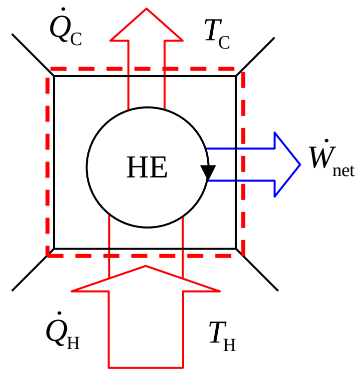
2-5-1 [LE] A steam power plant produces 500 MW of electricity with an overall thermal efficiency of 35%. Determine (a) the rate at which heat is supplied to the boiler, and (b) the waste heat that is rejected by the plant. (c) If the heating value of coal (heat that is realeased when 1 kg of coal is burned) is 30 MJ/kg, determine the rate of consumption of coal in tons(US)/day. Assume that 100% of heat released goes to the cycle. (d) ***What-if Scenario*** How would the fuel consumption rate change if the thermal efficiency were to increase to 40%?

SOLUTION



(a) The efficiency of a heat engine is defined as:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H};$$

$$\Rightarrow \dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \qquad \Rightarrow \dot{Q}_H = \frac{500}{0.35};$$

$$\Rightarrow \dot{Q}_H = 1429 \text{ MW}$$

(b) The energy balance equation is given as:

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{net}}^{0} + \dot{Q} - \dot{W}_{ext} = \cancel{\dot{J}_{net}}^{0} + \left(\dot{Q}_H - \dot{Q}_C\right) - \left(\dot{W}_{net}\right);$$

$$\Rightarrow \dot{Q}_C = \dot{Q}_H - \dot{W}_{net}; \qquad \Rightarrow \dot{Q}_C = 1429 - 500; \qquad \Rightarrow \dot{Q}_C = 929 \text{ MW}$$

(c) The amount of coal (U.S. tons per day) is calculated as:

$$\dot{m}_F = \left(1429\frac{\text{MJ}}{\text{s}}\right)\left(\frac{1}{30}\frac{\text{kg}}{\text{MJ}}\right)\left(\frac{1}{907.2}\frac{\text{ton}}{\text{kg}}\right)\left(86400\frac{\text{s}}{\text{day}}\right); \qquad \Rightarrow \dot{m}_F = 4536.5\ \frac{\text{tons}}{\text{day}}$$

(d) With an increase in efficiency, a reduction in heat input is expected. T_H

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \qquad \Rightarrow \dot{Q}_H = \frac{500}{0.4}; \qquad \Rightarrow \dot{Q}_H = 1250 \text{ MW};$$

Since the fuel consumption is directly proportional to the heat input, the new rate of fuel consumption can be expressed as:

$$\dot{m}_F = (4536.5)\frac{0.35}{0.4}; \qquad \Rightarrow \dot{m}_F = 3969.4\ \frac{\text{tons}}{\text{day}}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-2 [LI] A utility company charges its residential customers 12 cents/kW.h for electricity and $1.20 per Therm for natural gas. Fed up with the high cost of electricity, a customer decides to generate his own electricity by using a natural gas fired engine that has a thermal efficiency of 35%. Determine the fuel cost per kWh of electricity produced by the customer. Do you think electricity and natural gas are fairly priced by your utility company?

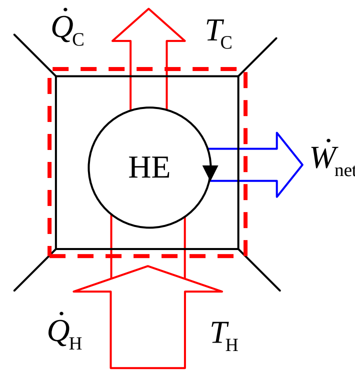
SOLUTION

The heat required to generate 1 kWh = 3.6 MJ of electricity can be calculated from the thermal efficiency.

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \quad [\text{kW}]$$

$$\Rightarrow Q_H = \frac{W_{net}}{\eta_{th}}; \qquad \Rightarrow Q_H = \frac{3.6}{0.35};$$

$$\Rightarrow Q_H = 10.28 \text{ MJ};$$



Since 1 Therm = 105.5 MJ, the cost of heat to generate 1 kWh of electricity is:

$$\Rightarrow \frac{10.28}{105.5}(120) = 11.7 \text{ cents}$$

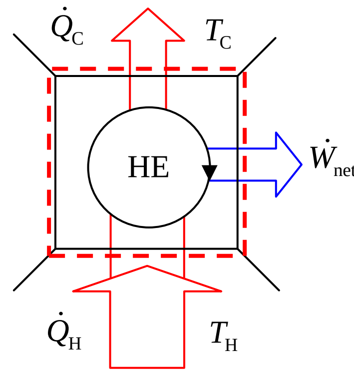
With only a 2.5% difference between the price of electricity and the cost to generate it, the electricity price seems to be fair.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-3 [LL] A sport utility vehicle with a thermal efficiency (η_{th}) of 20% produces 250 hp of engine output while traveling at a velocity of 80 mph. (a) Determine the rate of fuel consumption in kg/s if the heating value of the fuel is 43 MJ/kg. (b) If the density of the fuel is 800 kg/m^3, determine the fuel mileage of the vehicle in the unit of miles/gallon.

SOLUTION



Using the Engineering Converter TESTcalc we obtain:
250 hp = 186.4 kW;
80 mph = 35.76 m/s;
1 mile = 1609.3 m;
1 gallon = 0.003785 m^3;

(a) $\eta_{th} = \dfrac{\dot{W}_{net}}{\dot{Q}_H};$

$$\Rightarrow \dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \qquad \Rightarrow \dot{Q}_H = \frac{186.4}{0.2};$$

$$\Rightarrow \dot{Q}_H = 932 \text{ kW};$$

The fuel consumption rate is calculated as:

$$\left(932 \frac{\text{kJ}}{\text{s}}\right)\left(\frac{1}{43000} \frac{\text{kg}}{\text{kJ}}\right) = 0.021677 \ \frac{\text{kg}}{\text{s}}$$

(b) 1 gallon of fuel is consumed in:

$$t = \frac{m_F}{\dot{m}_F}; \qquad \Rightarrow t = \frac{\rho_F \mathcal{V}_F}{\dot{m}_F}; \qquad \Rightarrow t = \frac{(800)(0.003785)}{0.021677}; \qquad \Rightarrow t = 139.7 \text{ s};$$

The distance traveled by the vehicle in that time is:

$$x = Vt; \qquad \Rightarrow x = (35.76)(139.7); \qquad \Rightarrow x = 4995.2 \text{ m}; \qquad \Rightarrow x = 3.1 \text{ mile};$$

Therefore, the fuel mileage $= 3.1 \ \dfrac{\text{miles}}{\text{gallon}}$

2-5-4 [LG] A truck engine consumes diesel at a rate of 30 L/h and delivers 65 kW of power to the wheels. If the fuel has a heating value of 43.5 MJ/kg and a density of 800 kg/m^3, determine (a) the thermal efficiency of the engine and (b) the waste heat rejected by the engine. (c) How does the engine discard the waste heat?

SOLUTION

(a) The heat input can be calculated as:

$$\dot{Q}_H = \dot{m}_F; \quad (\text{Heating Value})$$

$$\Rightarrow \dot{Q}_H = \rho_F \dot{V}_F; \quad \Rightarrow \dot{Q}_H = (800)\left(\frac{30}{(1000)(3600)}\right)(43500); \quad \Rightarrow \dot{Q}_H = 290 \text{ kW};$$

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}; \quad \Rightarrow \eta_{th} = \frac{65}{290}; \quad \Rightarrow \eta_{th} = 22.4\%$$

(b) $\dot{Q}_C = \dot{Q}_H - \dot{W}_{net}; \quad \Rightarrow \dot{Q}_C = 290 - 65; \quad \Rightarrow \dot{Q}_C = 225 \text{ kW}$

(c) The waste heat is picked up by circulating water around the engine. The warm water is cooled in the radiator where the heat is ultimately rejected into the atmosphere. In cold climates, some of this waste heat can be recycled by reusing it to regulate the cab temperature.

2-5-5 [LZ] Determine the rate of coal consumption by a thermal power plant with a power output of 350 MW in tons/hr. The thermal efficiency (η_{th}) of the plant is 35% and the heating value of the coal is 30 MJ/kg.

SOLUTION

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}; \qquad \Rightarrow \dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}};$$

$$\Rightarrow \dot{Q}_H = \frac{350}{0.35}; \qquad \Rightarrow \dot{Q}_H = 1000 \text{ MW};$$

The rate of coal consumption can now be calculated as:

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating Value})}; \qquad \Rightarrow \dot{m}_F = \frac{1000}{30}\ \frac{\text{kg}}{\text{s}}; \qquad \Rightarrow \dot{m}_F = \frac{(1000)(3600)}{(30)(907.2)}\ \frac{\text{tons}}{\text{hr}};$$

$$\Rightarrow \dot{m}_F = 132.275\ \frac{\text{tons}}{\text{hr}}$$

2-5-6 [LK] In 2003 the United States generated 3.88 trillion kWh of electricity, 51% of which came from coal-fired power plants. (a) Assuming an average thermal efficiency of 34% and the heating value of coal as 30 MJ/kg, determine the coal consumption in 2003 in tons. (b) What-if Scenario: What would the coal consumption be if the average thermal efficiency were 35% instead?

SOLUTION

(a) Electricity generated from coal:

$$W_{\text{net}} = (0.51)(3.88\times10^{12}); \quad \Rightarrow W_{\text{net}} = 1.98\times10^{12} \text{ kWh}; \quad \Rightarrow W_{\text{net}} = (3.6)(1.98\times10^{12}) \text{ MJ};$$

$$\Rightarrow W_{\text{net}} = 7.128\times10^{12} \text{ MJ};$$

The required heat input:

$$Q_{\text{H}} = \frac{W_{\text{net}}}{\eta_{th}}; \quad \Rightarrow Q_{\text{H}} = \frac{7.128\times10^{12}}{0.34}; \quad \Rightarrow Q_{\text{H}} = 20.96\times10^{12} \text{ MJ};$$

The required coal consumed in 2003:

$$m_F = \frac{Q_{\text{H}}}{(\text{Heating Value})}; \quad \Rightarrow m_F = \frac{20.96\times10^{12}}{30} \text{ kg}; \quad \Rightarrow m_F = \frac{20.96\times10^{12}}{(30)(907.2)} \text{ tons};$$

$$\Rightarrow m_F = 7.7\times10^{8} \text{ tons}$$

(b) If the thermal efficiency was 35% instead:

$$\Rightarrow Q_{\text{H}} = \frac{W_{\text{net}}}{\eta_{th}}; \quad \Rightarrow Q_{\text{H}} = \frac{7.128\times10^{12}}{0.35}; \quad \Rightarrow Q_{\text{H}} = 20.365\times10^{12} \text{ MJ};$$

$$m_F = \frac{Q_{\text{H}}}{(\text{Heating Value})}; \quad \Rightarrow m_F = \frac{20.365\times10^{12}}{30} \text{ kg}; \quad \Rightarrow m_F = \frac{20.365\times10^{12}}{(30)(907.2)} \text{ tons};$$

$$\Rightarrow m_F = 7.48\times10^{8} \text{ tons}$$

2-5-7 [LP] Determine the fuel cost per kWh of electricity produced by a heat engine with a thermal efficiency of 40% if it uses diesel as the source of heat. The following data is supplied for diesel: price = $2.00 per gallon; heating value = 42.8 MJ/kg; density = 850 kg/m^3.

SOLUTION

$$W_{\text{net}} = 1 \text{ kWh}; \qquad \Rightarrow W_{\text{net}} = 3.6 \text{ MJ};$$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \qquad \Rightarrow \eta_{th} = \frac{W_{\text{net}}}{Q_{\text{H}}};$$

$$\Rightarrow Q_{\text{H}} = \frac{W_{\text{net}}}{\eta_{th}}; \qquad \Rightarrow Q_{\text{H}} = \frac{3.6}{0.4}; \qquad \Rightarrow Q_{\text{H}} = 9 \text{ MJ};$$

$$m_{\text{F}} = \frac{Q_H}{(\text{Heating value})}; \qquad \Rightarrow m_{\text{F}} = \frac{9}{42.8}; \qquad \Rightarrow m_{\text{F}} = 0.21 \text{ kg};$$

$$\cancel{V}_{\text{F}} = \frac{m_F}{\rho_F}; \qquad \Rightarrow \cancel{V}_{\text{F}} = \frac{0.21}{850} \text{ m}^3; \qquad \Rightarrow \cancel{V} = \frac{0.21}{850}(264.2) \text{ gallon}; \qquad \Rightarrow \cancel{V} = 0.0653 \text{ gallon};$$

The fuel cost:

$$\Rightarrow (0.0653)(200) = 13.1 \ \frac{\text{cents}}{\text{kWh}}$$

2-5-8 [LU] A gas turbine with a thermal efficiency (η_{th}) of 21% develops a power output of 8 MW. Determine (a) the fuel consumption rate in kg/min if the heating value of the fuel is 50 MJ/kg. (b) If the maximum temperature achieved during the combustion of diesel is 1700 K, determine the maximum thermal efficiency possible. Assume the atmospheric temperature to be 300 K.

