1977) aggregator places less weight on the dispersion in quality across products. As a result, the dilution effect of low-quality entrants on the average quality index is weaker, whereas when products are more differentiated (low θ), each entrant commands more market share regardless of its quality, and the drag on aggregate quality is stronger.

Let us now consider the expression for γ, which is where market incentives and R&D policy have their bite. First, a subsidy (τ^D) to development expenditures directly increases γ by reducing the cost of improving a product's quality. In contrast, a research subsidy (τ^R) lowers γ as it encourages entry, intensifying competition, and eroding expected profits per firm. This, in turn, dampens firms' incentives to invest in develop-

[5] Note that it is an equilibrium outcome that the product quality drift γ is constant over time and common across all firms.

ment. Finally, γ is neutral to corporate income taxes (τ^C) in the long run. While higher corporate taxes reduce post-tax profits, they also make entry less attractive. Through free entry, the number of firms in the market endogenously adjusts until expected profits per firm are restored, as discussed in Peretto (1998).

The fraction $\frac{n+d}{n+\epsilon}$ of non-obsolete firms is itself endogenous. It is decreasing in the arrival rate ϵ of obsolescence shocks and increasing in the endogenous exit rate $d = \frac{\epsilon g^Q}{\gamma}$ of firms that have already received this shock. The exit rate is increasing in g^Q: faster growth of the average quality index intensifies the race against the exit threshold, so obsolete firms hit that threshold sooner. By contrast, it is decreasing in γ: faster product-quality growth raises the likelihood that firms upgrade enough to move away from the threshold. Substituting the expression for this exit rate in equation (11) and rearranging yields

$$g^Q = \left[\frac{(\theta-1)(1-\tau^R)c_R}{(1-\tau^D)c_D\underline{q}^{\theta-1}}\right]^{1/\zeta} - \frac{(n+\epsilon)(1-\underline{q}^{\theta-1})}{\theta-1},$$

which only depends on exogenous parameters and taxes. In particular, this expression makes clear that even with zero population growth ($n = 0$), the economy can sustain constant long-run growth in average quality, and hence in per-capita consumption (see (10)).

Finally, these R&D and obsolescence forces also pin down the cross-sectional distribution of product quality. Indeed, the stationary firm size distribution is Pareto with shape parameter λ given by:

$$\lambda \equiv \frac{1}{1-\underline{q}^{\theta-1}} > 1.$$

Therefore, this distribution displays a fatter right tail when the initial quality of entrants $\underline{q}$ is lower (close to zero). Conversely, when entrants are nearly as good as incumbents (i.e., $\underline{q}$ close to one), the distribution becomes thinner-tailed. This is because a lower initial quality of entrants implies that incumbents have more time to improve their products before facing comparable competition from new entrants, leading to a wider dispersion in product quality and, consequently, firm size.

3 Discussion

3.1 Modeling assumptions

The predictions of our model hinge on two important assumptions. First, we assume that entrants draw an initial level of product quality that is proportional to the contemporaneous average productivity index Q_t. This proportionality is what sustains quality-led growth on a BGP. Without it (i.e., if entrants started producing at a fixed level of product quality) average quality growth would eventually stall. Although individual incumbent firms might temporarily improve their products' quality, these gains would be lost when they exit. New entrants would consistently reset the quality level to the same starting point, preventing any cumulative progress. The model would then revert to a semi-endogenous growth framework where per-capita consumption growth would be fueled exclusively by the introduction of new products, through a taste for variety.

This first assumption makes our model consistent with evidence from Garcia-Macia et al. (2019) who find that about three quarters of productivity growth in the U.S. between 2003 and 2013 occurred through incumbents' improvements to their existing products rather than the introduction of entirely new products. Therefore, without this quality-scaling at entry, our model would attribute too much of long-run growth to net entry and too little to incremental improvements, contrary to their findings.

The second key assumption we make is that entry costs are constant in units of labor and therefore rise with the wage at the same rate as firm-level profits. This keeps the free-entry margin "balanced": if entry costs grew faster than firm-level profits, entry would be choked off, the firm count would lag population growth, and development labor per firm would rise over time, delivering explosive growth with strong scale effects. This outcome would be at odds with empirical evidence and with our objective of neutralizing strong scale effects. As in second-generation endogenous growth models, by tying entry costs to the wage, the number of firms can track population, and development effort per firm remains constant.

Two sets of facts support this assumption. Direct evidence in Klenow and Li (2024) indicates that revenue per firm in the U.S. increases with the level of productivity (both over time and across states). If higher revenues are associated with higher profits, entry costs must also rise with productivity to satisfy the free-entry condition. Therefore, they conclude that entry costs rise with growth, as in our model. Complementarily, Laincz and Peretto (2006) document that in the U.S. the number of firms scales with population and the number of R&D workers per firm is roughly stable over time. Together, these observations justify our treatment of entry costs and underscore why this assumption is central to ruling out strong scale effects in the model.

3.2 Constrained-optimal allocation

In Appendix A.2, we also pose the problem of a planner that maximizes social welfare while taking the incumbent-to-incumbent and incumbent-to-entrant technology spillovers as given. More precisely, the planner takes the average quality index Q_t, which appears in the research and development technologies, as given. Because the planner does not internalize such spillovers, we refer to the solution to this problem as *constrained*-optimal. Interestingly, we show that the market equilibrium allocation exactly coincides with this constrained-optimal allocation, echoing the findings of Dhingra and Morrow (2019).

