9-2-1 [OUY] Water is the working fluid in a cogeneration cycle that generates electricity and provides heat for campus buildings. Steam at 2.5 MPa, 320°C, with a mass flow rate of 1 kg/s, expands through a two-stage turbine. Steam at 0.2 MPa is extracted with a mass flow rate of 0.3 kg/s between the two stages and provided for heating. The remaining steam expands through the second stage to the condenser at a pressure of 6 kPa. The condensate returns from the campus buildings at 0.1 MPa, 60°C and passes through a trap into the condenser. Each turbine stage has an isentropic efficiency of 80%. Determine (a) the net heat transfer rate to the working fluid passing through the steam generator, (b) the net power developed and (c) the rate of heat transfer ($\dot{Q}$) for building heating. (d)**What-if Scenario:** What would the rate of heat transfer be if the inlet conditions at the turbine were 3 MPa and 400°C?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3055.76\frac{\text{kJ}}{\text{kg}};\ s_1 = 6.7223\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 2547.30\frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_T$):

$$h_3 = h_1 - \eta_T\left(h_1 - h_2\right) = 3055.76 - (0.80)(3055.76 - 2547.30) = 2648.99\frac{\text{kJ}}{\text{kg}}$$

$$s_3 = 6.9808\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-4 (given $p_4, s_4 = s_3$):

$$h_{f@6\text{kPa}} = 151.51\frac{\text{kJ}}{\text{kg}};\ h_{fg@6\text{kPa}} = 2415.87\frac{\text{kJ}}{\text{kg}};\ s_{f@6\text{kPa}} = 0.5209\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@6\text{kPa}} = 7.8100\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_4 = \frac{s_4 - s_{f@6\text{kPa}}}{s_{fg@6\text{kPa}}} = \frac{6.9808 - 0.5209}{7.8100} = 0.8271$$

$$h_4 = h_{f@6\text{kPa}} + x_4 h_{fg@6\text{kPa}} = 151.51 + (0.8271)(2415.87) = 2149.68\frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, \eta_T$):

$$h_5 = h_3 - \eta_T\left(h_3 - h_4\right) = 2648.99 - (0.80)(2648.99 - 2149.68) = 2249.54\frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_4, x_6$):

$$h_6 = h_{f@6\text{kPa}} = 151.51\,\frac{\text{kJ}}{\text{kg}};\ v_6 = v_{f@6\text{kPa}} = 0.001007\,\frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_1, s_7 = s_6$):
Assuming that $T_7 \cong T_6$

$$h_7 = h_6 + v_{f@T_6}\left(p_7 - p_6\right) = 151.51 + (0.001007)(2500-6) = 154.02\,\frac{\text{kJ}}{\text{kg}}$$

State-8 (given p_8, T_8):
Subcooled liquid, therefore

$$u_{f@60°\text{C}} = 251.11\,\frac{\text{kJ}}{\text{kg}};\ v_{f@60°\text{C}} = 0.001017\,\frac{\text{m}^3}{\text{kg}}$$

$$h_8 = u_{f@60°\text{C}} + p_8 v_{f@60°\text{C}} = 251.11 + (100)(0.001017) = 251.21\,\frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_8 = p_4, h_9 = h_8$):

$$h_9 = h_8 = 251.21\,\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.
Device-A (1-3,5):

$$\dot{W}_T = \dot{m}_2\left(h_1 - h_3\right) + \dot{m}_3\left(h_1 - h_5\right);$$

$$\dot{W}_T = (0.3)(3055.76 - 2648.99) + (0.7)(3055.76 - 2249.54) = 686.39\ \text{kW}$$

Device-B (5,9-6):

$$\dot{Q}_{\text{out}} = \dot{m}_2 h_9 + \dot{m}_3 h_5 - \dot{m}_1 h_6;$$

$$\dot{Q}_{\text{out}} = (0.3)(251.21) + (0.7)(2249.54) - (1)(151.51) = 1498.15\ \text{kW}$$

Device-C (6-7): $\dot{W}_P = \dot{m}_1\left(h_7 - h_6\right) = (1)(154.02 - 151.51) = 2.51\ \text{kW}$

Device-D (7-1): $\dot{Q}_{\text{in}} = \dot{m}_1\left(h_1 - h_7\right) = (1)(3055.76 - 154.02) = 2901.74\ \text{kW}$

The net power developed

$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_P = 686.39 - 2.51 = 683.88\ \text{kW}$

The rate of heat transfer for the building heating

$\dot{Q} = \dot{m}_2\left(h_3 - h_8\right) = (0.3)(2648.99 - 251.21) = 719.33\ \text{kW}$

Verification and What-if Scenario: Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-2 [OUF] Water is the working fluid in a cogeneration cycle. Steam generator provides 280 kg/s of steam at 9 MPa, 500°C, of which 110 kg/s is extracted between the first and second stages at 1.5 MPa and diverted to a process heating load. Condensate returns from the process heating load at 1 MPa, 120°C and is mixed with the liquid exiting the lower pressure pump at 1 MPa. The entire flow is then pumped to the steam generator pressure. Saturated liquid at 8 kPa leaves the condenser. The turbine stages and pumps operate with isentropic efficiencies of 90% and 80%, respectively. Determine (a) the net heat transfer rate to the working fluid passing through the steam generator, (b) the net power developed and (c) the heating load.

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3386.02 \frac{\text{kJ}}{\text{kg}}; \; s_1 = 6.6575 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 2897.73 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_T$):

$$h_3 = h_1 - \eta_T \left(h_1 - h_2\right) = 3386.02 - (0.90)(3386.02 - 2897.73) = 2946.56 \frac{\text{kJ}}{\text{kg}}$$

$$s_3 = 6.7514 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-4 (given $p_4, s_4 = s_3$):

$$h_{f@8\text{kPa}} = 173.88 \frac{\text{kJ}}{\text{kg}}; \; h_{fg@8\text{kPa}} = 2403.12 \frac{\text{kJ}}{\text{kg}}; \; s_{f@8\text{kPa}} = 0.5926 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \; s_{fg@8\text{kPa}} = 7.6361 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$x_4 = \frac{s_3 - s_{f@8\text{kPa}}}{s_{fg@8\text{kPa}}} = \frac{6.7514 - 0.5926}{7.6361} = 0.8065$$

$$h_4 = h_{f@8\text{kPa}} + x_4 h_{fg@8\text{kPa}} = 173.88 + (0.8065)(2403.12) = 2112.00 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, \eta_T$):

$$h_5 = h_3 - \eta_T \left(h_3 - h_4\right) = 2946.56 - (0.90)(2946.56 - 2112.00) = 2195.46 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_4, x_6$):

$$h_6 = h_{f@8\text{kPa}} = 173.88 \frac{\text{kJ}}{\text{kg}}; \; v_6 = v_{f@8\text{kPa}} = 0.001008 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7, s_7 = s_6$):

Assuming that $T_7 \cong T_6$

$$h_7 = h_6 + v_{f@T_6}\left(p_7 - p_6\right) = 173.88 + \left(0.001008\right)\left(1000 - 8\right) = 174.88\,\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_7, \eta_P$):

$$h_8 = h_6 + \frac{h_7 - h_6}{\eta_T} = 173.88 + \frac{174.88 - 173.88}{0.80} = 175.13\,\frac{\text{kJ}}{\text{kg}}$$

State-9 (given p_9, T_9):

Subcooled liquid, therefore

$$u_{f@120°\text{C}} = 503.50\,\frac{\text{kJ}}{\text{kg}};\ v_{f@120°\text{C}} = 0.001060\,\frac{\text{m}^3}{\text{kg}}$$

$$h_9 = u_{f@120°\text{C}} + p_9 v_{f@120°\text{C}} = 503.50 + \left(1000\right)\left(0.001060\right) = 504.56\,\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_7$):

$$h_{10} = \frac{\dot{m}_2 h_9 + \dot{m}_3 h_8}{\dot{m}_1} = \frac{\left(110\right)\left(504.56\right) + \left(170\right)\left(175.13\right)}{280} = 304.55\,\frac{\text{kJ}}{\text{kg}}$$

$$v_{10} = v_{f@72.53°\text{C}} = 0.001025\,\frac{\text{m}^3}{\text{kg}}$$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):

Assuming that $T_{11} \cong T_{10}$

$$h_{11} = h_{10} + v_{f@T_{10}}\left(p_{11} - p_{10}\right) = 304.55 + \left(0.001025\right)\left(9000 - 1000\right) = 312.75\,\frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_1, \eta_P$):

$$h_{12} = h_{10} + \frac{h_{11} - h_{10}}{\eta_T} = 304.55 + \frac{312.75 - 304.55}{0.80} = 314.80\,\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3,5):

$$\dot{W}_T = \dot{m}_2\left(h_1 - h_3\right) + \dot{m}_3\left(h_1 - h_5\right);$$

$$\dot{W}_T = \left(110\right)\left(3386.02 - 2946.56\right) + \left(170\right)\left(3386.02 - 2195.46\right) = 250.74\ \text{MW}$$

Device-B (5-6): $\dot{Q}_{\text{out}} = \dot{m}_3\left(h_5 - h_6\right) = \left(170\right)\left(2195.46 - 173.88\right) = 343.67\ \text{MW}$

Device-C (6-8): $\dot{W}_{P,\text{I}} = \dot{m}_3\left(h_8 - h_6\right) = \left(170\right)\left(175.13 - 173.88\right) = 0.21\ \text{MW}$

Device-D (10-12): $\dot{W}_{P,\text{II}} = \dot{m}_1\left(h_{12} - h_{10}\right) = (280)(314.80 - 304.55) = 2.87 \text{ MW}$

Device-E (12-1): $\dot{Q}_{\text{in}} = \dot{m}_1\left(h_1 - h_{12}\right) = (280)(3386.02 - 314.80) = 859.94 \text{ MW}$

The net power developed

$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_{P,\text{I}} - \dot{W}_{P,\text{II}} = 250.74 - 0.21 - 2.87 = 247.66 \text{ MW}$

