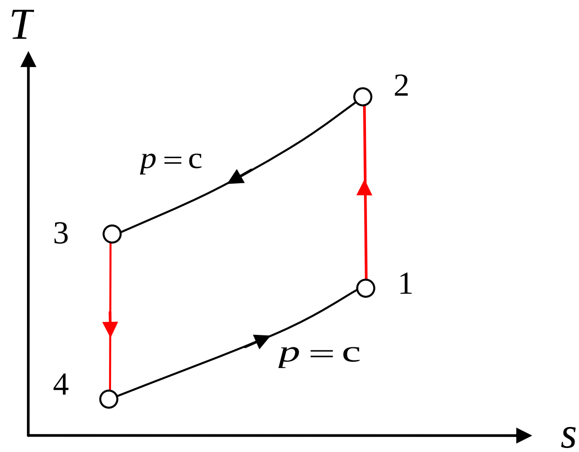
10-2-1 [OVR] In a gas refrigeration system air enters the compressor at 10°C, 50 kPa and the turbine at 50°C, 250 kPa. The mass flow rate is 0.08 kg/s. Assuming variable specific heat, determine (a) the rate of cooling, (b) the net power input and (c) the COP. (d) What-if Scenario: What would the COP be if the compressor inlet temperature were 15°C?

SOLUTION



State-1 (given p_1, T_1):

$$h_1 = 282.90 \frac{\text{kJ}}{\text{kg}}; \; s_1 = 7.0338 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given $p_2 = p_3, s_2 = s_1$):

$$h_2 = 448.75 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given p_3, T_3):

$$h_3 = 323.08 \frac{\text{kJ}}{\text{kg}}; \; s_3 = 6.7047 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$h_4 = 203.44 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m}(h_2 - h_1) = (0.08)(448.75 - 282.90) = 13.29 \text{ kW}$

Device-B (2-3): $\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.08)(448.75 - 323.08) = 10.05 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}(h_3 - h_4) = (0.08)(323.08 - 203.44) = 9.57 \text{ kW}$

Device-D (4-1): $\dot{Q}_C = \dot{m}(h_1 - h_4) = (0.08)(282.90 - 203.44) = 6.36 \text{ kW}$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|9.57 - 13.29\right| = 3.72\ \text{kW}$$

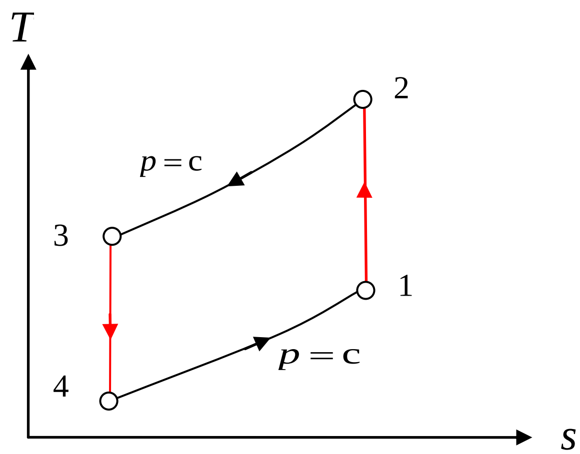
Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{6.36}{3.72} = 1.71$$

TEST Solution and What-if Scenario: Use IG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-2 [OVB] In an ideal Brayton refrigeration cycle air enters the compressor at 100 kPa and 300 K. The compression ratio is 4, and air enters the turbine inlet at 350 K. The mass flow rate of air is 0.05 kg/s. Determine (a) the rate of cooling, (b) the net power input, (c) the COP, and (d) the Carnot COP. (e) What-if Scenario: What would the answers be if air enters the turbine inlet at 370 K instead?

SOLUTION



Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $s_2 = s_1, r$):

$$T_2 = T_1 r^{\frac{k-1}{k}} = (300)(4)^{\frac{1.4-1}{1.4}} = 445.80 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}} = (350)\left(\frac{100}{400}\right)^{\frac{1.4-1}{1.4}} = 235.53 \text{ K}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m} c_p (T_2 - T_1) = (0.05)(1.005)(445.80 - 300) = 7.33 \text{ kW}$

Device-B (2-3): $\dot{Q}_H = \dot{m} c_p (T_2 - T_3) = (0.05)(1.005)(445.80 - 350) = 4.81 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m} c_p (T_3 - T_4) = (0.05)(1.005)(350 - 235.53) = 5.75 \text{ kW}$

Device-D (4-1): $\dot{Q}_C = \dot{m}c_p\left(T_1 - T_4\right) = (0.05)(1.005)(300 - 235.53) = 3.24\ \text{kW}$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|5.75 - 7.33\right| = 1.58\ \text{kW}$$

Therefore, the COP can be obtained as

$$\text{COP}_{\text{R}} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{3.24}{1.58} = 2.05$$

