8-2-1 [OKZ] Liquid water flows through a pipe at a mass flow rate of 100 kg/s. If the cross-sectional area of the pipe is 0.01 m^2, determine (a) the flow rate of momentum through the pipe.

SOLUTION

The momentum conservation equation is given as

$$\sum \frac{dM_x}{dt} = \sum \dot{m}_i \frac{V_{x,i}}{1000} - \sum \dot{m}_e \frac{V_{x,e}}{1000} + \sum F_x;$$

$$\Rightarrow 0 = 0 - \sum \dot{m}_e \frac{V_{x,e}}{1000} + \sum F_x;$$

$$\Rightarrow F_x = \frac{\dot{m} V_{x,e}}{1000}$$

$$V_{x,e} = \frac{\dot{m}}{\rho A_e};$$

$$\Rightarrow V_{x,e} = \frac{(100)}{(1000)(0.01)};$$

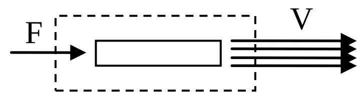
$$\Rightarrow V_{x,e} = 10 \frac{\text{m}}{\text{s}}$$

$$F_x = \frac{\dot{m} V_{x,e}}{1000};$$

$$\Rightarrow F_x = \frac{(100)(10)}{1000};$$

$$\Rightarrow F_x = 1 \text{ kN}$$

8-2-2 [OKP] A firefighter is trying to hold a fire hose steady while spraying water. If the jet of water (ρ = 997 kg/m^3) is coming from the 6.5-cm diameter fire hose at 400 GPM (0.025 m^3/s), what is the force required by the firefighter to hold the hose steady?



SOLUTION

The momentum conservation equation is given as

$$\sum \frac{dM_x}{dt} = \sum \dot{m}_i \frac{V_{x,i}}{1000} - \sum \dot{m}_e \frac{V_{x,e}}{1000} + \sum F_x;$$

$$\Rightarrow 0 = 0 - \sum \dot{m}_e \frac{V_{x,e}}{1000} + \sum F_x;$$

$$\Rightarrow F_x = \frac{\dot{m}V_{x,e}}{1000}$$

$$\dot{m} = \rho \dot{\cancel{V}};$$

$$\Rightarrow \dot{m} = \left(997 \text{ kg/m}^3\right)\left(0.025 \text{ m}^3/\text{s}\right);$$

$$\Rightarrow \dot{m} = 24.9 \text{ kg/s}$$

$$V_{x,e} = \frac{\dot{\cancel{V}}}{A_e};$$

$$\Rightarrow V_{x,e} = \frac{\dot{\cancel{V}}}{\frac{\pi}{4}D^2};$$

$$\Rightarrow V_{x,e} = \frac{(0.025)}{\left(\frac{\pi}{4}\right)(0.065)^2};$$

$$\Rightarrow V_{x,e} = 7.53 \frac{\text{m}}{\text{s}}$$

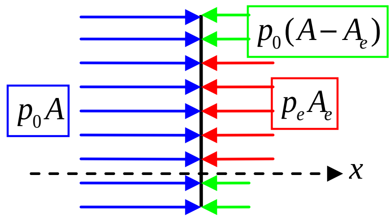
$$F_x = \frac{\dot{m}V_{x,e}}{1000};$$

$$\Rightarrow F_x = \frac{(24.9)(7.53)}{1000};$$

$$\Rightarrow F_x = 0.1875 \text{ kN} = 42 \text{ lb}$$

8-2-3 [OKU] A rocket motor is fired on a test stand. Hot exhaust gases leave the exit with a velocity of 700 m/s at a mass flow rate ($\dot{m}$) of 10 kg/s. The exit area is 0.01 m^2 and the exit pressure is 50 kPa. For an ambient pressure of 100 kPa, determine the rocket motor thrust that is transmitted to the stand. Assume steady state and one-dimensional flow.

SOLUTION



The momentum equation for rocket thrust produces

$$T=\frac{\dot{m}V_e}{1000}+A_t\left(p_e-p_0\right);$$

$$\Rightarrow T=\frac{(10)(700)}{1000}+(0.01)(50-100);$$

$$\Rightarrow T=6.5\text{ kN}$$

8-2-4 [OKX] A jet engine is traveling through the air with a velocity of 150 m/s. The exhaust gas (model as air using the PG model) leaves the nozzle with an exit velocity of 450 m/s with respect to the nozzle. Pressure at the inlet and exit are 30 kPa and 100 kPa respectively, and the ambient pressure is 30 kPa. If the mass flow rate ($\dot{m}$) is 10 kg/s and the exit area is 0.2 m^2, determine the jet thrust.

SOLUTION

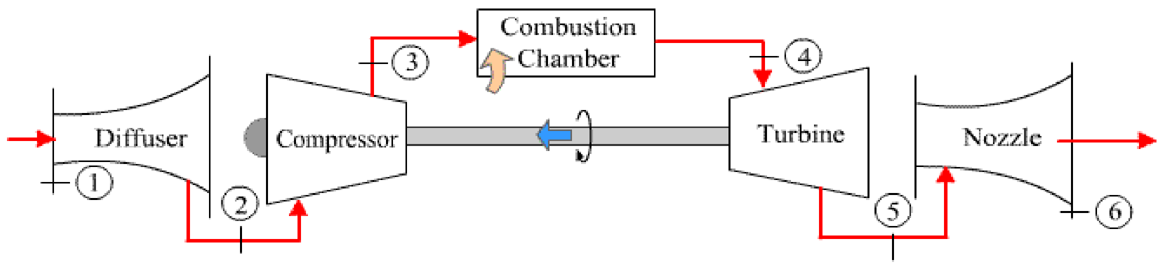
The momentum equation produces

$$T = \frac{\dot{m}}{1000}\left(V_j - V_a\right) + A_e\left(p_e - p_0\right) - A_i\cancel{\left(p_i - p_0\right)}^{0};$$

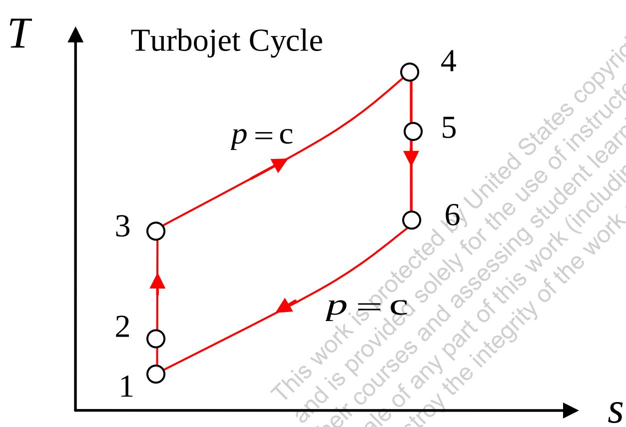
$$\Rightarrow T = \left(\frac{10}{1000}\right)(450-150) + (0.2)(100-30);$$

$$\Rightarrow T = 17 \text{ kN}$$

8-2-5 [OKC] A turbojet aircraft is flying with a velocity of 300 m/s at an altitude of 6000 m, the ambient conditions are 45 kPa and -15°C. The compressor pressure ratio is 14 and the turbine inlet temperature is 1500 K. Assuming ideal operation of all components and constant specific heats, determine (a) the pressure at the turbine exit, (b) the velocity of the exhaust gases and (c) the propulsive efficiency (η_P). (d) **What-if Scenario:** What would the velocity of the exhaust gases be if the aircraft velocity were 200 m/s?



