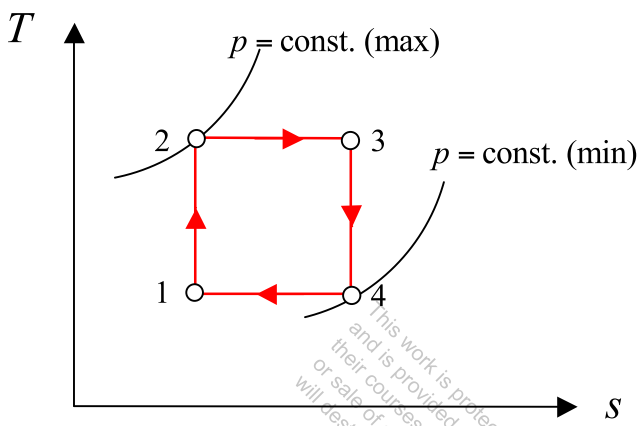
7-2-1 [OIY] A Carnot cycle running on a closed system has 1.5 kg of air. The temperature limits are 300 K and 1000 K, and the pressure limits are 20 kPa and 1900 kPa. Determine (a) the efficiency and (b) the net work output. Use the PG model.
(c) What-if Scenario: How would the answer in part (b) change if the IG model were used instead?

SOLUTION



Given:

$R = 0.287 \dfrac{\text{kJ}}{\text{kg} \cdot \text{K}}$

$k = 1.4005$

The highest pressure occurs at state-2 and the lowest at state-4.

For an isentropic process,

$$p_3 = p_4 \left(\frac{T_3}{T_4}\right)^{\frac{k}{k-1}};$$

$$\Rightarrow p_3 = (20)\left(\frac{1000}{300}\right)^{\frac{1.4005}{0.4005}};$$

$$\Rightarrow p_3 = 1347 \text{ kPa}$$

Therefore, the net work and efficiency are calculated as

$$s_3 - s_2 = c_p \ln\cancel{\frac{T_3}{T_2}}^{0} - R\ln\frac{p_3}{p_2};$$

$$\Rightarrow s_3 - s_2 = -(0.287)\ln\frac{1347}{1900};$$

$$\Rightarrow s_3 - s_2 = 0.0987\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$W_{\text{net}} = Q_{\text{net}};$$

$$\Rightarrow W_{\text{net}} = m(T_2 - T_1)(s_3 - s_2);$$

$$\Rightarrow W_{\text{net}} = (1.5)(1000 - 300)(0.0987);$$

$$\Rightarrow W_{\text{net}} = 103.7 \text{ kJ}$$

$$\eta_{\text{th},C} = 1 - \frac{T_1}{T_2};$$

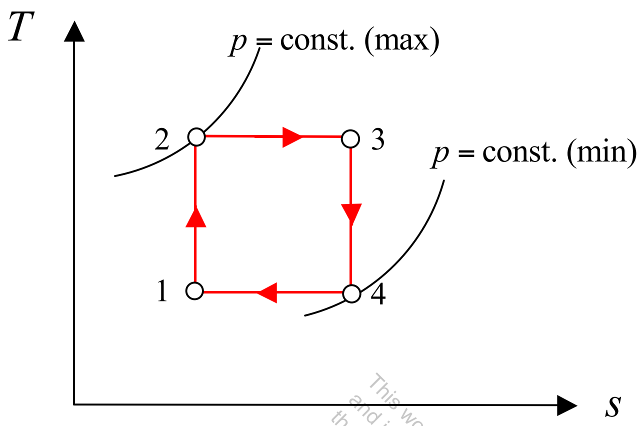
$$\Rightarrow \eta_{\text{th},C} = 1 - \frac{300}{1000};$$

$$\Rightarrow \eta_{\text{th},C} = 70\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) reciprocating closed-cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

7-2-2 [OIF] Consider a Carnot cycle executed in a closed system with 0.003 kg of air. The temperature limits are 25°C and 730°C, and the pressure limits are 15 kPa and 1700 kPa. Determine (a) the efficiency and (b) the net work output per cycle. Use the PG model for air.

SOLUTION



Given:

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4005$$

The highest pressure occurs at state-2 and the lowest at state-4.