The heating load

$\dot{Q} = \dot{m}_2\left(h_3 - h_9\right) = (110)(2946.56 - 504.56) = 268.62 \text{ MW}$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-3 [OUD] A large food processing plant requires 3.5 kg/s of saturated or slightly superheated steam at 550 kPa, which is extracted from the turbine of a cogeneration plant. The boiler generates steam at 7 MPa, 540°C at a rate of 9 kg/s, and the condenser pressure is 14 kPa. Steam leaves the process heater as saturated liquid. It is then mixed with the feedwater at the same pressure and this mixture is pumped to the boiler pressure. Assuming both the pumps and the turbine have adiabatic efficiencies of 86%, determine (a) the net heat transfer rate to the working fluid passing through the steam generator and (b) the power output of the cogeneration plant. (c) **What-if Scenario:** What would the power output be if the efficiencies of both the pumps and the turbine were 100%?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3506.73 \frac{\text{kJ}}{\text{kg}};\ s_1 = 6.9183 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 2810.93 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_T$):

$$h_3 = h_1 - \eta_T (h_1 - h_2) = 3506.73 - (0.86)(3506.73 - 2810.93) = 2907.61 \frac{\text{kJ}}{\text{kg}}$$

$$s_3 = 7.1225 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-4 (given $p_4, s_4 = s_3$):

$$h_{f@14\text{kPa}} = 219.99 \frac{\text{kJ}}{\text{kg}};\ h_{fg@14\text{kPa}} = 2376.60 \frac{\text{kJ}}{\text{kg}};\ s_{f@14\text{kPa}} = 0.7365 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@14\text{kPa}} = 7.2965 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_4 = \frac{s_3 - s_{f@14\text{kPa}}}{s_{fg@14\text{kPa}}} = \frac{7.1225 - 0.7365}{7.2965} = 0.8752$$

$$h_4 = h_{f@14\text{kPa}} + x_4 h_{fg@14\text{kPa}} = 219.99 + (0.8752)(2376.60) = 2299.99 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, \eta_T$):

$$h_5 = h_3 - \eta_T (h_3 - h_4) = 2907.61 - (0.86)(2907.61 - 2299.99) = 2385.06 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_4, x_6$):

$$h_6 = h_{f@14\text{kPa}} = 219.99 \frac{\text{kJ}}{\text{kg}};\ v_6 = v_{f@14\text{kPa}} = 0.001014 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_2, s_7 = s_6$):
Assuming that $T_7 \cong T_6$

$$h_7 = h_6 + v_{f@T_6}\left(p_7 - p_6\right) = 219.99 + \left(0.001014\right)\left(550 - 14\right) = 220.53\,\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_7, \eta_P$):

$$h_8 = h_6 + \frac{h_7 - h_6}{\eta_T} = 219.99 + \frac{220.53 - 219.99}{0.86} = 220.62\,\frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_2, x_9$):

$$h_9 = h_{f@550\text{kPa}} = 655.82\,\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_7$):

$$h_{10} = \frac{\dot{m}_2 h_9 + \dot{m}_3 h_8}{\dot{m}_1} = \frac{\left(3.5\right)\left(655.82\right) + \left(5.5\right)\left(220.62\right)}{9} = 389.86\,\frac{\text{kJ}}{\text{kg}}$$

$$v_{10} = v_{f@92.96^\circ\text{C}} = 0.001038\,\frac{\text{m}^3}{\text{kg}}$$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):
Assuming that $T_{11} \cong T_{10}$

$$h_{11} = h_{10} + v_{f@T_{10}}\left(p_{11} - p_{10}\right) = 389.86 + \left(0.001038\right)\left(7000 - 550\right) = 396.56\,\frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_1, \eta_P$):

$$h_{12} = h_{10} + \frac{h_{11} - h_{10}}{\eta_T} = 389.86 + \frac{396.56 - 389.86}{0.86} = 397.65\,\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.
Device-A (1-3,5):

$$\dot{W}_T = \dot{m}_2\left(h_1 - h_3\right) + \dot{m}_3\left(h_1 - h_5\right);$$

$$\dot{W}_T = \left(3.5\right)\left(3506.73 - 2907.61\right) + \left(5.5\right)\left(3506.73 - 2385.06\right) = 8266.10\text{ kW}$$

Device-B (5-6): $\dot{Q}_{\text{out}} = \dot{m}_3\left(h_5 - h_6\right) = \left(5.5\right)\left(2385.06 - 219.99\right) = 11907.89\text{ kW}$

Device-C (6-8): $\dot{W}_{P,\text{I}} = \dot{m}_3\left(h_8 - h_6\right) = \left(5.5\right)\left(220.62 - 219.99\right) = 3.47\text{ kW}$

Device-D (10-12): $\dot{W}_{P,\text{II}} = \dot{m}_1\left(h_{12} - h_{10}\right) = \left(9\right)\left(397.65 - 389.86\right) = 70.11\text{ kW}$

Device-E (12-1): $\dot{Q}_{\text{in}} = \dot{m}_1\left(h_1 - h_{12}\right) = \left(9\right)\left(3506.73 - 397.65\right) = 27981.72\text{ kW} = 27.98\text{ MW}$

The net power developed

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_{P,\text{I}} - \dot{W}_{P,\text{II}} = 8266.10 - 3.47 - 70.11 = 8192.52\ \text{kW} = 8.19\ \text{MW}$$

Verification and What-if Scenario: Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-4 [OUW] Consider a cogeneration plant. Steam enters the turbine at 8 MPa and 600°C. 20% of the steam is extracted before it enters the turbine and 60% of the steam is extracted from the turbine at 500 kPa for process heating . The remaining steam continues to expand to 6 kPa. Steam is then condensed at constant pressure and pumped to the boiler pressure of 8 MPa. Steam leaves the process heater as a saturated liquid at 500 kPa. The mass flow rate of steam through the boiler is 20 kg/s. Determine (a) the rate of process heat supply and (b) the net power developed. (c) **What-if Scenario:** What would the net power developed be if no process heat were to be supplied?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3642.01 \frac{\text{kJ}}{\text{kg}}; \; s_1 = 7.0205 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2 = p_1, T_2 = T_1$):

$$h_2 = h_1 = 3642.01 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_1, T_3 = T_1$):

$$h_3 = h_1 = 3642.01 \frac{\text{kJ}}{\text{kg}}; \; s_3 = s_1 = 7.0205 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-4 (given $p_4, h_4 = h_2$):

$$h_4 = h_2 = 3642.01 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, s_5 = s_3$):

$$h_5 = 2837.94 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6, s_6 = s_4$):

$$h_{f@6\text{kPa}} = 151.51 \frac{\text{kJ}}{\text{kg}}; \; h_{fg@6\text{kPa}} = 2415.87 \frac{\text{kJ}}{\text{kg}}; \; s_{f@6\text{kPa}} = 0.5209 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \; s_{fg@6\text{kPa}} = 7.8100 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_6 = \frac{s_6 - s_{f@6\text{kPa}}}{s_{fg@6\text{kPa}}} = \frac{7.0205 - 0.5209}{7.8100} = 0.8322$$

$$h_6 = h_{f@6\text{kPa}} + x_6 h_{fg@6\text{kPa}} = 151.51 + (0.8322)(2415.87) = 2162.00 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_5, x_7$):

$$h_7 = h_{f\,@500\text{kPa}} = 640.05\,\frac{\text{kJ}}{\text{kg}};\; v_7 = v_{f\,@500\text{kPa}} = 0.001093\,\frac{\text{m}^3}{\text{kg}}$$

State-8 (given $p_8 = p_6, x_8$):

$$h_8 = h_{f\,@6\text{kPa}} = 151.51\,\frac{\text{kJ}}{\text{kg}};\; v_8 = v_{f\,@6\text{kPa}} = 0.001007\,\frac{\text{m}^3}{\text{kg}}$$

State-9 (given $p_9 = p_1, s_9 = s_8$):

Assuming that $T_9 \cong T_8$

$$h_9 = h_8 + v_{f\,@T_8}\left(p_9 - p_8\right) = 151.51 + (0.001007)(8000-6) = 159.56\,\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_1, s_{10} = s_7$):

Assuming that $T_{10} \cong T_7$

$$h_{10} = h_7 + v_{f\,@T_7}\left(p_{10} - p_7\right) = 640.05 + (0.001093)(8000-500) = 648.25\,\frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_1$):

$$h_{11} = \frac{\dot{m}_9 h_9 + \dot{m}_{10} h_{10}}{\dot{m}_{11}} = \frac{(6.4)(159.56) + (13.6)(648.25)}{20} = 491.87\,\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (3-5,6):

$$\dot{W}_T = \dot{m}_5\left(h_3 - h_5\right) + \dot{m}_6\left(h_3 - h_6\right);$$

$$\dot{W}_T = (9.6)(3642.01 - 2837.94) + (6.4)(3642.01 - 2162.00) = 17191.14\text{ kW}$$

Device-B (6-8): $\dot{Q}_{\text{out}} = \dot{m}_6\left(h_6 - h_8\right) = (6.4)(2162.00 - 151.51) = 12867.14\text{ kW}$

Device-C (6-8): $\dot{W}_{P,\text{I}} = \dot{m}_6\left(h_9 - h_8\right) = (6.4)(159.56 - 151.51) = 51.52\text{ kW}$

Device-D (7-10): $\dot{W}_{P,\text{II}} = \dot{m}_7\left(h_{10} - h_7\right) = (13.6)(648.25 - 640.05) = 111.52\text{ kW}$

Device-E (11-1):

$$\dot{Q}_{\text{in}} = \dot{m}_1\left(h_1 - h_{11}\right) = (20)(3642.01 - 491.87) = 63002.80\text{ kW} = 63.00\text{ MW}$$

The net power developed

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_{P,\text{I}} - \dot{W}_{P,\text{II}} = 17191.14 - 51.52 - 111.52 = 17028.10\text{ kW} = 17.03\text{ MW}$$

The rate of process heat supply

$$\dot{Q} = \dot{m}_4 h_4 + \dot{m}_5 h_5 - \dot{m}_7 h_7;$$

$$\dot{Q} = (4)(3642.01) + (9.6)(2837.94) - (13.6)(640.05) = 33107.58\text{ kW} = 33.11\text{ MW}$$

Verification and What-if Scenario: Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-5 [OUM] Repeat problem 9-2-4 [OUW] to determine (a) the maximum rate at which process heat can be supplied.

SOLUTION

The maximum rate of process heat supply is achieved when 100% of the steam flows into the process heater. Using the same state numbering as in 9-2-4 [OUW], the problem simplifies to:

State-1 (given p_1, T_1):

$$h_1 = 3642.01\frac{\text{kJ}}{\text{kg}};\ s_1 = 7.0205\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2 = p_1, T_2 = T_1$):

$$h_2 = h_1 = 3642.01\frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4, h_4 = h_2$):

$$h_4 = h_2 = 3642.01\frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_5, x_7$):

$$h_7 = h_{f@500\text{kPa}} = 640.05\frac{\text{kJ}}{\text{kg}};\ v_7 = v_{f@500\text{kPa}} = 0.001093\frac{\text{m}^3}{\text{kg}}$$

State-10 (given $p_{10} = p_1, s_{10} = s_7$):
Assuming that $T_{10} \cong T_7$

$$h_{10} = h_7 + v_{f@T_7}\left(p_{10} - p_7\right) = 640.05 + (0.001093)(8000 - 500) = 648.25\frac{\text{kJ}}{\text{kg}}$$

The rate of process heat supply

$$\dot{Q} = \dot{m}\left(h_4 - h_7\right) = (20)(3642.01 - 640.05) = 60039.20\ \text{kW} = 60.04\ \text{MW}$$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-6 [OUJ] Steam is generated in the boiler of a cogeneration plant at 4 MPa and 480°C at a rate of 7 kg/s. The plant is to produce power while meeting the process steam requirements for a certain industrial application. One-third of the steam leaving the boiler is throttled to a pressure of 820 kPa and is routed to the process heater. The rest of the steam is expanded in an isentropic turbine to a pressure of 820 kPa and is also routed to the process heater. Steam leaves the process heater as saturated liquid. Determine (a) the net power produced and (b) the rate of process heat supply. (c) **What-if Scenario:** What would the net power produced be if one-half instead of one-third of the steam leaving the boiler were throttled?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3399.19 \frac{\text{kJ}}{\text{kg}};\ s_1 = 7.0285 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2 = p_1, T_2 = T_1$):

$$h_2 = h_1 = 3399.19 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_1, T_3 = T_1$):

$$h_3 = h_1 = 3399.19 \frac{\text{kJ}}{\text{kg}};\ s_3 = s_1 = 7.0285 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-4 (given $p_4, h_4 = h_2$):

$$h_4 = h_2 = 3399.19 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, s_5 = s_3$):

$$h_5 = 2950.68 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_4, x_6$):

$$h_6 = h_{f\,@820\text{kPa}} = 725.55 \frac{\text{kJ}}{\text{kg}};\ v_6 = v_{f\,@820\text{kPa}} = 0.001116 \frac{\text{m}^3}{\text{kg}}$$

