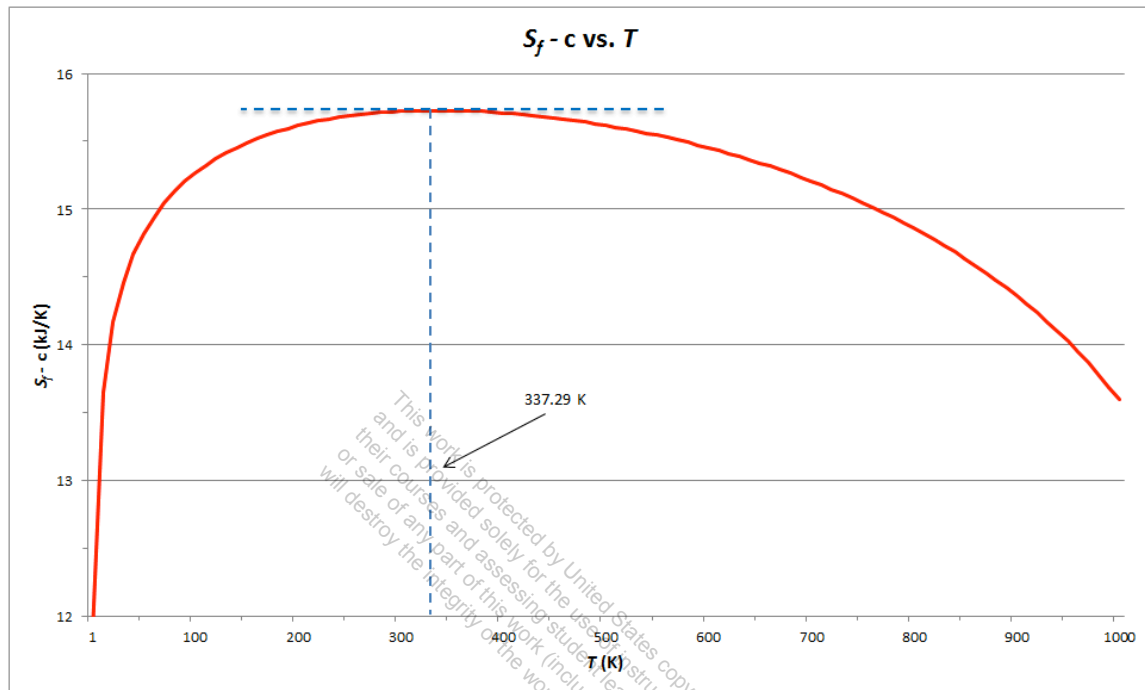


**14-1-1 [BRL]** A 2 kg copper block at 100°C is brought in thermal contact with a 5 kg copper block at 50°C. Treating the combined system as an isolated system, show that at equilibrium the entropy ( $S$ ) of the combined system reaches a maxima.

**SOLUTION**



Define the 2 kg and 5 kg blocks as systems 1 and 2, respectively.

Given:

$$T_{b1} = 373 \text{ K}$$

$$T_{b2} = 323 \text{ K}$$

$$c_{v1} = c_{v2} = c_v = 0.386 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

The energy equation can be simplified as follows:

$$\Delta U = \cancel{\phi^0} - \cancel{W_{\text{ext}}^0};$$

$$\Rightarrow \Delta(U_1 + U_2) = 0;$$

$$\Rightarrow \Delta U_1 = -\Delta U_2;$$

$$\Rightarrow m_1 c_v (T_{f1} - T_{b1}) = m_2 c_v (T_{b2} - T_{f2});$$

$$\Rightarrow m_1 (T_{f1} - T_{b1}) = m_2 (T_{b2} - T_{f2})$$

Substituting the known variables, the energy equation reduces to

$$m_1(T_{f1} - T_{b1}) = m_2(T_{b2} - T_{f2});$$

$$\Rightarrow (2)(T_{f1} - 373) = (5)(323 - T_{f2});$$

$$T_{f2} = 472.2 - 0.4T_{f1}$$

The entropy of the combined system at the final state can be written as  
 $\Delta S = \Delta S_1 + \Delta S_2;$

$$\Rightarrow S_f = S_b + m_1 c_v \ln\left(\frac{T_{f1}}{T_{b1}}\right) + m_2 c_v \ln\left(\frac{T_{f2}}{T_{b2}}\right);$$

$$\Rightarrow S_f = S_b + m_1 c_v \ln(T_{f1}) + m_2 c_v \ln(T_{f2}) - (m_1 c_v \ln(T_{b1}) + m_2 c_v \ln(T_{b2}))$$

Plugging in the previous expression for  $T_{f2}$

$$S_f = S_b + m_1 c_v \ln(T_{f1}) + m_2 c_v \ln(472.2 - 0.4T_{f1}) - (m_1 c_v \ln(T_{b1}) + m_2 c_v \ln(T_{b2}));$$

$$\Rightarrow S_f = m_1 c_v \ln(T_{f1}) + m_2 c_v \ln(472.2 - 0.4T_{f1}) - (m_1 c_v \ln(T_{b1}) + m_2 c_v \ln(T_{b2})) + S_b;$$

$$\Rightarrow S_f = m_1 c_v \ln(T_{f1}) + m_2 c_v \ln(472.2 - 0.4T_{f1}) + \text{constant}$$

Differentiating the expression with respect to the unknown final temperature  $T_{f1}$

$$\frac{d}{dT_{f1}} [S_f] = \frac{d}{dT_{f1}} [m_1 c_v \ln(T_{f1}) + m_2 c_v \ln(472.2 - 0.4T_{f1}) + \text{constant}];$$

$$\frac{dS_f}{dT_{f1}} = \frac{m_1 c_v}{T_{f1}} - \frac{0.4m_2 c_v}{472.2 - 0.4T_{f1}}$$

