**9-1-1** [OPE] Water is the working fluid in a Carnot vapor power cycle. Saturated liquid enters the boiler at a pressure of 10 MPa, and enters the turbine as saturated vapor. The condenser pressure is 9 kPa. Determine (a) the thermal efficiency ( $\eta_{th}$ ), (b) the back-work ratio, (c) the heat transfer to the working fluid per unit mass ( $q_{in}$ ) passing through the boiler in kJ/kg.

#### **SOLUTION**

State-1 (given  $p_1 = p_4, x_1$ ):

$$h_1 = h_{g@10\text{MPa}} = 2724.67 \frac{\text{kJ}}{\text{kg}}; \ s_1 = s_{g@10\text{MPa}} = 5.6141 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@9kPa} = 183.27 \frac{kJ}{kg}; \ h_{fg@9kPa} = 2397.72 \frac{kJ}{kg}; \ s_{f@9kPa} = 0.6227 \frac{kJ}{kg \cdot K}; \ s_{fg@9kPa} = 7.5649 \frac{kJ}{kg \cdot K}$$

$$x_2 = \frac{s_2 - s_{f@9\text{kPa}}}{s_{fg@9\text{kPa}}} = \frac{5.6141 - 0.6227}{7.5649} = 0.660$$

$$h_2 = h_{f@9kPa} + x_2 h_{fg@9kPa} = 183.27 + (0.660)(2397.72) = 1765.77 \frac{kJ}{kg}$$

State-3 (given  $p_3 = p_2, s_3 = s_4$ ):

$$h_{f@9kPa} = 183.27 \frac{kJ}{kg}; h_{fg@9kPa} = 2397.72 \frac{kJ}{kg}; s_{f@9kPa} = 0.6227 \frac{kJ}{kg \cdot K}; s_{fg@9kPa} = 7.5649 \frac{kJ}{kg \cdot K}$$

$$x_3 = \frac{s_3 - s_{f@9kPa}}{s_{fg@9kPa}} = \frac{3.3596 - 0.6227}{7.5649} = 0.362$$

$$h_3 = h_{f@9kPa} + x_3 h_{fg@9kPa} = 183.27 + (0.362)(2397.72) = 1051.24 \frac{kJ}{kg}$$

State-4 (given  $p_4, x_4$ ):

$$h_4 = h_{f@10\text{MPa}} = 1407.59 \frac{\text{kJ}}{\text{kg}}; \ s_4 = s_{f@10\text{MPa}} = 3.3596 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_T = h_1 - h_2 = 2724.67 - 1765.77 = 958.90 \frac{\text{kJ}}{\text{kg}}$$

Device-B (2-3): 
$$q_{\text{out}} = h_2 - h_3 = 1765.77 - 1051.24 = 714.53 \frac{\text{kJ}}{\text{kg}}$$

Device-C (3-4): 
$$w_P = h_4 - h_3 = 1407.59 - 1051.24 = 356.35 \frac{\text{kJ}}{\text{kg}}$$
  
Device-D (4-1):  $q_{\text{in}} = h_1 - h_4 = 2724.67 - 1407.59 = 1317.08 \frac{\text{kJ}}{\text{kg}}$ 

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 958.90 - 356.35 = 602.55 \frac{\text{kJ}}{\text{kg}}$$
  
$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{602.55}{1317.08} = 45.7\%$$

Back work ratio is

BWR = 
$$\frac{w_P}{w_T} = \frac{356.35}{958.90} = 37.2\%$$

9-1-2 [OPI] Water enters the boiler of a steady flow Carnot engine as a saturated liquid at 800 kPa, and leaves with a quality of 0.95. Steam leaves the turbine at a pressure of 100 kPa. Determine (a) the thermal efficiency ( $\eta_{th}$ ) and (b) the net work ( $w_{net}$ ) output per unit mass.

# **SOLUTION**

State-1 (given  $p_1 = p_4, x_1$ ):

$$\begin{split} h_{f @ 800 \text{kPa}} &= 721.07 \frac{\text{kJ}}{\text{kg}}; \ h_{fg @ 800 \text{kPa}} = 2048.04 \frac{\text{kJ}}{\text{kg}}; \ s_{f @ 800 \text{kPa}} = 2.0461 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg @ 800 \text{kPa}} = 4.6168 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ h_{1} &= h_{f @ 800 \text{kPa}} + x_{2} h_{fg @ 800 \text{kPa}} = 721.07 + (0.95) (2048.04) = 2666.71 \frac{\text{kJ}}{\text{kg}} \\ s_{1} &= s_{f @ 800 \text{kPa}} + x_{2} s_{fg @ 800 \text{kPa}} = 2.0461 + (0.95) (4.6168) = 6.4321 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{split}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$\begin{split} h_{f@100\text{kPa}} &= 417.44 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@100\text{kPa}} = 2258.02 \frac{\text{kJ}}{\text{kg}}; \ s_{f@100\text{kPa}} = 1.3025 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@100\text{kPa}} = 6.0568 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ x_2 &= \frac{s_2 - s_{f@100\text{kPa}}}{s_{fg@100\text{kPa}}} = \frac{6.4321 - 1.3025}{6.0568} = 0.847 \end{split}$$

$$h_2 = h_{f @ 100 \text{kPa}} + x_2 h_{fg @ 100 \text{kPa}} = 417.44 + (0.847)(2258.02) = 2329.98 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given 
$$p_3 = p_2, s_3 = s_4$$
).

$$h_{f@100\text{kPa}} = 417.44 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@100\text{kPa}} = 2258.02 \frac{\text{kJ}}{\text{kg}}; \ s_{f@100\text{kPa}} = 1.3025 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@100\text{kPa}} = 6.0568 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_3 = \frac{s_3 - s_{f@100\text{kPa}}}{s_{fg@100\text{kPa}}} = \frac{2.0461 - 1.3025}{6.0568} = 0.123$$

$$h_3 = h_{f@100\text{kPa}} + x_2 h_{fg@100\text{kPa}} = 417.44 + (0.123)(2258.02) = 695.18 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4, x_4$ ):

$$h_4 = h_{f @ 800 \text{kPa}} = 721.07 \frac{\text{kJ}}{\text{kg}}; \ s_1 = s_{f @ 800 \text{kPa}} = 2.0461 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

A steady-state energy analysis is carried out for each device as follows.

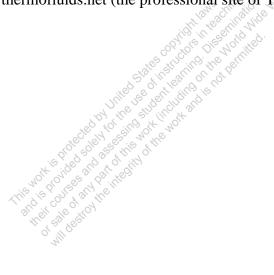
Device-A (1-2): 
$$w_T = h_1 - h_2 = 2666.71 - 2329.98 = 336.73 \frac{\text{kJ}}{\text{kg}}$$

Device-B (2-3): 
$$q_{\text{out}} = h_2 - h_3 = 2329.98 - 695.18 = 1634.80 \frac{\text{kJ}}{\text{kg}}$$
  
Device-C (3-4):  $w_P = h_4 - h_3 = 721.07 - 695.18 = 25.89 \frac{\text{kJ}}{\text{kg}}$   
Device-D (4-1):  $q_{\text{in}} = h_1 - h_4 = 2666.71 - 721.07 = 1945.64 \frac{\text{kJ}}{\text{kg}}$ 

The thermal efficiency and net work per unit mass are

$$w_{\text{net}} = w_T - w_P = 336.73 - 25.89 = 310.84 \frac{\text{kJ}}{\text{kg}}$$
  
$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{310.84}{1945.64} = 15.9\%$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).



9-1-3 [OPL] Water is the working fluid in a Carnot vapor power cycle. Saturated liquid enters the boiler at a pressure of 10 MPa, and saturated vapor enters the turbine. The condenser pressure is 8 kPa. The effects of irreversibilities in the adiabatic expansion and the compression processes are taken into consideration. The turbine and pump efficiencies are 80% and 75%, respectively. Determine (a) the thermal efficiency  $(\eta_{th})$ , (b) the back-work ratio, (c) the heat transfer (q) to the working fluid per unit mass passing through the boiler in kJ/kg and (d) the heat transfer (q) from the working fluid per unit mass passing through the condenser in kJ/kg.

## **SOLUTION**

State-1 (given  $p_1 = p_5, x_1$ ):

$$h_1 = h_{g@10\text{MPa}} = 2724.67 \frac{\text{kJ}}{\text{kg}}; \ s_1 = s_{g@10\text{MPa}} = 5.6141 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@8kPa} = 173.88 \frac{kJ}{kg}; \ h_{fg@8kPa} = 2403.12 \frac{kJ}{kg}; \ s_{f@8kPa} = 0.5926 \frac{kJ}{kg \cdot K}; \ s_{fg@8kPa} = 7.6361 \frac{kJ}{kg \cdot K}$$

$$x_2 = \frac{s_2 - s_{f@8kPa}}{s_{fg@8kPa}} = \frac{5.6141 - 0.5926}{7.6361} = 0.658$$

$$h_2 = h_{f@8kPa} + x_2 h_{fg@8kPa} = 173.88 + (0.658)(2403.12) = 1755.13 \frac{kJ}{kg}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 2724.67 - (0.80)(2724.67 - 1755.13) = 1949.04 \frac{\text{kJ}}{\text{kg}}$$
State-4 (given  $p_4, s_4 = s_5$ ):

$$h_{f@8kPa} = 173.88 \frac{kJ}{kg}; \ h_{fg@8kPa} = 2403.12 \frac{kJ}{kg}; \ s_{f@8kPa} = 0.5926 \frac{kJ}{kg \cdot K}; \ s_{fg@8kPa} = 7.6361 \frac{kJ}{kg \cdot K}$$

$$x_4 = \frac{s_4 - s_{f@8kPa}}{s_{f@8kPa}} = \frac{3.3596 - 0.5926}{7.6361} = 0.362$$

$$h_4 = h_{f@8kPa} + x_2 h_{fg@8kPa} = 173.88 + (0.362)(2403.12) = 1043.81 \frac{kJ}{kg}$$

State-5 (given  $p_5, x_5$ ):

$$h_5 = h_{f@10\text{MPa}} = 1407.59 \frac{\text{kJ}}{\text{kg}}; \ s_5 = s_{f@10\text{MPa}} = 3.3596 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-6 (given  $p_6 = p_5, \eta_P$ ):

$$h_6 = h_4 + \frac{\left(h_5 - h_4\right)}{\eta_P} = 1043.81 + \frac{\left(1407.59 - 1043.81\right)}{\left(0.75\right)} = 1528.85 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_T = h_1 - h_3 = 2724.67 - 1949.04 = 775.63 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = h_3 - h_4 = 1949.04 - 1043.81 = \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-6): 
$$w_P = h_6 - h_4 = 1528.85 - 1043.81 = 485.04 \frac{\text{kJ}}{\text{kg}}$$

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 2724.67 - 1528.85 = 1195.82 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 775.63 - 485.04 = 290.59 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{290.59}{1195.82} = 24.3\%$$

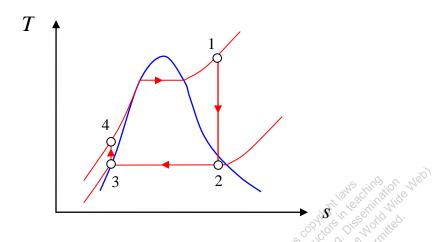
Back work ratio is

BWR = 
$$\frac{w_P}{w_T} = \frac{485.04}{775.63} = 62.5\%$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

**9-1-4** [OPG] Consider a steam power plant operating on the simple ideal Rankine cycle. The steam enters the turbine at 4 MPa,  $400^{\circ}$ C and is condensed in the condenser at a pressure of 100 kPa. Determine (a) the thermal efficiency ( $\eta_{\text{th,Rankine}}$ ) of the cycle. (b) **What-if Scenario:** What would the thermal efficiency be if steam entered the turbine at 5 MPa and the condenser pressure were 90 kPa?

#### **SOLUTION**



State-1 (given  $p_1, T_1$ ):

$$h_1 = 3213.49 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.7689 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@100\text{kPa}} = 417.44 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@100\text{kPa}} = 2258.02 \frac{\text{kJ}}{\text{kg}}; \ s_{f@100\text{kPa}} = 1.3025 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@100\text{kPa}} = 6.0568 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f @ 100\text{kPa}}}{s_{fg @ 100\text{kPa}}} = \frac{6.7689 - 1.3025}{6.0568} = 0.9025$$

$$h_2 = h_{f@100\text{kPa}} + x_2 h_{fg@100\text{kPa}} = 417.44 + (0.9025)(2258.02) = 2455.30 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, x_3$ ):

$$h_3 = h_{f@100\text{kPa}} = 417.44 \frac{\text{kJ}}{\text{kg}}; \ v_3 = v_{f@100\text{kPa}} = 0.001043 \frac{\text{m}^3}{\text{kg}}$$

State-4 (given  $p_4 = p_1, s_4 = s_1$ ):

Assuming that  $T_4 \cong T_3$ 

$$h_4 = h_3 + v_{f@T_3} (p_4 - p_3) = 417.44 + (0.001043)(4000 - 100) = 421.51 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_T = h_1 - h_2 = 3213.49 - 2455.30 = 758.19 \frac{\text{kJ}}{\text{kg}}$$

Device-B (2-3): 
$$q_{\text{out}} = h_2 - h_3 = 2455.30 - 417.44 = 2037.86 \frac{\text{kJ}}{\text{kg}}$$

Device-C (3-4): 
$$w_P = h_4 - h_3 = 421.51 - 417.44 = 4.07 \frac{\text{kJ}}{\text{kg}}$$

Device-D (4-1): 
$$q_{in} = h_1 - h_4 = 3213.49 - 421.51 = 2791.98 \frac{kJ}{kg}$$

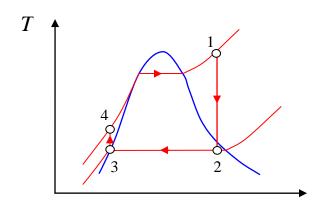
The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 758.19 - 4.07 = 754.12 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{754.12}{2791.98} = 27.01\%$$

9-1-5 [OPZ] A steam power plant operates on the simple ideal Rankine cycle. Steam enters the turbine at 4 MPa,  $500^{\circ}$ C and is condensed in the condenser at a temperature of  $40^{\circ}$ C. (a) Show the cycle on a T-s diagram. If the mass flow rate is 10 kg/s, determine (b) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (c) the net power output in MW.

# **SOLUTION**



State-1 (given  $p_1, T_1$ ):

$$h_1 = 3445.19 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 7.0900 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $T_2, s_2 = s_1$ ):

$$h_{f@40^{\circ}\mathrm{C}} = 167.57 \, \frac{\mathrm{kJ}}{\mathrm{kg}}; \ h_{fg@40^{\circ}\mathrm{C}} = 2406.72 \, \frac{\mathrm{kJ}}{\mathrm{kg}}; \ s_{f@40^{\circ}\mathrm{C}} = 0.5725 \, \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}; \ s_{fg@40^{\circ}\mathrm{C}} = 7.6847 \, \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}$$

$$x_2 = \frac{s_2 - s_{f@40^{\circ}C}}{s_{f@40^{\circ}C}} = \frac{7.0900 - 0.5725}{7.6847} = 0.8481$$

$$h_2 = h_{f @ 40^{\circ}\text{C}} + x_2 h_{fg @ 40^{\circ}\text{C}} = 167.57 + (0.8481)(2406.72) = 2208.71 \frac{\text{kJ}}{\text{kg}}$$

$$p_2 = p_{\text{sat @ 40°C}} = 7.38 \text{ kPa}$$

State-3 (given  $p_3 = p_2, x_3$ ):

$$h_3 = h_{f @ 40^{\circ}\text{C}} = 167.57 \frac{\text{kJ}}{\text{kg}}; \ v_3 = v_{f @ 40^{\circ}\text{C}} = 0.001008 \frac{\text{m}^3}{\text{kg}}$$

State-4 (given  $p_4 = p_1, s_4 = s_1$ ):

Assuming that  $T_4 \cong T_3$ 

$$h_4 = h_3 + v_{f@T_3} (p_4 - p_3) = 167.57 + (0.001008)(4000 - 7.38) = 171.59 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$\dot{W}_T = \dot{m}(h_1 - h_2) = (10)(3445.19 - 2208.71) = 12364.80 \text{ kW}$$

Device-B (2-3): 
$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) = (10)(2208.71 - 167.57) = 20411.40 \text{ kW}$$

Device-C (3-4): 
$$\dot{W}_P = \dot{m}(h_4 - h_3) = (10)(171.59 - 167.57) = 40.20 \text{ kW}$$

Device-D (4-1): 
$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (10)(3445.19 - 171.59) = 32736.00 \text{ kW}$$

The thermal efficiency is

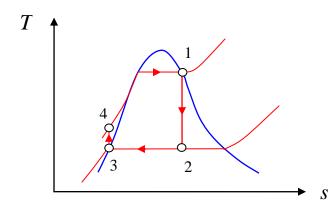
$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_P = 12364.80 - 40.20 = 12324.60 \text{ kW} = 12.324 \text{ MW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{12324.60}{32736.00} = 37.65\%$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-1-6 [OPK] Water is the working fluid in an ideal Rankine cycle. Saturated vapor enters the turbine at 6.9 MPa. The condenser pressure is 6.9 kPa. Determine (a) the net work per unit mass  $(w_{\text{net}})$  of steam flow in kJ/kg, (b) the heat transfer (q) to the steam passing through the boiler in kJ/kg, (c) the thermal efficiency ( $\eta_{th}$ ) and (d) the back-work ratio.

# **SOLUTION**



State-1 (given  $p_1, x_1$ ):

$$h_1 = h_{g@6.9\text{MPa}} = 2773.33 \frac{\text{kJ}}{\text{kg}}; \ s_1 = s_{g@6.9\text{MPa}} = 5.8205 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

State-2 (given 
$$p_2$$
,  $s_2 = s_1$ ):  
 $h_{f@6.9\text{kPa}} = 162.21 \frac{\text{kJ}}{\text{kg}}$ ;  $h_{fg@6.9\text{kPa}} = 2409.78 \frac{\text{kJ}}{\text{kg}}$ ;  $s_{f@6.9\text{kPa}} = 0.5554 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ ;  $s_{fg@6.9\text{kPa}} = 7.7261 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ 

$$x_2 = \frac{s_2 - s_{f@6.9\text{kPa}}}{s_{f@6.9\text{kPa}}} = \frac{5.8205 - 0.5554}{7.7261} = 0.6815$$

$$h_2 = h_{f @ 6.9 \text{kPa}} + x_2 h_{fg @ 6.9 \text{kPa}} = 162.21 + (0.6815)(2409.78) = 1804.48 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, x_3$ ):

$$h_3 = h_{f@6.9\text{kPa}} = 162.21 \frac{\text{kJ}}{\text{kg}}; \ v_3 = v_{f@6.9\text{kPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-4 (given  $p_4 = p_1, s_4 = s_1$ ):

Assuming that  $T_4 \cong T_3$ 

$$h_4 = h_3 + v_{f@T_3} (p_4 - p_3) = 162.21 + (0.001007)(6900 - 6.9) = 169.15 \frac{kJ}{kg}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_T = h_1 - h_2 = 2773.33 - 1804.48 = 968.85 \frac{\text{kJ}}{\text{kg}}$$

Device-B (2-3): 
$$q_{\text{out}} = h_2 - h_3 = 1804.48 - 162.21 = 1642.27 \frac{\text{kJ}}{\text{kg}}$$

Device-C (3-4): 
$$w_P = h_4 - h_3 = 169.15 - 162.21 = 6.94 \frac{\text{kJ}}{\text{kg}}$$

Device-D (4-1): 
$$q_{in} = h_1 - h_4 = 2773.33 - 169.15 = 2604.18 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 968.85 - 6.94 = 961.91 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{961.91}{2604.18} = \frac{36.94\%}{6000}$$

Back work ratio is

BWR = 
$$\frac{w_P}{w_T} = \frac{6.94}{968.85} = 0.72\%$$

**Verification:** Use PC vapor-power cycle TEST calc to verify this answer. TEST-code for this problem can be found in thermofluids net (the professional site of TEST).

