Solutions to 2008 Physics GRE

Detailed solutions to GR0877

Version 2.0

July 24, 2023

Problem 1

Let us say that the car is traveling in the $\hat{x}$ direction with velocity u and the ball is thrown in the $\hat{y}$ direction with velocity v. Since u, $v \ll c$ (not relativistic), the velocities are additive and $v_f = v\hat{x} + u\hat{y}$.

Answer: **B**

Problem 2

Because the object was thrown in the horizontal ($\hat{x}$) direction, the initial velocity in the vertical ($\hat{y}$) direction is zero. Therefore, we use the equation

$$y = y_0 + v_{y,0}t + \frac{1}{2}at^2 \tag{1}$$

$$y_0 = \frac{1}{2}gt^2 = \frac{1}{2}(9.81)(2.0)^2 = \boxed{19.6\,\mathrm{m}}$$

In this problem it is easier to approximate the acceleration due to gravity, g, as 10 to get an approximate solution ($y_0 = 20\,\mathrm{m}$).

Answer: **D**

Problem 3

The power dissipated by a resistor is

$$P = \frac{V^2}{R} = IV \tag{2}$$

Therefore, doubling the voltage across the resistor would give

$$P' = \frac{(2V)^2}{R} = \boxed{4P}$$

Answer: **E**

Problem 4

The Force on a moving charged particle is determined from the Lorentz force law:

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{3}$$

The magnetic field created by the current I_1 in the straight wire is in the same direction as the velocity of the charged particles in the loop, v. Therefore, $\vec{B} \times \vec{v} = 0$ and the magnetic force on the loop is zero.

Answer: **E**

Problem 5

The de Broglie wavelength is

$$\lambda = \frac{h}{p} \tag{4}$$

where h is Planck's Constant, $6.62 \times 10^{-34}\,\mathrm{m}^2\ \mathrm{kg\ s}^{-1}$

Answer: **A**

Problem 6

$n = 1$ and $n = 2$ levels correspond to the K and L shells of the atom. The L shell has one subshell denoted as *1s*; The K shell has two subshells denoted as *2s* and *2p*. s subshells can hold a maximum of 2 electrons and p subshells can hold 6.

Therefore, an atom with $n = 1$ and $n = 2$ levels filled is denoted as $1s^2 2s^2 2p^6$ and has 10 electrons.

There is also a nifty trick to solve this problem, since all the shells are full: $\sum 2n^2 = 2(1^2 + 2^2) = 10$.

This equation can be used for any atom where all the levels are filled.

Answer: **E**

Problem 7

The average kinetic energy of an ideal gas molecule is $\langle KE \rangle = \frac{3}{2} k_B T$. Equating this to the translational kinetic energy equation (using the average instead of the instantaneous velocity) we get

$$\langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T \tag{5}$$

$$\sqrt{\langle v^2 \rangle} = v_{rms} = \boxed{\sqrt{\frac{3 k_B T}{m}}} \tag{6}$$

Answer: **C**

Problem 8

The best way to approach this problem is to consider the cavity to be a blackbody. The rate at which a blackbody emits radiation is described by the Stefan-Boltzmann Law:

$$\frac{\Delta Q}{\Delta t} = \epsilon \sigma A T^4 \tag{7}$$

Doubling the temperature, $T' \to 2T$, would result in $\left(\frac{\Delta Q}{\Delta t}\right)' = 16\left(\frac{\Delta Q}{\Delta t}\right)$

Answer: **D**

Problem 9

Kepler's Three laws of planetary motion are:

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the period T of any planet is proportional to the cube of the semimajor axis a of its orbit.

$$T^2 = \frac{4\pi^2}{GM} a^3 \tag{8}$$

$$T^2 \propto a^3 \tag{9}$$

Statements I, II and III correspond to Kepler's 2nd, 1st, and 3rd Law, respectively.

Answer: **E**

Problem 10

The energy stored in a spring is determined using the equation $U = \frac{1}{2}kx^2$. Using energy conservation laws, we can equate this to the kinetic energy and solve for the displacement, x.

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \quad \to \quad \boxed{x = v\sqrt{\frac{m}{k}}} \tag{10}$$

Answer: **B**

Problem 11

The energy levels of a harmonic oscillator are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \tag{11}$$

$$\therefore \quad E_0 = \boxed{\frac{\hbar\omega}{2}}$$

Answer: **C**

Problem 12

The angular momentum of the Bohr atom is $L = n\hbar$. Equating this value to the classical equation for angular momentum we get:

$$L = m(r \times v) = mrv = n\hbar \quad \rightarrow \quad mv = p = \boxed{\frac{n\hbar}{r}} \tag{12}$$

here we have assumed v to be tangential so that $m(r \times v) = mrv \sin{(90^\circ)} = mrv$

Answer: **C**

Problem 13

A straight line on a log-log plot has the functional form: $y = ax^m$ where m is the slope and a is a constant. The line clearly passes through points $(x_1, y_1) = (3, 10)$ and $(x_2, y_2) = (300, 100)$. Therefore, we can find m using the equation:

$$m = \frac{\Delta y}{\Delta x} = \frac{\log_{10} y_2 - \log_{10} y_1}{\log_{10} x_2 - \log_{10} x_1} = \frac{\log_{10}\left(\frac{y_2}{y_1}\right)}{\log_{10}\left(\frac{x_2}{x_1}\right)} = \frac{\log_{10}\left(\frac{100}{10}\right)}{\log_{10}\left(\frac{300}{3}\right)} = \frac{\log_{10} 10}{\log_{10} 100} = \frac{1}{2}$$

We can then find a by using $x = 1$ so that $y = a = 6$ (extrapolating from (x_1, y_1)). Therefore, $y = 6\sqrt{x}$ must be the solution.

