Solutions to 1996 Physics GRE

Detailed Solutions to GR9677

Version 2.0

January 17, 2024

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Q	**A**	**%**	**Q**	**A**	**%**	**Q**	**A**	**%**	**Q**	**A**	**%**	**Q**	**A**	**%**
1	B	73	21	C	27	41	A	28	61	B	26	81	B	27
2	B	29	22	C	26	42	B	52	62	C	13	82	E	15
3	B	55	23	D	24	43	C	17	63	A	56	83	D	15
4	A	34	24	D	70	44	A	48	64	C	26	84	D	20
5	B	29	25	E	38	45	E	45	65	D	44	85	B	15
6	B	43	26	C	13	46	C	36	66	E	25	86	B	36
7	A	22	27	D	49	47	C	26	67	C	28	87	A	6
8	A	37	28	D	40	48	C	30	68	E	61	88	B	57
9	A	40	29	E	58	49	C	22	69	A	14	89	D	18
10	B	47	30	C	28	50	C	33	70	D	14	90	$*$	$*$
11	D	36	31	E	65	51	B	70	71	A	20	91	E	25
12	C	36	32	B	41	52	C	15	72	A	29	92	D	15
13	B	37	33	C	56	53	C	34	73	C	34	93	D	26
14	D	66	34	D	31	54	B	16	74	B	21	94	D	28
15	E	12	35	A	79	55	A	30	75	D	27	95	E	23
16	B	20	36	D	46	56	C	42	76	B	52	96	A	28
17	E	40	37	D	53	57	C	41	77	D	12	97	E	11
18	C	77	38	D	39	58	E	22	78	E	36	98	D	39
19	B	17	39	E	54	59	B	36	79	D	22	99	B	44
20	D	20	40	A	21	60	B	9	80	C	31	100	C	51

Q: Question **A**: Answer **%**: Percent Correct in 1996

Problem 1

The capacitor will discharge when disconnected from the voltage source which mean that the current will have an exponential decay curve. Therefore, we can immediately eliminate choices (A), (C), and (E).

Kirchoff's loop tells us that the initial circuit (when connected to a) is

$$V - I(t)r - \frac{Q(t)}{C} = 0 \tag{1}$$

Therefore, when the switch is connected to b we have

$$0 - I(t)R - \frac{Q(t)}{C} = 0 \quad \rightarrow \quad I(t) = \frac{Q(t)}{RC}$$

At $t = 0$ the capacitor has charge $Q_0 = CV$. Therefore,

$$\boxed{I(0) = \frac{Q_0}{RC} = \frac{V}{R}}$$

Answer: **B**

Problem 2

According to Faraday's Law the emf generated in the circuit is

$$\mathcal{E} = -\frac{\partial B}{\partial t}A \tag{2}$$

the problem tells us that the magnetic field, B, is **decreasing** in magnitude at the rate of $150\,\mathrm{T/s}$. We calculate the area, A, of the circuit as $0.01\,\mathrm{m}^2$

$$\mathcal{E} = -(-150\,\mathrm{T/s}) \cdot 0.01\,\mathrm{m}^2 = 1.5\,\mathrm{V}$$

We can then use Ohm's law to calculate the current through the circuit:

$$IR = V - \mathcal{E} \tag{3}$$

$$I(10\,\Omega) = 5.0\,\text{V} - 1.5\,\text{V} = 3.5\,\text{V} \quad \rightarrow \quad \boxed{I = \frac{3.5\,\text{V}}{10\,\Omega} = 0.35\,\text{A}}$$

Answer: **B**

Problem 3

The equation for electrostatic potential is

$$V = \int \vec{E} \cdot d\vec{l} = \int \frac{dq}{4\pi\epsilon_0 r} \tag{4}$$

where r is the distance from the charged ring to point P, which can be found via symmetry and the pythagorean theorem:

$$r^2 = R^2 + x^2 \quad \rightarrow \quad r = \sqrt{R^2 + x^2}$$

Therefore, the electric potential at point P is

$$V = \int \frac{dq}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + x^2}} = \frac{1}{4\pi\epsilon_0\sqrt{R^2 + x^2}} \int dq = \frac{Q}{4\pi\epsilon_0\sqrt{R^2 + x^2}}$$

Answer: **B**

Problem 4

The equation for angular frequency in SHM, ω, is

$$\omega = \sqrt{\frac{k}{m}} \tag{5}$$

where k is the spring constant and m is the mass of the particle. We can solve for k using the equation for force (equating Hooke's law with Coulomb's Law) and the potential found in the previous problem:

$$F = -kx = qE = -q \cdot \nabla V = -q\frac{dV}{dx} \tag{6}$$

$$kx = \frac{qQ}{4\pi\epsilon_0} \cdot \frac{d}{dx}\left(\frac{1}{\sqrt{R^2+x^2}}\right) = \frac{qQ}{4\pi\epsilon_0}\frac{x}{(R^2+x^2)^{3/2}}$$

$$\therefore \quad k = \frac{qQ}{4\pi\epsilon_0(R^2+x^2)^{3/2}}$$

Plugging this into our equation for angular frequency we get the the solution

$$\omega = \sqrt{\frac{1}{m}\frac{qQ}{4\pi\epsilon_0(R^2+x^2)^{3/2}}}$$

Since $R \gg x$ we can cancel out x which give us the answer

$$\boxed{\omega = \sqrt{\frac{qQ}{4\pi\epsilon_0 mR^3}}}$$

Answer: **A**

Problem 5

In order for the car to be traveling at a constant speed with F_{air} opposing its direction of travel, it must have a tangential acceleration in the direction of F_C. As it travels around the circular road, the car experiences a centripetal acceleration in the direction of F_A. Therefore, to find the force of the road on the tires we must add the forces due to the car's tangential and centripetal acceleration. Using simple vector addition, we find that

$$\boxed{F_{tires} = F_A + F_C = F_B}$$

Answer: **B**

Problem 6

The problem states that the block travels down the incline at a constant speed. Therefore, there is no change in the kinetic energy of the block. Since the block has potential energy $U = mgh$ at the top and $U = 0$ at the bottom of the incline, the energy dissipated by friction must be equal to $\boxed{mgh}$ (since ΔU is not transformed into kinetic energy).

Answer: **B**

Problem 7

The velocity of the center of mass follows the equation

$$V_{CM} = \frac{\sum_i m_i v_i}{\sum_i m_i} \tag{7}$$

$$\therefore \quad V_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m v_1 + 0}{3m} = \frac{v_1}{3}$$

We can find v_1 through conservation of energy, equating potential energy at the starting height, h, to the kinetic energy at the time on the collision:

$$\frac{1}{2} m v_1^2 = mgh \quad \rightarrow \quad v_1 = \sqrt{2gh}$$

In the CM frame, when dealing with elastic collisions, $v_i = v_f$. The kinetic energy right at collision, T, is therefore

$$T = \frac{1}{2} m V_{CM}^2 = \frac{1}{2} m \frac{v_1^2}{9} = \frac{mgh}{9}$$

We are looking for the height that the ball rises to after the collision, h'. Because energy is conserved the kinetic energy, T, is equal to the potential energy at h':

$$\frac{mgh}{9} = mgh' \quad \rightarrow \quad \boxed{h' = \frac{h}{9}}$$

Answer: **A**

Problem 8

The equation for simple harmonic motion comes from restoring force of the particle (Hooke's Law):

$$F = F_{rest} \quad \rightarrow \quad F - F_{rest} = m\ddot{x} + kx = 0 \tag{8}$$

Adding in the dampening force given in the problem statement give us

$$F = F_{rest} + f \quad \rightarrow \quad F - f - F_{rest} = m\ddot{x} + b\dot{x} + kx = 0 \tag{9}$$

Which has the characteristic equation:

$$m\omega^2 + b\omega + k = 0$$

using the quadratic equation we can find solutions for the frequency, ω

$$\omega = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

(notice that when $b = 0$ we get the SHM frequency $w = \sqrt{m/k}$)

The first term of this equation, $\left(-\frac{b}{2m}\right)$ is an exponentially decaying envelope. Therefore, in the presence of drag force the frequency of the oscillation is decreased which means that the period is increased (because $\omega \propto \frac{1}{T}$).

