```
(*Compute Jacobian of ODE system assuming constant bcr0 and cd0
with eqs S1 - S5 from Martinez2012

References:
https://resources.wolframcloud.com/FunctionRepository/resources/JacobianMatrix/
https://mathematica.stackexchange.com/questions/5790/how-to-make-jacobian-automatically-in-m
*)

Clear["Global`*"]
dissociation[k_, ui_] := k^2/(k^2+ui^2) ;
protein[k_, ui_] := ui^2/(k^2+ui^2);
```

In[∘]:=

```
(*Equations S1 - S5 in Martinez2012

NOTE: I use capital subscripts to avoid mathematica treating subscripts as state vars
*)
BCR = bcr_0*dissociation[k_B, b];
CD40 = cd_0*dissociation[k_B, b];
u = {p, b, r};
ds = {
    μ_P + σ_P*dissociation[k_B, b] + σ_P*protein[k_R, r] - λ_P*p,
    μ_B + σ_B*dissociation[k_P, p]*dissociation[k_B, b]*dissociation[k_R, r] - (λ_B + BCR)*b,
    μ_R + σ_R*protein[k_R, r] + CD40 - λ_R*r
};
jacobianDs = Grad[ds, u];
jacobianDs // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\lambda_P & -\frac{2\, b\, k_B^2\, \sigma_P}{(b^2+k_B^2)^2} & -\frac{2\, r^3\, \sigma_P}{(r^2+k_R^2)^2} + \frac{2\, r\, \sigma_P}{r^2+k_R^2} \\ -\frac{2\, p\, k_B^2\, k_P^2\, k_R^2\, \sigma_B}{(b^2+k_B^2)\,(p^2+k_P^2)^2\,(r^2+k_R^2)} & \frac{2\, b^2\, bcr_0\, k_B^2}{(b^2+k_B^2)^2} - \frac{bcr_0\, k_B^2}{b^2+k_B^2} - \lambda_B - \frac{2\, b\, k_B^2\, k_P^2\, k_R^2\, \sigma_B}{(b^2+k_B^2)^2\,(p^2+k_P^2)\,(r^2+k_R^2)} & -\frac{2\, r\, k_B^2\, k_P^2\, k_R^2\, \sigma_B}{(b^2+k_B^2)\,(p^2+k_P^2)\,(r^2+k_R^2)^2} \\ 0 & -\frac{2\, b\, cd_0\, k_B^2}{(b^2+k_B^2)^2} & -\lambda_R - \frac{2\, r^3\, \sigma_R}{(r^2+k_R^2)^2} + \frac{2\, r\, \sigma_R}{r^2+k_R^2} \end{pmatrix}$$

In[∘]:=

```
(*Define parameters to check a scalar output*)
μ_P = 10*^-6;
μ_B = 2;
μ_R = 0.1;

σ_P = 9;
```

```
σB = 100;
σR = 2.6;

kP = 1;
kB = 1;
kR = 1;

λP = 1;
λB = 1;
λR = 1;

bcr0 = 0.05;
cd0 = 0.015;

p = 0.2;
b = 5.0;
r = 0.2;

jacobianDs // MatrixForm

(*Clear symbols*)
μP =.
μB =.
μR =.

σP =.
σB =.
σR =.

kP =.
kB =.
kR =.

λP =.
λB =.
λR =.

bcr0 =.
cd0 =.
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & -0.133136 & 3.3284 \\ -1.36769 & -2.36591 & -1.36769 \\ 0 & -0.000221893 & -0.0384615 \end{pmatrix}$$