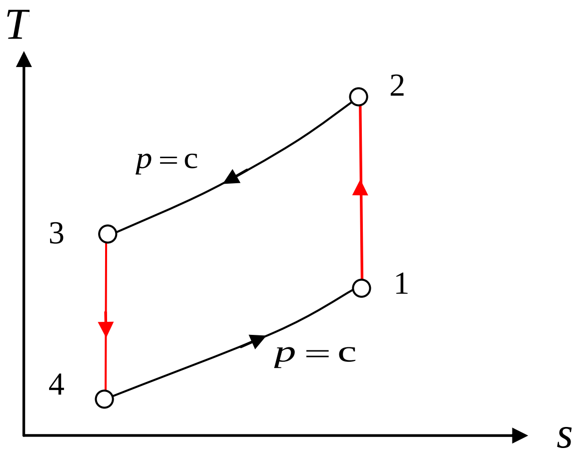
The Carnot COP can be obtained as

$$\text{COP}_{\text{R,Carnot}} = \frac{T_C}{T_H - T_C} = \frac{300}{350 - 300} = 6$$

TEST Solution and What-if Scenario: Use PG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-3 [OVS] Air enters the compressor of a perfect-gas refrigeration cycle at 45°F, 10 psia and the turbine at 120°F, 30 psia. The mass flow rate of air through the cycle is 0.5 lbm/s. Determine (a) the rate of refrigeration, (b) the net power input and (c) the coefficient of performance (COP_R).

SOLUTION



Given:

$$c_p = 0.240 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $p_2 = p_3, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = (504.67) \left(\frac{30}{10} \right)^{\frac{1.4-1}{1.4}} = 690.76\text{°R}$$

State-3 (given p_3, T_3)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3} \right)^{\frac{k-1}{k}} = (579.67) \left(\frac{10}{30} \right)^{\frac{1.4-1}{1.4}} = 423.51\text{°R}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m} c_p (T_2 - T_1) = (30)(0.240)(690.76 - 504.67) = 1339.85 \frac{\text{Btu}}{\text{min}}$

Device-B (2-3): $\dot{Q}_H = \dot{m} c_p (T_2 - T_3) = (30)(0.240)(690.76 - 579.67) = 799.85 \frac{\text{Btu}}{\text{min}}$

Device-C (3-4): $\dot{W}_T = \dot{m}c_p\left(T_3 - T_4\right) = (30)(0.240)(579.67 - 423.51) = 1124.35\,\frac{\text{Btu}}{\text{min}}$

Device-D (4-1): $\dot{Q}_C = \dot{m}c_p\left(T_1 - T_4\right) = (30)(0.240)(504.67 - 423.51) = 584.35\,\frac{\text{Btu}}{\text{min}}$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|1124.35 - 1339.85\right| = 215.50\,\frac{\text{Btu}}{\text{min}}$$

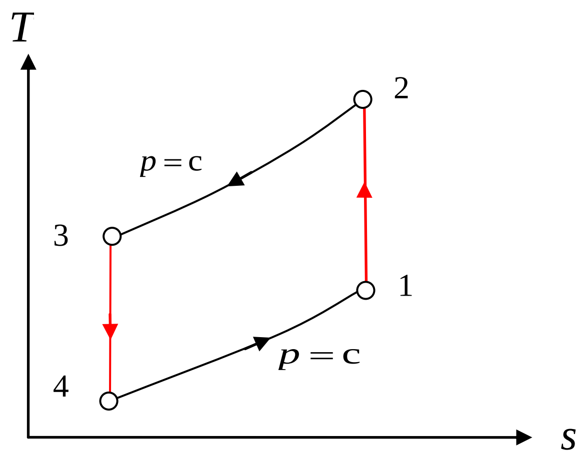
Therefore, the COP can be obtained as

$$\text{COP}_{\text{R}} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{584.35}{215.50} = 2.71$$

TEST Solution and What-if Scenario: Use PG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-4 [OVH] Air enters the compressor of a perfect-gas refrigeration cycle at 15°C, 50 kPa and the turbine at 50°C, 300 kPa. The mass flow rate through the cycle is 0.25 kg/s. Assuming constant specific heats for air (PG model), determine (a) the rate of refrigeration, (b) the net power input and (c) the coefficient of performance (COP_R). (d) What-if Scenario: What would the COP be if the IG model were used for air?

SOLUTION



Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $p_2 = p_3, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = (288)\left(\frac{300}{50} \right)^{\frac{1.4-1}{1.4}} = 480.53 \text{ K}$$

State-3 (given p_3, T_3)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3} \right)^{\frac{k-1}{k}} = (323)\left(\frac{50}{300} \right)^{\frac{1.4-1}{1.4}} = 193.59 \text{ K}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m}c_p (T_2 - T_1) = (0.25)(1.005)(480.53 - 288) = 48.37 \text{ kW}$

Device-B (2-3): $\dot{Q}_H = \dot{m}c_p (T_2 - T_3) = (0.25)(1.005)(480.53 - 323) = 39.58 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}c_p\left(T_3 - T_4\right) = \left(0.25\right)\left(1.005\right)\left(323 - 193.59\right) = 32.51\ \text{kW}$

Device-D (4-1): $\dot{Q}_C = \dot{m}c_p\left(T_1 - T_4\right) = \left(0.25\right)\left(1.005\right)\left(288 - 193.59\right) = 23.72\ \text{kW}$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|32.51 - 48.37\right| = 15.86\ \text{kW}$$