Answer: **A**

Problem 9

The problem calls for the use of the Rydberg formula for hydrogen:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{10}$$

The **Lyman** series has $n_f = 1$ (given) and $n_i = 2 \rightarrow \infty$ (not given)

The **Balmer** series has $n_f = 2$ (given) and $n_i = 3 \to \infty$ (not given)

The problem is asking about the longest wavelength. The longest wavelength is produced when n_i is at its minimum (because then the parenthetical part of the Rydberg formula is at a minimum). The ratio is therefore

$$\frac{\lambda_L}{\lambda_B} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} = \frac{\left(\frac{1}{4} - \frac{1}{9}\right)}{\left(\frac{1}{1} - \frac{1}{4}\right)} = \frac{\left(\frac{9}{36} - \frac{4}{36}\right)}{\left(\frac{3}{4}\right)} = \frac{\frac{5}{36}}{\frac{3}{4}} = \frac{20}{108} = \boxed{\frac{5}{27} = \frac{\lambda_L}{\lambda_B}}$$

Answer: **A**

Problem 10

Internal conversion is a radioactive decay process where the nucleus interacts with an orbital electron electromagnetically and causes that electron to be emitted. This is different from other processes in which the nucleus emits a particle after a nucleon decays. Since the problem expressly states that this is internal conversion, we can eliminate (C), (D), and (E).

The emitted electron leaves a hole in the electron shell which is subsequently filled by other electrons. By doing so, the electrons emit X-rays or Auger electrons (an outer-shell electron that is ejected due to the filling of a inner-shell vacancy, see Auger Effect for more). With this information it is clear that (B) is the best choice.

Answer: **B**

Problem 11

In 1922, German physicists Otto Stern and Walther Gerlach conducted an experiment which showed the quantization of electron spin had two orientations. In this experiment, the Stern-Gerlach experiment, a beam of silver atoms was passed through an inhomogeneous magnetic field and deflected **vertically into two beams** before hitting a detector screen. Silver atoms have the electron configuration:

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^1 \quad \text{or} \quad [\text{Kr}]\ 4d^{10} 5s^1$$

and neutral Hydrogen has the electron configuration:

$$1s^1$$

notice that both configurations have only one electron in the outer s orbital and therefore should behave similarly when passed through a non-uniform magnetic field.

Answer: **D**

Problem 12

The ground state energy of hydrogen is

$$E_{0,H} = -13.6\,\mathrm{eV} \propto \mu \tag{11}$$

where μ is the reduced mass of the system given by the equation

$$\mu = \frac{m_1 \cdot m_2}{m_1 + m_2} \tag{12}$$

For a normal hydrogen atom, $\mu \approx m_e$ (because $m_p \gg m_e$). For positronium:

$$\mu = \frac{m_e \cdot m_e}{m_e + m_e} = \frac{me}{2}$$

or 1/2 the reduced mass of hydrogen. Therefore the ground state energy of positronium is

$$E_{0,p} = \frac{-13.6\,\mathrm{eV}}{2} = \boxed{-6.8\,\mathrm{eV}}$$

Answer: **C**

Problem 13

This problem calls for the use of the specific heat equation

$$Q = cm\Delta T = Pt \tag{13}$$

where c is the specific heat, m is the mass, ΔT is the change in temperature, P is the power of the heating element, and t is the time. To find the mass of the water we use the equation density equation, knowing that the density of water is $\rho = 1000\,\mathrm{kg/m^3}$.

$$m = \rho V \tag{14}$$

$$m = 1000\,\mathrm{kg/m^3} \cdot 1\,\mathrm{L} = 1000\,\mathrm{kg/m^3} \cdot 0.001\,\mathrm{m^3} = 1\,\mathrm{kg}$$

Plugging this and the other given values into equation (13) and solving for time:

$$Pt = 4200\,\mathrm{J/kg} \cdot 1\,\mathrm{kg} \cdot 1\,\mathrm{K} = 100\,\mathrm{W} \cdot t$$

$$\rightarrow \quad t = \frac{4200\,\mathrm{J\,K}}{100\,\mathrm{J{\cdot}s}} = 42\,\mathrm{K/s} \approx 40\,\mathrm{K/s}$$

Therefore is takes approximately $40\,\mathrm{s}$ for the water to cool by $1^\circ\,\mathrm{C}$

Answer: **B**

Problem 14

The equilibrium temperature is the arithmetic mean of the two blocks, 50°C. The heat energy (equation (13)) transferred to the cold block from the hot block is therefore

$$Q = cm\Delta T = 0.1\,\mathrm{kcal/(kg\,K)} \cdot 1\,\mathrm{kg} \cdot 50\,\mathrm{K} = \boxed{5\,\mathrm{kcal}}$$

Answer: **D**

Problem 15

We need to look at each leg of the cycle individually, remembering the first law of thermodynamics

$$\Delta U = Q - W \tag{15}$$

and that ideal gasses follow the equations:

$$PV = nRT \tag{16}$$
$$\Delta U = C_v \Delta T \tag{17}$$
$$W = PdV \tag{18}$$

$A \to B$**:**

$$\Delta U = C_v \Delta T = 0$$

$$\therefore \quad Q_{AB} = W = PdV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \frac{V_2}{V_1}$$

$B \to C$**:**

$$\Delta U = C_v (T_c - T_h)$$

$$W = P(V_1 - V_2) = PV_1 - PV_2 = \frac{nRT_c V_1}{V_1} - \frac{nRT_h V_2}{V_2} = R(T_c - T_h)$$

$$\therefore \quad Q_{BC} = \Delta U + W = C_v(T_c - T_h) + R(T_c - T_h)$$

$C \to A$**:**

$$W = PdV = 0$$

$$\Delta U = C_v(T_h - T_c)$$

$$\therefore \quad Q_{CA} = C_v(T_h - T_c)$$

Therefore, the summation of all of the added heat energy is

$$Q_{AB} + Q_{BC} + Q_{CA} = nRT \ln \frac{V_2}{V_1} + C_v(T_c - T_h) + R(T_c - T_h) + C_v(T_h - T_c)$$

$$= nRT \ln \frac{V_2}{V_1} + C_v(T_c - T_h) + R(T_c - T_h) - C_v(T_c - T_h)$$

$$= nRT \ln \frac{V_2}{V_1} + R(T_c - T_h) = \boxed{nRT \ln \frac{V_2}{V_1} - R(T_h - T_c)}$$

This is a reversible, cycle process so $\Delta U_{ABCA} = 0$. Therefore, just adding up the work done during each leg would produce the same answer.

Answer: **E**

Problem 16

This problem is actually just a simple exercise in common sense. Knowing that the radius of an atom is about $10^{-10}\,\mathrm{m}$ eliminates choices (C), (D), and (E) since they are all less than or equal to this radius. Choice (A) is close to the width of human hair and thus is much larger than the expected mean free path at standard temperature and pressure, leaving us with (B).