**9-1-7** [OPX] Consider a steam power plant operating on the ideal Rankine cycle. The steam enters the turbine at 5 MPa, 350°C and is condensed in the condenser at a pressure of 15 kPa. Determine (a) the thermal efficiency ( $\eta_{\text{th,Rankine}}$ ) of the cycle. (b) **What-if Scenario:** What would the thermal efficiency be if steam were superheated to 750°C instead of 350°C?

#### **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3068.36 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.4492 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@15\text{kPa}} = 225.81 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@15\text{kPa}} = 2373.23 \frac{\text{kJ}}{\text{kg}}; \ s_{f@15\text{kPa}} = 0.7543 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@15\text{kPa}} = 7.2550 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f@15\text{kPa}}}{s_{f@015\text{kPa}}} = \frac{6.4492 - 0.7543}{7.2550} = 0.7850$$

$$h_2 = h_{f@15\text{kPa}} + x_2 h_{fg@15\text{kPa}} = 225.81 + (0.7850)(2373.23) = 2088.80 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, x_3$ ):

$$h_3 = h_{f@15\text{kPa}} = 225.81 \frac{\text{kJ}}{\text{kg}}; \ v_3 = v_{f@15\text{kPa}} = 0.001014 \frac{\text{m}^3}{\text{kg}}$$

State-4 (given  $p_4 = p_1, s_4 = s_1$ ):

Assuming that  $T_4 \cong T_3$ 

$$h_4 = h_3 + v_{f@T_3} (p_4 - p_3) = 225.81 + (0.001014)(5000 - 15) = 230.86 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_T = h_1 - h_2 = 3068.36 - 2088.80 = 979.56 \frac{\text{kJ}}{\text{kg}}$$

Device-B (2-3): 
$$q_{\text{out}} = h_2 - h_3 = 2088.80 - 225.81 = 1862.99 \frac{\text{kJ}}{\text{kg}}$$

Device-C (3-4): 
$$w_P = h_4 - h_3 = 230.86 - 225.81 = 5.05 \frac{\text{kJ}}{\text{kg}}$$

Device-D (4-1): 
$$q_{in} = h_1 - h_4 = 3068.36 - 230.86 = 2837.50 \frac{kJ}{kg}$$

The thermal efficiency is

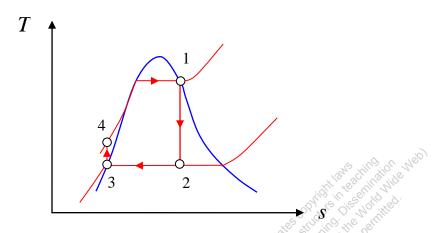
$$w_{\text{net}} = w_T - w_P = 979.56 - 5.05 = 974.51 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{974.51}{2837.50} = 34.34\%$$



9-1-8 [OPP] Steam is the working fluid in an ideal Rankine cycle. Saturated vapor enters the turbine at 9 MPa and saturated liquid exits the condenser at 0.009 MPa. The net power output of the cycle is 100 MW. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the back-work ratio, (c) the mass flow rate of steam, (d) the heat transfer into the working fluid as it passes through the boiler and (e) the heat transfer (Q) from the condenser to the steam as it passes through the condenser.

#### **SOLUTION**



State-1 (given  $p_1, x_1$ ):

$$h_1 = h_{g@9MPa} = 2742.11 \frac{\text{kJ}}{\text{kg}}; \ s_1 = s_{g@9MPa} = 5.6771 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@9\text{kPa}} = 183.27 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@9\text{kPa}} = 2397.72 \frac{\text{kJ}}{\text{kg}}; \ s_{f@9\text{kPa}} = 0.6227 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@9\text{kPa}} = 7.5649 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f@9\text{kPa}}}{s_{fg@9\text{kPa}}} = \frac{5.6771 - 0.6227}{7.5649} = 0.6681$$

$$h_2 = h_{f@9kPa} + x_2 h_{fg@9kPa} = 183.27 + (0.6681)(2397.72) = 1785.19 \frac{kJ}{kg}$$

State-3 (given  $p_3 = p_2, x_3$ ):

$$h_3 = h_{f@9kPa} = 183.27 \frac{kJ}{kg}; \ v_3 = v_{f@9kPa} = 0.001009 \frac{m^3}{kg}$$

State-4 (given  $p_4 = p_1, s_4 = s_1$ ):

Assuming that  $T_4 \cong T_3$ 

$$h_4 = h_3 + v_{f@T_3} (p_4 - p_3) = 183.27 + (0.001009)(9000 - 9) = 192.34 \frac{kJ}{kg}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_T = h_1 - h_2 = 2742.11 - 1785.19 = 956.92 \frac{\text{kJ}}{\text{kg}}$$

Device-B (2-3): 
$$q_{\text{out}} = h_2 - h_3 = 1785.19 - 183.27 = 1601.92 \frac{\text{kJ}}{\text{kg}}$$

Device-C (3-4): 
$$w_P = h_4 - h_3 = 192.34 - 183.27 = 9.07 \frac{\text{kJ}}{\text{kg}}$$

Device-D (4-1): 
$$q_{in} = h_1 - h_4 = 2742.11 - 192.34 = 2549.77 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 956.92 - 9.07 = 947.85 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{947.85}{2549.77} = 37.17\%$$

Back work ratio is

BWR = 
$$\frac{w_P}{w_T} = \frac{9.07}{956.92} = 0.95\%$$

The mass flow rate is

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{W_{\text{net}}} = \frac{100000}{947.85} = 105.50 \frac{\text{kg}}{\text{s}}$$

Now that mass flow rate is known

$$\dot{Q}_{\rm in} = \dot{m}q_{\rm in} = (105.50)(2549.77) = 269000.74 \text{ kW} = 269.0 \text{ MW}$$

$$\dot{Q}_{\text{out}} = \dot{m}q_{\text{out}} = (105.50)(1601.92) = 169002.56 \text{ kW} = 169.0 \text{ MW}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-1-9 [OPU] Steam is the working fluid in an ideal Rankine cycle. Saturated vapor enters the turbine at 10 MPa and saturated liquid exits the condenser at 0.01 MPa. The net power output of the cycle is 150 MW. The turbine and the pump each have an isentropic efficiency of 85%. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the mass flow rate of steam, (c) the heat transfer into the working fluid as it passes through the boiler and (d) the heat transfer from the condenser to the steam as it passes through the condenser.

#### **SOLUTION**

State-1 (given  $p_1, x_1$ ):

$$h_1 = h_{g@10\text{MPa}} = 2724.67 \frac{\text{kJ}}{\text{kg}}; \ s_1 = s_{g@10\text{MPa}} = 5.6141 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@10\mathrm{kPa}} = 191.83 \frac{\mathrm{kJ}}{\mathrm{kg}}; \ h_{fg@10\mathrm{kPa}} = 2392.80 \frac{\mathrm{kJ}}{\mathrm{kg}}; \ s_{f@10\mathrm{kPa}} = 0.6493 \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}; \ s_{fg@10\mathrm{kPa}} = 7.5008 \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}$$

$$x_2 = \frac{s_2 - s_{f@10\text{kPa}}}{s_{fg@10\text{kPa}}} = \frac{5.6141 - 0.6493}{7.5008} = 0.6619$$

$$h_2 = h_{f@10\text{kPa}} + x_2 h_{fg@10\text{kPa}} = 191.83 + (0.6619)(2392.80) = 1775.62 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given 
$$p_3 = p_2, \eta_T$$
):  

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 2724.67 - (0.85)(2724.67 - 1775.62) = 1917.98 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_2, x_4$ ):

$$h_{f@10\text{kPa}} = 191.83 \frac{\text{kJ}}{\text{kg}}; \ v_{f@10\text{kPa}} = 0.001010 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4} (p_5 - p_4) = 191.83 + (0.001010)(10000 - 10) = 201.92 \frac{kJ}{kg}$$

State-6 (given  $p_6 = p_1, \eta_P$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_P} = 191.83 + \frac{201.92 - 191.83}{0.85} = 203.70 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_T = h_1 - h_3 = 2724.67 - 1917.98 = 806.69 \frac{\text{kJ}}{\text{kg}}$$
  
Device-B (3-4):  $q_{\text{out}} = h_3 - h_4 = 1917.98 - 191.83 = 1726.15 \frac{\text{kJ}}{\text{kg}}$   
Device-C (4-6):  $w_P = h_6 - h_4 = 203.70 - 191.83 = 11.87 \frac{\text{kJ}}{\text{kg}}$ 

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 2724.67 - 203.70 = 2520.97 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 806.69 - 11.87 = 794.82 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{794.82}{2520.97} = 31.53\%$$

The mass flow rate is

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{150000}{794.82} = 188.72 \frac{\text{kg}}{\text{s}}$$

Now that mass flow rate is known

$$\dot{Q}_{\text{in}} = \dot{m}q_{\text{in}} = (188.72)(2520.97) = 475757.46 \text{ kW} = 475.8 \text{ MW}$$
  
 $\dot{Q}_{\text{out}} = \dot{m}q_{\text{out}} = (188.72)(1726.15) = 325759.03 \text{ kW} = 325.8 \text{ MW}$ 

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

**9-1-10** [OPQ] Propane is the working fluid in a supercritical power plant. The turbine inlet pressure is 10 MPa, the temperature is  $150^{\circ}$ C and it exits at  $-30^{\circ}$ C. The net power output of the cycle is 2 kW. The turbine and the pump have isentropic efficiencies of 90% and 80%, respectively. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the mass flow rate of propane.

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 369.63 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 5.5559 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $T_2$ ,  $s_2 = s_1$ ):

$$h_{f@-30^{\circ}\mathrm{C}} = -171.58 \frac{\mathrm{kJ}}{\mathrm{kg}}; \ h_{fg@-30^{\circ}\mathrm{C}} = 411.98 \frac{\mathrm{kJ}}{\mathrm{kg}}; \ s_{f@-30^{\circ}\mathrm{C}} = 3.9963 \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}; \ s_{fg@-30^{\circ}\mathrm{C}} = 1.6968 \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}$$

$$x_2 = \frac{s_2 - s_{f@-30^{\circ}C}}{s_{f@-30^{\circ}C}} = \frac{5.5559 - 3.9963}{1.6968} = 0.9191$$

$$h_2 = h_{f@-30^{\circ}\text{C}} + x_2 h_{fg@-30^{\circ}\text{C}} = -171.58 + (0.9191)(411.98) = 207.07 \frac{\text{kJ}}{\text{kg}}$$

$$p_2 = p_{\text{sat @-30°C}} = 164.77 \text{ kPa}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 369.63 - (0.90)(369.63 - 207.07) = 223.33 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_2, x_4$ ):

$$h_4 = h_{f@-30^{\circ}\text{C}} = -171.58 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f@-30^{\circ}\text{C}} = 0.001764 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Propane is now a supercritical liquid

$$h_5 = -151.80 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_1, \eta_P$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_P} = -171.58 + \frac{-151.80 + 171.58}{0.80} = -146.86 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_T = h_1 - h_3 = 369.63 - 223.33 = 146.30 \frac{\text{kJ}}{\text{kg}}$$
  
Device-B (3-4):  $q_{\text{out}} = h_3 - h_4 = 223.33 + 171.58 = 394.91 \frac{\text{kJ}}{\text{kg}}$   
Device-C (4-6):  $w_P = h_6 - h_4 = -146.86 + 171.58 = 24.72 \frac{\text{kJ}}{\text{kg}}$   
Device-D (6-1):  $q_{\text{in}} = h_1 - h_6 = 369.63 + 146.86 = 516.49 \frac{\text{kJ}}{\text{kg}}$ 

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 146.30 - 24.72 = 121.58 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{121.58}{516.49} = \frac{23.54\%}{600}$$

The mass flow rate is

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{2}{121.58} = 0.016 \frac{\text{kg}}{\text{s}}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids net (the professional site of TEST).

**9-1-11** [OPC] Water is the working fluid in an ideal Rankine cycle. Superheated vapor enters the turbine at 12 MPa and  $500^{\circ}$ C. The condenser pressure is 8 kPa. The turbine and the pump have isentropic efficiencies of 85% and 75%, respectively. Determine, for a flow of unit mass, (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the net power output (c) the heat transfer into the working fluid as it passes through the boiler and (d) the heat transfer from the condenser to the steam as it passes through the condenser. (e) **What-if Scenario:** What would the net power output be if the mass flow rate of the working fluid were 100 kg/s?

# **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3348.07 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.4863 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@8k\text{Pa}} = 173.88 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@8k\text{Pa}} = 2403.12 \frac{\text{kJ}}{\text{kg}}; \ s_{f@8k\text{Pa}} = 0.5926 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@8k\text{Pa}} = 7.6361 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f@8kPa}}{s_{f@8kPa}} = \frac{6.4863 - 0.5926}{7.6361} = 0.7718$$

$$h_2 = h_{f @ 8kPa} + x_2 h_{fg @ 8kPa} = 173.88 + (0.7718)(2403.12) = 2028.61 \frac{kJ}{kg}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3348.07 - (0.85)(3348.07 - 2028.61) = 2226.53 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_2, x_4$ ):

$$h_4 = h_{f@8kPa} = 173.88 \frac{kJ}{kg}; \ v_4 = v_{f@8kPa} = 0.001008 \frac{m^3}{kg}$$

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 173.88 + (0.001008)(12000 - 8) = 185.97 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_1, \eta_P$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_P} = 173.88 + \frac{185.97 - 173.88}{0.75} = 190.00 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_T = h_1 - h_3 = 3348.07 - 2226.53 = 1121.54 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = h_3 - h_4 = 2226.53 - 173.88 = 2052.65 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-6): 
$$w_P = h_6 - h_4 = 190.00 - 173.88 = 16.12 \frac{\text{kJ}}{\text{kg}}$$

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 3348.07 - 190.00 = 3158.07 \frac{kJ}{kg}$$

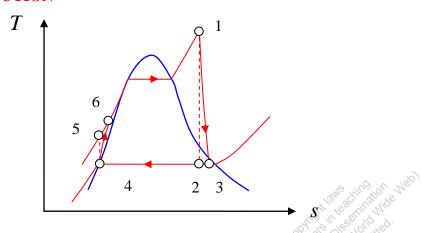
The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 1121.54 - 16.12 = 1105.42 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1105.42}{3158.07} = 35.00\%$$

**9-1-12** [OPV] In a steam power plant operating on a Rankine cycle, steam enters the turbine at 3 MPa,  $350^{\circ}$ C and is condensed in the condenser at a pressure of 75 kPa. If the adiabatic efficiencies of the pump and turbine are 80% each, determine (a) the thermal efficiency ( $\eta_{\text{th,Rankine}}$ ) of the cycle. (b) **What-if Scenario:** What would the thermal efficiency be if the boiler pressure were increased to 5 MPa?

## **SOLUTION**



State-1 (given  $p_1, T_1$ ):

$$h_1 = 3115.23 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.7427 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@75\text{kPa}} = 384.31 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@75\text{kPa}} = 2278.60 \frac{\text{kJ}}{\text{kg}}; \ s_{f@75\text{kPa}} = 1.2127 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@75\text{kPa}} = 6.2442 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f@75\text{kPa}}}{s_{fa@75\text{kPa}}} = \frac{6.7427 - 1.2127}{6.2442} = 0.8856$$

$$h_2 = h_{f @ 75 \text{kPa}} + x_2 h_{fg @ 75 \text{kPa}} = 384.31 + (0.8856)(2278.60) = 2402.24 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3115.23 - (0.80)(3115.23 - 2402.24) = 2544.84 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_2, x_4$ ):

$$h_4 = h_{f @ 75 \text{kPa}} = 384.31 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f @ 75 \text{kPa}} = 0.001037 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4} (p_5 - p_4) = 384.31 + (0.001037)(3000 - 75) = 387.34 \frac{kJ}{kg}$$

State-6 (given  $p_6 = p_1, \eta_P$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_P} = 384.31 + \frac{387.34 - 384.31}{0.80} = 388.10 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_T = h_1 - h_3 = 3115.23 - 2544.84 = 570.39 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = h_3 - h_4 = 2544.84 - 384.31 = 2160.53 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-6): 
$$w_P = h_6 - h_4 = 388.10 - 384.31 = 3.89 \frac{kJ}{kg}$$

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 3115.23 - 388.10 = 2727.13 \frac{kJ}{kg}$$

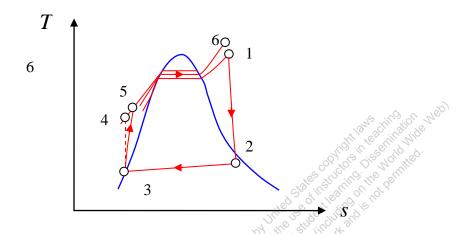
The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 570.39 - 3.89 = 566.50 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{566.50}{2727.13} = 20.77\%$$

**9-1-13** [OPT] Water is the working fluid in a vapor power plant. Superheated steam leaves the steam generator at 8.2 MPa, 540°C and enters the turbine at 7.5 MPa, 500°C. The steam expands through the turbine, exiting at 8 kPa with a quality of 94%. Condensate leaves the condenser at 5 kPa, 30°C and is pumped to 9 MPa before entering the steam generator. The pump efficiency is 80%. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the net power developed. (c) **What-if Scenario:** What would the net power developed be if the mass flow rate of steam were 15 kg/s?