Answer: **A**

Problem 14

The weighted average of N separate measurements of x $(x_1 \pm \sigma_1, x_2 \pm \sigma_2, \ldots, x_N \pm \sigma_N)$ is $x_{wavg} = \frac{\sum_{i=1}^{N} \omega_i x_i}{\sum_{i=1}^{N} \omega_i}$, where $\omega_i = \frac{1}{\sigma_i^2}$. The uncertainty for the weighted average is $\sigma_{wavg} =$

$\left(\sum_{i=1}^{N}\omega_i\right)^{-\frac{1}{2}}$. Therefore, the uncertainty of the weighted average is equal to

$$\left(\frac{1}{1^2}+\frac{1}{2^2}\right)^{-\frac{1}{2}} = \sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}}}$$

Answer: **B**

Problem 15

Diverging lenses have negative focal lengths. Converging lenses have positive focal lengths. Because lens (E) has the largest curvature, it has the shortest focal length. This can be seen directly from the equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \tag{13}$$

Answer: **E**

Problem 16

Malus law, $I = I_0\cos^2(\theta)$, tells us the transmitted intensity light through a linear polarizer. Since $\cos(45°) = \frac{1}{\sqrt{2}}$ we calculate the final intensity after both polarizers to be:

$$I_1 = \frac{I_0}{2} \quad \text{(Definition of intensity through a single linear polarizer)} \tag{14}$$

$$I_f = I_1\cos^2(45°) = \left(\frac{I_0}{2}\right)\cdot\left(\frac{1}{2}\right) = \frac{I_0}{4} = \boxed{0.25\%\ I_0}$$

Answer: **D**

Problem 17

Using Gauss' Law, $\int E\cdot dA = \frac{Q}{\epsilon_0}$ and imagining a cylindrical Gaussian surface with radius r:

$$E\cdot 2\pi r l = \frac{Q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$E = \boxed{\frac{\lambda}{2\pi\epsilon_0 r}}$$

It is also worth noting that (A) is the only solution with the correct dimensions.

Answer: **A**

Problem 18

As the magnet enters the loop, the flux through the loop increases. According to Lenz's law, $\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$, the induced current generates a magnetic field that is opposing the bar magnet's field. This current is counter-clockwise (b to a).

As the magnet leaves the loop, the flux decreases and the current flows clockwise (a to b).

Answer: **E**

Problem 19

Wein's Law stats the the maximum wavelength of the blackbody is inversely proportional to the temperature:

$$\lambda_{\max} = \frac{2.897 \times 10^{-}3}{T} \tag{15}$$

$$\begin{aligned} \therefore \quad \frac{\lambda_1}{\lambda_2} &= \frac{T_2}{T_1} \\ \rightarrow \quad \lambda_2 &= \frac{\lambda_1 T_1}{T_2} = \frac{500\,\text{nm} \cdot 6000\,\text{K}}{300\,\text{K}} = \boxed{10\,\mu\text{m}} \end{aligned}$$

Answer: **A**

Problem 20

The wavelength of a CMB photon is proportional to Freidmann-Robertson-Walker scale factor a. From Wein's Law:

$$\lambda_{\max} \propto a \propto \frac{1}{T} \tag{16}$$

$$\frac{a_{now}}{a_{then}} = \frac{12[K]}{3[K]} = 4 \quad \rightarrow \quad a_{then} = \boxed{\frac{1}{4}}$$

Answer: **A**

Problem 21

For an adiabatic process $PV^\gamma = const$ and plugging in $P \propto \frac{T}{V}$ (from the ideal gas law) yields $\left(\frac{T}{V}\right) V^\gamma = TV^{\gamma-1} = const$

Answer: **C**

Problem 22

The rest energy of an electron is $m_e c^2$ and so the total energy of this electron is $4m_e c^2$. From the relativistic energy-momentum equation:

$$E^2 = p^2c^2 + m_e^2c^4 \tag{17}$$

$$p^2 = 16m_e^2c^4 - m_e^2c^4 = 15m_e^2c^4 \quad \rightarrow \quad p = \boxed{\sqrt{15}m_e c^2}$$