3.3 Modeling choices and possible alternatives

Our baseline model is deliberately parsimonious. Here, we outline several directions in which the model could be extended without affecting its main insights. First, equation (5) specifies a deterministic process for product quality but one could instead allow for stochastic dynamics (e.g., Brownian or Poisson quality shocks), which would deliver additional cross-sectional dispersion. Second, equation (6) scales development costs by $(q_{it}/Q_t)^{\theta-1}$ to ensure Gibrat-type firm-level growth and tractability, but it is not a necessary condition for our main mechanism to operate. Third, we model technological obsolescence as a Poisson shock that shuts down development, but we could instead assume that development efficiency gradually declines with product quality. Fourth, we impose an exogenous exit threshold $\underline{q}_t = \underline{q} \cdot Q_t$, but we could instead replace it by an endogenous exit rule in the presence of overhead costs. Fifth, in the baseline model, entrants draw their initial quality at that exit threshold $\underline{q}_t$, but one could entertain an alternative learning process by new entrants (e.g., Yao 2024). These last two modifications would affect the contributions of entry and exit to productivity growth in equation (11), but would not otherwise affect our main results.

4 A simple calibration

In this section, we present a straightforward calibration of the parameters in our model to illustrate its mechanisms. Although this exercise is not meant as a rigorous empirical quantification for direct comparison with economic data, it provides insight into the potential magnitude of different forces.

We set the pure rate of time preference, ρ, to 0.04 and assume an annual population growth rate, n, of 1%. Consistent with Garcia-Macia et al. (2019), the elasticity of substitution across products, θ, is set to 4. We fix ζ, which captures the degree of decreasing returns to development labor, at 1, which aligns with the preferred value in Acemoglu et al. (2018).

Four additional parameters require calibration: the development cost parameter c_D, the research cost parameter c_R, the obsolescence shock arrival rate ϵ, and the initial relative quality of new products $\underline{q}$. Since these parameters are less conventional in the literature or not directly observable, we calibrate them by jointly matching a growth rate of per capita consumption of 2% per year, as well as the following three establishment-level moments, which we calculate from the U.S. Business Dynamics Statistics (BDS) between 2015 and 2019:[6]

1. The (Pareto) tail index of the establishment size distribution is 2.95.[7]

2. The establishment-level exit rate is 8.7%.

3. The average establishment has 18.1 employees.

The calibrated parameter values are reported in Table 1 and all four of the above empirical moments are matched exactly.

Table 1: Calibration

Parameter	Value	Source
ρ	0.04	Standard
n	0.01	Population growth
θ	4	Garcia-Macia et al. (2019)
ζ	1	Acemoglu et al. (2018)
c_D	$\exp(9.17)$	Per capita consumption growth of 2%
c_R	$\exp(4.52)$	Average establishment employment (U.S. BDS)
$\underline{q}$	0.87	Pareto tail index (U.S. BDS)
ϵ	0.23	Establishment exit rate (U.S. BDS)

While the parameters $\{c_D, c_R, \epsilon, \underline{q}\}$ are jointly calibrated to match the aforementioned moments, we provide some intuition for their identification. The development cost parameter c_D is primarily identified by the growth rate of per capita consumption, as it governs the pace at which firms improve their product's quality and, in turn, economic growth. The research cost parameter c_R is pinned down by the average establishment size, since it influences the cost of entry and thereby the equilibrium number of firms. The obsolescence shock arrival rate, ϵ, is identified from the establishment-level exit

[6] We use establishment- rather than firm-level data since every firm produces a single product in our model. We view a firm as a set of products and proxy products by establishments, following Garcia-Macia et al. (2019).

[7] The distribution of employment across U.S. establishments is better approximated by a lognormal distribution. Therefore, we fit a lognormal distribution and calculate the local Pareto tail index at 250 employees.

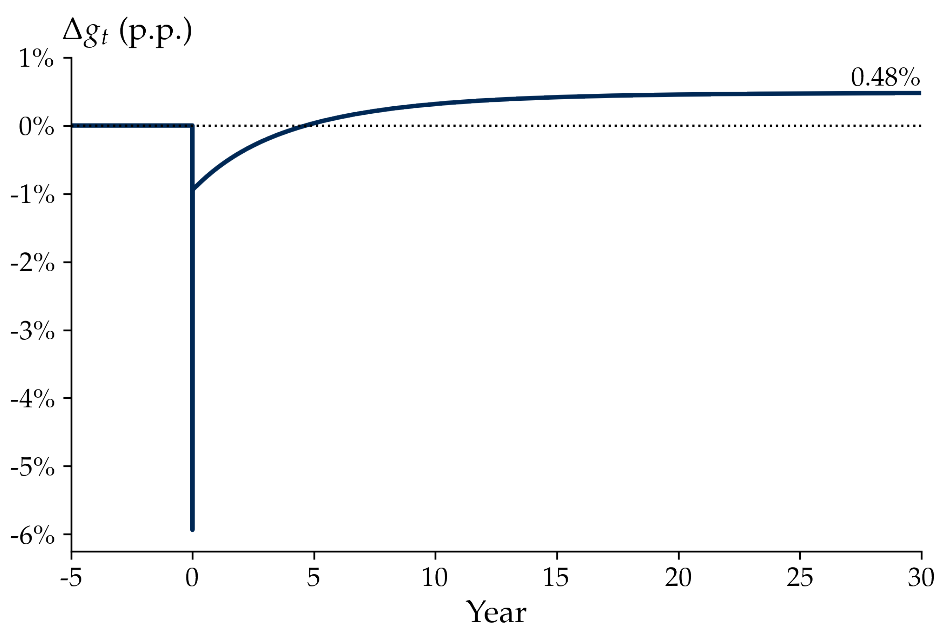
rate because it governs how quickly products become obsolete and ultimately exit the market. Finally, as discussed in Section 2.3, the initial relative quality of new products, $\underline{q}$, directly determines the Pareto tail index of the establishment size distribution.