The problem can also be solved through rigorous calculations: The number density, η, is the number of atoms per volume. We can calculate this value using the ideal gas law

$$\eta = \frac{N}{V} = \frac{P}{kT} \tag{19}$$

and using standard temperature and pressure values $T = 300\,\mathrm{K}$ and $P = 10^5\,\mathrm{Pa}$ (Boltzmann's constant should be given to you on your equation sheet, $k = 1.38 \times 10^{-23}\,\mathrm{m^2\,kg/(s^2\,K)}$)

$$\eta = \frac{N}{V} = \frac{P}{kT} = \frac{10^5\,\mathrm{kg/(m\,s^2)}}{1.38 \times 10^{-23}\,\mathrm{m^2\,kg/(s^2\,K)} \cdot 300\,\mathrm{K}} = 2.415 \times 10^{25}\,\mathrm{m^{-3}}$$

Now, the collision cross section is

$$\sigma = \pi r^2 \tag{20}$$

where r is the radius of an atom, $\sim 10^{-10}\,\mathrm{m}$.

$$\sigma = \pi r^2 = 3.141 \cdot 10^{-20}\,\mathrm{m^2} = 3.141 \times 10^{-20}\,\mathrm{m^2}$$

Therefore the mean free path is

$$\frac{1}{\eta\sigma} = \frac{1}{2.415 \times 10^{25}\,\mathrm{m^{-3}} \cdot 3.141 \times 10^{-20}\,\mathrm{m^2}} = \frac{1}{758839\,\mathrm{m^{-1}}} \approx \boxed{10^{-7}\,\mathrm{m}}$$

Answer: **B**

Problem 17

The probability that the particle in the range $0 < x < 5$ is

$$\psi * \psi\Big|_0^5 = |\psi|^2\Big|_0^5 = 1^2 + 1^2 + 2^2 + 3^2 + 1^2 = 1 + 1 + 4 + 9 + 1 = 16$$

Notice that this is not the same as adding up the squares and squaring it. The probability that the particle in the range $2 < x < 4$ is

$$|\psi|^2\Big|_2^4 = 2^2 + 3^2 4 + 9 = 13$$

Therefore the probability is $\boxed{\frac{13}{16}}$

Answer: **E**

Problem 18

Lets look at each of these answers individually

(A) The wave function of a particle oscillates in free space. This diagram is ground state particle in an infinite square well. This is **incorrect**.

(B) As expected, the amplitude of the wave function is decreased inside the potential. However, since $E < V_0$ the transmitted particle's wave function should have a lower amplitude than the incident wave function. This diagram is correct for the case where $E > V_0$ but is an **incorrect** solution to this problem.

(C) This is the only diagram in which the wave function is oscillating before incidence, decaying while inside the potential, and have a transmitted wave function which oscillates with a smaller amplitude. This is **correct** because it carries the asymmetry expected for the case where $E < V_0$.

(D) This picture states that the particle is most likely to be found inside the potential which is **incorrect** for a typical particle.

(E) The potential barrier is not changing the wave function of the particle in anyway and so must be **incorrect**.

Answer: **C**

Problem 19

The best way to get the correct answer is to eliminate incorrect solutions. For instance, Since the backscatter is such a large angle we know that the coulomb repulsion much be very large which means that the distance of closest approach is going to very small, probably around the same order as a nucleus ($\sim 10^{-15}\,\text{m}$). Therefore, we can eliminate solutions (C), (D), and (E).

Now, (A) could probably be eliminated because it is so strange but lets actually figure out the value: $50^{1/3}$ is between 3 and 4 (closer to 4: $3^3 = 27, 4^3 = 64$) so it is somewhere around $1.22 \cdot 4 \approx 5$. Therefore (A) is about $5\,\text{fm} = 5 \times 10^{-15}\,\text{m}$ which is less than the size of a large nucleus like silver. Therefore, (B) is the best choice.

Another, tougher, solution to the problem: Coulomb's law states that the potential electric potential energy is

$$V = \frac{(Z_1 q_1)(Z_2 q_2)}{4\pi\epsilon_0 r} \tag{21}$$

Because the alpha particle is scattered at an angle of 180° we know that energy is conserved so that

$$E = 5\,\text{MeV} = 5 \times 10^6\,\text{eV} = \frac{(Z_1 q_1)(Z_2 q_2)}{4\pi\epsilon_0 r} = \frac{Z_\alpha Z_{Ag} q^2}{4\pi\epsilon_0 r} = \frac{2 \cdot 50 \cdot q^2}{4\pi\epsilon_0 r} = \frac{100 \cdot q^2}{4\pi\epsilon_0 r}$$

Now, an electron volt is exactly what it should like: the electron charge multiplied by one volt. So, because $q = |e|$ we can rewrite this equation as

$$5 \times 10^6\,\text{V} = \frac{100 \cdot q}{4\pi\epsilon_0 r}$$

Now, the term $(4\pi\epsilon_0)^{-1}$ is sometimes written as the constant k which is equal to $9 \times 10^9\,\text{s}^4\,\text{A}^2/(\text{m}^3\,\text{kg})$ (NOT the Boltzmann constant). Lastly, the charge of an proton is $q = 1.6 \times 10^{-19}\,\text{C}$. Now we can solve for r:

$$r = \frac{100 \cdot k \cdot q}{5 \times 10^6\,\text{V}} = \frac{100 \cdot 9 \times 10^9\,\text{s}^4\,\text{A}^2/(\text{m}^3\,\text{kg}) \cdot 1.6 \times 10^{-19}\,\text{C}}{5 \times 10^6\,\text{V}} = 2.88 \times 10^{-14}\,\text{m} \approx 2.9 \times 10^{-14}\,\text{m}$$

Answer: **B**

Problem 20

This is an elastic collision and so the kinetic energy beforehand is the same as the kinetic energy afterwards.

$$T_{H,i} + T_{A,i} = T_{H,f} + T_{A,f}$$

$$\frac{1}{2}m_H v^2 + 0 = \frac{1}{2}m_H(0.6v)^2 + \frac{1}{2}m_A v'^2$$

$$(1 - 0.6^2)m_H v^2 = (1 - 0.36)m_H v^2 = 0.64 m_H v^2 = m_A v'^2$$

No matter what type of collision, the momentum of the system is conserved. We can use this fact to determine v':

$$m_H v = m_A v' - 0.6 m_H v \quad \rightarrow \quad m_A v' = 0.6 m_H v + m_H v$$

$$v' = \frac{1.6 m_H v}{m_A}$$

Therefore,

$$0.64 m_H v^2 = m_A v'^2 = m_A \left(\frac{1.6 m_H v}{m_A}\right)^2 = \frac{(1.6 m_H v)^2}{m_A} \quad \rightarrow \quad 0.64 m_A = 1.6^2 m_H$$

$$\therefore \quad m_A = \frac{1.6^2 m_H}{0.64} = \frac{1.6^2 m_H}{0.8^2} = 4 m_H$$

The problem states that $m_H = 4u$. Therefore, $m_A = 4(4u) = \boxed{16u}$

Answer: **D**

Problem 21

Recall the parallel axis theorem:

$$I = I_{CM} + mr^2 \tag{22}$$

Where r is the distance from the from the point to the center of mass. In this problem, the circular hoop is hanging form a nail so that $r = d = 20\,\mathrm{cm}$. The moment of inertia for a hoop (thin hollow cylinder) is $I_{hoop} = mr^2$.Therefore, the moment of inertia for the set up is

$$I = I_{hoop} + mr^2 = md^2 + md^2 = 2md^2$$

Plugging this into the given equation for the period we get

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{2md^2}{mgd}} = 2\pi\sqrt{\frac{2d}{g}}$$