State-7 (given $p_7 = p_1, s_7 = s_6$):

Assuming that $T_7 \cong T_6$

$$h_7 = h_6 + v_{f\,@T_6}\left(p_7 - p_6\right) = 725.55 + (0.001116)(4000 - 820) = 729.10 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (3-5): $\dot{W}_T = \dot{m}_3\left(h_3 - h_5\right) = \left(4.67\right)\left(3399.19 - 2950.68\right) = 2094.54\ \text{kW}$

Device-B (6-7): $\dot{W}_P = \dot{m}_1\left(h_7 - h_6\right) = \left(7\right)\left(729.10 - 725.55\right) = 24.85\ \text{kW}$

Device-C (7-1): $\dot{Q}_{\text{in}} = \dot{m}_1\left(h_1 - h_7\right) = \left(7\right)\left(3399.19 - 729.10\right) = 18690.63\ \text{kW}$

The net power developed

$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_P = 2094.54 - 24.85 = 2069.69\ \text{kW} = 2.07\ \text{MW}$

The rate of process heat supply

$\dot{Q} = \dot{m}_4 h_4 + \dot{m}_5 h_5 - \dot{m}_6 h_6;$

$\dot{Q} = \left(2.33\right)\left(3399.19\right) + \left(4.67\right)\left(2950.68\right) - \left(7\right)\left(725.55\right) = 16620.94\ \text{kW} = 16.62\ \text{MW}$

Verification and What-if Scenario: Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-7 [OXO] Consider a cogeneration power plant modified with regeneration. Steam enters the turbine at 5 MPa, 450°C and expands to a pressure of 0.6 MPa. At this pressure, 65% of the steam is extracted from the turbine, and the remainder expands to 10 kPa. Part of the extracted steam is used to heat the feedwater in an open feedwater heater. The rest of the extracted steam is used for process heating and leaves the process heater as saturated liquid at 0.6 MPa. It is subsequently mixed with the feedwater leaving the feedwater heater, and the mixture is pumped to the boiler pressure. Assuming the turbines and the pumps to be isentropic, (a) determine the mass flow rate of the steam through the boiler for a net power output of 15 MW. (b) **What-if Scenario:** What would the mass flow rate of steam be if only 50% of steam were extracted from the turbine?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3316.13 \frac{\text{kJ}}{\text{kg}};\ s_1 = 6.8185 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 2783.22 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, h_3 = h_2$):

$$h_3 = h_2 = 2783.22 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, h_4 = h_2$):

$$h_4 = h_2 = 2783.22 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5, s_5 = s_1$):

$$h_{f@10\text{kPa}} = 191.83 \frac{\text{kJ}}{\text{kg}};\ h_{fg@10\text{kPa}} = 2392.80 \frac{\text{kJ}}{\text{kg}};\ s_{f@10\text{kPa}} = 0.6493 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@10\text{kPa}} = 7.5008 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_5 = \frac{s_5 - s_{f@10\text{kPa}}}{s_{fg@10\text{kPa}}} = \frac{6.8185 - 0.6493}{7.5008} = 0.8225$$

$$h_5 = h_{f@10\text{kPa}} + x_5 h_{fg@10\text{kPa}} = 191.83 + (0.8225)(2392.80) = 2159.91 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, x_6$):

$$h_6 = h_{f@10\text{kPa}} = 191.83 \frac{\text{kJ}}{\text{kg}};\ v_6 = v_{f@10\text{kPa}} = 0.001010 \frac{\text{m}^3}{\text{kg}}$$

State-7 (given $p_7 = p_2, s_7 = s_6$):
Assuming that $T_7 \cong T_6$

$$h_7 = h_6 + v_{f\,@T_6}\left(p_7 - p_6\right) = 191.83 + \left(0.001010\right)\left(600 - 10\right) = 192.43\,\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_2, x_8$):

$$h_8 = h_{f\,@0.6\text{MPa}} = 670.56\,\frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_2, x_9$):

$$h_9 = h_{f\,@0.6\text{MPa}} = 670.56\,\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_2$):

$$h_{10} = \frac{\dot{m}_8 h_8 + \dot{m}_9 h_9}{\dot{m}_{10}} = \frac{\left(\dot{m}_8 + \dot{m}_9\right) h_8}{\dot{m}_{10}} = \frac{\dot{m}_{10} h_8}{\dot{m}_{10}} = h_8 = 670.56\,\frac{\text{kJ}}{\text{kg}}$$

$$T_{10} = 158.85°\text{C}$$

$$v_{f\,@158.85°\text{C}} = 0.001101\,\frac{\text{m}^3}{\text{kg}}$$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):
Assuming that $T_{11} \cong T_{10}$

$$h_{11} = h_{10} + v_{f\,@T_{10}}\left(p_{11} - p_{10}\right) = 670.56 + \left(0.001101\right)\left(5000 - 600\right) = 675.40\,\frac{\text{kJ}}{\text{kg}}$$

The problem statement provides the mass flow fractions, with subscript denoting state number

$$r_2 = 0.65$$

$$r_5 = 1 - r_1 = 0.35$$

An analysis of the open feedwater heater

$$\dot{m}_8 h_8 = \dot{m}_4 h_4 + \dot{m}_7 h_7;$$
$$\Rightarrow \dot{m}_8 h_8 = \dot{m}_4 h_4 + \dot{m}_5 h_7;$$
$$\Rightarrow \dot{m}_8 h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$$
$$\Rightarrow \left(\dot{m}_{10} - \dot{m}_9\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$$
$$\Rightarrow \left(\dot{m}_1 - \dot{m}_2 + \dot{m}_4\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$$
$$\Rightarrow \left(\dot{m}_1 - r_2 \dot{m}_1 + \dot{m}_4\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$$
$$\Rightarrow \left[\left(1 - r_2\right) \dot{m}_1 + \dot{m}_4\right] h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$$
$$\Rightarrow \left(r_5 \dot{m}_1 + \dot{m}_4\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$$
$$\Rightarrow \left(r_5 \dot{m}_1\right)\left(h_8 - h_7\right) = \dot{m}_4 \left(h_4 - h_8\right);$$
$$\Rightarrow r_5 \left(h_8 - h_7\right) = r_4 \left(h_4 - h_8\right);$$
$$\Rightarrow r_4 = \frac{r_5\left(h_8 - h_7\right)}{h_4 - h_8} = \frac{(0.35)(670.56 - 192.43)}{2783.22 - 670.56} = 0.0792$$

Knowing the percentage of mass flow at state-4
$r_3 = r_2 - r_4 = 0.65 - 0.0792 = 0.5708$

A steady-state energy analysis is carried out for each device as follows.
Device-A (1-2,5):

$$w_T = r_2\left(h_1 - h_2\right) + r_5\left(h_1 - h_5\right);$$
$$w_T = (0.65)(3316.13 - 2783.22) + (0.35)(3316.13 - 2159.91) = 751.07\,\frac{\text{kJ}}{\text{kg}}$$

Device-B (5-6): $q_{\text{out}} = r_5\left(h_5 - h_6\right) = (0.35)(2159.91 - 191.83) = 688.83\,\frac{\text{kJ}}{\text{kg}}$

Device-C (6-7): $w_{P,\text{I}} = r_5\left(h_7 - h_6\right) = (0.35)(192.43 - 191.83) = 0.21\,\frac{\text{kJ}}{\text{kg}}$

Device-D (10-11): $w_{P,\text{II}} = h_{11} - h_{10} = 675.40 - 670.56 = 4.84\,\frac{\text{kJ}}{\text{kg}}$

Device-E (11-1): $q_{\text{in}} = h_1 - h_{11} = 3316.13 - 675.40 = 2640.73\,\frac{\text{kJ}}{\text{kg}}$

The net work per unit of mass flow

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 751.07 - 0.21 - 4.84 = 746.02\,\frac{\text{kJ}}{\text{kg}}$$

The mass flow to develop the given net power

$$\dot{m}_1 = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{15000}{746.02} = 20.11\,\frac{\text{kg}}{\text{s}}$$

Verification and What-if Scenario: Use PC vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-8 [OXB] Consider a cogeneration power plant modified with regeneration. Steam enters the turbine at 7 MPa, 440^{o}C at a rate of 20 kg/s and expands to a pressure of 0.4 MPa. At this pressure 60% of the steam is extracted from the turbine, and the remainder expands to 10 kPa. Part of the extracted steam is used to heat the feedwater in an open feedwater heater. The rest of the extracted steam is used for process heating and leaves the process heater as a saturated liquid at 0.4 MPa. It is subsequently mixed with the feedwater leaving the feedwater heater, and the mixture is pumped to the boiler pressure. Assuming the turbines and the pumps to be isentropic, determine (a) the total power output of the turbine, (b) the mass flow rate of the steam through the process heater, (c) the rate of heat supply from the process heater per unit mass of steam passing through it and (d) the rate of heat transfer to the steam boiler.

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3261.22 \frac{\text{kJ}}{\text{kg}};\ s_1 = 6.5956 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_{f@0.4\text{MPa}} = 604.72 \frac{\text{kJ}}{\text{kg}};\ h_{fg@0.4\text{MPa}} = 2133.75 \frac{\text{kJ}}{\text{kg}};\ s_{f@0.4\text{MPa}} = 1.7764 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@0.4\text{MPa}} = 5.1200 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_3 = \frac{s_3 - s_{f@0.4\text{MPa}}}{s_{fg@0.4\text{MPa}}} = \frac{6.5956 - 1.7764}{5.1200} = 0.9413$$

$$h_3 = h_{f@0.4\text{MPa}} + x_3 h_{fg@0.4\text{MPa}} = 604.72 + (0.9413)(2133.75) = 2613.22 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, h_3 = h_2$):

$$h_3 = h_2 = 2613.22 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, h_4 = h_2$):

$$h_4 = h_2 = 2613.22 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5, s_5 = s_1$):

$$h_{f@10\text{kPa}} = 191.83\,\frac{\text{kJ}}{\text{kg}};\; h_{fg@10\text{kPa}} = 2392.80\,\frac{\text{kJ}}{\text{kg}};\; s_{f@10\text{kPa}} = 0.6493\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\; s_{fg@10\text{kPa}} = 7.5008\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_5 = \frac{s_5 - s_{f@10\text{kPa}}}{s_{fg@10\text{kPa}}} = \frac{6.5956 - 0.6493}{7.5008} = 0.7928$$

$$h_5 = h_{f@10\text{kPa}} + x_5 h_{fg@10\text{kPa}} = 191.83 + (0.7928)(2392.80) = 2088.84\,\frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, x_6$):

$$h_6 = h_{f@10\text{kPa}} = 191.83\,\frac{\text{kJ}}{\text{kg}};\; v_6 = v_{f@10\text{kPa}} = 0.001010\,\frac{\text{m}^3}{\text{kg}}$$

