APPENDIX 9

DESIGN OF A SOLAR COLLECTOR

It is necessary to design a flat plate solar collector to provide a required air flow of 1 m^3s^{-1} at a temperature of 40°C.

The following information is known:

(i) The drying season runs from the beginning of November to the end of January

(ii) Average ambient temperature during the drying season is 30°C.

(iii) The dryer is to be situated at a latitude of 10°N.

(iv) Available insolation data is limited but the following values for average total daily radiation on a horizontal surface are known:

November	20.9 MJ $m^{-2}day^{-1}$
December	19.2 MJ $m^{-2}day^{-1}$
January	20.1 MJ $m^{-2}day^{-1}$

Solution

For this example the design procedure will consist of the following steps;

(i) Determine the optimum collector slope,

(ii) Calculate the intensity of insolation on a surface of this slope,

(iii) Decide which of the types of collectors appears most suitable for this operation,

(iv) Determine the collector area required,

(v) Bearing in mind the effect of air velocity on collector performance determine the dimensions (width, depth and length) of the collector,

(i) Determining optimum collector slope.

From a general knowledge of the apparent movement of the sun it can be seen that during the drying season the sun will be overhead at or near the Tropic of Capricorn. Thus for a dryer situated in the northern hemisphere a south facing collector will receive most insolation.

To calculate the optimum slope angle, take December 15 as the midpoint of the drying season.

Angle of declination, δ is calculated from the equation

$$\delta = 23.45 \sin (0.9863.(284+n)) \qquad \text{(A9.1)}$$

For December 15, n = 349

Hence $\delta = 23.45 \sin (0.9863(284 + 349))$

$= 23.45 \sin (624)$

$= 23.45 \times (-0.995) = -23.3°$

To maximise the level of insolation on a collector the simplest approach is to situate the collector so that it is perpendicular to insolation at mid-day in the middle of the dry season.

Though equation 5.3 could be used to calculate the optimum slope, a consideration of the geometry of the situation offers a simple solution.

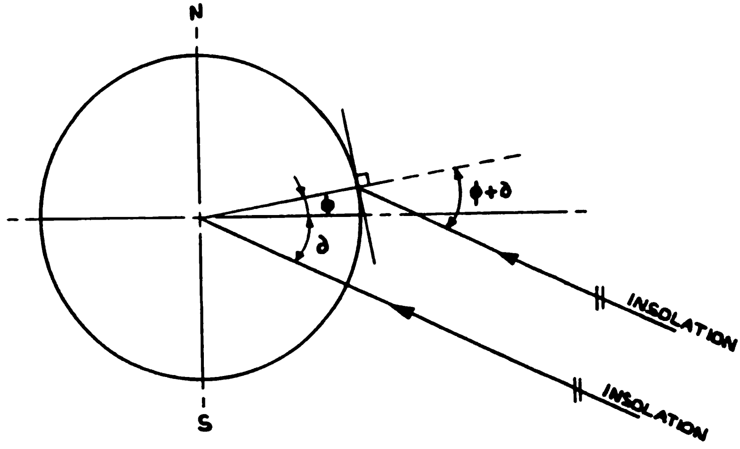

Figure A9.1 Angle of Incidence on a South Facing Roof at Mid-day

Looking at the angles subtended at the centre of the globe in Figure A9.1 it can be seen that the angle between the incident radiation and a perpendicular to the earth's surface at a latitude of 10°N is the sun of the declination angle δ and the angle of latitude Φ. Thus for a horizontal surface at this latitude, the angle of incidence θ, at solar noon is given by,

$$\theta = \delta + \Phi \qquad \text{(A9.2)}$$
$$= 10 + 23.3 = 33.3° \text{ (as a slope from the vertical)}$$

and so for a surface to be perpendicular to insolation at this time it must face south at a slope from the horizontal of 33.3°, as shown in Figure A9.1.

(It is important to note that this approach to determining θ, the angle of incidence, is only valid at solar noon and for a collector facing either due north or south.)