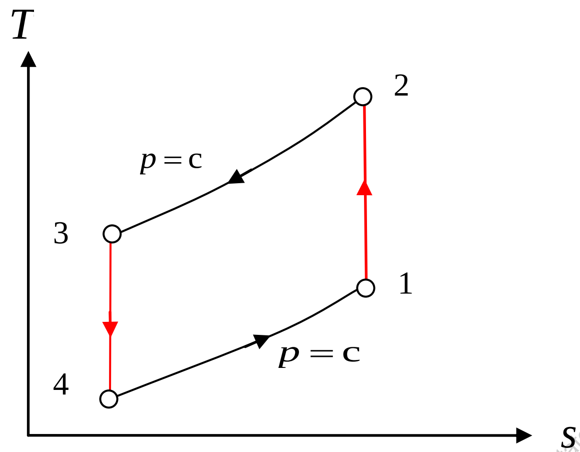
Therefore, the COP can be obtained as

$$\text{COP}_{\text{R}} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{23.72}{15.86} = 1.50$$

TEST Solution and What-if Scenario: Use PG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-5 [OVE] Air enters the compressor of an ideal Brayton refrigeration cycle at 200 kPa, 270 K, with a volumetric flow rate of 1 m^3/s, and is compressed to 600 kPa. The temperature at the turbine inlet is 330 K. Treating air as a perfect gas, determine (a) the net power input in kW, (b) the refrigeration capacity in kW and tons and (c) the coefficient of performance (COP_R). (d) What-if Scenario: What would the COP be if a reversible cycle could be operated between the highest and lowest temperatures of the cycle?

SOLUTION



Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1):

$$\dot{m} = \frac{p_1 \dot{V}_1}{RT_1} = \frac{(200)(1)}{(0.287)(270)} = 2.58 \frac{\text{kg}}{\text{s}}$$

State-2 (given $p_2, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = (270)\left(\frac{600}{200}\right)^{\frac{1.4-1}{1.4}} = 369.56 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}} = (330)\left(\frac{200}{600}\right)^{\frac{1.4-1}{1.4}} = 241.10\text{ K}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m}c_p(T_2 - T_1) = (2.58)(1.005)(369.56 - 270) = 258.15\text{ kW}$

Device-B (2-3): $\dot{Q}_H = \dot{m}c_p(T_2 - T_3) = (2.58)(1.005)(369.56 - 330) = 102.58\text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}c_p(T_3 - T_4) = (2.58)(1.005)(330 - 241.10) = 230.51\text{ kW}$

Device-D (4-1):

$$\dot{Q}_C = \dot{m}c_p(T_1 - T_4) = (2.58)(1.005)(270 - 241.10) = 74.93\text{ kW} = 21.31\text{ ton}$$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|230.51 - 258.15\right| = 27.64\text{ kW}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{74.93}{27.64} = 2.71$$

TEST Solution and What-if Scenario: Use PG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-6 [OVA] Repeat 10-2-5 [OVE] if the compressor and turbine have an isentropic efficiency of 80%.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1):

$$\dot{m} = \frac{p_1 \dot{V}_1}{RT_1} = \frac{(200)(1)}{(0.287)(270)} = 2.58 \frac{\text{kg}}{\text{s}}$$

State-2 (given $p_2, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = (270)\left(\frac{600}{200}\right)^{\frac{1.4-1}{1.4}} = 369.56 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 270 + \frac{369.56 - 270}{0.80} = 394.45 \text{ K}$$

State-4 (given $p_4 = p_2, T_4$)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = T_4 \left(\frac{p_5}{p_4}\right)^{\frac{k-1}{k}} = (330)\left(\frac{200}{600}\right)^{\frac{1.4-1}{1.4}} = 241.10 \text{ K}$$

State-6 (given $p_5 = p_6, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 330 - (0.80)(330 - 241.10) = 258.88 \text{ K}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $\dot{W}_C = \dot{m} c_p (T_3 - T_1) = (2.58)(1.005)(394.45 - 270) = 322.69 \text{ kW}$

Device-B (3-4): $\dot{Q}_H = \dot{m} c_p (T_3 - T_4) = (2.58)(1.005)(394.45 - 330) = 167.11 \text{ kW}$

Device-C (4-6): $\dot{W}_T = \dot{m} c_p (T_4 - T_6) = (2.58)(1.005)(330 - 258.88) = 184.41 \text{ kW}$

Device-D (6-1):

$$\dot{Q}_C = \dot{m}c_p\left(T_1 - T_4\right) = (2.58)(1.005)(270 - 258.88) = 28.83\text{ kW} = 8.20\text{ ton}$$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|322.69 - 184.41\right| = 138.28\text{ kW}$$

Therefore, the COP can be obtained as

$$\text{COP}_{\text{R}} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{28.83}{138.28} = 0.21$$

TEST Solution and What-if Scenario: Use PG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-7 [OVN] An ideal-gas refrigeration cycle uses air as the working fluid to maintain a refrigerated space at -30°C while rejecting heat to the surrounding medium at 30°C. If the pressure ratio of the compressor is 4, determine (a) the maximum and minimum temperatures in the cycle, (b) the coefficient of performance (COP_R) and (c) the rate of refrigeration for a mass flow rate of 0.05 kg/s. (d) What-if Scenario: What would the COP be if the PG model were used for air?

