```
(*Compute Jacobian of ODE system assuming Gaussian bcr0 and cd0 signaling
with eqs S1 - S5 from Martinez2012

References:
https://resources.wolframcloud.com/FunctionRepository/resources/JacobianMatrix/
https://mathematica.stackexchange.com/questions/5790/how-to-make-jacobian-automatically-in-m
*)

Clear["Global`*"]
dissociation[k_, ui_] := k^2/(k^2+ui^2) ;
protein[k_, ui_] := ui^2/(k^2+ui^2);
```

In[•]:=

```
(*Equations S1 - S5 in Martinez2012

NOTE: I use capital subscripts to avoid mathematica treating subscripts as state vars
*)
bcr0 = bcr0MaxSignal*PDF[NormalDistribution[
    bcr0MaxSignalCenteredOnTimestep,
    bcr0MaxSignalTimestepStd], t];
BCR = bcr0*dissociation[kb, b];
cd0 = cd0MaxSignal*PDF[
    NormalDistribution[
    cd0MaxSignalCenteredOnTimestep,
    cd0MaxSignalTimestepStd], t];
CD40 = cd0*dissociation[kb, b];

u = {p, b, r};
ds = {
    μp + σp*dissociation[kb, b] + σp*protein[kr, r] - λp*p,
    μb + σb*dissociation[kp, p]*dissociation[kb, b]*
        dissociation[kr, r] - (λb + BCR)*b,
    μr + σr*protein[kr, r] + CD40 - λr*r
};
jacobianDs = Grad[ds, u];
```

In[•]:=
```
jacobianDs[[1]]
```

Out[•]=
$$\left\{-\lambda p,\ -\frac{2\, b\, kb^2\, \sigma p}{\left(b^2+kb^2\right)^2},\ -\frac{2\, r^3\, \sigma p}{\left(kr^2+r^2\right)^2}+\frac{2\, r\, \sigma p}{kr^2+r^2}\right\}$$

In[•]:=
```
jacobianDs[[2]]
```

Out[•]=
$$\left\{-\frac{2\, kb^2\, kp^2\, kr^2\, p\, \sigma b}{\left(b^2+kb^2\right)\left(kp^2+p^2\right)^2\left(kr^2+r^2\right)},\ \frac{\sqrt{2}\ b^2\ bcr0MaxSignal\ e^{-\frac{\left(-bcr0MaxSignalCenteredOnTimestep+t\right)^2}{2\, bcr0MaxSignalTimestepStd^2}}\ kb^2}{bcr0MaxSignalTimestepStd\left(b^2+kb^2\right)^2\sqrt{\pi}}-\right.$$
$$\frac{bcr0MaxSignal\ e^{-\frac{\left(-bcr0MaxSignalCenteredOnTimestep+t\right)^2}{2\, bcr0MaxSignalTimestepStd^2}}\ kb^2}{bcr0MaxSignalTimestepStd\left(b^2+kb^2\right)\sqrt{2\,\pi}}-\lambda b-$$
$$\left.\frac{2\, b\, kb^2\, kp^2\, kr^2\, \sigma b}{\left(b^2+kb^2\right)^2\left(kp^2+p^2\right)\left(kr^2+r^2\right)},\ -\frac{2\, kb^2\, kp^2\, kr^2\, r\, \sigma b}{\left(b^2+kb^2\right)\left(kp^2+p^2\right)\left(kr^2+r^2\right)^2}\right\}$$

In[•]:=
```
jacobianDs[[3]]
```

Out[•]=
$$\left\{0,\ -\frac{\sqrt{2}\ b\ cd0MaxSignal\ e^{-\frac{\left(-cd0MaxSignalCenteredOnTimestep+t\right)^2}{2\, cd0MaxSignalTimestepStd^2}}\ kb^2}{cd0MaxSignalTimestepStd\left(b^2+kb^2\right)^2\sqrt{\pi}},\ -\lambda r-\frac{2\, r^3\, \sigma r}{\left(kr^2+r^2\right)^2}+\frac{2\, r\, \sigma r}{kr^2+r^2}\right\}$$

```
(*To be converted to prettier equations later*)
TeXForm[jacobianDs // MatrixForm]
```

Out[•]//TeXForm=
```
\left(
\begin{array}{ccc}
 -\text{$\lambda $p} & -\frac{2 b \text{kb}^2
   \text{$\sigma
   $p}}{\left(b^2+\text{kb}^2\right)^2} &
   \frac{2 r \text{$\sigma
   $p}}{\text{kr}^2+r^2}-\frac{2 r^3
   \text{$\sigma
   $p}}{\left(\text{kr}^2+r^2\right)^2} \\
 -\frac{2 \text{kb}^2 \text{kp}^2 \text{kr}^2
   p \text{$\sigma
   $b}}{\left(b^2+\text{kb}^2\right)
   \left(\text{kp}^2+p^2\right)^2
   \left(\text{kr}^2+r^2\right)} &
   -\frac{\text{bcr0MaxSignal} \text{kb}^2
   e^{-\frac{(t-\text{bcr0MaxSignalCenteredOn
   Timestep})^2}{2
   \text{bcr0MaxSignalTimestepStd}^2}}}{\sqrt
   {2 \pi } \text{bcr0MaxSignalTimestepStd}
   \left(b^2+\text{kb}^2\right)}+\frac{\sqrt{
```

```
   2} b^2 \text{bcr0MaxSignal} \text{kb}^2
   e^{-\frac{(t-\text{bcr0MaxSignalCenteredOn
   Timestep})^2}{2
   \text{bcr0MaxSignalTimestepStd}^2}}}{\sqrt
   {\pi } \text{bcr0MaxSignalTimestepStd}
   \left(b^2+\text{kb}^2\right)^2}-\frac{2 b
   \text{kb}^2 \text{kp}^2 \text{kr}^2
   \text{$\sigma
   $b}}{\left(b^2+\text{kb}^2\right)^2
   \left(\text{kp}^2+p^2\right)
   \left(\text{kr}^2+r^2\right)}-\text{$\lamb
   da $b} & -\frac{2 \text{kb}^2 \text{kp}^2
   \text{kr}^2 r \text{$\sigma
   $b}}{\left(b^2+\text{kb}^2\right)
   \left(\text{kp}^2+p^2\right)
   \left(\text{kr}^2+r^2\right)^2} \\
 0 & -\frac{\sqrt{2} b \text{cd0MaxSignal}
   \text{kb}^2
   e^{-\frac{(t-\text{cd0MaxSignalCenteredOnT
   imestep})^2}{2
   \text{cd0MaxSignalTimestepStd}^2}}}{\sqrt{
   \pi } \text{cd0MaxSignalTimestepStd}
   \left(b^2+\text{kb}^2\right)^2} & \frac{2
   r \text{$\sigma
   $r}}{\text{kr}^2+r^2}-\frac{2 r^3
   \text{$\sigma
   $r}}{\left(\text{kr}^2+r^2\right)^2}-\text
   {$\lambda $r} \\
\end{array}
\right)
```