State-7 (given $p_7 = p_2, s_7 = s_6$):
Assuming that $T_7 \cong T_6$

$$h_7 = h_6 + v_{f@T_6}\left(p_7 - p_6\right) = 191.83 + (0.001010)(400 - 10) = 192.22\,\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_2, x_8$):

$$h_8 = h_{f@0.4\text{MPa}} = 604.72\,\frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_2, x_9$):

$$h_9 = h_{f@0.4\text{MPa}} = 604.72\,\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_2$):

$$h_{10} = \frac{\dot{m}_8 h_8 + \dot{m}_9 h_9}{\dot{m}_{10}} = \frac{\left(\dot{m}_8 + \dot{m}_9\right) h_8}{\dot{m}_{10}} = \frac{\dot{m}_{10} h_8}{\dot{m}_{10}} = h_8 = 604.72\,\frac{\text{kJ}}{\text{kg}}$$

$$T_{10} = 143.61°\text{C}$$

$$v_{f@143.61°\text{C}} = 0.001084\,\frac{\text{m}^3}{\text{kg}}$$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):
Assuming that $T_{11} \cong T_{10}$

$$h_{11} = h_{10} + v_{f@T_{10}}\left(p_{11} - p_{10}\right) = 604.72 + (0.001084)(7000 - 400) = 611.87\,\frac{\text{kJ}}{\text{kg}}$$

The problem statement provides the mass flow fractions, with subscript denoting state number

$r_2 = 0.60$

$r_5 = 1 - r_1 = 0.40$

An analysis of the open feedwater heater

$\dot{m}_8 h_8 = \dot{m}_4 h_4 + \dot{m}_7 h_7;$

$\Rightarrow \dot{m}_8 h_8 = \dot{m}_4 h_4 + \dot{m}_5 h_7;$

$\Rightarrow \dot{m}_8 h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$

$\Rightarrow \left(\dot{m}_{10} - \dot{m}_9\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$

$\Rightarrow \left(\dot{m}_1 - \dot{m}_2 + \dot{m}_4\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$

$\Rightarrow \left(\dot{m}_1 - r_2 \dot{m}_1 + \dot{m}_4\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$

$\Rightarrow \left[\left(1 - r_2\right) \dot{m}_1 + \dot{m}_4\right] h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$

$\Rightarrow \left(r_5 \dot{m}_1 + \dot{m}_4\right) h_8 = \dot{m}_4 h_4 + r_5 \dot{m}_1 h_7;$

$\Rightarrow \left(r_5 \dot{m}_1\right)\left(h_8 - h_7\right) = \dot{m}_4 \left(h_4 - h_8\right);$

$\Rightarrow r_5 \left(h_8 - h_7\right) = r_4 \left(h_4 - h_8\right);$

$$\Rightarrow r_4 = \frac{r_5 \left(h_8 - h_7\right)}{h_4 - h_8} = \frac{(0.40)(604.72 - 192.22)}{2613.22 - 604.72} = 0.0822$$

Knowing the percentage of mass flow at state-4

$r_3 = r_2 - r_4 = 0.60 - 0.0822 = 0.5178$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2,5):

$$w_T = r_2 \left(h_1 - h_2\right) + r_5 \left(h_1 - h_5\right);$$

$$w_T = (0.60)(3261.22 - 2613.22) + (0.40)(3261.22 - 2088.84) = 857.75 \frac{\text{kJ}}{\text{kg}}$$

Device-B (5-6): $q_{\text{out}} = r_5 \left(h_5 - h_6\right) = (0.40)(2088.84 - 191.83) = 758.80 \frac{\text{kJ}}{\text{kg}}$

Device-C (6-7): $w_{P,\text{I}} = r_5 \left(h_7 - h_6\right) = (0.40)(192.22 - 191.83) = 0.16 \frac{\text{kJ}}{\text{kg}}$

Device-D (10-11): $w_{P,\text{II}} = h_{11} - h_{10} = 611.87 - 604.72 = 7.15 \frac{\text{kJ}}{\text{kg}}$

Device-E (11-1): $q_{\text{in}} = h_1 - h_{11} = 3316.13 - 675.40 = 2640.73 \frac{\text{kJ}}{\text{kg}}$

The net power developed

$$w_{\text{net}} = w_T - w_{P,\text{I}} - w_{P,\text{II}} = 857.75 - 0.16 - 7.15 = 850.44\ \frac{\text{kJ}}{\text{kg}}$$

$$\dot{W}_{\text{net}} = \dot{m}_1 w_{\text{net}} = (20)(850.44) = 17008.80\ \text{kW} = 17.01\ \text{MW}$$

The mass flow through the process heater

$$\dot{m}_3 = r_3 \dot{m}_1 = (0.5178)(20) = 10.36\ \frac{\text{kg}}{\text{s}}$$

The rate of heat supply by the process heater per unit flow mass

$$q = h_3 - h_9 = 2613.22 - 604.72 = 2008.50\ \frac{\text{kJ}}{\text{kg}}$$

The rate of heat transfer to the steam in the boiler

$$\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in}} = (20)(2640.73) = 52814.60\ \text{kW} = 52.81\ \text{MW}$$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-9 [OXR] The gas-turbine portion of a combined gas-steam power plant has a pressure ratio of 15. Air enters the compressor at 300 K and 1 atm at a rate of 13 kg/s and is heated to 1500 K in the combustion chamber. The combustion gases leaving the gas turbine are used to heat the steam to 400°C at 10 MPa in a heat exchanger. The combustion gases leave the heat exchanger at 420 K. The steam leaving the turbine is condensed at 15 kPa. Assuming all the compression and expansion processes to be isentropic,determine (a) the mass flow rate of steam, (b) the net power output and (c) the thermal efficiency (η_{th}) of the combined cycle.
(d) **What-if Scenario:** What would the thermal efficiency be if the compression ratio were increased to 17?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 299.81\frac{\text{kJ}}{\text{kg}};\ s_1 = 6.8891\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $s_2 = s_1, r_p$):

$$h_2 = 649.39\frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 1636.75\frac{\text{kJ}}{\text{kg}};\ s_3 = 7.8584\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$h_4 = 783.74\frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_1, T_5$):

$$h_5 = 421.39\frac{\text{kJ}}{\text{kg}}$$

State-6 (given p_6, T_6):

$$h_6 = 3096.43\frac{\text{kJ}}{\text{kg}};\ s_6 = 6.2119\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-7 (given $p_7, s_7 = s_6$):

$$h_{f@15\text{kPa}} = 225.81\frac{\text{kJ}}{\text{kg}};\; h_{fg@15\text{kPa}} = 2373.23\frac{\text{kJ}}{\text{kg}};\; s_{f@15\text{kPa}} = 0.7543\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\; s_{fg@15\text{kPa}} = 7.2550\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_7 = \frac{s_7 - s_{f@15\text{kPa}}}{s_{fg@15\text{kPa}}} = \frac{6.2119 - 0.7543}{7.2550} = 0.7523$$

$$h_7 = h_{f@15\text{kPa}} + x_7 h_{fg@15\text{kPa}} = 225.81 + (0.7523)(2373.23) = 2011.19\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_7, x_8$):

$$h_8 = h_{f@15\text{kPa}} = 225.81\frac{\text{kJ}}{\text{kg}};\; v_8 = v_{f@15\text{kPa}} = 0.001014\frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_6, s_9 = s_8$):

Assuming that $T_9 \cong T_8$

$$h_9 = h_8 + v_{f@T_8}(p_9 - p_8) = 225.81 + (0.001014)(10000 - 15) = 235.93\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $w_C = h_2 - h_1 = 649.39 - 299.81 = 349.58\frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = h_3 - h_2 = 1636.75 - 649.39 = 987.36\frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_{T,\text{I}} = h_3 - h_4 = 1636.75 - 783.74 = 853.01\frac{\text{kJ}}{\text{kg}}$

Device-D (6-7): $w_{T,\text{II}} = h_6 - h_7 = 3096.43 - 2011.19 = 1085.24\frac{\text{kJ}}{\text{kg}}$

Device-E (7-8): $q_{\text{out}} = h_7 - h_8 = 2011.19 - 225.81 = 1785.38\frac{\text{kJ}}{\text{kg}}$

Device-F (8-9): $w_P = h_9 - h_8 = 235.93 - 225.81 = 10.12\frac{\text{kJ}}{\text{kg}}$

An energy balance on the heat exchanger provides the mas flow of water

$$\dot{m}_1(h_4 - h_5) = \dot{m}_6(h_6 - h_9);$$

$$\Rightarrow \dot{m}_6 = \left(\frac{h_4 - h_5}{h_6 - h_9}\right)\dot{m}_1 = \left(\frac{783.74 - 421.39}{3096.43 - 235.93}\right)(13) = 1.65\frac{\text{kg}}{\text{s}}$$

The net power output

$$\dot{W}_{\text{net}} = \dot{m}_1(w_{T,\text{I}} - w_C) + \dot{m}_6(w_{T,\text{II}} - w_P);$$

$$\dot{W}_{\text{net}} = (13)(853.01 - 349.58) + (1.65)(1085.24 - 10.12) = 8318.54\text{ kW} = 8.32\text{ MW}$$

The thermal efficiency

$$\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in}} = (13)(987.36) = 12835.68 \text{ kW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{8318.54}{12835.68} = 64.81\%$$

Verification and What-if Scenario: Use PC/IG vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-10 [OXA] Consider a combined gas-steam power plant that has a net power output of 600 MW. The pressure ratio of the gas turbine cycle is 16. Air enters the compressor at 300 K and the turbine at 1600 K. The combustion gases leaving the gas turbine are used to heat the steam to 400°C at 10 MPa in a heat exchanger. The combustion gases leave the heat exchanger at 400 K. An open feedwater heater incorporated with the steam cycle operates at a pressure of 0.6 MPa. The condenser pressure is 15 kPa. Assuming all the compression and expansion processes to be isentropic, determine (a) the mass flow rate of steam and (b) the thermal efficiency (η_{th}) of the combined cycle. (c) **What-if Scenario:** What would the thermal efficiency be if the compression ratio were increased to 17?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 299.81\frac{\text{kJ}}{\text{kg}};\ s_1 = 6.8891\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $s_2 = s_1, r_p$):

$$h_2 = 661.33\frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 1758.84\frac{\text{kJ}}{\text{kg}};\ s_3 = 7.9186\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$h_4 = 831.05\frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_1, T_5$):

$$h_5 = 400.96\frac{\text{kJ}}{\text{kg}}$$

State-6 (given p_6, T_6):

$$h_6 = 3096.43\frac{\text{kJ}}{\text{kg}};\ s_6 = 6.2119\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-7 (given $p_7, s_7 = s_6$):

$$h_7 = 2519.98\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8, s_8 = s_6$):

$$h_{f@15\text{kPa}} = 225.81\frac{\text{kJ}}{\text{kg}};\ h_{fg@15\text{kPa}} = 2373.23\frac{\text{kJ}}{\text{kg}};\ s_{f@15\text{kPa}} = 0.7543\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@15\text{kPa}} = 7.2550\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_8 = \frac{s_8 - s_{f@15\text{kPa}}}{s_{fg@15\text{kPa}}} = \frac{6.2119 - 0.7543}{7.2550} = 0.7523$$

$$h_8 = h_{f@15\text{kPa}} + x_8 h_{fg@15\text{kPa}} = 225.81 + (0.7523)(2373.23) = 2011.19\frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_8, x_9$):

$$h_9 = h_{f@15\text{kPa}} = 225.81\frac{\text{kJ}}{\text{kg}};\ v_9 = v_{f@15\text{kPa}} = 0.001014\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_7, s_{10} = s_9$):