SOLUTION



Given:

$c_p = 1.005 \dfrac{\text{kJ}}{\text{kg} \cdot \text{K}}$

$k = 1.4$

State-1 (given p_1, T_1, V_1)

State-2 (given $s_2 = s_1, V_2$):

$$\frac{\cancel{dE}^{0}}{\cancel{dt}} = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_1 = j_2;$$

$$\Rightarrow h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{0} = h_2 + \cancel{\text{ke}_2}^{0} + \cancel{\text{pe}_2}^{0};$$

$$\Rightarrow h_2 - h_1 = \frac{V_1^2}{2000};$$

$$\Rightarrow c_p\left(T_2 - T_1\right) = \frac{V_1^2}{2000};$$

$$\Rightarrow T_2 = T_1 + \frac{V_1^2}{2000\, c_p};$$

$$\Rightarrow T_2 = 258 + \frac{(300)^2}{(2000)(1.005)} = 302.8\text{ K}$$

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = (45)\left(\frac{302.8}{258}\right)^{\frac{1.4}{0.4}} = 78.8\,\text{kPa}$$

State-3 (given $s_3 = s_2, r_p$):

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = T_2 r_p^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = (302.8)(14)^{\frac{0.4}{1.4}};$$

$$\Rightarrow T_3 = 643.6\,\text{K}$$

State-4 (given $p_4 = p_3, T_4$)

State-5 (given $s_5 = s_4$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_4 - j_5 = j_3 - j_2;$$

$$\Rightarrow h_4 - h_5 = h_3 - h_2;$$

$$\Rightarrow c_p\left(T_4 - T_5\right) = c_p\left(T_3 - T_2\right);$$

$$\Rightarrow T_5 = T_2 - T_3 + T_4;$$

$$\Rightarrow T_5 = 302.8 - 643.6 + 1500;$$

$$\Rightarrow T_5 = 1159.2\,\text{K}$$

$$p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} = (1103.2)\left(\frac{1159.2}{1500}\right)^{\frac{1.4}{0.4}} = 447.6\ \text{kPa}$$

State-6 (given $p_6, s_6 = s_5$):

$$T_6 = T_5\left(\frac{p_6}{p_5}\right)^{\frac{k-1}{k}} = (1159.2)\left(\frac{45}{447.6}\right)^{\frac{0.4}{1.4}} = 601.3\,\text{K}$$

The velocity of exhaust gases

$$\frac{d\cancel{E}^{\,0}}{\cancel{dt}} = \dot{m}_5 j_5 - \dot{m}_6 j_6 + \cancel{\dot{Q}}^{\,0} - \cancel{\dot{W}_{\text{ext}}}^{\,0};$$

$$\Rightarrow j_5 = j_6;$$

$$\Rightarrow h_5 + \cancel{\text{ke}_5}^{\,0} + \cancel{\text{pe}_5}^{\,0} = h_6 + \text{ke}_6 + \cancel{\text{pe}_6}^{\,0};$$

$$\Rightarrow h_5 - h_6 = +\frac{V_6^2}{2000};$$

$$\Rightarrow V_6 = \sqrt{2000 c_p (T_5 - T_6)};$$

$$\Rightarrow V_6 = \sqrt{(2000)(1.005)(1159.2 - 601.3)};$$

$$\Rightarrow V_6 = 1058.95\,\frac{\text{m}}{\text{s}}$$

The propulsive efficiency

$$T = \frac{\dot{m}}{1000}\left(V_j - V_a\right) = \frac{\dot{m}\left(V_6 - V_1\right)}{1000}$$

$$\dot{W}_P = TV_a = \frac{\dot{m}\left(V_6 - V_1\right)V_1}{1000}$$

$$\dot{\text{KE}}_{\text{out}} = \frac{\dot{m}\left(V_6 - V_1\right)^2}{2000}$$

$$\eta_P = \frac{\dot{W}_P}{\dot{W}_P + \dot{\text{KE}}_{\text{out}}};$$

$$\Rightarrow \eta_P = \frac{1}{1 + \dfrac{\dot{\text{KE}}_{\text{out}}}{\dot{W}_P}};$$

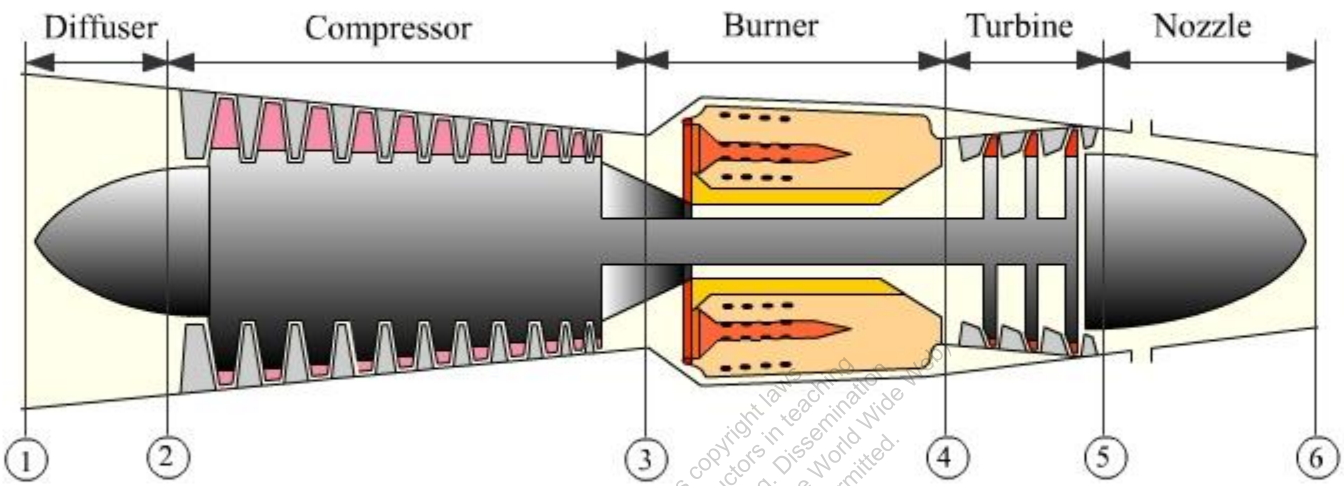
$$\Rightarrow \eta_P = \frac{1}{1 + \dfrac{V_6 - V_1}{2V_1}};$$

$$\Rightarrow \eta_P = \frac{1}{1 + \dfrac{1058.95 - 300}{(2)(300)}};$$

$$\Rightarrow \eta_P = 44.2\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-6 [OKV] A jet engine is being tested on a test stand. The inlet area to the diffuser is 0.2 m^2 and air enters the diffuser at 90 kPa, 100 m/s. The pressure of the atmosphere is 100 kPa. The exit area of the engine is 0.1 m^2, and the products of combustion leave the exit at a pressure of 130 kPa and a temperature of 1000 K. The air fuel ratio is 45 kg air/kg fuel, and the fuel enters with a low velocity. The mass flow rate of air entering the engine is 20 kg/s. Determine (a) the inlet temperature, (b) the exit velocity of combustion products and (c) the thrust on the engine.



SOLUTION

In accordance with the diagram, the problem can be solved by looking at states 1 and 6.