## **SOLUTION**



State-1 (given  $p_1, T_1$ ):

$$h_1 = 3404.26 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.7594 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, x_2$ ):

$$\begin{split} h_{f@8k\text{Pa}} &= 173.88 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@8k\text{Pa}} = 2403.12 \frac{\text{kJ}}{\text{kg}}; \ s_{f@8k\text{Pa}} = 0.5926 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@8k\text{Pa}} = 7.6361 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ h_2 &= h_{f@8k\text{Pa}} + x_2 h_{fg@8k\text{Pa}} = 173.88 + \big(0.94\big) \big(2403.12\big) = 2432.81 \frac{\text{kJ}}{\text{kg}} \end{split}$$

State-3 (given  $p_3, T_3$ ):

$$h_3 = 125.79 \frac{\text{kJ}}{\text{kg}}; \ v_{f@30^{\circ}\text{C}} = 0.001004 \frac{\text{m}^3}{\text{kg}}$$

State-4 (given  $p_4, s_4 = s_3$ ):

Assuming that  $T_4 \cong T_3$ 

$$h_4 = h_3 + v_{f@T_3} (p_4 - p_3) = 125.79 + (0.001004)(9000 - 5) = 134.82 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given  $p_5 = p_4, \eta_P$ ):

$$h_5 = h_3 + \frac{h_4 - h_3}{\eta_P} = 125.79 + \frac{134.82 - 125.79}{0.80} = 137.08 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6, T_6$ ):

$$h_6 = 3494.35 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_T = h_1 - h_2 = 3404.26 - 2432.81 = 971.45 \frac{\text{kJ}}{\text{kg}}$$

Device-B (2-3): 
$$q_{\text{out}} = h_2 - h_3 = 2432.81 - 125.79 = 2307.02 \frac{\text{kJ}}{\text{kg}}$$

Device-C (3-5): 
$$w_P = h_5 - h_3 = 137.08 - 125.79 = 11.29 \frac{\text{kJ}}{\text{kg}}$$

Device-D (5-6): 
$$q_{in} = h_6 - h_5 = 3494.35 - 137.08 = 3357.27 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 971.45 - 11.29 = 960.16 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{960.16}{3357.27} = 28.60\%$$

Assuming a unit mass flow rate

$$\dot{W}_{\text{net}} = \dot{m}w_{\text{net}} = (1)(960.16) = 960.16 \text{ kW}$$

**9-1-14** [OPY] Water is the working fluid in a vapor power plant. Superheated steam enters the turbine at 18 MPa and 580°C. Steam expands through the turbine, exiting at 6 kPa and the turbine efficiency is 82%. Condensate leaves the condenser at 4.5 kPa, 25°C and is pumped to 18.5 MPa before entering the steam generator. The pump efficiency is 77%. Determine (a) the net work per unit mass ( $w_{net}$ ) of steam flow, (b) the heat transfer per unit mass ( $w_{net}$ ) of steam passing through the boiler, (c) the thermal efficiency ( $y_{th}$ ) and (d) the heat transfer per unit mass ( $y_{th}$ ) of steam passing through the condenser. (e) **What-if Scenario:** What would the thermal efficiency be if efficiencies of both the turbine and pump were 99% each?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3499.66 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.5033 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@6kPa} = 151.51 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@6kPa} = 2415.87 \frac{\text{kJ}}{\text{kg}}; \ s_{f@6kPa} = 0.5209 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@6kPa} = 7.8100 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f@6kPa}}{s_{f@6kPa}} = \frac{6.5033 - 0.5209}{7.8100} = 0.7660$$

$$h_2 = h_{f @ 6kPa} + x_2 h_{fg @ 6kPa} = 151.51 + (0.7660)(2415.87) = 2002.07 \frac{kJ}{kg}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3499.66 - (0.82)(3499.66 - 2002.07) = 2271.64 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4, T_4$ ):

$$h_4 = 104.88 \frac{\text{kJ}}{\text{kg}}; \ v_{f@25^{\circ}\text{C}} = 0.001003 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 104.88 + (0.001003)(18500 - 4.5) = 123.43 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_5, \eta_P$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_B} = 104.88 + \frac{123.43 - 104.88}{0.77} = 128.97 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$W_T = h_1 - h_3 = 3499.66 - 2271.64 = 1228.02 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = h_3 - h_4 = 2271.64 - 104.88 = 2166.76 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-6): 
$$w_p = h_6 - h_4 = 128.97 - 104.88 = 24.09 \frac{\text{kJ}}{\text{kg}}$$

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 3499.66 - 128.97 = 3370.69 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 1228.02 - 24.09 = \frac{1203.93}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1203.93}{3370.69} = 35.72\%$$

**9-1-15** [OPF] A steam power plant operates on the following cycle producing a net power of 25 MW. Steam enters the turbine at 16 MPa, 550°C and enters the condenser as saturated mixture at 10 kPa. Subcooled liquid enters the pump at 9 kPa, 35°C and leaves at 17 MPa, which then enters the boiler at 16.8 MPa, 33°C and exits at 16.2 MPa, 575°C. If the isentropic efficiency of the turbine is 90% and that of the pump is 83%, determine (a) the mass flow rate of steam and (b) the mass flow rate of cooling water in the condenser in which temperature rises from 20°C to 30°C.

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3437.69 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.4792 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@10\text{kPa}} = 191.83 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@10\text{kPa}} = 2392.80 \frac{\text{kJ}}{\text{kg}}; \ s_{f@10\text{kPa}} = 0.6493 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@10\text{kPa}} = 7.5008 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f@10kPa}}{s_{f@10kPa}} = \frac{6.4792 - 0.6493}{7.5008} = 0.7772$$

$$h_2 = h_{f@10\text{kPa}} + x_2 h_{fg@10\text{kPa}} = 191.83 + (0.7772)(2392.80) = 2051.51 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3437.69 - (0.90)(3437.69 - 2051.51) = 2190.13 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $T_4$  and  $p_4$ ):

$$h_4 = 146.68 \frac{\text{kJ}}{\text{kg}}; \ v_{f@35^{\circ}\text{C}} = 0.001006 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 146.68 + (0.001006)(17000 - 9) = 163.77 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_5, \eta_P$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_P} = 146.68 + \frac{163.77 - 146.68}{0.83} = 167.27 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given  $p_7, T_7$ ):

$$h_7 = 155.21 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given  $p_8, T_8$ ):

$$h_8 = 3503.57 \frac{\text{kJ}}{\text{kg}}$$

Assuming the cooling water is at atmospheric pressure State-9 (given  $p_9, T_3$ ):

$$h_9 = 84.06 \frac{\text{kJ}}{\text{kg}}; \ s_9 = 0.2966 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-10 (given  $p_{10} = p_9, T_{10}$ ):

$$h_{10} = 125.89 \frac{\text{kJ}}{\text{kg}}; \ s_{10} = 0.4369 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_T = h_1 - h_3 = 3437.69 - 2190.13 = 1247.56 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = h_3 - h_4 = 2190.13 - 146.68 = 2043.45 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-6): 
$$w_P = h_6 - h_4 = 167.27 - 146.68 = 20.59 \frac{\text{kJ}}{\text{kg}}$$

Device-D (7-8): 
$$q_{in} = h_8 - h_7 = 3503.57 - 155.21 = 3348.36 \frac{kJ}{kg}$$

The net work is

$$w_{\text{net}} = w_T - w_P = 1247.56 - 20.59 = 1226.97 \frac{\text{kJ}}{\text{kg}}$$

The mass flow rate of the steam is

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{25000}{1226.97} = \frac{20.38 \text{ kg}}{\text{s}}$$

Assuming the SL model can be used for the cooling water in the condenser

$$\dot{m}_{\text{cooling}} = \frac{\dot{m}(h_3 - h_4)}{c_p(T_{10} - T_9)} = \frac{(20.38)(2043.45)}{(4.184)(30 - 20)} = \frac{995.35}{\text{s}} \frac{\text{kg}}{\text{s}}$$

The accuracy of this assumption can be checked by using the values from the PC model

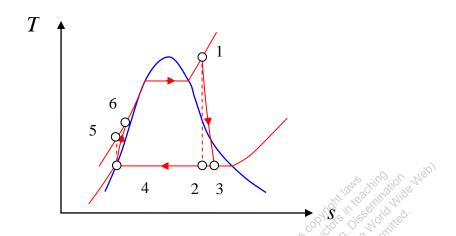
$$\dot{m}_{\text{cooling}} = \frac{\dot{m}(h_3 - h_4)}{h_{10} - h_9} = \frac{(20.38)(2043.45)}{125.89 - 84.06} = 995.59 \frac{\text{kg}}{\text{s}}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).



**9-1-16** [OPD] Water is the working fluid in a vapor power plant. Steam enters the turbine at 4 MPa, 540°C and exits the turbine as a two-phase liquid vapor mixture at 27°C. The condensate exits the condenser at 25°C. The turbine efficiency is 90% and the pump efficiency is 80%. If the power developed is 1 MW, determine (a) the steam quality at the turbine exit, (b) the mass flow rate and (c) the thermal efficiency ( $\eta_{th}$ ).

# **SOLUTION**



State-1 (given  $p_1, T_1$ ):

$$h_1 = 3536.88 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 7.2025 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $T_2, s_2 = s_1$ ):

$$h_{f@27^{\circ}\text{C}} = 113.24 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@27^{\circ}\text{C}} = 2437.61 \frac{\text{kJ}}{\text{kg}}; \ s_{f@27^{\circ}\text{C}} = 0.3952 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@27^{\circ}\text{C}} = 8.1212 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f @ 27^{\circ}C}}{s_{f_{R} @ 27^{\circ}C}} = \frac{7.2025 - 0.3952}{8.1212} = 0.8382$$

$$h_2 = h_{f @ 27^{\circ}\text{C}} + x_2 h_{fg @ 27^{\circ}\text{C}} = 113.24 + (0.8382)(2437.61) = 2156.44 \frac{\text{kJ}}{\text{kg}}$$

$$p_2 = p_{\text{sat @ 27^{\circ}C}} = 3.57 \text{ kPa}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3536.88 - (0.90)(3536.88 - 2156.44) = 2294.48 \frac{\text{kJ}}{\text{kg}}$$

$$x_3 = \frac{h_3 - h_{f@27^{\circ}C}}{h_{fg@27^{\circ}C}} = \frac{2294.48 - 113.24}{2437.61} = 0.8948$$

State-4 (given  $T_4, x_4$ ):

$$h_4 = h_{f @ 25^{\circ}C} = 104.88 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f @ 25^{\circ}C} = 0.001003 \frac{\text{m}^3}{\text{kg}}$$
  
 $p_4 = p_{\text{sat @ 25^{\circ}C}} = 3.16 \text{ kPa}$ 

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 104.88 + (0.001003)(4000 - 3.16) = 108.89 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_5, \eta_P$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_P} = 104.88 + \frac{108.89 - 104.88}{0.80} = 109.89 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_T = h_1 - h_3 = 3536.88 - 2294.48 = 1242.40 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = h_3 - h_4 = 2294.48 - 104.88 = 2189.60 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-6): 
$$w_P = h_6 - h_4 = 109.89 - 104.88 = 5.01 \frac{\text{kJ}}{\text{kg}}$$

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 3536.88 - 109.89 = 3426.99 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 1242.40 - 5.01 = 1237.39 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1237.39}{3426.99} = 36.11\%$$

The mass flow rate if power developed is 1 MW

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{1000}{1237.39} = 0.81 \frac{\text{kg}}{\text{s}}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

**9-1-17** [OPJ] Water is the working fluid in an ideal Rankine cycle. The pressure and temperature at the exit of the steam generator is 9 MPa and  $480^{\circ}$ C. A throttle valve placed between the steam generator and the turbine reduces the turbine inlet pressure to 7 MPa. The condenser pressure is 7 kPa, and the mass flow rate of the steam is 170 kg/s. The turbine and the pump have an isentropic efficiency of 90% each. Determine (a) the net power developed, (b) the heat transfer to the steam passing through the boiler and (c) the thermal efficiency ( $\eta_{th}$ ). (d) **What-if Scenario:** What would the net power developed be if the mass flow rate of steam were 100 kg/s?

# **SOLUTION**

State-1 (given  $p_1, h_1 = h_7$ ):

$$h_1 = 3334.24 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.6957 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@7\mathrm{kPa}} = 163.33 \frac{\mathrm{kJ}}{\mathrm{kg}}; \ h_{fg@7\mathrm{kPa}} = 2409.14 \frac{\mathrm{kJ}}{\mathrm{kg}}; \ s_{f@7\mathrm{kPa}} = 0.5589 \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}; \ s_{fg@7\mathrm{kPa}} = 7.7175 \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}$$

$$x_2 = \frac{s_2 - s_{f@7kPa}}{s_{fg@7kPa}} = \frac{6.6957 - 0.5589}{7.7175} = 0.7952$$

$$h_2 = h_{f @ 7 \text{kPa}} + x_2 h_{fg @ 7 \text{kPa}} = 163.33 + (0.7952)(2409.14) = 2079.08 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3334.24 - (0.90)(3334.24 - 2079.08) = 2204.60 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_2, x_4$ ):

$$h_4 = h_{f @ 7 \text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ v_{f @ 7 \text{kPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5 = p_7, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 163.33 + (0.001007)(9000 - 7) = 172.39 \frac{kJ}{kg}$$

State-6 (given  $p_6 = p_5, \eta_T$ ):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_P} = 163.33 + \frac{172.39 - 163.33}{0.90} = 173.40 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given  $p_7, T_7$ ):

$$h_7 = 3334.24 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$\dot{W}_T = \dot{m}(h_1 - h_3) = (170)(3334.24 - 2204.60) = 192038.80 \text{ kW} = 192.04 \text{ MW}$$

Device-B (3-4): 
$$\dot{Q}_{\text{out}} = \dot{m}(h_3 - h_4) = (170)(2204.60 - 163.33) = 347015.90 \text{ kW} = 347.02 \text{ MW}$$

Device-C (4-6): 
$$\dot{W}_P = \dot{m}(h_6 - h_4) = (170)(173.40 - 163.33) = 1711.90 \text{ kW} = 1.71 \text{ MW}$$

Device-D (6-1): 
$$\dot{Q}_{in} = \dot{m}(h_1 - h_6) = (170)(3334.24 - 173.40) = 537342.80 \text{ kW} = 537.34 \text{ MW}$$

The thermal efficiency is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_P = 192.04 - 1.71 = 190.33 \text{ MW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{190.33}{537.34} = 35.42\%$$

**9-1-18** [OUR] Consider a steam power plant operating on a reheat Rankine cycle. Steam enters the high pressure turbine at 16 MPa,  $550^{\circ}$ C and is condensed in the condenser at 10 kPa. If the moisture content of the steam at the exit of the low pressure turbine is not to exceed 5%, determine (a) the pressure at which the steam should be reheated and (b) the thermal efficiency ( $\eta_{\text{th,Rankine}}$ ) of the cycle. Assume the steam is reheated to the inlet temperature of the high pressure turbine. (c) **What-if Scenario:** What would the thermal efficiency be if the moisture tolerance of the turbine were increased to 10%?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3437.69 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.4792 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

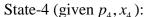
State-2 (given  $p_2 = p_3, s_2 = s_1$ ):

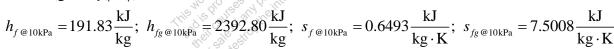
$$h_2 = 2778.62 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $T_3 = T_1, s_3 = s_4$ ):

$$p_3 = 1.29 \text{ MPa}$$

$$h_3 = 3585.44 \frac{\text{kJ}}{\text{kg}}$$





$$h_4 = h_{f@10\text{kPa}} + x_4 h_{fg@10\text{kPa}} = 191.83 + (0.95)(2392.80) = 2464.99 \frac{\text{kJ}}{\text{kg}}$$

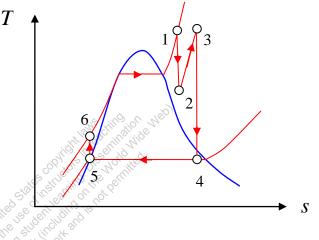
$$s_4 = s_{f@10\text{kPa}} + x_4 s_{fg@10\text{kPa}} = 0.6493 + (0.95)(7.5008) = 7.7751 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-5 (given  $p_5 = p_4, x_5$ ):

$$h_5 = h_{f@10kPa} = 191.83 \frac{kJ}{kg}; \ v_5 = v_{f@10kPa} = 0.001010 \frac{kJ}{kg \cdot K}$$

State-6 (given  $p_6 = p_1, s_6 = s_5$ ):

Assuming that  $T_6 \cong T_5$ 



$$h_6 = h_5 + v_{f@T_5} (p_6 - p_5) = 191.83 + (0.001010)(16000 - 10) = 207.98 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_{T,I} = h_1 - h_2 = 3437.69 - 2778.62 = 659.07 \frac{\text{kJ}}{\text{kg}}$$
  
Device-B (2-3):  $q_{\text{in,th}} = h_3 - h_2 = 3585.44 - 2778.62 = 806.82 \frac{\text{kJ}}{\text{kg}}$ 

Device-C (3-4): 
$$w_{T,II} = h_3 - h_4 = 3585.44 - 2464.99 = 1120.45 \frac{\text{kJ}}{\text{kg}}$$

Device-D (4-5): 
$$q_{\text{out}} = h_4 - h_5 = 2464.99 - 191.83 = 2273.16 \frac{\text{kJ}}{\text{kg}}$$

Device-E (5-6): 
$$w_p = h_6 - h_5 = 207.98 - 191.83 = 16.15 \frac{\text{kJ}}{\text{kg}}$$

Device-F (6-1): 
$$q_{\text{in,b}} = h_1 - h_6 = 3437.69 - 207.98 = 3229.71 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$w_{\text{net}} = w_{T,I} + w_{T,II} - w_P = 659.07 + 1120.45 - 16.15 = 1763.37 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{in}} = q_{\text{in,th}} + q_{\text{in,b}} = 806.82 + 3229.71 = 4036.53 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1763.37}{4036.53} = 43.69\%$$

9-1-19 [OPM] Consider a steam power plant that operates on a reheat Rankine cycle. Steam enters the high pressure turbine at 9 MPa, 600°C and leaves as a saturated vapor. The steam is then reheated to 500°C before entering the low pressure turbine, and is condensed in a condenser at 7 kPa. The mass flow rate is 150 kg/s. Determine (a) the net power developed, (b) the rate of heat transfer to the working fluid in the reheat process and (c) the thermal efficiency ( $\eta_{th}$ ). (d) What-if Scenario: What would the rate of heat transfer be if steam were reheated to 550°C?

# **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3633.71 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.9588 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $s_2 = s_1, x_2$ ):

$$p_2 = p_{s_p = s_1} = 331.20 \text{ kPa}$$

$$h_2 = h_{g @ 331.20 \text{kPa}} = 2729.86 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, T_3$ ):

$$h_3 = 3485.61 \frac{\text{kJ}}{\text{kg}}; \ s_3 = 8.2790 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-4 (given  $p_4$ ,  $s_4 = s_3$ ):

$$h_4 = 2580.85 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given  $p_5 = p_4, x_5$ ):

$$h_5 = h_{f@7\text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ v_5 = v_{f@7\text{kPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-6 (given  $p_6 = p_1, s_6 = s_5$ ):

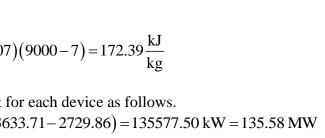
Assuming that  $T_6 \cong T_5$ 

$$h_6 = h_5 + v_{f@T_5} (p_6 - p_5) = 163.33 + (0.001007)(9000 - 7) = 172.39 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$\dot{W}_{T,1} = \dot{m}(h_1 - h_2) = (150)(3633.71 - 2729.86) = 135577.50 \text{ kW} = 135.58 \text{ MW}$$

Device-B (2-3): 
$$\dot{Q}_{\text{in,th}} = \dot{m}(h_3 - h_2) = (150)(3485.61 - 2729.86) = 113362.50 \text{ kW} = 113.36 \text{ MW}$$



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Device-C (3-4): 
$$\dot{W}_{T,II} = \dot{m}(h_3 - h_4) = (150)(3485.61 - 2580.85) = 135714.00 \text{ kW} = 135.71 \text{ MW}$$
  
Device-D (4-5):  $\dot{Q}_{out} = \dot{m}(h_4 - h_5) = (150)(2580.85 - 163.33) = 362628.00 \text{ kW} = 362.63 \text{ MW}$   
Device-E (5-6):  $\dot{W}_p = \dot{m}(h_6 - h_5) = (150)(172.39 - 163.33) = 1359.00 \text{ kW} = 1.36 \text{ MW}$   
Device-F (6-1):  $\dot{Q}_{in,b} = \dot{m}(h_1 - h_6) = (150)(3633.71 - 172.39) = 519198.00 \text{ kW} = 519.20 \text{ MW}$ 

The thermal efficiency is

$$\dot{W}_{\text{net}} = \dot{W}_{T,\text{I}} + \dot{W}_{T,\text{II}} - \dot{W}_{P} = 135.58 + 135.71 - 1.36 = 269.93 \text{ MW}$$

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{in,rh}} + \dot{Q}_{\text{in,b}} = 113.36 + 519.20 = 632.56 \text{ MW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{269.93}{632.56} = 42.67\%$$

**9-1-20** [OPW] Consider a steam power plant operating on an ideal Rankine cycle that has reheat at a pressure of one-fifth the pressure entering the high pressure turbine. Steam enters the high pressure turbine at 17 MPa and  $500^{\circ}$ C. The steam is reheated to  $500^{\circ}$ C before entering the low pressure turbine, and is condensed in a condenser at 10 kPa. Determine (a) the thermal efficiency ( $\eta_{th}$ ) and (b) the steam quality at the exit of the second turbine stage.