Setting the derivative to zero,

$$0 = \frac{m_1 c_v}{T_{f1}} - \frac{0.4m_2 c_v}{472.2 - 0.4T_{f1}};$$

$$\Rightarrow 0 = \frac{m_1}{T_{f1}} - \frac{0.4m_2}{472.2 - 0.4T_{f1}};$$

$$\Rightarrow 0 = \frac{2}{T_{f1}} - \frac{(0.4)(5)}{472.2 - 0.4T_{f1}};$$

$$\Rightarrow T_{f1} = 337.286 \text{ K} = 64.29^\circ \text{C}$$

Now plugging  $T_{f1}$  into the energy equation,

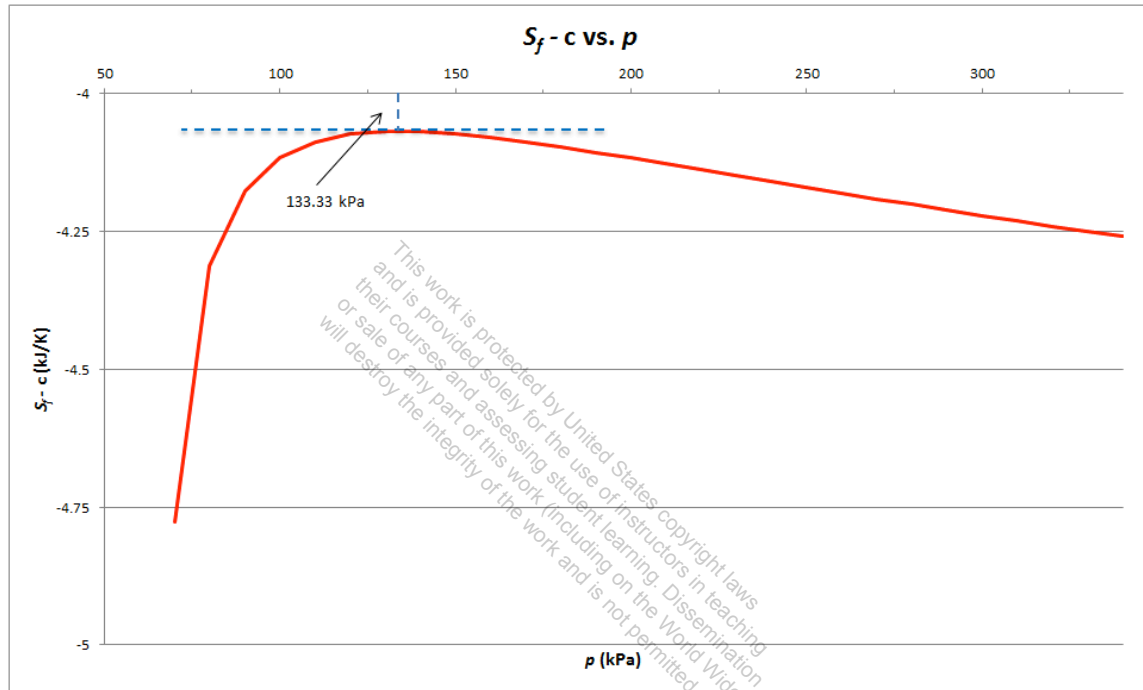
$$T_{f2} = 472.2 - (0.4)(337.29) = 337.286 = 64.29^\circ \text{C}$$

Thus we have proved that maximum entropy occurs at the final equilibrium temperature

$$T_{f1} = T_{f2} = 64.29^\circ \text{C}.$$

**14-1-2 [BRK]** An isolated cylindrical container is separated into two chambers separated by a pinned piston. The left chamber contains 0.1 kg of H<sub>2</sub> at 200 kPa, 20°C and the right chamber contains 0.2 kg of He at 100 kPa, 20°C. After the pin is removed, the piston oscillates and eventually comes to equilibrium. (a) Show that the entropy ( $S$ ) of the combined isolated system reaches a maximum when the pressure ( $p$ ) on two sides are equal. Assume the piston to be thermally conductive and use the PG model. (b) Can the equilibrium be reached if friction is assumed to be absent?

### SOLUTION



Define the left and right chambers as systems 1 and 2, respectively.

Given:

$$p_{b1} = 200 \text{ kPa}$$

$$p_{b2} = 100 \text{ kPa}$$

$$T_{b1} = T_{b2} = 293 \text{ K}$$

$$\bar{R} = 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\bar{M}_{\text{H}_2} = 2 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{He}} = 4 \frac{\text{kg}}{\text{kmol}}$$

Due to the thermally conductive piston, internal heat transfer will ensure that

$T_{f1} = T_{f2} = T_{b1} = T_{b2} = T = 293 \text{ K}$ , so the energy equation is not considered.