Given:

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-1 (given $p_1, A_1, V_1, \dot{m}$):

$$\rho_1 = \frac{\dot{m}}{A_1 V_1} = \frac{20}{(0.2)(100)} = 1 \frac{\text{kg}}{\text{m}^3}$$

$$T_1 = \frac{p_1}{\rho_1 R} = \frac{90}{(1)(0.287)} = 314 \text{ K}$$

State-6 (given p_6, T_6, A_2):

$$\dot{m}_F = \frac{\dot{m}_{\text{air}}}{45} = \frac{20}{45} = 0.44\ \frac{\text{kg}}{\text{s}}$$

$$\dot{m}_{\text{out}} = \dot{m}_{\text{in}} + \dot{m}_F = 20 + 0.44 = 20.44\frac{\text{kg}}{\text{s}}$$

$$\rho_6 = \frac{p_6}{RT_6} = \frac{130}{(0.287)(1000)} = 0.453\frac{\text{kg}}{\text{m}^3}$$

$$\dot{m}_{\text{out}} = \rho_6 A_6 V_6;$$

$$\Rightarrow V_6 = \frac{\dot{m}_{\text{out}}}{\rho_6 A_6} = \frac{20.44}{(0.453)(0.1)} = 451.2\frac{\text{m}}{\text{s}}$$

Neglecting the slight change in mass flow rate between the inlet and exit, the thrust produced can be calculated as:

$$T = \frac{\dot{m}}{1000}\left(V_j - V_a\right) + \left(p_e - p_0\right)A_e - \left(p_i - p_0\right)A_i$$

$$\Rightarrow T = \left(\frac{40}{1000}\right)(451-100) + (130-100)(0.1) - (90-100)(0.2) = 10.03\text{ kN}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-7 [OKQ] Consider an ideal jet propulsion cycle in which air enters the compressor at 100 kPa and 20°C. The pressure leaving the compressor is 1100 kPa, and the maximum temperature in the cycle is 1200°C. Air expands in the turbine at which the turbine work (W_T) is equal to the compressor work (W_C). On leaving the turbine, air expands in a nozzle to 100 kPa. The process is reversible and adiabatic. Determine (a) the velocity of the air leaving the nozzle. Use the PG model.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (unknown)

State-2 (given p_2, T_2)

State-3 (given $p_3, s_3 = s_2$):

$$T_3 = T_2 \left(\frac{p_3}{p_2} \right)^{\frac{k-1}{k}} = (293)\left(\frac{1100}{100} \right)^{\frac{0.4}{1.4}} = 581.3\,\text{K}$$

State-4 (given $p_4 = p_3, T_4$)

State-5 (given $s_5 = s_4$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_4 - j_5 = j_3 - j_2;$$
$$\Rightarrow h_4 - h_5 = h_3 - h_2;$$
$$\Rightarrow c_p (T_4 - T_5) = c_p (T_3 - T_2);$$
$$\Rightarrow T_5 = T_2 - T_3 + T_4;$$
$$\Rightarrow T_5 = 293 - 581.3 + 1473;$$
$$\Rightarrow T_5 = 1184.7\,\text{K}$$

$$p_5 = p_4 \left(\frac{T_5}{T_4} \right)^{\frac{k}{k-1}} = (1100)\left(\frac{1184.7}{1473} \right)^{\frac{1.4}{0.4}} = 513.2\,\text{kPa}$$

State-6 (given $p_6, s_6 = s_5$):

$$T_6 = T_5 \left(\frac{p_6}{p_5} \right)^{\frac{k-1}{k}} = (1184.7)\left(\frac{100}{513.2} \right)^{\frac{0.4}{1.4}} = 742.5\,\text{K}$$

The velocity of exhaust gases

$$\frac{d\cancel{E}^{0}}{\cancel{dt}} = \dot{m}_5 j_5 - \dot{m}_6 j_6 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_5 = j_6;$$

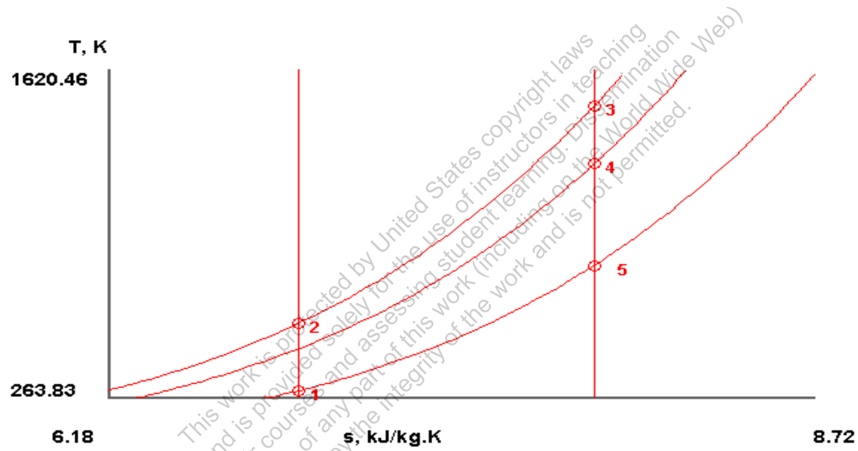
$$\Rightarrow h_5 + \cancel{\text{ke}_5}^{0} + \cancel{\text{pe}_5}^{0} = h_6 + \text{ke}_6 + \cancel{\text{pe}_6}^{0};$$

$$\Rightarrow h_5 - h_6 = +\frac{V_6^2}{2000};$$

$$\Rightarrow V_6 = \sqrt{2000 c_p (T_5 - T_6)};$$

$$\Rightarrow V_6 = \sqrt{(2000)(1.005)(1184.7 - 742.5)};$$

$$\Rightarrow V_6 = 942.8 \frac{\text{m}}{\text{s}}$$



TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-8 [OKT] Consider an ideal jet propulsion cycle in which air enters the compressor at 100 kPa and 25°C. The pressure leaving the compressor is 1 MPa, and the maximum temperature in the cycle is 1000°C. Air expands in the turbine at which the turbine work is just equal to the compressor work. On leaving the turbine, air expands in a nozzle to 100 kPa. The process is reversible and adiabatic. Determine (a) the velocity of the air leaving the nozzle. Use the PG model. (b) **What-if Scenario:** What would the exit velocity be if the maximum temperature achieved in the cycle were 800°C?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (unknown)

State-2 (given p_2, T_2)

State-3 (given $p_3, s_3 = s_2$):

$$T_3 = T_2 \left(\frac{p_3}{p_2} \right)^{\frac{k-1}{k}} = (298)\left(\frac{1000}{100} \right)^{\frac{0.4}{1.4}} = 575.3\,\text{K}$$

State-4 (given $p_4 = p_3, T_4$)

State-5 (given $s_5 = s_4$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_4 - j_5 = j_3 - j_2;$$

$$\Rightarrow h_4 - h_5 = h_3 - h_2;$$

$$\Rightarrow c_p (T_4 - T_5) = c_p (T_3 - T_2);$$

$$\Rightarrow T_5 = T_2 - T_3 + T_4;$$

$$\Rightarrow T_5 = 298 - 575.3 + 1273;$$

$$\Rightarrow T_5 = 995.7\,\text{K}$$

$$p_5 = p_4 \left(\frac{T_5}{T_4} \right)^{\frac{k}{k-1}} = (1000)\left(\frac{995.7}{1273} \right)^{\frac{1.4}{0.4}} = 423.2\text{ kPa}$$

State-6 (given $p_6, s_6 = s_5$):

$$T_6 = T_5\left(\frac{p_6}{p_5}\right)^{\frac{k-1}{k}} = (995.7)\left(\frac{100}{423.2}\right)^{\frac{0.4}{1.4}} = 659.3\,\text{K}$$

The velocity of exhaust gases

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_5 j_5 - \dot{m}_6 j_6 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_5 = j_6;$$

$$\Rightarrow h_5 + \cancel{\text{ke}_5}^{0} + \cancel{\text{pe}_5}^{0} = h_6 + \text{ke}_6 + \cancel{\text{pe}_6}^{0};$$

$$\Rightarrow h_5 - h_6 = +\frac{V_6^2}{2000};$$

$$\Rightarrow V_6 = \sqrt{2000 c_p (T_5 - T_6)};$$

$$\Rightarrow V_6 = \sqrt{(2000)(1.005)(995.7 - 659.3)};$$

$$\Rightarrow V_6 = 822.3\,\frac{\text{m}}{\text{s}}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-9 [OKY] A turbojet aircraft is flying with a velocity of 350 m/s at an altitude of 9150 m, the ambient conditions are 30 kPa and -30°C. The pressure ratio across the compressor is 10, and the temperature at the turbine inlet is 1200 K. Air enters the compressor at a rate of 30 kg/s, and the jet fuel has a heating value of 42000 kJ/kg. Using the PG model, determine (a) the velocity of the exhaust gases, (b) the propulsive power developed and (c) the rate of fuel consumption.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1, V_1)