For an isentropic process,

$$p_3 = p_4 \left(\frac{T_3}{T_4} \right)^{\frac{k}{k-1}};$$

$$\Rightarrow p_3 = (15) \left(\frac{1003}{298} \right)^{\frac{1.4005}{0.4005}};$$

$$\Rightarrow p_3 = 1045.3 \text{ kPa}$$

Therefore, the net work and efficiency are calculated as

$$s_3 - s_2 = c_p \cancel{\ln \frac{T_3}{T_2}}^{\,0} - R \ln \frac{p_3}{p_2};$$

$$\Rightarrow s_3 - s_2 = -(0.287) \ln \frac{1045.3}{1700};$$

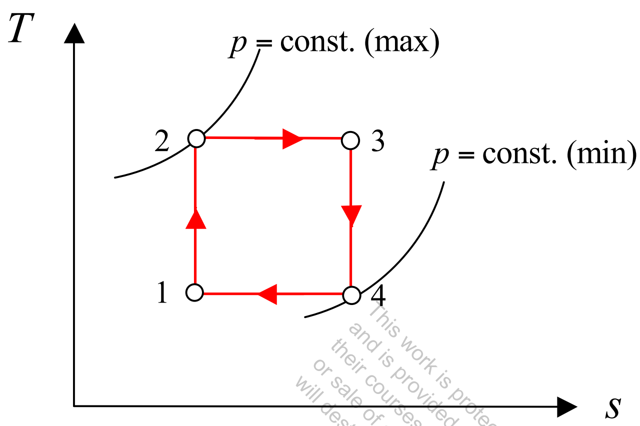
$$\Rightarrow s_3 - s_2 = 0.1396 \ \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$Q_{\text{in}} = mT_1(s_3 - s_2);$$
$$\Rightarrow Q_{\text{in}} = (0.003)(1003)(0.1396);$$
$$\Rightarrow Q_{\text{in}} = 0.4201 \text{ kJ}$$
$$\eta_{\text{th},C} = 1 - \frac{T_4}{T_1};$$
$$\Rightarrow \eta_{\text{th},C} = 1 - \frac{298}{1003};$$
$$\Rightarrow \eta_{\text{th},C} = 70.3\%$$
$$W_{\text{net}} = \eta_{\text{th},C} Q_{\text{in}};$$
$$\Rightarrow W_{\text{net}} = (0.703)(0.4201);$$
$$\Rightarrow W_{\text{net}} = 0.295 \text{ kJ}$$

TEST Solution Use the PG (or IG based on problem statement) reciprocating closed-cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

7-2-3 [OID] An air standard Carnot cycle is executed in a closed system between the temperature limits of 300 K and 1000 K. The pressure before and after the isothermal compression are 100 kPa and 300 kPa, respectively. If the net work output per cycle is 0.22 kJ, determine (a) the maximum pressure in the cycle, (b) the heat transfer to air, and (c) the mass of air. Use the PG model. (d) What-if Scenario: What would the mass of air be if the IG model were used?

SOLUTION



Given:

$$R = 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4005$$

The highest pressure occurs at state-2 and the lowest at state-4.

For an isentropic process,

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}};$$

$$\Rightarrow p_2 = (300)\left(\frac{1000}{300}\right)^{\frac{1.4005}{0.4005}};$$

$$\Rightarrow p_2 = 20.2\text{ MPa}$$

The efficiency of the Carnot cycle is

$$\eta_{\text{th},C} = 1-\frac{T_1}{T_2};$$

$$\Rightarrow \eta_{\text{th},C} = 1-\frac{300}{1000};$$

$$\Rightarrow \eta_{\text{th},C} = 70\%$$

$$Q_{\text{in}} = \frac{W_{\text{net}}}{\eta_{\text{th},C}};$$

$$Q_{\text{in}} = \frac{0.22}{0.7};$$

$$\Rightarrow Q_{\text{in}} = 0.314 \text{ kJ}$$

An energy analysis for the heat addition processes yields:

$$s_3 - s_2 = s_4 - s_1$$

$$s_4 - s_1 = c_p \cancelto{0}{\ln\frac{T_4}{T_1}} - R\ln\frac{p_4}{p_1};$$

$$\Rightarrow s_4 - s_1 = -(0.287)\ln\frac{100}{300};$$

$$\Rightarrow s_4 - s_1 = 0.315 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Therefore