Unfortunately, we now need to plug in numbers. Using $\pi \approx 3$ and $g \approx 10$ we can approximate the solution

$$T = 2 \cdot 3\sqrt{\frac{2 \cdot 0.2\,\mathrm{m}}{10\,\mathrm{m/s^2}}} = 6\sqrt{0.04\,\mathrm{s}^2} = 1.2\,\mathrm{s} \approx \boxed{1.3\,\mathrm{s}}$$

Answer: **C**

Problem 22

We need to start by calculating the radius of Mars. We are told that for ever 3600 m the surface drops 2 m. We can picture this as a right triangle with legs 3600 m, $R - 2\,\mathrm{m}$ and a hypotenuse R (which is the radius of the planet). Using the pythagorean theorem to solve for R:

$$R^2 = (3600\,\mathrm{m})^2 + (R - 2\,\mathrm{m})^2 = 3600^2\,\mathrm{m}^2 + R^2 - 4\,\mathrm{m} \cdot R + 4\,\mathrm{m}^2$$

$$\rightarrow \quad 4\,\mathrm{m} \cdot R = 3600^2\,\mathrm{m}^2 + 4\,\mathrm{m}^2$$

$$\rightarrow \quad R = \frac{3600^2\,\mathrm{m}^2}{4\,\mathrm{m}} + 1\,\mathrm{m}^2 \approx \frac{3600^2\,\mathrm{m}}{4}$$

In order to get a golf ball to orbit Mars the force of gravity and the centripetal force must be equal.

$$F_{cent} = F_{grav,M} \quad \rightarrow \quad m\frac{v^2}{R} = mg_M$$

$$\therefore \quad v = \sqrt{g_M R} = \sqrt{0.4gR} = \sqrt{4R} = \sqrt{4\,\mathrm{m/s^2} \cdot \frac{3600^2\,\mathrm{m}}{4}} = 3600\,\mathrm{m/s} = \boxed{3.6\,\mathrm{km/s}}$$

Answer: **C**

Problem 23

Process of elimination:

(A) The new gravitational force law does not introduce any damping or frictional forces so the total mechanical energy should still be conserved. This is **not necessarily false**.

(B) Angular momentum is always conserved in planetary orbits. The problem's proposed gravitational force law does not change that. This is **not necessarily false**.

(C) Kepler's Third Law states that the square of planet's period, T, is proportional to the the cube of the orbit's semi-major axis.

$$T^2 \propto a^3 \quad \rightarrow \quad T \propto a^{3/2} \tag{23}$$

For a circular orbit, $a = r$. Therefore, tith the addition of the small positive number, ϵ, Kepler's Third Law reads as

$$T^2 \propto r^{3+\epsilon} \quad \rightarrow \quad T \propto r^{(3+\epsilon)/2}$$

This is **not false**.

(D) There are only two types of central force potentials what allow **all** stable non-circular orbits. These are inverse square central force potentials where

$$V \propto \frac{1}{r}$$

or radial harmonic oscillator potentials where

$$V = \frac{1}{2}kr^2$$

This is a statement of Bertrand's theorem.

The problem's given central force equation is not an inverse-square potential or a radial harmonic oscillator so it can not have a stable non-circular orbit about the sun. This is **false**.

(E) A stable circular orbit can exist in a potential, V, if the following conditions are satisfied:

$$V'_{eff} = 0 \tag{24}$$
$$V''_{eff} > 0 \tag{25}$$

where

$$V_{eff} = \frac{L^2}{2Mr^2} + V(r) = \frac{L^2}{2Mr^2} - \int F dx \tag{26}$$

From here, one could derive the V_{eff} and see that both of the above conditions are met. This, however, would be a huge waste of time on the test. Since we already know that (D) is the correct answer and that circular orbits can exist for basically any potential we should not do more work than is needed.

Answer: **D**

Problem 24

The equation for the coulomb force is

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = k\frac{q^2}{r^2}$$

When sphere C comes in contact with A, it is imbued with half of q so that $q_{C,1} = q_A = \frac{q}{2}$

When C is then touched to B both spheres come away with half of the total charge on each sphere is

$$q_B = q_{C,2} = \frac{1}{2}(q + q_{C,1}) = \frac{1}{2}\left(\frac{2q}{2} + \frac{q}{2}\right) = \frac{3q}{4}$$

Therefore, the force between A and B is now

$$F = k\frac{q_A \cdot q_B}{r^2} = k\frac{3q^2}{8r^2} = \boxed{\frac{3F}{8}}$$

Answer: **D**

Problem 25

Lets look at each answer individually:

(A) The voltage across two capacitors in parallel is equivalent. When the switch is open C_1 carries the charge Q_0 and C_2 is uncharged. Therefore, conservation of charge tells us that this must be **correct**

(B) V is equivalent across both capacitors and $C_1 = C_2$ so the equation

$$Q = CV \tag{27}$$

Tells us that this must be **correct**.

(C) Once again, the voltage across two capacitors in parallel is equivalent. This is therefore **correct**.

(D) The equation for energy stored in a capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} \tag{28}$$

From (A), (B), and (C) we know that this must be **correct** which leaves

(E) From equation (28) we know that $U_0 = \frac{1}{2}CV^2$. When the switch is closed, each capacitor has energy $U = \frac{1}{2}CV^2$ so that

$$U_{tot} = U_1 + U_2 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = 2U_0 \neq U_0$$

So this is the only **incorrect** statement.

Answer: **E**

Problem 26

The hardest part of this problem is the fact that you must do all the calculations without a calculator. The resonance frequency of an LRC circuit is

$$\omega^2 = \frac{1}{LC} \tag{29}$$

The problem tells us that the frequency of broadcast is $f = 103.7\,\mathrm{MHz}$ which means that the resonance frequency is

$$\omega = 2\pi f = 2\pi \cdot 103.7\,\mathrm{MHz} \approx 6.2 \cdot 103.7 \times 10^6\,\mathrm{Hz} \approx 650 \times 10^6\,\mathrm{Hz}$$

solving equation (29) for C and plugging in ω we get

$$C = \frac{1}{L\omega^2} = \frac{1}{2.0 \times 10^{-6}\,\mathrm{H} \cdot (650 \times 10^6\,\mathrm{Hz})^2} = \frac{1}{8.45 \times 10^{11}\,\mathrm{m}^2\,\mathrm{kg}/(\mathrm{s}^4\,\mathrm{A}^2} = 1.18 \times 10^{-12}\,\mathrm{F} \approx \boxed{1\,\mathrm{pf}}$$

Notice that we don't care about the resistance because a radio circuit has all of its components in parallel.

Answer: **C**

Problem 27

Any physical phenomenon that can be described using an exponential is best expressed using a Log or semi-log diagram. Therefore, we can eliminate (A), (C), and (E).

Choice (B) can be rewritten into the form

$$f = \frac{e}{h}V_s + \frac{W}{h}$$

which is in the form $fx = mx + b$, a linear function. Therefore (D) is the only graph that is not appropriate for the mathematical relation.

Answer: **D**

Problem 28

The simple way to solve this problem is to find the beat frequency, the difference between the two interfering waves' frequencies. From the graph, we see that the wavelength is $\lambda \approx 1\,\mathrm{cm}$. Since we are given the velocity, v, of the spot we can use the equation $v = \lambda f$ to find the beat frequency:

$$f = \frac{v}{\lambda} = \frac{0.5\,\mathrm{cm/ms}}{1\,\mathrm{cm}} = 0.5\,\mathrm{ms}^{-1} = 500\,\mathrm{Hz}$$

Which is closest to the difference in frequencies of (D).