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

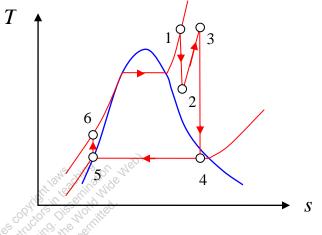
$$h_1 = 3280.89 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.2581 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2 = p_1 / 5$ ,  $s_2 = s_1$ ):

$$h_2 = 2868.47 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, T_3$ ):

$$h_3 = 3451.97 \frac{\text{kJ}}{\text{kg}}; \ s_3 = 7.1715 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$



State-4 (given  $p_4, s_4 = s_3$ ):

$$h_{f@10\text{kPa}} = 191.83 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@10\text{kPa}} = 2392.80 \frac{\text{kJ}}{\text{kg}}; \ s_{f@10\text{kPa}} = 0.6493 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@10\text{kPa}} = 7.5008 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_4 = \frac{s_3 - s_{f@10\text{kPa}}}{s_{f@10\text{kPa}}} = \frac{7.1715 - 0.6493}{7.5008} = 0.8695$$

$$h_4 = h_{f @ 10 \text{kPa}} + x_4 h_{fg @ 10 \text{kPa}} = 191.83 + (0.8695)(2392.80) = 2272.37 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given  $p_5 = p_4, x_5$ ):

$$h_5 = h_{f@10\text{kPa}} = 191.83 \frac{\text{kJ}}{\text{kg}}; \ v_5 = v_{f@10\text{kPa}} = 0.001010 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-6 (given  $p_6 = p_1, s_6 = s_5$ ):

Assuming that  $T_6 \cong T_5$ 

$$h_6 = h_5 + v_{f@T_5} (p_6 - p_5) = 191.83 + (0.001010)(17000 - 10) = 208.99 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_{T,I} = h_1 - h_2 = 3280.89 - 2868.47 = 412.42 \frac{\text{kJ}}{\text{kg}}$$
  
Device-B (2-3):  $q_{\text{in,rh}} = h_3 - h_2 = 3451.97 - 2868.47 = 583.50 \frac{\text{kJ}}{\text{kg}}$   
Device-C (3-4):  $w_{T,II} = h_3 - h_4 = 3451.97 - 2272.37 = 1179.60 \frac{\text{kJ}}{\text{kg}}$   
Device-D (4-5):  $q_{\text{out}} = h_4 - h_5 = 2272.37 - 191.83 = 2080.54 \frac{\text{kJ}}{\text{kg}}$   
Device-E (5-6):  $w_p = h_6 - h_5 = 208.99 - 191.83 = 17.16 \frac{\text{kJ}}{\text{kg}}$   
Device-F (6-1):  $q_{\text{in,b}} = h_1 - h_6 = 3280.89 - 208.99 = 3071.90 \frac{\text{kJ}}{\text{kg}}$ 

The thermal efficiency is

$$\begin{split} w_{\text{net}} &= w_{T,\text{I}} + w_{T,\text{II}} - w_P = 412.42 + 1179.60 - 17.16 = 1574.86 \frac{\text{kJ}}{\text{kg}} \\ q_{\text{in}} &= q_{\text{in,rh}} + q_{\text{in,b}} = 583.50 + 3071.90 = 3655.40 \frac{\text{kJ}}{\text{kg}} \\ \eta_{\text{th}} &= \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1763.37}{4036.53} = 43.69\% \end{split}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

**9-1-21** [OUO] An ideal reheat cycle operates with steam as the working fluid. The reheat pressure is 2 MPa. Steam enters the high pressure turbine at 13 MPa and  $600^{\circ}$ C. The steam is reheated to  $600^{\circ}$ C before entering the low pressure turbine, and is condensed in a condenser at 6 kPa. Determine (a) the thermal efficiency ( $\eta_{th}$ ) and (b) the steam quality (x) at the exit of the second turbine stage. (c)**What-if Scenario:** How would the answer in (b) change if the reheat pressure were 7 MPa?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

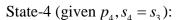
$$h_1 = 3599.67 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.7587 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 3019.34 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, T_3$ ):

$$h_3 = 3690.12 \frac{\text{kJ}}{\text{kg}}; \ s_3 = 7.7023 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$



$$h_{f@\,6\text{kPa}} = 151.51 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@\,6\text{kPa}} = 2415.87 \frac{\text{kJ}}{\text{kg}}; \ s_{f@\,6\text{kPa}} = 0.5209 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@\,6\text{kPa}} = 7.8100 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

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$$x_4 = \frac{s_3 - s_{f@6kPa}}{s_{fg@6kPa}} = \frac{7.7023 - 0.5209}{7.8100} = 0.9195$$

$$h_4 = h_{f @ 6kPa} + x_4 h_{fg @ 6kPa} = 151.51 + (0.9195)(2415.87) = 2372.90 \frac{kJ}{kg}$$

State-5 (given  $p_5 = p_4, x_5$ ):

$$h_5 = h_{f@6kPa} = 151.51 \frac{kJ}{kg}; \ v_5 = v_{f@10kPa} = 0.001007 \frac{kJ}{kg \cdot K}$$

State-6 (given  $p_6 = p_1, s_6 = s_5$ ):

Assuming that  $T_6 \cong T_5$ 

$$h_6 = h_5 + v_{f@T_5} (p_6 - p_5) = 151.51 + (0.001007)(13000 - 6) = 164.59 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_{T,I} = h_1 - h_2 = 3599.67 - 3019.34 = 580.33 \frac{kJ}{kg}$$
  
Device-B (2-3):  $q_{in,rh} = h_3 - h_2 = 3690.12 - 3019.34 = 670.78 \frac{kJ}{kg}$   
Device-C (3-4):  $w_{T,II} = h_3 - h_4 = 3690.12 - 2372.90 = 1317.22 \frac{kJ}{kg}$   
Device-D (4-5):  $q_{out} = h_4 - h_5 = 2372.90 - 151.51 = 2221.39 \frac{kJ}{kg}$   
Device-E (5-6):  $w_p = h_6 - h_5 = 164.59 - 151.51 = 13.08 \frac{kJ}{kg}$   
Device-F (6-1):  $q_{in,b} = h_1 - h_6 = 3599.67 - 164.59 = 3435.38 \frac{kJ}{kg}$ 

The thermal efficiency is

$$w_{\text{net}} = w_{T,\text{I}} + w_{T,\text{II}} - w_P = 580.33 + 1317.22 - 13.08 = 1884.47 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{in}} = q_{\text{in,rh}} + q_{\text{in,b}} = 670.78 + 3435.38 = 4106.16 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1884.47}{4106.16} = 45.89\%$$

9-1-22 [OUB] In a steam power plant operating on a reheat Rankine cycle, steam enters the high pressure turbine at 15 MPa, 620°C and is condensed in the condenser at a pressure of 15 kPa. If the moisture content in the turbine is not to exceed 10%, determine (a) the reheat pressure and (b) the thermal efficiency ( $\eta_{th}$ ) of the cycle. (c) **What-if Scenario:** What would the thermal efficiency be if the moisture tolerance of the turbine were increased to 15%?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3634.28 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.7354 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_3 = p_2, s_2 = s_1$ ):

$$h_2 = 3273.90 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $T_3, s_3 = s_4$ ):

$$p_3 = 5.26 \text{ MPa}$$

$$h_3 = 3711.17 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_A, x_A$ ):

$$h_{f @ 15 \text{kPa}} = 225.81 \frac{\text{kJ}}{\text{kg}}; \ h_{fg @ 15 \text{kPa}} = 2373.23 \frac{\text{kJ}}{\text{kg}}; \ s_{f @ 15 \text{kPa}} = 0.7543 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg @ 15 \text{kPa}} = 7.2550 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$h_4 = h_{f@15\text{kPa}} + x_4 h_{fg@15\text{kPa}} = 225.81 + (0.90)(2373.23) = 2361.72 \frac{\text{kJ}}{\text{kg}}$$

$$h_4 = h_{f @ 15 \text{kPa}} + x_4 h_{fg @ 15 \text{kPa}} = 0.7543 + (0.90)(7.2550) = 7.2838 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

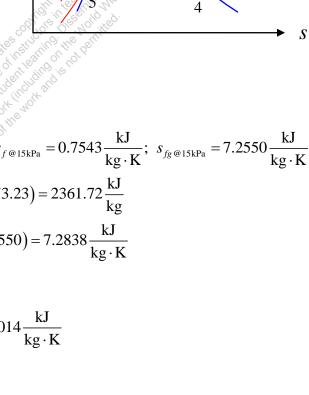
State-5 (given  $p_5 = p_4, x_5$ ):

$$h_5 = h_{f@15\text{kPa}} = 225.81 \frac{\text{kJ}}{\text{kg}}; \ v_5 = v_{f@15\text{kPa}} = 0.001014 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-6 (given  $p_6 = p_1, s_6 = s_5$ ):

Assuming that  $T_6 \cong T_5$ 

$$h_6 = h_5 + v_{f@T_5} (p_6 - p_5) = 225.81 + (0.001014)(15000 - 15) = 241.00 \frac{\text{kJ}}{\text{kg}}$$



A steady-state energy analysis is carried out for each device as follows.

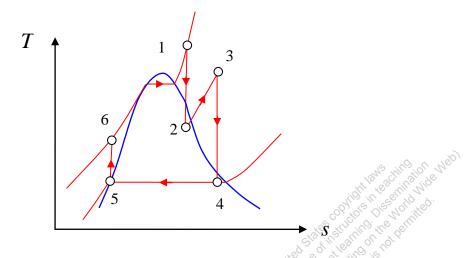
Device-A (1-2): 
$$w_{T,I} = h_1 - h_2 = 3634.28 - 3273.90 = 360.38 \frac{kJ}{kg}$$
  
Device-B (2-3):  $q_{in,rh} = h_3 - h_2 = 3711.17 - 3273.90 = 437.27 \frac{kJ}{kg}$   
Device-C (3-4):  $w_{T,II} = h_3 - h_4 = 3711.17 - 2361.72 = 1349.45 \frac{kJ}{kg}$   
Device-D (4-5):  $q_{out} = h_4 - h_5 = 2361.72 - 225.81 = 2135.91 \frac{kJ}{kg}$   
Device-E (5-6):  $w_p = h_6 - h_5 = 241.00 - 225.81 = 15.19 \frac{kJ}{kg}$   
Device-F (6-1):  $q_{in,b} = h_1 - h_6 = 3634.28 - 241.00 = 3393.28 \frac{kJ}{kg}$ 

The thermal efficiency is

$$\begin{split} w_{\text{net}} &= w_{T,\text{I}} + w_{T,\text{II}} - w_{P} = 360.38 + 1349.45 - 15.19 = 1694.64 \frac{\text{kJ}}{\text{kg}} \\ q_{\text{in}} &= q_{\text{in,rh}} + q_{\text{in,b}} = 437.27 + 3393.28 = 3830.55 \frac{\text{kJ}}{\text{kg}} \\ \eta_{\text{th}} &= \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1694.64}{3830.55} = 44.24\% \end{split}$$

9-1-23 [OUS] Steam is the working fluid in an ideal Rankine cycle with superheat and reheat. Steam enters the first stage turbine at 10 MPa,  $500^{\circ}$ C and expands to 700 kPa. It is then reheated to  $450^{\circ}$ C before entering the second stage turbine, where it expands to the condenser pressure of 8 kPa. The net power output is 100 MW. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the mass flow rate of steam and (c) the rate of heat transfer from the condensing steam as it passes through the condenser.

## **SOLUTION**



State-1 (given  $p_1, T_1$ ):

$$h_1 = 3373.60 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.5965 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_{f@700\text{kPa}} = 697.22 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@700\text{kPa}} = 2066.28 \frac{\text{kJ}}{\text{kg}}; \ s_{f@700\text{kPa}} = 1.9922 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@700\text{kPa}} = 4.7158 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_2 = \frac{s_2 - s_{f@700\text{kPa}}}{s_{f@700\text{kPa}}} = \frac{6.5965 - 1.9922}{4.7158} = 0.9764$$

$$h_2 = h_{f @ 700 \text{kPa}} + x_2 h_{fg @ 700 \text{kPa}} = 697.22 + (0.9764)(2066.28) = 2714.74 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, T_3$ ):

$$h_3 = 3375.15 \frac{\text{kJ}}{\text{kg}}$$
;  $s_3 = 7.7822 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ 

State-4 (given  $p_4, s_4 = s_3$ ):

$$\begin{split} h_{f@8k\text{Pa}} &= 173.88 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@8k\text{Pa}} = 2403.12 \frac{\text{kJ}}{\text{kg}}; \ s_{f@8k\text{Pa}} = 0.5926 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@8k\text{Pa}} = 7.6361 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ x_4 &= \frac{s_3 - s_{f@8k\text{Pa}}}{s_{fg@8k\text{Pa}}} = \frac{7.7822 - 0.5926}{7.6361} = 0.9415 \\ h_4 &= h_{f@8k\text{Pa}} + x_4 h_{fg@8k\text{Pa}} = 173.88 + (0.9415)(2403.12) = 2436.42 \frac{\text{kJ}}{\text{kg}} \end{split}$$

State-5 (given  $p_5 = p_4, x_5$ ):

$$h_5 = h_{f@8kPa} = 173.88 \frac{kJ}{kg}; \ v_5 = v_{f@8kPa} = 0.001008 \frac{m^3}{kg}$$

State-6 (given  $p_6 = p_1, s_6 = s_5$ ):

Assuming that  $T_6 \cong T_5$ 

$$h_6 = h_5 + v_{f@T_5} (p_6 - p_5) = 173.88 + (0.001008)(10000 - 8) = 183.95 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_{T,I} = h_1 - h_2 = 3373.60 - 2714.74 = 658.86 \frac{\text{kJ}}{\text{kg}}$$
  
Device-B (2-3):  $q_{\text{in,th}} = h_3 - h_2 = 3375.15 - 2714.74 = 660.41 \frac{\text{kJ}}{\text{kg}}$   
Device-C (3-4):  $w_{T,II} = h_3 - h_4 = 3375.15 - 2436.42 = 938.73 \frac{\text{kJ}}{\text{kg}}$   
Device-D (4-5):  $q_{\text{out}} = h_4 - h_5 = 2436.42 - 173.88 = 2262.54 \frac{\text{kJ}}{\text{kg}}$   
Device-E (5-6):  $w_p = h_6 - h_5 = 183.95 - 173.88 = 10.07 \frac{\text{kJ}}{\text{kg}}$   
Device-F (6-1):  $q_{\text{in,b}} = h_1 - h_6 = 3373.60 - 183.95 = 3189.65 \frac{\text{kJ}}{\text{kg}}$ 

The thermal efficiency is

$$w_{\text{net}} = w_{T,\text{I}} + w_{T,\text{II}} - w_P = 658.86 + 938.73 - 10.07 = 1587.52 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{in}} = q_{\text{in,rh}} + q_{\text{in,b}} = 660.41 + 3189.65 = 3850.06 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1587.52}{3850.06} = 41.23\%$$

The mass flow rate is

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{100000}{1587.52} = 62.99 \frac{\text{kg}}{\text{s}}$$

The heat transfer in the condenser is

$$\dot{Q}_{\text{out}} = \dot{m}(h_4 - h_5) = (62.99)(2262.54) = 142517.39 \text{ kW} = 142.52 \text{ MW}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-1-24 [OUA] In a steam power plant operating on the ideal regenerative Rankine cycle with one open feedwater heater, steam enters the turbine at 9 MPa,  $480^{\circ}$ C and is condensed in the condenser at a pressure of 7 kPa. Bleeding from the turbine to the FWH occurs at 0.7 MPa. The net power output of the cycle is 100 MW. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the mass flow rate entering the turbine and (c) the rate of heat transfer to the working fluid passing through the steam generator. (d)What-if Scenario: What would the net power developed be if the bleeding pressure were increased to 1.2 MPa?

#### **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3334.24 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.5882 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 2711.00 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):

$$h_{f@7\text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@7\text{kPa}} = 2409.14 \frac{\text{kJ}}{\text{kg}}; \ s_{f@7\text{kPa}} = 0.5589 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@7\text{kPa}} = 7.7175 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_3 = \frac{s_1 - s_{f@7kPa}}{s_{fg@7kPa}} = \frac{6.5882 - 0.5589}{7.7175} = 0.7813$$

$$h_3 = h_{f@7\text{kPa}} + x_3 h_{fg@7\text{kPa}} = 163.33 + (0.7813)(2409.14) = 2045.59 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_3, x_4$ ):

$$h_4 = h_{f @ 7kPa} = 163.33 \frac{kJ}{kg}; \ v_4 = v_{f @ 7kPa} = 0.001007 \frac{m^3}{kg}$$

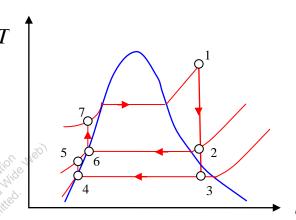
State-5 (given  $p_5 = p_2, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 163.33 + (0.001007)(700 - 7) = 164.03 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_2, x_6$ ):

$$h_6 = h_{f@0.7\text{MPa}} = 697.22 \frac{\text{kJ}}{\text{kg}}; \ v_6 = v_{f@0.7\text{MPa}} = 0.001108 \frac{\text{m}^3}{\text{kg}}$$



State-7 (given  $p_7 = p_1, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 697.22 + (0.001108)(9000 - 700) = 706.42 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_6 = \dot{m}_1$ ,  $\dot{m}_5 = \dot{m}_3$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_5 h_5;$$

$$\Rightarrow \dot{m}_1 h_6 = \dot{m}_2 h_2 + \left( \dot{m}_1 - \dot{m}_2 \right) h_5;$$

$$\Rightarrow h_6 = rh_2 + (1-r)h_5;$$

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_5} = \frac{705.64 - 164.03}{2711.00 - 164.03} = 0.2090$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-2,3):

$$W_T = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$

$$w_T = (0.2090)(3334.24 - 2711.00) + (1 - 0.2090)(3334.24 - 2045.59) = 1149.58 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = (1-r)(h_3 - h_4) = (1-0.2090)(2045.59 - 163.33) = 1488.87 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_{P,I} = (1-r)(h_5 - h_4) = (1-0.2090)(164.03-163.33) = 0.55 \frac{kJ}{kg}$$

Device-D (6-7): 
$$w_{P,II} = h_7 - h_6 = 706.42 - 697.22 = 9.20 \frac{\text{kJ}}{\text{kg}}$$

Device-E (7-1): 
$$q_{in} = h_1 - h_7 = 3334.24 - 706.42 = 2627.82 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,I} - w_{P,II} = 1149.58 - 0.55 - 9.20 = 1139.83 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1139.83}{2627.82} = 43.38\%$$

The mass flow into the turbine

$$\dot{m}_1 = \frac{W_{\text{net}}}{W_{\text{net}}} = \frac{100000}{1139.83} = 87.73 \frac{\text{kg}}{\text{s}}$$

The heat transfer to the fluid passing through the steam generator  $\dot{Q}_{\rm in} = \dot{m}_1 q_{\rm in} = (87.73)(2627.82) = 230538.65 \text{ kW} = 230.54 \text{ MW}$ 



**9-1-25** [OUE] In a steam power plant operating on the ideal regenerative Rankine cycle with one open feedwater heater, steam enters the turbine at 15 MPa,  $620^{\circ}$ C and is condensed in the condenser at a pressure of 15 kPa. Bleeding from the turbine to the FWH occurs at 1 MPa. Determine (a) the fraction of steam extracted and (b) the thermal efficiency ( $\eta_{th}$ ) of the cycle. (c) **What-if Scenario:** What would the thermal efficiency be if the bleeding pressure were increased to 1.5 MPa?