4.1 Policy experiments

Using this calibration, we can conduct simple policy experiments to illustrate the effects of different R&D and tax policies. We consider three types of policies: a 10% subsidy to development expenditures (τ^D), a 10% subsidy to research expenditures (τ^R), and a 10% cut in the corporate income tax rate (τ^C).

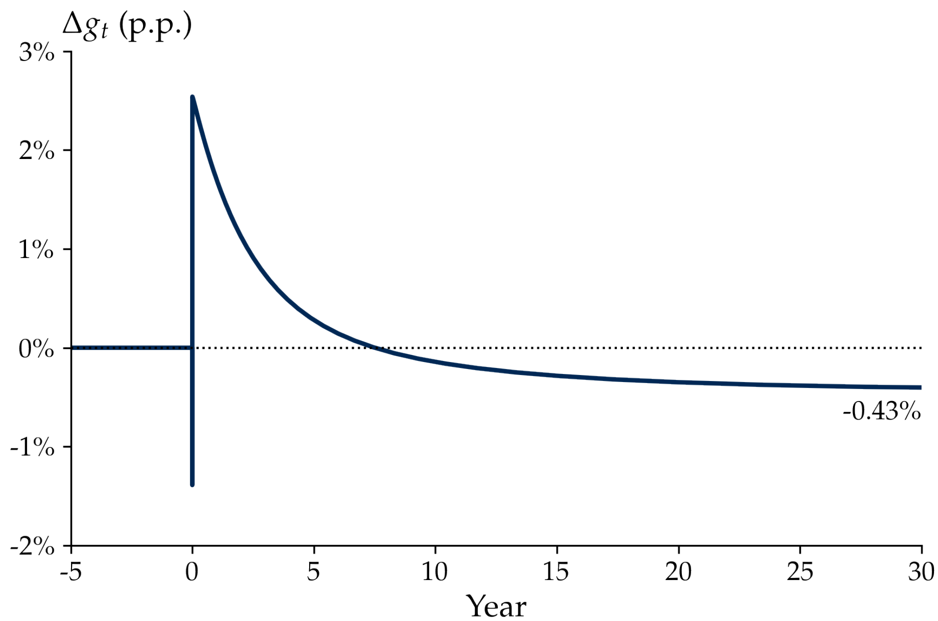
Development subsidy. Figure 1 shows the effect of a 10% subsidy to development expenditures on the growth rate of consumption per capita (in percentage points). This policy has a powerful positive effect on long-run growth, increasing it by almost 50 basis points. This is because it directly encourages firms to invest in improving their products, which raises the growth rate of consumption per capita through equation (10). However, this policy is costly in the first few years of its implementation, as it diverts labor away from production (which explains the growth dip of about 6 percentage points on impact) and research, which reduces the variety of products available to consumers.

Figure 1: 10% Development Subsidy



Research subsidy. Figure 2 reports the growth effect of a 10% research subsidy. This policy reduces long-run consumption per capita growth by roughly 43 basis points, as it promotes entry and competition, which erodes profits and discourages quality-improving development. In the short run, however, the policy increases consumption growth by expanding product variety, although the reallocation of labor away from production generates an instantaneous growth dip of about 1 percentage point. It is worth noting that our model abstracts from the possibility that research could deliver positive technological spillovers, in which case subsidies may enhance long-run growth.

