**14-3-1** [BOY] For the reaction A + B  $\leftrightarrow$  C + D,  $\Delta g^{-0}$  is calculated to be 0 at 500 K. Starting with a mixture of 1 kmol of A and 1 kmol of B, (a) evaluate what percent of the mixture is converted to products at equilibrium at 500 kPa, 500 K. (b) What would be the answer if the pressure (*p*) were 1000 kPa?

## **SOLUTION**

$$A+B \rightleftharpoons C+D$$

$$A+B \rightarrow a(A+B)+b(C+D)$$

$$y_{A+B} = \frac{a}{a+b+c+d}; \quad y_{C+D} = \frac{b}{a+b+c+d};$$

(a) 
$$-\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{y_{p}^{v_{p}}}{y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{0}} \right)^{\sum_{p} \sum_{r} v_{r}} \right];$$

$$\Rightarrow -\frac{0}{\overline{R}(500)} = \ln \left[ \left( \frac{y_{C+D}^{v_{C+D}}}{y_{A+B}^{v_{A+B}}} \right) \left( \frac{p}{p_{0}} \right)^{v_{C+D} - v_{A+B}} \right];$$

$$\Rightarrow 0 = \ln \left[ \left( \frac{y_{C+D}^{1}}{y_{A+B}^{1}} \right) \left( \frac{500}{p_{0}} \right)^{0} \right];$$

$$\Rightarrow 1 = \left[ \left( \frac{b}{a} \right) \right];$$

$$\Rightarrow a = b;$$

Therefore.

$$1 = a + b;$$
  $\Rightarrow 1 = a + a;$   $\Rightarrow 1 = 2a;$   $\Rightarrow a = 0.5;$   
 $\Rightarrow b = 1 - a = 1 - 0.5 = 0.5 = 50\%$ 

$$(b) -\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{y_{p}^{v_{p}}}{y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{0}} \right)^{\sum_{p} - \sum_{r} v_{r}} \right];$$

$$\Rightarrow -\frac{0}{\overline{R}(500)} = \ln \left[ \left( \frac{y_{C+D}^{v_{C+D}}}{y_{A+B}^{v_{A+B}}} \right) \left( \frac{p}{p_{0}} \right)^{v_{C+D} - v_{A+B}} \right]; \Rightarrow 0 = \ln \left[ \left( \frac{y_{C+D}^{1}}{y_{A+B}^{1}} \right) \left( \frac{1000}{p_{0}} \right)^{1-1} \right];$$

$$\Rightarrow 0 = \ln \left[ \left( \frac{b}{a+b+c+d} \right) \left( \frac{1000}{p_{0}} \right)^{0} \right]; \Rightarrow 1 = \left[ \left( \frac{b}{a} \right) \right]; \Rightarrow a = b;$$

$$1 = a + b;$$
  $\Rightarrow 1 = a + a;$   $\Rightarrow 1 = 2a;$   $\Rightarrow a = 0.5;$ 



**14-3-2** [BOF] For an elementary step A + B  $\leftrightarrow$  C + D,  $\Delta g^{-o}$  is tabulated as 22.819 at 298 K, 0 at 1000 K, and -229.724 MJ/kmol at 3000 K. Starting with a mixture of 1 kmol of A and 1 kmol of B, evaluate what percent of the mixture is converted to products at equilibrium at (a) 298 K, (b) 1000 K, and (c) 3000 K.

## **SOLUTION**

$$A+B \rightleftharpoons C+D$$

$$A+B \rightarrow a(A+B)+b(C+D)$$

$$y_{A+B} = \frac{a}{a+b+c+d}; \quad y_{C+D} = \frac{b}{a+b+c+d};$$

(a) 
$$-\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{y_{p}^{v_{p}}}{y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{0}} \right)^{\sum v_{p} - \sum v_{r}} \right];$$

$$\Rightarrow -\frac{22819}{(8.314)(298)} = \ln \left[ \left( \frac{y_{C+D}^{v_{C+D}}}{y_{A+B}^{v_{A+B}}} \right) \left( \frac{p}{p_{0}} \right)^{v_{C+D} - v_{A+B}} \right]; \Rightarrow -9.210227 = \ln \left[ \left( \frac{y_{C+D}^{1}}{y_{A+B}^{1}} \right) \left( \frac{1}{p_{0}} \right)^{1-1} \right];$$

$$\Rightarrow -9.210227 = \ln \left[ \left( \frac{b}{a+b+c+d} \right) \left( \frac{1}{p_{0}} \right)^{0} \right]; \Rightarrow 1.00011 \times 10^{-4} = \left[ \frac{b}{a} \right];$$

$$\Rightarrow 1.00011 \times 10^{-4} a = b;$$

$$1 = a + b;$$
  $\Rightarrow 1 = a + 1.00011 \times 10^{-4} a;$   $\Rightarrow 1 = 1.0001a;$   $\Rightarrow a = 0.99;$   $\Rightarrow b = 1 - a = 1 - 0.99 = 0.01 = 1\%$ 

(b) 
$$-\frac{\Delta \overline{g}_T^o}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{y_p^{v_p}}{y_r^{v_r}} \right) \left( \frac{p}{p_o} \right)^{\sum_{p} v_p - \sum_{r} v_r} \right];$$

$$\Rightarrow -\frac{0}{(8.314)(1000)} = \ln \left[ \left( \frac{y_{C+D}^{v_{C+D}}}{y_{A+B}^{v_{A+B}}} \right) \left( \frac{p}{p_o} \right)^{v_{C+D}-v_{A+B}} \right]; \Rightarrow 0 = \ln \left[ \left( \frac{y_{C+D}^1}{y_{A+B}^1} \right) \left( \frac{1}{p_o} \right)^{1-1} \right];$$

$$\Rightarrow 0 = \ln \left[ \left( \frac{b}{a+b+c+d} \right) \left( \frac{1}{p_o} \right)^0 \right]; \Rightarrow 1 = \left[ \frac{b}{a} \right];$$

$$\Rightarrow a = b;$$

Therefore,

$$1 = a + b;$$
  $\Rightarrow 1 = a + a;$   $\Rightarrow 1 = 2a;$   $\Rightarrow a = 0.5;$   
 $\Rightarrow b = 1 - a = 1 - 0.5 = 0.5 = 50\%$ 

(c) 
$$-\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{y_{p}^{v_{p}}}{y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{o}} \right)^{\sum_{p} v_{p} - \sum_{r} v_{r}} \right];$$

$$\Rightarrow -\frac{-229724}{(8.314)(3000)} = \ln \left[ \left( \frac{y_{C+D}^{v_{C+D}}}{y_{A+B}^{v_{A+B}}} \right) \left( \frac{p}{p_{o}} \right)^{v_{C+D} - v_{A+B}} \right]; \Rightarrow 9.210327961 = \ln \left[ \left( \frac{y_{C+D}^{1}}{y_{A+B}^{1}} \right) \left( \frac{1}{p_{o}} \right)^{1-1} \right];$$

$$\Rightarrow 9.210327961 = \ln \left[ \left( \frac{b}{a+b+c+d} \right) \left( \frac{a}{a+b+c+d} \right) \right]; \Rightarrow 9999.79 = \left[ \frac{b}{a} \right];$$

$$\Rightarrow 9999.79a = b;$$

$$1 = a + b;$$
  $\Rightarrow 1 = a + 9999.79a;$   $\Rightarrow 1 = 10000.79a;$   $\Rightarrow a = 9.99 \times 10^{-5};$   
 $\Rightarrow b = 1 - a = 1 - (9.99 \times 10^{-5}) = 0.99 = 99\%$ 

**14-3-3** [BOD] Evaluate  $\Delta g^{-0}$  at 298 K for the reaction  $H_2 + (1/2)O_2 \leftrightarrow H_2O$  using (a) fundamental definition using formation enthalpy ( $h^{-0}_f$ ) and entropy ( $s^{-0}$ ) values, and (b) using formation Gibbs function from Table G-1.

#### **SOLUTION**

$$H_2 + 0.5O_2 \rightleftharpoons H_2O$$

Given: T = 298;

(a) 
$$\overline{g}_{T}^{o} = \Delta \left[ \overline{h} \left( p_{o}, T \right) - T \overline{s} \left( p_{o}, T \right) \right] = \Delta \overline{h} \left( T \right) - T \Delta \overline{s}^{o} \left( T \right);$$

Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} \nu_{p} \bar{h}_{p}(T) - \sum_{r} \nu_{r} \bar{h}_{r}(T);$$

$$\Rightarrow \Delta \bar{h}(T) = \nu_{H_{2}O} \bar{h}_{f,H_{2}O} - \nu_{H_{2}} \bar{h}_{f,H_{2}} - \nu_{O_{2}} \bar{h}_{f,O_{2}};$$

$$\Rightarrow \Delta \bar{h}(T) = (1)(-241,820) - (1)(0) - (0.5)(0);$$

$$\Rightarrow \Delta \bar{h}(T) = -241,820 \frac{kJ}{kmol}$$

And then evaluating entropy and using the Table G-1,

$$\Delta \overline{s}^{\circ}(T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\circ}(T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\circ}(T);$$

$$\Rightarrow \Delta \overline{s}^{\circ}(T) = \nu_{\text{H}_{2}\text{O}} \overline{s}_{\text{H}_{2}\text{O@298 K}}^{\circ} - \nu_{\text{H}_{2}} \overline{s}_{\text{H}_{2}@298 K}^{\circ} - \nu_{\text{O}_{2}} \overline{s}_{\text{O}_{2}@298 K}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ}(T) = (1)(188.83) - (1)(130.68) - (0.5)(205.04);$$

$$\Rightarrow \Delta \overline{s}^{\circ}(T) = -44.37 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Combining all the terms now we obtain,

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -241,820 - (298)(-44.37);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -228,597 \frac{\text{kJ}}{\text{kmol}}$$

(b) Now using Gibb's formation function from Table G.1 and realizing that we only need to look up one value for water vapor we find that  $\bar{g}_f^{\,\circ} = -228,590 \, \frac{\text{kJ}}{\text{kmol}}$ , which is very close to what we had before in part (a).

**14-3-4** [BOM] Evaluate (a)  $\Delta h^{-0}$ , (b)  $\Delta g^{-0}$ , and (c)  $\ln K$  at 298 K for the reaction CO +  $1/2O_2 \leftrightarrow CO_2$  at 1 atm.

## **SOLUTION**

$$CO + 0.5O_2 \rightleftharpoons CO_2$$

Given: T = 298 K;  $p_0 = 1 \text{ atm}$ 

(a) Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} \nu_{p} \bar{h}_{p}(T) - \sum_{r} \nu_{r} \bar{h}_{r}(T);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = \nu_{\text{CO}_{2}} \bar{h}_{f,\text{CO}_{2}} - \nu_{\text{CO}} \bar{h}_{f,\text{CO}} - \nu_{\text{O}_{2}} \bar{h}_{f,\text{O}_{2}};$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-393,520) - (1)(-110,530) - (0.5)(0);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = -282990 \frac{\text{kJ}}{\text{kmol}} = -282.99 \frac{\text{MJ}}{\text{kmol}}$$

(b) 
$$\Delta \overline{g}_{T}^{o} = \Delta \left[ \overline{h} \left( p_{o}, T \right) - T \overline{s} \left( p_{o}, T \right) \right] = \Delta \overline{h} \left( T \right) - T \Delta \overline{s}^{o} \left( T \right);$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = \nu_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2}@298 \,\text{K}}^{\circ} - \nu_{\text{CO}} \overline{s}_{\text{CO}@298 \,\text{K}}^{\circ} - \nu_{\text{O}_{2}} \overline{s}_{\text{O}_{2}@298 \,\text{K}}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(213.685) - (1)(197.543) - (0.5)(205.0);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = -86.358 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -282990 - (298)(-86.358);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -257,255 = -257.255 \frac{\text{MJ}}{\text{kmol}}$$

(c) 
$$\ln K = -\frac{\Delta \overline{g}_{T}^{\circ}}{\overline{R}T};$$
  

$$\Rightarrow \ln K = -\frac{-257, 255}{(8.314)(298)};$$

$$\Rightarrow \ln K = 103.83$$

**14-3-5** [BOW] A mixture of  $CO_2$ , CO and  $O_2$  is in equilibrium at a specified temperature and pressure. Now the pressure is tripled, (a) will the equilibrium constant K change? (b) Will the number of moles of  $CO_2$ , CO and  $O_2$  change? How? (Enter 1 if increases, 2 if decreases, and 3 if no change)

## **SOLUTION**

$$CO_2 \rightarrow (1-\varepsilon)CO_2 + aCO + bO_2$$

C balance: 
$$1 = (1 - \varepsilon) + a$$
;  $\Rightarrow a = \varepsilon$ 

O balance: 
$$2 = 2(1 - \varepsilon) + a + 2b$$
;  $\Rightarrow b = \frac{\varepsilon}{2}$ 

$$CO_2 \rightarrow (1-\varepsilon)CO_2 + \varepsilon CO + \frac{\varepsilon}{2}O_2$$

Therefore,

$$(1-\varepsilon)+a+b=1-\varepsilon+\varepsilon+\frac{\varepsilon}{2}=\frac{2+\varepsilon}{2}$$

$$y_{\text{CO}_2}=\frac{2(1-\varepsilon)}{2+\varepsilon}; \quad y_{\text{CO}}=\frac{2\varepsilon}{2+\varepsilon}; \quad y_{\text{O}_2}=\frac{\varepsilon}{2+\varepsilon}$$

From the equilibrium equation:

$$-\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{\prod_{p} y_{p}^{v_{p}}}{\prod_{r} y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{o}} \right)^{\sum_{p} v_{p}} \sum_{r} v_{r}}, \right];$$

The associated elementary step,

$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$$

$$K = \left| \left( \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \right) \left( \frac{p}{p_{0}} \right)^{1.5-1} \right|;$$

(a) If pressure is tripled and temperature and number of moles is held constant:

The equilibrium constant is independent of the pressure at a given temperature, any change in pressure is absorbed by a change in mole fraction.

Therefore, *K* remains the same: 3

(b) If pressure is tripled and temperature and *K* is held constant:

$$K_{1} = \left[ \left( \frac{\prod_{p} y_{p}^{v_{p}}}{\prod_{r} y_{r}^{v_{r}}} \right) \left( \frac{3p}{p_{0}} \right)^{\sum_{p} v_{p} - \sum_{r} v_{r}} \right] = (3)^{\sum_{p} v_{p} - \sum_{r} v_{r}} \left[ \left( \frac{\prod_{p} y_{p}^{v_{p}}}{\prod_{r} y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{0}} \right)^{\sum_{p} v_{p} - \sum_{r} v_{r}} \right]$$

Since we know K remains constant, the change in right hand side of the above equation due to the change in the  $(p/p_0)^{\sum_p v_p - \sum_r v_r}$  term has to be canceled out. For the given reaction  $v_p$  and  $v_r$  are fixed quantities, and  $\sum_p v_p - \sum_r v_r = 0.5$ , which is positive, so any increase in p will result in a larger pressure term. Tripling the pressure results in  $(3)^{0.5} (p/p_0)^{0.5}$ . In order to nullify this, the product term must decrease in response, and since  $v_p$  and  $v_r$  are fixed,  $v_p$  and  $v_r$  are the only terms that can change. Therefore to keep the right hand side of the equation constant by cancelling out the  $v_p$  and  $v_r$  are fixed,  $v_p$  and  $v_r$  are the oncentration of the product has to decrease. Therefore,

 $CO_2$  = increases: 1 CO = decreases: 2  $O_2$  = decreases: 2 **14-3-6** [BBR] Suppose the equilibrium constant of the dissociation reaction  $H_2 \rightleftharpoons 2H$  at 2000 K and 1 atm is  $K_1$ . Express the equilibrium constants of the following reactions at 2000 K in terms of  $K_1$ : (a)  $H_2 \rightleftharpoons 2H$  at 4 atm, (b)  $2H \rightleftharpoons H_2$  at 1 atm, (c)  $2H_2 \rightleftharpoons 4H$  at 1 atm, (d)  $H_2 + 2N_2 \rightleftharpoons 2H + 2N_2$  at 2 atm and (e)  $6H \rightleftharpoons 3H_2$  at 4 atm.

## **SOLUTION**

Using the ideal gas model assumption, a specific equation for calculating the equilibrium constant will be used in finding the new equilibrium constant with respect to K1. The temperature of parts a through e do not change so no major calculations of enthalpy nor entropy need to be made.

The equation used is:

$$K = \left(\frac{y_p^{v_p}}{y_r^{v_r}}\right) \left(\frac{p}{p_0}\right)^{\sum v_p - \sum v_r}$$

Where  $p_0 = 1$  atm,  $y_k =$  mole fraction

$$y_i = \frac{n_i}{n_{\text{total}}}$$

The inverse reaction is the reciprocal of K, therefore:

$$K_{\text{inverse}} = \frac{1}{K}$$

In another form:

$$\log_{10} K_{\text{inverse}} = -\log_{10} K$$

For the case of  $K_1$ :

$$y_r = \frac{1}{3}$$

$$y_p = \frac{2}{3}$$

$$\ln K_1 = \ln \left| \left( \frac{0.66^2}{0.33^1} \right) \left( \frac{1 \text{ atm}}{1 \text{ atm}} \right)^{2-1} \right| = \ln \left[ \frac{4}{3} \right]$$

$$K_1 = \frac{4}{3}$$

With these equations, part a though e can be solved.