$$m = \frac{Q_{\text{in}}}{T_2(s_3 - s_2)};$$

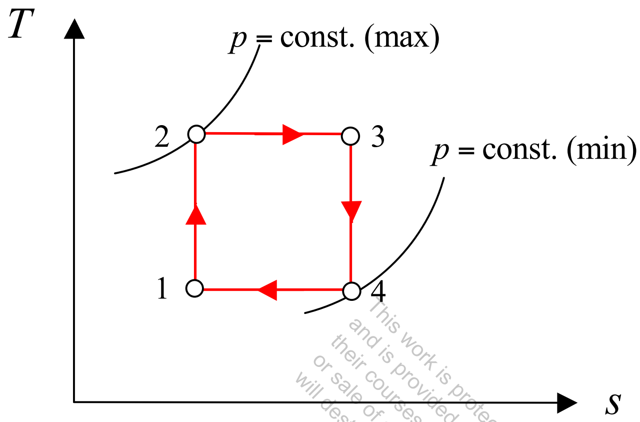
$$\Rightarrow m = \frac{0.314}{(1000)(0.315)};$$

$$\Rightarrow m = 0.001 \text{ kg}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) reciprocating closed-cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

7-2-4 [OIM] An air standard Carnot cycle is executed in a closed system between the temperature limits of 350 K and 1200 K. The pressure before and after the isothermal compression are 150 kPa and 300 kPa respectively. If the net work output per cycle is 0.5 kJ, determine (a) the maximum pressure in the cycle, (b) the heat transfer to air and (c) the mass of air. Use the IG model for air.

SOLUTION



Given:

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4005$$

The highest pressure occurs at state-2 and the lowest at state-4.

From the ideal gas table for air

$$s^o(T_1) = 1.85708 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$s^o(T_2) = 3.17888 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\cancel{\Delta s}^{\,0} = \int_{T_1}^{T_2} c_p \frac{dT}{T} - T \ln \frac{p_2}{p_1};$$

$$\Rightarrow 0 = s^o(T_2) - s^o(T_1) - R \ln \frac{p_2}{p_1};$$

$$\Rightarrow p_2 = p_1 e^{\frac{s^o(T_2) - s^o(T_1)}{R}};$$

$$\Rightarrow p_2 = (300) e^{\frac{3.17888 - 1.85708}{0.287}};$$

$$\Rightarrow p_2 = 300 e^{4.61}$$

$$\Rightarrow p_2 = 30.13 \text{ MPa}$$

An energy analysis for the heat addition processes yields:

$$\eta_{\text{th}} = 1 - \frac{T_2}{T_1};$$

$$\Rightarrow \eta_{\text{th}} = 1 - \frac{350}{1200};$$

$$\Rightarrow \eta_{\text{th}} = 70.83\%$$

$$Q_{\text{in}} = \frac{W_{\text{net}}}{\eta_{\text{th}}};$$

$$\Rightarrow Q_{\text{in}} = \frac{0.5}{0.7083};$$

$$\Rightarrow Q_{\text{in}} = 0.706 \text{ kJ}$$

The mass of air

$$s_3 - s_2 = s_4 - s_1$$

$$s_4 - s_1 = \cancel{\left[s^o\left(T_4\right) - s^o\left(T_1\right)\right]}^{0} - R\ln\frac{p_4}{p_1};$$

$$\Rightarrow s_4 - s_1 = -\left(0.287\right)\ln\frac{150}{300};$$

$$\Rightarrow s_4 - s_1 = 0.199\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$w_{\text{net}} = \left(s_3 - s_2\right)\left(T_2 - T_1\right);$$

$$\Rightarrow w_{\text{net}} = \left(0.199\right)\left(1200 - 350\right);$$

$$\Rightarrow w_{\text{net}} = 169.15\ \frac{\text{kJ}}{\text{kg}}$$

$$m = \frac{W_{\text{net}}}{w_{\text{net}}};$$

$$\Rightarrow m = \frac{0.5}{169.15};$$

$$\Rightarrow m = 0.003\ \text{kg}$$

TEST Solution Use the PG (or IG based on problem statement) reciprocating closed-cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.