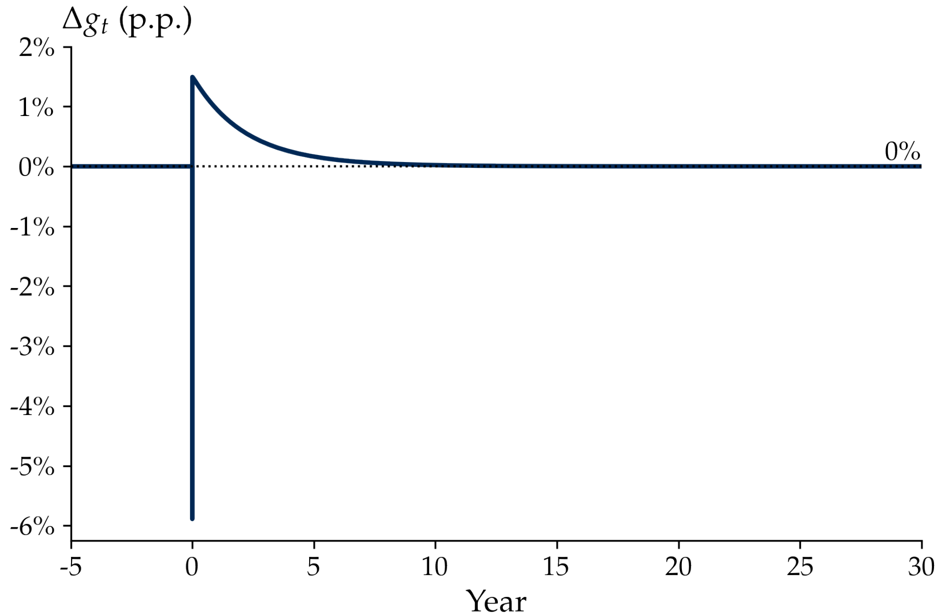
Figure 2: 10% Research Subsidy



Corporate income tax cut. Finally, Figure 3 shows the effect of a 10% corporate income tax cut on per capita consumption growth. In steady state, this policy has no effect since the equilibrium number of products adjusts to restore average profitability.[8] During the transition, however, higher after-tax profits encourage entry and expand the variety of products available to consumers. The impact effect again features a growth dip of around 6 percentage points, reflecting the reallocation of labor toward research and away from production.

[8]Note, however, that if entry costs were increasing in the number of products per capita (M_t/N_t), such a policy could increase long-run growth by discouraging entry and increasing the return to investments in development. In that sense, a small modification of our model could accommodate long-run market size effects on economic growth.

Figure 3: 10% Corporate Income Tax Cut



These experiments highlight the different transitional and long-run consequences of R&D and tax policies. They should be interpreted as stylized illustrations, and it is important to note that, in our model, the market equilibrium allocation is constrained-efficient. Thus, none of these interventions would improve welfare once we abstract from technological spillovers. As such, the welfare relevance of these policies ultimately depends on forces that lie outside the model's scope.

5 Conclusion

In this paper, we propose a new framework for endogenous economic growth that captures two key empirical facts: the absence of strong scale effects and the evidence that ideas are becoming harder to find at the micro level. Our model distinguishes between research (the creation of new products) and development (the improvement of existing products). Diminishing returns to development cause progress upon any given product to eventually stall. However, the continuous entry of new products via research "resets the innovation clock," creating a perpetual stream of new development opportunities that sustains aggregate growth.

This turnover-driven mechanism allows the model to generate a constant, endogenous rate of long-run economic growth while remaining consistent with the evidence of diminishing returns to innovation at both the macro and micro levels. Our calibration, disciplined by U.S. establishment-level data, demonstrates that policies directly targeting incumbent innovation, such as development subsidies, can have a powerful positive effect on long-run growth, whereas policies encouraging entry can backfire by intensifying product-market competition and weakening development incentives.

The implications of our framework are particularly important in light of the projected slowdown in global population growth. A standard semi-endogenous growth model, where per capita growth is entirely determined by population growth, would predict "the end of economic growth" as the latter falls to zero (Jones, 2022). Our model, by contrast, offers a more optimistic outlook. Because long-run growth is sustained by a policy-sensitive engine of quality growth insulated from population dynamics, our framework shows that sustained prosperity is possible even with a constant population. By providing a theory of endogenous growth that is consistent with the key empirical regularities of the U.S. economy, our model suggests that the future of economic growth need not be bleak.

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Appendix

A Theoretical appendix

This section of the appendix provides derivations and proofs for the results presented in the paper.

A.1 The market equilibrium allocation

Hamilton-Jacobi-Bellman equations. The old firm's value function satisfies the HJB equation:

$$r_t V_t^O(q) = (1-\tau^C)(q/Q_t)^{\theta-1} c_t N_t/(\theta M_t) + \dot{V}_t^O(q).$$

Defining $x_t \equiv \ln(q_t/Q_t)$, we can rewrite:

$$r_t V_t^O(x) = (1-\tau^C)\exp[(\theta-1)x] c_t N_t/(\theta M_t) - g_t^Q \partial_x V_t^O(x) + \dot{V}_t^O(x)$$

where $g_t^Q \equiv \dot{Q}_t/Q_t$ denotes the growth rate of the average quality index. Let us guess that this value function takes the following form:

$$V_t^O(x) = V_t^O \exp[(\theta-1)x].$$

Substituting this guess into the HJB equation, we obtain the following ordinary differential equation (ODE):

$$\dot{V}_t^O = [r_t + (\theta-1)g_t^Q]V_t^O - (1-\tau^C)c_t N_t/(\theta M_t),$$

which verifies our guess. The new firm's value function satisfies the HJB equation:

$$\begin{aligned}(r_t+\epsilon)V_t^N(q) = \max_{l_t^D(q)} \{ &(1-\tau^C)(q/Q_t)^{\theta-1} c_t N_t/(\theta M_t) - (1-\tau^D) w_t l_t^D(q) \\ &+ \gamma_t(q) q \partial_q V_t^N(q)\} + \epsilon V_t^O(q) + \dot{V}_t^N(q).\end{aligned}$$

Using the change of variable defined above, we can rewrite:

$$\begin{aligned}(r_t+\epsilon)V_t^N(x) = \max_{l_t^D(x)} \{ &(1-\tau^C)\exp[(\theta-1)x] c_t N_t/(\theta M_t) - (1-\tau^D) w_t l_t^D(x) \\ &+ [\gamma_t(x) - g_t^Q]\partial_x V_t^N(x)\} + \epsilon V_t^O(x) + \dot{V}_t^N(x).\end{aligned}$$

