2-4-1 [ID] A tank contains 50 kg of water, which is stirred by a paddle wheel at 300 rpm while transmitting a torque of 0.2 kNm. After the tank achieves steady state, determine (a) the rate of heat transfer, (b) the rate of entropy transfer into the atmosphere and (c) the rate of entropy generation in the tank's universe. Assume the atmospheric temperature to be 25°C.

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{sh,in}} = -2\pi NT; \quad \Rightarrow \dot{W}_{\text{ext}} = -2\pi \frac{300}{60} 0.2; \quad \Rightarrow \dot{W}_{\text{ext}} = -6.28 \text{ kW};$

The energy balance equation simplifies as:

$$\frac{\cancel{dE}^{\,0}}{\cancel{dt}} = \sum \cancel{\dot{m}_i j_i}^{\,0} - \sum \cancel{\dot{m}_e j_e}^{\,0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -6.28 \text{ kW}$$

(b) The entropy balance equation is given as:

$$\frac{\cancel{dS}^{\,0}}{\cancel{dt}} = \sum \cancel{\dot{m}_i s_i}^{\,0} - \sum \cancel{\dot{m}_e s_e}^{\,0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{\text{loss}}}{T_B} + \dot{S}_{\text{gen}}; \quad \Rightarrow \dot{S}_{\text{gen}} = \frac{\dot{Q}_{\text{loss}}}{T_B};$$

Since entropy is transferred by heat transfer to atmosphere:

$$\frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{6.28}{(273+25)}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 0.021 \frac{\text{kW}}{\text{K}}$$

(c) From the entropy balance equation:

$$\dot{S}_{gen} = \frac{\dot{Q}_{\text{loss}}}{T_B}; \quad \Rightarrow \dot{S}_{gen} = 0.021 \frac{\text{kW}}{\text{K}}$$

2-4-2 [IM] A tank contains 1 kg of air at 500 K and 500 kPa. A 1 kW internal heater operates inside the tank at steady state to make up for the heat lost to the atmosphere which is at 300 K. Determine (a) the rate of entropy transfer into the atmosphere, (b) the rate of entropy generation in the system's universe, and (c) the internal rate of entropy generation.

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = -1\ \text{kW};$

The energy balance equation simplifies as:

$$\cancel{\frac{dE}{dt}}^{0} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \qquad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \qquad \Rightarrow \dot{Q} = -1\ \text{kW};$$

The entropy balance equation is given as:

$$\cancel{\frac{dS}{dt}}^{0} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{\text{loss}}}{T_B} + \dot{S}_{\text{gen}}; \qquad \Rightarrow \dot{S}_{\text{gen}} = \frac{\dot{Q}_{\text{loss}}}{T_B};$$

Since entropy is transferred into the atmosphere by heat:

$$\frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{\dot{Q}_{\text{loss}}}{T_0}; \qquad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{1}{300}; \qquad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 0.00333\ \frac{\text{kW}}{\text{K}}$$

(b) From the entropy balance equation:

$$\dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_{\text{loss}}}{T_B}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{1}{300}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.00333\ \frac{\text{kW}}{\text{K}}$$

(c) By choosing a boundary just inside of the system's surface:

$$\dot{S}_{\text{gen, int}} = \frac{\dot{Q}_{\text{loss}}}{T_B}; \qquad \Rightarrow \dot{S}_{\text{gen, int}} = \frac{1}{500}; \qquad \Rightarrow \dot{S}_{\text{gen, int}} = 0.002\ \frac{\text{kW}}{\text{K}}$$

2-4-3 [IJ] A rigid tank contains 1 kg of air initially at 300 K and 100 kPa. A 1 kW internal heater is turned on. After the tank achieves steady state, determine (a) the rate of heat transfer, (b) the rate of entropy transfer into the atmoshere and (c) the rate of entropy generation in the tank's universe. Assume the atmospheric temperature to be 0°C.