SOLUTION

(a) $\dot{W}_{\mathrm{net}} = 8 \text{ MW};$

$$\eta_{th} = \frac{\dot{W}_{\mathrm{net}}}{\dot{Q}_{\mathrm{H}}}; \quad \Rightarrow \dot{Q}_{\mathrm{H}} = \frac{\dot{W}_{\mathrm{net}}}{\eta_{th}};$$

$$\Rightarrow \dot{Q}_{\mathrm{H}} = \frac{8}{0.21}; \quad \Rightarrow \dot{Q}_{\mathrm{H}} = 38.09 \text{ MW};$$

$$\dot{m}_{\mathrm{F}} = \frac{Q_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_{\mathrm{F}} = \frac{38.09}{50} \frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_{\mathrm{F}} = \frac{38.09}{50}(60) \frac{\text{kg}}{\text{min}};$$

$$\Rightarrow \dot{m}_{\mathrm{F}} = 45.72 \frac{\text{kg}}{\text{min}}$$

(b) The maximum efficiency can be found using the Carnot efficiency model:

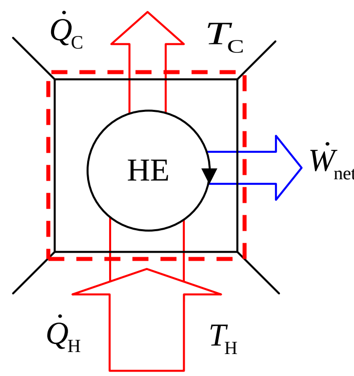
$$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{1700}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 0.8235;$$

$$\Rightarrow \eta_{\text{th, Carnot}} = 82.35\%$$

2-5-9 [LX] Two different fuels are being considered for a 1 MW (net output) heat engine which can operate between the highest temperature produced during the burning of the fuel and the atmospheric temperature of 300 K. Fuel A burns at 2500 K, delivering 50 MJ/kg (heating value) and costs \$2 per kilogram. Fuel B burns at 1500 K, delivering 40 MJ/kg and costs \$1.50 per kilogram. Determine the minimum fuel cost per hour for (a) fuel A and (b) fuel B.

SOLUTION

(a) For Fuel A: The minimum cost per hour will be achieved when the heat engine is at a maximum or Carnot efficiency.



$$\eta_{\text{th, Carnot}} = 1 - \frac{T_{C,A}}{T_{H,A}}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{2500};$$

$$\Rightarrow \eta_{\text{th, Carnot}} = 0.88; \quad \Rightarrow \eta_{\text{th, Carnot}} = 88\%;$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = \eta_{\text{th, Carnot}} = 0.88;$$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th, Carnot}}}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{1}{0.88}; \quad \Rightarrow \dot{Q}_{\text{H}} = 1.136 \text{ MW};$$

The cost per hour of fuel A can now be calculated as:

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{1.136}{50}\frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = \frac{(1.136)(3600)}{50}\frac{\text{kg}}{\text{h}};$$

$$\frac{\text{cost}}{\text{hour}} = \frac{(1.136)(3600)}{50}(2); \quad \Rightarrow \frac{\text{cost}}{\text{hour}} = \frac{\$163.58}{\text{h}}$$

(b) For Fuel B: The same process can be used to determine the cost per hour of fuel B.

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_{C,B}}{T_{H,B}}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{1500}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 0.8; \quad \Rightarrow \eta_{\text{th, Carnot}} = 80\%;$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = \eta_{\text{th, Carnot}} = 0.8;$$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{1}{0.8}; \quad \Rightarrow \dot{Q}_{\text{H}} = 1.25 \text{ MW};$$

The cost per hour of fuel B can now be calculated as:

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{1.25}{40}\frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = \frac{(1.25)(3600)}{40}\frac{\text{kg}}{\text{h}};$$

$$\frac{\text{cost}}{\text{hour}} = \frac{(1.25)(3600)}{40}(1.50); \quad \Rightarrow \frac{\text{cost}}{\text{hour}} = \frac{\$168.75}{\text{h}}$$

2-5-10 [LC] A heat engine receives heat from a source at 2000 K at a rate of 500 kW, and rejects the waste heat to a medium at 300 K. The net output from the engine is 300 kW. (a) Determine the maximum power that could be generated by the engine for the same heat input. (b) Determine the thermal efficiency of the engine.

SOLUTION

(a) The maximum efficiency is the Carnot efficiency given as:

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{200}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 0.85; \quad \Rightarrow \eta_{\text{th, Carnot}} = 85\%;$$

$$\eta_{\text{th, Carnot}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H}} \eta_{\text{th, Carnot}};$$

$$\Rightarrow \dot{W}_{\text{net}} = (500)(0.85); \quad \Rightarrow \dot{W}_{\text{net}} = 425 \text{ kW}$$

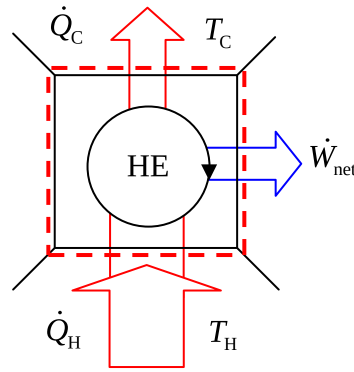
(b) The actual thermal efficiency of this device can be calculated from the given data as:

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = \frac{300}{500}; \quad \Rightarrow \eta_{\text{th}} = 0.6; \quad \Rightarrow \eta_{\text{th}} = 60\%$$

2-5-11 [LV] A Carnot heat engine with a thermal efficiency of 60% receives heat from a source at a rate of 3000 kJ/min, and rejects the waste heat to a medium at 300 K. Determine (a) the power that is generated by the engine and (b) the source temperature.

SOLUTION

(a) Knowing the efficiency and heat input of this Carnot heat engine, the power generated by this engine can be easily calculated.



$$\eta_{\text{th, Carnot}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \qquad \Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H}}\eta_{\text{th, Carnot}};$$

$$\Rightarrow \dot{W}_{\text{net}} = \left(\frac{3000}{60}\right)(0.6); \qquad \Rightarrow \dot{W}_{\text{net}} = 30 \text{ kW}$$

(b) Using the Carnot formula:

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H};$$

$$\Rightarrow T_H = \frac{T_C}{1-\eta_{\text{th, Carnot}}}; \qquad \Rightarrow T_H = \frac{300}{1-0.6}; \qquad \Rightarrow T_H = 750 \text{ K}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-12 [LQ] A heat engine operates between a reservoir at 2000 K and an ambient temperature of 300 K. It produces 10 MW of shaft power. If it has a thermal efficiency (η_{th}) of 40%, (a) determine the rate of fuel consumption in kg/h if the heating value of the fuel is 40 MJ/kg. (b) If the heat engine is replaced by the most efficient engine possible, what is the minimum possible fuel consumption rate for the same power output?

SOLUTION

(a) $\eta_{\text{th}} = \dfrac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \qquad \Rightarrow \eta_{\text{th}} = 0.4;$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \qquad \Rightarrow \dot{Q}_{\text{H}} = \frac{10}{0.4}; \qquad \Rightarrow \dot{Q}_{\text{H}} = 25 \text{ MW};$$

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \qquad \Rightarrow \dot{m}_F = \frac{25}{40}\ \frac{\text{kg}}{\text{s}}; \qquad \Rightarrow \dot{m}_F = \frac{(25)(3600)}{40}\ \frac{\text{kg}}{\text{h}};$$

$$\Rightarrow \dot{m}_F = 2250\ \frac{\text{kg}}{\text{h}}$$

(b) $\eta_{\max} = \eta_{\text{th,Carnot}}; \qquad \Rightarrow \eta_{\max} = 1 - \dfrac{T_C}{T_H}; \qquad \Rightarrow \eta_{\max} = 1 - \dfrac{300}{2000}; \qquad \Rightarrow \eta_{\max} = 0.85;$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \qquad \Rightarrow \dot{Q}_{\text{H}} = \frac{10}{0.85}; \qquad \Rightarrow \dot{Q}_{\text{H}} = 11.76 \text{ MW};$$

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \qquad \Rightarrow \dot{m}_F = \frac{11.76}{40}\ \frac{\text{kg}}{\text{s}}; \qquad \Rightarrow \dot{m}_F = \frac{(11.76)(3600)}{40}\ \frac{\text{kg}}{\text{h}};$$

$$\Rightarrow \dot{m}_F = 1058.8\ \frac{\text{kg}}{\text{h}}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-13 [LT] A heat engine, operating between two reservoirs at 1500 K and 300 K, produces an output of 100 MW. If the thermal efficiency of the engine is measured at 50%, determine

(a) The Carnot efficiency of the engine (in percent),

(b) the rate of heat transfer into the engine from the hot source in MW,

(c) the rate of fuel consumption (in kg/s) if the heating value of fuel is 40 MJ/kg.

SOLUTION

(a) $\eta_{\text{th,Carnot}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\text{th,Carnot}} = 1 - \frac{300}{1500}; \quad \Rightarrow \eta_{\text{th,Carnot}} = 80\%$

(b) $\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = 0.5;$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{100}{0.5}; \quad \Rightarrow \dot{Q} = 200 \text{ MW}$$

(c) $\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{200}{40}; \quad \Rightarrow \dot{m}_F = 5 \frac{\text{kg}}{\text{s}}$

TEST Solution:
Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-14 [LY] A heat engine receives heat from two reservoirs: 50 MW from a reservoir at 500 K and 100 MW from a reservoir at 1000 K. If it rejects 90 MW to atmosphere at 300 K, (a) determine the thermal efficiency of the engine. (b) Calculate the entropy generated in kW/K in the engine's universe.

SOLUTION

(a) $\dot{Q}_{\text{in}} = \dot{Q}_1 + \dot{Q}_2; \quad \Rightarrow \dot{Q}_{\text{in}} = 50 + 100; \quad \Rightarrow \dot{Q}_{\text{in}} = 150 \text{ MW};$

$\dot{W}_{\text{net}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}; \quad \Rightarrow \dot{W}_{\text{net}} = 150 - 90; \quad \Rightarrow \dot{W}_{\text{net}} = 60 \text{ MW};$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}; \quad \Rightarrow \eta_{th} = \frac{60}{150}; \quad \Rightarrow \eta_{th} = 40\%$$

(b) $$\cancelto{0}{\frac{dS}{dt}} = \cancelto{0}{\dot{S}_{\text{net}}} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_1}{T_1}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{90}{300} - \frac{100}{1000} - \frac{50}{500}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.1 \frac{\text{MW}}{\text{K}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = 100 \frac{\text{kW}}{\text{K}}$$

2-5-15 [LF] A heat engine produces 1000 kW of power while receiving heat from two reservoirs: 1000 kW from a 1000 K source and 2000 kW from a 2000 K source. Heat is rejected to the atmosphere at 300 K. Determine (a) the waste heat (heat rejected) in kW. (b)the entropy generation rate in kW/K in the system's universe.

SOLUTION

(a) $\dot{Q}_{\text{in}} = \dot{Q}_1 + \dot{Q}_2; \quad \Rightarrow \dot{Q}_{\text{in}} = 1000 + 2000; \quad \Rightarrow \dot{Q}_{\text{in}} = 3000\ \text{kW};$

$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{out}} = 3000 - 1000; \quad \Rightarrow \dot{Q}_{\text{out}} = 2000\ \text{kW}$

(b) $$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_1}{T_1}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{2000}{300} - \frac{2000}{2000} - \frac{1000}{1000};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = 4.67\ \frac{\text{kW}}{\text{K}}$$

2-5-16 [LD] A heat engine, operating between two reservoirs at 1500 K and 300 K, produces 150 kW of net power. If the rate of heat transfer from the hot reservoir to the engine is measured at 350 kW, determine (a) the thermal efficiency of the engine, (b) the rate of fuel consumption in kg/h to maintain the hot reservoir at steady state (assume the heating value of the fuel to be 45 MJ/kg), (c) the minimum possible fuel consumption rate (in kg/h) for the same output.

SOLUTION

(a) $\eta_{th} = \dfrac{W_{\text{net}}}{\dot{Q}_{\text{in}}}; \quad \Rightarrow \eta_{th} = \dfrac{150}{350}; \quad \Rightarrow \eta_{th} = 42.85\%$

(b) $\dot{m}_F = \dfrac{\dot{Q}_{\text{in}}}{\text{Heating value}}; \quad \Rightarrow \dot{m}_F = \dfrac{350}{45000}; \quad \Rightarrow \dot{m}_F = 0.00778 \dfrac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = 28 \dfrac{\text{kg}}{\text{h}}$

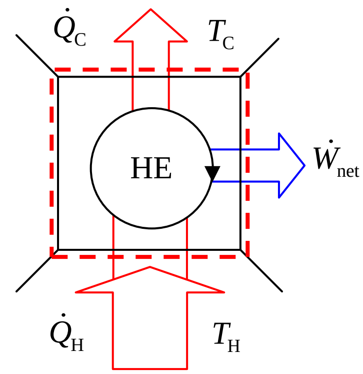
(c) $\eta_{\max} = \eta_{th,\text{Carnot}}; \quad \Rightarrow \eta_{\max} = 1 - \dfrac{T_C}{T_H}; \quad \Rightarrow \eta_{\max} = 1 - \dfrac{300}{1500}; \quad \Rightarrow \eta_{\max} = 80\%;$

$Q_{\text{in,min}} = \dfrac{W_{\text{net}}}{\eta_{\max}}; \quad \Rightarrow Q_{\text{in,min}} = \dfrac{150}{0.8}; \quad \Rightarrow Q_{\text{in,min}} = 187.5 \text{ kW};$

$\dot{m}_{F,\min} = \dfrac{\dot{Q}_{in,\min}}{\text{Heating Value}}; \quad \Rightarrow \dot{m}_{F,\min} = \dfrac{187.5}{45000}; \quad \Rightarrow \dot{m}_{F,\min} = 0.00417 \dfrac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_{F,\min} = 15 \dfrac{\text{kg}}{\text{h}}$

2-5-17 [LM] A solar-energy collector produces a maximum temperature of 100°C. The collected energy is used in a cyclic heat engine that operates in a 5°C environment. (a) What is the maximum thermal efficiency? (b) What-if Scenario: What would the maximum efficiency be if the collector were redesigned to focus the incoming light to enhance the maximum temperature to 400°C?