Answer: **C**

Problem 23

This requires the relativistic velocity addition formula:

$$w = \frac{v+u}{1+\frac{v\cdot u}{c^2}} \tag{18}$$

where w is the speed of the ship in Earth's reference frame, v is the speed of ship 1, and u is the speed of ship 2. From the description we know that $u = v$ so that the equation can be written as

$$w = \frac{2v}{1+\frac{v^2}{c^2}}$$

To find v we will use the length contraction formula $L = \frac{L_0}{\gamma}$:

$$\gamma = \frac{L_0}{L} = \frac{1\,\mathrm{m}}{0.6\,\mathrm{m}} = \frac{5}{3}$$

Using the w as the velocity in the Lorentz Factor, γ, we can solve for v (set $c = 1$ to make calculations simpler):

$$\begin{aligned}\frac{5}{3} = \frac{1}{\sqrt{1-\frac{w^2}{c^2}}} &= \frac{1}{\sqrt{1-\frac{4v^2}{(1+v^2)^2}}} \\ &= \frac{1+v^2}{\sqrt{(1+v^2)^2-4v^2}} \\ &= \frac{1+v^2}{\sqrt{1-2v^2+v^4}} = \frac{1+v^2}{1-v^2}\end{aligned}$$

$$5(1-v^2) = 3(1+v^2) \rightarrow v = 0.5 \text{ or } \boxed{v = 0.5c}$$

Answer: **B**

Problem 24

In order to find the time it takes to pass the observer, we must first find the Lorentz Factor, $\gamma(v = 0.8c)$:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{4}{5}\right)^2}} = \frac{5}{3} \tag{19}$$

(I strongly suggest that you memorize this solution)

Since $L = \frac{L_0}{\gamma}$ and $\Delta t = \frac{\Delta x}{v}$ we can solve for Δt by plugging in $\Delta x = L$:

$$\Delta t = \frac{L_0}{\gamma v} = \frac{1\,\mathrm{m} \cdot 3}{5 \cdot 0.8 \cdot 3 \times 10^8} = \boxed{2.5\,\mathrm{ns}}$$

Answer: **B**

Problem 25

For a wavefunction to be **normalized** it must satisfy the equation

$$\int_{\infty}^{-\infty} |\Psi(x)|^2 dx = 1 \tag{20}$$

For a wavefunction to be **orthogonal** when $i \neq j$ it must satisfy the equation

$$\int_{\infty}^{-\infty} \psi_i(x)\psi_j(x) dx = 0 \tag{21}$$

(E) is the only choice which satisfies both of the eqautions

Answer: **E**

Problem 26

The probability that the electron would be found between r and $r + dr$ is $P = |\psi|^2 dV = |\Psi(x)|^2 4\pi r^2 dr = p(r)dr$. The most probable value for P is when $p(r)$ is at a maximum and the maximum can be found by setting the first derivative equal to zero.

$$\frac{dP}{dr} = \frac{d|\psi|^2}{dr} \cdot 4\pi r^2 + |\psi|^2 \cdot 8\pi r = 0 \tag{22}$$

because ψ_{100} has no complex terms $|\psi|^2 = \frac{1}{\pi a_0^3} e^{-\frac{2r}{a_0}}$. Plugging this in:

$$\frac{dP}{dr} = -\frac{4r^2}{a_0^3}\frac{2}{a_0} e^{-\frac{2r}{a_0}} + \frac{8r^2}{a_0^3} e^{-\frac{2r}{a_0}} = 0$$

$$\therefore \quad \frac{4r^2}{a_0^3}\frac{2}{a_0} e^{-\frac{2r}{a_0}} = \frac{8r}{a_0^3} e^{-\frac{2r}{a_0}} \quad \rightarrow \quad \boxed{r = a_0}$$

Answer: **D**

Problem 27

The order of magnitude estimate of the time-energy uncertainty principle, $\Delta E \Delta t \geq h$, and planck's equation, $E = h\nu$, can be used together:

$$\Delta E \Delta t = h\nu \Delta t \geq h$$
$$\nu \geq \frac{1}{\Delta t} = \frac{1}{1.6 \times 10^{-9}\,\mathrm{s}} \sim \boxed{600\,\mathrm{MHz}}$$

Since this is a gross approximation calculation the order of magnitude, [MHz], is all that we need

Answer: **C**

Problem 28

Work is equal to the change in kinetic energy of a system. Since this is a spring, we use Hooke's energy equation:

$$W_1 = \frac{1}{2}k_1 x_1^2$$
$$W_2 = \frac{1}{2}k_2 x_2^2 = \frac{1}{2}k_2\left(\frac{x_1}{2}\right)^2$$
$$W_2 = 2W_1$$

$$k_1 x_1^2 = \frac{1}{8}k_2 x_1^2 \quad \rightarrow \quad \boxed{k_2 = 8k_1}$$

Answer: **D**

Problem 29

In an elastic collision kinetic energy is conserved.

$$T_0 = T_1 + T_2$$

$$\frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{v}{2}\right)^2 + \frac{1}{2}Mu^2$$

$$\rightarrow \quad u^2 = v^2\left(1 - \frac{1}{4}\right)$$
$$u = \boxed{\frac{\sqrt{3}}{2}v}$$

Answer: **C**

Problem 30

Hamilton's canonical equations of motion are

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \quad \text{and} \quad \dot{q}_i = \frac{\partial H}{\partial p_i} \tag{23}$$

Be careful! Answer (C) is a perfect example of a typical GRE trap since it is very similar to (D).

