15-1-1 [BAS] Air flows at Mach 0.2 through a circular duct with an internal diameter of 50 cm. The total pressure of the flow is 500 kPa and the total temperature is 200^{o}C. (a) Calculate the mass flow rate (m) through the channel. (b) What-if Scenario: What would the mass flow rate be if the gas were helium?

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc, obtain:

$$R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$$

$$\text{Given: }(\text{Air}, M = 0.2,\ d = 50\text{ cm}, T_t = 200°\text{C},\ p_t = 500\text{ kPa})$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.5\text{m})^2}{4} = 0.1963\text{ m}^2$$

(Alternative methods: obtain values from the isentropic table for air (Table H-1) in the textbook, or from table panel of the gas dynamics TESTcalc.)

$$\frac{T_t}{T} = 1 + \frac{k-1}{2}M^2 = 1 + \frac{1.4-1}{2}(0.2)^2$$

$$\Rightarrow T = \frac{473.15}{1.008} = 469.40\text{ K}$$

$$\frac{p_t}{p} = \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} = \left(\frac{473.15}{469.398}\right)^{\frac{1.4}{1.4-1}}$$

$$\Rightarrow p = \frac{500\text{ kPa}}{1.028} = 486.26\text{ kPa}$$

$$\dot{m} = \rho AV = \frac{p}{RT}AM\sqrt{1000kRT} = 61.54\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-2 [BAA] Determine (a) the velocity (V) of sound, (b) the Mach number, (c) the total temperature (T) and (d) the total pressure (p) of air that is flowing at 50 kPa, 250 K and 500 m/s. (e) What-if Scenario: What would the Mach number be if the gas were helium?

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc, obtain:

$$R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ c_p = 1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$$

Given: (Air, $p = 50$ kPa, $T = 250$ K, $V = 500$ m/s)

(a) $$c = \sqrt{1000kRT} = \sqrt{(1000)(1.4)(0.287)(250)} = 316.9\ \frac{\text{m}}{\text{s}}$$

(b) $$M = \frac{V}{c} = \frac{500}{316.9} = 1.58$$

(c) and (d) can be calculated from the isentropic relation equations for PG Model:

(c) $$T_t = T + \frac{V^2}{c_p(2000)} = 250 + \frac{500^2}{(1.005)(2000)} = 374.4\ \text{K}$$

(d) $$p_t = p\cdot\left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} = 50\cdot\left(\frac{374.4}{250}\right)^{\frac{1.4}{1.4-1}} = 205.5\ \text{kPa}$$

TEST Solution:

Launch the gas dynamics TESTcalc and select air/He as the working fluid to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-3 [BAH] A subsonic airplane is flying at a 3500 m altitude where the atmospheric conditions are 72 kPa and 260 K. A pitot tube measures the difference between the static and total pressures to be 10 kPa. Determine (a) the speed of the airplane and (b) the flight Mach number.

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc, obtain:

$$R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ c_p = 1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$$

Given: $(p = 72\text{ kPa}, T = 260\text{ K},\ p_t - p = 10\text{ kPa})$

$$\frac{T_t}{T} = \left(\frac{p_t}{p}\right)^{\frac{k-1}{k}};\quad \Rightarrow T_t = (260\text{K})\left(\frac{82\text{ kPa}}{72\text{ kPa}}\right)^{\frac{0.4}{1.4}} = 269.843 = 270\text{ K}$$

$$\frac{T_t}{T} = 1 + \frac{k-1}{2}M^2;\quad \Rightarrow M = \sqrt{\left(\frac{T_t}{T} - 1\right)\left(\frac{2}{k-1}\right)} = 0.435069 = 0.435\ \text{(subsonic flow)}$$

$$V = Mc = M\left(\sqrt{(1000)kRT}\right) = 0.435\sqrt{(1000)(1.4)(0.286)(260\text{K})} = 140.4 = 140\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-4 [BAN] Saturated steam at 200 kPa is flowing with a velocity of 500 m/s. A pitot tube brings the flow to stagnation and the stagnation pressure is measured as 350 kPa. Determine (a) the total pressure (p), (b) the total temperature (T) and (c) the stagnation temperature. Use the PC model.

SOLUTION:

For PC Model, we can use the PC flow-state TESTcalc or the manual approach (Sec.3.4) to evaluate the steam properties.