SOLUTION



(a) $\eta_{max} = \eta_{th,\ Carnot}; \quad \Rightarrow \eta_{max} = 1 - \dfrac{T_C}{T_H};$

$$\Rightarrow \eta_{max} = 1 - \frac{5+273}{100+273}; \quad \Rightarrow \eta_{max} = 0.255;$$

$$\Rightarrow \eta_{max} = 25.5\%$$

(b) With the maximum temperature increased to 400°C:

$$\eta_{th,\ Carnot} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{th,\ Carnot} = 1 - \frac{5+273}{400+273};$$

$$\Rightarrow \eta_{th,\ Carnot} = 0.587; \quad \Rightarrow \eta_{th,\ Carnot} = 58.7\%$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-18 [LJ] The Ocean Thermal Energy Conversion (OTEC) system in Hawaii utilizes the surface water and deep water as thermal energy reservoirs. Assume the ocean temperature at the surface to be 20^{o}C and at some depth to be 5^{o}C; determine (a) the maximum possible thermal efficiency achievable by a heat engine. (b) What-if Scenario: What would the maximum efficiency be if the surface water temperature increased to 25^{o}C?

SOLUTION

(a) $\eta_{\max} = \eta_{\text{th, Carnot}}; \quad \Rightarrow \eta_{\max} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\max} = 1 - \frac{5+273}{20+273}; \quad \Rightarrow \eta_{\max} = 0.0512;$

$\Rightarrow \eta_{\max} = 5.12\%$

(b) With the surface water temperature increased to 25^{o}C:

$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{5+273}{25+273}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 0.0671;$

$\Rightarrow \eta_{\text{th, Carnot}} = 6.71\%$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-19 [LW] You have been hired by a venture capitalist to evaluate a concept engine proposed by an inventor, who claims that the engine consumes 100 MW at a temperature of 500 K, rejects 40 MW at a temperature of 300 K, and delivers 50 MW of mechanical work. Does this claim violates the first law of thermodynamics (answer 1), second law (answer 2), both (answer 3), or none (answer 4)?

SOLUTION

A heat engine, by definition, must operate at steady state. The energy balance equation for this engine reduces to:

$$\frac{\cancel{dE}^{0}}{\cancel{dt}} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{Q};$$

$$\Rightarrow \dot{W}_{\text{net}} = \left(\dot{Q}_{\text{H}} - \dot{Q}_{\text{C}}\right); \qquad \Rightarrow \dot{W}_{\text{net}} = 100 - 40; \qquad \Rightarrow \dot{W}_{\text{net}} = 60 \text{ MW};$$

The engine, by producing only 50 MW of power violates the energy balance, that is, the first law of thermodynamics.

The entropy balance equation can be simplified as:

$$\frac{\cancel{dS}^{0}}{\cancel{dt}} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}; \qquad \Rightarrow \frac{\cancel{dS}^{0}}{\cancel{dt}} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{40}{300} - \frac{100}{500}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = -0.0667 \ \frac{\text{kW}}{\text{K}};$$

A negative entropy generation means that the engine violates the second law of thermodynamics.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-20 [GR] A heat engine produces 40 kW of power while consuming 40 kW of heat from a source at 1200 K, 50 kW of heat from a source at 1500 K, and rejecting the waste heat to atmosphere at 300 K. Determine (a) the thermal efficiency of the engine. (b) What-if Scenario: What would the thermal efficiency be if all the irreversibility could be magically eliminated? Assume no change in heat input from the two sources.

SOLUTION

(a) $\dot{Q}_{\text{in}} = \dot{Q}_1 + \dot{Q}_2; \quad \Rightarrow \dot{Q}_{\text{in}} = 40 + 50; \quad \Rightarrow \dot{Q}_{\text{in}} = 90 \text{ kW};$

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{out}} = 90 - 40; \quad \Rightarrow \dot{Q}_{\text{out}} = 50 \text{ kW};$$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{th} = \frac{40}{90}; \quad \Rightarrow \eta_{th} = 0.444; \quad \Rightarrow \eta_{th} = 44.4\%$$

(b) The entropy balance equation for the reversible engine produces the heat rejected to atmosphere:

$$\cancelto{0}{\frac{dS}{dt}} = \cancelto{0}{\dot{S}_{\text{net}}} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_{\text{out}}}{T_0} + \cancelto{0,\ \text{reversible}}{\dot{S}_{\text{gen}}};$$

$$\Rightarrow \dot{Q}_{\text{out}} = T_0\left(\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2}\right); \quad \Rightarrow \dot{Q}_{\text{out}} = 300\left(\frac{40}{1200} + \frac{50}{1500}\right); \quad \Rightarrow \dot{Q}_{\text{out}} = 20 \text{ kW};$$

With the heat input unchanged, the net work for this engine can be obtained from the energy balance equation:

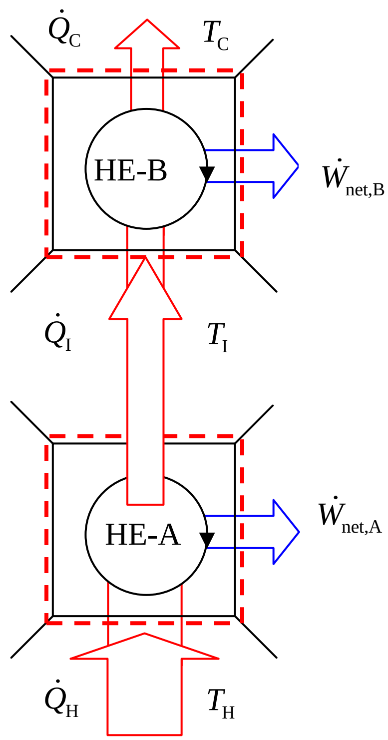
$$\dot{W}_{\text{net}} = 90 - \dot{Q}_{\text{out}}; \quad \Rightarrow \dot{W}_{\text{net}} = 90 - 20; \quad \Rightarrow \dot{W}_{\text{net}} = 70 \text{ kW};$$

The efficiency of this reversible engine:

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{th} = \frac{70}{(40+50)}; \quad \Rightarrow \eta_{th} = 77.7\%$$

2-5-21 [GO] Two reversible engines A and B are arranged in series with the waste heat of engine A used to drive engine B. Engine A receives 200 MJ from a hot source at a temperature of 420°C. Engine B is in communication with a heat sink at a temperature of 4.4°C. If the work output of A is twice that of B, determine (a) the intermediate temperature between A and B, and (b) the thermal efficiency of each engine.

SOLUTION



(a) In terms of the unknown intermediate reservoir temperature T_I:

$$\dot{W}_{net,A} = \eta_{th,A}\dot{Q}_H; \quad \Rightarrow \dot{W}_{net,A} = \left(1-\frac{T_I}{T_H}\right)\dot{Q}_H;$$

$$\dot{W}_{net,B} = \eta_{th,B}\dot{Q}_I; \quad \Rightarrow \dot{W}_{net,B} = \left(1-\frac{T_C}{T_I}\right)\dot{Q}_I;$$

$$\Rightarrow \dot{W}_{net,B} = \left(1-\frac{T_C}{T_I}\right)\left(\dot{Q}_H - \dot{W}_{net,A}\right);$$

$$\dot{W}_{net,A} = 2\dot{W}_{net,B};$$

$$\Rightarrow \eta_{th,A}\dot{Q}_H = 2\eta_{th,B}\left(\dot{Q}_H - \dot{W}_{net,A}\right);$$

$$\Rightarrow \dot{Q}_H\left(2\eta_{th,B} - \eta_{th,A}\right) = 2\eta_{th,B}\dot{W}_{net,A};$$

$$\Rightarrow \dot{Q}_H\left(2\eta_{th,B} - \eta_{th,A}\right) = 2\eta_{th,B}\eta_{th,A}\dot{Q}_H;$$

$$\Rightarrow 2\left(1-\frac{T_C}{T_I}\right) - \left(1-\frac{T_I}{T_H}\right) = 2\left(1-\frac{T_C}{T_I}\right)\left(1-\frac{T_I}{T_H}\right);$$

$$\Rightarrow 1 - 2\frac{T_C}{T_I} + \frac{T_I}{T_H} = 2 - 2\frac{T_C}{T_I} - 2\frac{T_I}{T_H} + 2\frac{T_C}{T_H};$$

$$\Rightarrow 3\frac{T_I}{T_H} = 1 + 2\frac{T_C}{T_H}; \quad \Rightarrow T_I = \frac{T_H}{3} + \frac{2T_C}{3};$$

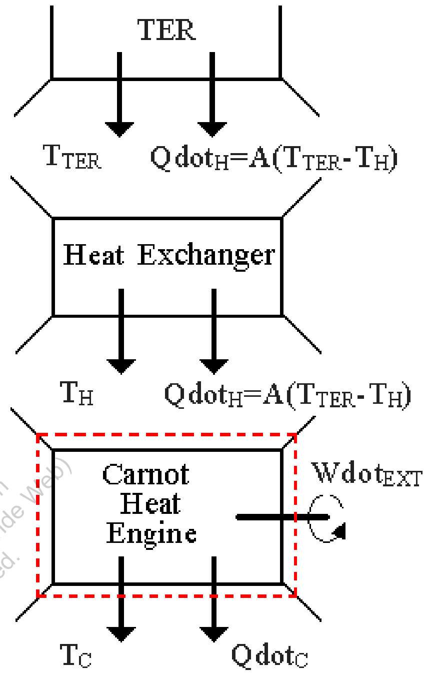
$$\Rightarrow T_I = \frac{(420+273) + 2(4.4+273)}{3}; \quad \Rightarrow T_I = 415.9 \text{ K}$$

(b.1) $\eta_{th,\,A} = \dfrac{\dot{W}_{net,A}}{\dot{Q}_H}; \quad \Rightarrow \eta_{th,\,A} = 1 - \dfrac{T_I}{T_H}; \quad \Rightarrow \eta_{th,\,A} = 1 - \dfrac{415.9}{(420+273)}; \quad \Rightarrow \eta_{th,\,A} = 40\%$

(b.2) $\eta_{th,\,B} = \dfrac{\dot{W}_{net,B}}{\dot{Q}_H}; \quad \Rightarrow \eta_{th,\,B} = 1 - \dfrac{T_C}{T_I}; \quad \Rightarrow \eta_{th,\,B} = 1 - \dfrac{(4.4+273)}{415.9}; \quad \Rightarrow \eta_{th,\,B} = 33.3\%$

2-5-22 [GB] A Carnot heat engine receives heat from a TER at T_{TER} through a heat exchanger where the heat transfer rate is proportional to the temperature difference as $Q_H = A(T_{TER}-T_H)$. It rejects heat to a cold reservoir at T_C. If the heat engine is to maximize the work output, show that the high temperature in the cycle should be selected as $T_H = sqrt(T_{TER}T_C)$.

SOLUTION



$$\dot{W}_{\text{net}} = \dot{Q}_{\text{H}}\eta_{\text{th,Carnot}}; \qquad \Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H}}\left(1-\frac{T_{\text{C}}}{T_{\text{H}}}\right);$$

$$\Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H}} - \dot{Q}_{\text{H}}\frac{T_{\text{C}}}{T_{\text{H}}};$$

To increase $\dot{Q}_{\text{H}} = A\left(T_{\text{TER}} - T_H\right)$, T_H must be reduced, which reduces the thermal efficiency. To maximize $\dot{W}_{\text{net}}$, we express it as a function of T_{H}.

$$\dot{W}_{\text{net}} = \dot{Q}_{\text{H}}\left(1-\frac{T_{\text{C}}}{T_{\text{H}}}\right); \qquad \Rightarrow \dot{W}_{\text{net}} = A\left(T_{\text{TER}} - T_H\right)\left(1-\frac{T_{\text{C}}}{T_{\text{H}}}\right);$$

$$\Rightarrow \dot{W}_{\text{net}} = AT_{\text{TER}} - AT_{\text{H}} - AT_{\text{TER}}\frac{T_{\text{C}}}{T_{\text{H}}} + AT_{\text{C}};$$

To optimize the work output, the derivative expression is set equal to zero.