Assuming that $T_{10} \cong T_9$

$$h_{10} = h_9 + v_{f@T_9}(p_{10} - p_9) = 225.81 + (0.001014)(600 - 15) = 226.40\frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_7, x_{11}$):

$$h_{11} = h_{f@0.6\text{MPa}} = 670.56\frac{\text{kJ}}{\text{kg}};\ v_{11} = v_{f@0.6\text{MPa}} = 0.001010\frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_6, s_{12} = s_{11}$):

Assuming that $T_{12} \cong T_{11}$

$$h_{12} = h_{11} + v_{f@T_{11}}(p_{12} - p_{11}) = 670.56 + (0.001101)(10000 - 600) = 680.91\frac{\text{kJ}}{\text{kg}}$$

An analysis of the open feedwater heater provides

$$\dot{m}_6 h_{11} = \dot{m}_7 h_7 + \dot{m}_8 h_{10}$$

$$\Rightarrow h_{11} = r h_7 + (1 - r) h_{10}$$

$$\Rightarrow r = \frac{h_{11} - h_{10}}{h_7 - h_{10}} = \frac{670.56 - 226.40}{2519.98 - 226.40} = 0.1937$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $w_C = h_2 - h_1 = 661.32 - 299.81 = 361.51\frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = h_3 - h_2 = 1758.84 - 661.32 = 1097.52\frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_{T,\text{I}} = h_3 - h_4 = 1758.84 - 831.05 = 927.79\frac{\text{kJ}}{\text{kg}}$

Device-D (6-7,8):

$$w_{T,\mathrm{II}} = r(h_6 - h_7) + (1-r)(h_6 - h_8);$$

$$w_{T,\mathrm{II}} = (0.1937)(3096.43 - 2519.98) + (1 - 0.1937)(3096.43 - 2011.19) = 986.69 \frac{\mathrm{kJ}}{\mathrm{kg}}$$

Device-E (8-9): $q_{\mathrm{out}} = (1-r)(h_8 - h_9) = (1 - 0.1937)(2011.19 - 225.81) = 1439.55 \frac{\mathrm{kJ}}{\mathrm{kg}}$

Device-F (9-10): $w_{P,\mathrm{I}} = (1-r)(h_{10} - h_9) = (1 - 0.1937)(226.40 - 225.81) = 0.48 \frac{\mathrm{kJ}}{\mathrm{kg}}$

Device-G (11-12): $w_{P,\mathrm{II}} = h_{12} - h_{11} = 680.91 - 670.56 = 10.35 \frac{\mathrm{kJ}}{\mathrm{kg}}$

An energy balance on the heat exchanger

$$\dot{m}_1 (h_4 - h_5) = \dot{m}_6 (h_6 - h_{12});$$

$$\Rightarrow \dot{m}_1 = \left(\frac{h_6 - h_{12}}{h_4 - h_5}\right)\dot{m}_6 = \left(\frac{3096.43 - 680.91}{831.05 - 400.96}\right)\dot{m}_6 = 5.616\dot{m}_6$$

The mass flow rate of steam

$$\dot{W}_{\mathrm{net}} = \dot{m}_1 (w_{T,\mathrm{I}} - w_C) + \dot{m}_6 (w_{T,\mathrm{II}} - w_{P,\mathrm{I}} - w_{P,\mathrm{II}});$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = 5.616\dot{m}_6 (w_{T,\mathrm{I}} - w_C) + \dot{m}_6 (w_{T,\mathrm{II}} - w_{P,\mathrm{I}} - w_{P,\mathrm{II}});$$

$$\Rightarrow \dot{m}_6 = \frac{\dot{W}_{\mathrm{net}}}{5.616(w_{T,\mathrm{I}} - w_C) + (w_{T,\mathrm{II}} - w_{P,\mathrm{I}} - w_{P,\mathrm{II}})};$$

$$\Rightarrow \dot{m}_6 = \frac{600000}{5.616(927.79 - 361.51) + (986.69 - 0.48 - 10.35)} = 144.37 \frac{\mathrm{kg}}{\mathrm{s}}$$

The mass flow rate of air

$$\dot{m}_1 = 5.616\dot{m}_6 = (5.616)(144.37) = 810.78 \frac{\mathrm{kg}}{\mathrm{s}}$$

The thermal efficiency

$$\dot{Q}_{\mathrm{in}} = \dot{m}_1 q_{\mathrm{in}} = (810.78)(1097.52) = 889847.27 \text{ kW}$$

$$\eta_{\mathrm{th}} = \frac{\dot{W}_{\mathrm{net}}}{\dot{Q}_{\mathrm{in}}} = \frac{600000}{889847.27} = 67.43\%$$

Verification and What-if Scenario: Use PC/IG vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-11 Repeat problem 9-2-10 [OXA] assuming isentropic efficiencies of 100% for the pump, 82% for the compressor, 86% for the gas and steam turbines. Determine (a) the mass flow rate of steam and (b) the thermal efficiency of the combined cycle.

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 299.81 \frac{\text{kJ}}{\text{kg}};\; s_1 = 6.8891 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $s_2 = s_1, r_p$):

$$h_2 = 661.33 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 299.81 + \frac{661.33 - 299.81}{0.82} = 740.69 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$$h_4 = 1758.84 \frac{\text{kJ}}{\text{kg}};\; s_4 = 7.9186 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$h_5 = 831.05 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$h_6 = h_4 - \eta_T\left(h_4 - h_5\right) = 1758.84 - (0.86)(1758.84 - 831.05) = 960.94 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_1, T_7$):

$$h_7 = 400.96 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given p_8, T_8):

$$h_8 = 3096.43 \frac{\text{kJ}}{\text{kg}};\; s_8 = 6.2119 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-9 (given $p_9, s_9 = s_8$):

$$h_9 = 2519.98 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_9, \eta_T$):

$$h_{10} = h_8 - \eta_T\left(h_8 - h_9\right) = 3096.43 - (0.86)(3096.43 - 2519.98) = 2600.68\,\frac{\text{kJ}}{\text{kg}}$$

$$s_{10} = 6.3986\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-11 (given $p_{11}, s_{11} = s_{10}$):

$$h_{f\,@15\text{kPa}} = 225.81\,\frac{\text{kJ}}{\text{kg}};\;\; h_{fg\,@15\text{kPa}} = 2373.23\,\frac{\text{kJ}}{\text{kg}};\;\; s_{f\,@15\text{kPa}} = 0.7543\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\;\; s_{fg\,@15\text{kPa}} = 7.2550\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_{11} = \frac{s_{11} - s_{f\,@15\text{kPa}}}{s_{fg\,@15\text{kPa}}} = \frac{6.3986 - 0.7543}{7.2550} = 0.7780$$

$$h_{11} = h_{f\,@15\text{kPa}} + x_{11}h_{fg\,@15\text{kPa}} = 225.81 + (0.7780)(2373.23) = 2072.18\,\frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_{11}, \eta_T$):

$$h_{12} = h_{10} - \eta_T\left(h_{10} - h_{11}\right) = 2600.68 - (0.86)(2600.68 - 2072.18) = 2146.17\,\frac{\text{kJ}}{\text{kg}}$$

State-13 (given $p_{13} = p_{11}, x_{12}$):

$$h_{13} = h_{f\,@15\text{kPa}} = 225.81\,\frac{\text{kJ}}{\text{kg}};\;\; v_{13} = v_{f\,@15\text{kPa}} = 0.001014\,\frac{\text{kJ}}{\text{kg}}$$

State-14 (given $p_{14} = p_9, s_{14} = s_{13}$):
Assuming that $T_{10} \cong T_9$

$$h_{14} = h_{13} + v_{f\,@T_{13}}\left(p_{14} - p_{13}\right) = 225.81 + (0.001014)(600 - 15) = 226.40\,\frac{\text{kJ}}{\text{kg}}$$

State-15 (given $p_{15} = p_9, x_{15}$):

$$h_{15} = h_{f\,@0.6\text{MPa}} = 670.56\,\frac{\text{kJ}}{\text{kg}};\;\; v_{15} = v_{f\,@0.6\text{MPa}} = 0.001010\,\frac{\text{kJ}}{\text{kg}}$$

State-16 (given $p_{16} = p_8, s_{16} = s_{15}$):
Assuming that $T_{16} \cong T_{15}$

$$h_{16} = h_{15} + v_{f\,@T_{15}}\left(p_{16} - p_{15}\right) = 670.56 + (0.001101)(10000 - 600) = 680.91\,\frac{\text{kJ}}{\text{kg}}$$

An analysis of the open feedwater heater provides

$$\dot{m}_8 h_{15} = \dot{m}_{10} h_{10} + \dot{m}_{12} h_{14}$$
$$\Rightarrow h_{15} = r h_{10} + (1-r) h_{14}$$
$$\Rightarrow r = \frac{h_{15} - h_{14}}{h_{10} - h_{14}} = \frac{670.56 - 226.40}{2600.68 - 226.40} = 0.1871$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = h_3 - h_1 = 740.69 - 299.81 = 440.88 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in}} = h_4 - h_3 = 1758.84 - 740.69 = 1018.15 \frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_{T,\text{I}} = h_4 - h_6 = 1758.84 - 960.94 = 797.90 \frac{\text{kJ}}{\text{kg}}$

Device-D (8-10,12):

$$w_{T,\text{II}} = r(h_8 - h_{10}) + (1-r)(h_8 - h_{12});$$
$$w_{T,\text{II}} = (0.1871)(3096.43 - 2600.68) + (1 - 0.1871)(3096.43 - 2146.17) = 865.22 \frac{\text{kJ}}{\text{kg}}$$

Device-E (12-13): $q_{\text{out}} = (1-r)(h_{12} - h_{13}) = (1 - 0.1871)(2146.17 - 225.81) = 1561.06 \frac{\text{kJ}}{\text{kg}}$

Device-F (13-14): $w_{P,\text{I}} = (1-r)(h_{14} - h_3) = (1 - 0.1871)(226.40 - 225.81) = 0.48 \frac{\text{kJ}}{\text{kg}}$

Device-G (15-16): $w_{P,\text{II}} = h_{16} - h_{15} = 680.91 - 670.56 = 10.35 \frac{\text{kJ}}{\text{kg}}$

An energy balance on the heat exchanger

$$\dot{m}_1 (h_6 - h_7) = \dot{m}_8 (h_8 - h_{16});$$
$$\Rightarrow \dot{m}_1 = \left(\frac{h_8 - h_{16}}{h_6 - h_7}\right)\dot{m}_8 = \left(\frac{3096.43 - 680.91}{960.94 - 400.96}\right)\dot{m}_8 = 4.314\dot{m}_8$$

The mass flow rate of steam

$$\dot{W}_{\text{net}} = \dot{m}_1 (w_{T,\text{I}} - w_C) + \dot{m}_8 (w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}});$$
$$\Rightarrow \dot{W}_{\text{net}} = 4.314\dot{m}_8 (w_{T,\text{I}} - w_C) + \dot{m}_8 (w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}});$$
$$\Rightarrow \dot{m}_8 = \frac{\dot{W}_{\text{net}}}{4.314(w_{T,\text{I}} - w_C) + (w_{T,\text{II}} - w_{P,\text{I}} - w_{P,\text{II}})};$$
$$\Rightarrow \dot{m}_6 = \frac{600000}{4.314(797.90 - 440.88) + (865.22 - 0.48 - 10.35)} = 250.57 \frac{\text{kg}}{\text{s}}$$

The mass flow rate of air

$$\dot{m}_1 = 4.314\dot{m}_8 = (4.314)(250.57) = 1080.96\,\frac{\text{kg}}{\text{s}}$$

The thermal efficiency

$$\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in}} = (1080.96)(1018.15) = 1100579.42\text{ kW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{600000}{1100579.42} = 54.52\%$$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-12 [OXG] A combined gas turbine-vapor power plant has a net power output of 15 MW. Air enters the compressor of the gas turbine at 100 kPa, 290 K and is compressed to 1100 kPa. The isentropic efficiency of the compressor is 80%. The conditions at the inlet to the turbine are 1100 kPa and 1400 K. Air expands through the turbine, that has an isentropic efficiency of 88%, to a pressure of 100 kPa. Air then passes through the interconnecting heat exchanger, and is finally discharged at 420 K. Steam enters the turbine of the vapor power cycle at 8 MPa, 390°C and expands to the condenser at a pressure of 8 kPa. Water enters the pump as saturated liquid at 8 kPa. The turbine and pump have isentropic efficiencies of 90% and 80%, respectively. Determine (a) the thermal efficiency (η_{th}) of the combined cycle, the mass flow rates of (b) air and (c) water, and (d) the rate of heat transfer to the combined cycle.