State-1 (given $p_1 = 200 \text{ KPa}, \; x = 1\,(\text{sat.steam}), V_1 = 500 \text{ m/s}$)

$$h_1 = 2706.6 \frac{\text{kJ}}{\text{kg}}, \; s_1 = 7.127 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 is the stagnation state, in which the specific flow energy remains constant:

(given $p_2 = 350 \text{ kPa}, \; j_2 = j_1, V_2 = 0$)

$$h_2 = h_1 + \text{ke}_1 = h_1 + \frac{V_1^2}{2000} = 2831.6 \frac{\text{kJ}}{\text{kg}}$$

The stagnation temperature can be manually interpolated as shown in example 3-6 in the textbook, or evaluated by using the PC flow-state TESTcalc:

$$T_2 = T_s = 185.2 \text{ °C}$$

State-3 is the isentropic stagnation state, in which the specific flow energy and entropy remain constant:

(given $j_3 = j_1, \; s_3 = s_1, V_3 = 0$)

$$\text{since} \quad j_3 = j_1 \Rightarrow h_3 = 2831.6 \frac{\text{kJ}}{\text{kg}}$$

$$s_3 = s_1 = 7.127 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

From the known enthalpy and entropy, state-3 can be determined using the manual approach (Sec.3.4 in the textbook), or using a PC state TESTcalc.

Total pressure and total temperature: $p_t = p_3 = 381.4 \text{ kPa}, \; T_t = T_3 = 186.1\text{°C}$

TEST Solution:

Use the PC system state TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-5 [BAE] Air flowing at 100 kPa and 298 K is brought to rest, and the stagnation pressure and temperature are measured as 130 kPa and 329 K, respectively. Determine (a) the flow velocity and (b) the total pressure (p) of the flow.

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; R = 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad k = 1.4$$

State-1: Given $(p_1 = 100\text{ kPa}, T_1 = 298\text{ K})$

State-2: Given $(p_2 = 130\text{ kPa}, T_2 = 329\text{ K}, \text{V}_2 = 0)$

(a) From the conservation of flow energy:

$$j_1 = j_2; \Rightarrow h_1 + \cancel{\text{pe}_1} + \text{ke}_1 = h_2 + \cancel{\text{pe}_2} + \cancel{\text{ke}_2}$$

$$\Rightarrow h_2 - h_1 = \text{ke}_1$$

$$\Rightarrow c_p(T_2 - T_1) = \frac{V_1^2}{2000} \quad \text{(PG Model)}$$

$$V_1 = \sqrt{2000c_p(T_2 - T_1)} = 249.6\text{ m/s}$$

(b) Since the total state is defined as the isentropic stagnation state, $T_{t1} = 329\text{ K}$ and remains constant in the flow(conservation of energy)

p_{t1} can be calculated by using the isentropic relation for PG Model:

$$\frac{p_{t1}}{p_1} = \left(\frac{T_{t1}}{T_1}\right)^{\frac{k}{k-1}}$$

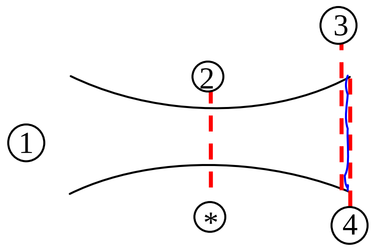
$$\Rightarrow p_{t1} = p_1\left(\frac{T_{t1}}{T_1}\right)^{\frac{k}{k-1}} = 100\text{ kPa}\left(\frac{329}{298}\right)^{\frac{1.4}{0.4}} = 141.4\text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-6 [BNG] Determine (a) the back pressure necessary for a normal shock to appear at the exit of a converging-diverging nozzle, with an exit to throat area ratio of 2, if the reservoir conditions are 1 MPa and 850 K. (b) What-if Scenario: What would the back pressure be if the working gas were helium instead?