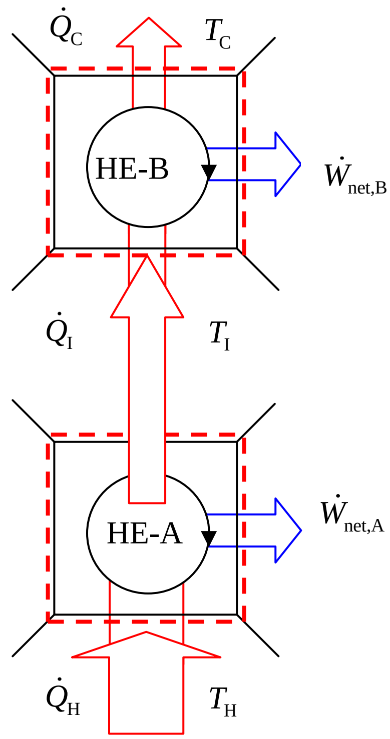
$$\frac{d\dot{W}_{\text{net}}}{dT_{\text{H}}} = 0 - A + \frac{AT_{\text{TER}}T_{\text{C}}}{\left(T_{\text{H}}\right)^2};$$

$$\Rightarrow 0 = -A + \frac{AT_{\text{TER}}T_{\text{C}}}{\left(T_{\text{H}}\right)^2};$$

$$\Rightarrow T_{\text{H}} = \sqrt{T_{\text{TER}}T_{\text{C}}}$$

2-5-23 [GS] Two Carnot engines operate in series. The first one receives heat from a TER at 2500 K and rejects the waste heat to another TER at a temperature *T*. The second engine receives this energy rejected by the first one, converts some of it to work, and rejects the rest to a TER at 300 K. If the thermal efficiency of both the engines are the same, (a) determine the temperature (*T*). What-if Scenario: (b) What would the temperature be if the two engines produced the same output instead?

SOLUTION



(a) Equating the Carnot efficiencies of these two heat engines yields the intermediate temperature *T*.

$$\eta_{\text{th, Carnot, A}} = \eta_{\text{th, Carnot, B}};$$

$$\Rightarrow 1-\frac{T_I}{T_H} = 1-\frac{T_B}{T_I};$$

$$\Rightarrow T_I = \sqrt{T_C T_H}; \qquad \Rightarrow T_I = \sqrt{(300)(2500)};$$

$$\Rightarrow T_I = 866 \text{ K}$$

(b) If the work output are the same, we obtain:

$$\dot{W}_{\text{net,A}} = \eta_{\text{th,A}}\dot{Q}_H; \qquad \Rightarrow \dot{W}_{\text{net,A}} = \left(1-\frac{T_I}{T_H}\right)\dot{Q}_H;$$

$$\dot{W}_{\text{net,B}} = \eta_{\text{th,B}}\dot{Q}_I; \qquad \Rightarrow \dot{W}_{\text{net,B}} = \left(1-\frac{T_C}{T_I}\right)\dot{Q}_I;$$

$$\Rightarrow \dot{W}_{\text{net,B}} = \left(1-\frac{T_C}{T_I}\right)\left(\dot{Q}_H - \dot{W}_{\text{net,A}}\right);$$

$$\dot{W}_{\text{net,A}} = \dot{W}_{\text{net,B}};$$

$$\Rightarrow \eta_{\text{th,A}}\dot{Q}_H = \eta_{\text{th,B}}\left(\dot{Q}_H - \dot{W}_{\text{net,A}}\right);$$

$$\Rightarrow \dot{Q}_H\left(\eta_{\text{th,B}} - \eta_{\text{th,A}}\right) = \eta_{\text{th,B}}\dot{W}_{\text{net,A}}; \qquad \Rightarrow \dot{Q}_H\left(\eta_{\text{th,B}} - \eta_{\text{th,A}}\right) = \eta_{\text{th,B}}\eta_{\text{th,A}}\dot{Q}_H;$$

$$\Rightarrow \left(1-\frac{T_C}{T_I}\right) - \left(1-\frac{T_I}{T_H}\right) = \left(1-\frac{T_C}{T_I}\right)\left(1-\frac{T_I}{T_H}\right);$$

$$\Rightarrow -\frac{T_C}{T_I} + \frac{T_I}{T_H} = 1 - \frac{T_C}{T_I} - \frac{T_I}{T_H} + \frac{T_C}{T_H}; \qquad \Rightarrow 2\frac{T_I}{T_H} = 1 + \frac{T_C}{T_H};$$

$$\Rightarrow T_I = \frac{T_H}{2} + \frac{T_C}{2}; \qquad \Rightarrow T_I = \frac{2500+300}{2}; \qquad \Rightarrow T_I = 1400 \text{ K}$$

2-5-24 [GA] A reversible heat engine operates in outer space. The only way heat can be rejected is by radiation, which is proportional to the fourth power of the temperature and the area of the radiating surface. Show that for a given power output and a given source temperature (T_1), the area of the radiator is minimized when the radiating surface temperature is $T_2 = 0.75\, T_1$.

SOLUTION

$$\dot{W}_{\text{net}} = \dot{Q}_{\text{H}} - \dot{Q}_{\text{C}};$$

For the reversible engine the entropy balance equation produces:

$$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}_H}{T_1} - \frac{\dot{Q}_C}{T_2} + \cancel{\dot{S}_{\text{gen,univ}}}^{0}; \qquad \Rightarrow \dot{Q}_{\text{H}} = \frac{T_1}{T_2}\dot{Q}_{\text{C}};$$

Therefore,

$$\dot{W}_{\text{net}} = \frac{T_1}{T_2}\dot{Q}_{\text{C}} - \dot{Q}_{\text{C}}; \qquad \Rightarrow \dot{W}_{\text{net}} = \left(\frac{T_1}{T_2} - 1\right)\dot{Q}_{\text{C}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \left(\frac{T_1}{T_2} - 1\right)kAT_2^4;$$

For minimum area:

$$\frac{dA}{dT_2} = 0;$$

$$A = \frac{\dot{W}_{\text{ext}}}{(T_1 - T_2)kT_2^3};$$

$$\frac{dA}{dT_2} = -\left(\frac{\dot{W}_{\text{ext}}}{(T_1 - T_2)kT_2^3}\right)\frac{d\left[(T_1 - T_2)kT_2^3\right]}{dT_2}; \qquad \Rightarrow \frac{dA}{dT_2} = 0;$$

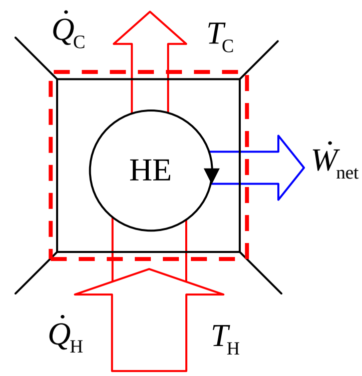
$$\Rightarrow (T_1 - T_2)\frac{d\left(kT_2^3\right)}{dT_2} + \left(kT_2^3\right)\frac{d\left(T_1 - T_2\right)}{dT_2} = 0;$$

$$\Rightarrow (T_1 - T_2)\left(3kT_2^2\right) + \left(kT_2^3\right)(-1) = 0;$$

$$\Rightarrow 3\left(T_1 - T_2\right) = T_2; \qquad \Rightarrow T_2 = 0.75T_1$$

2-5-25 [GH] A heat engine receives heat at a rate of 3000 kJ/min from a reservoir at 1000 K and rejects the waste heat to the atmosphere at 300 K. If the engine produces 20 kW of power, determine (a) the thermal efficiency and (b) the entropy generated in the engine's universe.

SOLUTION



(a) $\dot{Q}_H = \left(3000\frac{\text{kJ}}{\text{min}}\right)\left(\frac{1}{60}\frac{\text{min}}{\text{s}}\right); \quad \Rightarrow \dot{Q}_H = 50\ \frac{\text{kJ}}{\text{s}};$

$\Rightarrow \dot{Q}_H = 50\ \text{kW};$

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}; \quad \Rightarrow \eta_{th} = \frac{20}{50}; \quad \Rightarrow \eta_{th} = 0.4;$$

$\Rightarrow \eta_{th} = 40\%$

(b) From the energy balance equation we obtain:

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{net}}^{0} + \dot{Q} - \dot{W}_{ext} = \left(\dot{Q}_H - \dot{Q}_C\right) - \dot{W}_{net};$$

$$\Rightarrow \dot{Q}_C = \dot{Q}_H - \dot{W}_{net}; \quad \Rightarrow \dot{Q}_C = 50 - 20; \quad \Rightarrow \dot{Q}_C = 30\ \text{kW};$$

The entropy balance equation yields:

$$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{net}}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \quad \Rightarrow \cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{gen,univ};$$

$$\dot{S}_{gen,univ} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}; \quad \Rightarrow \dot{S}_{gen,univ} = \frac{30}{300} - \frac{50}{1000}; \quad \Rightarrow \dot{S}_{gen,univ} = 0.05\ \frac{\text{kW}}{\text{K}}$$

2-5-26 [GN] A household freezer operates in a kitchen at 25°C. Heat must be transferred from the cold space at a rate of 2.5 kW to maintain its temperature at -25°C. What is the smallest (power) motor required to operate the freezer.

SOLUTION

The minimum power input requirement occurs when the COP of this freezer is equal to the Carnot COP.

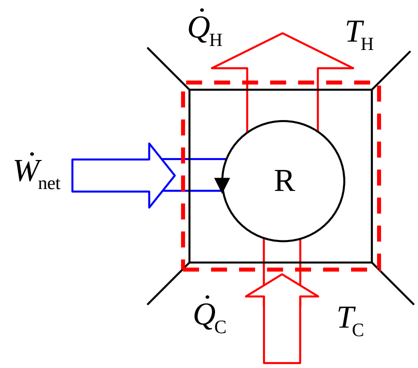
$$\mathrm{COP_R} = \mathrm{COP_{R,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\mathrm{net}}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{(2.5)\left(25-(-25)\right)}{(273-25)};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = 0.504\ \mathrm{kW}$$



TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-27 [GE] To keep a refrigerator in steady state at 2^{o}C, heat has to be removed from it at a rate of 200 kJ/min. If the surrounding air is at 27^{o}C, determine (a) the minimum power input to the refrigerator and (b) the maximum COP.

SOLUTION

(a) The minimum power input requirement occurs when the COP of this refrigerator is equal to the Carnot COP.

$$\text{COP}_\text{R} = \text{COP}_\text{R,Carnot};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_\text{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_\text{net} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \quad \Rightarrow \dot{W}_\text{net} = \frac{(200)(27-2)}{(273+2)};$$

$$\Rightarrow \dot{W}_\text{net} = 18\frac{\text{kJ}}{\text{min}}; \quad \Rightarrow \dot{W}_\text{net} = 0.3 \text{ kW}$$

(b) The maximum COP can be determined by evaluating the Carnot Cop.

$$\text{COP}_\text{R,Carnot} = \frac{T_C}{T_H - T_C}; \quad \Rightarrow \text{COP}_\text{R,Carnot} = \frac{(273+2)}{(27-2)}; \quad \Rightarrow \text{COP}_\text{R,Carnot} = 11$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-28 [GI] A Carnot refrigerator consumes 2 kW of power while operating in a room at 20°C. If the food compartment of the refrigerator is to be maintained at 3°C, determine the rate of heat removal in kJ/min from the compartment.

SOLUTION

The COP of the refrigerator is the Carnot COP.

$$\mathrm{COP_R} = \frac{\dot{Q}_C}{\dot{W}_{\mathrm{net}}}; \quad \Rightarrow \mathrm{COP_R} = \mathrm{COP_{R,Carnot}}; \quad \Rightarrow \mathrm{COP_R} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{Q}_C = \frac{\dot{W}_{\mathrm{net}} T_C}{T_H - T_C}; \quad \Rightarrow \dot{Q}_C = \frac{(2)(273+3)}{20-3}; \quad \Rightarrow \dot{Q}_C = 32.47 \text{ kW};$$

$$\Rightarrow \dot{Q}_C = 1948.2 \ \frac{\text{kJ}}{\text{min}}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-29 [BEQ] An actual refrigerator operates with a COP that is half the Carnot COP. It removes 10 kW of heat from a cold reservoir at 250 K and dumps the waste heat into the atmosphere at 300 K. (a) Determine the net work consumed by the refrigerator. (b) What-if Scenario: How would the answer change if the cold storage were to be maintained at 200 K without altering the rate of heat transfer?

SOLUTION

(a) The COP of the refrigerator is half that of the Carnot COP. Therefore,

$$\mathrm{COP_R} = \frac{\dot{Q}_C}{\dot{W}_\mathrm{net}}; \qquad \Rightarrow \mathrm{COP_R} = \frac{1}{2}\mathrm{COP_{R,Carnot}}; \qquad \Rightarrow \mathrm{COP_R} = \frac{1}{2}\frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_\mathrm{net} = \frac{2\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \qquad \Rightarrow \dot{W}_\mathrm{net} = \frac{(2)(10)(300-250)}{250};$$

$$\Rightarrow \dot{W}_\mathrm{net} = 4\ \mathrm{kW}$$

(b) For $T_C = 200$ K:

$$\Rightarrow \dot{W}_\mathrm{net} = \frac{2\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \qquad \Rightarrow \dot{W}_\mathrm{net} = \frac{(2)(10)(300-200)}{200};$$

$$\Rightarrow \dot{W}_\mathrm{net} = 10\ \mathrm{kW}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-30 [GL] An inventor claims to have developed a refrigerator with a COP of 10 that maintains a cold space at -10^{o}C, while operating in a 25^{o}C kitchen. Is this claim plausible? (1:Yes; 0:No)

SOLUTION

This claim can be evaluated by comparing the claimed COP with the Carnot COP of this refrigerator.

$$\mathrm{COP}_{\mathrm{R,Carnot}} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow \mathrm{COP}_{\mathrm{R,Carnot}} = \frac{(273-10)}{\{25-(-10)\}}; \qquad \Rightarrow \mathrm{COP}_{\mathrm{R,Carnot}} = 7.51;$$

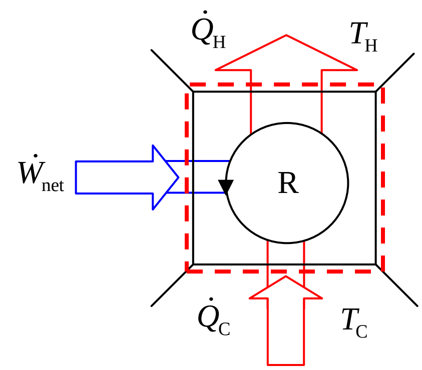
The maximum COP or Carnot COP is found to be less than the claimed COP. Therefore, this claim is not plausible.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-31 [GG] A refrigeration cycle removes heat at a rate of 250 kJ/min from a cold space maintained at -10°C while rejecting heat to the atmosphere at 25°C. If the power consumption rate is 0.75 kW, determine if the cycle is (1: reversible; 2: irreversible; 3: impossible).