Answer: **D**

Problem 31

Archimedes' principle states that $F_{bouyant} = \rho V g$. Since the block is in equilibrium with part of its volume in the water and the other part in oil, we know that $F_{net} = 0$ and can therefore use the equation

$$\begin{aligned}
\rho_{block} V g &= \rho_{water}\left(\frac{3V}{4}\right) g + \rho_{oil}\left(\frac{V}{4}\right) g \\
\rho_{block} &= \frac{3}{4}\rho_{water} + \frac{1}{4}\rho_{oil} \\
&= \frac{3}{4}1000\,\mathrm{kg\,m^{-3}} + \frac{1}{4}\,\mathrm{kg\,m^{-3}} \\
&= 750\,\mathrm{kg\ m^{-3}} + 200\,\mathrm{kg\,m^{-3}} = \boxed{950\,\mathrm{kg\,m^{-3}}}
\end{aligned}$$

Answer: **C**

Problem 32

According to Bernoulli's principle:

$$\frac{v_0^2}{2} + gz_0 + \frac{P_0}{\rho_0} = \frac{v_f^2}{2} + gz_f + \frac{P_f}{\rho_f} \tag{24}$$

Since both sections are centered at the same height, $z_0 = z_f$, the middle term is cancelled out. In addition, The fluid is incompressible so that $\rho_0 = \rho_f = \rho$ leaving us with:

$$P_f = P_0 + \frac{\rho v_0^2}{2} - \frac{\rho v_f^2}{2}$$

Conservation of mass tells us that $\rho v A \Delta t = const$ where A is the cross-sectional area of the pipe. We can now find the velocity of the fluid at the constriction, v_f:

$$\begin{aligned} \rho v_0 A_0 \Delta t &= \rho v_f A_f \Delta t \\ \rightarrow \quad v_f &= \frac{v_0 A_0}{A_f} = v_0 \frac{\pi r^2}{\pi (r/2)^2} = 4v_0 \end{aligned}$$

Plugging this in to Bernoulli's equation:

$$\begin{aligned} P_f &= P_0 + \frac{\rho v_0^2}{2} - \frac{\rho (4v_0)^2}{2} \\ &= P_0 + \frac{\rho v_0^2}{2}(1 - 16) = \boxed{P_0 - \frac{15}{2}\rho v_0^2} \end{aligned}$$

Answer: **A**

Problem 33

We can calculate the change in entropy using the equation

$$\Delta S \geq \int \frac{dQ}{T} \tag{25}$$

where $\Delta Q = mc\Delta T$ so that

$$\Delta S \geq \int_{T_1}^{T_2} mc \frac{dT}{T} = \boxed{mc \ln \frac{T_2}{T_1}}$$

Answer: **E**

Problem 34

From the first law of thermodynamics, we know that $\Delta U = \Delta Q - \Delta W = \Delta Q - P\Delta V$. Using this equation, we can calculate the specific heat at constant volume and pressure, C_V and C_P:

$$C_V = \left(\frac{dQ}{dT}\right)_V = \left(\frac{dU + dW}{dT}\right) = \left(\frac{dU}{dT}\right) \tag{26}$$

$$C_P = \left(\frac{dQ}{dT}\right)_P = \left(\frac{dU + dW}{dT}\right) = \left(\frac{dU}{dT}\right) + P\left(\frac{dV}{dT}\right) \tag{27}$$

For an nonatomic ideal gas $U = \frac{3}{2}NkT$ and $V = \frac{NkT}{P}$ so that C_V and C_P can be written as

$$C_V = \left(\frac{dU}{dT}\right) = \frac{3}{2}Nk$$

$$C_P = \left(\frac{dU}{dT}\right) + P\left(\frac{dV}{dT}\right) = \frac{3}{2}Nk + Nk = \frac{5}{2}Nk$$

Let us say that Q_V is the heat required to to change the temperature at constant volume and Q_P the the heat required for constant pressure. Rewriting the specific heat equations to fit our needs, we have

$$\frac{\Delta Q_P}{C_P} = \Delta T = \frac{\Delta Q_V}{C_V}$$

$$\Delta Q_P = \Delta Q_V \frac{C_P}{C_V} = \boxed{\frac{5}{3}\Delta Q} \tag{28}$$

Answer: **C**

Problem 35

An idealized heat pump has the efficiency of a Carnot engine:

$$e = \left|\frac{W}{Q_H}\right| = 1 - \left|\frac{T_C}{T_H}\right| \tag{29}$$

$$W = Q_H\left(1 - \left|\frac{T_C}{T_H}\right|\right) = 15000\,\mathrm{J}\left(1 - \frac{280\,\mathrm{K}}{300\,\mathrm{C}}\right)$$

$$= \frac{15000\,\mathrm{J}}{15} = \boxed{1000\,\mathrm{J}}$$

This calculation made use of the fact that $T([K]) = T([C]) + 273$

Answer: **B**

Problem 36

From Kirchhoff's second law we can write the equation for an LC circuit:

$$\sum V = V_L + V_C = L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0 \tag{30}$$

V_L comes from the induced EMF of an inductor ($V = |\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right|$ where $\Phi_B = LI = L\frac{dQ}{dt}$) and V_C comes from $Q = CV$ for a capacitor.