Answer: **D**

Problem 29

The plank length is

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \tag{30}$$

For students that have not learned this in their undergraduate career: dimensional analysis will also give you the correct answer, (E).

$$[G] = \left[\frac{m^3}{kg \cdot s^2}\right] \tag{31}$$

$$[\hbar] = \left[\frac{m^2 \cdot kg}{s}\right] \tag{32}$$

$$[c] = \left[\frac{m}{s}\right] \tag{33}$$

But don't waste your time looking at the dimensions of each solution! Knowing that l_p must have units of meters tells us that the kilograms in both G and $\hbar$ must cancel out. This eliminates (B),(C), and (E). Dimensional analysis of (A) gives us:

$$[G\hbar c] = \left[\frac{m^3}{kg \cdot s^2}\right] \cdot \left[\frac{m^2 \cdot kg}{s}\right] \cdot \left[\frac{m}{s}\right] = \left[\frac{m^5}{s^4}\right]$$

Which is incorrect. Therefore, that (E) is the correct answer. For the sake of completeness:

$$\left[\frac{G\hbar}{c^3}\right]^{\frac{1}{2}} = \left(\left[\frac{m^3}{kg \cdot s^2}\right] \cdot \left[\frac{m^2 \cdot kg}{s}\right] \cdot \left[\frac{s^3}{m^3}\right]\right)^{\frac{1}{2}} = \left[m^2\right]^{\frac{1}{2}} = [m]$$

Answer: **E**

Problem 30

The pressure due to a fluid follows the equation

$$P = \rho g h \tag{34}$$

Let us say that x is the distance that the water is displaced after the dark fluid is introduced to the system. h_1 can be defined as:

$$h_1 = h_d + (h_{1,i} - x) = 5\,\text{cm} + (20\,\text{cm} - x)$$

Then h_2 is

$$h_2 = h_{2,i} + x = 20\,\text{cm} + x$$

We then combine these two equations to get

$$h_1 + h_2 = [5\,\text{cm} + (20\,\text{cm} - x)] + [20\,\text{cm} + x] = 45\,\text{cm}$$

$$h_1 - h_2 = [5\,\text{cm} + (20\,\text{cm} - x)] - [20\,\text{cm} + x] = 5\,\text{cm} - 2x$$

Therefore:

$$2 \cdot h_1 = 50\,\text{cm} - 2x \quad \rightarrow \quad h_1 = 25\,\text{cm} - x$$

$$2 \cdot h_2 = 40 + 2x \quad \rightarrow \quad h_2 = 20\,\text{cm} + x$$

Then assign any number to x (I'll use $x = 10\,\text{cm}$) and solve h_2/h_1:

$$\frac{h_2}{h_1} = \frac{20\,\text{cm} + 10\,\text{cm}}{25\,\text{cm} - 10\,\text{cm}} = \frac{30\,\text{cm}}{15\,\text{cm}} = \boxed{\frac{2}{1}}$$

Answer: **C**

Problem 31

Process of elimination:

(A) As the sphere falls under the force of gravity, it is velocity is increased due to the acceleration of gravity. As the velocity increases the kinetic energy increases. So this is **incorrect**.

(B) The retarding force keeps the sphere from exceeding the terminal velocity but does not stop it. Therefore, the kinetic energy is not decreased to zero due to the retarding force. This is **incorrect**.

(C) The terminal velocity is the maximum velocity the sphere can achieve in the presence of a retarding force. Therefore, this solution makes no physical sense and is **incorrect**.

(D) The force equation is

$$F_{net} = ma = bv_t - mg \quad \rightarrow \quad v_t = \frac{ma + mg}{b}$$

Obviously, the terminal speed, v_t, does depend on both b and m. So this is **incorrect**.

(E) This is the **correct** answer. See argument in (D)

Answer: **E**

Problem 32

The rotational kinetic energy follows the equation:

$$T = \frac{1}{2}I\omega^2 \tag{35}$$

For this problem we must compare the kinetic energy at A and B:

$$\frac{T_B}{T_A} = \frac{I_B}{I_A} \tag{36}$$

Therefore, we must find I_A and I_B. For I_A, simply use the summation:

$$I_A = \sum_i m_i r_i = 3mr$$

where r is the distance from the center of the triangle to the corner: $l \cos\left(\frac{\pi}{3}\right) = \frac{l}{3}$. To find I_B we must use the parallel axis theorem:

$$I_B = I_{cm} + \sum_i m_i r_i = I_A + 3mr = 2I_A$$

Therefore,

$$\frac{T_B}{T_A} = \frac{I_B}{I_A} = \frac{2I_A}{I_A} = \boxed{2}$$

Answer: **B**

Problem 33

Quantum mechanical probability is always found using the equation

$$P = \int |\langle\psi|\psi\rangle|^2 dx \tag{37}$$

The question asks for the probability of obtaining the result $l = 5$ which has the wavefunction

$$\langle\psi| = \frac{(3Y_5^1 + 2Y_5^{-1})}{\sqrt{38}}$$

which then gives us the probability

$$P_{l=5} = \frac{3^2 + 2^2}{38} = \frac{9+4}{38} = \boxed{\frac{13}{38}}$$

Answer: **B**

Problem 34

We can and should immediately eliminate choices (A) and (B) since gauge and and time invariance stem from the conservation of energy and charge, respectively. The violation of either of these is therefore physically impossible.

Translational and rotational invariance deal with conservation of momentum and angular momentum. Therefore, the answer must be (D) since it is the only solution that does not break any conservation laws.

Answer: **D**

Problem 35

You should go into the test knowing that the Pauli-Exclusion principle states that no two fermions can occupy the same quantum state. This comes from the fact that the total wave function of two identical fermions is antisymmetric under particle exchange.

Answer: **A**

Problem 36

This calls for the use of the relativistic energy equation:

$$E = mc^2 = \gamma m_0 c^2 \tag{38}$$

where γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{39}$$

Since the lumps of clay are traveling at $v = \frac{3c}{5}$, they both have

$$\gamma = \left(1 - \frac{9}{25}\right)^{-1} = \left(\frac{16}{25}\right)^{-1} = \frac{5}{4}$$

when the masses collide and are at rest they have total mass M and energy Mc^2. Setting this equal to the summation of each lump's energy we get

$$2\gamma m_0 c^2 = Mc^2 \quad \rightarrow \quad 2\gamma m_0 = M = 2 \cdot \frac{5}{4} \cdot 4\,\mathrm{kg} = \boxed{10\,\mathrm{kg}}$$

Answer: **D**

Problem 37

This is a simple application of the relativistic velocity addition formula:

$$u' = \frac{u + v}{1 + \frac{uv}{c^2}} \tag{40}$$

where v is the speed of the atom, u is the speed of the emitted electron, and u' is the speed of the electron in the lab frame.

$$u' = \frac{0.6c + 0.3c}{1 + 0.18 \cdot \frac{c^2}{c^2}} = \frac{0.9c}{1.18} = \frac{90}{118}c = \boxed{0.76c}$$

you don't actually need to now that $90/118 = 0.76$, you can just look at the answers. it must be less than (E) but greater than (A), (B), and (C). So it must be (D).