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

State-1: Resevoir ($p_{t1} = 1\text{ MPa}, T_{t1} = 850\text{ K}$)

State-2: Throat ($A_2 = A_*$)

State-3: At the exit before the normal shock ($A_3 = 2A_2 = 2A_*,\ p_{t3} = p_{t1}$)

$A_3/A_* = A_3/A_2 = 2$,

$$\Rightarrow M_3 = 2.2, \frac{p_3}{p_{t3}} = 0.0939 \text{ (from isentropic table)}$$

$$p_3 = \left(\frac{p_3}{p_{t3}}\right) p_{t3} = (0.09397)(1\text{ MPa}) = 94\text{ kPa}$$

State-4: At the exit after the shock ($A_4 = A_3$)

From the shock table, we can obtain the static pressure ratio for $M_3 = 2.2$:

$$\text{Back pressure} = p_4 = \left(\frac{p_4}{p_3}\right) p_3 = \left(\frac{p_e}{p_i}\right)_{@M_3=2.2} p_3 = (5.46)(94) = 513\text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-7 [BAI] Carbon dioxide enters an adiabatic nozzle at 1200 K with a velocity of 100 m/s and leaves at 500 K. Determine the Mach number (a) at the inlet and (b) at the exit of the nozzle.

SOLUTION:

Working fluid: carbon dioxide; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 0.846 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; R = 0.189 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad k = 1.288$$

Given: $(T_1 = 1200 \text{ K}, V_1 = 100 \text{ m/s}, T_2 = 500 \text{ K})$

(a) State-1:

Mach number can be calculated from the sound wave equation for PG Model:

$$c_1 = \sqrt{1000kRT} = \sqrt{1000(1.288)(0.189)(1200)} = 540.5 \frac{\text{m}}{\text{s}}$$

$$M_1 = \frac{V_1}{c_1} = \frac{100}{540.5} = 0.185$$

We will also need to determine T_{t1} in order to calculate Mach number at state-2:

$$T_{t1} = T_1 + \frac{V_1^2}{2000c_p} = 1200 + \frac{100^2}{(2000)(0.846)} = 1205.9 \text{ K}$$

(b) State-2:

From the conservation of flow energy, $T_{t2} = T_{t1} = 1205.9 \text{ K}$

$$M_2 = \sqrt{\frac{2}{k-1}\left(\frac{T_{t2}}{T_2} - 1\right)} = \sqrt{\frac{2}{1.28758-1}\left(\frac{1205.9}{500} - 1\right)} = 3.13$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-8 [BAL] Air enters an adiabatic nozzle at 1200 K with a velocity of 100 m/s and leaves at 500 K. Determine (a) the Mach number at the inlet, (b) the velocity (V_2) and (c) the Mach number at the exit of the nozzle.

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad k = 1.4$$

Let States 1 and 2 represent the inlet and exit states.

State-1:

Given: $(T_1 = 1200 \text{ K}, V_1 = 100 \text{ m/s}, T_2 = 500 \text{ K})$

$$c_1 = \sqrt{1000kRT} = \sqrt{1000(1.4)(0.287)(1200)} = 694.4 \ \frac{\text{m}}{\text{s}}$$

$$M_1 = \frac{V_1}{c_1} = \frac{100}{694.4} = 0.144$$

$$T_{t1} = T_1 + \frac{V_1^2}{2000c_p} = 1200 + \frac{100^2}{(2000)(1.005)} = 1205 \text{ K}$$

State-2:

From the conservation of flow energy: $T_{t2} = T_{t1} = 1205$ K

$T_2 = 500$ K

$$M_2 = \sqrt{\frac{2}{k-1}\left(\frac{T_{t2}}{T_2} - 1\right)} = \sqrt{\frac{2}{1.4-1}\left(\frac{1205}{500} - 1\right)} = 2.655$$

$$V_2 = M_2\sqrt{1000kRT} = 2.655\sqrt{1000(1.4)(0.287)(500)} = 1190.1 \ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-9[BAK] Determine the velocity of sound in air (a) at 273 K and (b) at 1000 K?

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad k = 1.4$$

The velocity of sound can be calculated from the sound wave equation for PG Model:

(a) $c = \sqrt{(1000)kRT} = \sqrt{1000(1.4)(0.287)(273)} = 331.2 \ \frac{\text{m}}{\text{s}}$

(b) $c = \sqrt{(1000)kRT} = \sqrt{1000(1.4)(0.287)(1000)} = 633.9 \ \frac{\text{m}}{\text{s}}$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-10 [BAG] Determine the velocity of sound in steam at 800 kPa and 350°C assuming (a) 1% variation of temperature across the wave and using the PC model, (b) steam to behave as a perfect gas. Use the PC Flow State TESTcalc to evaluate steam properties.