Solving the second order differential equation we get

$$Q = Q_0 \cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$\rightarrow \quad I = \frac{dQ}{dt} = -\frac{Q_0}{\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

The energy stored in an inductor can be found using the equation

$$U = \frac{1}{2}LI^2 = \boxed{\frac{Q_0^2}{2C}\sin^2\left(\frac{t}{\sqrt{LC}}\right)}$$

Answer: **A**

Problem 37

From geometry we can easily see that the electric field is in the $-\hat{x}$ direction (basic vector addition) which leaves us with choices (C) and (E). The electric field at P is $E = 2E_q\cos(\theta)$.

$$\cos(\theta) = \frac{adj}{hyp} = \frac{l/2}{\sqrt{(l/2)^2 + r^2}} = \frac{l}{\sqrt{(l)^2 + (2r)^2}}$$

$$\therefore \quad \vec{E} = 2 \cdot \frac{1}{4\pi\epsilon_0}\frac{4q}{l^2 + 4r^2}\frac{l}{\sqrt{(l)^2 + 4r^2}}(-\hat{x})$$

for $r \gg l$:

$$\vec{E} = 2 \cdot \frac{1}{4\pi\epsilon_0}\frac{4q}{4r^2}\frac{l}{2r}(-\hat{x}) = \boxed{\frac{ql}{4\pi\epsilon_0 r^3}(-\hat{x})}$$

Answer: **E**

Problem 38

Using the right hand rule (thumb in direction of current, curled figures point in direction on B-Field) it is easy to see that at point P the magnetic fields are equal and opposite resulting in destructive interference and a magnetic field of zero.

Answer: **E**

Problem 39

The lifetime of a muon in the lab frame can be calculated using the Lorentz factor for particles moving at $0.8c$

$$\gamma = \left(1 - \frac{16}{25}\right)^{-\frac{1}{2}} = \left(\frac{9}{25}\right)^{-\frac{1}{2}} = \frac{5}{3}$$

$$t_{lab} = \gamma\tau = \left(\frac{5}{3}\right) 2.2 \times 10^{-6}\,\mathrm{s}$$

$$\Delta x_{lab} = v \cdot t_{lab} = \frac{4c}{5} \cdot \frac{5}{3} \cdot 2.2 \times 10^{-6}\,\mathrm{s} = \frac{4}{3} \cdot 2.2 \times 10^{-6}\,\mathrm{s} \cdot 3 \times 10^{8}\,\mathrm{m/s} = \boxed{880\,\mathrm{m}}$$

Answer: **C**

Problem 40

The relativistic energy-momentum equation states that

$$E^2 = p^2c^2 + m^2c^4 \tag{31}$$

The four momentum of the massless particle is $\vec{P}_1 = (\frac{E}{c}, p, 0, 0,) = (\sqrt{p^2 + m^2c^2}, p, 0, 0)$. The four momentum of the first particle is $\vec{P}_0 = (Mc^2, 0, 0, 0)$. Equate the momentums:

$$\begin{aligned} P_0 &= P_1 \\ Mc^2 &= \sqrt{p^2 + m^2c^4} + p \\ Mc^2 - p &= \sqrt{p^2 + m^2c^4} \end{aligned}$$

and square both sides:

$$M^2c^4 - 2pMc^2 + p^2 = p^2 + m^2c^4$$

$$2pMc^2 = c^4(M^2 - m^2)$$

$$\rightarrow \quad p = \frac{c^4(M^2 - m^2)}{2Mc^2} = \boxed{\frac{c^2(M^2 - m^2)}{2M}}$$

Answer: **B**

Problem 41

The photoelectric effect is summarized by the eqaution

$$h\nu = \phi + K_{\text{max}} = \phi + eV \tag{32}$$

$$\rightarrow \quad eV = h\nu - \phi$$
$$V = \frac{h}{e}\nu - \frac{\phi}{e}$$

The last equation is in the form $y = mx + b$ where the slope, m, is equal to $\frac{h}{e}$ (with a V intercept at $b = -\frac{\phi}{e}$)

Answer: **B**

Problem 42

The horizontal distance from crest to crest for either waveforms is $\sim$ 6 cm. The phase between two points of equal voltages for the two waveforms is $\sim$ 2 cm so that the phase difference can be read as:

$$2\,\text{cm}\frac{2\pi}{6\,\text{cm}} = \frac{2\pi}{3} = \boxed{120^\circ}$$

Answer: **E**

Problem 43

This question is pure fact recall from. The diamond structure of elemental carbon is a covalent network in the shape of a tetrahedron.

Answer: **D**

Problem 44

The BCS theory describes superconductivity as a microscopic effect caused by Cooper pairs condensing into a boson-like state. The two fermions in a Cooper pair are both attracted to a positive ion between them. With this information, (A), (B), and (D) can easily be discarded and (E) is also incorrect because the Casimir effect is a tiny attractive force that acts between two close parallel uncharged conducting plates.

