Random Variable Cheat Sheet

Advanced Statistics

$E[X] = \mu_X = \sum_{i=1}^{n} p(x_i) \cdot x_i$

$E[f(x)] = \sum_{i=1}^{n} p(x_i) \cdot f(x_i)$

$Var[X] = E[(X - \mu_X)^2] = E[X^2] - (\mu_X)^2 = \sigma_x^2 = \sum_{i=1}^{n} p(x_i) \cdot (x_i - \mu_X)^2$

Continuous Random Variables

$\rho(X) =$ "the probability density of X" (for continuous random variables)

$F(X) =$ "the cumulative distribution of X"

$F[x] = P(X \leq x)$

Sum of Random Variables

$E[X + Y] = E[X] + E[Y]$ (expected values add)

when X and Y are independent: $Var[X + Y] = Var[X] + Var[Y]$

more generally: $Var[X + Y] = Var[X] + Var[Y] + 2 \cdot Cov[X, Y]$

$Corr[X, Y] = \frac{Cov[X,Y]}{\sigma_x \cdot \sigma_y}$

Linear Transformations of Random Variables:

If $Y = aX + b$ then $E[Y] = aE[X] + b$ and $Var[Y] = a^2 Var[X]$

Binomial Random Variables

X ~ binomial(n, p)

X is the number of succesess in n independent trials each with p probability of success

$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$

$E[X] = np$

$Var[X] = np(1 - p)$

Uniform Random Variables

X ~ unif(a,b)

X is a randomly selected value from the interval (a,b) where all values have the same probability density.

$\rho[x] = \frac{1}{b-a}$

$E[X] = \frac{a+b}{2}$

$Var[X] = \frac{(b-a)^2}{12}$

Normal Random Variables

$X \sim N(\mu, \sigma^2)$, where μ is the mean and σ is the standard deviation

$\rho(x) = \sqrt{\frac{1}{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

Central Limit Theorem

The probability distribution of means or sums of a large number of independent draws from ANY distribution (with a finite expected value) can be approximated by a normal distribution. Outcomes that are affected by a large number of individually small, mostly independent, factors tend to be roughly normally distributed.

Z-Scores

The Z-score of an observation is the number of standard deviations it falls above or below the mean. We compute the Z-score for an observation x that follows a distribution with mean μ and standard deviation σ using

$$Z = \frac{x-\mu}{\sigma}$$

Z ~ N(0,1)

$\rho(z) = \sqrt{\frac{1}{2\pi}} e^{\frac{-z^2}{2}}$

$F(z) = \ldots$ look on a standard normal table!

Exponential Random Variables

$X \sim exp(\lambda)$ where λ is the rate at which events occur in a process in which events occur continuously and independently.

$\rho(X) = \lambda e^{-\lambda x}$

$F(x) = 1 - e^{-\lambda x}$

$E[X] = \frac{1}{\lambda}$

$Var[X] = \frac{1}{\lambda^2}$