## **SOLUTION**

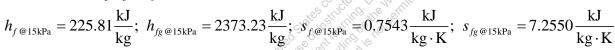
State-1 (given  $p_1, T_1$ ):

$$h_1 = 3634.28 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.7354 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 2848.49 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):



$$x_3 = \frac{s_3 - s_{f@15\text{kPa}}}{s_{f@15\text{kPa}}} = \frac{6.7354 - 0.7543}{7.2550} = 0.8244$$

$$h_3 = h_{f@15\text{kPa}} + x_3 h_{fg@15\text{kPa}} = 225.81 + (0.8244)(2373.23) = 2182.30 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_3, x_4$ ):

$$h_4 = h_{f@15\text{kPa}} = 225.81 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f@15\text{kPa}} = 0.001014 \frac{\text{m}^3}{\text{kg}}$$

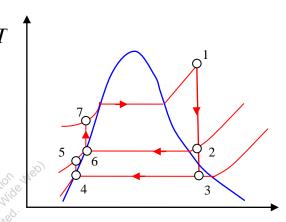
State-5 (given  $p_5 = p_2, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4} (p_5 - p_4) = 225.81 + (0.001014)(1000 - 15) = 226.81 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_2, x_6$ ):

$$h_6 = h_{f@1\text{MPa}} = 762.83 \frac{\text{kJ}}{\text{kg}}; \ v_6 = v_{f@1\text{MPa}} = 0.001127 \frac{\text{m}^3}{\text{kg}}$$



State-7 (given  $p_7 = p_1, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 762.83 + (0.001127)(15000 - 1000) = 778.61 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_6 = \dot{m}_1$ ,  $\dot{m}_5 = \dot{m}_3$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_5 h_5;$$

$$\Rightarrow \dot{m}_1 h_6 = \dot{m}_2 h_2 + (\dot{m}_1 - \dot{m}_2) h_5;$$

$$\Rightarrow h_6 = rh_2 + (1-r)h_5;$$

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.83 - 226.81}{2848.49 - 226.81} = 0.2045$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-2,3):

$$w_{\tau} = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$

$$w_T = (0.2045)(3634.28 - 2848.49) + (1 - 0.2045)(3634.28 - 2182.30) = 1315.74 \frac{kJ}{kg}$$

Device-B (3-4): 
$$q_{\text{out}} = (1-r)(h_3 - h_4) = (1-0.2045)(2182.30 - 225.81) = 1556.39 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_{P,I} = (1-r)(h_5 - h_4) = (1-0.2045)(226.81-225.81) = 0.80 \frac{kJ}{kg}$$

Device-D (6-7): 
$$w_{P,II} = h_7 - h_6 = 778.61 - 762.83 = 15.78 \frac{\text{kJ}}{\text{kg}}$$

Device-E (7-1): 
$$q_{in} = h_1 - h_7 = 3634.28 - 778.61 = 2855.67 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 1315.74 - 0.80 - 15.78 = 1299.16 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1299.16}{2855.67} = 45.49\%$$

**9-1-26** [OUH] A power plant operates on a regenerative vapor power cycle with one open feedwater heater. Steam enters the first turbine stage at 11 MPa,  $600^{\circ}$ C and expands to 1 MPa, where some of the steam is extracted and diverted to the open feedwater heater operating at 1 MPa. The remaining steam expands through the second turbine stage to a condenser pressure of 6 kPa. Saturated liquid exits the open feedwater heater at 1 MPa. The net power output is 264 MW. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the mass flow rate into the first turbine stage and (c) the fraction of flow extracted where bleeding occurs. (d) **What-if Scenario:** What would the net power developed be if the bleeding pressure were increased to 1.2 MPa?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3616.80 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.8508 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 2905.86 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):

$$h_{f@6kPa} = 151.51 \frac{kJ}{kg}; \ h_{fg@6kPa} = 2415.87 \frac{kJ}{kg}; \ s_{f@6kPa} = 0.5209 \frac{kJ}{kg \cdot K}; \ s_{fg@6kPa} = 7.8100 \frac{kJ}{kg \cdot K}$$

$$x_3 = \frac{s_3 - s_{f@6kPa}}{s_{f@6kPa}} = \frac{6.8508 - 0.5209}{7.8100} = 0.8105$$

$$h_3 = h_{f@6kPa} + x_3 h_{fg@6kPa} = 151.51 + (0.8105)(2415.87) = 2109.57 \frac{kJ}{kg}$$

State-4 (given  $p_4 = p_3, x_4$ ):

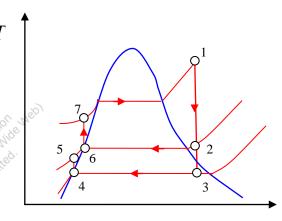
$$h_4 = h_{f@6\text{kPa}} = 151.51 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f@6\text{kPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5 = p_2, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4} (p_5 - p_4) = 151.51 + (0.001007)(1000 - 6) = 152.51 \frac{kJ}{kg}$$

State-6 (given  $p_6 = p_2, x_6$ ):



$$h_6 = h_{f@1\text{MPa}} = 762.83 \frac{\text{kJ}}{\text{kg}}; \ v_6 = v_{f@1\text{MPa}} = 0.001127 \frac{\text{m}^3}{\text{kg}}$$

State-7 (given  $p_7 = p_1, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 762.83 + (0.001127)(11000 - 1000) = 774.10 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_6 = \dot{m}_1$ ,  $\dot{m}_5 = \dot{m}_3$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_5 h_5;$$

$$\Rightarrow \dot{m}_1 h_6 = \dot{m}_2 h_2 + (\dot{m}_1 - \dot{m}_2) h_5;$$

$$\Rightarrow h_6 = rh_2 + (1-r)h_5;$$

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.83 - 152.51}{2905.86 - 152.51} = 0.2217$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-2,3):

$$w_T = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$

$$w_T = (0.2217)(3616.80 - 2905.86) + (1 - 0.2217)(3616.80 - 2109.57) = 1330.69 \frac{kJ}{kg}$$

Device-B (3-4): 
$$q_{\text{out}} = (1-r)(h_3 - h_4) = (1-0.2217)(2109.57 - 151.51) = 1523.96 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_{P,I} = (1-r)(h_5 - h_4) = (1-0.2217)(152.51-151.51) = 0.78 \frac{kJ}{kg}$$

Device-D (6-7): 
$$w_{P,II} = h_7 - h_6 = 774.10 - 762.83 = 11.27 \frac{kJ}{kg}$$

Device-E (7-1): 
$$q_{in} = h_1 - h_7 = 3616.80 - 774.10 = 2842.70 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 1330.69 - 0.78 - 11.27 = 1318.64 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{a_{\text{th}}} = \frac{1318.64}{2842.70} = 46.39\%$$

The mass flow into the turbine

$$\dot{m}_1 = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{264000}{1318.64} = \frac{200.21 \frac{\text{kg}}{\text{s}}}{\text{s}}$$



9-1-27 [OUN] Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 14 MPa,  $610^{\circ}$ C and is condensed in the condenser at a pressure of 12 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open feedwater heater. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the fraction of flow extracted where bleeding occurs. (c) **What-if Scenario:** What would the answer in (b) be if the bleeding pressure were increased to 1.5 MPa?

## **SOLUTION**

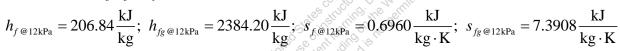
State-1 (given  $p_1, T_1$ ):

$$h_1 = 3616.69 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.7453 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 2893.23 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3$ ,  $s_3 = s_1$ ):



$$x_3 = \frac{s_3 - s_{f@12\text{kPa}}}{s_{f@012\text{kPa}}} = \frac{6.7453 - 0.6960}{7.3908} = 0.8185$$

$$h_3 = h_{f@12\text{kPa}} + x_3 h_{fg@12\text{kPa}} = 206.84 + (0.8185)(2384.20) = 2158.31 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_3, x_4$ ):

$$h_4 = h_{f@12\text{kPa}} = 206.84 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f@12\text{kPa}} = 0.001012 \frac{\text{m}^3}{\text{kg}}$$

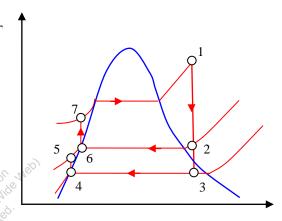
State-5 (given  $p_5 = p_2, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 206.84 + (0.001012)(1200 - 12) = 208.04 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_2, x_6$ ):

$$h_6 = h_{f@1.2\text{MPa}} = 798.64 \frac{\text{kJ}}{\text{kg}}; \ v_6 = v_{f@1.2\text{MPa}} = 0.001139 \frac{\text{m}^3}{\text{kg}}$$



State-7 (given  $p_7 = p_1, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 798.64 + (0.001139)(14000 - 1200) = 813.22 \frac{kJ}{kg}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_6 = \dot{m}_1$ ,  $\dot{m}_5 = \dot{m}_3$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_5 h_5;$$

$$\Rightarrow \dot{m}_1 h_6 = \dot{m}_2 h_2 + \left( \dot{m}_1 - \dot{m}_2 \right) h_5;$$

$$\Rightarrow h_6 = rh_2 + (1-r)h_5$$
;

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_5} = \frac{798.64 - 208.04}{2893.23 - 208.04} = 0.2199$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-2,3):

$$W_T = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$

$$w_T = (0.2199)(3616.69 - 2893.23) + (1 - 0.2199)(3616.69 - 2158.31) = 1296.77 \frac{kJ}{kg}$$

Device-B (3-4): 
$$q_{\text{out}} = (1-r)(h_3 - h_4) = (1-0.2199)(2158.31 - 206.84) = 1522.34 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_{P,I} = (1-r)(h_5 - h_4) = (1-0.2199)(208.04 - 206.84) = 0.94 \frac{kJ}{kg}$$

Device-D (6-7): 
$$w_{P,II} = h_7 - h_6 = 813.22 - 798.64 = 14.58 \frac{\text{kJ}}{\text{kg}}$$

Device-E (7-1): 
$$q_{in} = h_1 - h_7 = 3616.69 - 813.22 = 2803.47 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 1296.77 - 0.94 - 14.58 = 1281.25 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1281.25}{2803.47} = 45.70\%$$

9-1-28 [OUG] A steam power plant operates on an ideal regenerative Rankine cycle. Steam enters the turbine at 5 MPa, 450°C and is condensed in the condenser at 15 kPa. Steam is extracted from the turbine at a pressure of 0.4 MPa and enters the open feedwater heater. Water leaves the feedwater heater as a saturated liquid. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the net work output per kilogram of steam ( $w_{net}$ ) flowing through the boiler.

## **SOLUTION**

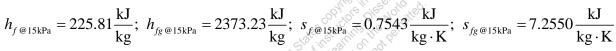
State-1 (given  $p_1, T_1$ ):

$$h_1 = 3316.13 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.8185 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 2706.01 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):



$$x_3 = \frac{s_1 - s_{f@15\text{kPa}}}{s_{fg@15\text{kPa}}} = \frac{6.8185 - 0.7543}{7.2550} = 0.8359$$

$$h_3 = h_{f@15\text{kPa}} + x_3 h_{fg@15\text{kPa}} = 225.81 + (0.8359)(2373.23) = 2209.59 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_3, x_4$ ):

$$h_4 = h_{f@15\text{kPa}} = 225.81 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f@15\text{kPa}} = 0.001014 \frac{\text{m}^3}{\text{kg}}$$

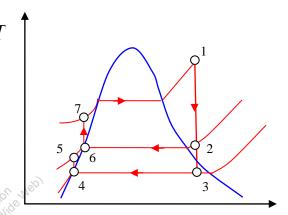
State-5 (given  $p_5 = p_2, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 225.81 + (0.001014)(400 - 15) = 226.20 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_2, x_6$ ):

$$h_6 = h_{f@0.4\text{MPa}} = 604.72 \frac{\text{kJ}}{\text{kg}}; \ v_6 = v_{f@0.4\text{MPa}} = 0.001084 \frac{\text{m}^3}{\text{kg}}$$



State-7 (given  $p_7 = p_1, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 604.72 + (0.001084)(5000 - 400) = 609.74 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_6 = \dot{m}_1$ ,  $\dot{m}_5 = \dot{m}_3$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_5 h_5;$$

$$\Rightarrow \dot{m}_1 h_6 = \dot{m}_2 h_2 + \left( \dot{m}_1 - \dot{m}_2 \right) h_5;$$

$$\Rightarrow h_6 = rh_2 + (1-r)h_5$$
;

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_5} = \frac{604.72 - 226.20}{2706.01 - 226.20} = 0.1526$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-2,3):

$$W_T = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$

$$w_T = (0.1526)(3316.13 - 2706.01) + (1 - 0.1526)(3316.13 - 2209.59) = 1030.79 \frac{kJ}{kg}$$

Device-B (3-4): 
$$q_{\text{out}} = (1-r)(h_3 - h_4) = (1-0.1526)(2209.59 - 225.81) = 1681.06 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_{P,I} = (1-r)(h_5 - h_4) = (1-0.1526)(226.20 - 225.81) = 0.33 \frac{kJ}{kg}$$

Device-D (6-7): 
$$w_{P,II} = h_7 - h_6 = 609.74 - 604.72 = 5.02 \frac{\text{kJ}}{\text{kg}}$$

Device-E (7-1): 
$$q_{in} = h_1 - h_7 = 3316.13 - 609.74 = 2706.39 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 1030.79 - 0.33 - 5.02 = 1025.44 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1025.44}{2706.39} = 37.89\%$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-1-29 [OUI] A regenerative vapor power cycle has two turbine stages with steam entering the first turbine stage at 8 MPa,  $550^{\circ}$ C and expanding to 700 kPa, where some of the steam is extracted and diverted to the open feedwater heater operating at 700 kPa. The remaining steam expands through the second turbine stage to the condenser at a pressure of 7 kPa. Saturated liquid exits the open feedwater heater at 700 kPa. Each turbine stage has an isentropic efficiency of 88% and each pump has an isentropic efficiency of 80%. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the net work developed per kilogram of steam ( $w_{net}$ ) and (c) the fraction of flow extracted where bleeding occurs. (d) **What-if Scenario:** What would the net power developed be if the mass flow rate of steam entering the first stage of turbine were 170 kg/s?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3520.99 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.8778 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given 
$$p_2, s_2 = s_1$$
):

$$h_2 = 2841.04 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3520.99 - (0.88)(3520.99 - 2841.04) = 2922.63 \frac{kJ}{kg}$$

$$s_3 = 7.0426 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-4 (given  $p_4, s_4 = s_3$ ):

$$h_{f@7k\text{Pa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@7k\text{Pa}} = 2409.14 \frac{\text{kJ}}{\text{kg}}; \ s_{f@7k\text{Pa}} = 0.5589 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@7k\text{Pa}} = 7.7175 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_4 = \frac{s_4 - s_{f@7\text{kPa}}}{s_{fg@7\text{kPa}}} = \frac{7.0426 - 0.5589}{7.7175} = 0.8401$$

$$h_4 = h_{f @ 7 \text{kPa}} + x_4 h_{fg @ 7 \text{kPa}} = 163.33 + (0.8401)(2409.14) = 2187.25 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given  $p_5 = p_4, \eta_T$ ):

$$h_5 = h_3 - \eta_T (h_3 - h_4) = 2922.63 - (0.88)(2922.63 - 2187.25) = 2275.50 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_4, x_6$ ):

$$h_6 = h_{f@7\text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ v_6 = v_{f@7\text{kPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-7 (given  $p_7 = p_2, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 163.33 + (0.001007)(700 - 7) = 164.03 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given  $p_8 = p_7, \eta_P$ ):

$$h_8 = h_6 + \frac{h_7 - h_6}{\eta_P} = 163.33 + \frac{164.03 - 163.33}{0.80} = 164.21 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_2, x_9$ ):

$$h_9 = h_{f@700\text{kPa}} = 697.22 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f@700\text{kPa}} = 0.001108 \frac{\text{m}^3}{\text{kg}}$$

State-10 (given  $p_{10} = p_1, s_{10} = s_9$ ):

Assuming that  $T_{10} \cong T_9$ 

$$h_{10} = h_9 + v_{f@T_9} (p_{10} - p_9) = 697.22 + (0.001108)(8000 - 700) = 705.31 \frac{kJ}{kg}$$

State-11 (given  $p_{11} = p_{10}, \eta_P$ ):

$$h_{11} = h_9 + \frac{h_{10} - h_9}{\eta_P} = 697.22 + \frac{705.31 - 697.22}{0.80} = 707.33 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_3 + \dot{m}_5$ ,  $\dot{m}_9 = \dot{m}_1$ ,  $\dot{m}_8 = \dot{m}_5$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_9 h_9 = \dot{m}_3 h_3 + \dot{m}_8 h_8;$$

$$\Rightarrow \dot{m}_1 h_0 = \dot{m}_3 h_3 + (\dot{m}_1 - \dot{m}_3) h_8;$$

$$\Rightarrow h_9 = rh_3 + (1-r)h_8;$$

$$\Rightarrow r = \frac{h_9 - h_8}{h_3 - h_8} = \frac{697.22 - 164.21}{2922.63 - 164.21} = 0.1932$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-3,5):

$$w_T = r(h_1 - h_3) + (1 - r)(h_1 - h_5);$$

$$w_T = (0.1932)(3520.99 - 2922.63) + (1 - 0.1932)(3520.99 - 2275.50) = 1120.46 \frac{kJ}{kg}$$
Device-B (5-6):  $q_{\text{out}} = (1 - r)(h_5 - h_6) = (1 - 0.1932)(2275.50 - 163.33) = 1704.10 \frac{kJ}{kg}$ 
Device-C (6-8):  $w_{P,I} = (1 - r)(h_8 - h_6) = (1 - 0.1932)(164.21 - 163.33) = 0.71 \frac{kJ}{kg}$ 
Device-D (9-11):  $w_{P,II} = h_{11} - h_9 = 707.33 - 697.22 = 10.11 \frac{kJ}{kg}$ 
Device-E (11-1):  $q_{\text{in}} = h_1 - h_{11} = 3520.99 - 707.33 = 2813.66 \frac{kJ}{kg}$ 

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 1120.46 - 0.71 - 10.11 = 1109.64 \frac{\text{kJ}}{\text{kg}} = 1.11 \frac{\text{MJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1109.64}{2813.66} = 39.44\%$$

**9-1-30** [OUL] A regenerative vapor power cycle has two turbine stages with steam entering the first turbine stage at 12 MPa,  $600^{\circ}$ C and expands to 1 MPa, where some of the steam is extracted and diverted to the open feedwater heater operating at 1 MPa. The remaining steam expands through the second turbine stage to a condenser at pressure of 6 kPa. Saturated liquid exits the open feedwater heater at 6 kPa. Each turbine stage and pump has an isentropic efficiency of 80%. The mass flow rate into the first turbine stage is 100 kg/s. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle, (b) the net power developed and (c) the heat transfer to the steam in the steam generator. (d) **What-if Scenario:** What would the net power developed be if the feedwater pressure were 1.4 MPa?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3608.28 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.8033 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given 
$$p_2, s_2 = s_1$$
):

$$h_2 = 2882.24 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3608.28 - (0.80)(3608.28 - 2882.24) = 3027.45 \frac{kJ}{kg}$$

$$s_3 = 7.0795 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-4 (given  $p_4$ ,  $s_4 = s_3$ ):

$$h_{f@6kPa} = 151.51 \frac{kJ}{kg}; \ h_{fg@6kPa} = 2415.87 \frac{kJ}{kg}; \ s_{f@6kPa} = 0.5209 \frac{kJ}{kg \cdot K}; \ s_{fg@6kPa} = 7.8100 \frac{kJ}{kg \cdot K}$$

$$x_4 = \frac{s_4 - s_{f@6kPa}}{s_{fg@6kPa}} = \frac{7.0795 - 0.5209}{7.8100} = 0.8398$$

$$h_4 = h_{f @ 6kPa} + x_4 h_{fg @ 6kPa} = 151.51 + (0.8398)(2415.87) = 2180.36 \frac{kJ}{kg}$$

State-5 (given  $p_5 = p_4, \eta_T$ ):

$$h_5 = h_3 - \eta_T (h_3 - h_4) = 3027.45 - (0.80)(3027.45 - 2180.36) = 2349.78 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_4, x_6$ ):

$$h_6 = h_{f @ 6kPa} = 151.51 \frac{kJ}{kg}; \ v_6 = v_{f @ 6kPa} = 0.001007 \frac{m^3}{kg}$$