SOLUTION

State-1 (given p_1, T_1):
Assume an inlet pressure of 1 kPa

$$h_1 = 242.76 \frac{\text{kJ}}{\text{kg}}; \; s_1 = 8.0037 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given $s_2 = s_1, r$):

$$T_2 = 87.99°\text{C}$$

$$h_2 = 361.46 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 302.97 \frac{\text{kJ}}{\text{kg}}; \; s_3 = 7.8272 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = -69.22°\text{C}$$

$$h_4 = 203.40 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m}(h_2 - h_1) = (0.05)(361.46 - 242.76) = 5.94 \text{ kW}$

Device-B (2-3): $\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.05)(361.46 - 302.97) = 2.92 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}(h_3 - h_4) = (0.05)(302.97 - 203.40) = 4.98 \text{ kW}$

Device-D (4-1): $\dot{Q}_C = \dot{m}(h_1 - h_4) = (0.05)(242.76 - 203.40) = 1.97 \text{ kW}$

The magnitude of net power input

$$|\dot{W}_{\text{net}}| = |\dot{W}_T - \dot{W}_C| = |4.98 - 5.94| = 0.96 \text{ kW}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{1.97}{0.96} = 2.05$$

TEST Solution and What-if Scenario: Use IG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-8 [OVG] In an ideal Brayton refrigeration cycle air enters the compressor at 18 lb_f/in^2 and 400°R. The compression ratio is 5, and air enters the turbine inlet at 600°R. The mass flow rate of air is 2 lb/min. Determine (a) the refrigeration capacity in tons, (b) the net power input in Btu/min, and (c) the COP. (d) What-if Scenario: What would the refrigeration capacity be if air enters the turbine inlet at 700°R instead?

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 95.35 \frac{\text{Btu}}{\text{lbm}};\ s_1 = 1.5596 \frac{\text{Btu}}{\text{lbm}\cdot\text{R}}$$

State-2 (given $s_2 = s_1, r$):

$$h_2 = 151.39 \frac{\text{Btu}}{\text{lbm}}$$

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 143.33 \frac{\text{Btu}}{\text{lbm}};\ s_3 = 1.5465 \frac{\text{Btu}}{\text{lbm}\cdot\text{R}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$h_4 = 90.26 \frac{\text{Btu}}{\text{lbm}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m}\left(h_2 - h_1\right) = (2)(151.39 - 95.35) = 112.08 \frac{\text{Btu}}{\text{min}}$

Device-B (2-3): $\dot{Q}_H = \dot{m}\left(h_2 - h_3\right) = (2)(151.39 - 143.33) = 16.12 \frac{\text{Btu}}{\text{min}}$

Device-C (3-4): $\dot{W}_T = \dot{m}\left(h_3 - h_4\right) = (0.05)(143.33 - 90.26) = 106.14 \frac{\text{Btu}}{\text{min}}$

Device-D (4-1): $\dot{Q}_C = \dot{m}\left(h_1 - h_4\right) = (2)(95.35 - 90.26) = 10.18 \frac{\text{Btu}}{\text{min}} = 0.05 \text{ ton}$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|106.14 - 112.08\right| = 5.94 \frac{\text{Btu}}{\text{min}}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{10.18}{5.94} = 1.71$$

TEST Solution and What-if Scenario: Use IG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-9 [OVI] An ideal Brayton refrigeration cycle has a compressor pressure ratio of 6. At the compressor inlet, the pressure and temperature of the entering air are 55 lbf/in^2 and 600^{o}R. The temperature at the exit of the turbine is 370^{o}R. For a refrigerating capacity of 15 tons, determine (a) the net power input in Btu/min, (b) the coefficient of performance (COP_R) and (c) the specific volumes of the air at the compressor and turbine inlets, each in ft^3/lb.

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 143.33 \frac{\text{Btu}}{\text{lbm}}; \;\; s_1 = 1.5803 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}}$$

$$v_1 = 4.04064 \frac{\text{ft}^3}{\text{lbm}}$$

State-2 (given $s_2 = s_1, r$):

$$h_2 = 239.16 \frac{\text{Btu}}{\text{lbm}}$$

State-3 (given $p_3 = p_2, s_3 = s_4$):

$$h_3 = 147.52 \frac{\text{Btu}}{\text{lbm}}$$

$$v_3 = 0.69294 \frac{\text{ft}^3}{\text{lbm}}$$

State-4 (given $p_4 = p_1, T_4$):

$$h_4 = 88.16 \frac{\text{Btu}}{\text{lbm}}; \;\; s_4 = 1.4643 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $w_C = h_2 - h_1 = 239.16 - 143.33 = 95.83 \frac{\text{Btu}}{\text{lbm}}$

Device-B (2-3): $q_H = h_2 - h_3 = 239.16 - 147.52 = 91.64 \frac{\text{Btu}}{\text{lbm}}$

Device-C (3-4): $w_T = h_3 - h_4 = 147.52 - 88.16 = 59.36 \frac{\text{Btu}}{\text{lbm}}$

Device-D (4-1): $q_C = h_1 - h_4 = 143.33 - 88.16 = 55.17 \frac{\text{Btu}}{\text{lbm}}$

Find the mass flow from the refrigeration capacity

$$\dot{m} = \frac{\dot{Q}_C}{q_C} = \frac{3000}{55.17} = 54.38 \frac{\text{lbm}}{\text{min}}$$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{m}\left(w_T - w_C\right)\right| = \left|(54.38)(59.36-95.83)\right| = 1983.24\ \frac{\text{Btu}}{\text{min}}$$