Answer: **D**

Problem 45

This question is a lesson in sign convention. The equation for nonrelativistic doppler shift is

$$f = \left(\frac{c \mp v_o}{c \pm v_s}\right) f_o \tag{33}$$

Relative to the medium, the velocity of the source, v_s, is positive if the source is moving *away* from the observer and the velocity of the observer, v_o, is is positive if the observer is moving *toward* the source. In this problem the siren is moving away from the observer and the observer is moving towards the siren **relative to the velocity of sound in the medium**. Therefore,

$$f = \left(\frac{c+55}{c+55}\right) f_o = f_o = \boxed{1200\,\mathrm{Hz}}$$

Answer: **C**

Problem 46

This is a single slit wave problem. The minimum of a single slit diffraction pattern is

$$a\sin(\theta) = m\lambda = m\left(\frac{c}{\nu}\right) \tag{34}$$

since we are looking for the first minimum, $m = 1$. Solving for ν (knowing that $\sin(45°) = \frac{1}{\sqrt{2}}$ and $\sqrt{2} \approx 1.14$):

$$\nu = \frac{c}{a\sin\theta} = \frac{350\,\mathrm{m/s}}{0.14\,\mathrm{m}\cdot\sin(45°)} = \frac{\sqrt{2}\cdot 350\,\mathrm{m/s}}{0.14\,\mathrm{m}} = \boxed{3500 Hz}$$

Answer: **D**

Problem 47

For a "closed" pipe (closed at one end, open at the other) of length L the resonant frequencies are given by the equation

$$f_n = \frac{v}{\lambda} = \frac{nv}{4L} \quad \text{where n = 1,3,5,\ldots} \tag{35}$$

The fundamental frequency ($n = 1$) is given as $f_1 = 131\,\mathrm{Hz}$ so the next higher harmonic ($n = 3$) is

$$f_3 = \frac{3v}{4L} = 3 \cdot 131\,\mathrm{Hz} = \boxed{393\,\mathrm{Hz}} \tag{36}$$

Answer: **D**

Problem 48

The logic gate equations are as follows:

$$\text{AND:} \quad A \cdot B \qquad \text{NAND:} \quad \overline{A \cdot B} \tag{37}$$

$$\text{OR:} \quad A + B \qquad \text{NOR:} \quad \overline{A + B} \tag{38}$$

Each equation can have a solution equal to 1 or 0. The overbar reverses the solution ($\overline{0} = 1$). This reversal is represented by a circle at the junction of the logic gate. Therefore A and B are reversed before passing through a NOR gate while C and D are not changed before passing through a NAND gate. Finally, each of the outputs is filtered through an AND gate to produce E. Therefore E is ($\overline{A}$ NOR $\overline{B}$) AND (C NAND D):

$$E = \boxed{\overline{\overline{A} + \overline{B}} \cdot \overline{C \cdot D}}$$

Answer: **C**

Problem 49

We need to know a little bit about lasers to solve this problem:

- A solid state laser and a diode laser both use transitions from the conduction band to the valence band of a semiconductor (eliminates (A) and (E))

- Free-electron lasers use the fact that free electrons (not in atoms) radiate when accelerated or decelerated (eliminates (C))
- A dye-laser works via transitions between different molecular states, *not* atomic states (eliminates (E))
- Gas laser is the is the only choice which involves free atoms.

Answer: **D**

Problem 50

The formula for Bohr energies is

$$E_n = \frac{mZ^2 (e^2)^2}{2\hbar^2 (4\pi\epsilon_0)^2 n^2} \qquad \text{or just remember} \qquad E_n \propto \frac{mZ^2 (e^2)^2}{n^2} \tag{39}$$

Dimensional analysis also can be used to show that (C) is the only answer with the correct units.

Answer: **C**

Problem 51

The Rydberg Formula,

$$\Delta E = R \left(\frac{1}{n_f^2} - \frac{1}{n_I^2} \right) \tag{40}$$

where the Rydberg constant, R, is equal to 2.178×10^{18} J, tells us that the light emitted from an atom must have energy equivalent to the energy difference between two quantum states. Therefore, (II) is false. The same logic tells us that (I) is true. (III) is also true because at low temperatures the atoms are in the ground state and can only absorb from the ground level.

Answer: **D**

Problem 52

The resulting diffraction from X rays on a crystal lattice is called Bragg Diffraction and is described by the equation

$$d \sin(\theta) = \frac{m\lambda}{2} \tag{41}$$

Solving for d with m = 1 (”smallest angle”):

$$d = \frac{m\lambda}{2\sin(\theta)} = \frac{0.250\,\text{nm}\cdot 4}{2} = \boxed{0.500\,\text{nm}}$$

Answer: **C**

Problem 53

Kepler’s third law tells us that $T^2 \propto a^3/M$. **This is a trap**. The M in that equation is the mass of the central body and not the individual planets!

The key to this problem is that fact that the angular momenta are have the same magnitude.

$$m_1 r_1 v1 = L = m_2 r_2 v_2$$

From the problem we know that $r_1 = r_2 = R$. The period of circular motion is

$$\begin{aligned} T &= \frac{2\pi}{\omega} = \frac{2\pi r}{v} \\ \rightarrow \quad v &= \frac{2\pi r}{T} \end{aligned} \tag{42}$$

plugging v into L:

$$L = \frac{2\pi R^2 m_1}{T_1} = \frac{2\pi R^2 m_2}{T_2} = \frac{6\pi R^2 m_2}{T_1}$$

$$\frac{m_1}{T_1} = \frac{3m_2}{T_1} \quad \rightarrow \quad \frac{m_1}{m_2} = \boxed{3}$$

Answer: **D**

Problem 54

Replacing the sun with a black hole of the same mass would exert no more gravitational force on its orbiting planets. This is because the gravitational field of any massive body can be be visualized as a point particle with the same mass located at the center of mass.