SOLUTION:

State-1 (given $p1 = 800$ kPa, $T1 = 350$ ºC)

(a) For PC Model, the velocity of sound is determined as:

$$c = \sqrt{(1000\,\text{N/kN})\left(\frac{\partial p}{\partial \rho}\right)}$$

Evaluate the steam properties for the PC Model by using PC Flow State in TESTcalc.

From the given information, density and entropy of steam can be found:

$$\rho_1 = 2.8218\frac{\text{kg}}{\text{m}^3}$$

$$s_1 = 7.40876\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Since the flow is assumed to be isentropic, the entropy remains constant:

State-2 ($s_2 = s_1$, $T_2 = 1.01T_1$)

From the above information, pressure and desity of state-2 can be found:

$$\rho_2 = 2.87278\frac{\text{kg}}{\text{m}^3}$$

$$p_2 = 818.97815\text{ kPa}$$

So

$$\left(\frac{\partial p}{\partial \rho}\right)_s \cong \left(\frac{\Delta p}{\Delta \rho}\right)_s = \frac{818.97815 - 800}{2.87278 - 2.8218} = 372.27\frac{\text{kPa}\cdot\text{m}^3}{\text{kg}}$$

$$c = \sqrt{(1000)\left(\frac{\partial p}{\partial \rho}\right)_s} = \sqrt{(1000)(372.27)\frac{\text{Pa}}{\text{kPa}}\frac{\text{kPa}\cdot\text{m}^3}{\text{kg}}} = 610.14\ \frac{\text{m}}{\text{s}}$$

(b) For PG Model, the velocity of sound is determined as:

$$c = \sqrt{(1000\,\text{N/kN})kRT}$$

The steam properties of PG Model can be found in table C-1, or evaluated by using PG Flow State in TESTcalc:

$k = 1.32749$

$R = 0.46189 \dfrac{\text{kJ}}{\text{kg} \cdot \text{K}}$

$c = \sqrt{1000kRT} = \sqrt{(1000)(1.32749)(0.46189)(350+273)} = 618.06 \ \dfrac{\text{m}}{\text{s}}$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-11 [BAZ] Determine the velocity of sound in steam at 800 kPa and 350^{o}C assuming (a) 1% variation of pressure across the wave and using the PC model, (b) steam to behave as a perfect gas. Use the PC Flow State TESTcalc to evaluate steam properties.

SOLUTION:

Using the PC system TESTcalc for H2O:

State 1 (given p1 = 800 kPa, T1 = 350 ºC)

$$\rho_1 = 2.8218\ \frac{\text{kg}}{\text{m}^3};\ s_1 = 7.40876\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State 2 (s2 = s1, p2 = 1.01*p1)

$$\rho_2 = 2.84336\frac{\text{kg}}{\text{m}^3};\ T_2 = 624.6\text{ K}$$

So $\left(\dfrac{\partial p}{\partial \rho}\right)_s \cong \left(\dfrac{\Delta p}{\Delta \rho}\right)_s = \dfrac{808-800}{2.84336-2.8218} = 371.1\ \dfrac{\text{kPa}\cdot\text{m}^3}{\text{kg}}$

$$\Rightarrow c = \sqrt{(1000)\left(\frac{\partial p}{\partial \rho}\right)_s} = \sqrt{(1000)(371.1)\frac{\text{Pa}}{\text{kPa}}\frac{\text{kPa}\cdot\text{m}^3}{\text{kg}}} = 609.2\ \frac{\text{m}}{\text{s}}$$

From Table C-1 or the gas dynamics TESTcalc obtain:

$$k = 1.32749\,;\ R = 0.46189\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\Rightarrow c = \sqrt{(1000)(k)(R)(T)} = \sqrt{(1000)(1.32749)(0.46189)(350+273)} = 618.06\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-12 [BAP] Refrigerant R-134a flows at 100 m/s at the exit of a nozzle where the pressure and temperature are 500 kPa and 30°C, respectively. Determine the Mach number of the flow state. Use the PC model for R-134a and assume a 1% variation in temperature across a sound wave.