State-7 (given  $p_7 = p_2, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6} (p_7 - p_6) = 151.51 + (0.001007)(1000 - 6) = 152.51 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given  $p_8 = p_7, \eta_P$ ):

$$h_8 = h_6 + \frac{h_7 - h_6}{\eta_P} = 151.51 + \frac{152.51 - 151.51}{0.80} = 152.76 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_2, x_9$ ):

$$h_9 = h_{f@1\text{MPa}} = 762.83 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f@1\text{MPa}} = 0.001127 \frac{\text{m}^3}{\text{kg}}$$

State-10 (given  $p_{10} = p_1, s_{10} = s_9$ ):

Assuming that  $T_{10} \cong T_9$ 

$$h_{10} = h_9 + v_{f@T_9} (p_{10} - p_9) = 762.83 + (0.001127)(12000 - 1000) = 775.23 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given  $p_{11} = p_{10}, \eta_P$ ):

$$h_{11} = h_9 + \frac{h_{10} - h_9}{\eta_P} = 762.83 + \frac{775.23 - 762.83}{0.80} = 778.33 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_3 + \dot{m}_5$ ,  $\dot{m}_9 = \dot{m}_1$ ,  $\dot{m}_8 = \dot{m}_5$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_9 h_9 = \dot{m}_3 h_3 + \dot{m}_8 h_8;$$

$$\Rightarrow \dot{m}_1 h_0 = \dot{m}_3 h_3 + (\dot{m}_1 - \dot{m}_3) h_8$$

$$\Rightarrow h_9 = rh_3 + (1-r)h_8;$$

$$\Rightarrow r = \frac{h_9 - h_8}{h_3 - h_8} = \frac{762.83 - 152.76}{3027.45 - 152.76} = 0.2122$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-3,5):

$$w_T = r(h_1 - h_3) + (1 - r)(h_1 - h_5);$$

$$w_T = (0.2122)(3608.28 - 3027.45) + (1 - 0.2122)(3608.28 - 2349.78) = 1114.70 \frac{kJ}{kg}$$
Device-B (5-6):  $q_{\text{out}} = (1 - r)(h_5 - h_6) = (1 - 0.2122)(2349.78 - 151.51) = 1731.80 \frac{kJ}{kg}$ 
Device-C (6-8):  $w_{P,I} = (1 - r)(h_8 - h_6) = (1 - 0.2122)(152.76 - 151.51) = 0.98 \frac{kJ}{kg}$ 
Device-D (9-11):  $w_{P,II} = h_{11} - h_9 = 778.33 - 762.83 = 15.50 \frac{kJ}{kg}$ 
Device-E (11-1):  $q_{\text{in}} = h_1 - h_{11} = 3608.28 - 778.33 = 2829.85 \frac{kJ}{kg}$ 

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 1114.70 - 0.98 - 15.50 = 1098.22 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1098.22}{2829.85} = 38.81\%$$

The net power developed and heat transfer in the steam generator  $\dot{W}_{\text{net}} = \dot{m}_1 w_{\text{net}} = (100)(1098.22) = 109822 \text{ kW} = 109.82 \text{ MW}$   $\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in}} = (100)(2829.85) = 2829.85 \text{ kW} = 282.99 \text{ MW}$ 

9-1-31 [OUZ] A power plant operates on a regenerative vapor power cycle with one closed feedwater heater. Steam enters the first turbine stage at 10 MPa,  $500^{\circ}$ C and expands to 1 MPa, where some of the steam is extracted and diverted to a closed feedwater heater. Condensate exiting the feedwater heater as saturated liquid at 1 MPa passes through a trap into the condenser. The feedwater exits the heater at 10 MPa with a temperature of  $175^{\circ}$ C. The condenser pressure is 6 kPa. The mass flow rate into the first stage turbine is 270 kg/s. For isentropic processes in each turbine stage and the pump, determine (a) the mass flow rate of steam extracted from the turbine, (b) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (c) the net power developed.

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3373.60 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.5965 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given 
$$p_2, s_2 = s_1$$
):

$$h_2 = 2782.77 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):

$$h_{f@6\text{kPa}} = 151.51 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@6\text{kPa}} = 2415.87 \frac{\text{kJ}}{\text{kg}}; \ s_{f@6\text{kPa}} = 0.5209 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@6\text{kPa}} = 7.8100 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_3 = \frac{s_3 - s_{f@6kPa}}{s_{f@6kPa}} = \frac{6.5965 - 0.5209}{7.8100} = 0.7779$$

$$h_3 = h_{f@6kPa} + x_3 h_{fg@6kPa} = 151.51 + (0.7779)(2415.87) = 2030.82 \frac{kJ}{kg}$$

State-4 (given  $p_4 = p_3, x_4$ ):

$$h_4 = h_{f@6\text{kPa}} = 151.51 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f@6\text{kPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 151.51 + (0.001007)(10000 - 6) = 161.57 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_5, T_6$ ):

$$\begin{split} u_{f@175^{\circ}\text{C}} &= 740.08 \frac{\text{kJ}}{\text{kg}}; \ v_{f@175^{\circ}\text{C}} = 0.001120 \frac{\text{m}^{3}}{\text{kg}} \\ h_{6} &= u_{f@175^{\circ}\text{C}} + p_{6}v_{f@175^{\circ}\text{C}} = 740.08 + \big(10000\big)\big(0.001120\big) = 751.28 \frac{\text{kJ}}{\text{kg}} \end{split}$$

State-7 (given 
$$p_7 = p_2, x_7$$
):

$$h_7 = h_{f@1\text{MPa}} = 762.83 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given 
$$p_8 = p_3, h_8 = h_7$$
):

$$h_8 = h_7 = 762.83 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_5 = \dot{m}_6 = \dot{m}_1$ ,  $\dot{m}_7 = \dot{m}_2$ , an energy balance for the closed feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 - \dot{m}_5 h_5 = \dot{m}_2 h_2 - \dot{m}_7 h_7;$$

$$\Rightarrow \dot{m}_1(h_6-h_5)=\dot{m}_2(h_2-h_7);$$

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_7} = \frac{751.28 - 161.57}{2782.77 - 762.83} = 0.2914$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2,3):

$$w_T = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$
  

$$w_T = (0.2914)(3373.60 - 2782.77) + (1 - 0.2914)(3373.60 - 2030.82) = 1123.66 \frac{kJ}{kg}$$

Device-B (3-4):

$$q_{\text{out}} = (1 - r)h_3 + rh_8 - h_4 = h_3 + r(h_8 - h_3) - h_4;$$
  

$$q_{\text{out}} = 2030.82 + (0.2914)(762.83 - 2030.82) - 151.51 = 1509.82 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_P = h_5 - h_4 = 161.57 - 151.51 = 10.06 \frac{\text{kJ}}{\text{kg}}$$

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 3373.60 - 751.28 = 2622.32 \frac{kJ}{kg}$$

The mass mass flow extracted from the turbine

$$\dot{m}_2 = r\dot{m}_1 = (0.2914)(270) = 78.68 \frac{\text{kg}}{\text{s}}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 1123.66 - 10.06 = 1113.60 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1113.60}{2622.32} = 42.47\%$$

The net power developed

$$\dot{W}_{\text{net}} = \dot{m}_1 w_{\text{net}} = (270)(1113.60) = 300672 \text{ kW} = 300.67 \text{ MW}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).



**9-1-32** [OUK] Repeat problem 9-1-31 [OUZ] above by replacing the trap with a pump and a mixing chamber as shown in the schematic below.

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3373.60 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.5965 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 2782.77 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):

$$h_{f@6kPa} = 151.51 \frac{kJ}{kg}; \ h_{fg@6kPa} = 2415.87 \frac{kJ}{kg}; \ s_{f@6kPa} = 0.5209 \frac{kJ}{kg \cdot K}; \ s_{fg@6kPa} = 7.8100 \frac{kJ}{kg \cdot K}$$

$$x_3 = \frac{s_3 - s_{f@6\text{kPa}}}{s_{f@6\text{kPa}}} = \frac{6.5965 - 0.5209}{7.8100} = 0.7779$$

$$h_3 = h_{f@6\text{kPa}} + x_3 h_{fg@6\text{kPa}} = 151.51 + (0.7779)(2415.87) = 2030.82 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_3, x_4$ ):

$$h_4 = h_{f@6kPa} = 151.51 \frac{kJ}{kg}; \ v_4 = v_{f@6kPa} = 0.001007 \frac{m^3}{kg}$$

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4}(p_5 - p_4) = 151.51 + (0.001007)(10000 - 6) = 161.57 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_5, T_6$ ):

$$u_{f@175^{\circ}\text{C}} = 740.08 \frac{\text{kJ}}{\text{kg}}; \ v_{f@175^{\circ}\text{C}} = 0.001120 \frac{\text{m}^3}{\text{kg}}$$

$$h_6 = u_{f@175^{\circ}C} + p_6 v_{f@175^{\circ}C} = 740.08 + (10000)(0.001120) = 751.28 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given  $p_7 = p_2, x_7$ ):

$$h_7 = h_{f@1\text{MPa}} = 762.83 \frac{\text{kJ}}{\text{kg}}; \ v_7 = v_{f@1\text{MPa}} = 0.001127 \frac{\text{m}^3}{\text{kg}}$$

State-8 (given  $p_8 = p_1, s_8 = s_7$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_8 = h_7 + v_{f@T_7}(p_8 - p_7) = 762.83 + (0.001127)(10000 - 1000) = 772.97 \frac{kJ}{kg}$$

State-9 (given  $p_0 = p_1$ ):

$$h_9 = (1-r)h_6 + rh_8 = (1-0.2260)(751.28) + (0.2260)(772.97) = 756.18 \frac{kJ}{kg}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_5 = \dot{m}_6 = \dot{m}_3$ ,  $\dot{m}_7 = \dot{m}_2$ , an energy balance for the closed feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 - \dot{m}_5 h_5 = \dot{m}_2 h_2 - \dot{m}_7 h_7;$$

$$\Rightarrow \dot{m}_3(h_6-h_5)=\dot{m}_2(h_2-h_7);$$

$$\Rightarrow$$
  $(1-r)(h_6-h_5)=r(h_2-h_7);$ 

$$\Rightarrow (1-r)(h_6 - h_5) = r(h_2 - h_7);$$

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_7 + h_6 - h_5} = \frac{751.28 - 161.57}{2782.77 - 762.83 + 751.28 - 161.57} = 0.2260$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-2,3):

$$w_T = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$

$$w_T = (0.2260)(3373.60 - 2782.77) + (1 - 0.2260)(3373.60 - 2030.82) = 1172.84 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4): 
$$q_{\text{out}} = (1-r)(h_3 - h_4) = (1-0.2260)(2030.82 - 151.51) = 1454.59 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_{P,I} = (1-r)(h_5 - h_4) = (1-0.2260)(161.57 - 151.51) = 7.79 \frac{kJ}{kg}$$

Device-D (7-8): 
$$w_{P,II} = r(h_8 - h_7) = (0.2260)(772.97 - 762.83) = 2.29 \frac{kJ}{kg}$$

Device-E (9-1): 
$$q_{in} = h_1 - h_9 = 3373.60 - 756.18 = 2617.42 \frac{kJ}{kg}$$

The mass mass flow extracted from the turbine

$$\dot{m}_2 = r\dot{m}_1 = (0.2260)(270) = 61.02 \frac{\text{kg}}{\text{s}}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 1172.84 - 7.79 - 2.29 = 1162.26 \frac{\text{kJ}}{\text{kg}}$$
$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1162.26}{2617.42} = 44.40\%$$

The net power developed

$$\dot{W}_{\text{net}} = \dot{m}_1 w_{\text{net}} = (270)(1162.26) = 313810.2 \text{ kW} = 313.81 \text{ MW}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).



9-1-33 [OUP] A power plant operates on a regenerative vapor power cycle with one closed feedwater heater. Steam enters the first turbine stage at 7 MPa,  $550^{\circ}$ C and expands to 700 kPa, where some of the steam is extracted and diverted to a closed feedwater heater. Condensate exiting the feedwater heater as saturated liquid at 700 kPa passes through a trap into the condenser. The feedwater exits the heater at 7 MPa with a temperature of  $175^{\circ}$ C. The condenser pressure is 8 kPa. If the power developed is 100 MW, determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the mass flow rate into the first stage turbine. (c) **What-if Scenario:** What would the thermal efficiency be if the extraction pressure were 600 kPa?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3530.85 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.9486 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given 
$$p_2, s_2 = s_1$$
):

$$h_2 = 2875.90 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):

$$h_{f@8kPa} = 173.88 \frac{kJ}{kg}; \ h_{fg@8kPa} = 2403.12 \frac{kJ}{kg}; \ s_{f@8kPa} = 0.5926 \frac{kJ}{kg \cdot K}; \ s_{fg@8kPa} = 7.6361 \frac{kJ}{kg \cdot K}$$

$$x_3 = \frac{s_3 - s_{f@8kPa}}{s_{fg@8kPa}} = \frac{6.9486 - 0.5926}{7.6361} = 0.8324$$

$$h_3 = h_{f@8kPa} + x_3 h_{fg@8kPa} = 173.88 + (0.8324)(2403.12) = 2174.24 \frac{kJ}{kg}$$

State-4 (given  $p_4 = p_3, x_4$ ):

$$h_4 = h_{f@8\text{kPa}} = 173.88 \frac{\text{kJ}}{\text{kg}}; \ v_4 = v_{f@8\text{kPa}} = 0.001008 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given  $p_5 = p_1, s_5 = s_4$ ):

Assuming that  $T_5 \cong T_4$ 

$$h_5 = h_4 + v_{f@T_4} (p_5 - p_4) = 173.88 + (0.001008)(7000 - 8) = 180.93 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_5, T_6$ ):

$$u_{f@175^{\circ}C} = 740.08 \frac{\text{kJ}}{\text{kg}}; \ v_{f@175^{\circ}C} = 0.001120 \frac{\text{m}^3}{\text{kg}}$$

$$h_6 = u_{f@175^{\circ}C} + p_6 v_{f@175^{\circ}C} = 740.08 + (7000)(0.001120) = 747.92 \frac{kJ}{kg}$$

State-7 (given  $p_7 = p_2, x_7$ ):

$$h_7 = h_{f@700\text{kPa}} = 697.22 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given  $p_8 = p_3, h_8 = h_7$ ):

$$h_8 = h_7 = 697.22 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_5 = \dot{m}_6 = \dot{m}_1$ ,  $\dot{m}_7 = \dot{m}_2$ , an energy balance for the closed feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_6 h_6 - \dot{m}_5 h_5 = \dot{m}_2 h_2 - \dot{m}_7 h_7;$$

$$\Rightarrow \dot{m}_1(h_6 - h_5) = \dot{m}_2(h_2 - h_7);$$

$$\Rightarrow r = \frac{h_6 - h_5}{h_2 - h_7} = \frac{747.92 - 180.93}{2875.90 - 697.22} = 0.2602$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2,3):

$$w_T = r(h_1 - h_2) + (1 - r)(h_1 - h_3);$$

$$w_T = (0.2602)(3530.85 - 2875.90) + (1 - 0.2602)(3530.85 - 2174.24) = 1174.04 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-4):

$$q_{\text{out}} = (1-r)h_3 + rh_8 - h_4 = h_3 + r(h_8 - h_3) - h_4;$$

$$q_{\text{out}} = 2174.24 + (0.2602)(697.22 - 2174.24) - 173.88 = 1616.04 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5): 
$$w_P = h_5 - h_4 = 180.93 - 173.88 = 7.04 \frac{\text{kJ}}{\text{kg}}$$

Device-D (6-1): 
$$q_{in} = h_1 - h_6 = 3530.85 - 747.92 = 2782.93 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_P = 1174.04 - 7.04 = 1167.00 \frac{\text{kJ}}{\text{kg}}$$
$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1167.00}{2782.93} = 41.93\%$$

The mass flow into the turbine

$$\dot{m}_1 = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{100000}{1167.00} = 85.69 \frac{\text{kg}}{\text{s}}$$

**Verification and What-if Scenario:** Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).



9-1-34 [OUX] A power plant operates on a regenerative vapor power cycle with one closed feedwater heater. Steam enters the first turbine stage at 12 MPa,  $520^{\circ}$ C and expands to 1200 kPa, where some of the steam is extracted and diverted to a closed feedwater heater. Condensate exiting the feedwater heater as saturated liquid at 120 kPa passes through a trap into the condenser. The feedwater exits the heater at 7 MPa with a temperature of  $170^{\circ}$ C. The condenser pressure is 12 kPa. If the mass flow rate into first stage of turbine is 300 kg/s and each turbine stage has an isentropic efficiency of 80%, determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the net power developed (c) **What-if Scenario:** What would the net power developed be if the condenser pressure were 9 kPa?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3400.95 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.5527 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given 
$$p_2, s_2 = s_1$$
):

$$h_2 = 2798.54 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, \eta_T$ ):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3400.95 - (0.80)(3400.95 - 2798.54) = 2919.02 \frac{kJ}{kg}$$

$$s_3 = 6.7972 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-4 (given  $p_4, s_4 = s_3$ ):

$$h_{f@12k\text{Pa}} = 206.84 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@12k\text{Pa}} = 2384.20 \frac{\text{kJ}}{\text{kg}}; \ s_{f@12k\text{Pa}} = 0.6960 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@12k\text{Pa}} = 7.3908 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_4 = \frac{s_4 - s_{f@12\text{kPa}}}{s_{fg@12\text{kPa}}} = \frac{6.7972 - 0.6960}{7.3908} = 0.8255$$

$$h_4 = h_{f @ 12 \text{kPa}} + x_4 h_{fg @ 12 \text{kPa}} = 206.84 + (0.8255)(2384.20) = 2175.00 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given  $p_5 = p_4, \eta_T$ ):

$$h_5 = h_3 - \eta_T (h_3 - h_4) = 2919.02 - (0.80)(2919.02 - 2175.00) = 2323.80 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_4, x_6$ ):

$$h_6 = h_{f@12\text{kPa}} = 206.84 \frac{\text{kJ}}{\text{kg}}; \ v_6 = v_{f@12\text{kPa}} = 0.001012 \frac{\text{m}^3}{\text{kg}}$$

State-7 (given  $p_7, s_7 = s_6$ ):

Assuming that  $T_7 \cong T_6$ 

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 206.84 + (0.001012)(7000 - 12) = 213.91 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given  $p_8 = p_7, T_8$ ):

$$u_{f@170^{\circ}\text{C}} = 718.33 \frac{\text{kJ}}{\text{kg}}; \ v_{f@170^{\circ}\text{C}} = 0.001114 \frac{\text{m}^3}{\text{kg}}$$

$$h_8 = u_{f@175^{\circ}C} + p_8 v_{f@175^{\circ}C} = 718.33 + (7000)(0.001114) = 726.13 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_1, s_9 = s_8$ ):

Assuming that  $T_9 \cong T_8$ 

$$h_9 = h_8 + v_{f@T_8} (p_9 - p_8) = 726.13 + (0.001114)(12000 - 7000) = 731.70 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given  $p_{10}, x_{10}$ ):

$$h_{10} = h_{f@120\text{kPa}} = 439.27 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given  $p_{11} = p_4, h_{11} = h_{10}$ ):

$$h_{11} = h_{10} = 439.27 \, \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_3 + \dot{m}_5$ ,  $\dot{m}_7 = \dot{m}_8 = \dot{m}_1$ ,  $\dot{m}_{10} = \dot{m}_3$ , an energy balance for the closed feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_8 h_8 - \dot{m}_7 h_7 = \dot{m}_3 h_3 - \dot{m}_{10} h_{10};$$

$$\Rightarrow \dot{m}_1(h_8-h_7)=\dot{m}_3(h_3-h_{10});$$

$$\Rightarrow r = \frac{h_8 - h_7}{h_3 - h_{10}} = \frac{726.13 - 213.91}{2919.02 - 439.27} = 0.2066$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-3,5):

$$w_T = r(h_1 - h_3) + (1 - r)(h_1 - h_5);$$
  

$$w_T = (0.2066)(3400.95 - 2919.02) + (1 - 0.2066)(3400.95 - 2323.80) = 954.18 \frac{kJ}{kg}$$

Device-B (5,11-6):

$$q_{\text{out}} = (1 - r)h_5 + rh_{11} - h_6 = h_5 + r(h_{11} - h_5) - h_6;$$
  
$$q_{\text{out}} = 2323.80 + (0.2066)(439.27 - 2323.80) - 206.84 = 1727.62 \frac{\text{kJ}}{\text{kg}}$$

Device-C (6-7): 
$$w_{P,I} = h_7 - h_6 = 213.91 - 206.84 = 7.07 \frac{\text{kJ}}{\text{kg}}$$

Device-D (8-9): 
$$w_{P,II} = h_9 - h_8 = 731.70 - 726.13 = 5.57 \frac{\text{kJ}}{\text{kg}}$$

Device-D (9-1): 
$$q_{in} = h_1 - h_9 = 3400.95 - 731.70 = 2669.25 \frac{kJ}{kg}$$

The thermal efficiency is

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 954.18 - 7.07 - 5.57 = 941.54 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{941.54}{2669.25} = 35.27\%$$

The net power developed

$$\dot{W}_{\text{net}} = \dot{m}_1 w_{\text{net}} = (300)(941.54) = 282462.00 \text{ kW} = 282.46 \text{ MW}$$

**Verification and What-if Scenario:** Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-1-35 [OUU] A power plant operates on an ideal reheat-regenerative Rankine cycle and has a net power output of 100 MW. Steam enters the high pressure turbine stage at 12 MPa, 550°C and leaves at 0.9 MPa. Some steam is extracted at 0.9 MPa to heat the feedwater in an open feedwater heater with the water leaving the FWH as saturated liquid. The rest of the steam is reheated to  $500^{\circ}$ C and is expanded in the low pressure turbine to the condenser at a pressure of 8 kPa. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the mass flow rate of steam through the boiler. (c) **What-if Scenario:** What would the thermal efficiency be if the steam entered the turbine at 15 MPa?