It is known that  $\mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2 = \text{constant}$ , and assuming PG model we use the relation

$$\mathcal{V} = \frac{mRT}{p}$$

Now plugging the expression above into the volume equation,

$$\mathcal{V} = \frac{m_1 R_{\text{H}_2} T_{b1}}{p_{b1}} + \frac{m_2 R_{\text{He}} T_{b2}}{p_{b2}};$$

$$\Rightarrow \mathcal{V} = \bar{R}T \left( \frac{m_1}{\bar{M}_{\text{H}_2} p_{b1}} + \frac{m_2}{\bar{M}_{\text{He}} p_{b2}} \right) = (8.314)(293) \left( \frac{0.1}{(2)(200)} + \frac{0.2}{(4)(100)} \right) = 1.827 \text{ m}^3$$

Now that the constant total volume has been found, the pressures are allowed to become variable, and since this relation must also hold true at the final states

$$\mathcal{V} = \bar{R}T \left( \frac{m_1}{\bar{M}_{\text{H}_2} p_1} + \frac{m_2}{\bar{M}_{\text{He}} p_2} \right) = \bar{R}T \left( \frac{m_1}{\bar{M}_{\text{H}_2} p_{f1}} + \frac{m_2}{\bar{M}_{\text{He}} p_{f2}} \right)$$

Substituting the known variables, the volume equation reduces to

$$1.827 = (8.314)(293) \left( \frac{0.1}{2p_{f1}} + \frac{0.2}{4p_{f2}} \right);$$

$$\Rightarrow 1.827 = (8.314)(293) \left( \frac{0.1}{2p_{f1}} + \frac{0.1}{2p_{f2}} \right);$$

$$\Rightarrow 1.827 = (8.314)(293) \left( \frac{0.1}{2} \right) \left( \frac{1}{p_{f1}} + \frac{1}{p_{f2}} \right);$$

$$\Rightarrow 0.015 = \frac{1}{p_{f1}} + \frac{1}{p_{f2}};$$

$$\Rightarrow p_{f2} = \frac{p_{f1}}{0.015p_{f1} - 1}$$

The entropy of the combined system at the final state can be written as

$$\Delta S = \Delta S_1 + \Delta S_2;$$

$$\Rightarrow S_f = S_b + m_1 \left[ c_{p, \text{H}_2} \ln \left( \frac{T_{f1}}{T_{b1}} \right) - R_{\text{H}_2} \ln \left( \frac{p_{f1}}{p_{b1}} \right) \right] + m_2 \left[ c_{p, \text{He}} \ln \left( \frac{T_{f2}}{T_{b2}} \right) - R_{\text{He}} \ln \left( \frac{p_{f2}}{p_{b2}} \right) \right];$$

$$\Rightarrow S_f = S_b + m_1 \left[ c_{p, \text{H}_2} \ln \left( \frac{T}{T} \right) - R_{\text{H}_2} \ln \left( \frac{p_{f1}}{p_{b1}} \right) \right] + m_2 \left[ c_{p, \text{He}} \ln \left( \frac{T}{T} \right) - R_{\text{He}} \ln \left( \frac{p_{f2}}{p_{b2}} \right) \right];$$

$$\Rightarrow S_f = S_b - \left[ m_1 R_{\text{H}_2} \ln \left( \frac{p_{f1}}{p_{b1}} \right) + m_2 R_{\text{He}} \ln \left( \frac{p_{f2}}{p_{b2}} \right) \right];$$

$$\Rightarrow S_f = S_b - \left[ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln \left( \frac{p_{f1}}{p_{b1}} \right) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln \left( \frac{p_{f2}}{p_{b2}} \right) \right];$$

$$\Rightarrow S_f = S_b - \left\{ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{f1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(p_{f2}) - \left[ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{b1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(p_{b2}) \right] \right\}$$

Plugging in the previous expression for  $p_{f2}$

$$S_f = S_b - \left\{ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{f1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln \left( \frac{p_{f1}}{0.015 p_{f1} - 1} \right) - \left[ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{b1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(p_{b2}) \right] \right\};$$

$$\Rightarrow S_f = - \left[ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{f1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln \left( \frac{p_{f1}}{0.015 p_{f1} - 1} \right) \right] + S_b + \left[ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{b1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(p_{b2}) \right];$$

$$\Rightarrow S_f = - \left[ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{f1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(p_{f1}) - m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(0.015 p_{f1} - 1) \right] + \text{constant}$$

Differentiating the expression with respect to the unknown final pressure  $p_{f1}$

$$\frac{d}{dp_{f1}} [S_f] = \frac{d}{dp_{f1}} \left\{ - \left[ m_1 \left( \frac{\bar{R}}{\bar{M}_{\text{H}_2}} \right) \ln(p_{f1}) + m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(p_{f1}) - m_2 \left( \frac{\bar{R}}{\bar{M}_{\text{He}}} \right) \ln(0.015 p_{f1} - 1) \right] + \text{constant} \right\}$$

$$\Rightarrow \frac{dS_f}{dp_{f1}} = - \left( \frac{m_1 \bar{R}}{\bar{M}_{\text{H}_2} p_{f1}} + \frac{m_2 \bar{R}}{\bar{M}_{\text{He}} p_{f1}} - \frac{0.015 m_2 \bar{R}}{\bar{M}_{\text{He}} (0.015 p_{f1} - 1)} \right);$$

$$\Rightarrow \frac{dS_f}{dp_{f1}} = \frac{0.015 m_2 \bar{R}}{\bar{M}_{\text{He}} (0.015 p_{f1} - 1)} - \left( \frac{m_1 \bar{R}}{\bar{M}_{\text{H}_2} p_{f1}} + \frac{m_2 \bar{R}}{\bar{M}_{\text{He}} p_{f1}} \right)$$