Answer: **D**

Problem 38

The total relativistic energy and momentum equations are

$$E = \gamma mc^2 \tag{41}$$
$$p = \gamma mv \tag{42}$$

We know that $p = 5\,\mathrm{MeV/c}$ and $E = 10\,\mathrm{MeV}$ so that. We can solve for v by dividing these two equations

$$\frac{p}{E} = \frac{\gamma mv}{\gamma mc^2} = \frac{v}{c^2} = \frac{5\,\mathrm{MeV/c}}{10\,\mathrm{MeV}} = \frac{1}{2c} \quad \rightarrow \quad \boxed{v = \frac{c}{2}}$$

Answer: **D**

Problem 39

Ionization potential is the amount of energy required to remove an electron from an atom of molecules. Noble gases (He, Ne, Ar, Kr, Xe, and Rn) all have full outer orbital shells which means that they have a high ionization potential. Therefore, since they are neither atom is an ion, we can immediately eliminate (A) and (D).

O and N have the majority of their outer shell filled and so they have a much higher potential then Cs which only have one electron in its outer shell. Because Cs is an alkali metal, it has a low ionization potential.

Answer: **E**

Problem 40

E_f of this singly ionized Helium atom (Z=2) can be calculated using the formula

$$E = E_f - E_i = \frac{Z^2 E_0}{n_f^2} - \frac{Z^2 E_0}{n_i^2} = Z^2 E_0 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = \tag{43}$$

where $E_0 = 13.6\,\text{eV}$ since this is a hydrogen-like atom. The total energy E is defined by the simple equation

$$E = \frac{hc}{\lambda} \tag{44}$$

You should memorize the fact that $hc = 1240\,\text{eV}\,\text{nm}$ becuase it is often used in these tests. With this number, and the given wavelength, we can solve for energy released:

$$E = \frac{1240\,\text{eV}\,\text{nm}}{470\,\text{nm}} \approx 2.6\,\text{eV}$$

Going back to equation (43) we can now solve for n_f

$$\frac{E}{Z^2 E_0} = \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad \rightarrow \quad \frac{E}{Z^2 E_0} + \frac{1}{n_i^2} = \frac{1}{n_f^2}$$

$$\frac{2.6\,\text{eV}}{2^2 \cdot 13.6\,\text{eV}} + \frac{1}{4^2} = \frac{2.6\,\text{eV}}{54.4\,\text{eV}} + \frac{1}{16} \approx 0.047 + 0.063 = 0.11 = \frac{1}{9} = \frac{1}{n_f^2} \quad \rightarrow \quad \boxed{n_f = 3}$$

The math is tricky to do in your head but making approximations like $1/16 \approx 6\%$ and $26/544 \approx 4\%$ will give you $1/n_f^2 = 1/10$ which is closest to $n_f = 3$. To check our work lets plug n_f to see what we get for E_f

$$E_f = \frac{Z^2 E_0}{n_f^2} = \frac{4 \cdot 13.6\,\text{eV}}{9} = \frac{54.4\,\text{eV}}{9} = \boxed{6}$$

Answer: **A**

Problem 41

Lets talk about spectroscopic notation: The general form is $N^{2s+1}L_j$ where N is the principle quantum number (often omitted), s is the total spin quantum number ($m = 2s + 1$ is the number of quantum states), L is the orbital angular momentum quantum number (but is written as $L = S,\ P,\ D,\ F,\ ...$ instead of $l = 0,\ 1,\ 2,\ 3,\ ...$), and j is the total angular momentum quantum number

Remembering the selection rules

$$\Delta l = \pm 1 \tag{45}$$
$$\Delta m = -1, 0, 1 \tag{46}$$

lets look at each of these problems individually:

(A) This is **allowed** because $\Delta l = -1$ ($P \to S$) and $\Delta m = -1$ ($3 \to 2$)

(B) While $\Delta m = -1$ works, $\Delta l = 0$ which is **not allowed**.

(C) Remember that $L = P$ is just another way of saying $l = 1$. Therefore this is **not an allowed solution** since $l = 3$ corresponds to the $L = F$

(D) An electron cannot have $j = l$ or $s = 3/2$ because it has spin value $s = 1/2$. So this is **not allowed**.

(E) While this could possibly be true, it would take far too much time to try and prove mathematically and solution (A) is known to be correct.

Answer: **A**

Problem 42

Einstein's equation for the photoelectric effect is

$$E = h\nu + \phi \tag{47}$$

where $h\nu$ is the kinetic energy of the of the ejected electrons. We can determine the energy of the incident photons using equation (44)

$$E = \frac{1240\,\mathrm{eV\,nm}}{500\,\mathrm{nm}} \approx 2.5\,\mathrm{eV}$$

$$\therefore \quad h\nu = E + \phi = 2.5\,\mathrm{eV} - 2.28\,\mathrm{eV} \approx \boxed{0.2\,\mathrm{eV}}$$

Answer: **B**

Problem 43

Remember that we can allows use Stokes Theorem to express a closed boundary line integral as an integral over its area:

$$\oint \vec{F} \cdot d\vec{r} = \iint \left(\nabla \times \vec{F}\right) d\vec{A} \tag{48}$$

So lets solve the right hand side of this equation using $\vec{F} = \vec{u}$:

$$\iint \left(\nabla \times \vec{F}\right) d\vec{A} = \left(\nabla \times \vec{F}\right) \vec{A} = (2) \cdot \pi R^2 = \boxed{2\pi R^2}$$

Answer: **C**

Problem 44

Acceleration is just the first derivative of the velocity with respect to time. Because v is not directly dependent on t so we must us the chain rule:

$$a = \dot{v} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v = -n\beta x^{-n-1} \cdot \beta x^{-n} = \boxed{-n\beta^2 x^{-2n-1}} \tag{49}$$

Answer: **A**

Problem 45

The problem is asking which set up is a high pass filer. High pass and low pass filters a created using a capacitor and resistor or an inductor and resistor in different configurations. Therefore, immediately eliminate (B) and (C).

A high-pass filter is a series combination of a **Capacitor followed by a Resistor** or a **Resistor followed by an Inductor** (CR or RL)

The inverse is a low-pass filter: a series combination of a**Resistor followed by a Capacitor** or an **Inductor followed by a Resistor** (RC or LR)

Therefore, (E) is the only high pass configuration.

Answer: **E**

Problem 46

Recall Faraday's law using the magnetic flux $\Phi = B \cdot dA$:

$$\mathcal{E} = -\frac{d\Phi}{dt} = -B\frac{dA}{dt} \tag{50}$$

Plugging in the give values gives us

$$\mathcal{E} = \varepsilon_0 \sin{(\omega t)} = -B\frac{d}{dt}\cos{(wt)}\pi R^2 = B\omega\sin{(\omega t)}\pi R^2$$

$$\therefore \quad \varepsilon_0 = B\omega\pi R^2 \quad \rightarrow \quad \boxed{\omega = \frac{\varepsilon_0}{B\pi R^2}}$$

Answer: **C**

Problem 47

Yet another application of Faraday's Law, equation (50). This time, $A = \pi R^2 n(t)$ where n(t) is the number of turns in the wire. Therefore, the voltage induced by the magnetic field (EMF) is

$$\mathcal{E} = -B\pi R^2\frac{dn}{dt} = -B\pi R^2 N \tag{51}$$

and so the potential difference between the two open ends in $\boxed{\pi NBR^2}$

Answer: **C**

Problem 48

Recall the invariance low of general relativity

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 \tag{52}$$

Let us say that S is the lab frame and S' is the frame of the π^+ meson.

$$-c^2\Delta t'^2 = \Delta x^2 - c^2\Delta t^2 \quad \rightarrow \quad c^2\Delta t^2 = c^2\Delta t'^2 + \Delta x^2 \tag{53}$$

