UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in GEO4310 – Stochastic methods in hydrology
Day of exam: 8 December 2015
Exam hours: 09:00 – 12:00
This examination paper consists of 4 pages (plus an appendix).
Appendices: Yes
Permitted materials: Calculator

Make sure that your copy of this examination paper is complete before answering.

Task 1 Definitions (10 points):

Give a brief definition for each of the following terms:

a) Coefficient of variation
b) Central limit theorem
c) Autocorrelation
d) Confidence interval
e) Homoscedasticity

Task 2 Distributions and exreme value statistics (~30 points)

a) Two catchments receive the same annual precipitation, 1000 mm/year. Based on 30 years of streamflow data, we have estimated the following statistics:

	Catchment 1	Catchment 2
Mean (m^3/s)	0.4	0.6
Standard deviation (m^3/s)	0.2	0.1
Skewness	3.5	0.0

Sketch the probability distributions (pdf) of catchment 1 and 2 in the same figure. Sketch the cumulative probabilitiy distributions (cdf) of the two catchments in a different figure. Which mathematical distributions could fit catchment 1 and which could fit catchment 2?

b) The probability that an annual maximum flood at a measuring site in a river exceeds 100 m^3/s is p=0.1, and the probability of not exceeding 100 m^3/s is q=1-p=0.9. Assume that annual maximum floods are independent of each other.

What is the probability that the annual maximum flood exceeds 100 m^3/s excactly three times during a period of 30 years?

c) Flood-frequency analysis is used to calculate the magnitude of a flood.

i) Describe and sketch how you may select data used for a flood-frequency analysis from a data series consisting of daily mean flow.

ii) How is the return period defined? What is the relationship between the return period (T) and the cumulative distribution function (F), and which non-exceedence probability does a 100-year flood have?

iii) For a certain catchment, extreme streamflow values (q [m^3/s]) may be described by a Gumbel distribution with parameters $\alpha = 0.234$ og $\beta = 0.369$.

$F(q) = \exp(-\exp(-(q-\beta)/\alpha))$

How large is a 100-year flood in this catchment?

Task 3 Hypothesis testing (~30 points)

a) Is there a significance difference between (a) the annual mean precipitation and (b) the variance of precipitation in point A and point B at a level of significance of 5% if for a 50-year period it has been found that:

	Point Precipitation	
	Mean	Standard deviation
A	720 mm	220 mm
B	660 mm	140 mm

b) Based on 90-year records of yearly maximum discharge data at two adjacent stations, the following statistics are calculated (with n=90). The mean value in station A is 130 m3/s, standard deviation in A is 50 m3/s. The mean value in station B is 120 m3/s, standard deviation in B is 50 m3/s. The significance level is 5%

	Q≤50	50<Q≤100	100<Q≤150	150<Q≤250	Q>250
Station A (# years)	6	24	35	15	10
Station B (# years)	5	25	20	26	14

Use Chi square method to test:

i) the hypothesis that data in Station A and Station B have the same distribution

ii) the hypothesis that data in Station A has normal distribution.

Task 4 Regression and time series (~30 points)

a) The precipitation and runoff data for 25 Storms on the Monocacy River are provided in the table below.

i) Plot the observed data for runoff vs. Precipitation

ii) Determine and draw the regression line of runoff on precipitation (that is, the mean runoff for given value of precipitation).

iii) Estimate the variance of runoff for a given precipitation. Assume that the variance of th runoff is constant with precipitation.

iv) Assume that the runoff corresponding to a given precipitation is a normal variate; what is the probability that the runoff will exceed 50 mm. During a storm with 100-mm. precipitation?

Storm ID	Precipitation (mm.)	Runoff (mm.)
1	28.194	13.208
2	29.718	10.16
3	45.466	24.638
4	142.748	74.168
5	28.702	4.318
6	39.116	4.826
7	81.026	19.304
8	43.942	16.764
9	53.086	19.812
10	69.85	31.496
11	30.48	9.906
12	25.654	7.62
13	41.656	17.78
14	39.878	19.558
15	39.116	14.986
16	53.086	24.13
17	89.916	25.908
18	29.718	9.906
19	29.21	5.842
20	65.278	11.43
21	90.678	40.386
22	129.794	44.196
23	38.608	14.224
24	74.422	28.448
25	29.464	16.256