Answer: **E**

Problem 55

The equation for relativistic doppler shift is

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}} \tag{43}$$

$$\left(\frac{580}{434}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{1+\beta}{1-\beta}$$
$$(9+9\beta) = 16 - 16\beta \quad \rightarrow \quad \beta = \frac{7}{25} = \boxed{0.28}$$

$\beta = v/c$ therefore $v = 0.28c$

Answer: **A**

Problem 56

Using vector addition along with the pythagorean theorem where v is the speed of the plane in still air, u is the speed of the wind, and v' is the final speed of the plane in the wind; we get

$$v'^2 = u^2 - v^2$$
$$v' = \sqrt{u^2 - v^2}$$

the question asks for the time it takes to fly 500 km

$$t = \frac{d}{v'} = \frac{500\,\text{km}}{\sqrt{u^2 - v^2}} = \frac{500\,\text{km}}{\sqrt{(200\,\text{km/h})^2 - (30\,\text{km/h})^2}}$$
$$= \frac{500\,\text{km}}{\sqrt{40000\,\text{km}^2/\text{h}^{-2} - 900\,\text{km}^2/\text{h}^{-2}}} = \frac{50\,\text{km}}{\sqrt{400\,\text{km}^2/\text{h}^{-2} - 9\,\text{km}^2/\text{h}^{-2}}} = \boxed{\frac{50}{\sqrt{391}}\,\text{h}}$$

Answer: **D**

Problem 57

The acceleration due to the applied force is the same in both figures

$$(2m+m)a = F \quad \rightarrow \quad a = \frac{F}{3m}$$

We can then find the force on the second block for the first figure:

$$F_{1,12} = ma = \boxed{\frac{F}{3}}$$

and the second figure:

$$F_{2,12} = 2ma = \boxed{\frac{2F}{3}}$$

Answer: **B**

Problem 58

Because B is resting on top of the A the net force on B is equal to the static frictional force (since the normal force and gravitational force cancel)

$$a\,m_B = 10\,\mathrm{kg} \cdot 2\,\mathrm{m/s^2} = \boxed{20\,\mathrm{N}}$$

Answer: **A**

Problem 59

The period of a pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{44}$$

Because the elevator is accelerating in the upward direction the total acceleration, a', is $a' = g + a$ so that the period can be expressed by the equation

$$T = 2\pi\sqrt{\frac{l}{a'}} = \boxed{2\pi\sqrt{\frac{l}{g+a}}}$$

Answer: **C**

Problem 60

The magnetic field at point $(x,\,0,\,0)$ due to the wire traveling along the z-axis is

$$B_1 = \frac{\mu_0 I}{2\pi x} \tag{45}$$

The wire directed at 45° **above** the positive-z produces a magnetic field of

$$B_2 = \frac{\mu_0 I}{2\pi x \sin 45^\circ} = \frac{\mu_0 I}{\sqrt{2}\pi x}$$

and the wire directed 45° **below** the positive-z is equivalent to B_2

$$B_3 = B_2 = \frac{\mu_0 I}{\sqrt{2}\pi x}$$

Therefore, the total magnetic field at point (x,0.0) is

$$B_{tot} = B_1 + B_2 + B_3 = \boxed{\frac{\mu_0 I}{2\pi x}(1 + 2\sqrt{2})}$$

(we know that the direction of the field is out of the page ($\hat{y}$) from the right hand rule)

Answer: **C**

Problem 61

Equating the Lorentz force to the centripetal force of the particle (using $r = \frac{d}{2}$) we get the equation for distance

$$m\frac{v^2}{r} = m\frac{2v^2}{d} = qvB \quad \rightarrow \quad d = \frac{m}{q}\frac{2v}{B}$$

Therefore, doubling the charge-to-mass ratio reduces the distance by a factor of 2 ($d' = \frac{d}{2}$)

Answer: **E**

Problem 62

This is a simple application of Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \Phi_{E,tot} = \Phi_{E,A} + \Phi_{E,other} = \frac{Q}{\epsilon_0} \tag{46}$$

$$\rightarrow \quad \Phi_{E,other} = \frac{Q}{\epsilon_0} - \Phi_{E,A} = \frac{1 \times 10^{-9}\,\mathrm{C}}{8.85 \times 10^{-12}\,\mathrm{F/m}} + 100\,\mathrm{N\ m^2\ C^{-1}} \approx \boxed{200\,\mathrm{N\ m^2\ C^{-1}}}$$

Answer: **E**

Problem 63

The fact that a positron (e^+) and electron neutrino ν_e are produced in this reaction tells us that this is a β^+ decay. β decays are mediated by interactions with the weak force.