Let us guess that this value function takes the following form:

$$V_t^N(x) = V_t^N \exp[(\theta-1)x].$$

Substituting this guess into the optimal product quality drift, we obtain:

$$\gamma_t = \left[\frac{(\theta - 1)V_t^N}{(1 - \tau^D)w_t c_D} \right]^{1/\zeta}$$

which is independent of a product's quality. Substituting this result into the HJB equation of the new firm, we obtain the following ODE, which verifies our guess:

$$\dot{V}_t^N = [r_t + \epsilon - (\theta - 1)(\gamma_t - g_t^Q)]V_t^N - \epsilon V_t^O - \frac{(1 - \tau^C)c_t N_t}{\theta M_t} - \frac{(1 - \tau^D)w_t c_D \gamma_t^{1+\zeta}}{1 + \zeta}.$$

Fokker-Planck equations. The Fokker-Planck (FP) equations describing the evolution of the density of log relative quality among new and old firms are given by:

$$\dot{m}_t^N(x) = -(\gamma_t - g_t^Q)\partial_x m_t^N(x) - \epsilon m_t^N(x),$$
$$\dot{m}_t^O(x) = g_t^Q \partial_x m_t^O(x) + \epsilon m_t^N(x),$$

where $\delta(\cdot)$ denotes the Dirac delta function, $\underline{x} \equiv \ln(\underline{q}_t / Q_t)$, and we have the following boundary condition:

$$(\gamma_t - g_t^Q) \lim_{x \to \underline{x}} m_t^N(x) = \delta(x - \underline{x})R_t.$$

Therefore, the law of motion for the measure of new and old products are given by:

$$\dot{M}_t^N = R_t - \epsilon M_t^N \quad \text{and} \quad \dot{M}_t^O = \epsilon M_t^N - d_t M_t$$

where d_t denotes the exit rate at the lower bound of the quality support:

$$d_t \equiv g_t^Q \lim_{x \to \underline{x}} m_t^O(x) / M_t.$$

Hence, the total measure of products evolves according to:

$$\dot{M}_t = (e_t - d_t)M_t$$

where $e_t \equiv R_t / M_t$ denotes the entry rate.

Equilibrium conditions. Using our previous results, the free-entry condition can be rewritten as:

$$V_t^N \underline{q}^{\theta - 1} = (1 - \tau^R)w_t c_R$$

the labor market clearing condition can be rewritten as:

$$N_t = \frac{c_t N_t}{\mu w_t} + \frac{c_D \gamma_t^{1+\zeta}}{1+\zeta} \cdot \int_{\underline{x}}^{\infty} \exp[(\theta-1)x] m_t^N(x) \mathrm{d}x + c_R e_t M_t.$$

The only endogenous variable for which a corresponding equation is missing is the growth rate of the average quality index. Using the change of variable defined above, the expression for this index implies:

$$M_t^{-1} \int_{\underline{x}}^{\infty} \exp[(\theta-1)x] m_t(x) \mathrm{d}x = 1.$$

Normalizations. Let us define normalized variables:

$$\mathcal{V}_t^O \equiv \frac{V_t^O}{c_t}, \quad \mathcal{V}_t^N \equiv \frac{V_t^N}{c_t}, \quad \mathcal{M}_t \equiv \frac{M_t}{N_t}, \quad \mathcal{S}_t^N \equiv \frac{M_t^N}{M_t}, \quad \mathcal{S}_t^O \equiv \frac{M_t^O}{M_t},$$

as well as the normalized distributions $f_t^N(x) = m_t^N(x)/M_t^N$ and $f_t^O(x) = m_t^O(x)/M_t^O$. With these definitions, we use the free-entry condition and the Euler equation to rewrite the HJB equations as:

$$\begin{aligned}
\dot{\mathcal{V}}_t^O &= [\rho + (\theta-1)g_t^Q]\mathcal{V}_t^O - \frac{1-\tau^C}{\theta \mathcal{M}_t}, \\
\dot{\mathcal{V}}_t^N &= [\rho + \epsilon - (\theta-1)(\gamma - g_t^Q)]\mathcal{V}_t^N - \epsilon \mathcal{V}_t^O - \frac{1-\tau^C}{\theta \mathcal{M}_t} - \frac{(\theta-1)\gamma \mathcal{V}_t^N}{1+\zeta}
\end{aligned}$$

where the product quality drift can be rewritten as follows using the free-entry condition:

$$\gamma = \left[\frac{(\theta-1)(1-\tau^R)c_R}{(1-\tau^D)c_D \underline{q}^{\theta-1}} \right]^{1/\zeta}.$$

Hence, the product quality drift is constant along any transition path. Similarly, the labor market clearing condition can be rewritten as:

$$1 = \frac{(1-\tau^R)c_R}{\mu \mathcal{V}_t^N \underline{q}^{\theta-1}} + \frac{c_D \gamma^{1+\zeta}}{1+\zeta} \cdot \mathcal{M}_t \mathcal{S}_t^N \int_{\underline{x}}^{\infty} \exp[(\theta-1)x] f_t^N(x) \mathrm{d}x + c_R e_t \mathcal{M}_t.$$