Setting the derivative to zero,

$$\begin{aligned}
 0 &= \frac{0.015m_2\bar{R}}{\bar{M}_{\text{He}}(0.015p_{f1}-1)} - \left( \frac{m_1\bar{R}}{\bar{M}_{\text{H}_2}p_{f1}} + \frac{m_2\bar{R}}{\bar{M}_{\text{He}}p_{f1}} \right); \\
 \Rightarrow 0 &= \frac{0.015m_2}{\bar{M}_{\text{He}}(0.015p_{f1}-1)} - \left( \frac{m_1}{\bar{M}_{\text{H}_2}p_{f1}} + \frac{m_2}{\bar{M}_{\text{He}}p_{f1}} \right); \\
 \Rightarrow 0 &= \frac{(0.015)(0.2)}{(4)(0.015p_{f1}-1)} - \left( \frac{0.1}{2p_{f1}} + \frac{0.2}{4p_{f1}} \right); \\
 \Rightarrow p_{f1} &= 133.333 \text{ kPa}
 \end{aligned}$$

Now plugging  $p_{f1}$  into the volume equation,

$$p_{f2} = \frac{133.333}{(0.015)(133.333)-1} = 133.334 \text{ kPa}$$

Thus we have proved that maximum entropy occurs at the final equilibrium pressure  $p_{f1} = p_{f2} = 133.33 \text{ kPa}$ . Additionally, if friction is absent equilibrium **cannot be reached**, as the piston will continuously oscillate.

**14-1-3 [BRG]** Since the beginning of atomic age, it is a practice to store uranium as gaseous uranium hexafluoride ( $\text{UF}_6$ , molar mass: 352 kg/kmol) in abandoned oil wells. The gas at the top of a 2 km deep well was sampled and found to be pure  $\text{UF}_6$  at 100 kPa, 300 K. Assuming equilibrium throughout the well, isothermal condition, and ideal gas behavior, determine the pressure ( $p$ ) at the bottom of the well.

### SOLUTION

Given:

$$\bar{R} = 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\bar{M}_{\text{UF}_6} = 352 \frac{\text{kg}}{\text{kmol}}$$

For an isothermal gas column using the ideal gas mixture, applying the equilibrium criterion yields

$$0 = \Delta \hat{h}^0 - T \Delta s + \Delta(\text{pe});$$

$$\Rightarrow \Delta(\text{pe}) = T \Delta s;$$

$$\Rightarrow T \Delta s = \Delta \left( \frac{gz}{1000} \right);$$

$$\Rightarrow T \left( c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right) = \frac{g(z_2 - z_1)}{1000};$$

$$\Rightarrow -RT \ln \frac{p_2}{p_1} = \frac{g(z_2 - z_1)}{1000};$$

$$\Rightarrow RT \ln \frac{p_1}{p_2} = \frac{g(z_2 - z_1)}{1000};$$

$$\Rightarrow \ln \frac{p_1}{p_2} = \frac{g(z_2 - z_1)}{1000RT};$$

$$\Rightarrow \frac{p_1}{p_2} = \exp \left[ \frac{g(z_2 - z_1)}{1000RT} \right];$$

$$\Rightarrow p_1 = p_2 \exp \left[ \frac{g(z_2 - z_1)}{1000 \frac{\bar{R}}{\bar{M}_{\text{UF}_6}} T} \right] = (100) \exp \left[ \frac{(9.81)(2000 - 0)}{(1000) \left( \frac{8.314}{352} \right) (300)} \right] = 1594.14 \text{ kPa}$$

**14-1-4 [BRZ]** A mixture of hydrogen and uranium hexafluoride ( $\text{UF}_6$ , molar mass: 352 kg/kmol) is stored in an abandoned oil well (in spherical chambers radioactive uranium can become critical leading to nuclear meltdown). After a long time, after thermodynamic equilibrium is achieved, a sampling at the top reveals that the mixture consists of 50-50 (by volume) mixture at 100 kPa and 300 K. Determine the (a) total pressure ( $p_t$ ) and (b) molar composition 3 km below the surface. Assume isothermal condition and ideal gas behavior.

### SOLUTION

Given:

$$\bar{R} = 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\bar{M}_{\text{H}_2} = 2 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{UF}_6} = 352 \frac{\text{kg}}{\text{kmol}}$$

Knowing the volume ratio of the two gases at the surface of the well

$$\frac{p_{\text{H}_2,2}}{p_2} = \frac{v_{\text{H}_2,2}}{v_2} = 0.5;$$

$$\Rightarrow p_{\text{H}_2,2} = 0.5 p_2 = (0.5)(100) = 50 \text{ kPa}$$

$$\frac{p_{\text{UF}_6,2}}{p_2} = \frac{v_{\text{UF}_6,2}}{v_2} = 0.5$$

$$\Rightarrow p_{\text{UF}_6,2} = 0.5 p_2 = (0.5)(100) = 50 \text{ kPa}$$

The partial pressures at the bottom of the well,

$$p_{\text{H}_2,1} = p_{\text{H}_2,2} \exp \left[ \frac{g(z_2 - z_1)}{1000 \frac{\bar{R}}{\bar{M}_{\text{H}_2}} T} \right] = (50) \exp \left[ \frac{(9.81)(3000 - 0)}{(1000) \left( \frac{8.314}{2} \right) (300)} \right] = 51.19 \text{ kPa}$$

$$p_{\text{UF}_6,1} = p_{\text{UF}_6,2} \exp \left[ \frac{g(z_2 - z_1)}{1000 \frac{\bar{R}}{\bar{M}_{\text{UF}_6}} T} \right] = (50) \exp \left[ \frac{(9.81)(3000 - 0)}{(1000) \left( \frac{8.314}{352} \right) (300)} \right] = 3182.44 \text{ kPa}$$

The total pressure at the bottom of the well

$$p_1 = p_{\text{H}_2,1} + p_{\text{UF}_6,1} = 51.19 + 3182.44 = 3233.63 \text{ kPa}$$