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 289.77\frac{\text{kJ}}{\text{kg}};\ s_1 = 6.8589\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 575.21\frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 289.77 + \frac{575.21 - 289.77}{0.80} = 646.57\frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$$h_4 = 1515.72\frac{\text{kJ}}{\text{kg}};\ \ s_4 = 7.8677\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$h_5 = 787.96\frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_{T,\text{I}}$):

$$h_6 = h_4 - \eta_{T,\text{I}}\left(h_4 - h_5\right) = 1515.72 - \left(0.88\right)\left(1515.72 - 787.95\right) = 875.28\frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_1, T_7$):

$$h_7 = 421.39\frac{\text{kJ}}{\text{kg}}$$

State-8 (given p_8, T_8):

$$h_8 = 3108.05\,\frac{\text{kJ}}{\text{kg}};\ s_8 = 6.3166\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-9 (given $p_9, s_9 = s_8$):

$$h_{f@8\text{kPa}} = 173.88\,\frac{\text{kJ}}{\text{kg}};\ h_{fg@8\text{kPa}} = 2403.12\,\frac{\text{kJ}}{\text{kg}};\ s_{f@8\text{kPa}} = 0.5926\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@8\text{kPa}} = 7.6361\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_9 = \frac{s_9 - s_{f@8\text{kPa}}}{s_{fg@8\text{kPa}}} = \frac{6.3166 - 0.5926}{7.6361} = 0.7496$$

$$h_9 = h_{f@8\text{kPa}} + x_9 h_{fg@8\text{kPa}} = 173.88 + (0.7496)(2403.12) = 1975.26\,\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_9, \eta_{T,\text{II}}$):

$$h_{10} = h_8 - \eta_{T,\text{II}}(h_8 - h_9) = 3108.05 - (0.90)(3108.05 - 1975.26) = 2088.54\,\frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_9, x_{11}$):

$$h_{11} = h_{f@8\text{kPa}} = 173.88\,\frac{\text{kJ}}{\text{kg}};\ v_{11} = v_{f@8\text{kPa}} = 0.001008\,\frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_8, s_{12} = s_{11}$):
Assuming that $T_{12} \cong T_{11}$

$$h_{12} = h_{11} + v_{f@T_{11}}(p_{12} - p_{11}) = 173.88 + (0.001008)(8000 - 8) = 181.94\,\frac{\text{kJ}}{\text{kg}}$$

State-13 (given $p_{13} = p_{12}, \eta_P$):

$$h_{13} = h_{11} + \frac{h_{12} - h_{11}}{\eta_P} = 173.88 + \frac{181.94 - 173.88}{0.80} = 183.96\,\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = h_3 - h_1 = 646.57 - 289.77 = 356.8\,\frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in}} = h_4 - h_3 = 1515.72 - 646.57 = 869.15\,\frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_{T,\text{I}} = h_4 - h_6 = 1515.72 - 875.28 = 640.44\,\frac{\text{kJ}}{\text{kg}}$

Device-D (8-10): $w_{T,\text{II}} = h_8 - h_{10} = 3108.05 - 2088.54 = 1019.51\,\frac{\text{kJ}}{\text{kg}}$

Device-E (10-11): $q_{\text{out}} = h_{10} - h_{11} = 2088.54 - 173.88 = 1914.66 \dfrac{\text{kJ}}{\text{kg}}$

Device-F (11-13): $w_P = h_{13} - h_{11} = 183.96 - 173.88 = 10.08 \dfrac{\text{kJ}}{\text{kg}}$

An energy balance on the heat exchanger

$$\dot{m}_1\left(h_6 - h_7\right) = \dot{m}_8\left(h_8 - h_{13}\right);$$

$$\Rightarrow \dot{m}_1 = \left(\frac{h_8 - h_{13}}{h_6 - h_7}\right)\dot{m}_8 = \left(\frac{3108.05 - 183.96}{875.28 - 421.39}\right)\dot{m}_8 = 6.442\dot{m}_8$$

The mass flow rate of steam

$$\dot{W}_{\text{net}} = \dot{m}_1\left(w_{T,\text{I}} - w_C\right) + \dot{m}_8\left(w_{T,\text{II}} - w_P\right);$$

$$\Rightarrow \dot{W}_{\text{net}} = 6.442\dot{m}_8\left(w_{T,\text{I}} - w_C\right) + \dot{m}_8\left(w_{T,\text{II}} - w_P\right);$$

$$\Rightarrow \dot{m}_8 = \frac{\dot{W}_{\text{net}}}{6.442\left(w_{T,\text{I}} - w_C\right) + \left(w_{T,\text{II}} - w_P\right)};$$

$$\Rightarrow \dot{m}_6 = \frac{15000}{6.442\left(640.44 - 356.8\right) + \left(1019.51 - 10.08\right)} = 5.29 \frac{\text{kg}}{\text{s}}$$

The mass flow rate of air

$$\dot{m}_1 = 6.442\dot{m}_8 = \left(6.442\right)\left(5.29\right) = 34.08 \frac{\text{kg}}{\text{s}}$$

The thermal efficiency

$$\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in}} = \left(34.08\right)\left(869.15\right) = 29620.32 \text{ kW} = 29.62 \text{ MW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{15000}{29620.32} = 50.64\%$$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-13 [OXH] A simple gas turbine is the topping cycle for a simple vapor power cycle. Air enters the compressor of the gas turbine at 101 kPa, 15^{o}C and mass flow rate of 23 kg/s. The compressor pressure ratio is 10 and the turbine inlet temperature is 1100^{o}C. The compressor and turbine have an isentropic efficiency of 85%. Air leaves the interconnecting heat exchanger at 200^{o}C and 101 kPa. Steam enters the turbine of the vapor power cycle at 7 MPa, 480^{o}C and expands to the condenser pressure of 7 kPa. Water enters the pump as saturated liquid at 7 kPa. The turbine and pump have isentropic efficiencies of 90% and 80%, respectively. Determine (a) the thermal efficiency (η_{th}) of the combined cycle, (b) the mass flow rate ($\dot{m}$) of water and (c) the net power output.

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 287.92 \frac{\text{kJ}}{\text{kg}}; \; s_1 = 6.8496 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 556.32 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 287.92 + \frac{556.32 - 287.92}{0.85} = 603.68 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$$h_4 = 1483.44 \frac{\text{kJ}}{\text{kg}}; \; s_4 = 7.8689 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$h_5 = 791.09 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_{T,\text{I}}$):

$$h_6 = h_4 - \eta_{T,\text{I}} \left(h_4 - h_5 \right) = 1483.44 - (0.85)(1483.44 - 791.09) = 894.94 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_1, T_7$):

$$h_7 = 476.01 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given p_8, T_8):

$$h_8 = 3360.97\frac{\text{kJ}}{\text{kg}};\ s_8 = 6.7315\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-9 (given $p_9, s_9 = s_8$):

$$h_{f@7\text{kPa}} = 163.33\frac{\text{kJ}}{\text{kg}};\ h_{fg@7\text{kPa}} = 2409.14\frac{\text{kJ}}{\text{kg}};\ s_{f@7\text{kPa}} = 0.5589\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@7\text{kPa}} = 7.7175\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_9 = \frac{s_9 - s_{f@7\text{kPa}}}{s_{fg@7\text{kPa}}} = \frac{6.7315 - 0.5589}{7.7175} = 0.7998$$

$$h_9 = h_{f@7\text{kPa}} + x_9 h_{fg@7\text{kPa}} = 163.33 + (0.7998)(2409.14) = 2090.16\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_9, \eta_{T,\text{II}}$):

$$h_{10} = h_8 - \eta_{T,\text{II}}(h_8 - h_9) = 3360.97 - (0.90)(3360.97 - 2090.16) = 2217.24\frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_9, x_{11}$):

$$h_{11} = h_{f@7\text{kPa}} = 163.33\frac{\text{kJ}}{\text{kg}};\ v_{11} = v_{f@7\text{kPa}} = 0.001007\frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_8, s_{12} = s_{11}$):

Assuming that $T_{12} \cong T_{11}$

$$h_{12} = h_{11} + v_{f@T_{11}}(p_{12} - p_{11}) = 163.33 + (0.001007)(7000 - 7) = 170.37\frac{\text{kJ}}{\text{kg}}$$

State-13 (given $p_{13} = p_{12}, \eta_P$):

$$h_{13} = h_{11} + \frac{h_{12} - h_{11}}{\eta_P} = 163.33 + \frac{170.37 - 163.33}{0.80} = 172.13\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = h_3 - h_1 = 603.68 - 287.92 = 315.76\frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in}} = h_4 - h_3 = 1483.44 - 603.68 = 879.76\frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_{T,\text{I}} = h_4 - h_6 = 1483.44 - 894.94 = 588.50\frac{\text{kJ}}{\text{kg}}$

Device-D (8-10): $w_{T,\text{II}} = h_8 - h_{10} = 3360.97 - 2217.24 = 1143.73\frac{\text{kJ}}{\text{kg}}$

Device-E (10-11): $q_{\text{out}} = h_{10} - h_{11} = 2217.24 - 163.33 = 2053.91 \dfrac{\text{kJ}}{\text{kg}}$

Device-F (11-13): $w_P = h_{13} - h_{11} = 172.13 - 163.33 = 8.80 \dfrac{\text{kJ}}{\text{kg}}$

An energy balance on the heat exchanger provides the mass flow of water

$$\dot{m}_1 \left(h_6 - h_7 \right) = \dot{m}_8 \left(h_8 - h_{13} \right);$$

$$\Rightarrow \dot{m}_8 = \left(\frac{h_6 - h_7}{h_8 - h_{13}} \right) \dot{m}_1 = \left(\frac{894.94 - 476.01}{3360.97 - 172.13} \right) (23) = 3.02 \frac{\text{kg}}{\text{s}}$$

The net power developed

$$\dot{W}_{\text{net}} = \dot{m}_1 \left(w_{T,\text{I}} - w_C \right) + \dot{m}_8 \left(w_{T,\text{II}} - w_P \right);$$

$$\dot{W}_{\text{net}} = (23)(588.50 - 315.76) + (3.02)(1143.73 - 8.80) = 9700.51 \text{ kW} = 9.70 \text{ MW}$$

