Covariance Example

Advanced Statistics

2024-10-04

Formulas from last time…

when X and Y are independent: $Var[X+Y] = Var[X] + Var[Y]$

more generally: $Var[X+Y] = Var[X] + Var[Y] + 2 \cdot Cov[X,Y]$

where:

$Cov[X,Y] = E[X \cdot Y] - E[X] \cdot [Y]$

From the last homework…

X = "The # of heads on the first flip"

Y = "The # of heads on the second flip"

For a FAIR coin we found:

- $E[X] = \frac{1}{2}$
- $E[Y] = \frac{1}{2}$
- $E[X \cdot Y] = \frac{1}{4}$

For a coin that has a half chance of being biased towards heads (2/3rds heads) and a half chance of being bised towards tails (2/3rds tails):

- $E[X] = \frac{1}{2}$
- $E[Y] = \frac{1}{2}$
- $E[X \cdot Y] = \frac{5}{18}$

It's also true that for both coins:

$Var[X] = Var[Y] = \frac{1}{4}$

For each coin, please find:

a. $Cov[X,Y]$
b. $Var[X+Y]$

Why is $Var[X+Y]$ larger for one of the coins?