Thus a surface of slope 33.3° facing south has been shown to be the optimum for receiving insolation. In practice a variation of a few degrees will make only a small difference in collector performance.

(ii) **Calculation of the Intensity of Insolation on the Collector Surface.**

Due to the limited amount of information available and the variation in intensity of insolation with climatic conditions precise prediction of the intensity of insolation on the collector surface is not possible. To provide data for the sizing of the solar collector an average value of insolation will be estimated. The precdure here is to first calculate the intensity of insolation on a horizontal surface and then upon the collector surface. The insolation data available is an average total amount of insolation over a period of a day on a horizontal surface. Instantaneous insolation on a surface is approximately proportional to the cosine of the angle of incidence, Θ. Equation 5.3 can be used to calculate the angle of incidence for insolation on a horizontal surface, Θ_h. Since the slope angle is zero, equation 5.3 simplifies to:

$$\cos \Theta_h = \sin\delta . \sin\phi + \cos\delta . \cos\phi . \cos\omega \qquad \text{(A9.3)}$$

δ is calculated from equation A9.1:

δ = -19.1° on November 15
δ = -23.3° on December 15
δ = -21.3° on January 15

A reasonable estimate of the mean value of Θ_h over the day can be made by assuming a 12 hour day and dividing it into four 3 hour periods and calculating Θ_h for each period. With a 12 hour day the hour angle, ω, varies from -90° at 0600 to +90° at 1800. For the 1st period 0600-0900, the mean value of w is -67.5°, and hence for November 15:

$$\begin{aligned}\cos \Theta_h &= \sin(-19,1).\sin(10) + \cos(-19.1).\cos(10).\cos(-67.5)\\ &= ((-0.327) \times 0.174) + (0.944 \times 0.984 \times 0.383)\\ &= -0.057 + 0.356\\ &= 0.299\end{aligned}$$

Similarly cos Θ for the period 0900-1200 on November 15 is calculated as 0.803. It cam be appreciated that by symmetry Θ_h for 0600-0900 is identical to Θ_h for 1500-1800, and Θ_h for 0900-1200 is identical to Θ_h for 1200-1500.

The insolation on a horizontal I_h surface for the period 0600-0900 on November 15 is calculated thus:

$$I_h = \frac{20.9 \times 0.299}{0.299 + 0.803 + 0.803 + 0.299}$$

$$= \frac{6.249}{2.204} = 2.84 \text{ MJ m}^{-2}$$

$$\text{and} \quad I_h = \frac{2.84 \times 1000 \times 1000}{3 \times 60 \times 60} = 263 \text{Wm}^{-2}$$

and for the period 0900-1200:

$$I_h = 7.61 \text{ MJ m}^{-2}$$

$$\text{and} \quad I_h = 705 \text{ W m}^{-2}$$

Data for December 15 and January 15 are likewise calculated and shown in Table A9.1.

Equation 5.3 is used again to calculate the angle of incidence upon the collector surface Θ. For the period 0600-0900 on November 15:

$$\begin{aligned}
\cos\Theta &= \sin(-19.1).\sin(10).\cos(33.3) \\
&\quad - \sin(-19.1).\cos(10).\sin(33.3).\cos(0) \\
&\quad + \cos(-19.1).\cos(10).\cos(33.3).\cos(-76.5) \\
&\quad + \cos(-19.1).\cos(33.3).\sin(0).\sin(-67.5) \\
&= (-0.327) \times 0.174 \times 0.836 \\
&\quad - (-0.327) \times 0.985 \times 0.549 \times 1 \\
&\quad + 0.945 \times 0.174 \times 0.549 \times 1 \\
&\quad + 0.945 \times 0.549 \times 0 \times 0.924 \\
&= -0.048 + 0.177 + 0.924 \\
&= 0.461
\end{aligned}$$

and for the period 0900-1200, $\cos\Theta = 0.931$.