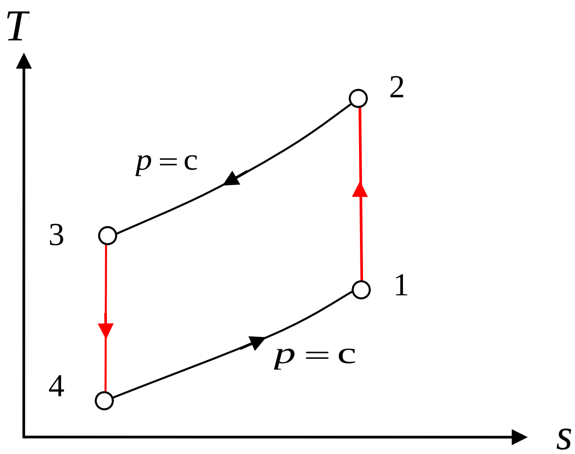
Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{3000}{1983.24} = 1.51$$

TEST Solution: Use IG gas-compression cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-10 [OVL] Air enters the compressor of an ideal Brayton refrigeration cycle at 120 kPa and 275 K. The compressor pressure ratio is 3, and the temperature at the turbine inlet is 325 K. Treating air as a perfect gas, determine (a) the net work input per unit mass of air flow (w_{net}), in kJ/kg, (b) the refrigeration capacity per unit mass of air flow, in kJ/kg and (c) the coefficient of performance (COP_R). (d) What-if Scenario: What would the COP be if the IG model were used for air?

SOLUTION



Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $s_2 = s_1, r$):

$$T_2 = T_1 r^{\frac{k-1}{k}} = (275)(3)^{\frac{1.4-1}{1.4}} = 376.40 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}} = (325)\left(\frac{120}{360}\right)^{\frac{1.4-1}{1.4}} = 237.45 \text{ K}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $w_C = c_p (T_2 - T_1) = (1.005)(376.40 - 275) = 101.91 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_H = c_p\left(T_2 - T_3\right) = (1.005)(376.40 - 325) = 51.66\,\dfrac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_T = c_p\left(T_3 - T_4\right) = (1.005)(325 - 237.45) = 87.99\,\dfrac{\text{kJ}}{\text{kg}}$

Device-D (4-1): $q_C = c_p\left(T_1 - T_4\right) = (1.005)(275 - 237.45) = 37.74\,\dfrac{\text{kJ}}{\text{kg}}$

The magnitude of net work input per unit mass

$$|w_{\text{net}}| = |w_T - w_C| = |87.99 - 101.91| = 13.92\,\frac{\text{kJ}}{\text{kg}}$$

Therefore, the COP can be obtained as

$$\text{COP}_{\text{R}} = \frac{q_C}{w_{\text{net}}} = \frac{37.74}{13.92} = 2.71$$

TEST Solution and What-if Scenario: Use PG and IG gas-compression cycle TESTcalcs to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-11 [OVZ] A gas refrigeration system uses helium as the working fluid operates with a pressure ratio of 3.5. The temperature of the helium is -10°C at the compressor inlet and 50°C at the turbine inlet. Assuming an adiabatic efficiency of 80% for both the compressor and the turbine, determine (a) the minimum temperature of the cycle, (b) mass flow rate for a refrigeration rate of 1 ton and (c) the COP. (d) What-if Scenario: What would the COP be if the adiabatic efficiency increased to 85%?

SOLUTION

State-1 (given p_1, T_1):

Assume an inlet pressure of 1 kPa

$$h_1 = -181.87\frac{\text{kJ}}{\text{kg}};\ s_1 = 40.4883\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $s_2 = s_1, r$):

$$h_2 = 707.66\frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = -181.87 + \frac{707.66 + 181.87}{0.80} = 930.04\frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$$h_4 = 129.92\frac{\text{kJ}}{\text{kg}};\ s_4 = 38.9518\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$h_5 = -531.92\frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$h_6 = h_4 - \eta_T\left(h_4 - h_5\right) = 129.92 - (0.80)(129.92 + 531.92) = -399.55\frac{\text{kJ}}{\text{kg}}$$

$$T_6 = -51.89°\text{C}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = h_3 - h_1 = 930.04 + 181.87 = 1111.91\frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_H = h_3 - h_4 = 930.04 - 129.92 = 800.12\frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_T = h_4 - h_6 = 129.92 + 399.55 = 529.47 \frac{\text{kJ}}{\text{kg}}$

Device-D (6-1): $q_C = h_1 - h_6 = -181.87 + 399.55 = 217.68 \frac{\text{kJ}}{\text{kg}}$

The mass flow to provide the required cooling load

$$\dot{m} = \frac{\dot{Q}_C}{q_C} = \frac{3.52}{217.68} = 0.016 \frac{\text{kg}}{\text{s}}$$

The magnitude of net work input per unit mass

$$|w_{\text{net}}| = |w_T - w_C| = |529.47 - 1111.91| = 582.44 \frac{\text{kJ}}{\text{kg}}$$

Therefore, the COP can be obtained as

$$\text{COP}_{\text{R}} = \frac{q_C}{w_{\text{net}}} = \frac{217.68}{582.44} = 0.37$$

TEST Solution and What-if Scenario: Use IG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-12 [OVP] In problem 10-2-9 [OVI] consider that the compressor and turbine each has an isentropic efficiency of 85%. Determine for the modified cycle, (a) the mass flow rate of air, in lb/s and (b) the coefficient of performance (COP_R). (c) What-if Scenario: Do a parametric study of how the COP would change if the isentropic efficiency varied from 50% to 100%.