The thermal efficiency

$$\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in}} = (23)(879.76) = 20234.48 \text{ kW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{9700.51}{20234.48} = 47.94\%$$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-14 [OXN] Consider a combined cycle power plant using helium and water as the working fluids. Helium enters the compressor of the gas turbine at 1.4 MPa, 350 K and is compressed to 5.5 MPa. The isentropic efficiency of the compressor is 80%. The conditions at the inlet to the turbine are 5.5 MPa and 760°C. Helium expands through the turbine to a pressure of 1.4 MPa. The turbine has an isentropic efficiency of 80%. The mass flow rate of the gas is 100 kg/s. Saturated vapor at 8 MPa exits the heat exchanger which is superheated to 425°C before it enters the turbine of the vapor power cycle, and expands to the condenser at a pressure of 7 kPa. The steam exits the turbine at a quality of 0.9. Water enters the pump as saturated liquid at 7 kPa. Determine (a) the thermal efficiency (η_{th}) of the combined cycle, (b) the mass flow rate of steam and (c) the net power developed. (d) **What-if Scenario:** What would the thermal efficiency be if air were used as working fluid for the gas phase?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 269.44\frac{\text{kJ}}{\text{kg}};\ s_1 = 26.9133\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 1594.51\frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 269.44 + \frac{1594.51 - 269.44}{0.80} = 1925.78\frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$$h_4 = 3819.44\frac{\text{kJ}}{\text{kg}};\ s_4 = 29.6942\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$h_5 = 1556.60\frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_{T,\text{I}}$):

$$h_6 = h_4 - \eta_{T,\text{I}}\left(h_4 - h_5\right) = 3819.44 - \left(0.80\right)\left(3819.44 - 1556.60\right) = 2009.17\frac{\text{kJ}}{\text{kg}}$$

State-7 (given p_7, x_7):

$$h_7 = h_{g@8\text{MPa}} = 2757.94\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_7, T_8$):

$$h_8 = 3205.11 \frac{\text{kJ}}{\text{kg}}; \; s_8 = 6.4591 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-9 (given p_9, x_9):

$$h_{f@7\text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \; h_{fg@7\text{kPa}} = 2409.14 \frac{\text{kJ}}{\text{kg}}; \; s_{f@7\text{kPa}} = 0.5589 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \; s_{fg@7\text{kPa}} = 7.7175 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$h_9 = h_{f@7\text{kPa}} + x_9 h_{fg@7\text{kPa}} = 163.33 + (0.9)(2409.14) = 2331.56 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_9, x_{10}$):

$$h_{10} = h_{f@7\text{kPa}} = 163.33 \frac{\text{kJ}}{\text{kg}}; \; v_{10} = v_{f@7\text{kPa}} = 0.001007 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_7, s_{11} = s_{10}$):

Assuming that $T_{11} \cong T_{10}$

$$h_{11} = h_{10} + v_{f@T_{10}}(p_{11} - p_{10}) = 163.33 + (0.001007)(8000 - 7) = 171.38 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = h_3 - h_1 = 1925.78 - 269.44 = 1656.34 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in,hx}} = h_4 - h_3 = 3819.44 - 1925.78 = 1893.66 \frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_{T,\text{I}} = h_4 - h_6 = 3819.44 - 2009.17 = 1810.27 \frac{\text{kJ}}{\text{kg}}$

Device-D (7-8): $q_{\text{in,sh}} = h_8 - h_7 = 3205.11 - 2757.94 = 447.17 \frac{\text{kJ}}{\text{kg}}$

Device-E (8-9): $w_{T,\text{II}} = h_8 - h_9 = 3205.11 - 2331.56 = 873.55 \frac{\text{kJ}}{\text{kg}}$

Device-F (9-10): $q_{\text{out}} = h_9 - h_{10} = 2331.56 - 163.33 = 2168.23 \frac{\text{kJ}}{\text{kg}}$

Device-F (10-11): $w_P = h_{11} - h_{10} = 171.38 - 163.33 = 8.05 \frac{\text{kJ}}{\text{kg}}$

An energy balance on the heat exchanger provides the mass flow of water

$$\dot{m}_1(h_6 - h_1) = \dot{m}_7(h_7 - h_{11});$$

$$\Rightarrow \dot{m}_7 = \left(\frac{h_6 - h_1}{h_7 - h_{11}}\right)\dot{m}_1 = \left(\frac{2009.17 - 269.44}{2757.94 - 171.38}\right)(100) = 67.26 \frac{\text{kg}}{\text{s}}$$

The net power developed

$$\dot{W}_{\text{net}} = \dot{m}_1\left(w_{T,\text{I}} - w_C\right) + \dot{m}_8\left(w_{T,\text{II}} - w_P\right);$$

$$\dot{W}_{\text{net}} = (100)(1810.27 - 1656.34) + (67.26)(873.55 - 8.05) = 73606.53 \text{ kW} = 73.61 \text{ MW}$$

The thermal efficiency

$$\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in,hx}} + \dot{m}_7 q_{\text{in,sh}} = (100)(1893.66) + (67.26)(447.17) = 219442.65 \text{ kW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{73606.53}{219442.65} = 33.54\%$$

Verification and What-if Scenario: Use PC/IG vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-15 [OXE] Steam and ammonia are the working fluids in a binary vapor power cycle consisting of two ideal Rankine cycles. The heat rejected from the steam cycle is provided to the ammonia cycle. In the steam cycle, steam at 6 MPa, 650^{o}C enters the turbine and exits at 60^{o}C. Saturated liquid at 60^{o}C enters the pump and is pumped to the steam generator pressure. Saturated vapor of ammonia enters the turbine at 50^{o}C and exits at 1 MPa which enters the condenser and condenses to saturated liquid. The saturated liquid is then pumped through the heat exchanger. The power output of the binary cycle is 25 MW. Determine (a) the mass flow rates of steam and ammonia , (b) the power outputs of the steam and ammonia turbines, (c) the rate of heat addition to the cycle and (d) the thermal efficiency (η_{th}).

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3776.32 \frac{\text{kJ}}{\text{kg}};\ s_1 = 7.2955 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $T_2, s_2 = s_1$):

$$h_{f@60°\text{C}} = 251.13 \frac{\text{kJ}}{\text{kg}};\ h_{fg@60°\text{C}} = 2358.47 \frac{\text{kJ}}{\text{kg}};\ s_{f@60°\text{C}} = 0.8312 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@60°\text{C}} = 7.0784 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_2 = \frac{s_2 - s_{f@60°\text{C}}}{s_{fg@60°\text{C}}} = \frac{7.2955 - 0.8312}{7.0784} = 0.9132$$

$$h_2 = h_{f@60°\text{C}} + x_2 h_{fg@60°\text{C}} = 251.13 + (0.9132)(2358.47) = 2404.88 \frac{\text{kJ}}{\text{kg}}$$

$$p_2 = p_{\text{sat}@60°\text{C}} = 19.94 \text{ kPa}$$

State-3 (given $p_3 = p_2, x_3$):

$$h_3 = h_{f@60°\text{C}} = 251.13 \frac{\text{kJ}}{\text{kg}};\ v_3 = v_{f@60°\text{C}} = 0.001017 \frac{\text{m}^3}{\text{kg}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

Assuming that $T_4 \cong T_3$

$$h_{11} = h_{10} + v_{f@T_{10}}(p_{11} - p_{10}) = 251.13 + (0.001017)(6000 - 19.94) = 257.21 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given T_5, x_5):

$$h_5 = h_{g@50°\text{C}} = 1471.50 \frac{\text{kJ}}{\text{kg}};\ s_5 = s_{g@50°\text{C}} = 4.7614 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$p_5 = p_{\text{sat}@50°\text{C}} = 2033.10 \text{ kPa}$$

State-6 (given $p_6, s_6 = s_5$):

$$h_{f@1\text{MPa}} = 297.74\frac{\text{kJ}}{\text{kg}};\ h_{fg@1\text{MPa}} = 1165.69\frac{\text{kJ}}{\text{kg}};\ s_{f@1\text{MPa}} = 1.1193\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@1\text{MPa}} = 3.9112\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_6 = \frac{s_6 - s_{f@1\text{MPa}}}{s_{fg@1\text{MPa}}} = \frac{4.7614 - 1.1193}{3.9112} = 0.9312$$

$$h_6 = h_{f@1\text{MPa}} + x_6 h_{fg@1\text{MPa}} = 297.74 + (0.9312)(1165.69) = 1383.23\frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6, x_7$):

$$h_7 = h_{f@1\text{MPa}} = 297.74\frac{\text{kJ}}{\text{kg}};\ v_7 = v_{f@1\text{MPa}} = 0.001658\frac{\text{m}^3}{\text{kg}}$$

State-8 (given $p_8 = p_5, x_7$):
Assuming that $T_8 \cong T_7$

$$h_8 = h_7 + v_{f@T_7}(p_8 - p_7) = 297.74 + (0.001658)(2033.10 - 1000) = 299.45\frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $w_{T,\text{I}} = h_1 - h_2 = 3776.32 - 2404.88 = 1371.44\frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $w_{P,\text{I}} = h_4 - h_3 = 257.21 - 251.13 = 6.08\frac{\text{kJ}}{\text{kg}}$

Device-C (4-1): $q_{\text{in}} = h_1 - h_4 = 3776.32 - 257.21 = 3519.11\frac{\text{kJ}}{\text{kg}}$

Device-D (5-6): $w_{T,\text{II}} = h_5 - h_6 = 1471.50 - 1383.23 = 88.27\frac{\text{kJ}}{\text{kg}}$

Device-E (6-7): $q_{\text{out}} = h_6 - h_7 = 1383.23 - 297.74 = 1085.49\frac{\text{kJ}}{\text{kg}}$

Device-F (7-8): $w_{P,\text{II}} = h_8 - h_7 = 299.45 - 297.74 = 1.71\frac{\text{kJ}}{\text{kg}}$

An energy balance on the heat exchanger

$$\dot{m}_1(h_2 - h_3) = \dot{m}_5(h_5 - h_8);$$

$$\Rightarrow \dot{m}_5 = \left(\frac{h_2 - h_3}{h_5 - h_8}\right)\dot{m}_1 = \left(\frac{2404.88 - 251.13}{1471.50 - 299.45}\right)\dot{m}_1 = 1.838\dot{m}_1$$

The mass flow rate of steam

$$\dot{W}_{\text{net}} = \dot{m}_1\left(w_{T,\text{I}} - w_{P,\text{I}}\right) + \dot{m}_5\left(w_{T,\text{II}} - w_{P,\text{II}}\right);$$

$$\Rightarrow \dot{W}_{\text{net}} = \dot{m}_1\left(w_{T,\text{I}} - w_{P,\text{I}}\right) + 1.838\dot{m}_1\left(w_{T,\text{II}} - w_{P,\text{II}}\right);$$

$$\Rightarrow \dot{m}_1 = \frac{\dot{W}_{\text{net}}}{\left(w_{T,\text{I}} - w_{P,\text{I}}\right) + 1.838\left(w_{T,\text{II}} - w_{P,\text{II}}\right)};$$

$$\Rightarrow \dot{m}_1 = \frac{25000}{(1371.44 - 6.08) + 1.838(88.27 - 1.71)} = 16.40\,\frac{\text{kg}}{\text{s}}$$