The molar composition at the bottom of the well



$$y_{\text{H}_2,1} = \frac{p_{\text{H}_2,1}}{p_1} = \left( \frac{51.19}{3233.63} \right) = 0.0158$$

$$y_{\text{UF}_6,1} = \frac{p_{\text{UF}_6,1}}{p_1} = \left( \frac{3182.44}{3233.63} \right) = 0.9842$$

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**14-1-5 [BRP]** Hydrogen is produced at 100 kPa and 298 K at a rate of 1 kg/s from a mixture of hydrogen and methane containing 20% hydrogen and 80% methane by volume. If the device works by raising the pressure of the mixture on one side of a semipermeable membrane, determine (a) the minimum power consumption ( $\dot{W}$ ) and (b) the minimum mixture pressure necessary to produce hydrogen.

**SOLUTION**

Given:

$$\dot{M}_{\text{H}_2} = 2 \frac{\text{kg}}{\text{kmol}}$$

$$y_{\text{H}_2} = \frac{v_{\text{H}_2}}{v} = \frac{20}{100} = 0.2$$

$$\dot{m}_{\text{H}_2} = 1 \frac{\text{kg}}{\text{s}}$$

The minimum power consumption for separation

$$\dot{W}_{\text{rev, separation}} = \dot{m}_{\text{H}_2} \bar{R} T \ln y_{\text{H}_2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = \frac{\dot{m}_{\text{H}_2}}{\dot{M}_{\text{H}_2}} \bar{R} T \ln y_{\text{H}_2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = \frac{(1)(8.314)(298) \ln(0.2)}{2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = -1993.75 \text{ kW}$$

Therefore, the power consumed is **1993.75 kW**.

The minimum mixture pressure necessary

$$p_{\text{H}_2} = y_{\text{H}_2} p;$$

$$\Rightarrow p = \frac{p_{\text{H}_2}}{y_{\text{H}_2}} = \frac{100}{0.2} = \mathbf{500 \text{ kPa}}$$

**14-1-6 [BRU]** An ideal gas mixture contains 9 kmol of argon and 1 kmol of helium at a total pressure of 100 kPa and a temperature of 25°C. Determine the chemical potential ( $\mu_k$ ) of argon and helium in the mixture.

**SOLUTION**

Given:

$$p_0 = 101.325 \text{ kPa}$$

The mole fractions of both argon and helium

$$n = n_{\text{Ar}} + n_{\text{He}} = 9 + 1 = 10 \text{ kmol}$$

$$y_{\text{Ar}} = \frac{n_{\text{Ar}}}{n} = \frac{9}{10} = 0.90$$

$$y_{\text{He}} = \frac{n_{\text{He}}}{n} = \frac{1}{10} = 0.10$$

Therefore, the partial pressures are

$$p_{\text{Ar}} = y_{\text{Ar}} p = (0.90)(100) = 90 \text{ kPa}$$

$$p_{\text{He}} = y_{\text{He}} p = (0.10)(100) = 10 \text{ kPa}$$

The chemical potential of the argon and helium will be their respective partial molar Gibbs functions. Using the n-IG Model system-state TESTcalc.

$$\bar{\mu}_k = \bar{g}_k(p_k, T) = \bar{h}_k(T) - T\bar{s}_k(p_k, T) = [\bar{h}_k(T) - T\bar{s}_k^\circ(T)] + \bar{R}T \ln \frac{p_k}{p_0}$$

$$\bar{\mu}_{\text{Ar}} = [\bar{h}_{\text{Ar}}(T) - T\bar{s}_{\text{Ar}}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{Ar}}}{p_0};$$

$$\Rightarrow \bar{\mu}_{\text{Ar}} = [0 - (298.15)(154.845)] + (8.314)(298.15) \ln \left( \frac{90}{101.325} \right) = -46460.84 \frac{\text{kJ}}{\text{kmol}}$$

$$\bar{\mu}_{\text{He}} = [\bar{h}_{\text{He}}(T) - T\bar{s}_{\text{He}}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{He}}}{p_0};$$

$$\Rightarrow \bar{\mu}_{\text{He}} = [0 - (298.15)(126.152)] + (8.314)(298.15) \ln \left( \frac{10}{101.325} \right) = -43352.54 \frac{\text{kJ}}{\text{kmol}}$$

**14-1-7 [BRX]** Determine the chemical potential (molar specific Gibbs function ( $g_k^-$ )) of pure oxygen at (a) 100 kPa, 300 K, (b) 1000 kPa, 300 K, and (c) 100 kPa, 3000 K. Verify your answers using the n-IG system state TESTcalc.

**SOLUTION**

Given:

$$p_0 = 101.325 \text{ kPa}$$

$$y_{\text{O}_2} = 1 \therefore p_{\text{O}_2} = p$$

The chemical potential will be the specific molar Gibbs function

$$\bar{\mu}_k = \bar{g}_k(p_k, T) = \bar{h}_k(T) - T\bar{s}_k(p_k, T) = [\bar{h}_k(T) - T\bar{s}_k^\circ(T)] + \bar{R}T \ln \frac{p_k}{p_0}$$

Using Table D-5 or the n-IG Model system-state TESTcalc.