(a)  $H_2 \rightleftharpoons 2H$  at 4 atm

$$K = \left[ \left( \frac{0.66^2}{0.33^1} \right) \left( \frac{4 \text{ atm}}{1 \text{ atm}} \right)^{2-1} \right] = \frac{16}{3}$$

Therefore,

$$K = 4K_1$$

(b)  $2H \rightleftharpoons H_2$  at 1 atm

This is the inverse of the original reaction. Therefore,

$$K = \frac{1}{K_1}$$

(c)  $2H_2 \rightleftharpoons 4H$  at 1 atm

The mole fractions of the reactants and products are still the same even though there are more kmols of products and reactants. Since the pressure ratio is equal to one the equilibrium constant doesn't change. Therefore,

$$K = \left[ \left( \frac{0.66^2}{0.33^1} \right) \left( \frac{1 \text{ atm}}{1 \text{ atm}} \right)^{4-2} \right] = \frac{4}{3}$$

Therefore,

$$K = K_1$$

(d)  $H_2 + 2N_2 \rightleftharpoons 2H + 2N_2$  at 2 atm

$$K = \left[ \frac{\left(\frac{2}{7}\right)^2 \left(\frac{2}{7}\right)^2}{\left(\frac{1}{7}\right)^1 \left(\frac{2}{7}\right)^2} \left(\frac{2 \text{ atm}}{1 \text{ atm}}\right)^{4-3} \right] = \frac{24}{21}$$

Therefore,

$$K = \frac{6}{7} K_1$$

(e)  $6H \rightleftharpoons 3H_2$  at 4 atm

$$K = \left\lfloor \left( \frac{\left(\frac{3}{9}\right)^3}{\left(\frac{6}{9}\right)^6} \right) \left( \frac{4 \text{ atm}}{1 \text{ atm}} \right)^{3-6} \right\rfloor = 0.0049 \left\lfloor \frac{4}{3} \right\rfloor$$

$$K = 0.0049 K_{1}$$



**14-3-7** [BBB] A mixture of NO,  $O_2$  and  $N_2$  is in equilibrium at a specified temperature and pressure. The pressure is doubled, (a) will the equilibrium constant K change? (b) Will the number of moles of NO,  $O_2$  and  $N_2$ , change? How?

#### **SOLUTION**

$$NO \rightarrow (1-\varepsilon)NO + aN_2 + bO_2$$

N balance: 
$$1 = (1 - \varepsilon) + 2a$$
;  $\Rightarrow a = \frac{\varepsilon}{2}$ 

O balance: 
$$1 = (1 - \varepsilon) + 2b$$
;  $\Rightarrow b = \frac{\varepsilon}{2}$ 

$$NO \rightarrow (1-\varepsilon)NO + \frac{\varepsilon}{2}N_2 + \frac{\varepsilon}{2}O_2$$

Therefore

$$(1-\varepsilon)+a+b=1-\varepsilon+\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=1$$

$$y_{\text{NO}} = 1 - \varepsilon;$$
  $y_{\text{NO}} = \frac{\varepsilon}{2};$   $y_{\text{O}_2} = \frac{\varepsilon}{2}$ 

From the equilibrium equation:

$$-\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{\prod_{p} y_{p}^{v_{p}}}{\prod_{r} y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{o}} \right)^{\sum_{p} v_{p} - \sum_{r} v_{r}} y_{r}^{v_{r}} \right];$$

The associated elementary step,

$$NO \rightleftharpoons \frac{1}{2}N_2 + \frac{1}{2}O_2$$

$$K = \left[ \left( \frac{y_{N_2}^{0.5} y_{O_2}^{0.5}}{y_{NO}^1} \right) \left( \frac{p}{p_0} \right)^{1-1} \right]$$

(a) If pressure is doubled and temperature and number of moles is held constant:

The equilibrium constant is independent of the pressure at a given temperature, any change in pressure is absorbed by a change in mole fraction.

Therefore, *K* remains the same: 3

(b) If pressure is doubled and temperature and K is held constant:

$$K_{1} = \left[ \left( \frac{\prod_{p} y_{p}^{v_{p}}}{\prod_{r} y_{r}^{v_{r}}} \right) \left( \frac{2p}{p_{0}} \right)^{\sum_{p} v_{p} - \sum_{r} v_{r}} \right] = (2)^{\sum_{p} v_{p} - \sum_{r} v_{r}} \left[ \left( \frac{\prod_{p} y_{p}^{v_{p}}}{\prod_{r} y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{0}} \right)^{\sum_{p} v_{p} - \sum_{r} v_{r}} \right]$$

We know that K is a constant, so the  $(2)_p^{\sum_p v_p - \sum_r v_r}$  has to be canceled out. We know that for this reaction  $v_p$  and  $v_r$  are fixed, and as such for this case  $\sum_p v_p - \sum_r v_r = 0$ , so  $(2)_p^{\sum_p v_p - \sum_r v_r} = 1$  and the change in pressure has no effect on the right hand side of the

 $(2)^{\sum_{p} v_{p} - \sum_{r} v_{r}}_{r} = 1$  and the change in pressure has no effect on the right hand side of the equation. Therefore the concentrations will not change.

NO = no change: 3  $N_2$  = no change: 3  $O_2$  = no change: 3



**14-3-8** [BOJ] Suppose the equilibrium constant of the reaction  $CO + 1/2O_2 \rightleftharpoons CO_2$  at 1500 K and 1 atm is  $K_1$ . Express the equilibrium constant of the following reactions at 1500 K in terms of  $K_1$ . (a)  $CO + (1/2)O_2 \rightleftharpoons CO_2$  at 2 atm, (b)  $CO_2 \rightleftharpoons CO + (1/2)O_2$  at 1 atm, (c)  $CO + O_2 \rightleftharpoons CO_2 + (1/2)O_2$  at 1 atm, (d)  $CO + 2O_2 + 6N_2 \rightleftharpoons CO_2 + 1.5O_2 + 6N_2$  at 5 atm and (e)  $2CO + O_2 \rightleftharpoons 2CO_2$ .

### **SOLUTION**

Using the ideal gas model assumption, a specific equation for calculating the equilibrium constant will be used in finding the new equilibrium constant with respect to K1. The temperature of parts a through e do not change so no major calculations of enthalpy or entropy need to be made.

The equation used is:

$$K = \left(\frac{y_p^{\nu_p}}{y_r^{\nu_r}}\right) \left(\frac{p}{p_0}\right)^{\sum \nu_p - \sum \nu_r}$$

Where  $p_0 = 1$  atm,  $y_k =$  mole fraction

$$y_i = \frac{n_i}{n_{\text{total}}};$$

The inverse reaction is the reciprocal of K, therefore:

$$K_{\text{inverse}} = \frac{1}{K}$$

In another form:

$$\log_{10} K_{\text{inverse}} = -\log_{10} K$$

For the case of  $K_1$ :

$$\ln K_1 = \ln \left[ \left( \frac{\left( \frac{1}{2.5} \right)^1}{\left( \frac{1}{2.5} \right)^1 \left( \frac{0.5}{2.5} \right)^{0.5}} \right] \left( \frac{1 \text{ atm}}{1 \text{ atm}} \right)^{1-1.5} \right] = \ln \left[ 2.236 \right]$$

$$K_1 = 2.236$$

With these equations, part a though e can be solved.

(a) 
$$CO + 0.5O_2 \rightleftharpoons CO_2$$
 at 2 atm

$$K = \left| \left( \frac{\left( \frac{1}{2.5} \right)^{1}}{\left( \frac{1}{2.5} \right)^{1} \left( \frac{0.5}{2.5} \right)^{0.5}} \right) \left( \frac{2 \text{ atm}}{1 \text{ atm}} \right)^{1-1.5} \right| = 3.549$$

Therefore,

$$K = 2^{-0.5} K_1$$

(b)  $CO_2 \rightleftharpoons CO + 0.5O_2$  at 1 atm

This is the inverse of the original reaction. Therefore,

$$K = \frac{1}{K_1}$$

(c)  $CO + O_2 \rightleftharpoons CO_2 + 0.5O_2$  at 1 atm

$$K = \left[ \left( \frac{\left(\frac{1}{3.5}\right)^{1} \left(\frac{0.5}{3.5}\right)^{0.5}}{\left(\frac{1}{3.5}\right)^{1} \left(\frac{1}{3.5}\right)^{1}} \left( \frac{1 \text{ atm}}{1 \text{ atm}} \right)^{1.5-2} \right] = 1.3229$$

Therefore,

$$K = 0.5916K_1$$

(d)  $CO + 2O_2 + 6N_2 \rightleftharpoons CO_2 + 1.5O_2 + 6N_2$  at 5 atm

$$K = \left[ \frac{\left(\frac{1}{17.5}\right)^{1} \left(\frac{1.5}{17.5}\right)^{1.5} \left(\frac{6}{17.5}\right)^{6}}{\left(\frac{1}{17.5}\right)^{1} \left(\frac{2}{17.5}\right)^{2} \left(\frac{6}{17.5}\right)^{6}} \left(\frac{5 \text{ atm}}{1 \text{ atm}}\right)^{8.5-9} \right] = 0.8592$$

Therefore,

$$K = 0.3843K_{\perp}$$

(e)  $2\text{CO} + \text{O}_2 \rightleftharpoons 2\text{CO}_2$ 

$$K = \left| \left( \frac{\left(\frac{2}{5}\right)^2}{\left(\frac{2}{5}\right)^2 \left(\frac{1}{5}\right)^1} \right) \left( \frac{1 \text{ atm}}{1 \text{ atm}} \right)^{2-3} \right| = 5$$

Therefore,  $K = K_1^2$ 



**14-3-9** [BBO] For the dissociation of nitrogen tetraoxide according to the reaction,  $N_2O_4 \Rightarrow 2NO_2$ , show that the degree of dissociation at equilibrium is given by  $\varepsilon = V_e/V_0-1$ , where  $V_0$  is the initial volume and  $V_e$  is the equilibrium volume.

## **SOLUTION**

$$N_2O_4 \rightleftharpoons 2NO_2$$

$$V_0 N_2 O_4 \rightleftharpoons V_0 (1-\varepsilon) N_2 O_4 + 2V_0 (\varepsilon) NO_2$$

$$V_e = V_0 - V_0 \varepsilon + 2V_0 \varepsilon = V_0 + V_0 \varepsilon;$$

$$\Rightarrow V_e = V_0 (1 + \varepsilon);$$

$$\Rightarrow \frac{\overline{V_e}}{\overline{V_0}} = (1 + \varepsilon);$$

$$\Rightarrow \varepsilon = \frac{V_e}{V_0} - 1$$

**14-3-10** [BBH] For the dissociation of nitrogen tetraoxide described in problem 14-3-9 [BBO], there is a 77.7% increase in volume when equilibrium is reached at  $50^{\circ}$ C, 125 kPa. Determine the value of the equilibrium constant (K).

## **SOLUTION**

$$N_2O_4 \rightleftharpoons 2NO_2$$

$$V_0 N_2 O_4 \rightleftharpoons V_0 (1-\varepsilon) N_2 O_4 + 2V_0 (\varepsilon) NO_2$$

Therefore,

$$\begin{aligned} & V_e = V_0 - V_0 \varepsilon + 2V_0 \varepsilon = V_0 + V_0 \varepsilon; \\ & \Rightarrow V_e = V_0 (1 + \varepsilon); \\ & \Rightarrow \frac{V_e}{V_0} = (1 + \varepsilon); \end{aligned}$$

$$\Rightarrow \varepsilon = \frac{V_e}{V_0} - 1$$

For this problem, we will assume that the initial volume is 1 m<sup>3</sup>, making the equilibrium volume equal to 1.77 m<sup>3</sup>. Solving for the dissociation,

$$\varepsilon = \frac{V_e}{V_0} - 1 = \frac{1.77}{1} - 1 = 0.77$$

The equilibrium equation now becomes:

$$N_2O_4 \rightleftharpoons 0.33N_2O_4 + 1.54NO_2$$

$$y_{N_2O_4} = \frac{0.33}{1.87}; \quad y_{NO_2} = \frac{1.54}{1.87}$$

$$\ln K = \ln \left[ \left( \frac{y_p^{\nu_p}}{y_r^{\nu_r}} \right) \left( \frac{p}{p_0} \right)^{\sum_{p} \nu_p - \sum_{r} \nu_r} \right] = \ln \left[ \left( \frac{y_{NO_2}^{\nu_{NO_2}}}{y_{N_2O_4}^{\nu_{N_2O_4}}} \right) \left( \frac{p}{p_0} \right)^{\nu_{NO_2} - \nu_{N_2O_4}} \right] = \ln \left[ \frac{\left( \frac{1.54}{1.87} \right)^2}{\left( \frac{0.33}{1.87} \right)} \left( \frac{125}{100} \right)^{2-1} \right];$$

$$\Rightarrow \ln K = \ln(4.81);$$

$$\Rightarrow K = 4.81$$

**14-3-11** [BBS] A chamber contains a mixture of  $CO_2$ , CO and  $O_2$  is in equilibrium at a specified temperature and pressure. How will (a) increasing the temperature (T) at constant pressure and (b) increasing the pressure (p) at constant temperature affect the number of moles of CO? (Enter 1 if increases, 2 if decreases, and 3 if no change)

## **SOLUTION**

$$CO_2 \rightarrow (1-\varepsilon)CO_2 + aCO + bO_2$$

C balance: 
$$1 = (1 - \varepsilon) + a$$
;  $\Rightarrow a = \varepsilon$ 

O balance: 
$$2 = 2(1 - \varepsilon) + a + 2b$$
;  $\Rightarrow b = \frac{\varepsilon}{2}$ 

$$CO_2 \rightarrow (1-\varepsilon)CO_2 + \varepsilon CO + \frac{\varepsilon}{2}O_2$$

Therefore,

$$(1-\varepsilon) + a + b = 1 - \varepsilon + \varepsilon + \frac{\varepsilon}{2} = \frac{2+\varepsilon}{2}$$

$$2(1-\varepsilon) \qquad 2\varepsilon \qquad \varepsilon$$

$$y_{\text{CO}_2} = \frac{2(1-\varepsilon)}{2+\varepsilon}; \quad y_{\text{CO}} = \frac{2\varepsilon}{2+\varepsilon}; \quad y_{\text{O}_2} = \frac{\varepsilon}{2+\varepsilon}$$

From the equilibrium equation:

$$-\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T} = \ln K = \ln \left[ \left( \frac{\prod_{p} y_{p}^{v_{p}}}{\prod_{r} y_{r}^{v_{r}}} \right) \left( \frac{p}{p_{o}} \right)^{\sum_{p} v_{p}} \sum_{r} v_{r}}, \right];$$

The associated elementary step,

$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$$

$$K = \left| \left( \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \right) \left( \frac{p}{p_{0}} \right)^{1.5-1} \right|;$$

(a) For this elementary step, checking against Table G-3, we can see that for

$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$$
 the natural logarithm of the equilibrium constant increases with

temperature. As such, the concentration of the products must increase (and the reactants decrease) to match this change since there is no pressure effect. Therefore, the moles of CO must increase: 1

(b) Again, looking at the above expression, it can be seen that when the temperature is at a constant specified value, in order to keep the value  $\ln K$  the same, the number of moles of products must decrease to offset the effect of the pressure increase since  $\sum_{p} v_{p} - \sum_{r} v_{r} = 0.5$ . This causes the number of moles of CO to decrease: 2



**14-3-12** [BBA] Determine the change in the Gibbs function  $\Delta g^{-o}$  at 25°C, in kJ/kmol, for the reaction  $CH_4(g) + 2O_2 \leftrightarrow CO_2 + 2H_2O(g)$  using enthalpy of formation and absolute entropy data.

## **SOLUTION**

$$\Delta \overline{g}_{T}^{\circ} = \Delta \left\lceil \overline{h} \left( p_{0}, T \right) - T \overline{s} \left( p_{0}, T \right) \right\rceil = \Delta \overline{h} \left( T \right) - T \Delta \overline{s}^{\circ} \left( T \right);$$

Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} v_{p} \bar{h}_{p}(T) - \sum_{r} v_{r} \bar{h}_{r}(T);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = v_{\text{CO}_{2}} \bar{h}_{f,\text{CO}_{2}} + v_{\text{H}_{2}\text{O}} \bar{h}_{f,\text{H}_{2}\text{O}} - v_{\text{CH}_{4}} \bar{h}_{f,\text{CH}_{4}} - v_{\text{O}_{2}} \bar{h}_{f,\text{O}_{2}};$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-393,520) + (2)(-241,820) - (1)(-74,850) - (2)(0);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = -802,310 \frac{\text{kJ}}{\text{kmol}}$$

Evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{o}(T) = \sum_{p} v_{p} \overline{s}_{p}^{o}(T) - \sum_{r} v_{r} \overline{s}_{r}^{o}(T);$$

$$\Rightarrow \Delta \overline{s}^{o} = v_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2}@298 \text{ K}}^{o} + v_{\text{H}_{2}\text{O}} \overline{s}_{\text{H}_{2}\text{O}@298 \text{ K}}^{o} - v_{\text{CH}_{4}} \overline{s}_{\text{CH}_{4}@298 \text{ K}}^{o} - v_{\text{O}_{2}} \overline{s}_{\text{O}_{2}@298 \text{ K}}^{o};$$

$$\Rightarrow \Delta \overline{s}^{o} = (1)(213.8) + (2)(188.83) - (1)(186.16) - (2)(205.04);$$

$$\Rightarrow \Delta \overline{s}^{o} = -4.78 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\Delta \overline{g}_{T}^{o} = \Delta \overline{h} (T) - T \Delta \overline{s}^{o} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{o} = -802310 - (298)(-4.78);$$

$$\Rightarrow \Delta \overline{g}_{T}^{o} = -800,886 \frac{\text{kJ}}{\text{kmol}}$$

**14-3-13** [BBN] Calculate the equilibrium constant (K) for  $CO_2 \rightleftharpoons CO + 1/2O_2$  at (a) 500 K and (b) 1000 K.