SOLUTION:

Given $p_1 = 500\text{ kPa},\; T_1 = 303\text{ K},\; \text{V}_1 = 100\ \dfrac{\text{m}}{\text{s}}$, and using the PC system TESTcalc for R-134a:

$$h_1 = 277.23\ \frac{\text{kJ}}{\text{kg}},\; \rho_1 = 22.48\ \frac{\text{kg}}{\text{m}^3}$$

Given the 1% temperature variation across a sound wave:

$$T_2 = (1.01)T_1 = (1.01)(303) = 306\text{ K}$$

The formula for the speed of a sound wave at constant entropy is given by: $a = \sqrt{\left(\dfrac{\partial p}{\partial \rho}\right)_s}$

Setting the PC ystem TESTcalc with $s_2 = s_1$ and $T_2 = 306\text{ K}$ yields

$$p_2 = 544.41\text{ kPa},\; \rho_2 = 24.412\text{ kPa}.$$

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \cong \sqrt{\left(\frac{\Delta p}{\Delta \rho}\right)_s} = \sqrt{\left(\frac{544.41-500}{24.41-22.49}\right)} = 152.09\ \frac{\text{m}}{\text{s}}$$

Therefore, the Mach number of the flow is given by: $M = \dfrac{V}{a} = \dfrac{100}{152.09} = 0.658$

15-1-13 [BAUA needle nose projectile traveling at a speed with M = 2 passes 250 m above the observer. Determine (a) the projectile's velocity (V), and (b) how far beyond the observer the projectile will first be heard. Assume the static temperature as 15^{o}C.

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.4$$

Given: (T = 288 K, M = 2 m/s)

(a) $$c = \sqrt{1000kRT} = \sqrt{(1000)(1.4)(0.287)(288)} = 340.2 \ \frac{\text{m}}{\text{s}}$$

Therefore, the projectile velocity is: $V = Mc = 2\left(340.2\right) = 680.4 \ \frac{\text{m}}{\text{s}}$

(b) The angle made by the mach wave (see Anim. 15.B.MachNumber) with the flight direction is:

$$\mu = \text{Sin}^{-1}\left(\frac{1}{M}\right) = \text{Sin}^{-1}\left(\frac{1}{2}\right) = 30^{o}$$

After the projectile covers a distance of x from the location of the observer,

$$\tan\mu = \frac{250}{x};\quad \Rightarrow x = \frac{250}{\tan\left(30\right)} = 433 \text{ m}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-14 [BAX] Air flows through a device such that the total pressure is 700 kPa, the total temperature is 300°C and the velocity is 500 m/s. Determine (a) the static pressure (p) and (b) the static temperature (T) of air at this state. (c) What-if Scenario: What would the static temperature be if the gas were oxygen?

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ R = 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.4$$

Given: ($p_t = 700\ \text{KPa}, T_t = 300\ °\text{C}, V = 500\ \text{m/s}$)

The static temperature can be determined by using the isentropic relation for a perfect gas:

$$T_t = T + \frac{V^2}{2(1000\,\text{J/kJ})c_p};\quad \Rightarrow T = T_t - \frac{V^2}{2(1000\,\text{J/kJ})c_p}$$

$$T = 300°\text{C} - \frac{(500)^2}{2(1000)(1.005)} = 175.6\ °\text{C}$$

$$\frac{p_t}{p} = \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}};\quad \Rightarrow p = p_t \Big/ \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}}$$

$$p = 700\ \text{kPa} \Big/ \left(\frac{300+273}{175.6+273}\right)^{\frac{1.4}{0.4}} = 297.2\ \text{kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-15 [BAC] Air enters a diffuser with a velocity of 180 m/s and a inlet temperature of 303 K. Determine (a) the velocity of sound and (b) the flow Mach number at the diffuser inlet. (c) What-if Scenario: What would the flow Mach number at the diffuser inlet be if the gas were nitrogen?

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \; R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad k = 1.4$$

Given: ($T = 303\,\text{K}$, V=180 m/s)

(a): Speed of sound: $c = \sqrt{(1000)kRT} = 348.88 \; \frac{\text{m}}{\text{s}}$

(b): Mach number: $M = \frac{V}{c} = 0.5159$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-16 [BAV] Determine the total temperature for the following substances flowing through a duct: (a) helium at 180 kPa, 40°C and 250 m/s, (b) nitrogen at 180 kPa, 40°C and 250 m/s, and (c) steam at 3 MPa, 300°C and 400 m/s.