State-2 (given $s_2 = s_1, V_2$):

$$\cancel{\frac{dE}{dt}}^0 = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^0 - \cancel{\dot{W}_{\text{ext}}}^0;$$

$$\Rightarrow j_1 = j_2;$$

$$\Rightarrow h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^0 = h_2 + \cancel{\text{ke}_2}^0 + \cancel{\text{pe}_2}^0;$$

$$\Rightarrow h_2 - h_1 = \frac{V_1^2}{2000};$$

$$\Rightarrow c_p (T_2 - T_1) = \frac{V_1^2}{2000};$$

$$\Rightarrow T_2 = T_1 + \frac{V_1^2}{2000 c_p};$$

$$\Rightarrow T_2 = 243 + \frac{(350)^2}{(2000)(1.005)} = 303.9 \text{ K}$$

$$p_2 = p_1 \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}} = (30) \left(\frac{303.9}{243} \right)^{\frac{1.4}{0.4}} = 65.6 \,\text{kPa}$$

State-3 (given $s_3 = s_2, r_p$):

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = T_2 r_p^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = (303.9)(10)^{\frac{0.4}{1.4}};$$

$$\Rightarrow T_3 = 586.7\,\text{K}$$

State-4 (given $p_4 = p_3, T_4$)

State-5 (given $s_5 = s_4$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_4 - j_5 = j_3 - j_2;$$

$$\Rightarrow h_4 - h_5 = h_3 - h_2;$$

$$\Rightarrow c_p(T_4 - T_5) = c_p(T_3 - T_2);$$

$$\Rightarrow T_5 = T_2 - T_3 + T_4;$$

$$\Rightarrow T_5 = 303.9 - 586.7 + 1200;$$

$$\Rightarrow T_5 = 917.2\,\text{K}$$

$$p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} = (656)\left(\frac{917.2}{1200}\right)^{\frac{1.4}{0.4}} = 256.1\,\text{kPa}$$

State-6 (given $p_6 = p_1, s_6 = s_5$):

$$T_6 = T_5\left(\frac{p_6}{p_5}\right)^{\frac{k-1}{k}} = (917.2)\left(\frac{30}{256.1}\right)^{\frac{0.4}{1.4}} = 497.0\,\text{K}$$

The velocity of exhaust gases

$$\cancel{\frac{dE}{dt}}^0 = \dot{m}_5 j_5 - \dot{m}_6 j_6 + \cancel{\dot{Q}}^0 - \cancel{\dot{W}_{\text{ext}}}^0;$$

$$\Rightarrow j_5 = j_6;$$

$$\Rightarrow h_5 + \cancel{\text{ke}_5}^0 + \cancel{\text{pe}_5}^0 = h_6 + \text{ke}_6 + \cancel{\text{pe}_6}^0;$$

$$\Rightarrow h_5 - h_6 = +\frac{V_6^2}{2000};$$

$$\Rightarrow V_6 = \sqrt{2000 c_p \left(T_5 - T_6\right)};$$

$$\Rightarrow V_6 = \sqrt{(2000)(1.005)(917.2 - 497.0)};$$

$$\Rightarrow V_6 = 919.0 \frac{\text{m}}{\text{s}}$$

The momentum equation produces

$$T = \frac{\dot{m}}{1000}\left(V_j - V_a\right) = \left(\frac{30}{1000}\right)(919 - 350) = 17.1 \text{ kN}$$

$$\dot{W}_P = TV_a = (17.1)(350) = 5985 \text{ kW}$$

The fuel consumption rate can be found from

$$\dot{Q}_{\text{in}} = \dot{m} c_p \left(T_4 - T_3\right) = (30)(1.005)(1200 - 586.7) = 18491.0 \text{ kW}$$

$$\dot{m}_F = \frac{\dot{Q}_{\text{in}}}{q_{\text{comb}}} = \frac{18491.0}{42000} = 0.44 \frac{\text{kg}}{\text{s}}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-10 [OKF] A turbojet aircraft is flying with a velocity of 250 m/s at an altitude where the ambient conditions are 20 kPa and -25°C. The pressure ratio across the compressor is 12, and the temperature at the turbine inlet is 1000°C. Air enters the compressor at a rate of 50 kg/s. Determine (a) the temperature (*T*) and pressure (*p*) of the gases at the turbine exit, (b) the velocity (*V*) of the gases at the nozzle exit and (c) the propulsive efficiency of the cycle. Use the PG model. (d) **What-if Scenario:** What would the exit velocity be if the aircraft velocity were 300 m/s?

SOLUTION

Given:

$$c_p = 1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1, V_1)

State-2 (given $s_2 = s_1, V_2, \dot{m}$):

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_1 = j_2;$$

$$\Rightarrow h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{0} = h_2 + \cancel{\text{ke}_2}^{0} + \cancel{\text{pe}_2}^{0};$$

$$\Rightarrow h_2 - h_1 = \frac{V_1^2}{2000};$$

$$\Rightarrow c_p\left(T_2 - T_1\right) = \frac{V_1^2}{2000};$$

$$\Rightarrow T_2 = T_1 + \frac{V_1^2}{2000\,c_p};$$

$$\Rightarrow T_2 = 248 + \frac{(250)^2}{(2000)(1.005)} = 279.1\,\text{K}$$

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = (20)\left(\frac{279.1}{248}\right)^{\frac{1.4}{0.4}} = 30.2\,\text{kPa}$$

State-3 (given $s_3 = s_2, r_p$):

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = T_2 r_p^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = (279.1)(12)^{\frac{0.4}{1.4}};$$

$$\Rightarrow T_3 = 567.7\,\mathrm{K}$$

State-4 (given $p_4 = p_3, T_4$)

State-5 (given $s_5 = s_4$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_4 - j_5 = j_3 - j_2;$$

$$\Rightarrow h_4 - h_5 = h_3 - h_2;$$

$$\Rightarrow c_p(T_4 - T_5) = c_p(T_3 - T_2);$$

$$\Rightarrow T_5 = T_2 - T_3 + T_4;$$

$$\Rightarrow T_5 = 279.1 - 567.7 + 1273;$$

$$\Rightarrow T_5 = 984.4\,\mathrm{K}$$

$$p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} = (362.4)\left(\frac{984.4}{1273}\right)^{\frac{1.4}{0.4}} = 147.4\ \mathrm{kPa}$$

State-6 (given $p_6 = p_1, s_6 = s_1$):

$$T_6 = T_5\left(\frac{p_6}{p_5}\right)^{\frac{k-1}{k}} = (984.4)\left(\frac{20}{147.4}\right)^{\frac{0.4}{1.4}} = 556.3\,\mathrm{K}$$