## **SOLUTION**

$$CO_2 \rightleftharpoons CO + 0.5O_2$$

(a) At 
$$T = 500 \text{ K}$$

Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} \nu_{p} \bar{h}_{p}(T) - \sum_{r} \nu_{r} \bar{h}_{r}(T);$$

$$\Delta \bar{h}^{\circ} = \nu_{\text{CO}}(\bar{h}_{f,\text{CO}} + \Delta \bar{h}_{\text{CO}}(T)) + \nu_{\text{O}_{2}}(\bar{h}_{f,\text{O}_{2}} + \Delta \bar{h}_{\text{O}_{2}}(T)) - \nu_{\text{CO}_{2}}(\bar{h}_{f,\text{CO}_{2}} + \Delta \bar{h}_{\text{CO}_{2}}(T));$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-110,527 + 5,931) + (0.5)(0 + 6,088) - (1)(-393,522 + 8,314);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = 283,656 \frac{\text{kJ}}{\text{kmol}}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = \nu_{\text{CO}} \overline{s}_{\text{CO}@500 \text{ K}}^{\circ} + \nu_{\text{O}_{2}} \overline{s}_{\text{O}_{2}@500 \text{ K}}^{\circ} - \nu_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2}@500 \text{ K}}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(212.719) + (0.5)(220.589) - (1)(234.814);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = 88.1995 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Combining all the terms now we obtain,

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 283,656 - (500)(88.1995);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 239,556.25 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_{T}^{\circ}}{RT};$$

$$\Rightarrow \ln K = -\frac{239,556.25}{(8.314)(500)};$$

$$\Rightarrow \ln K = -57.627$$

(b) At 
$$T = 1000 \text{ K}$$

Handling the enthalpy term first and using Table G-1 we get,

$$\begin{split} & \Delta \bar{h}\left(T\right) = \bar{h}_{p}\left(T\right) - \bar{h}_{r}\left(T\right) = \sum_{p} \nu_{p} \bar{h}_{p}\left(T\right) - \sum_{r} \nu_{r} \bar{h}_{r}\left(T\right); \\ & \Delta \bar{h}^{\circ} = \nu_{\text{CO}}\left(\bar{h}_{f,\text{CO}} + \Delta \bar{h}_{\text{CO}}\left(T\right)\right) + \nu_{\text{O}_{2}}\left(\bar{h}_{f,\text{O}_{2}} + \Delta \bar{h}_{\text{O}_{2}}\left(T\right)\right) - \nu_{\text{CO}_{2}}\left(\bar{h}_{f,\text{CO}_{2}} + \Delta \bar{h}_{\text{CO}_{2}}\left(T\right)\right); \\ & \Rightarrow \Delta \bar{h}^{\circ} = (1)\left(-110,527 + 21,686\right) + \left(0.5\right)\left(0 + 22,714\right) - \left(1\right)\left(-393,522 + 33,432\right); \\ & \Rightarrow \Delta \bar{h}^{\circ} = 282,606 \frac{\text{kJ}}{\text{kmol}} \end{split}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = \nu_{\text{CO}} \overline{s}_{\text{CO@1000 K}}^{\circ} + \nu_{\text{O}_{2}} \overline{s}_{\text{O}_{2@1000 K}}^{\circ} - \nu_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2@1000 K}}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(234.421) + (0.5)(243.634) - (1)(269.215);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = 87.023 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 282,606 - (1000)(87.023);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 195,583 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_{T}^{\circ}}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{195,583}{(8.314)(1000)};$$

$$\Rightarrow \ln K = -23.52$$

**14-3-14** [BBE] One kmol of  $CO_2$  is heated at a constant pressure of 100 kPa to 3000 K. (a) Calculate the equilibrium composition of  $CO_2$  and use the IGE system state TESTcalc to verify your answers. (b) Use the n-IG system state TESTcalc and calculate the Gibbs function of the mixture  $(1-x)CO_2 + xCO + (x/2)O_2$  at 100 kPa, 3000 K for x varying from 0 to 1. Show that for x = 0.438, Gibbs function reaches a minimum.

## **SOLUTION**

$$CO_2 \rightleftharpoons CO + 0.5O_2$$

Molar analysis:

$$CO_2 \rightarrow (1-\varepsilon)CO_2 + \varepsilon CO + aO_2$$

$$C:1=1-\varepsilon+\varepsilon=1$$

$$O: 2 = 2(1-\varepsilon) + \varepsilon + 2a; \quad \Rightarrow 2 = 2 - 2\varepsilon + \varepsilon + 2a; \quad \Rightarrow a = \frac{\varepsilon}{2}$$

Therefore,

$$a+b+c = a+1-a+\frac{1-a}{2} = \frac{3-a}{2}$$

$$y_{\text{CO}_2} = \frac{1-\varepsilon}{1+a};$$
  $y_{\text{CO}} = \frac{\varepsilon}{1+a};$   $y_{\text{O}_2} = \frac{a}{1+a};$ 

For  $CO_2 \rightleftharpoons CO + 0.5O_2$  on Table G-3 at 3000 K,  $\ln K_p = -1.111$ 

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{v_{\text{CO}}} y_{\text{O}_{2}}^{v_{\text{O}_{2}}}}{y_{\text{CO}_{2}}^{v_{\text{O}_{2}}}} \right) \left( \frac{p}{p_{0}} \right)^{v_{\text{O}} y_{\text{O}_{2}}^{0.5}} = \ln \left[ \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \times (1)^{1.5-1} \right] = \ln \left[ \frac{\varepsilon}{1+a} \right) \left( \frac{a}{1+a} \right)^{0.5} \right];$$

$$\Rightarrow \ln K_{p} = \ln \left[ \left( \frac{\varepsilon}{1-\varepsilon} \right) \left( \frac{a}{1+a} \right)^{0.5} \right]; \Rightarrow -1.111 = \ln \left[ \left( \frac{\varepsilon}{1-\varepsilon} \right) \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \right];$$

$$\Rightarrow e^{-1.111} = \left[ \left( \frac{\varepsilon}{1-\varepsilon} \right) \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \right] \Rightarrow 0.3292296 = \left[ \left( \frac{\varepsilon}{1-\varepsilon} \right) \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \right];$$

$$\Rightarrow 0.1083921077 = \left[ \left( \frac{\varepsilon}{1-\varepsilon} \right) \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \right]^{2}; \Rightarrow 0.1083921077 = \frac{\varepsilon^{3}}{\varepsilon^{3} - 3\varepsilon + 2};$$

$$\Rightarrow 0.1083921077 \varepsilon^{3} - 0.3251763231\varepsilon + 0.2167843154 = \varepsilon^{3};$$

$$\Rightarrow 0 = 0.8916078923\varepsilon^{3} + 0.3251763231\varepsilon - 0.2167843154;$$

$$\Rightarrow \varepsilon = 0.437328$$

$$\text{CO}_{2} = 1 - \varepsilon = 0.562672 \text{ kmol}$$

**14-3-15** [BBI] Calculate the equilibrium constant (K) for the water-gas reaction CO +  $H_2O(g) \leftrightarrow CO_2 + H_2$  at (a) 298 K and (b) 1000 K.

# **SOLUTION**

$$CO + H_2O \rightleftharpoons CO_2 + H_2$$

(a) Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} \nu_{p} \bar{h}_{p}(T) - \sum_{r} \nu_{r} \bar{h}_{r}(T);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = \nu_{\text{CO}_{2}} \bar{h}_{f,\text{CO}_{2}} + \nu_{\text{H}_{2}} \bar{h}_{f,\text{H}_{2}} - \nu_{\text{CO}} \bar{h}_{f,\text{CO}} - \nu_{\text{H}_{2}\text{O}} \bar{h}_{f,\text{H}_{2}\text{O}};$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-393,520) + (1)(0) - (1)(-110,530) - (1)(-241,820);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = -41,170 \frac{\text{kJ}}{\text{kmol}}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = \nu_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2}@298 \text{ K}}^{\circ} + \nu_{\text{H}_{2}} \overline{s}_{\text{H}_{2}@298 \text{ K}}^{\circ} - \nu_{\text{CO}} \overline{s}_{\text{CO}@298 \text{ K}}^{\circ} - \nu_{\text{H}_{2}\text{O}} \overline{s}_{\text{H}_{2}\text{O}@298 \text{ K}}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(213.8) + (1)(130.68) - (1)(197.65) - (1)(188.83);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = -42 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Combining all the terms now we obtain,

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -41,170 - (298)(-42);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -28,654 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_T^{\circ}}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{-28,654}{(8.314)(298)};$$

$$\Rightarrow \ln K = 11.57$$

$$\Rightarrow K = 105,873$$

(b) Handling the enthalpy term first and using Table G-1 we get,

$$\begin{split} & \Delta \overline{h}\left(T\right) = \overline{h}_{p}\left(T\right) - \overline{h}_{r}\left(T\right) = \sum_{p} \nu_{p} \overline{h}_{p}\left(T\right) - \sum_{r} \nu_{r} \overline{h}_{r}\left(T\right); \\ & \Delta \overline{h}^{\circ} = \nu_{\text{CO}_{2}}\left(\overline{h}_{f,\text{CO}_{2}} + \Delta \overline{h}_{\text{CO}_{2}}(T)\right) + \nu_{\text{H}_{2}}\left(\overline{h}_{f,\text{H}_{2}} + \Delta \overline{h}_{\text{H}_{2}}(T)\right) - \nu_{\text{CO}}\left(\overline{h}_{f,\text{CO}} + \Delta \overline{h}_{\text{CO}}(T)\right) - \nu_{\text{H}_{2}\text{O}}\left(\overline{h}_{f,\text{H}_{2}\text{O}} + \Delta \overline{h}_{\text{H}_{2}\text{O}}(T)\right); \end{split}$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-393,520+33,432)+(1)(20,686)-(1)(-110,530+21,686)-(1)(-241,820+25,978);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = -34,716 \frac{\text{kJ}}{\text{kmol}}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = \nu_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2}@1000 \text{ K}}^{\circ} + \nu_{\text{H}_{2}} \overline{s}_{\text{H}_{2}@1000 \text{ K}}^{\circ} - \nu_{\text{CO}} \overline{s}_{\text{CO}@1000 \text{ K}}^{\circ} - \nu_{\text{H}_{2}\text{O}} \overline{s}_{\text{H}_{2}\text{O}@1000 \text{ K}}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(269.215) + (1)(166.114) - (1)(234.421) - (1)(232.597);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = -31.689 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -34,716 - (1000)(-31.689);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -3,027 \frac{kJ}{kmol}$$

$$\ln K = -\frac{\Delta \overline{g}_{T}^{\circ}}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{-3,027}{(8.314)(1000)};$$

$$\Rightarrow \ln K = 0.364$$

$$\Rightarrow K = 1.44$$

**14-3-16** [BBZ] Determine the equilibrium constant (K) for the reaction  $H_2 + 1/2O_2 \leftrightarrow H_2O$  at (a) 298 K and (b) 2500 K.

## **SOLUTION**

$$H_2 + 0.5O_2 \rightleftharpoons H_2O$$

(a) At 
$$T = 298 \text{ K}$$

Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} v_{p} \bar{h}_{p}(T) - \sum_{r} v_{r} \bar{h}_{r}(T);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = v_{\text{H}_{2}\text{O}} \bar{h}_{f,\text{H}_{2}\text{O}} - v_{\text{H}_{2}} \bar{h}_{f,\text{H}_{2}} - v_{\text{O}_{2}} \bar{h}_{f,\text{O}_{2}};$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-241,826) - (1)(0) - (0.5)(0);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = -241,826 \frac{\text{kJ}}{\text{kmol}}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = \nu_{\text{H}_{2}\text{O}} \overline{s}_{\text{H}_{2}\text{O} @ 298 \text{ K}}^{\circ} - \nu_{\text{H}_{2}} \overline{s}_{\text{H}_{2}\text{@}298 \text{ K}}^{\circ} - \nu_{\text{O}_{2}} \overline{s}_{\text{O}_{2}\text{@}298 \text{ K}}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(188.72) - (1)(130.574) - (0.5)(205.033);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = -44.3705 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Combining all the terms now we obtain,

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -241,826 - (298)(-44.3705);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -228,603.59 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_{T}^{o}}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{-228,603.59}{(8.314)(298)};$$

$$\Rightarrow \ln K = 92.27$$

$$\Rightarrow K = e^{92.27} = 1.18 \times 10^{40}$$

(b) At 
$$T = 2500 \text{ K}$$

Handling the enthalpy term first and using Table G-1 we get,

$$\begin{split} & \Delta \overline{h} \left( T \right) = \overline{h}_{p} \left( T \right) - \overline{h}_{r} \left( T \right) = \sum_{p} \nu_{p} \overline{h}_{p} \left( T \right) - \sum_{r} \nu_{r} \overline{h}_{r} \left( T \right); \\ & \Delta \overline{h}^{\circ} = \nu_{\mathrm{H}_{2}\mathrm{O}} \left( \overline{h}_{f,\mathrm{H}_{2}\mathrm{O}} + \Delta \overline{h}_{\mathrm{H}_{2}\mathrm{O}} \left( T \right) \right) - \nu_{\mathrm{H}_{2}} \left( \overline{h}_{f,\mathrm{H}_{2}} + \Delta \overline{h}_{\mathrm{H}_{2}} \left( T \right) \right) - \nu_{\mathrm{O}_{2}} \left( \overline{h}_{f,\mathrm{O}_{2}} + \Delta \overline{h}_{\mathrm{O}_{2}} \left( T \right) \right); \\ & \Rightarrow \Delta \overline{h}^{\circ} = \left( 1 \right) \left( -241,826 + 98,964 \right) - \left( 1 \right) \left( 0 + 70,492 \right) - \left( 0.5 \right) \left( 0 + 78,375 \right); \\ & \Rightarrow \Delta \overline{h}^{\circ} = -252,541.5 \frac{\mathrm{kJ}}{\mathrm{kmol}} \end{split}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\,\circ}(T) = \sum_{p} \nu_{p} \overline{s}_{p}^{\,\circ}(T) - \sum_{r} \nu_{r} \overline{s}_{r}^{\,\circ}(T);$$

$$\Rightarrow \Delta \overline{s}^{\,\circ} = \nu_{\text{H}_{2}\text{O}} \overline{s}_{\text{H}_{2}\text{O} \oplus 2500 \,\text{K}}^{\,\circ} - \nu_{\text{H}_{2}} \overline{s}_{\text{H}_{2} \oplus 2500 \,\text{K}}^{\,\circ} - \nu_{\text{O}_{2}} \overline{s}_{\text{O}_{2} \oplus 2500 \,\text{K}}^{\,\circ};$$

$$\Rightarrow \Delta \overline{s}^{\,\circ} = (1)(276.286) - (1)(196.125) - (0.5)(277.207);$$

$$\Rightarrow \Delta \overline{s}^{\,\circ} = -58.4425 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h}(T) - T \Delta \overline{s}^{\circ}(T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -252,541.5 - (2500)(-58.4425);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -106,435.25 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_T^o}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{-106,435.25}{(8.314)(2500)};$$

$$\Rightarrow \ln K = 5.12$$

$$\Rightarrow K = 167.34$$

**14-3-17** [BBL] Determine the equilibrium constant (K) for the reaction  $CH_4 + 2O_2 \leftrightarrow CO_2 + 2H_2O$  at 25°C.

## **SOLUTION**

$$CH_4 + 2O_2 \rightleftharpoons CO_2 + 2H_2O$$

Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} \nu_{p} \bar{h}_{p}(T) - \sum_{r} \nu_{r} \bar{h}_{r}(T);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = \nu_{\text{CO}_{2}} \bar{h}_{f,\text{CO}_{2}} + \nu_{\text{H}_{2}\text{O}} \bar{h}_{f,\text{H}_{2}\text{O}} - \nu_{\text{CH}_{4}} \bar{h}_{f,\text{CH}_{4}} - \nu_{\text{O}_{2}} \bar{h}_{f,\text{O}_{2}};$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-393,520) + (2)(-241,820) - (1)(-74,850) - (2)(0);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = -802,310 \frac{\text{kJ}}{\text{kmol}}$$

And then evaluating entropy and using the IG tables:

$$\begin{split} \Delta \overline{s}^{\,o}\left(T\right) &= \sum_{p} \nu_{p} \overline{s}_{p}^{\,o}\left(T\right) - \sum_{r} \nu_{r} \overline{s}_{r}^{\,o}\left(T\right); \\ &\Rightarrow \Delta \overline{s}^{\,o} = \nu_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2}@298\,\text{K}}^{\,o} + \nu_{\text{H}_{2}\text{O}} \overline{s}_{\text{H}_{2}\text{O}@298\,\text{K}}^{\,o} - \nu_{\text{CH}_{4}} \overline{s}_{\text{CH}_{4}@298\,\text{K}}^{\,o} - \nu_{\text{O}_{2}} \overline{s}_{\text{O}_{2}@298\,\text{K}}^{\,o}; \\ &\Rightarrow \Delta \overline{s}^{\,o} = (1)(213.8) + (2)(188.83) - (1)(186.16) - (2)(205.04); \\ &\Rightarrow \Delta \overline{s}^{\,o} = -4.78 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \end{split}$$

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -802,310 - (298)(-4.78);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = -800,885.56 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_{T}^{\circ}}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{-800,885.56}{(8.314)(298)};$$

$$\Rightarrow \ln K = 323.25$$

$$\Rightarrow K = e^{323.25} = 2.43 \times 10^{140}$$

**14-3-18** [BBG] Determine the equilibrium constant (K) for the dissociation process  $CO_2 \leftrightarrow CO + 1/2O_2$  at (a) 298 K and (b) 2000 K.