The mass flow rate of ammonia

$$\dot{W}_{\text{net}} = \dot{m}_1\left(w_{T,\text{I}} - w_{P,\text{I}}\right) + \dot{m}_5\left(w_{T,\text{II}} - w_{P,\text{II}}\right);$$

$$\dot{m}_5 = 1.838\dot{m}_1 = (1.838)(16.40) = 30.14\,\frac{\text{kg}}{\text{s}}$$

The power output of the steam turbine

$$\dot{W}_{T,\text{I}} = \dot{m}_1 w_{T,\text{I}} = (16.40)(1371.44) = 22491.62\text{ kW} = 22.49\text{ MW}$$

The power output of the ammonia turbine

$$\dot{W}_{T,\text{II}} = \dot{m}_5 w_{T,\text{II}} = (30.14)(88.27) = 2660.46\text{ kW} = 2.66\text{ MW}$$

The rate of heat addition

$$\dot{Q}_{\text{in}} = \dot{m}_1 q_{\text{in}} = (16.40)(3519.11) = 57713.40\text{ kW} = 57.71\text{ MW}$$

The thermal efficiency

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{25000}{57713.40} = 43.32\%$$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-16 [OXI] Water and refrigerant R-134a are the working fluids in a binary cycle used for cogeneration of power and process steam. In the steam cycle, superheated vapor enters the turbine with a mass flow rate of 5 kg/s at 4 MPa, 470°C and expands isentropically to 150 kPa. Half of the flow is extracted at 150 kPa and is used for industrial process heating. The rest of the stream passes through a heat exchanger, which serves as the boiler for the refrigerant cycle and the condenser of the steam cycle. The condensate leaves the heat exchanger as saturated liquid at 100 kPa, which is combined with the return flow from the process, at 100 kPa and 65°C, before being pumped isentropically to the steam generator pressure. Refrigerant 134a is in an ideal Rankine cycle with refrigerant entering the turbine at 1.5 MPa, 101°C and saturated liquid leaving the condenser at 800 kPa. Determine (a) the rate of heat transfer to the working fluid passing through the steam generator of the steam cycle, (b) the net power output of the binary cycle, (c) the mass flow rate of the refrigerant and (d) the rate of heat transfer to the industrial process.
(e) **What-if Scenario:** What would the mass flow rate be if refrigerant R-12 were used instead of R-134a?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 3376.20 \frac{\text{kJ}}{\text{kg}};\ s_1 = 6.9977 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_{f@150\text{kPa}} = 467.00 \frac{\text{kJ}}{\text{kg}};\ h_{fg@150\text{kPa}} = 2226.52 \frac{\text{kJ}}{\text{kg}};\ s_{f@150\text{kPa}} = 1.4333 \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ s_{fg@150\text{kPa}} = 5.7904 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$x_2 = \frac{s_2 - s_{f@150\text{kPa}}}{s_{fg@150\text{kPa}}} = \frac{6.9977 - 1.4333}{5.7904} = 0.9610$$

$$h_2 = h_{f@150\text{kPa}} + x_2 h_{fg@150\text{kPa}} = 467.00 + (0.9610)(2226.52) = 2606.69 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, s_3 = s_1$):

$$h_3 = h_2 = 2606.69 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given p_4, x_4):

$$h_4 = h_{f@100\text{kPa}} = 417.44 \frac{\text{kJ}}{\text{kg}};\ v_4 = v_{f@100\text{kPa}} = 0.001043 \frac{\text{m}^3}{\text{kg}}$$

State-5 (given p_5, T_5):

Subcooled liquid, therefore

$$u_{f@100\text{kPa}} = 272.03\frac{\text{kJ}}{\text{kg}};\ v_{f@100\text{kPa}} = 0.001020\frac{\text{m}^3}{\text{kg}}$$

$$h_5 = u_{f@100\text{kPa}} + p_5 v_{f@100\text{kPa}} = 272.03 + (100)(0.001020) = 272.13\frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_4$):

$$h_6 = \frac{\dot{m}_4 h_4 + \dot{m}_5 h_5}{\dot{m}_6} = \frac{h_4 + h_5}{2} = \frac{417.44 + 272.13}{2} = 344.79\frac{\text{kJ}}{\text{kg}}$$

$$T_6 = 82.34°\text{C}$$

$$v_{f@82.34°\text{C}} = 0.001031\frac{\text{m}^3}{\text{kg}}$$

State-7 (given $p_7 = p_1, s_7 = s_6$):

Assuming that $T_7 \cong T_6$

$$h_7 = h_6 + v_{f@T_6}(p_7 - p_6) = 344.79 + (0.001031)(4000 - 100) = 348.81\frac{\text{kJ}}{\text{kg}}$$

State-8 (given p_8, T_8):

$$h_8 = 329.66\frac{\text{kJ}}{\text{kg}};\ s_8 = 1.0584\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-9 (given $p_9, s_9 = s_8$):

$$h_9 = 313.29\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_9, x_{10}$):

$$h_{10} = h_{f@800\text{kPa}} = 94.68\frac{\text{kJ}}{\text{kg}};\ v_{10} = v_{f@800\text{kPa}} = 0.000847\frac{\text{m}^3}{\text{kg}}$$

State-11 (given $p_{11} = p_8, s_{11} = s_{10}$):

Assuming that $T_{11} \cong T_{10}$

$$h_{11} = h_{10} + v_{f@T_{10}}(p_{11} - p_{10}) = 94.68 + (0.000847)(1500 - 800) = 95.28\frac{\text{kJ}}{\text{kg}}$$

Knowing that half of the steam is extracted for process heating, $r = 0.5$.

A steady-state energy analysis is carried out for each device as follows.
Device-A (1-2,3):

$$w_{T,\mathrm{I}} = r\left(h_1 - h_2\right) + \left(1-r\right)\left(h_1 - h_3\right);$$

$$w_{T,\mathrm{I}} = \left(0.5\right)\left(3376.20 - 2606.69\right) + \left(1-0.5\right)\left(3376.20 - 2606.69\right) = 769.51\,\frac{\mathrm{kJ}}{\mathrm{kg}}$$

Device-B (6-7): $w_{P,\mathrm{I}} = h_7 - h_6 = 348.81 - 344.79 = 4.02\,\frac{\mathrm{kJ}}{\mathrm{kg}}$

Device-C (7-1): $q_{\mathrm{in}} = h_1 - h_7 = 3376.20 - 348.81 = 3027.39\,\frac{\mathrm{kJ}}{\mathrm{kg}}$

Device-D (8-9): $w_{T,\mathrm{II}} = h_8 - h_9 = 329.66 - 313.29 = 16.37\,\frac{\mathrm{kJ}}{\mathrm{kg}}$

Device-E (9-10): $q_{\mathrm{out}} = h_9 - h_{10} = 313.29 - 94.68 = 218.61\,\frac{\mathrm{kJ}}{\mathrm{kg}}$

Device-F (10-11): $w_{P,\mathrm{II}} = h_{11} - h_{10} = 95.28 - 94.68 = 0.60\,\frac{\mathrm{kJ}}{\mathrm{kg}}$

An energy balance on the heat exchanger provides the mass flow of R-134a

$$\left(1-5\right)\dot{m}_1\left(h_3 - h_4\right) = \dot{m}_8\left(h_8 - h_{11}\right);$$

$$\Rightarrow \dot{m}_8 = \left(\frac{h_3 - h_4}{h_8 - h_{11}}\right)\left(1-r\right)\dot{m}_1 = \left(\frac{2606.69 - 417.44}{329.66 - 95.28}\right)\left(1-0.5\right)\left(5\right) = 23.35\,\frac{\mathrm{kg}}{\mathrm{s}}$$

The rate of heat addition

$$\dot{Q}_{\mathrm{in}} = \dot{m}_1 q_{\mathrm{in}} = \left(5\right)\left(3027.39\right) = 15136.95\ \mathrm{kW} = 15.14\ \mathrm{MW}$$

The power output of the steam cycle

$$\dot{W}_{\mathrm{H_2O}} = \dot{m}_1\left(w_{T,\mathrm{I}} - w_{P,\mathrm{I}}\right) = \left(5\right)\left(769.51 - 4.02\right) = 22391.90\ \mathrm{kW}$$

The power output of the R-134a cycle

$$\dot{W}_{\mathrm{R-134a}} = \dot{m}_5\left(w_{T,\mathrm{II}} - w_{P,\mathrm{II}}\right) = \left(23.35\right)\left(16.37 - 0.60\right) = 368.23\ \mathrm{kW}$$

The net power output

$$\dot{W}_{\mathrm{net}} = \dot{W}_{\mathrm{H_2O}} + \dot{W}_{\mathrm{R-134a}} = 22391.90 + 368.23 = 22760.13\ \mathrm{kW} = 2.27\ \mathrm{MW}$$

The rate of heat transfer to the industrial process

$$\dot{Q} = r\dot{m}_1\left(h_2 - h_5\right) = \left(0.5\right)\left(5\right)\left(2606.69 - 272.13\right) = 5836.4\ \mathrm{kW} = 5.84\ \mathrm{MW}$$

Verification and What-if Scenario: Use PC/IG vapor-power cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

9-2-17 [OXL] A geothermal resource exists as saturated liquid at 200°C. The geothermal liquid is withdrawn from a production well at a rate of 200 kg/s, and is flashed to a pressure of 500 kPa by an essentially isenthalpic flashing process, where the resulting vapor is separated from the liquid in a separator and is directed to the turbine. Steam leaves the turbine at 12 kPa with a moisture content of 14% and enters the condenser, where it is condensed and routed to a reinjection well, along with the liquid coming from the separator. Determine (a) the mass flow rate of steam through the turbine and (b) the power output of the turbine.

SOLUTION

State-1 (given p_1, x_1):

$$h_1 = 852.45 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $p_2, h_2 = h_1$):

$$h_{f@500\text{kPa}} = 640.05 \frac{\text{kJ}}{\text{kg}}; \; h_{fg@500\text{kPa}} = 2108.56 \frac{\text{kJ}}{\text{kg}}$$

$$x_2 = \frac{h_2 - h_{f@500\text{kPa}}}{h_{fg@500\text{kPa}}} = \frac{852.45 - 640.05}{2108.56} = 0.1007$$

State-3 (given $p_3 = p_2, x_3$):

$$h_3 = h_{g@500\text{kPa}} = 2748.61 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given p_4, x_4):

$$h_{f@12\text{kPa}} = 206.84 \frac{\text{kJ}}{\text{kg}}; \; h_{fg@12\text{kPa}} = 2384.20 \frac{\text{kJ}}{\text{kg}}$$

$$h_4 = h_{f@12\text{kPa}} + x_4 h_{fg@12\text{kPa}} = 206.84 + (0.86)(2384.20) = 2257.25 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_5, x_4$):

$$h_5 = h_{f@12\text{kPa}} = 206.84 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_2, x_6$):

$$h_6 = h_{f@500\text{kPa}} = 604.05 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_4, h_7 = h_6$):

$$h_7 = h_6 = 604.05 \frac{\text{kJ}}{\text{kg}}$$

The mass flow through the turbine

$$\dot{m}_3 = x_2 \dot{m}_1 = (0.1007)(200) = 20.14 \frac{\text{kg}}{\text{s}}$$

The power output of the turbine

$$\dot{W}_{\text{ext}} = \dot{m}_3 (h_3 - h_4) = (20.14)(2748.61 - 2257.25) = 9895.99\ \text{kW} = 9.90\ \text{MW}$$

Verification: Use PC vapor-power cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).