The problem tells us that $t' = 2.5 \times 10^{-8}\,\mathrm{s}$, $x = 15\,\mathrm{m}$, and you should know that $c = 3 \times 10^8\,\mathrm{m/s}$. Plugging in:

$$c^2\Delta t^2 = \left(3 \times 10^8\,\mathrm{m/s} \cdot 2.5 \times 10^{-8}\,\mathrm{s}\right)^2 + (15\,\mathrm{m})^2 = (7.5\,\mathrm{m})^2 + (15\,\mathrm{m})^2$$

$$= \left(\frac{15\,\mathrm{m}}{2}\right)^2 + (15\,\mathrm{m})^2 = (15\,\mathrm{m})^2\left(\frac{1}{2^2} + 1\right) = (15\,\mathrm{m})^2\left(\frac{5}{4}\right)$$

$$\therefore \quad \Delta t = 15\,\mathrm{m}\left(\frac{\sqrt{5}}{2c^2}\right) \tag{54}$$

But since we are looking for the velocity of the mesons in the lab frame we must divide the distance, x, by the time, t.

$$v = \frac{x}{t} = \frac{15\,\mathrm{m}}{15\,\mathrm{m}\left(\frac{\sqrt{5}}{2c^2}\right)} = \boxed{\frac{2c}{\sqrt{5}}} \tag{55}$$

Answer: **C**

Problem 49

Recall the equation for electric field of an infinite charged surface:

$$E = \frac{\sigma}{2\epsilon_0} \tag{56}$$

Now, the surface charge density is defined by the equation.

$$\sigma = \frac{Q}{A} = \frac{Q}{\hat{x} \times \hat{y}}$$

Here we are looking at a unit cell to represent the area, $A = 1\hat{x} \times 1\hat{y}$. Because second observer is moving in the x direction there is a length contraction in along the x-axis:

$$\hat{x}' = \frac{\hat{x}}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}\hat{x}$$

Therefore, the second observer measures an electric field equal to

$$E' = \frac{1}{2\epsilon_0} \frac{Q}{\sqrt{1 - \frac{v^2}{c^2}}\hat{x} \times \hat{y}} = \boxed{\frac{\sigma}{2\epsilon_0\sqrt{1 - \frac{v^2}{c^2}}}\hat{z}}$$

Answer: **C**

Problem 50

Going back to equation (52) we have

$$\Delta x^2 = \Delta x'^2 - c^2 \Delta t'^2$$

$$(3c)^2 = (5c)^2 - c^2 \Delta t'^2 \quad \rightarrow \quad \Delta t' = \sqrt{\Delta t'^2} = \sqrt{25 - 9} = \sqrt{16} = \boxed{4\,\text{min}}$$

Answer: **C**

Problem 51

Recall that the probability density is $|\psi|^2$ and the wave function for an infinite well of size l is

$$\psi_n = \sqrt{\frac{2}{l}} \sin\left(\frac{n\pi x}{l}\right) \tag{57}$$

because we are interested in the probability density in the middle of the well, let us set $x = l/2$ so that

$$\psi_n = \sqrt{\frac{2}{l}} \sin\left(\frac{n\pi}{2}\right)$$

the probability density vanishes when $\psi_n = 0$ and only even values for n meet this criteria (because $\sin(\pi) = 0$).

Answer: **B**

Problem 52

Like many of the physics GRE problems, there is both an easy way and a hard way to solve this problem. The easy way is to commit the first few spherical harmonics (Y_l^m) to memory.

$$Y_0^0(\theta, \phi) = \frac{1}{2}\sqrt{\frac{1}{\pi}} = const \tag{58}$$

$$Y_1^0(\theta, \phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cos\theta = const \cdot \cos\theta \tag{59}$$

$$Y_1^{\pm 1}(\theta, \phi) = \frac{1}{2}\sqrt{\frac{3}{4\pi}} \sin\theta \cdot e^{\pm i\phi} = const \cdot \cos\theta \cdot e^{\pm i\phi} \tag{60}$$

Noting the that given equation is equivalent to $Y_1^{\pm 1}$ tells us that $m = \pm 1$. We can then find the eigenstates of the z-component of angular momentum using the equation

$$L_z Y_l^m = m\hbar Y_l^m \tag{61}$$

So that $L_z = \pm\hbar$. Solution (C).

The harder way to solve this problem is to start with the equation for the angular momentum operator in the z-direction, L_z:

$$L_z = -i\hbar \frac{\partial}{\partial \phi} \tag{62}$$

combining this with equation (61) we get

$$-i\hbar \frac{\partial}{\partial \phi} Y_l^m = m\hbar Y_l^m \quad \rightarrow \quad \frac{\partial (Y_l^m)}{\partial \phi} = \frac{m}{-i} \cdot Y_l^m = imY_l^m$$

$$\therefore \quad Y_l^m = Ae^{im\phi}$$

Where A is some constant. Equating this to the given wave function we get

$$Y_l^m(\theta, \phi) = \psi(\theta, \phi)$$

$$Ae^{im\phi} = \sqrt{3/4\pi} \sin\theta \sin\phi$$

Therefore, all the information about m is contained in the $\sin\phi$ term. Recalling the identity

$$\sin\phi = \frac{e^{i\phi} - e^{-i\phi}}{2i} \tag{63}$$

we can see that $m = \pm 1$. Therefore, from equation (61), we see that $L_z = \pm\hbar$

Answer: **C**

Problem 53

Because positronium is unstable the two particles (electron and positron) will quickly annihilate with one another, producing photons. We can therefore eliminate choice (A). Due to conservation of momentum we know that the annihilation cannot produce only one photon, so we are left with choices (C), (D), and (E).

There are two types of positronium:

Para-positronium: Singlet state with antiparallel spins ($S = 0$, $M_s = 0$)

Ortho-positronium: Triplet state with parallel spins ($S = 1$, $M_s = -1, 0, 1$)

The problem is asking about Para-positronium which can only decay an even number of photons (due to C-symmetry rules). Therefore, we can have either 2 or 4 photons produced in this decay. However, the probability for decay into 2 photons is much higher than the probability of decay into 4 photons so the best answer is (C).

(In case you were wondering: ortho-positronium decays into an odd number of photons with 3 being the most probable number)

Answer: **C**

Problem 54

Lets ignore the phase shift π in the $\hat{y}$ term for now. Since $E_1 = E_2$ the slope of the trajectory is 1 and the so, from $y = mx$, the angle is $\theta = 45°$ (Quadrant I). With this information we can eliminate (C), (D), and (E).

Now, because the phase shift does occur, we are dealing with a trajectory the follows the equation $-y = mx$ since the shift moves the line second term from $\hat{y}$ to $-\hat{y}$. Therefore we now have $\theta = -45°$ (Quadrant IV).

Heres the tricky part: the final angle is $45°$ away from the $+x$-axis but in the negative direction and so it is actually $315°$ away from the $+x$-axis! This is because of convention, which has the angle increasing in the counter-clockwise direction. Because $315°$ is not an option we must continue the line through the origin and into Quadrant II where it is $135°$ ($90° + 45°$) from the $+x$-axis.