SOLUTION



The energy balance equation yields:

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{net}}^{0} + \dot{Q} - \dot{W}_{ext}; \qquad \Rightarrow \cancel{\frac{dE}{dt}}^{0} = \left(\dot{Q}_C - \dot{Q}_H\right) - \left(-\dot{W}_{net}\right);$$

$$\Rightarrow \dot{Q}_H = \dot{Q}_C + \dot{W}_{net}; \qquad \Rightarrow \dot{Q}_H = \frac{250}{60} + 0.75;$$

$$\Rightarrow \dot{Q}_H = 4.917 \text{ kW};$$

The entropy balance equation yields:

$$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{net}}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \qquad \Rightarrow \cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{gen,univ};$$

$$\Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C}; \qquad \Rightarrow \dot{S}_{gen,univ} = \frac{4.917}{273+25} - \frac{250}{60(273-10)};$$

$$\Rightarrow \dot{S}_{gen,univ} = 6.57 \times 10^{4} \frac{\text{kW}}{\text{K}};$$

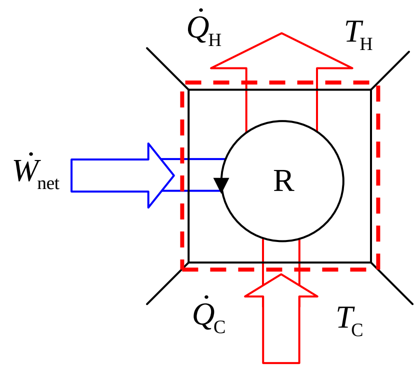
The cycle is 2: irreversible.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-32 [GZ] A refrigeration cycle removes heat at a rate of 250 kJ/min from a cold space maintained at -10^{o}C while rejecting heat to the atmosphere at 25^{o}C. If the power consumption rate is 1.5 kW, (a) do a first-law analysis to determine the rate of heat rejection to the atmosphere in kW. (b) Do a second-law analysis to determine the entropy generation rate in the refrigerator's universe.

SOLUTION



(a) $$\frac{d\cancel{E}^{0}}{dt} = \cancel{\dot{J}_{net}}^{0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \frac{d\cancel{E}^{0}}{dt} = \left(\dot{Q}_C - \dot{Q}_H\right) - \left(-\dot{W}_{net}\right);$$

$$\Rightarrow \dot{Q}_H = \dot{Q}_C + \dot{W}_{ext}; \qquad \Rightarrow \dot{Q}_H = \frac{250}{60} + 1.5;$$

$$\Rightarrow \dot{Q}_H = 5.67 \text{ kW}$$

(b) $$\frac{d\cancel{S}^{0}}{dt} = \cancel{\dot{S}_{net}}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \qquad \Rightarrow \frac{d\cancel{S}^{0}}{dt} = \cancel{\dot{S}_{net}}^{0} + \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{gen,univ};$$

$$\Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C}; \qquad \Rightarrow \dot{S}_{gen,univ} = \frac{5.67}{298} - \frac{4.167}{263}; \qquad \Rightarrow \dot{S}_{gen,univ} = 0.00318 \ \frac{\text{kW}}{\text{K}}$$

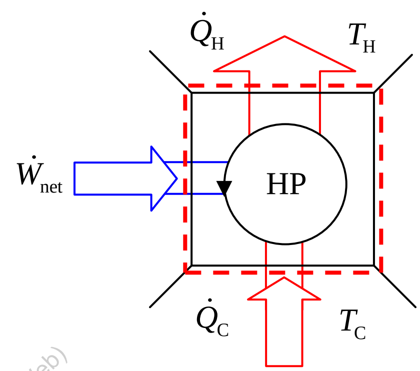
TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-33 [GK] In a cryogenic experiment a container is maintained at -120°C, although it gains 200 W due to heat transfer from the surroundings. What is the minimum power of a motor that is needed for a heat pump to absorb heat from the container and reject heat to the room at 25°C?

SOLUTION

The heat pump is working as a refrigerator to withdraw $\dot{Q}_C = 200$ W of heat from the container to maintain the container at steady state.



For minimum power consumption,

$\mathrm{COP_R} = \mathrm{COP_{R,Carnot}}$;

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{(200)\left(25-(-120)\right)}{(273-120)};$$

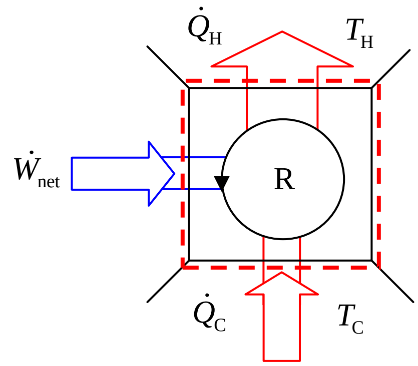
$$\Rightarrow \dot{W}_{net} = 189.542 \text{ W}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-34 [GP] An air-conditioning system maintains a house at a temperature of 20°C while the outside temperature is 40°C. If the cooling load on this house is 10 tons, determine (a) the minimum power requirement. (b) What-if Scenario: What would the minimum power requirement be if the interior were 5 degrees warmer?

SOLUTION



(a) From the problem statement,

$$\dot{Q}_C = 10 \text{ tons} = 35.1667 \text{ kW};$$

The minimum power input requirement occurs when the COP of this refrigerator is equal to the Carnot COP.

$$\text{COP}_\text{R} = \text{COP}_\text{R,Carnot};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_\text{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_\text{net} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \qquad \Rightarrow \dot{W}_\text{net} = \frac{(35.1667)(40-20)}{(273+20)};$$

$$\Rightarrow \dot{W}_\text{net} = 2.4 \text{ kW}$$

(b) $$\dot{W}_\text{net} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \qquad \Rightarrow \dot{W}_\text{net} = \frac{(35.1667)(40-(20+5))}{(273+(20+5))}; \qquad \Rightarrow \dot{W}_\text{net} = 1.77 \text{ kW}$$

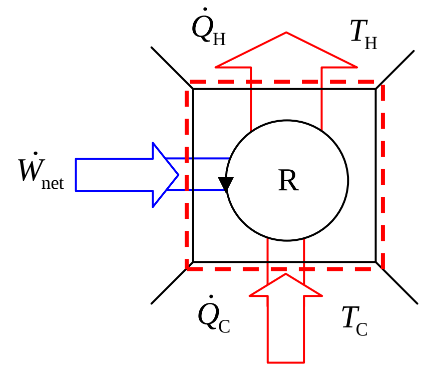
TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-35 [GU] An air-conditioning system is required to transfer heat from a house at a rate of 800 kJ/min to maintain its temperature at 20°C. (a) If the COP of the system is 3.7, determine the power required for air conditioning the house. (b) If the outdoor temperature is 35°C, determine the minimum possible power required.

SOLUTION

(a) From the definition of the COP of a refrigerator we obtain:



$$\mathrm{COP_R} = \frac{\dot{Q}_C}{\dot{W}_{net}};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C}{\mathrm{COP_R}}; \qquad \Rightarrow \dot{W}_{net} = \frac{800}{3.7};$$

$$\Rightarrow \dot{W}_{net} = 216.2 \frac{\mathrm{kJ}}{\mathrm{min}}; \qquad \Rightarrow \dot{W}_{net} = 3.6 \text{ kW}$$

(b) The minimum power input requirement occurs when the COP of this refrigerator is equal to the Carnot COP.

$$\mathrm{COP_R} = \mathrm{COP_{R,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C (T_H - T_C)}{T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\left(\frac{800}{60}\right)(35-20)}{(273+20)}; \qquad \Rightarrow \dot{W}_{net} = 0.68 \text{ kW}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-36 [GX] A solar-powered refrigeration system receives heat from a solar collector at T_H, rejects heat to the atmosphere at T_0 and extracts heat from a cold space at T_C. The three heat transfer rates are $\dot{Q}_H$, $\dot{Q}_0$ and $\dot{Q}_C$, respectively. (a) Do an energy and entropy analysis of the system to derive an expression for the maximum possible COP, defined as the ratio $\dot{Q}_C/\dot{Q}_H$. (b) Determine the COP for T_H= 400°C, T_0 = 30°C, and T_C = -20 °C.

SOLUTION

(a) For maximum possible COP, the system should be reversible. An energy and entropy analysis yields:

$$\cancel{\frac{dE}{dt}}^0 = \cancel{\dot{J}_{\text{net}}}^0 + \dot{Q} - \cancel{\dot{W}_{\text{ext}}}^0; \qquad \Rightarrow \cancel{\frac{dE}{dt}}^0 = \left(\dot{Q}_C + \dot{Q}_H - \dot{Q}_0\right);$$

$$\Rightarrow \dot{Q}_0 = \dot{Q}_H + \dot{Q}_C;$$

$$\cancel{\frac{dS}{dt}}^0 = \cancel{\dot{S}_{\text{net}}}^0 + \frac{\dot{Q}}{T_B} + \cancel{\dot{S}_{\text{gen,univ}}}^0; \qquad \Rightarrow \cancel{\frac{dS}{dt}}^0 = \cancel{\dot{S}_{\text{net}}}^0 + \frac{\dot{Q}_C}{T_C} + \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_0}{T_0};$$

$$\Rightarrow \frac{\dot{Q}_C}{T_C} + \frac{\dot{Q}_H}{T_H} = \frac{\dot{Q}_C + \dot{Q}_H}{T_0};$$

$$\Rightarrow \dot{Q}_H\left(\frac{1}{T_H} - \frac{1}{T_0}\right) = \dot{Q}_C\left(\frac{1}{T_0} - \frac{1}{T_C}\right);$$

$$\Rightarrow \text{COP}_{\text{max}} = \frac{\dot{Q}_{\text{C}}}{\dot{Q}_{\text{H}}}; \qquad \Rightarrow \text{COP}_{\text{max}} = \frac{T_{\text{C}}\left(T_{\text{H}} - T_0\right)}{T_{\text{H}}\left(T_0 - T_{\text{C}}\right)}$$

(b) Substituting the given values of temperature (in Kelvin), we obtain:

$$\text{COP}_{\text{max}} = \frac{\dot{Q}_{\text{C}}}{\dot{Q}_{\text{H}}}; \qquad \Rightarrow \text{COP}_{\text{max}} = \frac{T_{\text{C}}\left(T_{\text{H}} - T_0\right)}{T_{\text{H}}\left(T_0 - T_{\text{C}}\right)}; \qquad \Rightarrow \text{COP}_{\text{max}} = \frac{(273-20)(400-30)}{(273+400)(30-(-20))};$$

$$\Rightarrow \text{COP}_{\text{max}} = 2.78$$

2-5-37 [GC] Assume T_H = 425 K, T_0 = 298 K, T_C = 250 K and $\dot{Q}_C$ = 20 kW in the above system of problem 2-5-36[GX]. (a) Determine the maximum COP of the system. (b) If the collector captures 0.2 kW/m^2, determine the minimum collector area required.

SOLUTION

(a) Using the results from the previous problem:

$$\mathrm{COP}_{\max} = \frac{\dot{Q}_C}{\dot{Q}_H}; \qquad \Rightarrow \mathrm{COP}_{\max} = \frac{T_C\left(T_H - T_0\right)}{T_H\left(T_0 - T_C\right)};$$

$$\Rightarrow \mathrm{COP}_{\max} = \frac{250(425-298)}{425(298-250)}; \qquad \Rightarrow \mathrm{COP}_{\max} = 1.556$$

(b) To solve for the minimum collector area required to extract 20 kW of heat

$$0.2A = \dot{Q}_H;$$

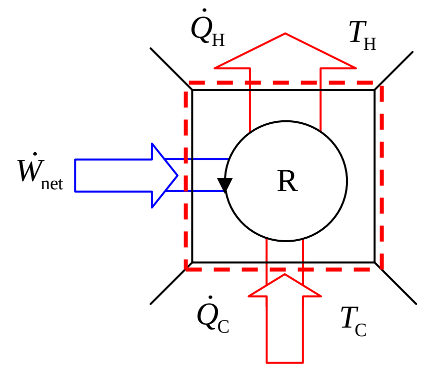
$$\Rightarrow A = \frac{\dot{Q}_H}{0.2}; \qquad \Rightarrow A = \frac{\dot{Q}_H}{\dot{Q}_C}\frac{\dot{Q}_C}{0.2}; \qquad \Rightarrow A = \frac{\dot{Q}_C}{(0.2)\mathrm{COP}_{\max}};$$

$$\Rightarrow A = \frac{20}{(0.2)(1.556)}; \qquad \Rightarrow A = 64.3\ \mathrm{m}^2$$

2-5-38 [GV] A refrigerator with a COP of 2.0 extracts heat from a cold chamber at 0^{o}C at a rate of 400 kJ/min. If the atmospheric temperature is 20^{o}C, determine (a) the power drawn by the refrigerator and (b) the rate of entropy generation in the refrigerator's universe.