## **SOLUTION**

$$CO_2 \rightleftharpoons CO + 0.5O_2$$

(a) At 
$$T = 298 \text{ K}$$

Handling the enthalpy term first and using Table G-1 we get,

$$\Delta \bar{h}(T) = \bar{h}_{p}(T) - \bar{h}_{r}(T) = \sum_{p} \nu_{p} \bar{h}_{p}(T) - \sum_{r} \nu_{r} \bar{h}_{r}(T);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = \nu_{\text{CO}} \bar{h}_{f,\text{CO}} + \nu_{\text{O}_{2}} \bar{h}_{f,\text{O}_{2}} - \nu_{\text{CO}_{2}} \bar{h}_{f,\text{CO}_{2}};$$

$$\Rightarrow \Delta \bar{h}^{\circ} = (1)(-110,530) + (0.5)(0) - (1)(-393,520);$$

$$\Rightarrow \Delta \bar{h}^{\circ} = 282,990 \frac{\text{kJ}}{\text{kmol}}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} v_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} v_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = v_{\text{CO}} \overline{s}_{\text{CO@298 K}}^{\circ} + v_{\text{O}_{2}} \overline{s}_{\text{O}_{2@298 K}}^{\circ} - v_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2@298 K}}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(197.543) + (0.5)(205.033) - (1)(213.685);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = 86.3745 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Combining all the terms now we obtain,

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 282,990 - (298)(86.3745);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 257,250.399 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_{T}^{\circ}}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{257, 250.399}{(8.314)(298)};$$

$$\Rightarrow \ln K = -103.83$$

(b) At 
$$T = 2000 \text{ K}$$

Handling the enthalpy term first and using Table G-1 we get,  $\Delta \overline{h}(T) = \overline{h}_p(T) - \overline{h}_r(T) = \sum_{p} v_p \overline{h}_p(T) - \sum_{r} v_r \overline{h}_r(T);$ 

$$\begin{split} \Delta \overline{h}^{\circ} &= \nu_{\text{CO}} \left( \overline{h}_{f,\text{CO}} + \Delta \overline{h}_{\text{CO}} \left( T \right) \right) + \nu_{\text{O}_{2}} \left( \overline{h}_{f,\text{O}_{2}} + \Delta \overline{h}_{\text{O}_{2}} \left( T \right) \right) - \nu_{\text{CO}_{2}} \left( \overline{h}_{f,\text{CO}_{2}} + \Delta \overline{h}_{\text{CO}_{2}} \left( T \right) \right); \\ &\Rightarrow \Delta \overline{h}^{\circ} = \left( 1 \right) \left( -110,530 + 56,739 \right) + \left( 0.5 \right) \left( 0 + 59,199 \right) - \left( 1 \right) \left( -393,520 + 91,440 \right); \\ &\Rightarrow \Delta \overline{h}^{\circ} = 277,888.5 \frac{\text{kJ}}{\text{kmol}} \end{split}$$

And then evaluating entropy and using the IG tables:

$$\Delta \overline{s}^{\circ} (T) = \sum_{p} v_{p} \overline{s}_{p}^{\circ} (T) - \sum_{r} v_{r} \overline{s}_{r}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = v_{\text{CO}} \overline{s}_{\text{CO@2000 K}}^{\circ} + v_{\text{O}_{2}} \overline{s}_{\text{O}_{2}@2000 K}^{\circ} - v_{\text{CO}_{2}} \overline{s}_{\text{CO}_{2}@2000 K}^{\circ};$$

$$\Rightarrow \Delta \overline{s}^{\circ} = (1)(258.6) + (0.5)(268.655) - (1)(309.21);$$

$$\Rightarrow \Delta \overline{s}^{\circ} = 83.7175 \frac{\text{kJ}}{\text{kmol. K}}$$

$$\Delta \overline{g}_{T}^{\circ} = \Delta \overline{h} (T) - T \Delta \overline{s}^{\circ} (T);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 277,888.5 - (2000)(83.7175);$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 110,453.5 \frac{\text{kJ}}{\text{kmol}}$$

$$\ln K = -\frac{\Delta \overline{g}_T^{\circ}}{\overline{R}T};$$

$$\Rightarrow \ln K = -\frac{110,453.5}{(8.314)(2000)};$$

$$\Rightarrow \ln K = -6.64$$

**14-3-19** [BBK] For the chemical reaction  $CO_2 + H_2 \leftrightarrow CO + H_2O$ , the equilibrium value of the degree of reaction (forward completion fraction) at 1200 K is 0.56. Determine the equilibrium constant and the change in Gibbs function.

#### **SOLUTION**

$$CO_2 + H_2 \rightleftharpoons CO + H_2O$$

Molar analysis:

$$CO_2 + H_2 \rightarrow (1-\varepsilon)CO_2 + aH_2 + bCO + cH_2O$$

$$C:1=(1-\varepsilon)+b; \Rightarrow b=\varepsilon$$

$$O: 2 = 2(1-\varepsilon) + b + c; \Rightarrow 2 = 2 - 2\varepsilon + \varepsilon + c; \Rightarrow c = \varepsilon$$

$$H: 2 = 2a + 2c; \Rightarrow 2 = 2a + 2\varepsilon; \Rightarrow a = 1 - \varepsilon$$

$$(1-\varepsilon)+a+b+c=(1-\varepsilon)+(1-\varepsilon)+\varepsilon+\varepsilon=2$$

$$y_{\text{CO}_2} = \frac{1-\varepsilon}{2} = 0.22;$$
  $y_{\text{H}_2} = \frac{a}{2} = \frac{1-\varepsilon}{2} = 0.22;$   $y_{\text{CO}} = \frac{b}{2} = \frac{\varepsilon}{2} = 0.28;$   $y_{\text{H}_2\text{O}} = \frac{c}{2} = \frac{\varepsilon}{2} = 0.28$ 

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{\nu_{\text{CO}}} y_{\text{H}_{2}\text{O}}^{\nu_{\text{H}_{2}\text{O}}}}{y_{\text{CO}_{2}}^{\nu_{\text{CO}_{2}}} y_{\text{H}_{2}}^{\nu_{\text{H}_{2}}}} \right) \left( \frac{p}{p_{0}} \right)^{\nu_{\text{CO}} + \nu_{\text{H}_{2}\text{O}} - \nu_{\text{CO}_{2}} - \nu_{\text{H}_{2}}} \right] = \ln \left[ \left( \frac{y_{\text{CO}}^{1} y_{\text{H}_{2}\text{O}}^{1}}{y_{\text{CO}_{2}}^{1} y_{\text{H}_{2}}^{1}} \right) \left( \frac{p}{p_{0}} \right)^{2 - 2} \right] = \ln \left[ \frac{(0.28)(0.28)}{(0.22)(0.22)} \right];$$

$$\Rightarrow \ln K_p = \ln[1.6198];$$

$$\Rightarrow \ln K_p = 0.4823;$$

$$\Rightarrow K_p = 1.6198$$

$$\ln K = -\frac{\Delta \overline{g}_{T}^{\circ}}{\overline{R}T};$$

$$\Rightarrow \ln (1.6198) = -\frac{\Delta \overline{g}_{T}^{\circ}}{(8.314)(1200)};$$

$$\Rightarrow \Delta \overline{g}_{T}^{\circ} = 4812.05$$

**14-3-20** [BBX] Determine the temperature at which 2% of diatomic oxygen (O<sub>2</sub>) dissociates into monatomic oxygen (O) at a pressure of 3 atm.

## **SOLUTION**

$$O_2 \rightleftharpoons 2O$$

Molar analysis:

$$O_2 \rightarrow (1-\varepsilon)O_2 + aO$$

$$O: 2 = 2(1-\varepsilon) + a; \Rightarrow a = 2\varepsilon$$

Therefore,

$$(1-\varepsilon)+a=1-\varepsilon+2\varepsilon=1+\varepsilon$$

$$y_{O_2} = \frac{(1-\varepsilon)}{1+\varepsilon} = \frac{0.98}{1.02}; \quad y_O = \frac{2\varepsilon}{1+\varepsilon} = \frac{0.04}{1.02}$$

$$\ln K_{p} = \ln \left[ \left( \frac{y_{O}^{v_{O}}}{y_{O_{2}}^{v_{O_{2}}}} \right) \left( \frac{p}{p_{0}} \right)^{v_{O} - v_{O_{2}}} \right] = \ln \left[ \frac{y_{O}^{2}}{y_{O_{2}}} \times (3)^{2-1} \right] = \ln \left[ \frac{\left( \frac{0.04}{1.02} \right)^{2}}{\left( \frac{0.98}{1.02} \right)} \times (3)^{2-1} \right] = \ln \left[ \left( \frac{0.04}{1.02} \right)^{2} \left( \frac{1.02}{0.98} \right) (3) \right];$$

$$\Rightarrow \ln K_p = \ln[0.048019]; \Rightarrow \ln K = -5.338739;$$

Looking at Table G-3:

Temperature  $\approx 2866 \text{ K}$ 

**14-3-21** [BBP] Oxygen  $(O_2)$  is heated to 3000 K at a constant pressure of 5 atm. Determine the percentage of  $O_2$  that will dissociate into O during this process.

## **SOLUTION**

Given: T = 3000 K; p = 5 atm

 $O_2 \rightleftharpoons 2O$ 

Molar analysis:

$$O_2 \rightarrow (1-\varepsilon)O_2 + aO$$

$$O: 2 = 2(1-\varepsilon) + a; \Rightarrow a = 2\varepsilon$$

Therefore,

$$(1-\varepsilon)+a=1-\varepsilon+2\varepsilon=1+\varepsilon$$

$$y_{O_2} = \frac{(1-\varepsilon)}{1+\varepsilon}; \quad y_O = \frac{2\varepsilon}{1+\varepsilon}$$

For  $O_2 \rightleftharpoons 2O$  in the  $\ln K_p$  Table at 3000K,  $\ln K_p = -4.357$ 

$$\ln K_{p} = \ln \left[ \left( \frac{y_{0}^{\nu_{0}}}{y_{0_{2}}^{\nu_{0}}} \right) \left( \frac{p}{p_{0}} \right)^{\nu_{0} - \nu_{0_{2}}} \right] = \ln \left[ \left( \frac{y_{0}^{2}}{y_{0_{2}}} \right) (5)^{2-1} \right] = \ln \left[ \left( \frac{2\varepsilon}{1+\varepsilon} \right)^{2} \left( \frac{1+\varepsilon}{1+\varepsilon} \right)^{2} \left( \frac{1+\varepsilon}{1+\varepsilon} \right) (5) \right];$$

$$\Rightarrow \ln K_{p} = \ln \left[ \frac{(5)(2\varepsilon)^{2}}{(1+\varepsilon)(1-\varepsilon)} \right] = \ln \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow -4.357 = \ln \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \Rightarrow e^{-4.357} = \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow 0.012817 = \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \Rightarrow 0.012817 - 0.012817\varepsilon^{2} = 20\varepsilon^{2}$$

$$\Rightarrow 20.012817\varepsilon^{2} = 0.012817; \Rightarrow \varepsilon = 0.0253;$$

Thus the percentage of  $O_2$  that dissociates is 2.53%.

**14-3-22** [BBU]  $H_2O(g)$  dissociates into an equilibrium mixture at 3000 K and 100 kPa. Assume the equilibrium mixture consists of  $H_2(g)$ ,  $H_2O$ ,  $O_2$ , and OH, determine the equilibrium composition of (a)  $H_2O$ , (b) OH (in kmols).

#### **SOLUTION**

Given: 
$$T = 3000 \text{ K}$$
;  $p = 100 \text{ kPa}$ 

Molar analysis:

$$H_2O \rightarrow aH_2 + bH_2O + cO_2 + dOH$$

H: 
$$2 = 2a + 2b + d$$
  $\Rightarrow d = 2 - 2b - 2a$   
O:  $1 = 1b + 2c + d$   $\Rightarrow c = \frac{2a + b - 1}{2}$   
 $a + b + c + d = a + b + \frac{2a + b - 1}{2} + 2 - 2b - 2a = 1.5 - 0.5b$ 

Plugging the relations for the stoichiometric coefficients into the equation above we get,

$$H_2O \rightarrow aH_2 + bH_2O + \left(\frac{2a+b-1}{2}\right)O_2 + (2-2b-2a)OH$$

$$y_{\text{H}_2} = \frac{a}{1.5 - 0.5b}; \quad y_{\text{H}_2\text{O}} = \frac{b}{1.5 - 0.5b}; \quad y_{\text{O}_2} = \frac{2a + b - 1}{3 - b}; \quad y_{\text{OH}} = \frac{2 - 2b - 2a}{1.5 - 0.5b}$$

Now we utilize the equilibrium relations constants from Table G-3

$$H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2; \ln K_{p1@T=3000 \text{ K}} = -3.086$$

$$H_2O \rightleftharpoons \frac{1}{2}H_2 + OH; \ln K_{p2@T=3000 \text{ K}} = -2.937$$

Now plugging in all the values into equation Eq. 14.53 to obtain

$$\ln K_{p_1@T=3000 \text{ K}} = \ln \left( \frac{\left(\frac{a}{1.5-0.5b}\right) \left(\frac{2a+b-1}{3-b}\right)^{\frac{1}{2}}}{\left(\frac{b}{1.5-0.5b}\right)}; \Rightarrow e^{-3.086} = \left(\frac{a}{b}\right) \left(\frac{2a+b-1}{3-b}\right)^{\frac{1}{2}};$$

$$\Rightarrow \left(e^{-3.086}\right)^2 = \left(\frac{a}{b}\right) \left(\frac{2a+b-1}{3-b}\right)^{\frac{1}{2}}; \Rightarrow \left(0.045684\right)^2 = \left(\frac{a^2}{b^2}\right) \left(\frac{2a+b-1}{3-b}\right);$$

$$\Rightarrow 0.00208706 = \frac{2a^3+ba^2-a^2}{3b^2-b^3}$$

$$\ln K_{p_2} = \ln \left( \frac{\left(\frac{a}{1.5 - 0.5b}\right)^{\frac{1}{2}} \left(\frac{2 - 2b - 2a}{1.5 - 0.5b}\right)}{\left(\frac{b}{1.5 - 0.5b}\right)}; \Rightarrow e^{-2.937} = \left(\frac{2 - 2b - 2a}{b}\right) \left(\frac{a}{1.5 - 0.5b}\right)^{\frac{1}{2}};$$

$$\Rightarrow \left(0.05302456\right)^2 = \left(\left(\frac{2 - 2b - 2a}{b}\right) \left(\frac{a}{1.5 - 0.5b}\right)^{\frac{1}{2}}\right)^2;$$

$$\Rightarrow 0.002811604 = \left(\frac{a}{1.5 - 0.5b}\right) \left(\frac{4 - 8b - 8a + 8ab + 4a^2 + 4b^2}{b^2}\right);$$

We now have two simultaneous equations to solve for a and b

Using the n-IGE TESTcalc, the following solution can be obtained:  $H_2O \rightarrow 0.1616H_2 + 0.7801H_2O + 0.0517O_2 + 0.1163OH$ 

- (a)  $H_2O = 0.7801$  kmol
- (b) OH = 0.1163 kmol

**14-3-23** [BBC] Nitrogen  $(N_2)$  is heated to 3000 K at a constant pressure of 5 atm. (a) Determine the percentage of  $N_2$  that will dissociate into N during this process. What-if Scenario: What would the answer be if the conditions were (b) 5 atm, 5000 K and (c) 0.1 atm, 5000 K?

## **SOLUTION**

$$N_2 \rightleftharpoons 2N$$

Molar analysis:

$$N_2 \rightarrow (1-\varepsilon)N_2 + aN$$

$$N: 2 = 2(1-\varepsilon) + a; \Rightarrow a = 2\varepsilon$$

Therefore.

$$(1-\varepsilon)+a=1-\varepsilon+2\varepsilon=1+\varepsilon$$

$$y_{N_2} = \frac{1-\varepsilon}{1+\varepsilon}; \quad y_N = \frac{2\varepsilon}{1+\varepsilon}$$

(a) For  $N_2 \rightleftharpoons 2N$  from Table G-3 at 3000K,  $\ln K_p = -22.359$  and p = 5 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{N}^{\nu_{N}}}{y_{N_{2}}^{\nu_{N_{2}}}} \right) \left( \frac{5}{p_{0}} \right)^{\nu_{N} - \nu_{N_{2}}} \right] = \ln \left[ \left( \frac{y_{N}^{2}}{y_{N_{2}}} \right) (5)^{2-1} \right] = \ln \left[ \frac{\left( \frac{1-\varepsilon}{1+\varepsilon} \right)^{2}}{\left( \frac{1-\varepsilon}{1+\varepsilon} \right)} \times (5)^{2-1} \right];$$

$$\Rightarrow \ln K_{p} = \ln \left[ \frac{(5)(2\varepsilon)^{2}}{(1+\varepsilon)(1-\varepsilon)} \right] = \ln \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow -22.359 = \ln \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \qquad \Rightarrow e^{-22.359} = \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow 1.95 \times 10^{-10} = \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \qquad \Rightarrow 1.95 \times 10^{-10} - \left( 1.95 \times 10^{-10} \right) \varepsilon^{2} = 20\varepsilon^{2};$$

$$\Rightarrow 20\varepsilon^{2} = 0; \qquad \Rightarrow \varepsilon = 0$$

Thus 0% of the  $N_2$  dissociates.

(b) For  $N_2 \rightleftharpoons 2N$  from Table G-3 at 5000 K,  $\ln K_p = -6.807$  and at p = 5 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{N}^{\nu_{N}}}{y_{N_{2}}^{\nu_{N_{2}}}} \right) \left( \frac{5}{p_{0}} \right)^{\nu_{N} - \nu_{N_{2}}} \right] = \ln \left[ \left( \frac{y_{N}^{2}}{y_{N_{2}}} \right) (5)^{2-1} \right] = \ln \left[ \frac{\left( \frac{2\varepsilon}{1+\varepsilon} \right)^{2}}{\left( \frac{1-\varepsilon}{1+\varepsilon} \right)} \times (5)^{2-1} \right];$$

$$\Rightarrow \ln K_{p} = \ln \left[ \frac{(5)(2\varepsilon)^{2}}{(1+\varepsilon)(1-\varepsilon)} \right] = \ln \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow -6.807 = \ln \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \qquad \Rightarrow e^{-6.807} = \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow 0.001106 = \left[ \frac{20\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \qquad \Rightarrow 0.001106 - 0.001106\varepsilon^{2} = 20\varepsilon^{2};$$

$$\Rightarrow 20.001106\varepsilon^{2} = 0.001106; \qquad \Rightarrow a = 0.00744$$

Thus 0.74% of the  $N_2$  dissociates.