Equation 5.1 is used to determine the intensity of insolation on the collector surface:

$$I_c = \frac{I_h.\cos\Theta}{\cos\Theta_h}$$

For the period 0600-0900 on November 15:

$$I_c = 263 \times \frac{0.461}{0.299} = 405 \text{ W m}^{-2}$$

and for the period 0900-1200, $I_c = 817 \text{ W m}^{-2}$.

Data for December 15 and January 15 are likewise calculated and shown in Table A9.2.

From Table A9.2 it is clear that there is a considerable variation in the level of insolation incident upon the collector. This will result in a significant variation in the amount of heat which the collector can impart to the air flowing through it. A compromise must be made between a collector which can provide the required air flowrate and temperature under the low insolation conditions (which would be large and thus expensive) and one which only meets the design requirement when insolation is high (which would have a lower cost, but might result in excessively long drying times). Factors which will govern how this compromise is made include:

TABLE A9.1

Time	Mean	$\cos \Theta$	I_h MJ m^{-2}	I_h W m^{-2}
Nov 15				
0600-0900	-67.5	0.299	2.84	263
0900-1200	-22.5	0.803	7.61	705
1200-1500	22.5	0.803	7.61	705
1500-1800	67.5	0.299	2.84	263
Dec 15				
0600-0900	-67.5	0.277	2.54	235
0900-1200	-22.5	0.767	7.05	653
1200-1500	22.5	0.767	7.05	653
1500-1800	67.5	0.277	2.54	235
Jan 15				
0600-0900	-67.5	0.288	2.70	250
0900-1200	-22.5	0.785	7.35	681
1200-1500	22.5	0.785	7.35	681
1500-1800	67.5	0.288	2.70	250

TABLE A9.2

Time	$\cos \Theta_h$	$\cos \Theta$	$\frac{\cos \Theta}{\cos \Theta_h}$	I_c W m^{-2}
Nov 15				
0600-0900	0.299	0.461	1.54	405
0900-1200	0.803	0.931	1.15	810
1200-1500	0.803	0.931	1.15	810
1500-1800	0.299	0.461	1.54	405
Dec 15				
0600-0900	0.277	0.480	1.73	409
0900-1200	0.767	0.937	1.22	797
1200-1500	0.767	0.937	1.22	797
1500-1800	0.277	0.480	1.73	406
Jan 15				
0600-0900	0.288	0.471	1.63	408
0900-1200	0.785	0.934	1.19	810
1200-1500	0.785	0.934	1.19	810
1500-1800	0.288	0.471	1.63	408

(i) cost of collector materials
(ii) effect of extended drying times on product quality
(iii) whether dryer is used continuously or intermittently

For the purpose of this exercise it will be taken that an average of 10°C temperature rise is required. Thus the average of the insolation levels I_c in Table A9.2 will be used as the basis for sizing, viz I_c (average) = 606 W m^{-2}

(iii) **Selection of Collector Type**

For this case selection of collector type is not a clear cut decision, the temperature elevation 10°C is quite high for a bare plate collector. A covered collector will be more expensive. In such a situation a design of both a bare plate and a single cover collector and an approximate costing will help in making a decision.

For this example it will be assumed that a single cover collector has been selected and that glass is available for the cover.

(iv) **Determination of collector area**

The procedure here is to first estimate the collection efficiency $_c$, and then the collector area.

The collection efficiency is determined from equation 6.4:

$$\eta_c = \frac{1}{1 + U_L/h} . (1 - \exp(-U_o/G_a.C_p)). \frac{G_a C_p}{U_o} . f_{ca}$$

where U_L = collector heat loss coefficient, 6.99 W m^{-2} K^{-1}

h = heat transfer coefficient between absorber and flowing air 22.7 W m^{-2} K^{-1}

U_o = overall heat transfer coefficient, 5.3 W m^{-2} K^{-1}

G_a = mass flowrate per unit collector area 40.8 kg $s^{-1} m^{-2}$

C_p = specific heat of air, 1.005 kJ.kg^{-1} K^{-1}

f_{ca} = effective transmissivity absorptivity product from Table 6.2.