SOLUTION

(a) The power drawn by this refrigerator can be found using the COP equation.



$$\mathrm{COP_R} = \frac{\dot{Q}_C}{\dot{W}_{net}}; \quad \Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C}{\mathrm{COP_R}};$$

$$\Rightarrow \dot{W}_{net} = \frac{400}{2}; \quad \Rightarrow \dot{W}_{net} = 200\frac{\mathrm{kJ}}{\mathrm{min}};$$

$$\Rightarrow \dot{W}_{net} = 3.33\ \mathrm{kW}$$

(b) The energy balance equation produces:

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{net}}^{0} + \dot{Q} - \dot{W}_{ext}; \quad \Rightarrow \cancel{\frac{dE}{dt}}^{0} = \left(\dot{Q}_H - \dot{Q}_C\right) - \dot{W}_{net};$$

$$\Rightarrow \dot{Q}_H = \dot{Q}_C - \dot{W}_{net}; \quad \Rightarrow \dot{Q}_H = \frac{400}{60} - 3.33; \quad \Rightarrow \dot{Q}_H = 3.33\ \mathrm{kW};$$

An entropy analysis of the refrigerator's universe produces:

$$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{net}}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \quad \Rightarrow \cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{gen,univ};$$

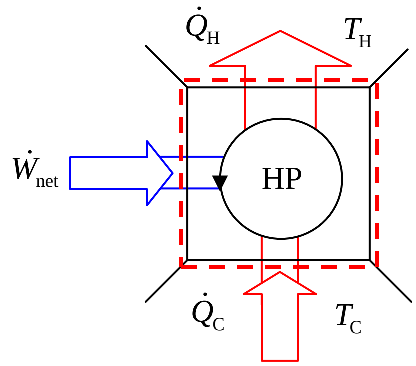
$$\Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}; \quad \Rightarrow \dot{S}_{gen,univ} = \frac{6.67}{273} - \frac{3.33}{(273+20)}; \quad \Rightarrow \dot{S}_{gen,univ} = 0.0131\ \frac{\mathrm{kW}}{\mathrm{K}}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-39 [GQ] On a cold night a house is losing heat at a rate of 15 kW. A reversible heat pump maintains the house at 20^{o}C while the outside temperature is 0^{o}C. (a) Determine the heating cost for the night (8 hours). (b) Also determine the heating cost if resistance heating were used instead. Assume the price of electricity to be 15 cents/kWh.

SOLUTION



(a) Since this heat pump is reversible, it's COP is equal to the Carnot COP.

$$\mathrm{COP}_{\mathrm{HP}} = \mathrm{COP}_{\mathrm{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\mathrm{net}}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{\dot{Q}_H (T_H - T_C)}{T_H};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{(15)(20-0)}{(273+20)};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = 1.023\ \mathrm{kW};$$

Knowing the power input, the cost per night can be calculated as:

$$\frac{\mathrm{cost}}{\mathrm{night}} = (1.022\ \mathrm{kW})(8\mathrm{hrs})\left(0.15\frac{\$}{\mathrm{kWh}}\right); \quad \Rightarrow \frac{\mathrm{cost}}{\mathrm{night}} = \$1.23$$

(b) With resistance heating (the energy equation can be used to show that, at steady state, the entire amount of electric work is converted to heat).

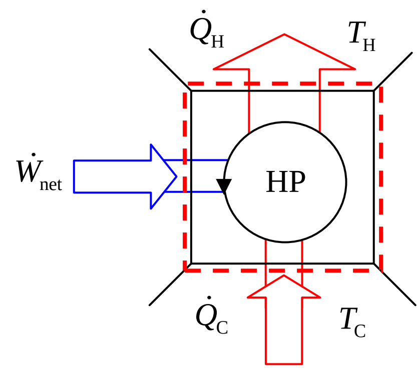
$$\frac{\mathrm{cost}}{\mathrm{night}} = (15\ \mathrm{kW})(8\mathrm{hrs})\left(0.15\frac{\$}{\mathrm{kWh}}\right); \quad \Rightarrow \frac{\mathrm{cost}}{\mathrm{night}} = \$18$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat pump radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-40 [GT] On a cold night a house is losing heat at a rate of 80,000 Btu/h. A reversible heat pump maintains the house at 70°F, while the outside temperature is 30°F. Determine (a) the heating cost for the night (8 hours) assuming the price of 10 cents/kWh for electricity. Also determine (b) the heating cost if resistance heating were used instead.

SOLUTION



(a) Using the Engineering Converter,

$$\dot{Q}_H = 80,000 \ \frac{\text{Btu}}{\text{h}} = 23.44 \text{ kW};$$

Since this heat pump is reversible,

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_H (T_H - T_C)}{T_H};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{(23.44)(294.26 - 272)}{294.26}; \qquad \Rightarrow \dot{W}_{\text{net}} = 1.77 \text{ kW};$$

Knowing the power input, the cost per night can be calculated as:

$$\frac{\text{cost}}{\text{night}} = (1.77 \text{ kW})(8\text{hrs})\left(0.1\frac{\$}{\text{kWh}}\right); \qquad \Rightarrow \frac{\text{cost}}{\text{night}} = \$1.42$$

(b) With resistance heating (the energy equation can be used to show that, at steady state, the entire amount of electric work is converted to heat).

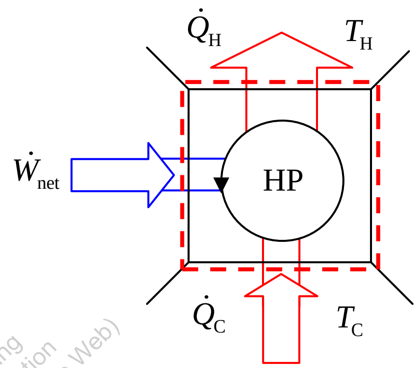
$$\frac{\text{cost}}{\text{night}} = (23.44 \text{ kW})(8\text{hrs})\left(0.1\frac{\$}{\text{kWh}}\right); \qquad \Rightarrow \frac{\text{cost}}{\text{night}} = \$18.75$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat pump radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-41 [GY] A house is maintained at a temperature of 20°C by a heat pump pumping heat from the atmosphere. Heat transfer rate through the wall and roof is estimated at 0.6 kW per unit temperature difference between inside and outside. (a) If the atmospheric temperature is -10°C, what is the minimum power required to drive the pump? (b) It is proposed to use the same pump to cool the house in the summer. For the same room temperature, the same heat transfer rate, and the same power input to the pump, determine the maximum permissible atmospheric temperature. (c) What-if Scenario: What would the answer in part (b) be if the heat transfer rate between the house and outside were estimated at 0.7 kW per unit temperature difference?

SOLUTION



(a) From the problem statement,

$$\dot{Q}_H = 0.6\Delta T; \qquad \Rightarrow \dot{Q}_H = 0.6(T_H - T_C);$$

$$\Rightarrow \dot{Q}_H = 0.6\big(20-(-10)\big); \qquad \Rightarrow \dot{Q}_H = 18 \text{ kW};$$

The minimum power requirement will occur when the pump's COP is equal to the Carnot COP.

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{T_H}{T_H - T_C}; \qquad \Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_H(T_H - T_C)}{T_H}; \qquad \Rightarrow \dot{W}_{\text{net}} = 1.84 \text{ kW}$$

(b) As a refrigerator,

$$\dot{Q}_C = 0.6\Delta T; \qquad \Rightarrow \dot{Q}_C = 0.6(T_H - T_C); \qquad \Rightarrow \dot{Q}_C = 0.6(T_H - 293);$$

The maximum outdoor temperature that the refrigerator can handle will occur when it's COP equals the Carnot COP.

$$\text{COP}_{\text{R}} = \text{COP}_{\text{R,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow \frac{0.6(T_H - 293)}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C};$$

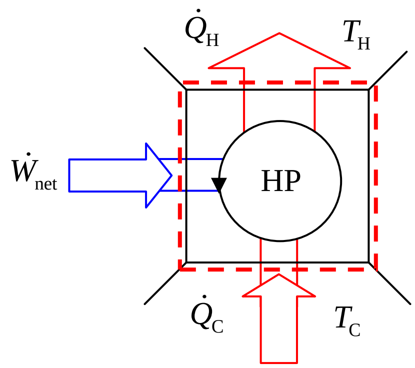
$$\Rightarrow T_H = 323 \text{ K}; \qquad \Rightarrow T_H = 50^{\circ}\text{C}$$

(c) $\dot{Q}_C = 0.7\Delta T; \qquad \Rightarrow \dot{Q}_C = 0.7(T_H - T_C); \qquad \Rightarrow \dot{Q}_C = 0.7(T_H - 293);$

$$\frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow \frac{0.7(T_H - 293)}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow T_H = 307 \text{ K}; \qquad \Rightarrow T_H = 34^{\circ}\text{C}$$

2-5-42 [GF] A house is maintained at a temperature T_H by a heat pump that is powered by an electric motor. The outside air at T_C is used as the low-temperature TER. Heat loss from the house to the surroundings is directly proportional to the temperature difference and is given by $\dot{Q}_{loss} = U(T_H - T_C)$. (a) Determine the minimum electric power to drive the heat pump as a function of the given variables. (b) The electric power consumption is calculated to be 10 kW for $T_H = 20$ deg-C and $T_{H=}$ 10 deg-C. Determine the power consumption if the outside temperature drops to 0 deg-C.

SOLUTION



(a) The minimum power requirement will occur when this heat pump runs as a reversible device.

$$\mathrm{COP_{HP}} = \mathrm{COP_{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\mathrm{net}}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{\dot{Q}_H (T_H - T_C)}{T_H};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{U(T_H - T_C)(T_H - T_C)}{T_H};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{U(T_H - T_C)^2}{T_H}$$

(b) The expression derived in part (a) shows that the power consumption is proportional to the square of temperature difference. Therefore, as the temperature difference doubles,

$$\dot{W}_{\mathrm{net,new}} = \frac{U(T_H - T_{C,new})^2}{T_H}; \quad \Rightarrow \dot{W}_{\mathrm{net,new}} = \frac{U(T_H - T_{C,old})^2}{T_H} \frac{(T_H - T_{C,new})^2}{(T_H - T_{C,old})^2};$$

$$\Rightarrow \dot{W}_{\mathrm{net,new}} = (10)\frac{(20-0)^2}{(20-10)^2}; \qquad \Rightarrow \dot{W}_{\mathrm{net,new}} = 40 \text{ kW}$$

2-5-43 [GD] A house is maintained at steady state (closed system) at 300 K while the outside temperature is 275 K. The heat loss ($\dot{Q}_C$) is measured at 2 kW. Two approaches are being considered: (A) electrical heating at 100% efficiency and (B) an ideal heat pump that operates with Carnot COP. The price of electricity is \$0.2/kWh. (a) Determine the cost of option A and (b) option B over a 10 hour operating period.

SOLUTION

(a) $\dot{W}_{\text{el,in}} = \dot{Q}_{\text{loss}} = 2 \text{ kW};$

$$\Rightarrow W_{\text{el}} = \dot{W}_{\text{el}} \Delta t = 20 \text{ kWh};$$

$$\text{Cost of Option A} = (0.2)(10)(2); \quad \Rightarrow \text{Cost} = \$4$$

(b) $\text{COP}_{\text{HP,Carnot}} = \dfrac{T_H}{T_H - T_c}; \quad \Rightarrow \text{COP}_{\text{HP,Carnot}} = \dfrac{300}{300-275}; \quad \Rightarrow \text{COP}_{\text{HP,Carnot}} = 12;$

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = 12;$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{2}{12};$$

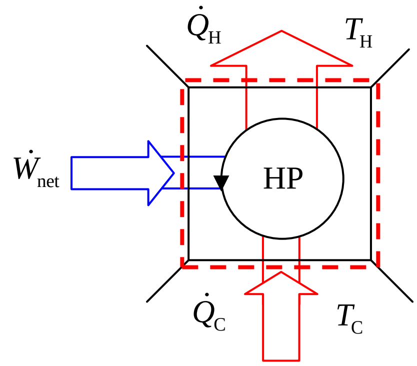
$$\Rightarrow \dot{W}_{\text{net}} = 1.84 \text{ kW};$$

$$\text{Cost of Option B} = (0.2)\dot{W}_{\text{el}} \Delta t; \quad \Rightarrow \text{Cost} = (0.2)\left(\frac{1}{6}\right)(10); \quad \Rightarrow \text{Cost} = \$0.33$$

2-5-44 [GM] A house is maintained at a temperature of 25°C by a reversible heat pump powered by an electric motor. The outside air at 10°C is used as the low-temperature TER. Determine the percent saving in electrical power consumption if the house is kept at 20°C instead. Assume that the heat loss from the house to the surroundings is directly proportional to the temperature difference.