(c) For  $N_2 \rightleftharpoons 2N$  from Table G-3 at 5000 K,  $\ln K_p = -6.807$  and at p=0.1 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{N}^{\nu_{N}}}{y_{N_{2}}^{\nu_{N_{2}}}} \right) \left( \frac{5}{p_{0}} \right)^{\nu_{N} - \nu_{N_{2}}} \right] = \ln \left[ \left( \frac{y_{N}^{2}}{y_{N_{2}}^{2}} \right) (0.1)^{2-1} \right] = \ln \left[ \frac{\left( \frac{1-\varepsilon}{1+\varepsilon} \right)^{2}}{\left( \frac{1-\varepsilon}{1+\varepsilon} \right)} \times (0.1)^{2-1} \right];$$

$$\Rightarrow \ln K_{p} = \ln \left[ \frac{(0.1)(2\varepsilon)^{2}}{(1+\varepsilon)(1+\varepsilon)} \right] = \ln \left[ \frac{0.4\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow -6.807 = \ln \left[ \frac{0.4\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \qquad \Rightarrow e^{-6.807} = \left[ \frac{0.4\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow 0.001106 = \left[ \frac{0.4\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \qquad \Rightarrow 0.001106 - 0.001106\varepsilon^{2} = 0.4\varepsilon^{2};$$

$$\Rightarrow 0.401106\varepsilon^{2} = 0.001106; \qquad \Rightarrow \varepsilon = 0.0525$$

Thus 5.25% of the  $N_2$  dissociates.

**14-3-24** [BBV] Carbon dioxide (CO<sub>2</sub>) is heated to 3200 K at a constant pressure of 2 atm. Determine the percentage of CO<sub>2</sub> that will dissociate into CO and O<sub>2</sub> during the process.

## **SOLUTION**

$$CO_2 \rightleftharpoons CO + 0.5O_2$$

Molar Analysis:

$$CO_2 \rightarrow (1-\varepsilon)CO_2 + aCO + bO_2$$

C: 
$$1 = (1 - \varepsilon) + a$$
;  $\Rightarrow a = \varepsilon$ 

O: 
$$2 = 2(1 - \varepsilon) + a + 2b$$
;  $\Rightarrow b = \frac{\varepsilon}{2}$ 

Therefore,

$$(1-\varepsilon)+a+b=1-\varepsilon+\varepsilon+\frac{\varepsilon}{2}=1+\frac{\varepsilon}{2}=\frac{2+\varepsilon}{2}$$

$$y_{\text{CO}_2} = \frac{1-\varepsilon}{\left(\frac{2+\varepsilon}{2}\right)} = \frac{2-2\varepsilon}{2+\varepsilon}; \quad y_{\text{CO}} = \frac{a}{\left(\frac{2+\varepsilon}{2}\right)} = \frac{2\varepsilon}{2+\varepsilon}; \quad y_{\text{O}_2} = \frac{b}{\left(\frac{2+\varepsilon}{2}\right)} = \frac{\varepsilon}{2+\varepsilon}$$

For  $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$  from Table G-3 at 3200 K,  $\ln K_p = -0.429$  and p = 2 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{\nu_{\text{CO}}} y_{\text{O}_{2}}^{\nu_{\text{O}_{2}}}}{y_{\text{CO}_{2}}^{\nu_{\text{CO}_{2}}}} \right) \left( \frac{2}{p_{0}} \right)^{\nu_{\text{CO}} + \nu_{\text{O}_{2}} + \nu_{\text{CO}_{2}}} = \ln \left[ \left( \frac{2\varepsilon}{2+\varepsilon} \right)^{1} \left( \frac{\varepsilon}{2+\varepsilon} \right)^{1} \right] \left( 2^{0.5} \right] = \ln \left[ \left( \frac{2\varepsilon}{2-2\varepsilon} \right)^{1} \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \times (2^{0.5}) \right];$$

$$\Rightarrow -0.429 = \ln \left[ \left( \frac{2\varepsilon}{2-2\varepsilon} \right)^{1} \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \times (2^{0.5}) \right]; \Rightarrow e^{-0.429} = \left[ \left( \frac{2\varepsilon}{2-2\varepsilon} \right)^{1} \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \times (2^{0.5}) \right];$$

$$\Rightarrow 0.6511599 = \left[ \left( \frac{2\varepsilon}{2-2\varepsilon} \right)^{1} \left( \frac{\varepsilon}{2+\varepsilon} \right)^{0.5} \times (2^{0.5}) \right]; \Rightarrow 0.42400925 = \left[ \left( \frac{2\varepsilon}{2-2\varepsilon} \right)^{2} \left( \frac{\varepsilon}{2+\varepsilon} \right)^{1} \times (2^{0.5}) \right];$$

$$\Rightarrow 0.21200463 = \left[ \frac{\varepsilon^{3}}{\varepsilon^{3} - 3\varepsilon + 2} \right];$$

$$\Rightarrow 0.21200463\varepsilon^{3} - 0.63601388\varepsilon + 0.42400925 = \varepsilon^{3};$$

$$\Rightarrow 0.42400925 - 0.63601388\varepsilon - 0.78799537\varepsilon^{3} = 0; \Rightarrow \varepsilon = 0.506079$$

Thus 50.61% of the  $CO_2$  dissociates.

**14-3-25** [BBQ] Consider the dissociation of 1 kmol of  $A_2$  through the elementary step  $A_2 \leftrightarrow 2A$ . If  $\varepsilon$  stands for the degree of dissociation (fraction of A2 by volume that is dissociated), the overall reaction can be represented by  $A_2 \leftrightarrow (1-\varepsilon)A_2 + 2\varepsilon A$ . At 500 K and 100 kPa, the equilibrium constant is calculated as 1. (a) Determine  $\varepsilon$ . (b) What would be the value of  $\varepsilon$  if the pressure is reduced to 0.01 kPa. (c) What-if Scenario: Plot how the degree of dissociation changes as the pressure is increased from 0.01 MPa to 1 MPa.

## **SOLUTION**

$$A_2 \rightleftharpoons 2A$$

Molar analysis:

$$A_2 \rightarrow (1-\varepsilon)A_2 + 2\varepsilon A$$

Therefore.

$$(1-\varepsilon)+2\varepsilon=1+\varepsilon$$

$$y_{A_2} = \frac{1-\varepsilon}{1+\varepsilon}; \quad y_A = \frac{2\varepsilon}{1+\varepsilon};$$

(a) For  $A_2 \rightleftharpoons 2A$  at 500K and p = 100 kPa,  $K_p = 1$ 

$$K_{p} = \left[ \left( \frac{y_{A}^{v_{A}}}{y_{A_{2}}^{v_{A_{2}}}} \right) \left( \frac{1}{p_{0}} \right)^{v_{A} - v_{A_{2}}} \right] = \left[ \left( \frac{y_{A}^{v_{A}}}{y_{A_{2}}^{v_{A_{2}}}} \right) \left( 1 \right)^{2+1} \right] = \left[ \left( \frac{2\varepsilon}{1+\varepsilon} \right)^{2} \left( \frac{1+\varepsilon}{1-\varepsilon} \right) \right] = \left[ \left( \frac{2\varepsilon}{1+\varepsilon} \right)^{2} \left( \frac{1+\varepsilon}{1-\varepsilon} \right) \right] = \left[ \frac{4\varepsilon^{2}}{1-\varepsilon^{2}} \right];$$

$$\Rightarrow 1 = \left[ \frac{4\varepsilon^{2}}{1-\varepsilon^{2}} \right]; \quad \Rightarrow 1 - \varepsilon^{2} = 4\varepsilon^{2}; \quad \Rightarrow 0 = 1 - 5\varepsilon^{2};$$

$$\Rightarrow \varepsilon = 0.44721 = 44.77\%$$

(b) For  $A_2 \rightleftharpoons 2A$  at 500 K and p = 0.01 kPa,  $K_p = 1$ 

$$K_{p} = \left[ \left( \frac{y_{A}^{\nu_{A}}}{y_{A_{2}}^{\nu_{A_{2}}}} \right) \left( \frac{0.01}{p_{0}} \right)^{\nu_{A} - \nu_{A_{2}}} \right] = \left[ \left( \frac{y_{A}^{2}}{y_{A_{2}}} \right) (0.0001)^{2-1} \right] = \left[ \left( \frac{2\varepsilon}{1+\varepsilon} \right)^{2} \times (0.0001) \right] = \left[ \left( \frac{2\varepsilon}{1+\varepsilon} \right)^{2} \left( \frac{1+\varepsilon}{1-\varepsilon} \right) \times (0.0001) \right];$$

$$\Rightarrow K_{p} = \left[ \frac{4\varepsilon^{2}}{1-\varepsilon^{2}} \times (0.0001) \right]; \quad \Rightarrow 1 = \left[ \frac{4\varepsilon^{2}}{1-\varepsilon^{2}} \right] \times (0.0001); \quad \Rightarrow 1 - \varepsilon^{2} = 0.0004\varepsilon^{2};$$

$$\Rightarrow 0 = 1 - 1.0004\varepsilon^{2}; \quad \Rightarrow \varepsilon = 0.99980006 = 99.98\%$$

(c) Plot how  $\varepsilon$  changes with pressure from 0.01 MPa to 1MPa.

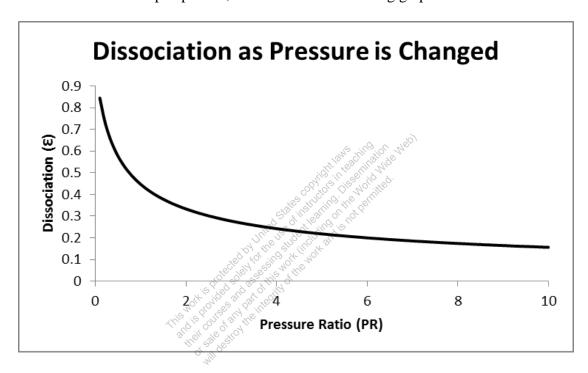
To solve this, we need to first solve the base equation:

$$1 = \frac{4\varepsilon^2}{1 - \varepsilon^2} \times PR$$

Where PR is the ratio of mixture pressure to ambient pressure. This simplifies to:

$$\varepsilon = \sqrt{\frac{1}{4PR + 1}}$$

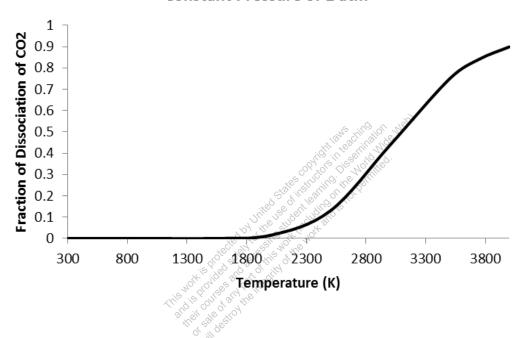
When this relationship is plotted, we achieve the following graph:



**14-3-26** [BBT] One kmol of carbon dioxide (CO<sub>2</sub>) is heated from 300 K to 4000 K at a constant pressure of 1 atm. Use the IGE system state TESTcalc, set up the reactants as 1 kmol of CO<sub>2</sub> and select CO<sub>2</sub>, CO, and O<sub>2</sub> as the possible products. Now calculate a series of equilibrium products states (in the state panel, select the Products radio-button) for several temperatures ranging from 300 K through 3000 K. Plot how the fraction of CO<sub>2</sub> dissociated changes with temperature.

## **TEST Solution:**

## Dissociation of CO2 over a Range of Temperatures at a Constant Pressure of 1 atm



**14-3-27** [BBY] A mixture of 1 kmol of CO and 2 kmol of  $O_2$  is heated to 2000 K at a pressure of 2 atm. Determine the equilibrium composition of (a)  $O_2$ , (b)  $CO_2$  (in kmols) assuming the mixture consists of  $CO_2$ , CO and  $O_2$ .

#### **SOLUTION**

Given: T = 2000 K; p = 2 atm

Molar Analysis:

$$CO + 2O_2 \rightarrow aCO_2 + bCO + cO_2$$

C: 
$$1 = a + b$$
;  $\Rightarrow b = 1 - a$ 

O: 
$$1+4 = 2a+b+2c; \Rightarrow c = \frac{4-a}{2}$$

Therefore:

$$a+b+c=1+\frac{4-a}{2}$$

$$y_{\text{CO}_2} = \frac{a}{1 + \frac{4 - a}{2}}; \quad y_{\text{CO}} = \frac{1 - a}{1 + \frac{4 - a}{2}}; \quad y_{\text{O}_2} = \frac{2 - a}{2};$$

For  $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$  from Table G-3 at 2000 K,  $\ln K = -6.635$  and p = 2 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{\nu_{\text{CO}}} y_{\text{O}_{2}}^{\nu_{\text{O}_{2}}}}{y_{\text{CO}_{2}}^{\nu_{\text{CO}_{2}}}} \right) \left( \frac{2}{p_{0}} \right)^{\nu_{\text{CO}} + \nu_{\text{O}_{2}} - \nu_{\text{CO}_{2}}} \right] = \ln \left[ \left( \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \right) (2)^{1.5-1} \right] = \ln \left[ \left( \frac{\frac{1-a}{2}}{1+\frac{4-a}{2}} \right)^{1} \left( \frac{\frac{4-a}{2}}{1+\frac{4-a}{2}} \right)^{0.5} \times (2)^{0.5} \right];$$

$$\ln K_p = \ln \left| \left( \frac{1-a}{1+\frac{4-a}{2}} \right) \left( \frac{1+\frac{4-a}{2}}{a} \right) \left( \frac{\frac{4-a}{2}}{1+\frac{4-a}{2}} \right)^{0.5} \times (2)^{0.5} \right|; \quad \Rightarrow -6.635 = \ln \left[ \left( \frac{1-a}{a} \right) \left( \frac{4-a}{6-a} \right)^{0.5} \times (2)^{0.5} \right];$$

$$\Rightarrow e^{-6.635} = \left[ \left( \frac{1-a}{a} \right) \left( \frac{4-a}{6-a} \right)^{0.5} \times (2)^{0.5} \right]; \Rightarrow 0.0013135787 = \left[ \left( \frac{1-a}{a} \right) \left( \frac{4-a}{6-a} \right)^{0.5} \times (2)^{0.5} \right];$$

$$\Rightarrow 1.725489 \times 10^{-6} = \left[ \left( \frac{1-a}{a} \right)^2 \left( \frac{4-a}{6-a} \right) (2) \right]; \Rightarrow 1.725489 \times 10^{-6} = \left[ \left( \frac{1-2a+a^2}{a^2} \right) \left( \frac{4-a}{6-a} \right) (2) \right];$$

$$\Rightarrow 8.627445599 \times 10^{-7} = \left[ \frac{4-9a+6a^2-a^3}{6a^2-a^3} \right];$$

$$\Rightarrow 5.176467359 \times 10^{-6} a^2 - 8.627445599 \times 10^{-7} a^3 = 4-9a+6a^2-a^3;$$

$$\Rightarrow 0 = 4-9a+5.999995a^2-0.9999991a^3; \Rightarrow a = 0.998832$$

(a) 
$$O_2: \frac{4-a}{2} = \frac{4-0.998832}{2} = 1.50058 \text{ kmol}$$

(b)  $CO_2$ : a = 0.998832 kmol



**14-3-28** [BBF] A mixture of 1 kmol of  $CO_2$ , 1/2 kmol of  $O_2$  and 1/2 kmol of  $N_2$  is heated to 2900 K at a pressure of 1 atm. Determine the equilibrium composition of (a)  $CO_2$ , (b)  $O_2$  assuming the mixture consists of  $CO_2$ , CO,  $O_2$  and  $O_2$ .

#### **SOLUTION**

Molar Analysis:

$$CO_2 + \frac{1}{2}O_2 + \frac{1}{2}N_2 \rightarrow (1 - \varepsilon)CO_2 + aCO + bO_2 + cN_2$$

C: 
$$1 = (1 - \varepsilon) + a; \Rightarrow a = \varepsilon$$

O: 
$$3 = 2(1 - \varepsilon) + a + 2b;$$
  $\Rightarrow b = \frac{1 + \varepsilon}{2}$ 

$$N: 1 = 2c;$$
  $\Rightarrow c = \frac{1}{2}$ 

Therefore:

$$(1-\varepsilon)+a+b+c=1-\varepsilon+\varepsilon+\frac{1+\varepsilon}{2}+\frac{1}{2}=\frac{4+\varepsilon}{2}$$

$$y_{\text{CO}_2} = \frac{\left(1 - \varepsilon\right)}{4 + \varepsilon} = \frac{2\left(1 - \varepsilon\right)}{4 + \varepsilon}; \quad y_{\text{CO}} = \frac{a}{4 + \varepsilon} = \frac{2\varepsilon}{4 + \varepsilon}; \quad y_{\text{O}_2} = \frac{b}{4 + \varepsilon} = \frac{1 + \varepsilon}{4 + \varepsilon}; \quad y_{\text{N}_2} = \frac{c}{4 + \varepsilon} = \frac{1}{4 + \varepsilon}$$

For  $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$  from Table G-3 at 2900 K,  $\ln K_p \approx -1.5025$  and p = 1 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{v_{\text{CO}}} y_{\text{O}_{2}}^{v_{\text{O}_{2}}}}{y_{\text{CO}_{2}}^{v_{\text{CO}_{2}}}} \right) \left( \frac{p}{p_{0}} \right)^{v_{\text{CO}} + v_{\text{O}_{2}} - v_{\text{CO}_{2}}} \right] = \ln \left[ \left( \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \right) (1)^{1.5-1} \right] = \ln \left[ \frac{\left( \frac{2\varepsilon}{4+\varepsilon} \right)^{1} \left( \frac{1+\varepsilon}{4+\varepsilon} \right)^{0.5}}{\left[ \frac{2(1-\varepsilon)}{4+\varepsilon} \right]^{1}} \right];$$

$$\ln K_p = \ln \left| \left( \frac{\varepsilon}{1 - \varepsilon} \right) \left( \frac{1 + \varepsilon}{4 + \varepsilon} \right)^{0.5} \right|; \qquad \Rightarrow -1.5025 = \ln \left| \left( \frac{\varepsilon}{1 - \varepsilon} \right) \left( \frac{1 + \varepsilon}{4 + \varepsilon} \right)^{0.5} \right|;$$

$$\Rightarrow e^{-1.5025} = \left(\frac{\varepsilon}{1-\varepsilon}\right) \left(\frac{1+\varepsilon}{4+\varepsilon}\right)^{0.5}; \quad \Rightarrow 0.22257303 = \left(\frac{\varepsilon}{1-\varepsilon}\right) \left(\frac{1+\varepsilon}{4+\varepsilon}\right)^{0.5};$$

$$\Rightarrow 0.04953875 = \left(\frac{\varepsilon}{1-\varepsilon}\right)^2 \left(\frac{1+\varepsilon}{4+\varepsilon}\right); \Rightarrow 0.04953875 = \frac{\varepsilon^3 + \varepsilon^2}{\varepsilon^3 + 2\varepsilon^2 - 7\varepsilon + 4};$$

$$\Rightarrow 0.04953875\varepsilon^3 + 0.0990775\varepsilon^2 - 0.34677125\varepsilon + 0.198155 = \varepsilon^3 + \varepsilon^2;$$

$$\Rightarrow$$
 0.95046125 $\varepsilon^3$  + 0.9009225 $\varepsilon^2$  + 0.34677125 $\varepsilon$  - 0.198155 = 0;

$$\Rightarrow \varepsilon = 0.28877$$

(a)  $CO_2: (1-\varepsilon) = (1-0.28877) = 0.71123 \text{ kmol}$ 

(b) 
$$O_2: \frac{1+\varepsilon}{2} = \frac{1+0.28877}{2} = 0.6444 \text{ kmol}$$



**14-3-29** [BBD] One kmol of carbon monoxide, CO, reacts with 1 kmol of oxygen, O<sub>2</sub>, to form an equilibrium mixture of CO<sub>2</sub>, CO and O<sub>2</sub> at 2800 K. Determine the equilibrium composition of CO<sub>2</sub> at (a) 1 atm, (b) 5 atm.