SOLUTION:

The total temperature and pressure can be calculated from the equations for isentropic relations of perfect gas:

$$T_t = T + \frac{V^2}{2(1000\text{ J/kJ})c_p}$$

$$\frac{p_t}{p} = \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}}$$

Values for c_p and k can be obtained from Table C-1 or gas dynamics TESTcalc:

(a)

(a) Working fluid: helium; $c_p = 5.1926 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.667$

$p = 180\text{ KPa}, T = 40\text{ °C}, V = 250\text{ m/s}$

$T_t = 46\text{ °C}; \quad p_t = 188.8\text{ KPa}$

(b) Working fluid: nitrogen; $c_p = 1.039 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

$p = 180\text{ KPa}, T = 40\text{ °C}, V = 250\text{ m/s}$

$T_t = 70.08\text{ °C}; \quad p_t = 248.11\text{ KPa}$

(c) Working fluid: H2O; $c_p = 1.8723 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.3275$

$p = 3\ M\text{Pa}, T = 300\text{ °C}, V = 400\text{ m/s}$

$T_t = 342.73\text{ °C}; \quad p_t = 4\text{ MPa}$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-17 [BAT] Air leaves a compressor in a pipe with a total temperature and pressure of 180°C, 350 kPa and a velocity of 150 m/s. The pipe has a cross sectional area of 0.02 m^2. Determine (a) the static pressure (p), (b) the static temperature (T) and (c) the mass flow rate ($\dot{m}$). (d) What-if Scenario: What would the mass flow rate be if the gas were hydrogen?

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.4$$

Given $(p_t = 350\ \text{kPa}, T_t = 180\ °\text{C}, A = 0.02\ \text{m}^2, V = 150\ \text{m/s})$

(a) and (b) can be calculated from the isentropic relation equations for PG Model:

$$T = T_t - \frac{V^2}{c_p \cdot 2000} = 180 - \frac{150^2}{(1.005)(2000)} = 168.8\ °\text{C}$$

$$\frac{p_t}{p} = \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} \Rightarrow p = p_t \bigg/ \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} = 350 \bigg/ \left(\frac{180+273.15}{168.8+273.15}\right)^{\frac{1.4}{0.4}} = 320.65\ \text{kPa}$$

$$\dot{m} = \rho A V = \frac{AV}{v} = \frac{AVp}{RT} = \frac{(0.02\ \text{m}^2)(150)(320.6)}{(0.287)(168.8+273.15)} = 7.58\ \text{kg/s};$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-18 [BAQ] Steam at 1.2 MPa and 450°C flows through a pipe with a velocity of 300 m/s. Treating the superheated steam as a perfect gas, determine (a) the velocity of sound and (b) the flow Mach number for the steam. (c) What-if Scenario: What would the flow Mach number for the steam be if the velocity of steam through the pipe were 480 m/s?

SOLUTION:

Working fluid: steam; From Table C-1 or gas dynamics TESTcalc:

$$R = 0.462 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad k = 1.33$$

$$\text{Given: } p = 1.2 \text{ MPa}; \; T = 450°\text{C}; \; V = 300 \frac{\text{m}}{\text{s}}$$

$$c = \sqrt{1000kRT}$$

$$c = \sqrt{1000(1.33)(0.462)(723)} = 665.8 \frac{\text{m}}{\text{s}}$$

$$M = \frac{V}{c} = \frac{300}{665.8} = 0.45$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-19 [BAX] In 15-1-16[BAV] determine the total pressure in each case.

SOLUTION:

The total temperature and pressure can be calculated from the equations for isentropic relations of perfect gas:

$$T_t = T + \frac{V^2}{2\left(1000\text{ J/kJ}\right)c_p}$$

$$\frac{p_t}{p} = \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}}$$

Values for c_p and k can be obtained from Table C-1 or gas dynamics TESTcalc:

(a) Given: (He, $p = 180\text{ KPa}, T = 40\text{ °C}, V = 250\text{ m/s}$)

$$c_p = 5.1926\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.667$$

$$T_t = 46\text{ °C};\quad p_t = 188.8\text{ KPa}$$

(b) Given: (N_2, $p = 180\text{ KPa}, T = 40\text{ °C}, V = 250\text{ m/s}$)

$$c_p = 1.039\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.4$$

$$T_t = 70.08\text{ °C};\quad p_t = 248.11\text{ KPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-20 [BAY] Air flows in a duct at a pressure of 200 kPa with a velocity of 200 m/s. The temperature of air is 300 K. Determine (a) the total pressure (*p*) and (b) the total temperature (*T*).