The velocity of exhaust gases

$$\frac{d\cancel{E}^{0}}{\cancel{dt}} = \dot{m}_5 j_5 - \dot{m}_6 j_6 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_5 = j_6;$$

$$\Rightarrow h_5 + \cancel{\mathrm{ke}_5}^{0} + \cancel{\mathrm{pe}_5}^{0} = h_6 + \mathrm{ke}_6 + \cancel{\mathrm{pe}_6}^{0};$$

$$\Rightarrow h_5 - h_6 = +\frac{V_6^2}{2000};$$

$$\Rightarrow V_6 = \sqrt{2000 c_p (T_5 - T_6)};$$

$$\Rightarrow V_6 = \sqrt{(2000)(1.005)(984.4 - 556.3)};$$

$$\Rightarrow V_6 = 927.6 \frac{\mathrm{m}}{\mathrm{s}}$$

The propulsive efficiency

$$T = \frac{\dot{m}}{1000}(V_j - V_a) = \left(\frac{50}{1000}\right)(927.6 - 250) = 33.9 \text{ kN}$$

$$\dot{W}_P = TV_a = (33.9)(250) = 8475 \text{ kW}$$

$$\dot{\mathrm{KE}}_{\text{out}} = \frac{\dot{m}(V_6 - V_1)^2}{2000} = \frac{(50)(927.6 - 250)^2}{2000} = 11478.5 \text{ kW}$$

$$\eta_P = \frac{\dot{W}_P}{\dot{W}_P + \dot{\mathrm{KE}}_{\text{out}}};$$

$$\Rightarrow \eta_P = \frac{8475}{8475 + 11478.5};$$

$$\Rightarrow \eta_P = 42.5\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-11 [OKM] Consider an aircraft powered by a turbojet engine that has a pressure ratio of 10. The aircraft is stationary on the ground, held in position by its brakes. The ambient air is 25^{o}C and 100 kPa and enters the engine at a rate of 20 kg/s. The jet fuel has a heating value of 42700 kJ/kg, and it is burned completely at a rate of 0.35 kg/s. Neglecting the effect of the diffuser and disregarding the slight change in the mass at the engine exit as well as the inefficiencies of the engine components, determine (a) the force that must be applied on the brakes to hold the plane stationary. Use IG model.

SOLUTION

State-1 (given p_1, T_1, V_1):

$$h_1 = 297.95 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $p_2 = p_1, T_2 = T_1$):

$$h_2 = 297.95 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $s_3 = s_2, r_p$):

$$h_3 = 575.48 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_3$):

$$\dot{Q}_{\text{in}} = \dot{m}_F q_{\text{comb}} = 14945 \text{ kW}$$

$$j_4 = j_3 + \frac{\dot{Q}_{\text{in}}}{\dot{m}};$$

$$\Rightarrow h_4 = h_3 + \frac{\dot{Q}_{\text{in}}}{\dot{m}};$$

$$\Rightarrow h_4 = 575.48 + \frac{14945}{20};$$

$$\Rightarrow h_4 = 1322.73 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $s_5 = s_4$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_4 - j_5 = j_3 - j_2;$$
$$\Rightarrow h_4 - h_5 = h_3 - h_2;$$
$$\Rightarrow h_5 = h_2 - h_3 + h_4;$$
$$\Rightarrow h_5 = 297.95 - 575.48 + 1322.73;$$
$$\Rightarrow h_5 = 1045.20 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_1, s_6 = s_1$):

$$h_6 = 701.40 \frac{\text{kJ}}{\text{kg}}$$

The velocity of exhaust gases

$$\cancel{\frac{dE}{dt}}^0 = \dot{m}_5 j_5 - \dot{m}_6 j_6 + \cancel{\dot{Q}}^0 - \cancel{\dot{W}_{\text{ext}}}^0;$$
$$\Rightarrow j_5 = j_6;$$
$$\Rightarrow h_5 + \cancel{\text{ke}_5}^0 + \cancel{\text{pe}_5}^0 = h_6 + \text{ke}_6 + \cancel{\text{pe}_6}^0;$$
$$\Rightarrow h_5 - h_6 = + \frac{V_6^2}{2000};$$
$$\Rightarrow V_6 = \sqrt{2000\left(h_5 - h_6\right)};$$
$$\Rightarrow V_6 = \sqrt{(2000)(1045.20 - 701.40)};$$
$$\Rightarrow V_6 = 829.2 \frac{\text{m}}{\text{s}}$$

The thrust is

$$T = \frac{\dot{m} V_6}{1000} = \frac{(20)(829.2)}{1000} = 16.584 \text{ kN}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-12 [OKD] Air at 10^{o}C enters a turbojet engine at a rate ($\dot{m}$) of 15 kg/s and at a velocity of 320 m/s (relative to the engine). Air is heated in the combustion chamber at a rate of 25000 kJ/s and it leaves the engine at 420^{o}C. Determine the momentum thrust produced by this turbojet engine. Use the IG model.

SOLUTION

This problem can be solved by simplifying to two states.

State-1 (given $T_1, V_1, \dot{m}$):

$$h_1 = 282.90 \frac{\text{kJ}}{\text{kg}}$$

$$j_1 = h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{0};$$

$$\Rightarrow j_1 = h_1 + \frac{V_1^2}{2000};$$

$$\Rightarrow j_1 = 282.90 + \frac{320^2}{2000};$$

$$\Rightarrow j_1 = 334.10 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $T_2, \dot{Q}_{\text{in}}$):

$$h_2 = 707.67 \frac{\text{kJ}}{\text{kg}}$$

$$j_2 = j_1 + \frac{\dot{Q}_{\text{in}}}{\dot{m}};$$

$$\Rightarrow h_2 + \text{ke}_2 + \cancel{\text{pe}_2}^{0} = j_1 + \frac{\dot{Q}_{\text{in}}}{\dot{m}};$$

$$\Rightarrow \text{ke}_2 = j_1 - h_2 + \frac{\dot{Q}_{\text{in}}}{\dot{m}};$$

$$\Rightarrow \frac{V_2^2}{2000} = j_1 - h_2 + \frac{\dot{Q}_{\text{in}}}{\dot{m}};$$

$$\Rightarrow V_2 = \sqrt{(2000)\left(j_1 - h_2 + \frac{\dot{Q}_{\text{in}}}{\dot{m}}\right)};$$

$$\Rightarrow V_2 = \sqrt{(2000)\left(334.10 - 707.67 + \frac{25000}{15}\right)};$$

$$\Rightarrow V_2 = 1608.18 \frac{\text{m}}{\text{s}}$$

The thrust is

$$T = \frac{\dot{m}}{1000}\left(V_j - V_a\right) = \left(\frac{15}{1000}\right)\left(1608.18 - 320\right) = 19.32 \text{ kN}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-13 [OKJ] Repeat problem 8-2-9 [OKY] using a compressor efficiency of 80% and turbine efficiency of 85%. Determine (a) the velocity (V_2) of the exhaust gases, (b) the propulsive power ($\dot{W}_P$) developed and (c) the rate of fuel consumption.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1, V_1)

State-2 (given $s_2 = s_1, V_2$):

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_1 = j_2;$$

$$\Rightarrow h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{0} = h_2 + \cancel{\text{ke}_2}^{0} + \cancel{\text{pe}_2}^{0};$$

$$\Rightarrow h_2 - h_1 = \frac{V_1^2}{2000};$$

$$\Rightarrow c_p\left(T_2 - T_1\right) = \frac{V_1^2}{2000};$$

$$\Rightarrow T_2 = T_1 + \frac{V_1^2}{2000\,c_p};$$

$$\Rightarrow T_2 = 243 + \frac{(350)^2}{(2000)(1.005)} = 303.9 \text{ K}$$

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = (30)\left(\frac{303.9}{243}\right)^{\frac{1.4}{0.4}} = 65.6\,\text{kPa}$$

State-3 (given $s_3 = s_2, r_p$):

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = T_2 r_p^{\frac{k-1}{k}};$$

$$\Rightarrow T_3 = (303.9)(10)^{\frac{0.4}{1.4}};$$

$$\Rightarrow T_3 = 586.7\,\text{K}$$

State-4 (given $p_4 = p_3, \eta_C$):

$$T_4 = T_2 + \frac{T_3 - T_2}{\eta_C} = 303.9 + \frac{586.7 - 303.9}{0.80} = 657.4 \text{ K}$$

State-5 (given $p_5 = p_3, T_5$)

State-6 (given $s_6 = s_5, \eta_T$):

$$T_6 = T_5 + \frac{T_7 - T_5}{\eta_T} = 1200 + \frac{(846.5 - 1200)}{(0.85)} = 784.1 \text{ K}$$

$$p_6 = p_5 \left(\frac{T_6}{T_5} \right)^{\frac{k}{k-1}} = (656) \left(\frac{784.1}{1200} \right)^{\frac{1.4}{0.4}} = 147.9 \text{ kPa}$$

State-7 (given $p_7 = p_6$):

The power output from the turbine can be equated to the power input to the compressor.