Answer: **D**

Problem 64

The eigenvalue equations for L^2 and L_z are

$$L^2 Y_l^m(\theta,\phi) = l(l+1)\hbar^2 Y_l^m(\theta,\phi) \tag{47}$$

$$L_z Y_l^m(\theta,\phi) = -i\hbar\frac{\partial}{\partial\phi} Y_l^m(\theta,\phi) = m\hbar Y_l^m(\theta,\phi) \tag{48}$$

therefore, with $L = \sqrt{2}\hbar$ we can solve for l:

$$L^2 = 2\hbar^2 = l(l+1)\hbar^2 \to l = 1$$

For any given l, there are $2l+1$ possible values for m (from $m = -l, -l+1, ..., l-1, l$). We can therefore eliminate (A), (B) and (C) since there are a total of 3 values for m ($m = -1, 0, 1$). Plugging the values for m into the L_z eigenvalue equation we get

$$\boxed{L_z = -\hbar, 0, \hbar}$$

Answer: **D**

Problem 65

Process of elimination:

I. Correct. The energy levels of a harmonic oscillator are $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$. Therefore they are evenly spaced with a step length of $\hbar\omega$.

II. Incorrect. The potential energy is $U = \frac{1}{2}m\omega^2 x^2$ which is quadratic, not linear.

III. Incorrect. The virial theorem tells us that the expectation value for both the potential and kinetic energy is half of the total energy. At $n = 1$:

$$\langle T \rangle = \langle V \rangle = \frac{E}{2} = \frac{\hbar\omega}{4} \neq 0$$

IV. Correct. There is a nonzero probability for the oscillator to be found at any x.

Answer: **C**

Problem 66

The energy levels for H are

$$E_n = -\frac{|E_0|}{n^2} \propto \frac{\mu}{n^2} \tag{49}$$

where the reduced mass, μ, is $\frac{m_e m_p}{m_e + m_p}$. Replacing the electron with the muon we get

$$E'_n \propto \frac{\mu'}{n^2} \quad \text{where} \quad \mu' = \frac{m_\mu m_p}{m_\mu + m_p}$$

Therefore,

$$\frac{1}{n^2} = \frac{E_n}{\mu} = \frac{E'_n}{\mu'}$$

$$\begin{aligned} \rightarrow \quad E'_n = \frac{E_n}{\mu} \cdot \mu' &= E_n \frac{m_e + m_p}{m_e m_p} \frac{m_\mu m_p}{m_\mu + m_p} \\ &= E_n \frac{m_\mu (m_e + m_p)}{m_e (m_\mu + m_p)} = \boxed{-\frac{|E_0|}{n^2} \frac{m_\mu (m_e + m_p)}{m_e (m_\mu + m_p)}} \end{aligned}$$

Answer: **D**

Problem 67

The electric field inside a parallel-plate capacitor is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} \quad (\text{using } \ Q = \sigma A)$$

Since we are looking for the change in electric field over time we will differentiate both sides with respect to time:

$$\frac{dE}{dt} = \frac{1}{A\epsilon_0} \frac{dQ}{dt} = \frac{I}{A\epsilon_0} = \frac{9\,\text{A}}{8.85 \times 10^{-12}\,\text{F/m} \cdot (0.5\,\text{m})^2} \sim \frac{10^{12}}{0.25}\,\text{V m}^{-1}\ \text{s}^{-1} = \boxed{4 \times 10^{12}\,\text{V m}^{-1}\ \text{s}^{-1}}$$

Answer: **D**

Problem 68

Using symmetry (and assuming an ideal circuit), we can see that the horizontal nodes have the same voltage and, therefore, there is no current through either of the two middle resistors

(Ohm's law). The problem is now greatly simplified since the circuit is equal to three sets of two resistors in series.

$$R_{left} = R_{right} = R_{middle} = R + R = 2R$$

$$R_{tot} = \left(\frac{1}{R_{left}} + \frac{1}{R_{right}} + \frac{1}{R_{middle}}\right)^{-1} = \frac{2R}{3}$$

Now we can find the current using Ohm's Law:

$$I = \frac{V}{R_{tot}} = \boxed{\frac{3V}{2R}}$$

Answer: **D**

Problem 69

The impedance for a resistor and a capacitor is

$$Z_R = R \ \ , \ \ Z_C = \frac{1}{i\omega C} \tag{50}$$
$$Z = R + \frac{1}{i\omega C}$$

The current through the circuit can be found using Ohm's law using the input voltage, V_i.

$$I = \frac{V_i}{Z} = \frac{V_i}{R + \frac{1}{i\omega C}}$$

We can then use this current to find the output voltage, V_0 (voltage after capacitor)

$$V_0 = IZ_C = \frac{V_i}{R + \frac{1}{i\omega C}} \cdot \frac{1}{i\omega C} = \frac{V_i}{i\omega CR + 1}$$

$$\therefore \ \ G = \frac{V_0}{V_i} = \boxed{\frac{1}{i\omega CR + 1}}$$

Looking at limiting cases, when $\omega \to \infty$, $G \to 0$ and when $\omega \to 0$, $G \to 1$.

Answer: **D**

Problem 70

Faraday's law of induction states that

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = IR = \frac{\Delta q}{\Delta t}R \tag{51}$$

Then we can use the equation for magnetic flux Φ_B to solve for Δq:

$$\Delta\Phi_B = B \cdot \Delta A \tag{52}$$

$$\therefore \quad \Delta q = -\frac{B \cdot \Delta A}{R} = -\frac{0.5\,\mathrm{T} \cdot 10 \times 10^{-4}\,\mathrm{m}^2}{5\,\Omega} = \boxed{10 \times 10^{-4}\,\mathrm{C}}$$

Answer: **A**