SOLUTION

The percent savings can be evaluated by examining the differences in power inputs as this heat pump runs as a reversible device.



$$\mathrm{COP}_{HP} = \mathrm{COP}_{HP,Carnot};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{net}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_H (T_H - T_C)}{T_H};$$

And $\dot{Q}_H = A(T_H - T_C);$

$$\Rightarrow \dot{W}_{net} = \frac{A(T_H - T_C)^2}{T_H};$$

$$\dot{W}_{net,25} = \frac{A(T_H - T_C)^2}{T_H}; \quad \Rightarrow \dot{W}_{net,25} = \frac{A(25-10)^2}{273+25}; \quad \Rightarrow \dot{W}_{net,25} = 0.755A \text{ kW};$$

$$\dot{W}_{net,20} = \frac{A(T_H - T_C)^2}{T_H}; \quad \Rightarrow \dot{W}_{net,20} = \frac{A(20-10)^2}{273+20}; \quad \Rightarrow \dot{W}_{net,20} = 0.341A \text{ kW};$$

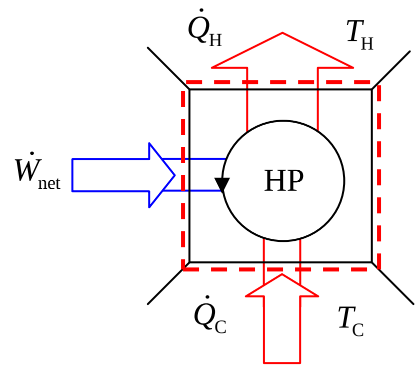
$$\%\text{Savings} = 1 - \frac{\dot{W}_{net,20}}{\dot{W}_{net,25}}; \quad \Rightarrow \%\text{Savings} = 1 - \frac{0.341\cancel{A}}{0.755\cancel{A}}; \quad \Rightarrow \%\text{Savings} = 0.548;$$

$$\Rightarrow \%\text{Savings} = 54.8\%$$

2-5-45 [GJ] A house is heated and maintained at 25°C by a heat pump. Determine the maximum possible COP if heat is extracted from the outside atmosphere at (a) 10°C, (b) 0°C, (c) -10°C and (d) -40°C. (e) Based on these results, would you recommend heat pumps at locations with a severe climate?

SOLUTION

The maximum possible COP occurs when the COP of the heat pump is equal to the Carnot COP.



$$\mathrm{COP}_{\mathrm{HP,max}} = \mathrm{COP}_{\mathrm{HP,Carnot}}; \qquad \Rightarrow \mathrm{COP}_{\mathrm{HP,max}} = \frac{T_H}{T_H - T_C};$$

(a) $$\mathrm{COP}_{10^\circ\mathrm{C}} = \frac{273+25}{25-10}; \qquad \Rightarrow \mathrm{COP}_{10^\circ\mathrm{C}} = 19.9$$

(b) $$\mathrm{COP}_{0^\circ\mathrm{C}} = \frac{273+25}{25-0}; \qquad \Rightarrow \mathrm{COP}_{0^\circ\mathrm{C}} = 11.92$$

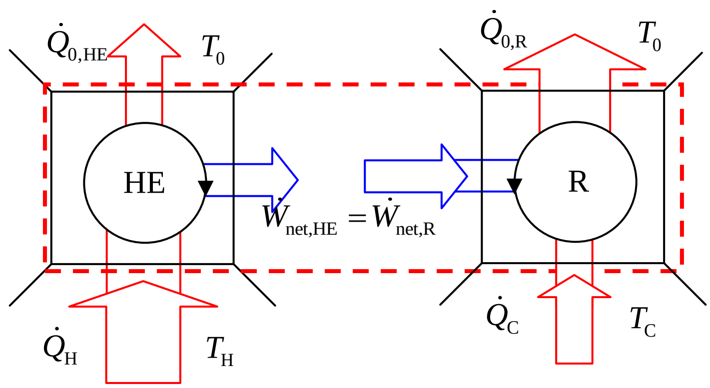
(c) $$\mathrm{COP}_{-10^\circ\mathrm{C}} = \frac{273+25}{25-(-10)}; \qquad \Rightarrow \mathrm{COP}_{-10^\circ\mathrm{C}} = 8.51$$

(d) $$\mathrm{COP}_{-40^\circ\mathrm{C}} = \frac{273+25}{25-(-40)}; \qquad \Rightarrow \mathrm{COP}_{-40^\circ\mathrm{C}} = 4.58$$

(e) Given that a real heat pump is significantly less efficient that a Carnot heat pump, the advantage of a heat pump over electric heating start diminishing rapidly when the outside temperature becomes really low. As the COP decreases, so does the heating power. Heat pump is ideal for milder climate.

2-5-46 [GW] A Carnot heat engine receives heat at 800 K and rejects the waste heat to the surroundings at 300 K. The output from the heat engine is used to drive a Carnot refrigerator that removes heat from the cooled space at -20°C at a rate of 400 kJ/min and rejects it to the same surroundings at 300 K. Determine (a) the rate of heat supplied to the heat engine and (b) the total rate of heat rejection to the surroundings. What-if Scenario: (c) What would the rate of heat supplied be if the temperature of the cooled space were -30°C?

SOLUTION



(a) The work required by the refrigerator is equal to the work produced by the heat engine.

$$\mathrm{COP}_{\mathrm{R,Carnot}} = \frac{\dot{Q}_C}{\dot{W}_{\mathrm{net,R}}}; \qquad \Rightarrow \mathrm{COP}_{\mathrm{R,Carnot}} = \frac{T_C}{T_H - T_C};$$

$$\dot{W}_{\mathrm{net,R}} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \qquad \Rightarrow \dot{W}_{\mathrm{net,R}} = \frac{\dot{Q}_C\left(T_0 - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net,R}} = \frac{400\left[300 - (273 - 20)\right]}{273 - 20}; \qquad \Rightarrow \dot{W}_{\mathrm{net,R}} = 74.3\ \frac{\mathrm{kJ}}{\mathrm{min}};$$

$$\eta_{th,\mathrm{Carnot}} = \frac{\dot{W}_{\mathrm{net,HE}}}{\dot{Q}_{\mathrm{H}}}; \qquad \Rightarrow \eta_{th,\mathrm{Carnot}} = 1 - \frac{T_C}{T_H};$$

$$\dot{Q}_{\mathrm{H}} = \frac{\dot{W}_{\mathrm{net,HE}}}{\left(1 - \frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{\mathrm{H}} = \frac{\dot{W}_{\mathrm{net,R}}}{\left(1 - \frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{\mathrm{H}} = \frac{74.3}{\left(1 - \frac{300}{800}\right)}; \qquad \Rightarrow \dot{Q}_{\mathrm{H}} = 118.88\ \frac{\mathrm{kJ}}{\mathrm{min}}$$

(b) An energy balance on the combined system produces:

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{\mathrm{net}}}^{0} + \dot{Q} - \cancel{\dot{W}_{\mathrm{ext}}}^{0}; \qquad \Rightarrow \cancel{\frac{dE}{dt}}^{0} = \dot{Q}_{\mathrm{H}} + \dot{Q}_{\mathrm{C}} - \dot{Q}_{0,\mathrm{HE}} - \dot{Q}_{0,\mathrm{R}}; \qquad \Rightarrow \cancel{\frac{dE}{dt}} = 118.88 + 400 - \dot{Q}_0;$$

$$\Rightarrow \dot{Q}_0 = \dot{Q}_{0,\mathrm{HE}} + \dot{Q}_{0,\mathrm{R}}; \qquad \Rightarrow \dot{Q}_0 = \dot{Q}_{\mathrm{H}} + \dot{Q}_{\mathrm{C}};$$

$$\Rightarrow \dot{Q}_0 = 118.88 + 400; \qquad \Rightarrow \dot{Q}_0 = 518.88\ \frac{\mathrm{kJ}}{\mathrm{min}}$$

(c) If the temperature of the cooled space were -30°C, $\mathrm{COP}_{\mathrm{R,Carnot}}$ would change.

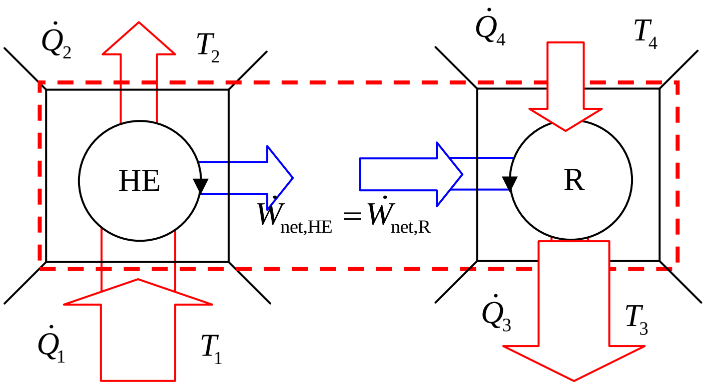
$$\dot{W}_{\text{net,R}} = \frac{\dot{Q}_C}{\text{COP}_{\text{R,Carnot}}}; \qquad \Rightarrow \dot{W}_{\text{net,R}} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \qquad \Rightarrow \dot{W}_{\text{net,R}} = \frac{\dot{Q}_C\left(T_0 - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{\text{net,R}} = \frac{400\left\lfloor 300 - \left(273 - 30\right)\right\rfloor}{273 - 30}; \qquad \Rightarrow \dot{W}_{\text{net,R}} = 93.8\ \frac{\text{kJ}}{\text{min}};$$

$$\dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net,HE}}}{\left(1 - \frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net,R}}}{\left(1 - \frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{\text{H}} = \frac{93.8}{\left(1 - \frac{300}{800}\right)}; \qquad \Rightarrow \dot{Q}_{\text{H}} = 150.08\ \frac{\text{kJ}}{\text{min}}$$

2-5-47 [ZR] A reversible heat engine is used to drive a reversible heat pump. The power cycle takes in Q_1 heat units at T_1 and rejects Q_2 heat units at T_2. The heat pump extracts Q_4 from a heat sink at T_4 and discharges Q_3 at T_3. (a) Develop an expression for Q_4/Q_1 in terms of the four given temperatures. (b) Evaluate the expression for T_1 = 500 K, T_2 = 300 K, T_3 = 400 K, and T_4 = 300 K.

SOLUTION



(a) An energy analysis of this total system produces:

$$\cancelto{0}{\frac{dE}{dt}} = \cancelto{0}{\dot{J}_{\text{net}}} + \dot{Q} - \cancelto{0}{\dot{W}_{\text{ext}}}; \qquad \Rightarrow \cancelto{0}{\frac{dE}{dt}} = \dot{Q}_1 - \dot{Q}_2 - \dot{Q}_3 + \dot{Q}_4; \qquad \Rightarrow \dot{Q}_1 - \dot{Q}_2 + \dot{Q}_4 = \dot{Q}_3;$$

The entropy equation for the engine and the heat pump yields:

$$\cancelto{0}{\frac{dS}{dt}} = \cancelto{0}{\dot{S}_{net}} + \frac{\dot{Q}}{T_B} + \cancelto{0}{\dot{S}_{gen}}; \qquad \Rightarrow \frac{\dot{Q}}{T_B} = 0;$$

Heat engine: $\dfrac{\dot{Q}_1}{T_1} - \dfrac{\dot{Q}_2}{T_2} = 0; \quad \Rightarrow \dot{Q}_2 = \dfrac{T_2}{T_1}\dot{Q}_1;$

Heat pump: $\dfrac{\dot{Q}_4}{T_4} - \dfrac{\dot{Q}_3}{T_3} = 0; \quad \Rightarrow \dot{Q}_3 = \dfrac{T_3}{T_4}\dot{Q}_4;$

By substituting these equations into the result of the energy balance equation:

$$\Rightarrow \dot{Q}_1 - \frac{T_2}{T_1}\dot{Q}_1 + \dot{Q}_4 = \frac{T_3}{T_4}\dot{Q}_4; \qquad \Rightarrow \dot{Q}_1\left(1 - \frac{T_2}{T_1}\right) = \dot{Q}_4\left(\frac{T_3}{T_4} - 1\right);$$

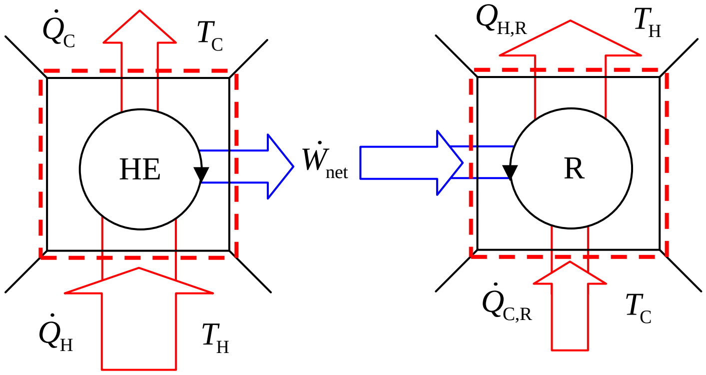
$$\Rightarrow \frac{\dot{Q}_4}{\dot{Q}_1} = \frac{\left(1 - \frac{T_2}{T_1}\right)}{\left(\frac{T_3}{T_4} - 1\right)}; \qquad \Rightarrow \frac{\dot{Q}_4}{\dot{Q}_1} = \frac{T_4\left(T_1 - T_2\right)}{T_1\left(T_3 - T_4\right)}$$

(b) Substituting the given values:

$$\frac{\dot{Q}_1}{\dot{Q}_4} = \frac{T_4\left(T_1 - T_2\right)}{T_1\left(T_3 - T_4\right)}; \qquad \Rightarrow \frac{\dot{Q}_1}{\dot{Q}_4} = \frac{(300)(500-300)}{(500)(400-300)}; \qquad \Rightarrow \frac{\dot{Q}_1}{\dot{Q}_4} = 1.2$$

2-5-48 [ZO] A heat engine with a thermal efficiency (η_{th}) of 35% is used to drive a refrigerator having a COP of 4. (a) What is the heat input to the engine for each MJ removed from the cold region by the refrigerator? (b) If the system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input into the engine?