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = -1\text{ kW};$

The energy balance equation simplifies as:

$$\cancel{\frac{dE}{dt}}^{0} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \qquad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \qquad \Rightarrow \dot{Q} = -1\text{ kW}$$

(b) The entropy balance equation is given as:

$$\cancel{\frac{dS}{dt}}^{0} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{\text{loss}}}{T_B} + \dot{S}_{\text{gen,univ}}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_{\text{loss}}}{T_0};$$

Since entropy is only transferred by heat into the atmosphere,

$$\frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{\dot{Q}_{\text{loss}}}{T_0}; \qquad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{1}{273}; \qquad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 0.003663\ \frac{\text{kW}}{\text{K}}$$

(c) From the entropy balance equation:

$$\dot{S}_{\text{gen, univ}} = \frac{\dot{Q}_{\text{loss}}}{T_B}; \qquad \Rightarrow \dot{S}_{\text{gen, univ}} = \frac{\dot{Q}_{\text{loss}}}{T_0}; \qquad \Rightarrow \dot{S}_{\text{gen, univ}} = \frac{1}{273}; \qquad \Rightarrow \dot{S}_{\text{gen, univ}} = 0.003663\ \frac{\text{kW}}{\text{K}}$$

2-4-4 [IW] A closed chamber containing a gas is at steady state. The shaft transfers power at a rate of 2 kW to the paddle wheel and the electric lamp consumes electricity at a rate of 500 W. Using an energy balance determine (a) the rate of heat transfer. (b) If the surface temperature of the chamber is 400 K, determine the entropy generated within the chamber. (c) What-if Scenario: What would the entropy generation within the chamber be if the surface temperature increased to 500 K?

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{sh,in}} - \dot{W}_{\text{el,in}}; \quad \Rightarrow \dot{W}_{\text{ext}} = (-2) + (-0.5); \quad \Rightarrow \dot{W}_{\text{ext}} = -2.5 \text{ kW};$

The energy balance equation simplifies as:

$$\frac{d\cancel{E}^{\,0}}{\cancel{dt}} = \sum \cancel{\dot{m}_i j_i}^{\,0} - \sum \cancel{\dot{m}_e j_e}^{\,0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -2.5 \text{ kW}$$

(b) The entropy balance equation is given as:

$$\frac{d\cancel{S}^{\,0}}{\cancel{dt}} = \sum \cancel{\dot{m}_i s_i}^{\,0} - \sum \cancel{\dot{m}_e s_e}^{\,0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{\text{loss}}}{T_s} + \dot{S}_{\text{gen,int}}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = \frac{\dot{Q}_{\text{loss}}}{T_s}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = \frac{2.5}{400};$$

$$\Rightarrow \dot{S}_{\text{gen,int}} = 0.00625 \ \frac{\text{kW}}{\text{K}}$$

(c) $\dot{S}_{\text{gen, int}} = \frac{\dot{Q}_{\text{loss}}}{T_s}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = \frac{2.5}{500}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = 0.005 \ \frac{\text{kW}}{\text{K}}$

2-4-5 [LR] A copper block receives heat from two different sources: 5 kW from a source at 1500 K and 3 kW from a source at 1000 K. It loses heat to atmosphere at 300 K. Assuming the block to be at steady state, determine (a) the net rate of heat transfer in kW; (b) the rate of entropy generation in the system's universe. What-if Scenario: What would the entropy generation if the second source were also at 1500 K?

SOLUTION

(a) $$\frac{d\cancel{E}^{\,0}}{dt} = \cancel{\dot{J}_{\text{net}}}^{\,0} + \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \cancel{\dot{W}_{\text{ext}}}^{\,0};$$

$$\Rightarrow \dot{Q}_3 = \dot{Q}_1 + \dot{Q}_2; \qquad \Rightarrow \dot{Q}_3 = (5+3); \qquad \Rightarrow \dot{Q}_3 = 8 \text{ kW};$$

$$\Rightarrow \dot{Q}_{\text{net}} = \dot{Q}; \qquad \Rightarrow \dot{Q}_{\text{net}} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3; \qquad \Rightarrow \dot{Q}_{\text{net}} = 0 \text{ kW}$$