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3480.26 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.6521 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given 
$$p_2, s_2 = s_1$$
):

$$h_2 = 2787.57 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given 
$$p_3 = p_2, T_3 = T_2$$
):

$$h_3 = h_2 = 2787.57 \frac{\text{kJ}}{\text{kg}}; \ \dot{m}_3 = (1 - r) \dot{m}_1$$

State-4 (given 
$$p_4 = p_2, T_4 = T_2$$
):

$$h_4 = h_2 = 2787.57 \frac{\text{kJ}}{\text{kg}}; \ \dot{m}_4 = r\dot{m}_1$$

State-5 (given  $p_5 = p_3, T_5$ ):

$$h_5 = 3479.50 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 7.8117 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-6 (given  $p_6$ ,  $s_6 = s_5$ ):

$$h_{f@8k\text{Pa}} = 173.88 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@8k\text{Pa}} = 2403.12 \frac{\text{kJ}}{\text{kg}}; \ s_{f@8k\text{Pa}} = 0.5926 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@8k\text{Pa}} = 7.6361 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_6 = \frac{s_6 - s_{f@8kPa}}{s_{f@8kPa}} = \frac{7.8117 - 0.5926}{7.6361} = 0.9454$$

$$h_6 = h_{f@8kPa} + x_6 h_{fg@8kPa} = 173.88 + (0.9454)(2403.12) = 2445.79 \frac{kJ}{kg}$$

State-7 (given  $p_7 = p_6, x_7$ ):

$$h_7 = h_{f @ 8kPa} = 173.88 \frac{kJ}{kg}; \ v_7 = v_{f @ 8kPa} = 0.001008 \frac{m^3}{kg}$$

State-8 (given  $p_8 = p_2, s_8 = s_7$ ):

Assuming that  $T_8 \cong T_7$ 

$$h_8 = h_7 + v_{f@T_7} (p_8 - p_7) = 173.88 + (0.001008)(900 - 8) = 174.78 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_2, x_9$ ):

$$h_9 = h_{f@0.9\text{MPa}} = 742.83 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f@0.9\text{MPa}} = 0.001121 \frac{\text{m}^3}{\text{kg}}$$

State-10 (given  $p_{10} = p_1, s_{10} = s_9$ ):

Assuming that  $T_{10} \cong T_9$ 

$$h_{10} = h_9 + v_{f@T_9} (p_{10} - p_9) = 742.83 + (0.001121)(12000 - 900) = 755.27 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_3 + \dot{m}_4$ ,  $\dot{m}_9 = \dot{m}_1$ ,  $\dot{m}_8 = \dot{m}_3$ , an energy balance for the open feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_9 h_9 = \dot{m}_4 h_4 + \dot{m}_8 h_8;$$

$$\Rightarrow \dot{m}_1 h_9 = \dot{m}_4 h_4 + \left( \dot{m}_1 - \dot{m}_4 \right) h_8;$$

$$\Rightarrow h_9 = rh_4 + (1-r)h_8;$$

$$\Rightarrow r = \frac{h_9 - h_8}{h_4 - h_8} = \frac{742.83 - 174.78}{2787.57 - 174.78} = 0.2174$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_{T,I} = h_1 - h_2 = 3480.26 - 2787.57 = 692.69 \frac{\text{kJ}}{\text{kg}}$$

Device-B (3-5): 
$$q_{\text{in,rh}} = (1-r)(h_5 - h_3) = (1-0.2174)(3479.50 - 2787.57) = 541.50 \frac{\text{kJ}}{\text{kg}}$$

Device-C (5-6): 
$$w_{T,II} = (1-r)(h_5 - h_6) = (1-0.2174)(3479.50 - 2445.79) = 808.98 \frac{kJ}{kg}$$

Device-D (6-7): 
$$q_{\text{out}} = (1-r)(h_6 - h_7) = (1-0.2174)(2445.79 - 173.88) = 1778.00 \frac{\text{kJ}}{\text{kg}}$$

Device-E (7-8): 
$$w_{P,I} = (1-r)(h_8 - h_7) = (1-0.2174)(174.78 - 173.88) = 0.70 \frac{kJ}{kg}$$
  
Device-F (9-10):  $w_{P,II} = h_{10} - h_9 = 755.27 - 742.83 = 12.44 \frac{kJ}{kg}$   
Device-G (10-1):  $q_{in,b} = h_1 - h_{10} = 3480.26 - 755.27 = 2724.99 \frac{kJ}{kg}$ 

The thermal efficiency is

$$\begin{split} w_{\text{net}} &= w_{T,\text{I}} + w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}} = 692.69 + 808.98 - 0.70 - 12.44 = 1488.53 \frac{\text{kJ}}{\text{kg}} \\ q_{\text{in}} &= q_{\text{in,rh}} + q_{\text{in,b}} = 541.50 + 2724.99 = 3266.49 \frac{\text{kJ}}{\text{kg}} \\ \eta_{\text{th}} &= \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1488.53}{3266.49} = 45.57\% \end{split}$$

The mass flow rate in the boiler is equal to that entering the first turbine

$$\dot{m}_1 = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{100000}{1488.53} = 67.18 \frac{\text{kg}}{\text{s}}$$

**Verification and What-if Scenario:** Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

**9-1-36** [OUQ] Repeat problem 9-1-35 [OUU], but replace the open feedwater heater with closed feedwater heater. Assume that the feedwater leaves the heater at the condensation temperature of the extracted steam and that the extracted steam leaves the heater at state-10 as a saturated liquid before it is pumped to the line carrying the feedwater. Determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the mass flow rate of steam through the boiler.

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3480.26 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.6521 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 2787.57 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3 = p_2, T_3 = T_2$ ):

$$h_3 = h_2 = 2787.57 \frac{\text{kJ}}{\text{kg}}; \ \dot{m}_3 = (1 - r) \dot{m}_1$$

State-4 (given  $p_4 = p_2, T_4 = T_2$ ):

$$h_4 = h_2 = 2787.57 \frac{\text{kJ}}{\text{kg}}; \ \dot{m}_4 = r\dot{m}_1$$

State-5 (given  $p_5 = p_3, T_5$ ):

$$h_5 = 3479.50 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 7.8117 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-6 (given  $p_6, s_6 = s_5$ ):

$$h_{f@8kPa} = 173.88 \frac{kJ}{kg}; \ h_{fg@8kPa} = 2403.12 \frac{kJ}{kg}; \ s_{f@8kPa} = 0.5926 \frac{kJ}{kg \cdot K}; \ s_{fg@8kPa} = 7.6361 \frac{kJ}{kg \cdot K}$$

$$x_6 = \frac{s_6 - s_{f@8kPa}}{s_{fg@8kPa}} = \frac{7.8117 - 0.5926}{7.6361} = 0.9454$$

$$h_6 = h_{f@8kPa} + x_6 h_{fg@8kPa} = 173.88 + (0.9454)(2403.12) = 2445.79 \frac{kJ}{kg}$$

State-7 (given  $p_7 = p_6, x_7$ ):

$$h_7 = h_{f@8kPa} = 173.88 \frac{kJ}{kg}; \ v_7 = v_{f@8kPa} = 0.001008 \frac{m^3}{kg}$$

State-8 (given  $p_8 = p_1, s_8 = s_7$ ):

Assuming that  $T_8 \cong T_7$ 

$$h_8 = h_7 + v_{f@T_7}(p_8 - p_7) = 173.88 + (0.001008)(12000 - 8) = 185.97 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_1, T_9 = T_{\text{sat } @ 0.9 \text{MPa}}$ ):

$$u_{f@0.9\text{MPa}} = 741.82 \frac{\text{kJ}}{\text{kg}}; \ v_{f@0.9\text{MPa}} = 0.001121 \frac{\text{m}^3}{\text{kg}}$$

$$h_9 = u_{f@0.9\text{MPa}} + p_9 v_{f@0.9\text{MPa}} = 741.82 + (12000)(0.001121) = 755.27$$

State-10 (given  $p_{10} = p_2, x_{10}$ ):

$$h_{10} = h_{f@0.9\text{MPa}} = 742.83 \frac{\text{kJ}}{\text{kg}}; \ v_{10} = v_{f@0.9\text{MPa}} = 0.001124 \frac{\text{m}^3}{\text{kg}}$$

State-11 (given  $p_{11} = p_1, s_{11} = s_{10}$ ):

Assuming that  $T_{11} \cong T_{10}$ 

$$h_{11} = h_{10} + v_{f@T_{10}} (p_{11} - p_{10}) = 742.83 + (0.001121)(12000 - 900) = 755.27 \frac{kJ}{kg}$$

State-12 (given  $p_{12} = p_1$ ):

$$h_{12} = (1-r)h_9 + rh_{11} = (1-0.2178)(755.27) + (0.2178)(755.27) = 755.27 \frac{kJ}{kg}$$

Knowing that  $\dot{m}_1 = \dot{m}_3 + \dot{m}_4$ ,  $\dot{m}_8 = \dot{m}_9 = \dot{m}_3$ ,  $\dot{m}_{10} = \dot{m}_4$ , an energy balance for the closed feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_9 h_9 - \dot{m}_8 h_8 = \dot{m}_4 h_4 - \dot{m}_{10} h_{10};$$

$$\Rightarrow \dot{m}_3 (h_9 - h_8) = \dot{m}_4 (h_4 - h_{10});$$

$$\Rightarrow (\dot{m}_1 - \dot{m}_4)(h_9 - h_8) = \dot{m}_4(h_4 - h_{10});$$

$$\Rightarrow (1-r)(h_9-h_8)=r(h_4-h_{10});$$

$$\Rightarrow r = \frac{h_9 - h_8}{h_4 - h_{10} + h_9 - h_8} = \frac{755.27 - 185.97}{2787.57 - 742.83 + 755.27 - 185.97} = 0.2178$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): 
$$w_{T,I} = h_1 - h_2 = 3480.26 - 2787.57 = 692.69 \frac{\text{kJ}}{\text{kg}}$$
  
Device-B (3-5):  $q_{\text{in,rh}} = (1-r)(h_5 - h_3) = (1-0.2178)(3479.50 - 2787.57) = 541.22 \frac{\text{kJ}}{\text{kg}}$   
Device-C (5-6):  $w_{T,II} = (1-r)(h_5 - h_6) = (1-0.2178)(3479.50 - 2445.79) = 808.57 \frac{\text{kJ}}{\text{kg}}$   
Device-D (6-7):  $q_{\text{out}} = (1-r)(h_6 - h_7) = (1-0.2178)(2445.79 - 173.88) = 1777.10 \frac{\text{kJ}}{\text{kg}}$   
Device-E (7-8):  $w_{P,I} = (1-r)(h_8 - h_7) = (1-0.2174)(185.97 - 173.88) = 9.46 \frac{\text{kJ}}{\text{kg}}$   
Device-F (10-11):  $w_{P,II} = r(h_{10} - h_9) = (0.2178)(755.27 - 742.83) = 2.71 \frac{\text{kJ}}{\text{kg}}$   
Device-G (12-1):  $q_{\text{in,b}} = h_1 - h_{10} = 3480.26 - 755.27 = 2724.99 \frac{\text{kJ}}{\text{kg}}$ 

The thermal efficiency is

$$w_{\text{net}} = w_{T,\text{I}} + w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}} = 692.69 + 808.57 - 9.46 - 2.71 = 1489.09 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{in}} = q_{\text{in,rh}} + q_{\text{in,b}} = 541.22 + 2724.99 = 3266.21 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{th}}} = \frac{1489.09}{3266.21} = 45.59\%$$

The mass flow rate in the boiler is equal to that entering the first turbine

$$\dot{m}_1 = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{100000}{1489.09} = 67.16 \frac{\text{kg}}{\text{s}}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-1-37 [OUC] A steam power plant operates on a reheat-regenerative Rankine cycle with a closed feedwater heater. Steam enters the turbine at 12 MPa,  $500^{\circ}$ C at a rate of 25 kg/s and is condensed in the condenser at a pressure of 20 kPa. Steam is reheated at 5 MPa to  $500^{\circ}$ C. Steam at a rate of 5 kg/s is extracted from the high pressure turbine at 1.2 MPa, and is completely condensed in the closed feedwater heater, and pumped to 12 MPa before it mixes with the feedwater at the same pressure. Assuming an isentropic efficiency of 88% for both the turbine and the pump, determine (a) the temperature at the inlet of the closed feedwater heater, (b) the thermal efficiency ( $\eta_{\text{th,Rankine}}$ ) of the cycle and (c) the net power output.

## **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3348.07 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.4863 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given 
$$p_2, s_2 = s_1$$
):

$$h_2 = 3092.42 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given 
$$p_3 = p_2, \eta_T$$
):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3348.07 - (0.88)(3348.07 - 3092.42) = 3123.10 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given 
$$p_A = p_2, T_A$$
):

$$h_4 = 3433.74 \frac{\text{kJ}}{\text{kg}}; \ s_4 = 6.9758 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-5 (given 
$$p_5, s_5 = s_4$$
):

$$h_5 = 3015.22 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6 = p_5, \eta_T$ ):

$$h_6 = h_4 - \eta_T (h_4 - h_5) = 3433.74 - (0.88)(3433.74 - 3015.22) = 3065.44 \frac{\text{kJ}}{\text{kg}}$$

$$s_6 = 7.0645 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ T_6 = 309.12 \text{°C}$$

State-7 (given 
$$p_7, s_7 = s_6$$
):

$$h_{f@20\text{kPa}} = 251.39 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@20\text{kPa}} = 2358.32 \frac{\text{kJ}}{\text{kg}}; \ s_{f@20\text{kPa}} = 0.8320 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@20\text{kPa}} = 7.0766 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$s_7 - s_{f@20\text{kPa}} = 7.0645 - 0.8320$$

$$x_7 = \frac{s_7 - s_{f@20\text{kPa}}}{s_{fg@20\text{kPa}}} = \frac{7.0645 - 0.8320}{7.0766} = 0.8807$$

$$h_7 = h_{f @ 20 \text{kPa}} + x_7 h_{fg @ 20 \text{kPa}} = 251.39 + (0.8807)(2358.32) = 2328.36 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given  $p_8 = p_7, \eta_T$ ):

$$h_8 = h_6 - \eta_T (h_6 - h_7) = 3065.44 - (0.88)(3065.44 - 2328.36) = 2416.81 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_7, x_9$ ):

$$h_9 = h_{f @ 20 \text{kPa}} = 251.39 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f @ 20 \text{kPa}} = 0.001017 \frac{\text{m}^3}{\text{kg}}$$

State-10 (given  $p_{10} = p_1, s_{10} = s_9$ ):

Assuming that  $T_{10} \cong T_9$ 

$$h_{10} = h_9 + v_{f@T_9} (p_{10} - p_9) = 251.39 + (0.001017)(12000 - 20) = 263.57 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given  $p_{11} = p_{10}, \eta_P$ ):

$$h_{11} = h_9 + \frac{h_{10} - h_9}{\eta_P} = 251.39 + \frac{263.57 - 251.39}{0.88} = 265.23 \frac{\text{kJ}}{\text{kg}}$$

State-12 (given  $p_{12} = p_1$ ):

$$h_{12} = 831.93 \frac{\text{kJ}}{\text{kg}}$$

State-13 (given  $p_{13} = p_5, x_{13} = 0$ ):

$$h_{13} = h_{f@1.2\text{MPa}} = 798.64 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f@1.2\text{MPa}} = 0.001139 \frac{\text{m}^3}{\text{kg}}$$

State-14 (given  $p_{14} = p_1, s_{14} = s_{13}$ ):

Assuming that  $T_{14} \cong T_{13}$ 

$$h_{14} = h_{13} + v_{f@T_{13}} (p_{14} - p_{13}) = 798.64 + (0.001139)(12000 - 1200) = 810.94 \frac{\text{kJ}}{\text{kg}}$$

State-15 (given  $p_{15} = p_{14}, \eta_P$ ):

$$h_{15} = h_{13} + \frac{h_{14} - h_{13}}{\eta_P} = 798.64 + \frac{810.94 - 798.64}{0.88} = 812.62 \frac{\text{kJ}}{\text{kg}}$$

State-16 (given  $p_{16} = p_1$ ):

$$h_{16} = (1-r)h_{12} + rh_{15} = (1-0.2)(831.93) + (0.2)(812.62) = 828.07 \frac{\text{kJ}}{\text{kg}}$$

Since the initial mass flow rate and the bleed mass flow rate are given, the ratio between them can be found as

$$r = \frac{\dot{m}_6}{\dot{m}_1} = \frac{5}{25} = 0.2$$

An energy analysis on the closed feedwater heater provides the previously given enthalpy for state-12

$$\dot{m}_{12}(h_{12}-h_{11})=\dot{m}_{6}(h_{6}-h_{13});$$

$$\Rightarrow h_{12} = h_{11} + \left(\frac{\dot{m}_6}{\dot{m}_{12}}\right) (h_6 - h_{13}) = 265.23 + \left(\frac{5}{20}\right) (3065.44 - 798.64) = 831.93 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): 
$$w_{T,I} = h_1 - h_3 = 3348.07 - 3123.10 = 224.97 \frac{kJ}{kg}$$

Device-B (3-4): 
$$q_{\text{in,rh}} = h_4 - h_3 = 3433.74 - 3123.10 = 310.64 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-6,8):

$$W_{T,\Pi} = r(h_4 - h_6) + (1 - r)(h_4 - h_8);$$

$$w_{T,II} = (0.2)(3433.74 - 3065.44) + (1 - 0.2)(3433.74 - 2416.81) = 887.20 \frac{\text{kJ}}{\text{kg}}$$

Device-D (8-9): 
$$q_{\text{out}} = (1-r)(h_8 - h_9) = (1-0.2)(2416.81 - 251.39) = 1732.34 \frac{\text{kJ}}{\text{kg}}$$

Device-E (9-11): 
$$w_{P,I} = (1-r)(h_{11} - h_9) = (1-0.2)(265.23 - 251.39) = 11.07 \frac{\text{kJ}}{\text{kg}}$$

Device-F (13-15): 
$$w_{P,II} = r(h_{15} - h_{13}) = (0.2)(812.62 - 798.64) = 2.80 \frac{kJ}{kg}$$

Device-G (16-1): 
$$q_{\text{in,b}} = h_1 - h_{16} = 3348.07 - 828.07 = 2520.00 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\begin{split} w_{\text{net}} &= w_{T,\text{I}} + w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}} = 224.97 + 887.20 - 11.07 - 2.80 = 1098.30 \frac{\text{kJ}}{\text{kg}} \\ q_{\text{in}} &= q_{\text{in,th}} + q_{\text{in,b}} = 310.64 + 2520.00 = 2830.64 \frac{\text{kJ}}{\text{kg}} \\ \eta_{\text{th}} &= \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1098.30}{2830.64} = 38.80\% \end{split}$$

The net power output

$$\dot{W}_{\text{net}} = \dot{m}_1 w_{\text{net}} = (25)(1098.30) = 27457.50 \text{ kW} = 27.46 \text{ MW}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).