State-1:

$$p_1 = 100 \text{ kPa}, T_1 = 300 \text{ K}$$

$$\bar{\mu}_{\text{O}_2} = [\bar{h}_{\text{O}_2}(T) - T\bar{s}_{\text{O}_2}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{O}_2}}{p_0}$$

$$\Rightarrow \bar{\mu}_{\text{O}_2} = [34.2 - (300)(205.248)] + (8.314)(300) \ln \left( \frac{100}{101.325} \right) = -61573.03 \frac{\text{kJ}}{\text{kmol}}$$

State-2:

$$p_2 = 1000 \text{ kPa}, T_2 = 300 \text{ K}$$

$$\bar{\mu}_{\text{O}_2} = [\bar{h}_{\text{O}_2}(T) - T\bar{s}_{\text{O}_2}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{O}_2}}{p_0};$$

$$\Rightarrow \bar{\mu}_{\text{O}_2} = [34.2 - (300)(205.248)] + (8.314)(300) \ln \left( \frac{1000}{101.325} \right) = -55829.92 \frac{\text{kJ}}{\text{kmol}}$$

State-3:

$$p_3 = 100 \text{ kPa}, T_3 = 3000 \text{ K}$$

$$\bar{\mu}_{\text{O}_2} = [\bar{h}_{\text{O}_2}(T) - T\bar{s}_{\text{O}_2}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{O}_2}}{p_0};$$

$$\Rightarrow \bar{\mu}_{\text{O}_2} = [98067.4 - (3000)(284.478)] + (8.314)(3000) \ln \left( \frac{100}{101.325} \right) = -755694.91 \frac{\text{kJ}}{\text{kmol}}$$

**14-1-8 [BRC]** A membrane permeable to oxygen separates pure oxygen at 100kPa, 300 K from an ideal gas mixture of oxygen, nitrogen, and hydrogen of equal volume fractions at 300K. What is the minimum pressure ( $p$ ) of the mixture at which oxygen start separating from the mixture?

**SOLUTION**

Consider the pure oxygen to be represented by state-1 and the mixture by state-2.

State-1 (given  $p_1, T_1$ ):

$$y_{O_2,1} = 1$$

$$p_{O_2,1} = y_{O_2,1} p_1 = (1)(100) = 100 \text{ kPa}$$

State-2 (given  $T_2$ ):

$$y_{O_2,2} = \frac{p_{O_2,2}}{p_2} = \frac{1}{3}$$

$$p_{O_2,2} = y_{O_2,2} p_2 = \frac{p_2}{3}$$

The minimum mixture pressure necessary

$$p_{O_2,1} = p_{O_2,2};$$

$$\Rightarrow p_{O_2,1} = \frac{p_2}{3};$$

$$\Rightarrow p_2 = 3 p_{O_2,1} = (3)(100) = 300 \text{ kPa}$$

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**14-1-9 [BOS]** A two-phase liquid-vapor mixture of R-134a is in equilibrium at 22°C. Show that the specific Gibbs functions of the saturated liquid and saturated vapor are equal.

**SOLUTION**

Given:

$$\bar{M}_{\text{R-134a}} = 102.03 \frac{\text{kg}}{\text{kmol}}$$

State-1 (given  $T_1, x_1$ ):

$$p_1 = p_{\text{sat}@22^\circ\text{C}} = 608.54 \text{ kPa}$$

$$h_1 = h_{f@22^\circ\text{C}} = 81.35 \frac{\text{kJ}}{\text{kg}}; s_1 = s_{f@22^\circ\text{C}} = 0.3067 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $T_2, x_2$ ):

$$p_2 = p_{\text{sat}@22^\circ\text{C}} = 608.54 \text{ kPa}$$

$$h_2 = h_{g@22^\circ\text{C}} = 261.93 \frac{\text{kJ}}{\text{kg}}; s_2 = s_{g@22^\circ\text{C}} = 0.9186 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

In order for the saturated liquid and saturated vapor to be in equilibrium, their chemical potentials must be equal. Also working with a pure substance,

$$\bar{g}_f(p, T) = \bar{g}_g(p, T);$$

$$\Rightarrow g_f(p, T) = g_g(p, T);$$

$$\Rightarrow h_1 - T_1 s_1 = h_2 - T_2 s_2;$$

$$\Rightarrow 81.35 - (295.15)(0.3067) = 261.93 - (295.15)(0.9186);$$

$$\Rightarrow -9.19 \frac{\text{kJ}}{\text{kg}} \approx -9.17 \frac{\text{kJ}}{\text{kg}}$$

**14-1-10 [BRV]** Determine the chemical potential (molar specific Gibbs function) of pure hydrogen at (a) 100 kPa, 300 K, (b) 1000 kPa, 300 K, and (c) 100 kPa, 3000 K. Verify your answers using the n-IG system state TESTcalc.

### SOLUTION

Given:

$$p_0 = 101.325 \text{ kPa}$$

$$y_{\text{H}_2} = 1 \therefore p_{\text{H}_2} = p$$

The chemical potential will be the specific molar Gibbs function

$$\bar{\mu}_k = \bar{g}_k(p_k, T) = \bar{h}_k(T) - T\bar{s}_k(p_k, T) = [\bar{h}_k(T) - T\bar{s}_k^\circ(T)] + \bar{R}T \ln \frac{p_k}{p_0}$$

Using Table D-8 or the n-IG Model system-state TESTcalc.