(b) $$\frac{d\cancel{S}^{\,0}}{dt} = \cancel{\dot{S}_{\text{net}}}^{\,0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} + \dot{S}_{\text{gen,univ}};$$

$$\dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{8}{300} - \frac{5}{1500} - \frac{3}{1000};$$

$$\Rightarrow \dot{S}_{\text{gen}} = 0.0203 \ \frac{\text{kW}}{\text{K}}$$

2-4-6 [LO] An electric bulb consumes 500 W of electricity at steady state. The outer surface of the bulb is warmer than the surrounding atmosphere by 75°C. If the atmospheric temperature is 300 K, determine (a) the rate of heat transfer between the bulb (the system) and the atmosphere. Also determine the entropy generation rate (b) within the bulb, (c) in the system's universe and (d) in the immediate surroundings outside the bulb.

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = -0.5 \text{ kW};$

The energy balance equation simplifies as:

$$\cancel{\frac{dE}{dt}}^{0} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -0.5 \text{ kW}$$

(b) The entropy balance equation is given as:

$$\cancel{\frac{dS}{dt}}^{0} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow \dot{S}_{\text{gen,int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = -\frac{-0.5}{375}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = 0.00133 \frac{\text{kW}}{\text{K}}$$

(c) $$\dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_0}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{-0.5}{300}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.001667 \frac{\text{kW}}{\text{K}}$$

(d) $$\dot{S}_{\text{gen, surr}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen, surr}} = 0.001667 - 0.00133; \quad \Rightarrow \dot{S}_{\text{gen, surr}} = 0.00033 \frac{\text{kW}}{\text{K}}$$

2-4-7 [LB] An electric heater consumes 2 kW of electricity at steady state to keep a house at 27°C. The outside temperature is -10°C. Taking the heater inside the house as the system, determine (a) the maximum possible energetic efficiency of the heater, (b) the rate of entropy generation in the heater's universe in W/K. (c) If the surface of the heater is at 150°C, how much entropy is generated in the immediate surroundings of the the heater?

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = 2 \text{ kW};$

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{\text{net}}}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{out}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -2 \text{ kW};$$

$$\eta_{\text{I-max}} \equiv \frac{\text{Desired energy output}}{\text{Required energy input}}; \quad \Rightarrow \eta_{\text{I-max}} = \frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{el,in}}}; \quad \Rightarrow \eta_{\text{I-max}} = \frac{2}{2};$$

$$\Rightarrow \eta_{\text{I-max}} = 100\%$$

(b) $$\cancel{\frac{dS}{dt}}^{0} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{-2}{273+27}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.00667 \frac{\text{kW}}{\text{K}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = 6.67 \frac{\text{W}}{\text{K}}$$

(c) $$\dot{S}_{\text{gen, univ}} = 6.67 \frac{\text{kW}}{\text{K}};$$

$$\dot{S}_{\text{gen, int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = -\frac{-2000}{273+150}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = \frac{2000}{423}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 4.728 \frac{\text{W}}{\text{K}};$$

$$\dot{S}_{\text{gen,ext}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 6.67 - 4.728; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 1.94 \frac{\text{W}}{\text{K}}$$

2-4-8 [LS] An electric adaptor for a notebook computer (converting 110 volts to 19 volts) operates 10^{o}C warmer than the surroundings, which is at 20^{o}C. If the output current is measured at 3 amps and heat is lost from the adapter at a rate of 10 W, determine (a) the energetic efficiency of the device, (b) the rate of internal entropy generation and (c) the rate of external entropy generation.