The Fokker-Planck (FP) equations describing the evolution of the distribution of log relative quality among new and old firms are given by:

$$\dot{f}_t^N(x) = -(\gamma - g_t^Q)\partial_x f_t^N(x) - (e_t/\mathcal{S}_t^N) f_t^N(x),$$
$$\dot{f}_t^O(x) = g_t^Q \partial_x f_t^O(x) + (\epsilon \mathcal{S}_t^N/\mathcal{S}_t^O)[f_t^N(x) - f_t^O(x)] + (d_t/\mathcal{S}_t^O) f_t^O(x)$$

where $(\gamma - g_t^Q)\lim_{x\to\underline{x}} f_t^N(x) = e_t/\mathcal{S}_t^N$ and $d_t \equiv g_t^Q \mathcal{S}_t^O \lim_{x\to\underline{x}} f_t^O(x)$. The law of motion for the share of new and old products are given by:

$$\dot{\mathcal{S}}_t^N = e_t - \mathcal{S}_t^N(e_t + \epsilon - d_t) \quad \text{and} \quad \mathcal{S}_t^O = 1 - \mathcal{S}_t^N,$$

and the law of motion for the measure of products per capita is given by:

$$\dot{\mathcal{M}}_t = (e_t - d_t - n)\mathcal{M}_t.$$

Finally, the expression for the average quality index implies:

$$\int_{\underline{x}}^{\infty} \exp[(\theta - 1)x][f_t^N(x)\mathcal{S}_t^N + f_t^O(x)\mathcal{S}_t^O]\mathrm{d}x = 1.$$

Balanced growth path. On a BGP, the growth rate of the average quality index and the drift of product quality for new firms are both constant. Moreover, the measure of new and old firms both grow at the same rate as the population. This implies that the entry and exit rates are constant, and the former is equal to $e = n + d$. The share of new and old products are also constant and equal to:

$$\mathcal{S}_t^N = \frac{n + d}{n + \epsilon} \quad \text{and} \quad \mathcal{S}_t^O = \frac{\epsilon - d}{n + \epsilon}.$$

The stationary FP equation for the distribution of log relative quality among new firms is given by:

$$-(\gamma - g^Q)\partial_x f^N(x) - (n + \epsilon) f^N(x) = -(n + \epsilon)\delta(x - \underline{x}).$$

With parameter values such that $\gamma > g^Q$, the solution to this ODE is:

$$f^N(x) = \lambda_N \exp[-\lambda_N(x - \underline{x})] \quad \text{where} \quad \lambda_N \equiv \frac{n + \epsilon}{\gamma - g^Q}.$$

Similarly, the stationary FP equation for this distribution of log relative quality among old firms is given by:

$$g^Q \partial_x f^O(x) - n f^O(x) = -\frac{\epsilon(n+d)}{\epsilon - d} \cdot f^N(x).$$

Dividing through by g^Q, and multiplying by the integration factor $\exp(-\lambda_O x)$ where $\lambda_O \equiv n/g^Q$, we can rewrite:

$$\partial_x[\exp(-\lambda_O x) f^O(x)] = -\frac{\epsilon(n+d)}{(\epsilon - d)g^Q} \cdot \exp(-\lambda_O x) f^N(x).$$

Integrating this equation and solving for $f^O(x)$, we obtain:

$$f^O(x) = \left[C - \frac{\epsilon(n+d)}{(\epsilon - d)g^Q} \cdot \int_{\underline{x}}^{x} f^N(x) \exp(-\lambda_O x) \mathrm{d}x\right] \exp(\lambda_O x)$$

where C is an integration constant. For $f^O(x)$ to be integrable, we must have:

$$C = \frac{\epsilon(n+d)}{(\epsilon - d)g^Q} \cdot \int_{\underline{x}}^{\infty} f^N(x) \exp(-\lambda_O x) \mathrm{d}x.$$

Substituting this expression back in the solution for $f^O(x)$, we obtain:

$$f^O(x) = \frac{\epsilon(n+d) f^N(x)}{(\epsilon - d)(\lambda_N g^Q + n)}.$$

For $f^O(x)$ to be a probability distribution, we must verify that:

$$\epsilon(n+d) = (\epsilon - d)(\lambda_N g^Q + n).$$

Using the consistency condition of the exit rate, which delivers $d = \epsilon g^Q/\gamma$, it is straightforward to verify that this condition is satisfied. Using the ODE for the old firm's value function, we find:

$$\mathcal{V}^O = \frac{1 - \tau^C}{\theta \mathcal{M}[\rho + (\theta - 1)g^Q]}.$$

To obtain that last expression for the growth rate of the average quality index, we use the definition of the index itself:

$$g^Q = \gamma - \frac{(n+\epsilon)(1 - \underline{q}^{\theta - 1})}{\theta - 1}.$$

Using the ODE for the new firm's value function, we find:

$$\mathcal{V}^N = \frac{\frac{\epsilon+\rho+(\theta-1)g^Q}{\rho+(\theta-1)g^Q} \cdot \frac{1-\tau^C}{\theta \mathcal{M}}}{\rho + \epsilon - (n+\epsilon)(1-\underline{q}^{\theta-1}) - (\theta-1)\gamma/(1+\zeta)}.$$

Finally, we have the labor market clearing condition:

$$1 = \frac{(1-\tau^R)c_R}{\mu \mathcal{V}^N \underline{q}^{\theta-1}} + \frac{(n+d)c_D\gamma^{1+\zeta}}{(1+\zeta)(n+\epsilon)} \cdot \mathcal{M} + c_R(n+d)\mathcal{M}.$$