State-1:

$$p_1 = 100 \text{ kPa}, T_1 = 300 \text{ K}$$

$$\bar{\mu}_{\text{H}_2} = [\bar{h}_{\text{H}_2}(T) - T\bar{s}_{\text{H}_2}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{H}_2}}{p_0}$$

$$\Rightarrow \bar{\mu}_{\text{H}_2} = [53.4 - (300)(130.859)] + (8.314)(300) \ln \left( \frac{100}{101.325} \right) = -39237.13 \frac{\text{kJ}}{\text{kmol}}$$

State-2:

$$p_2 = 1000 \text{ kPa}, T_2 = 300 \text{ K}$$

$$\bar{\mu}_{\text{H}_2} = [\bar{h}_{\text{H}_2}(T) - T\bar{s}_{\text{H}_2}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{H}_2}}{p_0};$$

$$\Rightarrow \bar{\mu}_{\text{H}_2} = [53.4 - (300)(130.859)] + (8.314)(300) \ln \left( \frac{1000}{101.325} \right) = -33493.75 \frac{\text{kJ}}{\text{kmol}}$$

State-3:

$$p_3 = 100 \text{ kPa}, T_3 = 3000 \text{ K}$$

$$\bar{\mu}_{\text{H}_2} = [\bar{h}_{\text{H}_2}(T) - T\bar{s}_{\text{H}_2}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{H}_2}}{p_0};$$

$$\Rightarrow \bar{\mu}_{\text{H}_2} = [88737.4 - (3000)(202.891)] + (8.314)(3000) \ln \left( \frac{100}{101.325} \right) = -520263.91 \frac{\text{kJ}}{\text{kmol}}$$

**14-1-11 [BRY]** Use the n-IG system state TESTcalc to (a) determine the chemical potential ( $\mu_k^-$ ) of hydrogen in an equimolar mixture of hydrogen and carbon dioxide at 100 kPa and 300 K. Plot how the chemical potential ( $\mu_k^-$ ) of hydrogen varies with pressure (in a range 50 kPa through 10000 kPa) as the temperature is held constant at (b) 300 K, (c) 1000 K.

### SOLUTION

Given:

$$p_0 = 101.325 \text{ kPa}$$

$$y_{\text{H}_2} = 0.5 \therefore p_{\text{H}_2} = 0.5p$$

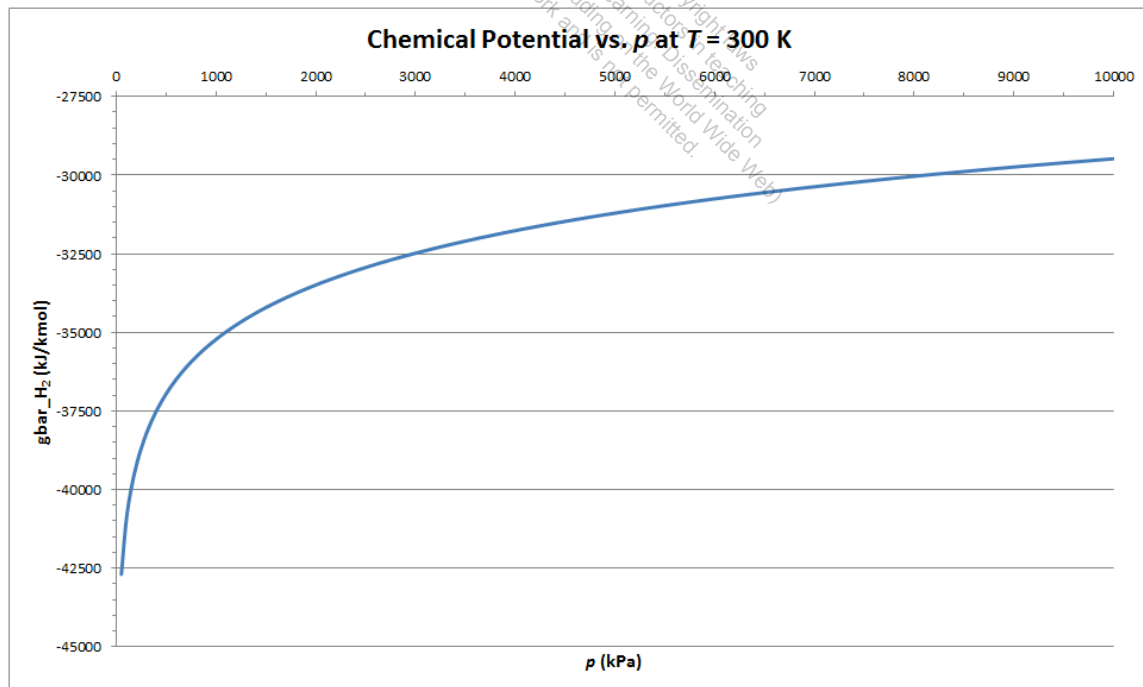
The chemical potential will be the specific molar Gibbs function

$$\bar{\mu}_k = \bar{g}_k(p_k, T) = \bar{h}_k(T) - T\bar{s}_k(p_k, T) = [\bar{h}_k(T) - T\bar{s}_k^\circ(T)] + \bar{R}T \ln \frac{p_k}{p_0}$$