SOLUTION

State-1 (given p_1, T_1):

$$h_1 = 143.33 \frac{\text{Btu}}{\text{lbm}};\ s_1 = 1.5803 \frac{\text{Btu}}{\text{lbm}\cdot\text{R}}$$

State-2 (given $s_2 = s_1, r$):

$$h_2 = 239.16 \frac{\text{Btu}}{\text{lbm}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 143.33 + \frac{239.16 - 143.33}{0.85} = 256.07 \frac{\text{Btu}}{\text{lbm}\cdot\text{R}}$$

State-4 (given $p_4 = p_2, s_4 = s_5$):

$$h_4 = 147.52 \frac{\text{Btu}}{\text{lbm}}$$

State-5 (given $p_5 = p_1, T_5$):

$$h_5 = 88.16 \frac{\text{Btu}}{\text{lbm}};\ s_5 = 1.4643 \frac{\text{Btu}}{\text{lbm}\cdot\text{R}}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$h_6 = h_4 - \eta_T (h_4 - h_5) = 147.52 - (0.85)(147.52 - 88.16) = 97.06 \frac{\text{Btu}}{\text{lbm}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = h_3 - h_1 = 256.07 - 143.33 = 112.74 \frac{\text{Btu}}{\text{lbm}}$

Device-B (3-4): $q_H = h_3 - h_4 = 239.16 - 147.52 = 108.55 \frac{\text{Btu}}{\text{lbm}}$

Device-C (4-6): $w_T = h_4 - h_6 = 147.52 - 97.06 = 50.46 \frac{\text{Btu}}{\text{lbm}}$

Device-D (6-1): $q_C = h_1 - h_6 = 143.33 - 97.06 = 46.27 \frac{\text{Btu}}{\text{lbm}}$

Find the mass flow from the refrigeration capacity

$$\dot{m} = \frac{\dot{Q}_C}{q_C} = \frac{3000}{46.27} = 64.84\,\frac{\text{lbm}}{\text{min}} = 1.08\,\frac{\text{lbm}}{\text{s}}$$

The magnitude of net work input per unit mass

$$\left|w_{\text{net}}\right| = \left|w_T - w_C\right| = \left|50.46 - 112.74\right| = 62.28\,\frac{\text{Btu}}{\text{min}}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{q_C}{w_{\text{net}}} = \frac{46.27}{62.28} = 0.74$$

TEST Solution and What-if Scenario: Use IG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-13 [OVK] In 10-2-10 [OVL] consider that the compressor and turbine have isentropic efficiencies of 80% and 90%, respectively. Determine for the modified cycle, (a) the coefficient of performance and (b) the irreversibility rates, per unit mass of air flow, in the compressor and turbine, each in kJ/kg, for T_0 = 300 K.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $s_2 = s_1, r$):

$$T_2 = T_1 r^{\frac{k-1}{k}} = (275)(3)^{\frac{1.4-1}{1.4}} = 376.40 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 275 + \frac{376.40 - 275}{0.80} = 401.75 \text{ K}$$

State-4 (given $p_4 = p_2, T_4$)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = T_4 \left(\frac{p_5}{p_4}\right)^{\frac{k-1}{k}} = (325)\left(\frac{120}{360}\right)^{\frac{1.4-1}{1.4}} = 237.45 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 325 - (0.90)(325 - 237.45) = 246.21 \text{ K}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = c_p (T_3 - T_1) = (1.005)(401.75 - 275) = 127.38 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_H = c_p (T_3 - T_4) = (1.005)(401.75 - 325) = 77.13 \frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_T = c_p (T_4 - T_6) = (1.005)(325 - 246.21) = 79.18 \frac{\text{kJ}}{\text{kg}}$

Device-D (6-1): $q_C = c_p\left(T_1 - T_6\right) = (1.005)(275 - 246.21) = 28.93\,\dfrac{\text{kJ}}{\text{kg}}$

The magnitude of net work input per unit mass

$$\left|w_{\text{net}}\right| = \left|w_T - w_C\right| = \left|79.18 - 127.38\right| = 48.20\,\frac{\text{kJ}}{\text{kg}}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{q_C}{w_{\text{net}}} = \frac{28.93}{48.20} = 0.60$$

The irreversibility generation rate in the compressor per unit mass

$$\cancel{\frac{ds}{dt}}^{\,0} = s_1 - s_3 + \cancel{\frac{q}{T_B}}^{\,0} + \dot{s}_{\text{gen}};$$

$$\Rightarrow \dot{s}_{\text{gen}} = s_3 - s_1;$$

$$\Rightarrow \dot{s}_{\text{gen}} = c_p \ln\frac{T_3}{T_1} - R\ln\frac{p_3}{p_1} = (1.005)\ln\left(\frac{401.75}{275}\right) - (0.287)\ln\left(\frac{360}{120}\right) = 0.0657\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