#### **SOLUTION**

Molar Analysis:

$$CO + O_2 \rightarrow aCO_2 + bCO + cO_2$$

$$C: 1 = a + b;$$
  $\Rightarrow b = 1 - a$ 

O: 
$$3 = 2a + b + 2c$$
;  $\Rightarrow c = \frac{2 - a}{2}$ 

Therefore,

$$a+b+c=a+1-a+\frac{2-a}{2}=\frac{4-a}{2}$$

$$y_{\text{CO}_2} = \frac{2a}{4-a};$$
  $y_{\text{CO}} = \frac{2-2a}{4-a};$   $y_{\text{O}_2} = \frac{\frac{2-a}{2}}{\frac{4-a}{2}} = \frac{2-a}{4-a};$ 

For  $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$  from Table G-3 at 2800 K,  $\ln K_p = -1.894$  and p = 1 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{v_{\text{CO}}} y_{\text{O}_{2}}^{v_{\text{O}_{2}}}}{y_{\text{CO}_{2}}^{v_{\text{CO}_{2}}}} \right) \left( \frac{1}{p_{0}} \right)^{v_{\text{CO}} + v_{\text{O}_{2}} + v_{\text{CO}_{2}}} \right] = \ln \left[ \left( \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \right) (1)^{1.5-1} \right] = \ln \left[ \left( \frac{\left( \frac{2-2a}{4-a} \right)^{1} \left( \frac{2-a}{4-a} \right)^{0.5}}{\left( \frac{2a}{4-a} \right)^{1}} \right];$$

$$\ln K_p = \ln \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \right|; \qquad \Rightarrow -1.894 = \ln \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \right|;$$

$$\Rightarrow e^{-1.894} = \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \right|; \quad \Rightarrow 0.1504687286 = \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \right|;$$

$$\Rightarrow 0.0226408383 = \left\lceil \left( \frac{2 - 2a}{2a} \right)^2 \left( \frac{2 - a}{4 - a} \right) \right\rceil; \quad \Rightarrow 0.0226408383 = \left\lceil \frac{8 - 20a + 16a^2 - 4a^3}{16a^2 - 4a^3} \right\rceil;$$

$$\Rightarrow$$
 0.3622534124 $a^2$  - 0.0905633531 $a^3$  = 8 - 20 $a$  + 16 $a^2$  - 4 $a^3$ ;

$$\Rightarrow 0 = 8 - 20a + 15.63774659a^2 - 3.909436647a^3;$$

$$\Rightarrow a = 0.802643$$

 $CO_2$ : a = 0.8026 kmol

For 
$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$$
 from Table G-3 at 2800 K,  $\ln K_p = -1.894$  and  $p = 5$  atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{\nu_{\text{CO}}} y_{\text{O}_{2}}^{\nu_{\text{O}_{2}}}}{y_{\text{CO}_{2}}^{\nu_{\text{CO}_{2}}}} \right) \left( \frac{5}{p_{0}} \right)^{\nu_{\text{CO}} + \nu_{\text{O}_{2}} - \nu_{\text{CO}_{2}}} \right] = \ln \left[ \left( \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \right) (5)^{1.5 - 1} \right] = \ln \left[ \left( \frac{2 - 2a}{4 - a} \right)^{1} \left( \frac{2 - a}{4 - a} \right)^{0.5} \times (5)^{0.5} \right];$$

$$\ln K_p = \ln \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \times (5)^{0.5} \right|; \qquad \Rightarrow -1.894 = \ln \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \times (5)^{0.5} \right|;$$

$$\Rightarrow e^{-1.894} = \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \times (5)^{0.5} \right|; \quad \Rightarrow 0.1504687286 = \left| \left( \frac{2 - 2a}{2a} \right) \left( \frac{2 - a}{4 - a} \right)^{0.5} \times (5)^{0.5} \right|;$$

$$\Rightarrow 0.0226408383 = \left[ \left( \frac{2 - 2a}{2a} \right)^2 \left( \frac{2 - a}{4 - a} \right) \times (5) \right]; \quad \Rightarrow 0.0226408383 = \left[ \frac{40 - 100a + 80a^2 - 20a^3}{16a^2 - 4a^3} \right];$$

$$\Rightarrow 0.3622534124a^2 - 0.0905633531a^3 = 40 - 100a + 80a^2 - 20a^3;$$

$$\Rightarrow$$
 0 = 40 - 100a + 79.63774659 $a^2$  - 19.909436647 $a^3$ ;

$$\Rightarrow a = 0.898539$$

 $CO_2$ : a = 0.8985 kmol

**14-3-30** [BBM] One kmol of carbon monoxide (CO), reacts with theoretical amount of air to form an equilibrium mixture of CO<sub>2</sub>, CO, O<sub>2</sub> and N<sub>2</sub> at 2200 K and 1 atm. Determine the equilibrium composition of (a) CO, (b) CO<sub>2</sub>.

#### **SOLUTION**

Molar Analysis:

$$CO + 0.5(O_2 + 3.76N_2) \rightarrow aCO_2 + bCO + cO_2 + dN_2$$

C: 
$$1 = a + b$$
;  $\Rightarrow b = 1 - a$ 

O: 
$$2 = 2a + b + 2c$$
;  $\Rightarrow c = \frac{1-a}{2}$ 

N: 
$$3.76 = 2d$$
;  $\Rightarrow 1.88 = d$ 

Therefore,

$$a+b+c+d = \frac{6.76 - a}{2}$$

$$y_{CO_2} = \frac{2a}{6.76 - a};$$
  $y_{CO_2} = \frac{2 - 2a}{6.76 - a};$   $y_{O_2} = \frac{\frac{1 - a}{2}}{\frac{6.76 - a}{2}};$   $y_{N_2} = \frac{3.76}{6.76 - a};$   $y_{N_2} = \frac{3.76}{6.76 - a};$ 

For  $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$  from Table G-3 at 2200 K,  $\ln K_p = -5.120$  and p = 1 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{\nu_{\text{CO}}} y_{\text{O}_{2}}^{\nu_{\text{O}_{2}}}}{y_{\text{CO}_{2}}^{\nu_{\text{CO}_{2}}}} \right) \left( \frac{1}{p_{0}} \right)^{\nu_{\text{CO}} + \nu_{\text{O}_{2}} - \nu_{\text{CO}_{2}}} = \ln \left[ \left( \frac{y_{\text{CO}}^{1} y_{\text{O}_{2}}^{0.5}}{y_{\text{CO}_{2}}^{1}} \right) (1)^{1.5-1} \right] = \ln \left[ \frac{\left( \frac{2-2a}{6.76-a} \right)^{1} \left( \frac{1-a}{6.76-a} \right)^{0.5}}{\left( \frac{2a}{6.76-a} \right)^{1}} \right];$$

$$\ln K_{p} = \ln \left[ \left( \frac{2 - 2a}{2a} \right) \left( \frac{1 - a}{6.76 - a} \right)^{0.5} \right]; \quad \Rightarrow -5.120 = \ln \left[ \left( \frac{2 - 2a}{2a} \right) \left( \frac{1 - a}{6.76 - a} \right)^{0.5} \right];$$

$$\Rightarrow e^{-5.120} = \left[ \left( \frac{2 - 2a}{2a} \right) \left( \frac{1 - a}{6.76 - a} \right)^{0.5} \right]; \quad \Rightarrow 0.0059760229 = \left[ \left( \frac{2 - 2a}{2a} \right) \left( \frac{1 - a}{6.76 - a} \right)^{0.5} \right];$$

$$\Rightarrow 3.571284964 \times 10^{-5} = \left[ \left( \frac{2 - 2a}{2a} \right)^2 \left( \frac{1 - a}{6.76 - a} \right) \right]; \qquad \Rightarrow 3.571284964 \times 10^{-5} = \left[ \frac{4 - 12a + 12a^2 - 4a^3}{27.04a^2 - 4a^3} \right];$$

$$\Rightarrow$$
 9.656754543×10<sup>-4</sup>  $a^2$  -1.428513986×10<sup>-4</sup>  $a^3$  = 4 -12 $a$  +12 $a^2$  -4 $a^3$ ;

$$\Rightarrow$$
 0 = 4-12a+11.99903432a<sup>2</sup>-3.999857149a<sup>3</sup>;

$$\Rightarrow a = 0.943045$$

(a) 
$$CO:1-a=1-0.943045=0.057$$
 kmol

(b) 
$$CO_2 : a = 0.943 \text{ kmol}$$

**14-3-31** [BSR] A mixture of 2 kmol of  $N_2$ , 1 kmol of  $O_2$  and 0.1 kmol of Ar is heated to 2500 K at a constant pressure of 10 atm. Assuming the equilibrium mixture consists of  $N_2$ ,  $O_2$ , Ar and NO, determine the equilibrium composition of (a)  $O_2$ , (b)  $N_2$  (in kmols).

#### **SOLUTION**

Molar Analysis:

$$2N_2 + 1O_2 + 0.1Ar \rightarrow aN_2 + bO_2 + cAr + dNO$$

N: 
$$4 = 2a + d$$
;  $d = 4 - 2a$ 

 $\Rightarrow a = 1.96012$ 

O: 
$$2 = 2b + d$$
;  $b = \frac{2 - d}{2} = \frac{-2 + 2a}{2}$ 

Ar: 
$$0.1 = c$$

Therefore.

$$a+b+c+d=3.1$$

$$y_{N_2} = \frac{a}{3.1};$$
  $y_{O_2} = \frac{\left(\frac{-2+2a}{2}\right)}{3.1} = \frac{-2+2a}{6.2};$   $y_{NO} = \frac{4-2a}{3.1};$   $y_{Ar} = \frac{0.1}{3.1}$ 

For  $\frac{1}{2}$  N<sub>2</sub> +  $\frac{1}{2}$  O<sub>2</sub>  $\rightleftharpoons$  NO from Table G-3 at 2500 K,  $\ln K_p = -2.845$  and p = 10 atm

$$\ln K_{p} = \ln \left[ \left( \frac{y_{NO}^{v_{NO}}}{y_{O_{2}}^{v_{O_{2}}} y_{N_{2}}^{v_{N_{2}}}} \right) \left( \frac{10}{p_{0}} \right)^{v_{NO}-v_{O_{2}}+v_{N_{2}}} \right] = \ln \left[ \left( \frac{y_{NO}^{1}}{y_{O_{2}}^{0.5} y_{N_{2}}^{0.5}} \right) (10)^{1-1} \right] = \ln \left[ \frac{\left( \frac{4-2a}{3.1} \right)^{1}}{\left( \frac{-2+2a}{6.2} \right)^{0.5}} \left( \frac{a}{3.1} \right)^{0.5} \times (10)^{0} \right];$$

$$\ln K_{p} = \ln \left[ \left( \frac{4-2a}{3.1} \right) \left( \frac{6.2}{-2+2a} \right)^{0.5} \left( \frac{3.1}{a} \right)^{0.5} \right]; \quad \Rightarrow -2.845 = \ln \left[ \left( \frac{4-2a}{3.1} \right) \left( \frac{6.2}{-2+2a} \right)^{0.5} \left( \frac{3.1}{a} \right)^{0.5} \right];$$

$$\Rightarrow e^{-2.845} = \left[ \left( \frac{4-2a}{3.1} \right) \left( \frac{6.2}{-2+2a} \right)^{0.5} \left( \frac{3.1}{a} \right)^{0.5} \right]; \quad \Rightarrow 0.0581342667 = \left[ \left( \frac{4-2a}{3.1} \right) \left( \frac{6.2}{-2+2a} \right)^{0.5} \left( \frac{3.1}{a} \right)^{0.5} \right];$$

$$\Rightarrow 0.003379593 = \left[ \left( \frac{4-2a}{3.1} \right)^{2} \left( \frac{6.2}{-2+2a} \right) \left( \frac{3.1}{a} \right) \right]; \quad \Rightarrow 0.003379593 = \left[ \frac{32-32a+8a^{2}}{-2a+2a^{2}} \right];$$

$$\Rightarrow -0.0067591859a + 0.0067591859a^{2} = 32-32a+8a^{2};$$

$$\Rightarrow 0 = 32-31.99324081a + 7.99324081a^{2};$$

- (a)  $O_2: \frac{-2+2a}{2} = \frac{-2+2(1.96012)}{2} = 0.96012 \text{ kmol}$
- (b)  $N_2: a = 1.96012 \text{ kmol}$



**14-3-32** [BBJ] A chamber initially contains a gaseous mixture consisting of 4 kmol of CO<sub>2</sub>, 8 kmol of CO and 2 kmol of H<sub>2</sub>. Assume an equilibrium mixture formed consists of CO<sub>2</sub>, CO, H<sub>2</sub>O, H<sub>2</sub> and O<sub>2</sub> at 2600 K and 100 kPa, determine the equilibrium composition of (a) CO<sub>2</sub>, (b) CO (in kmols).

## **SOLUTION**

Molar Analysis:

$$4\text{CO}_2 + 8\text{CO} + 2\text{H}_2 \rightarrow a\text{CO}_2 + b\text{CO} + c\text{H}_2\text{O} + d\text{H}_2 + e\text{O}_2$$

C: 
$$12 = a + b$$
;  $\Rightarrow b = 12 - a$ 

O: 
$$16 = 2a + b + c + 2e$$
;  $4 = a + c + 2e$ 

H: 
$$4 = 2c + 2d$$
;  $\Rightarrow d = 2 - c$ 

Therefore,

$$a+b+c+d+e=14+e$$

$$y_{\text{CO}_2} = \frac{a}{14 + e};$$
  $y_{\text{CO}} = \frac{b}{14 + e} = \frac{12 - a}{14 + e};$   $y_{\text{H}_2\text{O}} = \frac{c}{14 + e};$   $y_{\text{H}_2} = \frac{d}{14 + e} = \frac{2 - c}{14 + e};$   $y_{\text{O}_2} = \frac{e}{14 + e}$ 

For  $CO_2 + H_2 \rightleftharpoons CO + H_2O$  from Table G-3 at 2600 K,  $\ln K_p = 1.847$  and p = 100 kPa

$$\ln K_{p} = \ln \left[ \left( \frac{y_{\text{CO}}^{\text{V}_{\text{H}_{2}\text{O}}}}{y_{\text{CO}_{2}}^{\text{V}_{\text{H}_{2}\text{O}}}} \right) \left( \frac{p}{p_{0}} \right)^{\nu_{\text{CO}} + \nu_{\text{H}_{2}\text{O}} - \nu_{\text{CO}_{2}} + \nu_{\text{H}_{2}}} \right] = \ln \left[ \left( \frac{y_{\text{CO}}^{1} y_{\text{H}_{2}\text{O}}^{1}}{y_{\text{CO}_{2}}^{1} y_{\text{H}_{2}}^{1}} \right) (1)^{2-2} \right] = \ln \left[ \frac{\left( \frac{12-a}{14+e} \right)^{1} \left( \frac{c}{14+e} \right)^{1}}{\left( \frac{a}{14+e} \right)^{1} \left( \frac{12-a}{14+e} \right)^{1}} \right];$$

$$\ln K_{p} = \ln \left[ \left( \frac{14+e}{2-c} \right)^{1} \left( \frac{14+e}{a} \right)^{1} \left( \frac{12-a}{14+e} \right)^{1} \left( \frac{c}{14+e} \right)^{1} \right]; \Rightarrow 1.847 = \ln \left[ \frac{\left( 12-a \right)c}{\left( 2-c \right)a} \right];$$

$$\Rightarrow e^{1.847} = \left[ \frac{12c-ac}{2a-ac} \right]; \Rightarrow 6.340768655 = \left[ \frac{12c-ac}{2a-ac} \right];$$

$$\Rightarrow 12.68153731a - 6.340768655ac = 12c-ac; \Rightarrow 12.68153731a - 5.340768655ac = 12c;$$

$$\Rightarrow a \left( 12.68153731 - 5.340768655c \right) = 12c;$$

$$\Rightarrow a = \frac{12c}{\left( 12.68153731 - 5.340768655c \right)}$$

For 
$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$$
 from Table G-3 at 2600 K,  $\ln K_p = -2.801$  and  $p = 100$  kPa

$$\begin{split} \ln K_p &= \ln \left[ \left( \frac{y_{\text{CO}}^{v_{\text{CO}}} y_{\text{O}_2}^{v_{\text{O}_2}}}{y_{\text{CO}_2}^{v_{\text{CO}_2}}} \right) \left( \frac{p}{p_0} \right)^{v_{\text{CO}} + v_{\text{O}_2} - v_{\text{CO}_2}} \\ &= \ln \left[ \left( \frac{y_{\text{CO}}^1 y_{\text{O}_2}^{0.5}}{y_{\text{CO}_2}^1} \right) (1)^{1.5 - 1} \right] = \ln \left[ \frac{\left( \frac{12 - a}{14 + e} \right)^1 \left( \frac{e}{14 + e} \right)^{0.5}}{\left( \frac{a}{14 + e} \right)^1} \right]; \\ &= \ln \left[ \left( \frac{14 + e}{a} \right)^1 \left( \frac{12 - a}{14 + e} \right)^1 \left( \frac{e}{14 + e} \right)^{0.5} \right]; \\ &\Rightarrow -2.801 = \ln \left[ \left( \frac{12 - a}{a} \right)^1 \left( \frac{e}{14 + e} \right)^{0.5} \right]; \\ &\Rightarrow e^{-2.801} = \left[ \left( \frac{12 - a}{a} \right)^1 \left( \frac{e}{14 + e} \right)^{0.5} \right]; \\ &\Rightarrow 0.0036904754 = \left[ \left( \frac{12 - a}{a} \right)^2 \left( \frac{e}{14 + e} \right)^1 \right]; \\ &\Rightarrow 0.0036904754 = \left[ \left( \frac{12 - a}{a} \right)^2 \left( \frac{e}{14 + e} \right)^1 \right]; \\ &\Rightarrow 0.0516666553a^2 + 0.0036904754a^2e = 144e - 24ae + a^2e; \\ &\Rightarrow 0.0516666553a^2 - 0.9963095246a^2e + 24ae - 144e = 0; \end{split}$$

There are now three systems of equations and three variables to solve for.