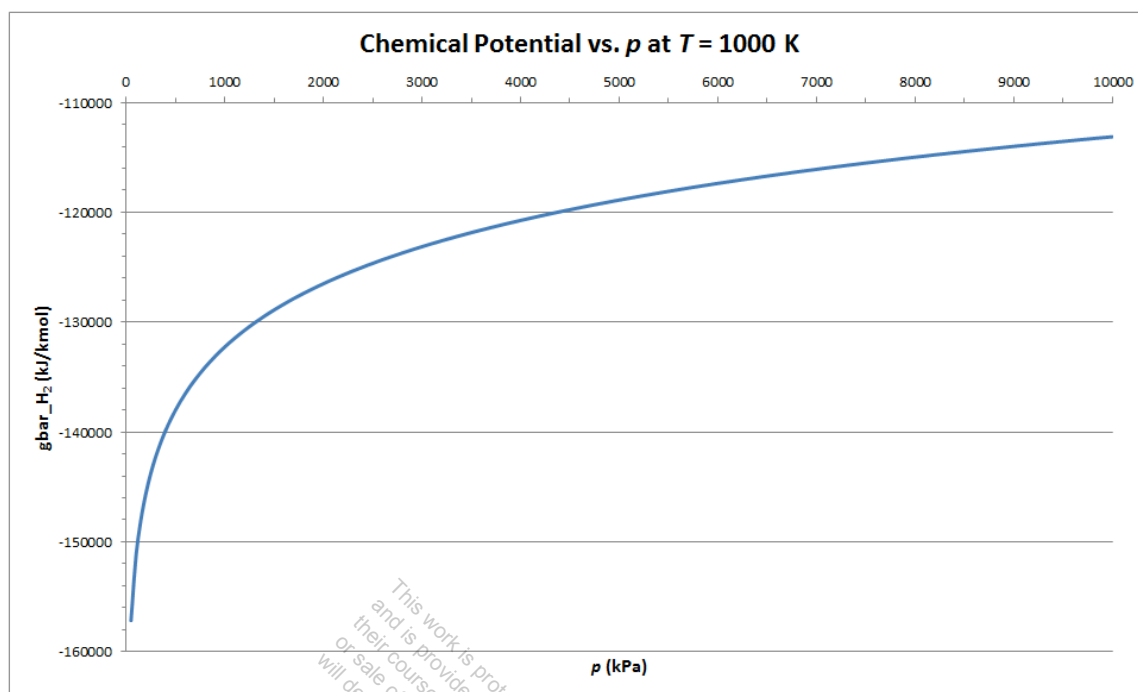
State-1 (given  $p_1, T_1$ ):

$$\bar{\mu}_{\text{H}_2} = [\bar{h}_{\text{H}_2}(T) - T\bar{s}_{\text{H}_2}^\circ(T)] + \bar{R}T \ln \frac{p_{\text{H}_2}}{p_0}$$

$$\Rightarrow \bar{\mu}_{\text{H}_2} = [53.5 - (300)(130.859)] + (8.314)(300) \ln \left( \frac{50}{101.325} \right) = -40965.88 \frac{\text{kJ}}{\text{kmol}}$$







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**14-1-12 [BRQ]** Oxygen is produced at 100 kPa and 298 K at a rate of 1 kg/s from air (21% oxygen and 79% nitrogen by volume). If the device works by raising the pressure of the mixture on one side of a semipermeable membrane, determine (a) the minimum power consumption ( $\dot{W}$ ), and (b) the minimum mixture pressure ( $p$ ) necessary to produce oxygen.

**SOLUTION**

Given:

$$\bar{M}_{\text{O}_2} = 32 \frac{\text{kg}}{\text{kmol}}$$

$$y_{\text{O}_2} = \frac{v_{\text{O}_2}}{v} = \frac{21}{100} = 0.21$$

$$\dot{m}_{\text{O}_2} = 1 \frac{\text{kg}}{\text{s}}$$

The minimum power consumption for separation

$$\dot{W}_{\text{rev, separation}} = \dot{m}_{\text{O}_2} \bar{R} T \ln y_{\text{O}_2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = \frac{\dot{m}_{\text{O}_2}}{\bar{M}_{\text{O}_2}} \bar{R} T \ln y_{\text{O}_2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = \frac{(1)(8.314)(298) \ln(0.21)}{32}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = -120.83 \text{ kW}$$

Therefore, the power consumed is **120.83 kW**.

The minimum mixture pressure necessary

$$p_{\text{O}_2} = y_{\text{O}_2} p;$$

$$\Rightarrow p = \frac{p_{\text{O}_2}}{y_{\text{O}_2}} = \frac{100}{0.21} = \mathbf{476.19 \text{ kPa}}$$

**14-1-13 [BRT]** Hydrogen is produced at 100 kPa and 298 K from a mixture of hydrogen and methane containing 10% hydrogen and 90% methane by volume. If the device has an exergetic efficiency of 20% (a) determine the power consumption ( $\dot{W}$ ) in kW to produce hydrogen at a rate of 1 kg/s. (b) What-if Scenario: What would be the power consumption if the temperature were 500 K instead?

**SOLUTION**

Given:

$$\dot{M}_{\text{H}_2} = 2 \frac{\text{kg}}{\text{kmol}}$$

$$y_{\text{H}_2} = \frac{v_{\text{H}_2}}{v} = \frac{10}{100} = 0.10$$

$$\dot{m}_{\text{H}_2} = 1 \frac{\text{kg}}{\text{s}}$$

The minimum power consumption for separation

$$\dot{W}_{\text{rev, separation}} = \dot{m}_{\text{H}_2} \bar{R} T \ln y_{\text{H}_2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = \frac{\dot{m}_{\text{H}_2}}{\dot{M}_{\text{H}_2}} \bar{R} T \ln y_{\text{H}_2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = \frac{(1)(8.314)(298) \ln(0.10)}{2}$$

$$\Rightarrow \dot{W}_{\text{rev, separation}} = -2852.41 \text{ kW}$$

Therefore, the reversible power consumed is 2852.41 kW

Applying the exergetic efficiency

$$\eta_{\text{II}} = \frac{\dot{W}_{\text{rev}}}{\dot{W}_{\text{net}}};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{\dot{W}_{\text{rev}}}{\eta_{\text{II}}} = \frac{2852.41}{0.20} = 14262.05 \text{ kW}$$