Answer: **B**

Problem 55

From Malus's Law we know that the transmitted intensity trhough a polarizer is equivalent to

$$I = I_0 \cos^2 \theta \tag{64}$$

Where I_0 is the incident intensity on the polarizer, $I_0 = E_0^2$. Since we are dealing with two polarizers with incident energy E_1 and E_2, the intensity of the final beam is

$$I = I_1 + I_2 = E_1^2 \cos^2\theta + E_2^2 \cos^2\theta = cos^2\theta(E_1^2 + E_2^2)$$

$$\therefore \quad \boxed{I \propto E_1^2 + E_2^2}$$

Answer: **A**

Problem 57

Recall Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{65}$$

Now, if all of the light is reflected at the surface then there is no transmitted light and $\theta_2 = 90°$. This is the definition of critical angle, θ_C.

$$n_1 \sin\theta_C = n_2 \sin 90° \quad \rightarrow \quad \sin\theta_C = \frac{n_2}{n_1} \quad \rightarrow \quad \theta_C = \arcsin\frac{n_2}{n_1} \tag{66}$$

Plugging in our numbers:

$$\theta_C = \arcsin\left(\frac{1}{1.33}\right) = \arcsin\left(\frac{3}{4}\right) = \arcsin\left(\frac{1.5}{2}\right) \approx \arcsin\left(\frac{1.4}{2}\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = 45°$$

Which is closest to 50°.

Answer: **C**

Problem 57

The formula for single slit diffraction is

$$d\sin\theta = m\lambda \tag{67}$$

For this problem $m = 1$ (first minimum), $\sin\theta = \theta = 4 \times 10^{-3}$ rads (small angle in radians), and $\lambda = 400\,\text{nm}$. Solving for d and plugging in our values we get

$$d = \frac{m\lambda}{\theta} = \frac{400\,\text{nm}}{4 \times 10^{-3}\,\text{rads}} = 100,000\,\text{nm} = \boxed{1 \times 10^{-4}\,\text{m}}$$

Answer: **C**

Problem 58

This is a beam expander which 1) has the focal length of both lenses located at the same point between them so that

$$f_1 + f_2 = d \tag{68}$$

and 2) has magnifying power based upon the focal length of the objective lens (f_2) and image lens (f_1).

$$M = \frac{f_2}{f_1} \tag{69}$$

Since the beam is expanded from 1 mm to 10 mm the magnifying power is $M = 10$.

$$M = 10 = \frac{f_2}{1.5\,\text{cm}} \quad \rightarrow \quad \boxed{f_2 = \ 15[cm]}$$

and from equation (68) we get

$$d = f_1 + f_2 = 1.5\,\text{cm} + 15\,\text{cm} = \boxed{16.5\,\text{cm}} \tag{70}$$

Answer: **E**

Problem 59

The energy of beam, E_b, is equivilent to the number of photons, n, in the beam times the energy of each photon, E_γ (equation 44).

$$E_b = nE_\gamma = n\frac{hc}{\lambda} \tag{71}$$

The power of a beam of light is the energy divided by the time. This is easy to remember since the unit for power, Watts, is the unit for energy, Joules, divided but seconds ($[W] = \left[\frac{J}{s}\right]$).

$$P = \frac{E_b}{t} = n\frac{hc}{\lambda t}$$

Solving for n and plugging in gives us

$$n = P\frac{\lambda t}{hc} = 10^4\,\mathrm{W} \cdot \frac{6 \times 10^{-7}\,\mathrm{m} \cdot 10^{-15}\,\mathrm{s}}{6.6 \times 10^{-34}\,\mathrm{J\,s} \cdot 3 \times 10^8\,\mathrm{m/s}}$$

$$= \frac{10^{-11}\,\mathrm{J} \cdot 6 \times 10^{-7}\,\mathrm{m}}{6.6 \times 10^{-34}\,\mathrm{J\,s} \cdot 3 \times 10^8\,\mathrm{m/s}} = \frac{6 \times 10^{-18}\,\mathrm{J\,m}}{(6.6 \cdot 3) \times 10^{-26}\,\mathrm{J\,m}} = \frac{2}{6.6} \cdot 10^8$$

Which is closest to $\boxed{10^7\text{ photons}}$

Answer: **B**

Problem 60

The two source are moving towards earth (λ_t) and away from earth (λ_a) at the same time so the doppler shift is

$$\lambda_t - \lambda_a = \Delta\lambda = \lambda\left(\sqrt{\frac{1+\beta}{1-\beta}} - \sqrt{\frac{1-\beta}{1+\beta}}\right) = \lambda\left(\sqrt{\frac{(1+\beta)^2 - (1-\beta)^2}{(1-\beta)(1+\beta)}}\right) = \lambda\left(\frac{2\beta}{\sqrt{1-\beta^2}}\right)$$

Because the sun is moving at non-relativistic speeds we can approximate $1 - \beta^2 \approx 1$ leaving us with

$$\frac{\Delta\lambda}{\lambda} = 2\beta = \frac{2v}{c}$$

$$\therefore \quad v = \frac{\Delta\lambda}{\lambda}\frac{c}{2} = \frac{1.8 \times 10^{-12}\,\mathrm{m}}{122 \times 10^{-9}\,\mathrm{m}}\frac{3 \times 10^8\,\mathrm{m/s}}{2} = \frac{180 \times 10^{-14}\,\mathrm{m}}{244 \times 10^{-9}\,\mathrm{m}} \cdot 3 \times 10^8\,\mathrm{m/s}$$

$$\approx 7.5 \times 10^{-6} \cdot 3 \times 10^8\,\mathrm{m/s} = 22 \times 10^2\,\mathrm{m/s} = \boxed{2.2\,\mathrm{km/s}}$$

Answer: **B**

Problem 61

This is a simple application of Gauss's Law:

$$\vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \tag{72}$$

Let us first solve for Q:

$$Q = \int_0^{2\pi} \int_0^{\pi} \int_0^{R/2} \rho r^2 \sin(\theta) dr d\theta d\phi = \int_0^{2\pi} d\phi \int_0^{\pi} \sin(\theta) d\theta \int_0^{R/2} \rho r^2 dr$$

$$= 4\pi \int_0^{R/2} A r^4 dr = 4\pi A \frac{r^5}{5}\bigg|_0^{R/2} = A\pi \frac{R^5}{2^3 \cdot 5} = A\pi \frac{R^5}{40}$$

Now solve for E (Remembering that $A = 4\pi(R/2)^2$ **NOT** $A = 4\pi R^2$):

$$E = \frac{Q}{\pi R^2 \epsilon_0} = \frac{A\pi R^5}{40\pi R^2 \epsilon_0} = \boxed{\frac{AR^3}{40\epsilon_0}}$$

Answer: **B**

Problem 62

Each capacitor is is initially charged using a 5 V battery so that C_1 and C_2 have charges:

$$Q_1 = C_1 V = 1.0\,\mu\text{F} \cdot 5\,\text{V} = 5\,\mu\text{C}$$
$$Q_2 = C_2 V = 2.0\,\mu\text{F} \cdot 5\,\text{V} = 10\,\mu\text{C}$$

When the capacitors are connected with plates of opposite charge connected together they will both have the same voltage so that

$$V_f = \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

From conservation of charge we know that

$$Q_1 - Q_2 = -5\,\mu\mathrm{C} = q_1 + q_2$$

(The LHS uses subtraction because they are connected with opposite charges together). Therefore, putting the last two equations together

$$-5\,\mu\mathrm{C} = q_1 + q_2 = q_2\frac{C_1}{C_2} + q_2 \quad \rightarrow \quad q_2(1.5) = -5\,\mu\mathrm{C} \quad \rightarrow \quad q_2 = -3.33\,\mu\mathrm{C}$$

And now we can find the voltage across C_2:

$$V = \frac{q_2}{C_2} = \frac{3.33\,\mu\mathrm{C}}{2.0\,\mu\mathrm{F}} = 10/6\,\mathrm{V} \approx \boxed{1.7\,\mathrm{V}}$$

Answer: **C**