(1) 
$$0.0516666553a^2 - 0.9963095246a^2e + 24ae - 144e = 0$$

(2) 
$$a = \frac{12c}{(12.68153731 - 5.340768655c)}$$

(3) 
$$4 = a + c + 2e$$

Combining equations 2 and 3 together:

Combining equations 2 and 3 together.
$$a = \frac{12(4 - a - 2e)}{(12.68153731 - 5.340768655(4 - a - 2e))} = \frac{48 - 12a - 24e}{-8.68153731 + 5.340768655a + 10.68153731e};$$

$$\Rightarrow -8.68153731a + 5.340768655a^2 + 10.68153731ea = 48 - 12a - 24e;$$

$$\Rightarrow 3.31846269a + 5.340768655a^2 - 48 = -24e - 10.68153731a;$$

$$\Rightarrow 3.31846269a + 5.340768655a^2 - 48 = \left(-24 - 10.68153731a\right)e;$$

$$\Rightarrow e = \frac{3.31846269a + 5.340768655a^2 - 48}{-24 - 10.68153731a}$$

This can now be plugged into equation 1 to solve for *a*:  $0.0516666553a^2 - 0.9963095246a^2e + 24ae - 144e = 0$ ;

$$\Rightarrow 0.05a^{2} - 0.996a^{2} \left( \frac{3.3a + 5.3a^{2} - 48}{-24 - 10.68a} \right) + 24a \left( \frac{3.3a + 5.3a^{2} - 48}{-24 - 10.68a} \right) - 144 \left( \frac{3.3a + 5.3a^{2} - 48}{-24 - 10.68a} \right) = 0;$$

$$\Rightarrow 0.05a^{2} \left( -24 - 10.7a \right) - 0.996a^{2} \left( 3a + 5a^{2} - 48 \right) + 24a \left( 3a + 5a^{2} - 48 \right) - 144 \left( 3a + 5a^{2} - 48 \right) = 0;$$

$$\Rightarrow -1.2a^{2} - 0.6a^{3} - 3.3a^{3} - 5.3a^{4} + 47.8a^{2} + 79.6a^{2} + 128.2a^{3} - 1152a - 477.9a - 769.1a^{2} + 6912 = 0;$$

$$\Rightarrow -5.32105868a^{4} + 124.3203524a^{3} - 642.8447243a^{2} - 1629.858627a + 6912 = 0;$$

$$\Rightarrow a = 2.69617$$

- (a)  $CO_2$ : a = 2.69617 kmol
- (b) CO: b = 12 a = 12 2.69617 = 9.30383 kmol



**14-3-33** [BBW] Butane ( $C_4H_{10}$ ) burns inside a vessel with 50% excess air to form an equilibrium mixture at 1400 K and 1 MPa. The equilibrium mixture is composed of  $CO_2$ ,  $O_2$ ,  $N_2$ ,  $H_2O(g)$ ,  $NO_2$ , and NO. Determine the balanced reaction equation, using the IGE open-steady equilibrium TESTcalc.

## **SOLUTION**

Using the IGE TESTcalc, the following equation is obtained:

 $C_4H_{10} + 9.75(O_2 + 3.76N_2) \rightarrow 3.999CO_2 + 3.239O_2 + 36.649N_2 + 4.999H_2O(g) + 0.0002NO_2 + 0.0211NO$ 



**14-3-34** [BSO] Methane (CH<sub>4</sub>) reacts with 125% of theoretical air inside a chamber to form an equilibrium mixture consisting of  $CO_2$ , CO,  $H_2O(g)$ ,  $H_2$ , and  $N_2$  at 1200 K, 100 kPa. Determine the equilibrium composition of (a)  $CO_2$ , (b)  $H_2O$  (in kmols) using the IGE opensteady equilibrium TESTcalc.

## **SOLUTION**

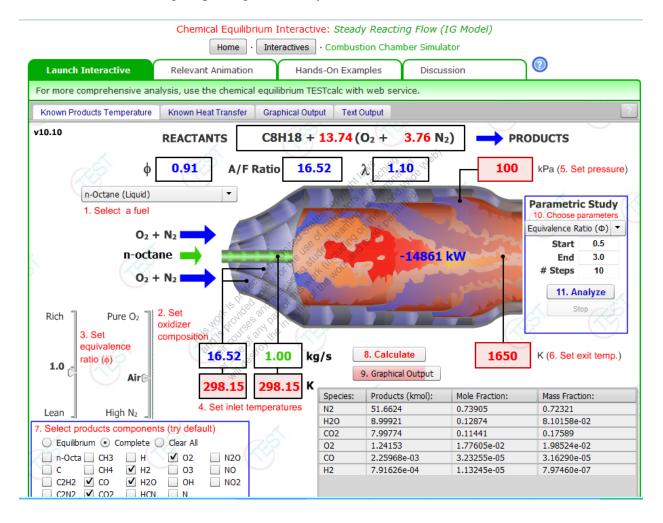
Using the IGE TESTcalc, the following equation is obtained:  $CH_4 + 2.5(O_2 + 3.76N_2) = ICO_2 + 0CO + 2H_2O + 0H_2 + 9.4N_2$ 



**14-3-35** [BSB] Octane ( $C_8H_{18}$ ) reacts with 110% of theoretical air inside a chamber to form an equilibrium mixture consists of  $CO_2$ , CO,  $H_2O(g)$ ,  $H_2$ ,  $O_2$ , and  $N_2$  at 1650 K, 100 kPa. Determine the composition of the equilibrium mixture using the equilibrium combustion interactive.

## **SOLUTION**

Start the equilibrium interactive. Adjust the air fuel ratio with the slide bar so that lambda becomes 1.1. Select the products species, change the exit temperature to 1650 K and click Calculate. The following output is generated by the interactive:



**14-3-36** [BSS] Starting with  $n_0$  kmol of NH<sub>3</sub>, which dissociates according to NH<sub>3</sub>  $\leftrightarrow$  (1/2)N<sub>2</sub> + (3/2)H<sub>2</sub>, evaluate an expression for K in terms of the degree of dissociation  $\varepsilon$  and pressure p.

#### **SOLUTION**

The theoretical reaction is given as:

$$NH_3 \rightleftharpoons 0.5N_2 + 1.5H_2$$

Molar Analysis:

$$n_o NH_3 \rightarrow (1-\varepsilon)n_o NH_3 + aN_2 + bH_2$$

N: 
$$n_0 = (1 - \varepsilon)n_0 + 2a; \implies a = \frac{n_0 \varepsilon}{2}$$

H: 
$$3 n_0 = 3(1-\varepsilon)n_0 + 2b; \implies b = \frac{3n_0\varepsilon}{2}$$

Therefore,

$$(1-\varepsilon)n_0 + a + b = (1-\varepsilon)n_0 + \frac{n_0\varepsilon}{2} + \frac{3n_0\varepsilon}{2};$$

$$\Rightarrow \frac{2(1-\varepsilon)n_0 + n_0\varepsilon + 3n_0\varepsilon}{2} = \frac{2n_0 + 2n_0\varepsilon}{2} = n_0 + n_0\varepsilon = (1+\varepsilon)n_0$$

$$y_{\mathrm{NH_{3}}} = \frac{\left(1-\varepsilon\right)n_{0}}{\left(1+\varepsilon\right)n_{0}} = \frac{1-\varepsilon}{1+\varepsilon}; \quad y_{\mathrm{N_{2}}} = \frac{a}{\left(1+\varepsilon\right)n_{0}} = \frac{n_{0}\varepsilon}{2\left(1+\varepsilon\right)n_{0}} = \frac{0.5\varepsilon}{1+\varepsilon}; \quad y_{\mathrm{CO_{2}}} = \frac{b}{\left(1+\varepsilon\right)n_{0}} = \frac{3n_{0}\varepsilon}{2\left(1+\varepsilon\right)n_{0}} = \frac{1.5\varepsilon}{1+\varepsilon}$$

$$K = \left(\frac{y_{\text{N}_2}^{\nu_{\text{N}_2}} y_{\text{H}_2}^{\nu_{\text{H}_2}}}{y_{\text{NH}_3}^{\nu_{\text{NH}_3}}}\right) \left(\frac{p}{p_0}\right)^{\nu_{\text{N}_2} + \nu_{\text{H}_2} - \nu_{\text{NH}_3}} = \left(\frac{y_{\text{N}_2}^{0.5} y_{\text{H}_2}^{1.5}}{y_{\text{NH}_3}^{1}}\right) \left(\frac{p}{p_0}\right)^{2-1};$$

$$K = \frac{\left(\frac{0.5\varepsilon}{1+\varepsilon}\right)^{0.5} \left(\frac{1.5\varepsilon}{1+\varepsilon}\right)^{1.5}}{\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{1}} \times \left(\frac{p}{p_{0}}\right) = \left(\frac{0.5\varepsilon}{1+\varepsilon}\right)^{0.5} \left(\frac{1.5\varepsilon}{1+\varepsilon}\right)^{1.5} \left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{1} \left(\frac{p}{p_{0}}\right);$$

$$K^{2} = \left(\frac{0.5\varepsilon}{1+\varepsilon}\right)^{1} \left(\frac{1.5\varepsilon}{1+\varepsilon}\right)^{3} \left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{2} \left(\frac{p}{p_{0}}\right)^{2} = \frac{\left(0.5\varepsilon\right)\left(1.5^{3}\varepsilon^{3}\right)}{\left(1+\varepsilon\right)^{2} \left(1-\varepsilon\right)^{2}} \left(\frac{p}{p_{0}}\right)^{2};$$

$$K^{2} = \frac{1.6875\varepsilon^{4}}{\left(1+\varepsilon\right)^{2}\left(1-\varepsilon\right)^{2}} \left(\frac{p}{p_{0}}\right)^{2};$$

$$K = \frac{1.299\varepsilon^2}{(1+\varepsilon)(1-\varepsilon)} \left(\frac{p}{p_0}\right) = \frac{1.299\varepsilon^2}{1-\varepsilon^2} \left(\frac{p}{p_0}\right)$$

**14-3-37** [BSA] Determine the mole fraction of sodium that ionizes according to the reaction Na  $\leftrightarrow$  Na<sup>+</sup> + e<sup>-</sup> at 2300 K and 0.5 atm (use K = 0.688 for this reaction).

## **SOLUTION**

Given: 
$$T = 2300 \text{ K}$$
;  $p = 0.5 \text{ atm}$ ;

$$K = 0.688$$
;

$$Na \rightleftharpoons Na^+ + e^-$$

Molar Analysis:

$$Na \rightarrow (1-\varepsilon)Na + \varepsilon Na^+ + \varepsilon e^-$$

Therefore,

$$1 - \varepsilon + \varepsilon + \varepsilon = 1 + \varepsilon$$

$$y_{\text{Na}} = \frac{1-\varepsilon}{1+\varepsilon}; \quad y_{\text{Na}^{+}} = \frac{\varepsilon}{1+\varepsilon}; \quad y_{\text{e}^{-}} = \frac{\varepsilon}{1+\varepsilon}$$

$$K = \left(\frac{y_{p}^{\nu_{p}}}{y_{r}^{\nu_{r}}}\right) \left(\frac{p}{p_{0}}\right)^{\sum \nu_{p} - \sum \nu_{r}} = \left(\frac{y_{\text{Na}^{+}}^{\nu_{\text{Na}^{+}}} y_{\text{e}^{-}}^{\nu_{\text{e}^{-}}}}{y_{\text{Na}}^{\nu_{\text{Na}}}}\right) \left(\frac{p}{p_{0}}\right)^{\nu_{\text{Na}^{+}} + \nu_{\text{e}^{-}} - \nu_{\text{Na}}} = \left(\frac{y_{\text{Na}^{+}}^{1} y_{\text{e}^{-}}^{1}}{y_{\text{Na}}^{1}}\right) \left(\frac{p}{p_{0}}\right)^{2-1};$$

$$K = \frac{\left(\frac{\varepsilon}{1+\varepsilon}\right) \left(\frac{\varepsilon}{1+\varepsilon}\right)}{\left(\frac{1-\varepsilon}{1+\varepsilon}\right)} \times \left(\frac{0.5}{1}\right) = \frac{0.5\varepsilon^{2}}{\left(1+\varepsilon\right) \left(1-\varepsilon\right)} = \frac{0.5\varepsilon^{2}}{1-\varepsilon^{2}};$$

$$\Rightarrow 1.376 = \frac{\varepsilon^2}{1 - \varepsilon^2}$$
;

$$\Rightarrow$$
 2.376 $\varepsilon^2$  -1.376 = 0;

$$\Rightarrow \varepsilon = 0.7610$$

**14-3-38** [BSH] Determine the percent ionization of cesium that ionizes according to the reaction  $Ce \leftrightarrow Ce^+ + e^-$  at 2000 K and 1 atm (use K = 15.63 for this reaction).

## **SOLUTION**

Given: T = 2000 K; p = 1 atm;

$$K = 15.63$$
;

$$Ce \rightleftharpoons Ce^+ + e^-$$

Molar Analysis:

$$Ce \rightarrow (1-\varepsilon)Ce + \varepsilon Ce^+ + \varepsilon e^-$$

Therefore

$$1-\varepsilon+\varepsilon+\varepsilon=1+\varepsilon$$

$$y_{\text{Ce}} = \frac{1-\varepsilon}{1+\varepsilon}; \quad y_{\text{Ce}^+} = \frac{\varepsilon}{1+\varepsilon}; \quad y_{\text{e}^-} = \frac{\varepsilon}{1+\varepsilon}$$

$$K = \left(\frac{y_{p}^{\nu_{p}}}{y_{r}^{\nu_{r}}}\right) \left(\frac{p}{p_{0}}\right)^{\sum \nu_{p} - \sum \nu_{r}} = \left(\frac{y_{\text{Ce}^{+}}^{\nu_{\text{Ce}^{+}}} y_{\text{e}^{-}}^{\nu_{\text{e}^{-}}}}{y_{\text{Ce}}^{\nu_{\text{Ce}^{+}}}}\right) \left(\frac{p}{p_{0}}\right)^{\nu_{\text{Ce}^{+}} + \nu_{\text{e}^{-}} - \nu_{\text{Ce}}} = \left(\frac{y_{\text{Ce}^{+}}^{\nu_{\text{e}^{-}}} y_{\text{e}^{-}}^{\nu_{\text{Ce}^{+}}}}{y_{\text{Ce}^{+}}^{\nu_{\text{Ce}^{+}}}}\right) \left(\frac{p}{p_{0}}\right)^{2-1};$$

$$K = \frac{\left(\frac{\varepsilon}{1+\varepsilon}\right)\left(\frac{\varepsilon}{1+\varepsilon}\right)}{\left(\frac{1-\varepsilon}{1+\varepsilon}\right)} \times \left(\frac{1}{1}\right) = \frac{\varepsilon^2}{\left(1+\varepsilon\right)\left(1-\varepsilon\right)} = \frac{\varepsilon^2}{1-\varepsilon^2};$$

$$\Rightarrow 15.63 = \frac{\varepsilon^2}{1 - \varepsilon^2};$$

$$\Rightarrow$$
 16.63 $\varepsilon^2$  -15.63 = 0;

$$\Rightarrow x = 0.9694 = 96.94\%$$

**14-3-39** [BSN] Determine (a) the pressure if the ionization of Ar is 80% complete according to the reaction Ar  $\leftrightarrow$  Ar<sup>+</sup> + e<sup>-</sup> at 10,000 K (use  $K = 4.2 \times 10^{-4}$  for this reaction). (b) What-if Scenario: What would the pressure be if the ionization of Ar is 50% complete instead?