$j_5 - j_7 = j_4 - j_2;$

$\Rightarrow h_5 - h_7 = h_4 - h_2;$

$\Rightarrow c_p (T_5 - T_7) = c_p (T_4 - T_2);$

$\Rightarrow T_7 = T_2 - T_4 + T_5;$

$\Rightarrow T_7 = 303.9 - 657.4 + 1200;$

$\Rightarrow T_7 = 846.5 \text{K}$

State-8 (given $p_8 = p_1, s_8 = s_7$):

$$T_8 = T_7 \left(\frac{p_8}{p_7} \right)^{\frac{k-1}{k}} = (846.5) \left(\frac{30}{147.9} \right)^{\frac{0.4}{1.4}} = 536.6 \text{K}$$

The velocity of exhaust gases

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_7 j_7 - \dot{m}_8 j_8 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_7 = j_8;$$

$$\Rightarrow h_7 + \cancel{\text{ke}_7}^{0} + \cancel{\text{pe}_7}^{0} = h_8 + \text{ke}_8 + \cancel{\text{pe}_8}^{0};$$

$$\Rightarrow h_7 - h_8 = +\frac{V_8^2}{2000};$$

$$\Rightarrow V_8 = \sqrt{2000 c_p \left(T_7 - T_8\right)};$$

$$\Rightarrow V_8 = \sqrt{(2000)(1.005)(846.5 - 536.6)};$$

$$\Rightarrow V_8 = 789.2 \frac{\text{m}}{\text{s}}$$

The momentum equation produces

$$T = \frac{\dot{m}}{1000}\left(V_j - V_a\right) = \left(\frac{30}{1000}\right)(789.2 - 350) = 13.2 \text{ kN}$$

$$\dot{W}_P = TV_a = (13.2)(350) = 4620 \text{ kW}$$

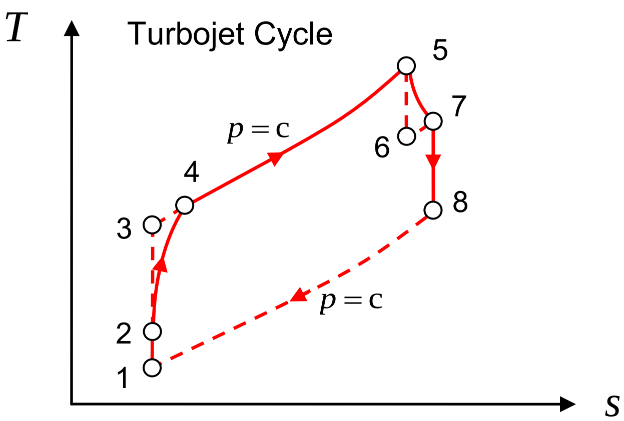
The fuel consumption rate can be found from

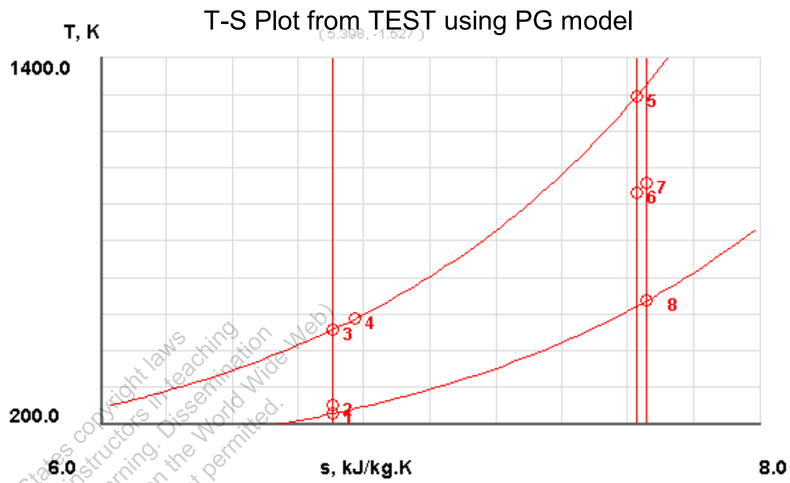
$$\dot{Q}_{\text{in}} = \dot{m} c_p \left(T_5 - T_4\right) = (30)(1.005)(1200 - 657.4) = 16359.4 \text{ kW}$$

$$\dot{m}_F = \frac{\dot{Q}_{\text{in}}}{q_{\text{comb}}} = \frac{16359.4}{42000} = 0.39 \frac{\text{kg}}{\text{s}}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-14 [OPR] Air enters the diffuser of a turbojet engine with a mass flow rate ($\dot{m}$) of 65 kg/s at 80 kPa, -40°C and a velocity of 245 m/s. The pressure ratio for the compressor is 10, and its isentropic efficiency is 87%. Air enters the turbine at 1000°C with the same pressure as the exit of the compressor. Air exits the nozzle at 80 kPa. The turbine has an isentropic efficiency of 90%. Determine (a) the rate of heat addition ($\dot{Q}$), (b) the compressor power input ($\dot{W}_{in}$) and (c) the velocity of the air leaving the nozzle. Use the IG model.





SOLUTION

State-1 (given $p_1, T_1, V_1, \dot{m}$):

$$h_1 = 232.72 \frac{\text{kJ}}{\text{kg}}$$

$$j_1 = h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{\,0};$$

$$\Rightarrow j_1 = h_1 + \frac{V_1^2}{2000};$$

$$\Rightarrow j_1 = 232.72 + \frac{(245)^2}{2000};$$

$$\Rightarrow j_1 = 262.73 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, V_2$):

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_1 = j_2;$$

$$j_2 = h_2 + \cancel{ke_2}^{0} + \cancel{pe_2}^{0};$$

$$\Rightarrow j_2 = h_2 = 262.73 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $s_3 = s_2, r_p$):

$$h_3 = 508.14 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_3, \eta_C$):

$$h_4 = h_2 + \frac{h_3 - h_2}{\eta_C} = 262.73 + \frac{508.14 - 262.73}{0.87} = 544.81 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, T_4$):

$$h_5 = 1364.13 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $s_6 = s_5, \eta_T$):

$$h_6 = h_5 + \frac{h_7 - h_5}{\eta_T} = 1364.13 + \frac{1082.04 - 1364.13}{0.90} = 1050.70 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_5 - j_7 = j_4 - j_2;$$

$$\Rightarrow h_5 - h_7 = h_4 - h_2;$$

$$\Rightarrow h_7 = h_2 - h_4 + h_5;$$

$$\Rightarrow h_7 = 262.72 - 544.81 + 1364.13;$$

$$\Rightarrow h_7 = 1082.04 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_1, s_8 = s_7$):

$$h_8 = 663.04 \frac{\text{kJ}}{\text{kg}}$$

The rate of heat addition

$$\dot{Q}_{\text{in}} = \dot{m} q_{\text{in}};$$
$$\Rightarrow \dot{Q}_{\text{in}} = \dot{m}\left(h_5 - h_4\right);$$
$$\Rightarrow \dot{Q}_{\text{in}} = (65)(1364.13 - 544.81);$$
$$\Rightarrow \dot{Q}_{\text{in}} = 53255.80 \text{ kW} = 53.3 \text{ MW}$$