SOLUTION



(a) $$\frac{\dot{Q}_{\text{H}}}{\dot{Q}_{\text{C,R}}}=\frac{\dot{Q}_{\text{H}}}{\dot{W}_{\text{net,HE}}}\frac{\dot{W}_{\text{net,HE}}}{\dot{Q}_{\text{C,R}}}=\frac{\dot{Q}_{\text{H}}}{\dot{W}_{\text{net,HE}}}\frac{\dot{W}_{\text{net,R}}}{\dot{Q}_{\text{C,R}}}=\frac{1}{\eta_{\text{th}}}\frac{1}{\text{COP}_{\text{R}}};$$

$$\Rightarrow\frac{\dot{Q}_{\text{H}}}{\dot{Q}_{\text{C,R}}}=\frac{1}{4(0.35)}=0.714;$$

$$\Rightarrow\dot{Q}_{\text{H}}=(0.714)\dot{Q}_{\text{C,R}}=(0.714)(1)=0.714\text{ MJ}$$

(b)

$$\text{COP}_{\text{HP}}=\frac{\dot{Q}_{\text{H,R}}}{\dot{W}_{\text{net,R}}}=\frac{\dot{W}_{\text{net,R}}+\dot{Q}_{\text{C,R}}}{\dot{W}_{\text{net,R}}}=1+\text{COP}_{\text{R}}=5$$

$$\frac{\dot{Q}_{\text{H}}}{\dot{Q}_{\text{H,R}}}=\frac{\dot{Q}_{\text{H}}}{\dot{W}_{\text{net,HE}}}\frac{\dot{W}_{\text{net,HE}}}{\dot{Q}_{\text{H,R}}}=\frac{\dot{Q}_{\text{H}}}{\dot{W}_{\text{net,HE}}}\frac{\dot{W}_{\text{net,R}}}{\dot{Q}_{\text{H,R}}}=\frac{1}{\eta_{\text{th}}}\frac{1}{\text{COP}_{\text{HP}}}=\frac{1}{5(0.35)}=0.571;$$

$$\Rightarrow\dot{Q}_{\text{H,R}}=\frac{\dot{Q}_{\text{H}}}{0.571}=\frac{1}{0.571}=1.75\text{ MJ}$$

2-5-49 [ZB] A heat engine operates between two TERs at 1000°C and 20°C respectively. Two-thirds of the work output is used to drive a heat pump that removes heat from the cold surroundings at 0°C and transfers it to a house kept at 20°C. If the house is losing heat at a rate of 60,000 kJ/h, determine (a) the minimum rate of heat supply to the heat engine. (b) What-if Scenario: What would the minimum heat supply be if the outside temperature dropped to -10°C?

SOLUTION

(a) For minimum heat supply, both the heat engine and heat pump must be reversible.

$$\mathrm{COP_{HP}} = \mathrm{COP_{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_{H,HP}}{\dot{W}_{\mathrm{net},HP}} = \frac{T_H}{T_H - T_C};$$

$$\dot{W}_{\mathrm{net,HP}} = \frac{2}{3}\dot{W}_{\mathrm{net,HE}}; \qquad \Rightarrow \dot{W}_{\mathrm{net,HE}} = \frac{\dot{Q}_{H,\mathrm{HP}}\left(T_H - T_C\right)}{T_H}\left(\frac{3}{2}\right); \qquad \Rightarrow \dot{W}_{\mathrm{net,HE}} = \frac{60,000\left(20-0\right)}{273+20}\left(\frac{3}{2}\right);$$

$$\Rightarrow \dot{W}_{\mathrm{net,HE}} = 6143.3\ \frac{\mathrm{kJ}}{\mathrm{h}}; \qquad \Rightarrow \dot{W}_{\mathrm{net,HE}} = 1.706\ \mathrm{kW};$$

The minimum heat amount of heat can be supplied when the engine is reversible.

$$\eta_{\mathrm{th}} = \eta_{\mathrm{th,\ Carnot}};$$

$$\Rightarrow \frac{\dot{W}_{\mathrm{net,HE}}}{\dot{Q}_{H,\mathrm{HE}}} = 1 - \frac{T_C}{T_H}; \qquad \Rightarrow \dot{Q}_{H,\mathrm{HE}} = \frac{\dot{W}_{\mathrm{net,HE}}}{\left(1-\frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{H,\mathrm{HE}} = \frac{1.706}{\left(1-\frac{273+20}{273+1000}\right)};$$

$$\Rightarrow \dot{Q}_{H,\mathrm{HE}} = 2.22\ \mathrm{kW}$$

(b) With a temperature drop of 10 degrees,

$$\dot{W}_{\mathrm{net,HE}} = \frac{\dot{Q}_{H,HP}\left(T_H - T_C\right)}{T_H}\left(\frac{3}{2}\right); \qquad \Rightarrow \dot{W}_{\mathrm{net,HE}} = \frac{60,000\left(20-(-10)\right)}{273+20}\left(\frac{3}{2}\right);$$

$$\Rightarrow \dot{W}_{\mathrm{net,HE}} = 9215.02\ \frac{\mathrm{kJ}}{\mathrm{hr}}; \qquad \Rightarrow \dot{W}_{\mathrm{net,HE}} = 2.56\ \mathrm{kW};$$

$$\Rightarrow \dot{Q}_{H,\mathrm{HE}} = \frac{\dot{W}_{\mathrm{net,HE}}}{\left(1-\frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{H,\mathrm{HE}} = \frac{2.56}{\left(1-\frac{273+20}{273+1000}\right)}; \qquad \Rightarrow \dot{Q}_{H,\mathrm{HE}} = 3.33\ \mathrm{kW}$$

2-5-50 [ZS] A heat engine is used to drive a heat pump. The waste heat from the heat engine and the heat transfer from the heat pump are used to heat the water circulating through the radiator of a building. The thermal efficiency of the heat engine is 30% and the COP of the heat pump is 4.2. Evaluate the COP of the combined system, defined as the ratio of the heat transfer to the circulating water to the heat transfer to the heat engine.

SOLUTION

The efficiency of a heat engine is defined as:

$$\eta_{\text{th,Engine}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H,Engine}}}; \quad \Rightarrow \eta_{\text{th,Engine}} = \frac{\dot{Q}_{\text{H,Engine}} - \dot{Q}_{\text{C,Engine}}}{\dot{Q}_{\text{H,Engine}}}; \quad \Rightarrow \eta_{\text{th,Engine}} = 1 - \frac{\dot{Q}_{\text{C,Engine}}}{\dot{Q}_{\text{H,Engine}}};$$

$$\Rightarrow \dot{Q}_{\text{H,Engine}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \quad \Rightarrow \dot{Q}_{\text{H,Engine}} = \frac{\dot{W}_{\text{net}}}{0.3};$$

And $\dot{Q}_{\text{C,Engine}} = \dot{Q}_{\text{H,Engine}} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{C,Engine}} = \frac{\dot{W}_{\text{net}}}{0.3} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{C,Engine}} = \frac{7}{3}\dot{W}_{\text{net}};$

The heat pump uses the net work produced by the engine. The COP of the heat pump is defined as:

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_{\text{H,HP}}}{\dot{W}_{\text{net}}};$$

$$\dot{Q}_{\text{H,HP}} = \text{COP}_{\text{HP}}\dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{H,HP}} = 4.2\dot{W}_{\text{net}};$$

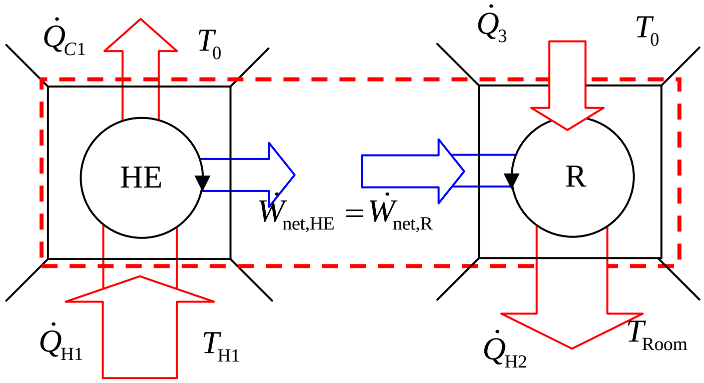
The COP of the combined system is calculated as:

$$\text{COP}_{\text{SYSTEM}} = \frac{\dot{Q}_{\text{C,Engine}} + \dot{Q}_{\text{H,HP}}}{\dot{Q}_{\text{H,Engine}}}; \quad \Rightarrow \text{COP}_{\text{SYSTEM}} = \frac{2.33\cancel{\dot{W}_{\text{net}}} + 4.2\cancel{\dot{W}_{\text{net}}}}{\left(\frac{\cancel{\dot{W}_{\text{net}}}}{0.3}\right)};$$

$$\Rightarrow \text{COP}_{\text{SYSTEM}} = 1.96$$

2-5-51 [ZA] A furnace delivers heat at a rate of $\dot{Q}_{H1}$ at T_{H1}. Instead of directly using this for room heating, it is used to drive a heat engine that rejects the waste heat to atmosphere at T_0. The heat engine drives a heat pump that delivers $\dot{Q}_{H2}$ at T_{room} using the atmosphere as the cold reservoir. Find the ratio $\dot{Q}_{H2}/\dot{Q}_{H1}$, the energetic efficiency of the system as a function of the given temperatures. Why is this a better set-up than direct room heating from the furnace?

SOLUTION



(a) For the heat engine:

$$\eta_{th} = \eta_{th,\text{Carnot}}; \qquad \Rightarrow \eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H1}}}; \qquad \Rightarrow \eta_{th} = 1 - \frac{T_0}{T_{\text{H1}}};$$

$$\Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H1}}\left(1 - \frac{T_0}{T_{\text{H1}}}\right);$$

For the heat pump:

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_{\text{H2}}}{\dot{W}_{\text{net}}} = \frac{T_{\text{Room}}}{T_{\text{Room}} - T_0}; \qquad \Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H2}}\left(\frac{T_{\text{Room}} - T_0}{T_{\text{Room}}}\right);$$

Equating the net work produced by the engine with the work consumed by the heat pump, we obtain:

$$\Rightarrow \frac{\dot{Q}_{\text{H2}}}{\dot{Q}_{\text{H1}}} = \frac{T_{\text{Room}}\left(T_{\text{H1}} - T_0\right)}{T_{\text{H1}}\left(T_{\text{Room}} - T_0\right)};$$

(b) Substituting the given temperatures, values: T_{H1}=2000K, T_{room}=300K and T_0=270K, the energetic efficiency of this system would be:

$$\frac{\dot{Q}_{\text{H2}}}{\dot{Q}_{\text{H1}}} = \frac{300(2000-270)}{2000(300-270)}; \qquad \Rightarrow \frac{\dot{Q}_{\text{H2}}}{\dot{Q}_{\text{H1}}} = 8.65$$

2-5-52 [ZH] A heat pump is used for heating a house in the winter and cooling it in the summer by reversing the flow of the refrigerant. The interior temperature should be 20°C in the winter and 25°C in the summer. Heat transfer through the walls and ceilings is estimated to be 2500 kJ per hour per °C temperature difference between the inside and outside. (a) If the winter outside temperature is 0°C, what is the minimum power required to drive the heat pump? (b) For the same power input as in part (a), what is the maximum outside summer temperature for which the house can be maintained at 25°C?

SOLUTION

(a) $\dot{Q}_H = \dot{Q}_{\text{winter}};\quad \Rightarrow \dot{Q}_H = \dfrac{2500}{3600}(20-0);\quad \Rightarrow \dot{Q}_H = 13.89\text{ kW};$

As a heat pump:

$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{net}} = \frac{T_H}{T_H - T_C};\qquad \Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_H}{\left(\dfrac{T_H}{T_H - T_C}\right)};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_{\text{winter}}}{\left(\dfrac{T_H}{T_H - T_C}\right)};\qquad \Rightarrow \dot{W}_{\text{net}} = \frac{13.89}{\left(\dfrac{273+20}{20-0}\right)};\qquad \Rightarrow \dot{W}_{\text{net}} = 0.948\text{ kW}$$

(b) $\dot{Q}_{\text{summer}} = \dfrac{2500}{3600}\Delta T;\quad \Rightarrow \dot{Q}_{\text{summer}} = \dfrac{2500}{3600}(T_H - T_C);$

As a refrigerator:

$\text{COP}_{\text{R}} = \text{COP}_{\text{R,Carnot}};$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{\dot{Q}_{\text{summer}}}{\dot{W}_{\text{net}}};\qquad \Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \frac{\dfrac{2500}{3600}(T_H - T_C)}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow T_H = \sqrt{T_C \dot{W}_{\text{net}} \frac{3600}{2500}} + 25;\qquad \Rightarrow T_H = \sqrt{(273+25)(0.948)\frac{3600}{2500}} + 25;$$

$$\Rightarrow T_H = 45.16^{\circ}\text{C}$$