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.4$$

$$T_t = T + \frac{V^2}{2000c_p} = 300 + \frac{(200)^2}{2000(1.005)} = 319.9\ \text{K};$$

$$p_t = p\left(\frac{T_t}{T}\right)^{k/(k-1)} = (200)\left(\frac{319.9}{300}\right)^{1.4/(1.4-1)} = 250.4\ \text{kPa};$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-21 [BAW] Steam at 250°C and quality 95% is flowing through a duct with a velocity of 250 m/s. Determine the total properties (a) temperature (T), (b) pressure (p), (c) quality and (d) density. Use the PC model for steam. (e) What-if Scenario: What would the temperature be if steam were treated as a perfect gas?

SOLUTION:

(a-d) For PC Model, we can use the PC flow-state TESTcalc or the manual approach (Sec.3.4) to evaluate the steam properties.

State-1 (given $T_1 = 250^{\circ}\text{C},\ x = 0.95\ ,V_1 = 500\ \text{m/s}$

$$h_1 = 2715\frac{\text{kJ}}{\text{kg}},\ s_1 = 5.91\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 is the total (isentropic) state, in which the specific flow energy and entropy remains constant:

(given $s_2 = s_1,\ j_2 = j_1, V_2 = 0$)

$$h_2 = h_1 + \text{ke}_1 = h_1 + \frac{V_1^2}{2000} = 2747\ \frac{\text{kJ}}{\text{kg}}$$

From the known enthalpy and entropy, the state can be be determined using the manual approach (Sec.3.4 in the textbook), or using a PC state TESTcalc:

$$T_2 = T_s = T_t = 260\ ^{\circ}\text{C};\ \ p_2 = 4677\ \text{kPa};\ \ x_2 = 0.97\ \text{kPa};\ \ \rho_2 = 24.35\frac{\text{kg}}{\text{m}^3};$$

(e)

$$T_{t,1} = T_1 + \frac{V_1^2}{2000c_p} = 250 + \frac{(250)^2}{2000(1.8677)} \Rightarrow T_{t,1} = 266.7^{\circ}\text{C};$$

TEST Solution:

Use the PC flow-state TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-22 [BAF] Determine (a) the total temperature (T) and (b) the total pressure (p) of air that is flowing at 40 kPa, -25°C and 400 m/s.

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.4$$

$$T_t = T + \frac{V^2}{2000c_p} = 248 + \frac{(400)^2}{2000(1.005)} = 327.8\ \text{K};$$

$$p_t = p_1\left(\frac{T_t}{T}\right)^{k/(k-1)} = (200)\left(\frac{327.8}{300}\right)^{1.4/(1.4-1)} = 106.0\ \text{kPa};$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-23 [BAD] Nitrogen is discharged from a large reservoir at 500 K and 150 kPa through an adiabatic nozzle. At the exit the pressure is 100 kPa, area is 10 cm^2 and the Mach number is 0.7. Determine (a) the exit temperature (T_2), (b) the mass flow rate ($\dot{m}$) and (c) the total pressure (p_2) at the exit. (d) Is the flow through the nozzle isentropic? (If Yes: 1 and if No: 2)

SOLUTION:

Working fluid: nitrogen; From Table C-1 or gas dynamics TESTcalc:
$R = 0.297$ kJ/kg K, $k = 1.4$

Given $p_r = p_{ti} = 150$ kPa, $T_r = T_t = 500$ K, $p_e = 100$ kPa, $A_e = 10$ cm^2, $M_e = 0.7$

While T_t remains constant (energy equation), p_t can decrease due to irreversibilities.

State-e (exit):

$$T_e = \frac{T_t}{1+\frac{k-1}{2}M^2} = \frac{500\text{ K}}{1+\frac{1.4-1}{2}(0.7^2)} = 455.4\text{ K}$$

$$\dot{m} = \frac{p_e}{RT_e}AM\sqrt{kRT_e} = \frac{100}{(0.297)(455.4)}\left(\frac{10}{10000}\right)(0.7)\sqrt{(1000)(1.4)(0.297)(455.4)} = 0.225\ \frac{\text{kg}}{\text{s}}$$

$$p_{te} = p_e\left(\frac{T_t}{T_e}\right)^{\frac{k}{k-1}} = (100)\left(\frac{500}{455.4}\right)^{\frac{1.4}{1.4-1}} = 138.7\text{ kPa}$$

(a) Because $p_{t,e} \neq p_{t,i}$ the flow is not isentropic

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-24 [BAM] Repeat problem 15-1-23 [BAD] with carbon dioxide as the working fluid.