SOLUTION

(a) $\dot{W}_{\text{ext}} = \dot{W}_{\text{el,out}} - \dot{W}_{\text{el,in}}; \quad \Rightarrow \dot{W}_{\text{ext}} = \dfrac{(19)(3)}{1000} - \dot{W}_{\text{el,in}}; \quad [\text{kW}]$

$\dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = -\dfrac{10}{1000}; \quad [\text{kW}]$

The energy balance equation is given as:

$$\cancelto{0}{\frac{dE}{dt}} = \sum \cancelto{0}{\dot{m}_i j_i} - \sum \cancelto{0}{\dot{m}_e j_e} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{Q}; \quad \Rightarrow \dot{W}_{\text{el,out}} - \dot{W}_{\text{el,in}} = -\dot{Q}_{\text{loss}};$$

$$\Rightarrow \dot{W}_{\text{el,in}} = \dot{W}_{\text{el,out}} + \dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{W}_{\text{el,in}} = \frac{(3)(19)}{1000} + 0.01; \quad \Rightarrow \dot{W}_{\text{el,in}} = 0.067 \text{ kW};$$

$$\eta_{\text{I}} \equiv \frac{\text{Desired energy output}}{\text{Required energy input}}; \quad \Rightarrow \eta_{\text{I}} = \frac{\dot{W}_{\text{el,out}}}{\dot{W}_{\text{el,in}}}; \quad \Rightarrow \eta_{\text{I}} = \frac{\left[\dfrac{(3)(19)}{1000}\right]}{0.067}; \quad \Rightarrow \eta_{\text{I}} = 0.85;$$

$$\Rightarrow \eta_{\text{I}} = 85\%$$

(b) The entropy balance equation is given as:

$$\cancelto{0}{\frac{dS}{dt}} = \sum \cancelto{0}{\dot{m}_i s_i} - \sum \cancelto{0}{\dot{m}_e s_e} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

$$\dot{S}_{\text{gen, int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = -\frac{-0.01}{(273+20+10)}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 0.000033 \ \frac{\text{kW}}{\text{K}};$$

$$\Rightarrow \dot{S}_{\text{gen, int}} = 0.033 \ \frac{\text{W}}{\text{K}}$$

(c) $\dot{S}_{\text{gen, univ}} = -\dfrac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = -\dfrac{-0.01}{(273+20)}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 0.0000341 \ \dfrac{\text{kW}}{\text{K}};$

$$\Rightarrow \dot{S}_{\text{gen, univ}} = 0.034 \ \frac{\text{W}}{\text{K}};$$

$$\dot{S}_{\text{gen, ext}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen, ext}} = 0.034 - 0.033; \quad \Rightarrow \dot{S}_{\text{gen, ext}} = 0.001 \ \frac{\text{W}}{\text{K}}$$

2-4-9 [LA] At steady state, the input shaft of a gearbox rotates at 2000 rpm while transmitting a torque of 0.2 kN-m. Due to friction, 1 kW of power is dissipated into heat and the rest is delivered to the output shaft. If the atmospheric temperature is 300 K and the surface of the gearbox maintains a constant temperature of 350 K, determine (a) the rate of entropy transfer into the atmosphere, (b) the rate of entropy generation in the system's universe, (c) the rate of entropy generation within the gearbox and (d) the rate of entropy generation in the immediate surroundings.

SOLUTION

(a) $\dot{W}_{\text{ext}} = \dot{W}_{\text{sh,out}} - \dot{W}_{\text{sh,in}}; \quad [\text{kW}]$

$\dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = -1; \quad [\text{kW}]$

The entropy balance equation is given as:

$$\cancelto{0}{\frac{dS}{dt}} = \sum \cancelto{0}{\dot{m}_i s_i} - \sum \cancelto{0}{\dot{m}_e s_e} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

The rate of entropy transfer into air is give as:

$$\frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{1}{300}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 0.00333\ \frac{\text{kW}}{\text{K}}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 3.33\ \frac{\text{W}}{\text{K}}$$

(b) From the entropy balance equation:

$$\dot{S}_{\text{gen, univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = -\frac{-1}{300}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 0.00333\ \frac{\text{kW}}{\text{K}}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 3.33\ \frac{\text{W}}{\text{K}}$$

(c) $$\dot{S}_{\text{gen, int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = -\frac{-1}{350}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 0.002857\ \frac{\text{kW}}{\text{K}}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 2.857\ \frac{\text{W}}{\text{K}}$$