A.2 The constrained-optimal allocation

Notation. We introduce the following notation to define the inner product between two square-integrable functions $f(x), g(x) : \Omega \to \mathbb{R}$ over their common domain:

$$\langle f(x), g(x) \rangle_{x\in\Omega} \equiv \int_\Omega f(x)g(x)\mathrm{d}x.$$

Second, let us denote the (partial) Gateaux derivative of a functional F with respect to the function $f(x)$ in direction $\varrho(x)$ as:

$$\delta F[f(x); \varrho(x)] \equiv \left.\frac{\partial F[f(x) + \varepsilon \cdot \varrho(x), .]}{\partial \varepsilon}\right|_{\varepsilon=0}$$

where the functional F can take additional arguments through the "dot" notation and the "test" function $\varrho(x)$ is assumed to vanish on the boundaries of the relevant integration domain.

The planner's problem. Consider the problem of a planner seeking to maximize the following objective:

$$U_0 = \int_0^\infty e^{-(\rho-n)t} \ln(C_t/N_t)\mathrm{d}t$$

subject to the constraints:[9]

$$
\begin{aligned}
C_t &= [\textstyle\sum_{S\in\{N,O\}}\langle (ql_t^{PS}(q))^{\frac{\theta-1}{\theta}}, m_t^S(q)\rangle_{q\in[\underline{q}_t,\infty)}]^{\frac{\theta}{\theta-1}},\\
N_t &\geq \textstyle\sum_{S\in\{N,O\}}\langle l_t^{PS}(q), m_t^S(q)\rangle_{q\in[\underline{q}_t,\infty)} + \langle l_t^D(q), m_t^N(q)\rangle_{q\in[\underline{q}_t,\infty)} + L_t^R,\\
\dot{m}_t^N(q) &= -\partial_q[\gamma_t(q)qm_t^N(q)] - \epsilon m_t^N(q) + \delta(q-\underline{q}_t)L_t^R/c_R,\\
\dot{m}_t^O(q) &= \epsilon m_t^N(q)
\end{aligned}
$$

by choosing $\{\{\{l_t^{PS}(q)\}_{S\in\{N,O\}}, l_t^D(q)\}_{q=\underline{q}_t}^{\infty}, L_t^R\}_{t=0}^{\infty}$. The solution to the planner's problem is "constrained-optimal" in the sense that the planner takes externalities across firms (technology spillovers) as given. More precisely, the planner takes Q_t as given in the development technology. Reformulating this problem using the current-value Hamiltonian, we obtain:

$$
\begin{aligned}
&\mathcal{H}_t = \ln(C_t/N_t) + \nu_t^L[N_t - \textstyle\sum_{S\in\{N,O\}}\langle l_t^{PS}(q), m_t^S(q)\rangle_{q\in[\underline{q}_t,\infty)} - \langle l_t^D(q), m_t^N(q)\rangle_{q\in[\underline{q}_t,\infty)} - L_t^R]\\
&- \langle \nu_t^N(q), \partial_q[\gamma_t(q)qm_t^N(q)]\rangle_{q\in[\underline{q}_t,\infty)} + \epsilon\langle \nu_t^O(q) - \nu_t^N(q), m_t^N(q)\rangle_{q\in[\underline{q}_t,\infty)}\\
&+ \nu_t^N(\underline{q}_t)L_t^R/c_R
\end{aligned}
$$

where $\{\nu_t^L, \{\nu_t^N(q), \nu_t^O(q)\}_{q=\underline{q}_t}^{\infty}\}_{t=0}^{\infty}$ are the costate functions. Using integration by parts, we can rewrite:

$$
\begin{aligned}
&\mathcal{H}_t = \ln(C_t/N_t) + \nu_t^L[N_t - \textstyle\sum_{S\in\{N,O\}}\langle l_t^{PS}(q), m_t^S(q)\rangle_{q\in[\underline{q}_t,\infty)} - \langle l_t^D(q), m_t^N(q)\rangle_{q\in[\underline{q}_t,\infty)} - L_t^R]\\
&+ \langle \gamma_t(q)q\partial_q\nu_t^N(q), m_t^N(q)\rangle_{q\in[\underline{q}_t,\infty)} + \epsilon\langle \nu_t^O(q) - \nu_t^N(q), m_t^N(q)\rangle_{q\in[\underline{q}_t,\infty)}\\
&+ \nu_t^N(\underline{q}_t)L_t^R/c_R.
\end{aligned}
$$

The first-order condition with respect to $l_t^{PS}(q)$ implies:

$$
\delta C_t[l_t^{PS}(q);\varrho(q)]/C_t = \nu_t^L\langle m_t^S(q), \varrho(q)\rangle_{q\in[\underline{q}_t,\infty)}
$$

where $\delta C_t[l_t^{PS}(q);\varrho(q)]$ is the Gateaux derivative of C_t with respect to $l_t^{PS}(q)$ in direction $\varrho(q)$, which is an arbitrary function that vanishes on the boundaries of $[\underline{q}_t,\infty)$:

$$
\delta C_t[l_t^{PS}(q);\varrho(q)] = C_t^{1/\theta}\langle q^{\frac{\theta-1}{\theta}} l_t^{PS}(q)^{-1/\theta} m_t^S(q), \varrho(q)\rangle_{q\in[\underline{q}_t,\infty)}.
$$

[9]Here, we substituted the product resource constraints in the labor resource constraint.