Hence

$$\eta_c = \frac{1}{1 + \frac{6.99}{22.7}} \times (1 - \exp \frac{-5.3}{40.8 \times 1.005}) \times \frac{40.8 \times 1005}{5.3} \times 0.88$$

$$= 0.765 \times 0.121 \times 7.737 \times 0.88$$
$$= 0.63$$

From equation 6.1

$$\eta_c = \frac{V.\rho.C_p.\Delta T}{A_c\ I_c}$$

and rearranging

$$A_c = \frac{V.\rho.C_p.\Delta T}{\eta_c.I_c}$$

$$= \frac{1 \times 1.28 \times 1005 \times 10}{0.63 \times 606}$$

$$= 33.7\ m^2$$

The mass flowrate of air per unit area of collector G_a is then calculated:

$$G_a = \frac{V.\rho}{A_c}$$

$$= \frac{1 \times 1.28}{33.7}$$

$$= 0.038\ kg\ s^{-1}\ m^{-2}$$

A correction factor for the air flow is now determined by linear interpolation from Table 6.3:

$$\text{Correction factor} = \frac{38.0 - 13.6}{40.8 - 13.6} . (1 - 0.88) + 0.88$$

$$= 0.99$$

$$\text{Thus corrected } \eta_c = 0.99 \times 0.63 = 0.62$$

$$\text{and corrected } A_c = \frac{1 \times 1.28 \times 1.005 \times 10 \times 1000}{0.62 \times 606} = 34.2\ m^2$$

$\therefore$ Collector Area = 34 m^2

(v) Determination of Collector Dimensions

This last step is to some extent an iterative procedure, dimensions are chosen and if these prove suitable when factors such as pressure drop and heat transfer coefficients are considered then the chosen dimensions can be used; if the chosen dimensions are not suitable then new values are selected until acceptable ones are found.

As a first estimate, assume collector dimensions of 8.5 m x 4 m x 0.05 m. Equation 6.5 is used to determine the heat transfer coefficient h;

$$Nu = 0.02\ Re^{0.8}$$

where

$$Re = \frac{\rho.v.L}{\mu}$$

$$\rho = 1.28 \text{ kg m}^{-3}$$

L = hydraulic diameter of the duct

$$= \frac{2 \times (\text{width} \times \text{depth})}{(\text{width} + \text{depth})}$$

$$= \frac{2 \times (4 \times 0.05)}{4 + 0.05} = 0.099 \text{ m}$$

$$v = \frac{\text{volumetric air flow}}{\text{cross-sectional area of flow}}$$

$$= \frac{1}{4 \times 0.05} = 5 \text{ ms}^{-1}$$

$$\mu = 1.8 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$$

$$\therefore Re = \frac{1.28 \times 5 \times 0.099}{1.8 \times 10^{-5}} = 35.2 \times 10^3$$

Hence $Nu = 0.02 \times (35.2 \times 10^3)\ 0.8 = 86.7$

Now $Nu = \frac{h.L}{k}$

$$\therefore h = \frac{k.Nu}{L} = \frac{0.025 \times 86.7}{0.099} = 21.9 \text{ W m}^{-2} \text{ K}^{-1}$$

The value of h is in good agreement with the assumed value made in calculating the collection efficiency.

The pressure drop through the collector ΔP, is calculated from equation 12.3

$$\Delta P = \frac{f.L.G_d^2}{2.\rho.R_h}$$

where f = friction factor, which is determined from Figure 12.2 as 0.007

G_d = mass flowrate of air per unit duct area, 6.4 kg s^{-1} m^{-2}

R_h = hydraulic radius

$$= \frac{0.099}{4} = 0.0248 \text{ m}$$

Hence $\Delta P = \frac{0.007 \times 8.5 \times 6.4^2}{2 \times 1.28 \times 0.0248} = 38 \text{ Pa}$

Even if the collector is to be situated on the suction side of the fan such a pressure drop should pose no problems for an axial flow fan.

The collector dimensions chosen are therefore appropriate for the duty in question.