The compressor power

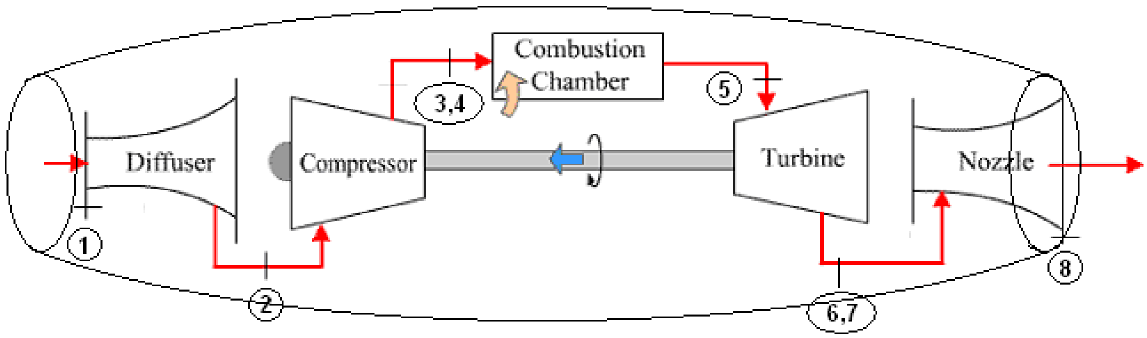
$$\dot{W}_C = \dot{m} w_C;$$
$$\Rightarrow \dot{W}_C = \dot{m}\left(h_4 - h_2\right);$$
$$\Rightarrow \dot{W}_C = (65)(544.81 - 262.73);$$
$$\Rightarrow \dot{W}_C = 18335.20 \text{ kW} = 18.3 \text{ MW}$$

The velocity of exhaust gases

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_7 j_7 - \dot{m}_8 j_8 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$
$$\Rightarrow j_7 = j_8;$$
$$\Rightarrow h_7 + \cancel{\text{ke}_7}^{0} + \cancel{\text{pe}_7}^{0} = h_8 + \text{ke}_8 + \cancel{\text{pe}_8}^{0};$$
$$\Rightarrow h_7 - h_8 = +\frac{V_8^2}{2000};$$
$$\Rightarrow V_8 = \sqrt{2000\left(h_7 - h_8\right)};$$
$$\Rightarrow V_8 = \sqrt{(2000)(1082.04 - 663.04)};$$
$$\Rightarrow V_8 = 915.42 \frac{\text{m}}{\text{s}}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-15 [OPB] Air at 30 kPa, 250 K and 250 m/s enters a turbojet engine in flight at an altitude of 10,000 m. The pressure ratio across the compressor is 12. The turbine inlet temperature is 1400 K, and the pressure at the nozzle exit is 30 kPa. The diffuser and nozzle processes are isentropic, the compressor and turbine have isentropic efficiencies of 85% and 80%, respectively, and there is no pressure drop for flow through the combustor. Using the IG model, determine (a) the velocity (V) of the exhaust gases.



SOLUTION

State-1 (given $p_1, T_1, V_1, \dot{m}$):

$$h_1 = 249.63 \frac{\text{kJ}}{\text{kg}}$$

$$j_1 = h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{0};$$

$$\Rightarrow j_1 = h_1 + \frac{V_1^2}{2000};$$

$$\Rightarrow j_1 = 249.63 + \frac{(250)^2}{2000};$$

$$\Rightarrow j_1 = 280.88 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, V_2$):

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_1 = j_2;$$

$$j_2 = h_2 + \cancel{\text{ke}_2}^{0} + \cancel{\text{pe}_2}^{0};$$

$$\Rightarrow j_2 = h_2 = 280.88 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $s_3 = s_2, r_p$):

$$h_3 = 571.72\,\frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_3, \eta_C$):

$$h_4 = h_2 + \frac{h_3 - h_2}{\eta_C} = 280.88 + \frac{571.72 - 280.88}{0.85} = 623.04\,\frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, T_4$):

$$h_5 = 1515.72\,\frac{\text{kJ}}{\text{kg}}$$

State-6 (given $s_6 = s_5, \eta_T$):

$$h_6 = h_5 + \frac{h_7 - h_5}{\eta_T} = 1515.72 + \frac{1173.56 - 1515.72}{0.80} = 1088.02\,\frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_5 - j_7 = j_4 - j_2;$$
$$\Rightarrow h_5 - h_7 = h_4 - h_2;$$
$$\Rightarrow h_7 = h_2 - h_4 + h_5;$$
$$\Rightarrow h_7 = 280.88 - 623.04 + 1515.72;$$
$$\Rightarrow h_7 = 1173.56\,\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_1, s_8 = s_7$):

$$h_8 = 740.95\,\frac{\text{kJ}}{\text{kg}}$$

The velocity of exhaust gases

$$\frac{\cancel{dE}^{0}}{\cancel{dt}} = \dot{m}_7 j_7 - \dot{m}_8 j_8 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_7 = j_8;$$

$$\Rightarrow h_7 + \cancel{\text{ke}_7}^{0} + \cancel{\text{pe}_7}^{0} = h_8 + \text{ke}_8 + \cancel{\text{pe}_8}^{0};$$

$$\Rightarrow h_7 - h_8 = +\frac{V_8^2}{2000};$$

$$\Rightarrow V_8 = \sqrt{2000\left(h_7 - h_8\right)};$$

$$\Rightarrow V_8 = \sqrt{\left(2000\right)\left(1173.56 - 740.95\right)};$$

$$\Rightarrow V_8 = 930.17 \frac{\text{m}}{\text{s}}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-16 [OKW] Consider the addition of an afterburner to the turbojet engine in 8-2-15 [OPB] that raises the temperature at the inlet of the nozzle to 1300 K. Using the IG model, determine (a) the velocity (*V*) at the nozzle exit. (b) **What-if Scenario:** What would the exit velocity be if the temperature at the inlet of the nozzle were 1200 K?

SOLUTION

State-1 (given $p_1, T_1, V_1, \dot{m}$):

$$h_1 = 249.63 \frac{\text{kJ}}{\text{kg}}$$

$$j_1 = h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{0};$$

$$\Rightarrow j_1 = h_1 + \frac{V_1^2}{2000};$$

$$\Rightarrow j_1 = 249.63 + \frac{(250)^2}{2000};$$

$$\Rightarrow j_1 = 280.88 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, V_2$):

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow j_1 = j_2;$$

$$j_2 = h_2 + \cancel{\text{ke}_2}^{0} + \cancel{\text{pe}_2}^{0};$$

$$\Rightarrow j_2 = h_2 = 280.88 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $s_3 = s_2, r_p$):

$$h_3 = 571.72 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_3, \eta_C$):

$$h_4 = h_2 + \frac{h_3 - h_2}{\eta_C} = 280.88 + \frac{571.72 - 280.88}{0.85} = 623.04 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, T_4$):

$$h_5 = 1515.72 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $s_6 = s_5, \eta_T$):

$$h_6 = h_5 + \frac{h_7 - h_5}{\eta_T} = 1515.72 + \frac{1173.56 - 1515.72}{0.80} = 1088.02\ \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6$):

The power output from the turbine can be equated to the power input to the compressor.

$$j_5 - j_7 = j_4 - j_2;$$
$$\Rightarrow h_5 - h_7 = h_4 - h_2;$$
$$\Rightarrow h_7 = h_2 - h_4 + h_5;$$
$$\Rightarrow h_7 = 280.88 - 623.04 + 1515.72;$$
$$\Rightarrow h_7 = 1173.56\ \frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_6, T_8$):

$$h_8 = 1396.02\ \frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_1, s_9 = s_8$):