Since that first-order condition must hold for any function $\varrho(q)$, we obtain:

$$l_t^{PS}(q) = (q/C_t)^{\theta-1}\nu_t^{L^{-\theta}}, \ \ \forall q \in [\underline{q}_t, \infty).$$

Integrating this expression, we find:

$$\nu_t^L = 1/L_t^P$$

such that we can rewrite:

$$l_t^{PS}(q) = (q/Q_t)^{\theta-1}L_t^P/M_t, \ \ \forall q \in [\underline{q}_t, \infty).$$

The first-order condition with respect to $l_t^D(q)$ implies:

$$\langle \gamma_t(q) q \partial_q \nu_t^N(q) m_t^N(q)/l_t^D(q), \varrho(q)\rangle_{q\in[\underline{q}_t,\infty)}/(1+\zeta) = \nu_t^L \langle m_t^N(q), \varrho(q)\rangle_{q\in[\underline{q}_t,\infty)}.$$

Since that first-order condition must hold for any function $\varrho(q)$, we obtain:

$$\gamma_t(q) = \left[\frac{q\partial_q \nu_t^N(q)}{\nu_t^L c_D (q/Q_t)^{\theta-1}}\right]^{1/\zeta}, \ \ \forall q \in [\underline{q}_t, \infty).$$

The first-order condition with respect to L_t^R implies:

$$\nu_t^L c_R = \nu_t^N(\underline{q}_t).$$

The first-order condition with respect to $m_t^N(q)$ implies:

$$\begin{aligned}&\langle (\rho - n)\nu_t^N(q) - \dot{\nu}_t^N(q), \varrho(q)\rangle_{q\in[\underline{q}_t,\infty)} = \langle (q/Q_t)^{\theta-1}, \varrho(q)\rangle_{q\in[\underline{q}_t,\infty)}/[(\theta-1)M_t]\\ &+ \langle \gamma_t(q) q\partial_q \nu_t^N(q) - \nu_t^L l_t^D(q) + \epsilon[\nu_t^O(q) - \nu_t^N(q)], \varrho(q)\rangle_{q\in[\underline{q}_t,\infty)}.\end{aligned}$$

Since this condition must hold for any function $\varrho(q)$, we have:

$$\begin{aligned}&(\rho - n)\nu_t^N(q) - \dot{\nu}_t^N(q) = (q/Q_t)^{\theta-1}/[(\theta-1)M_t] - \nu_t^L l_t^D(q)\\ &+ \gamma_t(q) q\partial_q \nu_t^N(q) + \epsilon[\nu_t^O(q) - \nu_t^N(q)], \ \ \forall q \in [\underline{q}_t, \infty).\end{aligned}$$

Similarly, the first-order condition with respect to $m_t^O(q)$ implies:

$$(\rho - n)\nu_t^O(q) - \dot{\nu}_t^O(q) = (q/Q_t)^{\theta-1}/[(\theta-1)M_t], \ \ \forall q \in [\underline{q}_t, \infty).$$

Defining the following functions:

$$V_t^{N*}(q) \equiv \nu_t^N(q)C_t/\mu, \;\; V_t^{O*}(q) \equiv \nu_t^O(q)C_t/\mu, \;\; w_t^* \equiv \nu_t^L C_t/\mu, \;\; r_t^* \equiv \dot{c}_t/c_t + \rho,$$

and substituting them in the new firm's social HJB equation, we obtain:

$$\begin{aligned} r_t^* V_t^{N*}(q) - \dot{V}_t^{N*}(q) = (q/Q_t)^{\theta-1} C_t/(\theta M_t) - w_t^* l_t^D(q) \\ + \gamma_t(q) q \partial_q V_t^{N*}(q) + \epsilon [V_t^{O*}(q) - V_t^{N*}(q)] \end{aligned}$$

where $\gamma_t(q)$ is given by:

$$\gamma_t(q) = \left[\frac{q \partial_q V_t^{N*}(q)}{c_D w_t^* (q/Q_t)^{\theta-1}} \right]^{1/\zeta}.$$

Doing so for the old firm's social HJB equation, we obtain:

$$r_t^* V_t^{O*}(q) - \dot{V}_t^{O*}(q) = (q/Q_t)^{\theta-1} C_t/(\theta M_t).$$

Finally, substituting these functions in the social free-entry condition, we obtain:

$$w_t^* c_R = V_t^{N*}(\underline{q}_t).$$

This demonstrates that market equilibrium allocation is constrained-optimal since it exactly coincides with the solution to the planner's problem, echoing the findings of Dhingra and Morrow (2019). This efficiency holds for both research (entry) and development (product quality improvements). In each case, it arises because two opposing forces exactly cancel each other out under a Dixit and Stiglitz (1977) demand system. For research, a positive consumer surplus externality (entrants do not internalize that they raise the demand for their competitors' products through a love-of-variety) is precisely offset by a negative business-stealing externality (entrants ignore the profits they divert from incumbents). Similarly, for development, the incentive to improve a product is dampened by market power, since firms must deploy those quality improvements at a suboptimal production scale. This push toward too little development is perfectly counteracted by the fact that this same underproduction frees up labor for development (away from production), creating a push toward too much. It is worth noting, however, that this efficiency result is sensitive to the model's structure; adding head-to-head creative destruction or a different demand system, for instance, could lead to constrained-inefficient R&D.