(d) $$\dot{S}_{\text{gen,ext}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 3.33 - 2.857; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 0.476\ \frac{\text{W}}{\text{K}}$$

2-4-10 [LH] A gearbox (a closed steady system that converts low-torque shaft power to high-torque shaft power) consumes 100 kW of shaft work Due to lack of proper lubrication, the frictional losses amounts to 5 kW, resulting in an output power of 95 kW. The surface of the greabox is measured to be 350 K while the surrounding temperature is 300 K. Determine (a) the first law efficiency (η) of the gearbox, (b) the heat transfer rate (c) the rate of entropy generation in kW/K in the system's universe, and (d) the external rate of entropy generation (entropy that is generated in the immediate surroundings of the gear box).

SOLUTION

(a) $\dot{W}_{\text{ext}} = \dot{W}_{\text{sh,out}} - \dot{W}_{\text{sh,in}}; \quad \Rightarrow \dot{W}_{\text{ext}} = 95 - 100; \quad \Rightarrow \dot{W}_{\text{ext}} = -5 \text{ kW};$

$$\eta_{\text{I}} \equiv \frac{\text{Desired energy output}}{\text{Required energy input}}; \quad \Rightarrow \eta_{\text{I}} = \frac{\dot{W}_{\text{el,out}}}{\dot{W}_{\text{el,in}}}; \quad \Rightarrow \eta_{\text{I}} = \frac{95}{100}; \quad \Rightarrow \eta_{\text{I}} = 0.95;$$

$$\Rightarrow \eta_{\text{I}} = 95\%$$

(b) The energy balance equation produces:

$$\cancel{\frac{dE}{dt}}^{0} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -5 \text{ kW}$$

(c) $$\cancel{\frac{dS}{dt}}^{0} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

$$\dot{S}_{\text{gen, univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = -\frac{-5}{300}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 0.01667 \ \frac{\text{kW}}{\text{K}}$$

(d) $$\dot{S}_{\text{gen, int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = -\frac{-5}{350}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 0.01428 \ \frac{\text{kW}}{\text{K}};$$

$$\dot{S}_{\text{gen,ext}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 0.01667 - 0.01428; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 0.00238 \ \frac{\text{kW}}{\text{K}}$$

2-4-11 [LN] A closed steady system receives 1000 kW of heat from a reservoir at 1000 K and 2000 kW of heat from a reservoir at 2000 K. Heat is rejected to the two reservoirs at 300 K and 3000 K, respectively. (a) Determine the maximum amount of heat that can be transferred to the reservoir at 3000 K. (b) The device clearly transfers heat to a high temperature TER without directly consuming external work. Is this a violation of the Clausius statement of the second law of thermodynamics?(1:Yes; 2:No)

SOLUTION

(a) Designating the reservoirs with the suffixes 1, 2, 3, and 4 respectively, the energy balance equation for the closed-steady system can be written as:

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{\text{net}}}^{0} + \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \dot{Q}_4 - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow \dot{Q}_3 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_4;$$

$$\Rightarrow \dot{Q}_3 = 3000 - \dot{Q}_4;$$

The entropy balance equation simplifies as:

$$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_4}{T_4} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \frac{\dot{Q}_3}{T_3} + \frac{\dot{Q}_4}{T_4} = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \dot{S}_{\text{gen,univ}};$$

Substituting $\dot{Q}_3$ form the energy equation,

$$\Rightarrow \frac{3000 - \dot{Q}_4}{300} + \frac{\dot{Q}_4}{3000} = \frac{1000}{1000} + \frac{2000}{2000} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow 10 - \frac{\dot{Q}_4}{300} + \frac{\dot{Q}_4}{3000} = 2 + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{Q}_4 = \frac{-8 + \dot{S}_{\text{gen,univ}}}{\left(\frac{1}{3000} - \frac{1}{300}\right)} = 2666.67 - 333.33\dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{Q}_{4,\max} = 2666.67 \text{ kW}$$

(b) 2: No. As long as entropy is not destroyed in the system's universe, the second law is not violated.