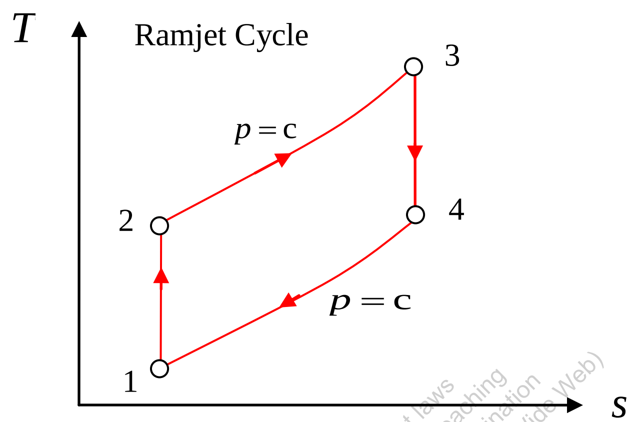
$$h_9 = 886.86\ \frac{\text{kJ}}{\text{kg}}$$

The velocity of exhaust gases

$$\cancel{\frac{dE}{dt}}^{0} = \dot{m}_8 j_8 - \dot{m}_9 j_9 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$
$$\Rightarrow j_8 = j_9;$$
$$\Rightarrow h_8 + \cancel{\text{ke}_8}^{0} + \cancel{\text{pe}_8}^{0} = h_9 + \text{ke}_9 + \cancel{\text{pe}_9}^{0};$$
$$\Rightarrow h_8 - h_9 = + \frac{V_9^2}{2000};$$
$$\Rightarrow V_9 = \sqrt{2000\left(h_8 - h_9\right)};$$
$$\Rightarrow V_9 = \sqrt{\left(2000\right)\left(1396.02 - 886.86\right)};$$
$$\Rightarrow V_9 = 1009.12\ \frac{\text{m}}{\text{s}}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-17 [OPO] Air enters the diffuser of a ramjet engine at 50 kPa, 230 K, with a velocity of 480 m/s, and is decelerated to a velocity of zero. After combustion, the gases reach a temperature of 1000 K before being discharged through the nozzle at 50 kPa. Using IG model, determine (a) the pressure (p) at the diffuser exit and (b) the velocity (V) at the nozzle exit.



SOLUTION

State-1 (given p_1, T_1, V_1):

$$h_1 = 229.56\frac{\text{kJ}}{\text{kg}}$$

$$j_1 = h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^{0};$$

$$\Rightarrow j_1 = h_1 + \frac{V_1^2}{2000};$$

$$\Rightarrow j_1 = 229.56 + \frac{(480)^2}{2000};$$

$$\Rightarrow j_1 = 344.76\frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, V_2$):

$$\frac{d\cancel{E}^{\,0}}{\cancel{dt}} = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^{\,0} - \cancel{\dot{W}_{\text{ext}}}^{\,0};$$

$$\Rightarrow j_1 = j_2;$$

$$j_2 = h_2 + \cancel{\text{ke}_2}^{\,0} + \cancel{\text{pe}_2}^{\,0};$$

$$\Rightarrow j_2 = h_2 = 344.76 \frac{\text{kJ}}{\text{kg}}$$

$p_2 = 205.99$ kPa

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 1046.85 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$h_4 = 706.37 \frac{\text{kJ}}{\text{kg}}$$

The velocity of exhaust gases

$$\frac{d\cancel{E}^{\,0}}{\cancel{dt}} = \dot{m}_3 j_3 - \dot{m}_4 j_4 + \cancel{\dot{Q}}^{\,0} - \cancel{\dot{W}_{\text{ext}}}^{\,0};$$

$$\Rightarrow j_3 = j_4;$$

$$\Rightarrow h_3 + \cancel{\text{ke}_3}^{\,0} + \cancel{\text{pe}_3}^{\,0} = h_4 + \text{ke}_4 + \cancel{\text{pe}_4}^{\,0};$$

$$\Rightarrow h_3 - h_4 = +\frac{V_4^2}{2000};$$

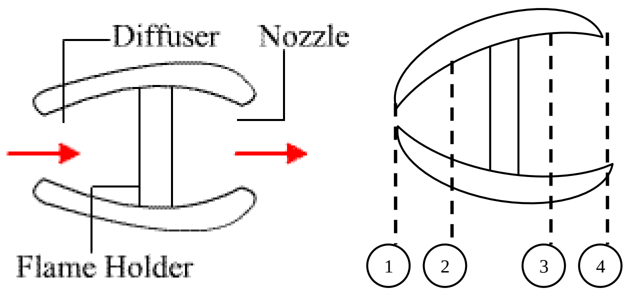
$$\Rightarrow V_4 = \sqrt{2000\left(h_3 - h_4\right)};$$

$$\Rightarrow V_4 = \sqrt{(2000)(1046.85 - 706.37)};$$

$$\Rightarrow V_4 = 825.20 \frac{\text{m}}{\text{s}}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-2-18 [OPA] Air enters the diffuser of a ramjet engine at 25 kPa, 200 K, with a velocity of 3000 km/h, and is decelerated to a negligible velocity. On the basis of an air standard analysis, the heat addition is 870 kJ/kg of air passing through the engine. Air exits the nozzle at 25 kPa. Using the IG model, determine (a) the pressure (p) at the diffuser exit, and (b) the velocity (V) at the nozzle exit. (c) **What-if Scenario:** What would the exit velocity be if the heat addition were 1200 kJ/kg?



SOLUTION

State-1 (given p_1, T_1, V_1):

$$h_1 = 199.46\frac{\text{kJ}}{\text{kg}}$$

$$j_1 = h_1 + \text{ke}_1 + \cancel{\text{pe}_1}^0;$$

$$\Rightarrow j_1 = h_1 + \frac{V_1^2}{2000};$$

$$\Rightarrow j_1 = 199.46 + \frac{\left[(3000)\left(\frac{1000}{3600}\right)\right]^2}{2000};$$

$$\Rightarrow j_1 = 546.68\frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, V_2$):

$$\cancel{\frac{dE}{dt}}^0 = \dot{m}_1 j_1 - \dot{m}_2 j_2 + \cancel{\dot{Q}}^0 - \cancel{\dot{W}_{\text{ext}}}^0;$$

$$\Rightarrow j_1 = j_2;$$

$$j_2 = h_2 + \cancel{\text{ke}_2}^0 + \cancel{\text{pe}_2}^0;$$

$$\Rightarrow j_2 = h_2 = 546.68\frac{\text{kJ}}{\text{kg}}$$

$$p_2 = 841.76\text{ kPa}$$

State-3 (given $p_3 = p_2, q_{in}$):

$$\frac{d\cancel{E}^{0}}{\cancel{dt}} = \dot{m}_2 j_2 - \dot{m}_3 j_3 + \dot{Q} - \cancel{\dot{W}_{ext}}^{0};$$

$$\Rightarrow j_3 = j_2 + q_{in};$$

$$\Rightarrow h_3 = h_2 + q_{in};$$

$$\Rightarrow h_3 = 546.68 + 870;$$

$$\Rightarrow h_3 = 1416.68 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$h_4 = 534.86 \frac{\text{kJ}}{\text{kg}}$$

The velocity of exhaust gases

$$\frac{d\cancel{E}^{0}}{\cancel{dt}} = \dot{m}_3 j_3 - \dot{m}_4 j_4 + \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{ext}}^{0};$$

$$\Rightarrow j_3 = j_4;$$

$$\Rightarrow h_3 + \cancel{\text{ke}_3}^{0} + \cancel{\text{pe}_3}^{0} = h_4 + \text{ke}_4 + \cancel{\text{pe}_4}^{0};$$

$$\Rightarrow h_3 - h_4 = +\frac{V_4^2}{2000};$$

$$\Rightarrow V_4 = \sqrt{2000(h_3 - h_4)};$$

$$\Rightarrow V_4 = \sqrt{(2000)(1416.68 - 534.86)};$$

$$\Rightarrow V_4 = 1328.02 \frac{\text{m}}{\text{s}}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.