The irreversibility generation rate in the turbine per unit mass

$$\cancel{\frac{ds}{dt}}^{\,0} = s_4 - s_6 + \cancel{\frac{q}{T_B}}^{\,0} + \dot{s}_{\text{gen}};$$

$$\Rightarrow \dot{s}_{\text{gen}} = s_6 - s_4;$$

$$\Rightarrow \dot{s}_{\text{gen}} = c_p \ln\frac{T_6}{T_4} - R\ln\frac{p_6}{p_4} = (1.005)\ln\left(\frac{246.21}{325}\right) - (0.287)\ln\left(\frac{120}{360}\right) = 0.0363\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

TEST Solution: Use PG gas-compression cycle TESTcalcs to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-14 [OVU] A gas refrigeration cycle with a pressure ratio of 3 uses helium as the working fluid. The temperature of the helium is -15°C at the compressor inlet at 50°C at the turbine inlet. Assuming adiabatic efficiencies of 85% for both the turbine and the compressor, determine (a) the minimum temperature in the cycle, (b) the coefficient of performance (COP_R) and (c) the mass flow rate of the helium for a refrigeration rate of 10 kW. (d) What-if Scenario: What would the COP be if the turbine inlet temperature were increased to 60°C?

SOLUTION

State-1 (given p_1, T_1):
Assume an inlet pressure of 1 kPa

$$h_1 = -207.86 \frac{\text{kJ}}{\text{kg}}; \; s_1 = 40.3886 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-2 (given $s_2 = s_1, r$):

$$h_2 = 532.39 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = -207.86 + \frac{532.39 + 207.86}{0.85} = 663.02 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$$h_4 = 129.92 \frac{\text{kJ}}{\text{kg}}; \; s_4 = 39.2722 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$h_5 = -467.21 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$h_6 = h_4 - \eta_T (h_4 - h_5) = 129.92 - (0.85)(129.92 + 467.21) = -377.64 \frac{\text{kJ}}{\text{kg}}$$

$$T_6 = -47.67°\text{C}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-3): $w_C = h_3 - h_1 = 663.02 + 207.86 = 870.88 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_H = h_3 - h_4 = 663.02 - 129.92 = 533.10 \frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_T = h_4 - h_6 = 129.92 + 377.64 = 507.56 \dfrac{\text{kJ}}{\text{kg}}$

Device-D (6-1): $q_C = h_1 - h_6 = -207.86 + 377.64 = 169.78 \dfrac{\text{kJ}}{\text{kg}}$

The mass flow to provide the required cooling load

$$\dot{m} = \frac{\dot{Q}_C}{q_C} = \frac{10}{169.78} = 0.059 \frac{\text{kg}}{\text{s}}$$

The magnitude of net work input per unit mass

$$|w_{\text{net}}| = |w_T - w_C| = |507.56 - 870.88| = 363.32 \frac{\text{kJ}}{\text{kg}}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{q_C}{w_{\text{net}}} = \frac{169.78}{363.32} = 0.47$$

TEST Solution and What-if Scenario: Use IG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-15 [OVV] An ideal gas refrigeration system with a regenerative HX uses air as the working fluid. Air enters the compressor at 270 K, 150 kPa at a flow rate of 0.1 m^3/min and exits at 400 kPa. Compressed air enters the regenerative HX at 300 K and is cooled down to 270 K as the turbine inlet. Determine (a) the cooling capacity, (b) the net power input, and (c) the COP.

SOLUTION

Given:

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-1 (given $p_1, T_1, \dot{V}_1$):

$$h_1 = 269.70 \frac{\text{kJ}}{\text{kg}}; \; s_1 = 6.6708 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\dot{m} = \frac{p_1 \dot{V}_1}{RT_1} = \frac{(150)(0.00167)}{(0.287)(270)} = 0.00323 \frac{\text{kg}}{\text{s}}$$

State-2 (given $p_2, s_2 = s_1$):

$$h_2 = 357.42 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 299.81 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_3, T_4$):

$$h_4 = 269.70 \frac{\text{kJ}}{\text{kg}}; \; s_4 = 6.3893 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$h_5 = 203.43 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5$):

$$h_6 = h_1 - (h_3 - h_4) = 269.70 - (299.81 - 269.70) = 239.59 \frac{\text{kJ}}{\text{kg}}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $\dot{W}_C = \dot{m}(h_2 - h_1) = (0.00323)(357.42 - 269.70) = 0.283 \text{ kW}$

Device-B (2-3): $\dot{Q}_H = \dot{m}\left(h_2 - h_3\right) = \left(0.00323\right)\left(357.42 - 299.81\right) = 0.186 \text{ kW}$

Device-C (4-5): $\dot{W}_T = \dot{m}\left(h_4 - h_5\right) = \left(0.00323\right)\left(269.70 - 203.43\right) = 0.214 \text{ kW}$

Device-D (5-6): $\dot{Q}_C = \dot{m}\left(h_6 - h_5\right) = \left(0.00323\right)\left(239.59 - 203.43\right) = 0.117 \text{ kW}$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{W}_T - \dot{W}_C\right| = \left|0.214 - 0.283\right| = 0.069 \text{ kW}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{0.117}{0.069} = 1.70$$