9-1-38 [OUV] A steam power plant operates on an ideal reheat-regenerative Rankine with one reheat and two open feedwater heaters. Steam enters the high pressure turbine at 10 MPa,  $600^{\circ}$ C and leaves the low pressure turbine at 7 kPa. Steam is extracted from the turbine at 2 MPa and 275 kPa, and it is reheated to  $540^{\circ}$ C at a pressure of 1 MPa. Water leaves both feedwater heaters as saturated liquid. Heat is transferred to the steam in the boiler at a rate of 6 MW. Determine (a) the mass flow rate of steam through the boiler, (b) the net power output and (c) the thermal efficiency ( $\eta_{th}$ ) of the cycle.

# **SOLUTION**

State-1 (given  $p_1, T_1$ ):

$$h_1 = 3625.32 \frac{\text{kJ}}{\text{kg}}; \ s_1 = 6.9028 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given  $p_2, s_2 = s_1$ ):

$$h_2 = 3105.04 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given  $p_3, s_3 = s_1$ ):

$$h_3 = 2931.73 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given  $p_4 = p_3, T_4$ ):

$$h_4 = 3566.18 \frac{\text{kJ}}{\text{kg}}; \ s_4 = 7.8688 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-5 (given  $p_5, s_5 = s_4$ ):

$$h_5 = 3148.11 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given  $p_6, s_6 = s_4$ ):

$$h_{f@7\text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@7\text{kPa}} = 2409.14 \frac{\text{kJ}}{\text{kg}}; \ s_{f@7\text{kPa}} = 0.5589 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@7\text{kPa}} = 7.7175 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_6 = \frac{s_6 - s_{f@7\text{kPa}}}{s_{fa@7\text{kPa}}} = \frac{7.8688 - 0.5589}{7.7175} = 0.9472$$

$$h_6 = h_{f @ 7 \text{kPa}} + x_6 h_{fg @ 7 \text{kPa}} = 163.33 + (0.9472)(2409.14) = 2445.27 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given  $p_7 = p_6, x_7$ ):

$$h_7 = h_{f@7\text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ v_7 = v_{f@7\text{kPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-8 (given  $p_8 = p_5, s_8 = s_7$ ):

Assuming that  $T_8 \cong T_7$ 

$$h_8 = h_7 + v_{f@T_7} (p_8 - p_7) = 163.33 + (0.001007)(275 - 7) = 163.60 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_5, x_9$ ):

$$h_9 = h_{f@275\text{kPa}} = 548.85 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f@275\text{kPa}} = 0.001071 \frac{\text{m}^3}{\text{kg}}$$

State-10 (given  $p_{10} = p_2, s_{10} = s_9$ ):

Assuming that  $T_{10} \cong T_9$ 

$$h_{10} = h_9 + v_{f@T_9} (p_{10} - p_9) = 548.85 + (0.001071)(2000 - 275) = 550.70 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given  $p_{11} = p_2, x_{11}$ ):

$$h_{11} = h_{f@2\text{MPa}} = 908.82 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f@2\text{MPa}} = 0.001177 \frac{\text{m}^3}{\text{kg}}$$

State-12 (given  $p_{12} = p_1, s_{12} = s_{11}$ ):

Assuming that  $T_{10} \cong T_9$ 

$$h_{12} = h_{11} + v_{f@T_{11}} (p_{12} - p_{11}) = 908.82 + (0.001177)(10000 - 2000) = 918.24 \frac{\text{kJ}}{\text{kg}}$$

An energy balance on the second open feedwater heater provides the mass fraction bled from the first turbine stage, knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_{11} = \dot{m}_1$ ,  $\dot{m}_{10} = \dot{m}_3$ ,

$$\dot{m}_1 h_{11} = \dot{m}_2 h_2 + \dot{m}_3 h_{10};$$

$$\Rightarrow h_{11} = r_1 h_2 + (1 - r_1) h_{10};$$

$$\Rightarrow r_1 = \frac{h_{11} - h_{10}}{h_2 - h_{10}} = \frac{908.82 - 550.70}{3105.04 - 550.70} = 0.1402$$

An energy balance on the first open feedwater heater provides the mass fraction bled from the second turbine stage, knowing that  $\dot{m}_3 = \dot{m}_5 + \dot{m}_6$ ,  $\dot{m}_3 = 1 - r_1$ ,  $\dot{m}_9 = \dot{m}_3$ ,  $\dot{m}_8 = \dot{m}_6$ ,

$$\dot{m}_{3}h_{9} = \dot{m}_{5}h_{5} + \dot{m}_{6}h_{8};$$

$$\Rightarrow h_{9} = \frac{\dot{m}_{5}}{\dot{m}_{3}}h_{5} + \left(1 - \frac{\dot{m}_{5}}{\dot{m}_{3}}\right)h_{8};$$

$$\Rightarrow h_{9} = \frac{\dot{m}_{5}/\dot{m}_{1}}{\dot{m}_{3}/\dot{m}_{1}}h_{5} + \left(1 - \frac{\dot{m}_{5}/\dot{m}_{1}}{\dot{m}_{3}/\dot{m}_{1}}\right)h_{8};$$

$$\Rightarrow h_{9} = \frac{r_{2}}{1 - r_{1}}h_{5} + \left(1 - \frac{r_{2}}{1 - r_{1}}\right)h_{8};$$

$$\Rightarrow r_{2} = (1 - r_{1})\frac{h_{9} - h_{8}}{h_{5} - h_{8}} = (1 - 0.1402)\left(\frac{548.85 - 163.60}{3148.11 - 163.60}\right) = 0.1110$$

A steady-state energy analysis is carried out for each device as follows. Device-A (1-2,3):

$$w_{T,I} = r_1 (h_1 - h_2) + (1 - r_1) (h_1 - h_3);$$

$$w_{T,I} = (0.1402) (3625.32 - 3105.04) + (1 - 0.1402) (3625.32 - 2931.73) = 669.29 \frac{\text{kJ}}{\text{kg}}$$
Device-B (3-4):  $q_{\text{in,rh}} = (1 - r_1) (h_4 - h_3) = (1 - 0.1402) (3566.18 - 2931.73) = 545.50 \frac{\text{kJ}}{\text{kg}}$ 

Device-C (4-5,6):

$$\begin{split} w_{T,\text{II}} &= r_2 \left( h_4 - h_5 \right) + \left( 1 - r_1 - r_2 \right) \left( h_4 - h_6 \right); \\ w_{T,\text{II}} &= \left( 0.1110 \right) \left( 3566.18 - 3148.11 \right) + \left( 1 - 0.1402 - 0.1110 \right) \left( 3566.18 - 2445.27 \right) = 885.74 \, \frac{\text{kJ}}{\text{kg}} \end{split}$$

Device-D (6-7):

$$q_{\text{out}} = (1 - r_1 - r_2)(h_6 - h_7);$$
  
 $q_{\text{out}} = (1 - 0.1402 - 0.1110)(2445.27 - 163.33) = 1708.72 \frac{\text{kJ}}{\text{kg}}$ 

Device-E (7-8): 
$$w_{P,I} = (1 - r_1 - r_2)(h_8 - h_7) = (1 - 0.1402 - 0.1110)(163.60 - 163.33) = 0.20 \frac{kJ}{kg}$$

Device-F (9-10): 
$$w_{P,II} = (1 - r_1)(h_{10} - h_9) = (1 - 0.1402)(550.70 - 548.85) = 1.59 \frac{kJ}{kg}$$

Device-G (11-12): 
$$w_{P,III} = h_{12} - h_{11} = 918.24 - 908.82 = 9.42 \frac{kJ}{kg}$$

Device-H (12-1): 
$$q_{\text{in,b}} = h_1 - h_{12} = 3625.32 - 918.24 = 2707.08 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\begin{split} w_{\text{net}} &= w_{T,\text{I}} + w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}} - w_{P,\text{III}} = 669.29 + 885.74 - 0.20 - 1.59 - 9.42 = 1543.82 \frac{\text{kJ}}{\text{kg}} \\ q_{\text{in}} &= q_{\text{in,rh}} + q_{\text{in,b}} = 545.50 + 2707.08 = 3252.58 \frac{\text{kJ}}{\text{kg}} \\ \eta_{\text{th}} &= \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1543.82}{3262.58} = 47.46\% \end{split}$$

The mass flow rate of steam through the boiler

$$\dot{m}_1 = \frac{\dot{Q}_{\rm in}}{q_{\rm in}} = \frac{6000}{3252.58} = 1.84 \frac{\rm kg}{\rm s}$$

The net power output

$$\dot{W}_{\text{net}} = \dot{m}_1 w_{\text{net}} = (1.84)(1543.82) = 2840.63 \text{ kW} = 2.84 \text{ MW}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-1-39 [OUT] Consider a reheat-regenerative vapor power cycle with two feedwater heaters, a closed feedwater heater and an open feedwater heater. Steam enters the first turbine at 10 MPa,  $500^{\circ}$ C and expands to 0.8 MPa. The steam is reheated to  $440^{\circ}$ C before entering the second turbine, where it expands to the condenser at a pressure of 0.007 MPa. Steam is extracted from the first turbine at 2 MPa and fed to the closed feedwater heater. Feedwater leaves the closed heater at  $205^{\circ}$ C, 10 MPa and condensate exits as saturated liquid at 2 MPa. The condensate is trapped into the open feedwater heater. Steam extracted from second turbine at 0.3 MPa is also fed into the open feedwater heater, which operates at 0.3 MPa. The steam exiting the open feedwater heater is saturated liquid at 0.3 MPa. The net power output of the cycle is 100 MW. There is no stray heat transfer from any component to its surroundings. If the working fluid experiences no irreversibilities as it passes through the turbines, pumps, steam generator, reheater and condenser, determine (a) the thermal efficiency ( $\eta_{th}$ ) of the cycle and (b) the mass flow rate of steam entering the first turbine.

## **SOLUTION**

State-6 (given  $p_6, s_6 = s_4$ ):

State-1 (given 
$$p_1, T_1$$
):  
 $h_1 = 3373.60 \frac{\text{kJ}}{\text{kg}}$ ;  $s_1 = 6.5965 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$   
State-2 (given  $p_2, s_2 = s_1$ ):  
 $h_2 = 2930.52 \frac{\text{kJ}}{\text{kg}}$   
State-3 (given  $p_3, s_3 = s_1$ ):  
 $h_3 = 2739.65 \frac{\text{kJ}}{\text{kg}}$   
State-4 (given  $p_4 = p_3, T_4$ ):  
 $h_4 = 3352.46 \frac{\text{kJ}}{\text{kg}}$ ;  $s_4 = 7.6898 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$   
State-5 (given  $p_5, s_5 = s_4$ ):  
 $h_5 = 3062.46 \frac{\text{kJ}}{\text{kg}}$ 

$$h_{f@0.007\text{MPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ h_{fg@0.007\text{MPa}} = 2409.14 \frac{\text{kJ}}{\text{kg}}; \ s_{f@0.007\text{MPa}} = 0.5589 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ s_{fg@0.007\text{MPa}} = 7.7175 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_6 = \frac{s_6 - s_{f@0.007\text{MPa}}}{s_{fg@0.007\text{MPa}}} = \frac{7.6898 - 0.5589}{7.7175} = 0.9240$$

$$h_6 = h_{f@0.007\text{MPa}} + x_6 h_{fg@0.007\text{MPa}} = 163.33 + (0.9240)(2409.14) = 2389.38 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given  $p_7 = p_6, x_7$ ):

$$h_7 = h_{f@0.007\text{MPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \ v_7 = v_{f@0.007\text{MPa}} = 0.001007 \frac{\text{m}^3}{\text{kg}}$$

State-8 (given  $p_8 = p_5, s_8 = s_7$ ):

Assuming that  $T_8 \cong T_7$ 

$$h_8 = h_7 + v_{f@T_7} (p_8 - p_7) = 163.33 + (0.001007)(300 - 7) = 163.63 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given  $p_9 = p_5, x_9$ ):

$$h_9 = h_{f@0.3\text{MPa}} = 561.47 \frac{\text{kJ}}{\text{kg}}; \ v_9 = v_{f@0.3\text{MPa}} = 0.001073 \frac{\text{m}^3}{\text{kg}}$$

State-10 (given  $p_{10} = p_1, s_{10} = s_9$ ):

Assuming that  $T_{10} \cong T_9$ 

$$h_{10} = h_9 + v_{f@T_9} (p_{10} - p_9) = 561.47 + (0.001073)(10000 - 300) = 571.88 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given  $p_{11} = p_1, T_{11}$ ):

$$u_{f@205^{\circ}\text{C}} = 873.10 \frac{\text{kJ}}{\text{kg}}; \ v_{f@205^{\circ}\text{C}} = 0.001165 \frac{\text{m}^3}{\text{kg}}$$

$$h_{11} = u_{f @ 205^{\circ}C} + p_{11}v_{f @ 205^{\circ}C} = 873.10 + (10000)(0.001165) = 884.75 \frac{kJ}{kg}$$

State-12 (given  $p_{12} = p_2, x_{12}$ ):

$$h_{12} = h_{f @ 2MPa} = 908.82 \frac{kJ}{kg}$$

State-13 (given  $p_{13} = p_5, h_{13} = h_{12}$ ):

$$h_{13} = h_{12} = 908.82 \frac{\text{kJ}}{\text{kg}}$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_{10} = \dot{m}_{11} = \dot{m}_1$ ,  $\dot{m}_{12} = \dot{m}_2$ , an energy balance for the closed feedwater heater provides the mass fraction bled from the first turbine stage

$$\dot{m}_{11}h_{11} - \dot{m}_{10}h_{10} = \dot{m}_{2}h_{2} - \dot{m}_{12}h_{12};$$

$$\Rightarrow \dot{m}_{1}(h_{11} - h_{10}) = \dot{m}_{2}(h_{2} - h_{12});$$

$$\Rightarrow h_{11} - h_{10} = r_{1}(h_{2} - h_{12});$$

$$\Rightarrow r_{1} = \frac{h_{11} - h_{10}}{h_{2} - h_{12}} = \frac{884.75 - 571.88}{2930.52 - 908.82} = 0.1548$$

Knowing that  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ ,  $\dot{m}_3 = \dot{m}_5 + \dot{m}_6$ ,  $\dot{m}_9 = \dot{m}_1$ ,  $\dot{m}_{13} = \dot{m}_2$ ,  $\dot{m}_8 = \dot{m}_6$ , an energy balance for the open feedwater heater provides the mass fraction bled from the second turbine stage

$$\dot{m}_9 h_9 = \dot{m}_5 h_5 + \dot{m}_8 h_8 - \dot{m}_{13} h_{13};$$

$$\Rightarrow \dot{m}_1 h_9 = \dot{m}_5 h_5 + \dot{m}_6 h_8 + \dot{m}_2 h_{13};$$

$$\Rightarrow \dot{m}_1 h_9 = \dot{m}_5 h_5 + (\dot{m}_3 - \dot{m}_5) h_8 + \dot{m}_2 h_{13};$$

$$\Rightarrow \dot{m}_1 h_9 = \dot{m}_5 h_5 + \left[ \left( \dot{m}_1 - \dot{m}_2 \right) - \dot{m}_5 \right] h_8 + \dot{m}_2 h_{13};$$

$$\Rightarrow \dot{m}_1 h_9 = \dot{m}_5 h_5 + \left[ \left( \dot{m}_1 - \dot{m}_2 \right) - \dot{m}_5 \right] h_8 + \dot{m}_2 h_{13};$$

$$\Rightarrow h_9 = r_2 h_5 + \lceil (1 - r_1) - r_2 \rceil h_8 + r_1 h_{13};$$

$$\Rightarrow r_2 = \frac{h_9 - h_8 - r_1 (h_{13} - h_8)}{h_5 - h_8} = \frac{561,47 - 163.63 - (0.1548)(908.82 - 163.63)}{3062.46 - 163.63} = 0.0974$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2,3):

$$W_{T,1} = r_1(h_1 - h_2) + (1 - r_1)(h_1 - h_3);$$

$$w_{T,I} = (0.1548)(3373.60 - 2930.52) + (1 - 0.1548)(3373.60 - 2739.65) = 604.40 \frac{kJ}{kg}$$

Device-B (3-4): 
$$q_{\text{in,rh}} = (1 - r_1)(h_4 - h_3) = (1 - 0.1548)(3352.46 - 2739.65) = 517.95 \frac{\text{kJ}}{\text{kg}}$$

Device-C (4-5.6):

$$W_{T,II} = r_2 (h_4 - h_5) + (1 - r_1 - r_2) (h_4 - h_6);$$

$$w_{T,II} = (0.0974)(3352.46 - 3062.46) + (1 - 0.1548 - 0.0974)(3352.46 - 2389.38) = 748.44 \frac{kJ}{kg}$$

Device-D (6-7):

$$q_{\text{out}} = (1 - r_1 - r_2)(h_6 - h_7);$$

$$q_{\text{out}} = (1 - 0.1548 - 0.0974)(2389.38 - 163.33) = 1664.64 \frac{\text{kJ}}{\text{kg}}$$
Device-E (7-8):  $w_{P,I} = (1 - r_1 - r_2)(h_8 - h_7) = (1 - 0.1548 - 0.0974)(163.63 - 163.33) = 0.22 \frac{\text{kJ}}{\text{kg}}$ 
Device-F (9-10):  $w_{P,II} = h_{10} - h_9 = 571.88 - 561.47 = 10.41 \frac{\text{kJ}}{\text{kg}}$ 
Device-G (11-1):  $q_{\text{in,b}} = h_1 - h_{11} = 3373.60 - 884.75 = 2488.85 \frac{\text{kJ}}{\text{kg}}$ 

The thermal efficiency is

$$\begin{split} w_{\text{net}} &= w_{T,\text{I}} + w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}} - w_{P,\text{III}} = 604.40 + 748.44 - 0.22 - 10.41 = 1342.21 \frac{\text{kJ}}{\text{kg}} \\ q_{\text{in}} &= q_{\text{in,rh}} + q_{\text{in,b}} = 517.95 + 2488.85 = 3006.80 \frac{\text{kJ}}{\text{kg}} \\ \eta_{\text{th}} &= \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1342.21}{3006.80} = 44.64\% \end{split}$$

The mass flow rate of steam entering the first turbine

$$\dot{m}_1 = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{100000}{1342.21} = 74.50 \frac{\text{kg}}{\text{s}}$$

**Verification:** Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).