## **SOLUTION**

Given: T = 10,000 K

$$Ar \rightleftharpoons Ar^+ + e^-$$

$$\varepsilon = 0.8$$

Molar Analysis:

$$Ar \rightarrow (1-\varepsilon)Ar + \varepsilon Ar^+ + \varepsilon e^-$$

$$Ar \rightarrow 0.2Ar + 0.8Ar^{+} + 0.8e^{-}$$

Therefore,

$$0.2 + 0.8 + 0.8 = 1.8$$

$$y_{Ar} = \frac{0.2}{1.8};$$
  $y_{Ar^{+}} = \frac{0.8}{1.8};$   $y_{e^{-}} = \frac{0.8}{1.8}$ 

$$K = \left(\frac{y_{p}^{\nu_{p}}}{y_{r}^{\nu_{r}}}\right) \left(\frac{p}{p_{0}}\right)^{\sum \nu_{p} - \sum \nu_{r}} = \left(\frac{y_{Ar^{+}}^{\nu_{Ar^{+}}} y_{e^{-}}^{\nu_{e^{-}}}}{y_{Ar}^{\nu_{Ar}}}\right) \left(\frac{p}{p_{0}}\right)^{\nu_{Ar^{+}} + \nu_{e^{-}} - \nu_{Ar}} = \left(\frac{y_{Ar^{+}}^{1} y_{e^{-}}^{1}}{y_{Ar}^{1}}\right) \left(\frac{p}{p_{0}}\right)^{2-1};$$

$$4.2 \times 10^{-4} = \frac{\left(\frac{0.2}{1.8}\right) \left(\frac{0.8}{1.8}\right)}{\left(\frac{0.8}{1.8}\right)} \left(\frac{p}{p_0}\right) = 0.1111111 \left(\frac{p}{p_0}\right);$$

$$\Rightarrow p = 0.00378$$
 atm

$$Ar \rightarrow 0.5Ar + 0.5Ar^{+} + 0.5e^{-}$$

Therefore.

$$0.5 + 0.5 + 0.5 = 1.5$$

$$y_{Ar} = \frac{0.5}{1.5};$$
  $y_{Ar^{+}} = \frac{0.5}{1.5};$   $y_{e^{-}} = \frac{0.5}{1.5}$ 

$$K = \left(\frac{y_{p}^{\nu_{p}}}{y_{r}^{\nu_{r}}}\right) \left(\frac{p}{p_{0}}\right)^{\sum \nu_{p} - \sum \nu_{r}} = \left(\frac{y_{Ar^{+}}^{\nu_{Ar^{+}}} y_{e^{-}}^{\nu_{e^{-}}}}{y_{Ar}^{\nu_{Ar}}}\right) \left(\frac{p}{p_{0}}\right)^{\nu_{Ar^{+}} + \nu_{e^{-}} - \nu_{Ar}} = \left(\frac{y_{Ar^{+}}^{1} y_{e^{-}}^{1}}{y_{Ar}^{1}}\right) \left(\frac{p}{p_{0}}\right)^{2-1};$$

$$4.2 \times 10^{-4} = \frac{\left(\frac{0.5}{1.5}\right) \left(\frac{0.5}{1.5}\right)}{\left(\frac{0.5}{1.5}\right)} \left(\frac{p}{p_{0}}\right) = 0.3333333 \left(\frac{p}{p_{0}}\right);$$

$$\Rightarrow p = 0.00126 \text{ atm}$$



**14-3-40** [BSE] One kmol of  $H_2O$  is heated to 3000 K at a pressure of 1 atm. Determine the equilibrium composition of  $H_2O$  (in kmols) assuming that only  $H_2O$ , OH,  $O_2$  and O are present.

## **SOLUTION**

$$T = 3000 \text{ K}; p = 1 \text{ atm}$$

Molar analysis:

$$H_2O \rightarrow aH_2O + bOH + cO_2 + dO$$

Looking at the molar analysis, if any of the  $H_2O$  were to dissociate, the water would only be able to dissociate into OH which would cause the reaction to be unbalanced due to a floating hydrogen atom. Therefore,

$$H_2O = 1$$
 kmol

## **TEST Solution:**

TEST-Specific, Closed Process, Chem. Equilibrium Daemon: IG model a = 0.9999978; b = 0.0000023;  $c = 7.565 \times 10^{-24}$ ;  $d = 9.79 \times 10^{-12}$ 

$$H_2O = 0.9999978$$
;  $OH = 0.0000022$ ;  $O_2 = 1.3434 \times 10^{-20}$ ;  $O = 8.6928 \times 10^{-21}$ 

Check the result from TEST in Balance Equation

 $H: 2 = 2a + b \rightarrow 2(0.9999978) + 0.0000023 = 1.9999979(1.05 \times 10^{-4})$ 

$$O: 1 = a + b + 2c + d \rightarrow 0.9999978 + 0.0000023 + 7.565 \times 10^{-21} + 9.79 \times 10^{-12} = 1.0000001$$

 $H_2O = 0.9999978$  kmol

**14-3-41** [BSI] A mixture of 2 kmol of  $CO_2$  and 1 kmol of  $O_2$  is heated to 3200 K at a pressure of 5 atm. Determine the equilibrium composition of (a)  $CO_2$  (in kmols) using the IGE open-steady equilibrium TESTcalc assuming that only  $CO_2$ ,  $CO_2$  and  $O_3$  are present.

## **SOLUTION**

Using the IGE TESTcalc, the following equation is obtained:  $2\text{CO}_2 + \text{O}_2 \rightarrow 1.346\text{CO}_2 + 0.654\text{CO} + 1.229\text{O}_2 + 0.196\text{O}$ 



**14-3-42** [BSG] Air (21%  $O_2$ , 79%  $N_2$ ) is heated to 2800 K at a pressure of 1 atm. Determine the equilibrium composition, assuming that only  $O_2$ ,  $N_2$ , O and NO are present. Can the presence of NO in the equilibrium mixture be neglected?

#### **SOLUTION**

Molar Analysis:

$$21 \text{ O}_2 + 79 \text{ N}_2 \rightarrow \text{a O}_2 + \text{b N}_2 + \text{c O} + \text{d NO}$$

O: 
$$42 = 2a + c + d$$
;  $\Rightarrow d = 42 - 2a - c$ 

N: 
$$158 = 2b + d$$
;  $\Rightarrow b = 79 - 0.5d$ ;  $\Rightarrow b = 79 - 21 + a + 0.5c$ ;  $\Rightarrow b = 58 + a + 0.5c$ 

Total: 
$$a+b+c+d=a+58+a+0.5c+c+42-2a-c=100+0.5c$$

$$y_{O_2} = \frac{a}{100 + 0.5c}; \quad y_{N_2} = \frac{b}{100 + 0.5c} = \frac{58 + a + 0.5c}{100 + 0.5c};$$
$$y_{O} = \frac{c}{100 + 0.5c}; \quad y_{NO} = \frac{d}{100 + 0.5c} = \frac{42 - 2a - c}{100 + 0.5c}$$

Now we use:

$$O_2 \rightleftharpoons 2O$$
 gives us  $\ln K_{p@2800\text{K}} = -5.826$ 

$$\frac{1}{2}$$
N<sub>2</sub> +  $\frac{1}{2}$ O<sub>2</sub>  $\Longrightarrow$  NO gives us  $\ln K_{p@2800K} = -2.372$ 

$$K_{p} = \frac{y_{0}^{2}}{y_{0_{2}}} \left(\frac{p}{p_{0}}\right)^{2-1} = \left[p = p_{0}\right] = \frac{y_{0}^{2}}{y_{0_{2}}} = \frac{\sum v_{p}}{\sum v_{p}} = \frac{c^{2}}{\left(21 - 0.5c - 0.5d\right)\left(100 + 0.5c\right)} = e^{-5.826} \Leftrightarrow$$

$$\Leftrightarrow f(c,d) = c^2 - e^{-5.826} (21 - 0.5c - 0.5d) (100 + 0.5c) = 0$$

$$K_{p} = \frac{y_{NO}}{\sqrt{y_{N_{2}}}\sqrt{y_{O_{2}}}} \left(\frac{p}{p_{0}}\right) = \left[p = p_{0}\right] = \frac{y_{NO}}{\sqrt{y_{N_{2}}}\sqrt{y_{O_{2}}}} = \frac{\frac{d}{\sum v_{p}}}{\sqrt{b}\sqrt{a}} = \frac{d}{\sqrt{21 - 0.5c - 0.5d}\sqrt{79 - 0.5d}} = e^{-2.372} \Leftrightarrow \frac{d}{\sum v_{p}}$$

$$\Leftrightarrow g(c,d) = d - e^{-2.372} \sqrt{21 - 0.5c - 0.5d} \sqrt{79 - 0.5d} = 0$$

TEST calculations gives:  $a \approx 18.08008$ ;  $b \approx 77.23361$ ;  $c \approx 2.30862$ ;  $d \approx 3.53065$  Let us now see that the equations we have above satisfy that:

$$f(c,d) \approx -0.065$$
  
 $g(c,d) \approx 0.044$ 

This is close enough to be true since  $K_p$  as well as a, b, c & d are not as exact as in TEST so it should be some discrepancy.

To find the equilibrium composition we must find the relative mole fraction:

$$y_{O_2} = \frac{a}{100 + 0.5c} \approx 0.17874; \ y_{N_2} = \frac{b}{100 + 0.5c} \approx 0.76352; \ y_O = \frac{c}{100 + 0.5c} \approx 0.02282;$$
$$y_{NO} = \frac{d}{100 + 0.5c} \approx 0.03490$$

If we don't assume we have any NO: 21 O<sub>2</sub> + 79 N<sub>2</sub>  $\rightarrow$  a O<sub>2</sub> + b N<sub>2</sub> + c O TEST gives us:  $a \approx 19.79194$ ;  $b \approx 78.99996$ ;  $c \approx 2.41610$ 

The equilibrium composition is 17.9%  $O_2$ , 76.4%  $N_2 + 2.3\%$  O + 3.5% NO As the TEST calculations above shows NO can **not** be neglected as a product. N,  $N_2O$ ,  $NO_2$  &  $O_3$  on the other hand can be neglected as products (*verified with a TEST calculation*)



14-3-43 [BSL] The equilibrium constant for dissociation of  $N_2O_4$  is 0.664 and 0.141 at 318 K and 298 K respectively. Calculate the average heat of reaction within this temperature range.

## **SOLUTION**

Given:

 $T_1 = 318 \text{ K};$ 

 $T_2 = 298 \text{ K};$ 

 $K_1 = 0.664;$ 

 $K_2 = 0.141;$ 

$$\ln \frac{K_{2}}{K_{1}} = \frac{-\left(\Delta \bar{h}_{T}^{\circ}\right)}{\bar{R}} \left[\frac{1}{T_{2}} - \frac{1}{T_{1}}\right]; \quad \Rightarrow \Delta \bar{h}_{T}^{\circ} = \bar{R} \frac{\ln \frac{K_{2}}{K_{1}}}{\left(\frac{1}{T_{1}} - \frac{1}{T_{2}}\right)};$$

$$\Delta \bar{h}_{T}^{\circ} = 8.314 \frac{\ln \frac{0.141}{0.664}}{\left(\frac{1}{318} - \frac{1}{298}\right)};$$

$$\Rightarrow \Delta \bar{h}_{T}^{\circ} = 61,040 \frac{\text{kJ}}{\text{kmol}}$$

**14-3-44** [BSK] At an average temperature of 2000 K, the slope of the graph of log K against 1/T for the dissociation of water vapor (into hydrogen and oxygen) is found to be -13000. (a) Determine the heat of dissociation. (b) Is this an exothermic reaction?

## **SOLUTION**

$$\ln \frac{K_{2}}{K_{1}} = \frac{-\left(\Delta \overline{h}_{T}^{\circ}\right)}{\overline{R}} \left[\frac{1}{T_{2}} - \frac{1}{T_{1}}\right];$$

$$\Rightarrow \frac{\ln \frac{K_{2}}{K_{1}}}{\frac{1}{T_{2}} - \frac{1}{T_{1}}} = -13000 = \frac{-\left(\Delta \overline{h}_{T}^{\circ}\right)}{\overline{R}};$$

$$\Rightarrow \Delta \overline{h}_{T}^{\circ} = 13000\overline{R} = (13000)(8.314) = 108082 \frac{\text{kJ}}{\text{kmol}}$$

No, a positive heat of dissociation means the reaction is endothermic.



**14-3-45** [BSU] Potassium is ionized according to the equation  $K \leftrightarrow K^+ + e^-$ . The values of equilibrium constants at 3000 K and 3500 K are measured to be  $8.33 \times 10^{-6}$  and  $1.33 \times 10^{-4}$  respectively. Determine the average heat of reaction in MJ/kmol.

## **SOLUTION**

Given:

$$T_1 = 3000 \text{ K};$$

$$T_2 = 3,500 \text{ K};$$

$$K_1 = 8.33 \times 10^{-6}$$
;

$$K_2 = 1.33 \times 10^{-4}$$
;

$$\ln \frac{K_{2}}{K_{1}} = \frac{-\left(\Delta \bar{h}_{T}^{\circ}\right)}{\bar{R}} \left[\frac{1}{T_{2}} - \frac{1}{T_{1}}\right]; \quad \Rightarrow \Delta \bar{h}_{T}^{\circ} = \bar{R} \frac{\ln \frac{K_{2}}{K_{1}}}{\left(\frac{1}{T_{1}} - \frac{1}{T_{2}}\right)};$$

$$\Delta \bar{h}_{T}^{\circ} = 8.314 \frac{\ln \frac{1.33E - 4}{8.33E - 6}}{\left(\frac{1}{T_{1}} - \frac{1}{T_{2}}\right)};$$

$$\Delta \bar{h}_{T}^{\circ} = 8.314 \frac{\ln \frac{1.33E - 4}{8.33E - 6}}{\left(\frac{1}{3,000} - \frac{1}{3,500}\right)};$$

$$\Rightarrow \Delta \bar{h}_{T}^{\circ} = 483.71 \frac{MJ}{kmol}$$

**14-3-46** [BSZ] The equilibrium constant for the reaction  $SO_3 \leftrightarrow SO_2 + O$  has the following values:

T	800 K	900 K	1000 K	1105 K
K	0.0319	0.153	0.540	1.59

Determine the average heat of dissociation using graphical method.

## **SOLUTION**

$$\ln \frac{K_2}{K_1} = \frac{-\left(\Delta \overline{h_T}^{\circ}\right)}{R} \left[\frac{1}{T_2} - \frac{1}{T_1}\right]; \qquad \Rightarrow \Delta \overline{h_T}^{\circ} = R \frac{\ln \frac{K_2}{K_1}}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)};$$

$$\Delta \overline{h}_{T,1\to 2}^{\circ} = 8.314 \frac{\ln\left(\frac{0.153}{0.0319}\right)}{\left(\frac{1}{800} - \frac{1}{900}\right)} = 93.851 \frac{\text{MJ}}{\text{kmol}}$$

$$\Delta \overline{h}_{T,2\to 3}^{\circ} = 8.314 \frac{\ln\left(\frac{0.540}{0.153}\right)}{\left(\frac{1}{900} - \frac{1}{1000}\right)} = 94.365 \frac{\text{MJ}}{\text{kmol}}$$

$$\Delta \overline{h}_{T,3\to 4}^{\circ} = 8.314 \frac{\ln\left(\frac{1.59}{0.540}\right)}{\left(\frac{1}{1000} - \frac{1}{1105}\right)} = 94.487 \frac{\text{MJ}}{\text{kmol}}$$

$$\Delta \overline{h}_{T}^{\circ} = \frac{\Delta \overline{h}_{T,1 \to 2}^{\circ} + \Delta \overline{h}_{T,2 \to 3}^{\circ} + \Delta \overline{h}_{T,3 \to 4}^{\circ}}{3} = \frac{93.851 + 94.365 + 94.487}{3} = \frac{94.234}{\text{kmol}}$$

**14-3-47** [BVT] 1 kmol of Carbon at 25°C, 0.1 MPa reacts with 2.5 kmol of oxygen at 25°C, 0.1 MPa form an equilibrium mixture of CO<sub>2</sub>, CO and O<sub>2</sub> at 3000 K, 0.1 MPa. Determine (a) the amount of CO<sub>2</sub> present in the products mixture and (b) heat transfer for the process.

## **SOLUTION**

Using the IGE open-steady equilibrium TESTcalc, the following equilibrium reaction is found:

$$C+2.5O_2 \Rightarrow 0.7043CO_2 + 0.2956CO + 0.09O_2$$

$$CO_2 = 0.7043 \text{ kmol}$$

The enthalpies for the products and the reactants are also found as

$$h_R = 7789.327 \frac{\text{kJ}}{\text{kg}}$$

$$h_P = -139.874 \frac{\text{kJ}}{\text{kg}}$$

The mass involved in the reaction,

$$m = m_{\rm C} + m_{\rm O_2} = n_{\rm C} \overline{M}_{\rm C} + n_{\rm O_2} \overline{M}_{\rm O_2} = (1)(12) + (2.5)(32) = 92 \text{ kg}$$

Therefore, the heat transfer in this process,

$$Q - W_{\text{ext}}^{0} = m(h_P - h_R) = (92)(-139.874 - 7789.327) = -729486.49 \text{ kJ} = -729.486 \text{ MW}$$

**14-3-48** [BSY] For the reaction in the previous problem, 14-3-47 [BVT], plot how heat transfer varies with equivalence ratio varying from 0.1 to 1, all other parameters remaining unchanged.

## **SOLUTION**

Keeping in mind that the equivalence ratio is the inverse of theoretical air  $\phi = 1/\lambda$ ,

Parameter		Reactants			Products		Results				
lambda	phi	n_C	n_O2	n_N2	n_CO2	n_O2	n_N2	m	h_R	h_P	Q
unitless	unitless	[kmol]	[kmol]	[kmol]	[kmol]	[kmol]	[kmol]	[kg]	[kJ/kg]	[kJ/kg]	[MW]
10	0.1	1	10	37.6	1	9	37.6	1385.302	62.2	2980.07	4042.132
5	0.2	1	5	18.8	1	4	18.8	698.6567	123.33	2712.1	1808.662
3.333333	0.3	1	3.333333	12.53333	1	2.333333	12.53333	469.7748	183.41	2448.69	1064.171
2.5	0.4	1	2.5	9.4	1	1.5	9.4	355.3339	242.49	2189.73	691.9203
2	0.5	1	2	7.52	1	1	7.52	286.6693	300.57	1935.12	468.5753
1.666667	0.6	1	1.666667	6.266667	1	0.666667	6.266667	240.8929	357.68	1684.73	319.6769
1.428571	0.7	1	1.428571	5.371429	1	0.428571	5.371429	208.1955	413.86	1438.47	213.3192
1.25	0.8	1	1.25	4.7	1	0.25	4.7	183.6724	469.12	1196.23	133.5501
1.111111	0.9	1	1.111111	4.177778	1	0.111111	4.177778	164.5989	523.48	957.92	71.50836
1	1	1	1	3.76	1	0	3.76	149.3401	576.97	723.6233	21.90122

# Heat Transfer per kmol of C vs Equivalence Ratio