TEST Solution and What-if Scenario: Use IG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-16 [OVX] A gas refrigeration system uses air as the working fluid has a pressure ratio of 4. Air enters the compressor at -7°C. The high-pressure air is cooled to 30°C by rejecting heat to the surroundings. It is further cooled to -15°C by regenerative cooling before it enters the turbine. Assuming both the turbine and the compressor to be isentropic and using the PG model for air, determine (a) the lowest temperature that can be obtained by this cycle, (b) the coefficient of performance (COP_R) of the cycle and (c) the mass flow rate of air for a refrigeration rate of 12 kW. (d) What-if Scenario: What would the COP be if the pressure ratio were 5?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1):

Assume an inlet pressure of 1 kPa

State-2 (given $s_2 = s_1, r$):

$$T_2 = T_1 r^{\frac{k-1}{k}} = (266.15)(4)^{\frac{1.4-1}{1.4}} = 395.50 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_3, T_4$)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = T_4 \left(\frac{p_5}{p_4}\right)^{\frac{k-1}{k}} = (258.15)\left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 173.72 \text{ K} = -99.43°\text{C}$$

State-6 (given $p_6 = p_5$):

$$T_6 = T_1 - (T_3 - T_4) = 266.15 - (303.15 - 258.15) = 221.15 \text{ K}$$

A steady-state energy analysis is carried out for each device as follows.

Device-A (1-2): $w_C = c_p (T_2 - T_1) = (1.005)(395.50 - 266.15) = 130.00 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_H = c_p (T_2 - T_3) = (1.005)(395.50 - 303.15) = 92.81 \frac{\text{kJ}}{\text{kg}}$

Device-C (4-5): $w_T = c_p\left(T_4 - T_5\right) = (1.005)(258.15 - 173.72) = 84.85\,\frac{\text{kJ}}{\text{kg}}$

Device-D (5-6): $q_C = c_p\left(T_6 - T_5\right) = (1.005)(221.15 - 173.72) = 47.67\,\frac{\text{kJ}}{\text{kg}}$

The mass flow to provide the required cooling load

$$\dot{m} = \frac{\dot{Q}_C}{q_C} = \frac{12}{47.67} = 0.25\,\frac{\text{kg}}{\text{s}}$$

The magnitude of net power input

$$\left|\dot{W}_{\text{net}}\right| = \left|\dot{m}\left(w_T - w_C\right)\right| = \left|(0.25)(84.85 - 130.00)\right| = 11.29\text{ kW}$$

Therefore, the COP can be obtained as

$$\text{COP}_{\text{R}} = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{12}{11.29} = 1.06$$

TEST Solution and What-if Scenario: Use PG gas-compression cycle TESTcalc to verify this answer and explore the what-if scenario. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).

10-2-17 [OVC] In problem 10-2-16 [OVX] evaluate the effect of regeneration on the COP by changing the turbine inlet temperature to (a) -10^oC, (b) 0^oC, (c) 10^oC and (d) 20^oC.

SOLUTION

Use the PG vapor-compression cycle TESTcalc to find the effect of regeneration on the COP due to changing turbine inlet temperatures. Then input the values from the TESTcalc into an Excel spreadsheet to generate a plot of the parametric study.

10-2-18 [OVQ] Helium undergoes a Stirling refrigeration cycle, which is a reverse Stirling power cycle. At the beginning of isothermal compression helium is at 100 kPa, 275 K. The compression ratio is 4 and during isothermal expansion the temperature is 150 K. Determine per kg of helium, (a) the net work per cycle, (b) the heat transfer during isothermal expansion, and (c) the COP.

SOLUTION

Given:

$$R = 2.079 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-1 (given p_1, T_1):

$$h_1 = -120.30 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $T_2 = T_1, r$):

$$h_2 = -120.30 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $T_3, v_3 = v_2$):

$$h_3 = -769.86 \frac{\text{kJ}}{\text{kg}}$$

$$p_3 = 218.18 \text{ kPa}$$

State-4 (given $T_4 = T_3, v_4 = v_1$):

$$h_4 = -769.86 \frac{\text{kJ}}{\text{kg}}$$

$$p_4 = 54.55 \text{ kPa}$$

A steady-state energy analysis is carried out for the compression and expansion strokes.

Process-A (1-2): $w_C = q_H = \left| RT \ln \frac{p_1}{p_2} \right| = \left| (2.079)(275) \ln \left(\frac{100}{400} \right) \right| = 792.58 \frac{\text{kJ}}{\text{kg}}$

Process-B (3-4): $w_T = q_C = \left| RT \ln \frac{p_3}{p_4} \right| = \left| (2.079)(150) \ln \left(\frac{218.18}{54.55} \right) \right| = 432.29 \frac{\text{kJ}}{\text{kg}}$

The regeneration process

$$q_{\text{reg}} = h_2 - h_3 = -120.30 + 769.86 = 649.56 \frac{\text{kJ}}{\text{kg}}$$

The magnitude of net work input per unit mass

$$\left|w_{\text{net}}\right| = \left|w_T - w_C\right| = \left|432.29 - 792.58\right| = 360.29\,\frac{\text{kJ}}{\text{kg}}$$

Therefore, the COP can be obtained as

$$\text{COP}_\text{R} = \frac{q_C}{w_{\text{net}}} = \frac{432.29}{360.29} = 1.20$$

TEST Solution: Use IG gas-compression cycle TESTcalc to verify this answer. TEST-code for this problem can be found in thermofluids.net (the professional site of TEST).