SOLUTION:

Working fluid: carbon dioxide; From Table C-1 or gas dynamics TESTcalc:
$R = 0.189$ kJ/kg K, $k = 1.29$

Given $p_r = p_{t,i} = 150$ kPa, $T_r = T_t = 500$ K, $p_e = 100$ kPa, $A_e = 10$ cm^2, $M_e = 0.7$

State-e (exit):

$$T_e = \frac{T_t}{1 + \frac{k-1}{2}M^2} = \frac{500\text{ K}}{1 + \frac{1.4-1}{2}(0.7^2)} = 467\text{ K}$$

$$\dot{m} = \frac{p_e}{RT_e} AM\sqrt{kRT_e} = \frac{100}{(0.189)(455.4)}\left(\frac{10}{10000}\right)(0.7)\sqrt{(1000)(1.29)(0.297)(467)} = 0.267\ \frac{\text{kg}}{\text{s}}$$

$$p_{te} = p_e\left(\frac{T_t}{T_e}\right)^{\frac{k}{k-1}} = (100)\left(\frac{500}{467}\right)^{\frac{1.4}{1.4-1}} = 135.6\text{ kPa}$$

(a) Because $p_{t,e} \neq p_{t,i}$ the flow is not isentropic

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-25 [BAJ] An aircraft is cruising at a velocity of 1000 km/h at an altitude of 10 km, where the static temperature and pressure are -50^{o}C and 26.5 kPa, respectively. Determine (a) the Mach number of the aircraft. Also determine (b) the pressure (p) and (c) temperature (T) of the air brought to rest isentropically by a diffuser. (d) What-if Scenario: How would the answer in part (c) change if the diffuser were adiabatic but irreversible?

SOLUTION:

Working fluid: air; From Table C-1 or gas dynamics TESTcalc:

$$c_p = 1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ R = 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad k = 1.4$$

Given: $(p = 26.5\ \text{kPa}, T = 223\ \text{K}, V = 1000\ \text{km/h} = 277.7\ \text{m/s})$

$$M = \frac{V}{c} = \frac{V}{\sqrt{(1000)kRT}} = \frac{277.7}{\sqrt{(1000)(1.4)(0.286)(223)}} = 0.928$$

$$T_t = T\left[1+\frac{k-1}{2}M^2\right] = (223)\left[1+\frac{1.4-1}{2}0.928^2\right] = 261.5\ \text{K}$$

$$\frac{T_t}{T} = \left(\frac{p_t}{p}\right)^{\frac{k-1}{k}};\quad \Rightarrow p_t = p\left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} = (26.5)\left(\frac{261.5}{223}\right)^{\frac{1.4}{0.4}} = 46.2\ \text{kPa}$$

The total temperature is the same as the stagnation temperature; therefore, it will not change regardless of irreversibilities.

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-1-26 [BHR] Steam at 450°C and 1 MPa is flowing through a nozzle with a velocity of 300 m/s. Determine (a) the total temperature (*T*) and (b) pressure (*p*) of the flow. Use the PG model for steam. (c) What-if Scenario: What would the total temperature and pressure of the flow be if the PC model were used instead?

SOLUTION:

Working fluid: steam; From Table C-1 or gas dynamics TESTcalc:
$R = 0.462$ kJ/kg K, $k = 1.327$

Given $(p = 1000 \text{ kPa}, T = 723 \text{ K}, V = 300 \text{ m/s})$

$$M = \frac{V}{c} = \frac{V}{\sqrt{(1000)kRT}} = \frac{300}{\sqrt{(1000)(1.327)(0.462)(723)}} = 0.451$$

$$T_t = T\left[1 + \frac{k-1}{2}M^2\right] = (723)\left[1 + \frac{1.327-1}{2}0.451^2\right] = 747.2 \text{ K}$$

$$\frac{T_t}{T} = \left(\frac{p_t}{p}\right)^{\frac{k-1}{k}}; \quad \Rightarrow p_t = p\left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} = (1000)\left(\frac{747.2}{723}\right)^{\frac{1.327}{0.327}} = 1142 \text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.