8-1-1 [OZA] Air enters the compressor of an ideal air standard Brayton cycle at 100 kPa, 25°C, with a volumetric flow rate of 8 m^3/s. The compressor pressure ratio is 12. The turbine inlet temperature is 1100°C. Determine (a) the thermal efficiency (η_{th}), (b) net power output and (c) back work ratio. Use the PG model for air. (d) **What-if Scenario:** What would the answers be if the IG model were used?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given $p_1, T_1, \dot{V}_1$):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(298)}{100} = 0.8553 \frac{\text{m}^3}{\text{kg}}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{8}{0.8553} = 9.35 \frac{\text{kg}}{\text{s}}$$

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (298)(12)^{(1.4-1)/1.4} = 606.11 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = \frac{T_3}{r_p^{(k-1)/k}} = \frac{1373}{12^{(1.4-1)/1.4}} = 675.04 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $\dot{W}_C = \dot{m} c_p (T_2 - T_1) = (9.35)(1.005)(606.11 - 298) = 2895.23 \text{ kW}$

Device-B (2-3): $\dot{Q}_{\text{in}} = \dot{m} c_p (T_3 - T_2) = (9.35)(1.005)(1373 - 606.11) = 7206.27 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m} c_p (T_3 - T_4) = (9.35)(1.005)(1373 - 675.04) = 6558.56 \text{ kW}$

Device-D (4-1): $\dot{Q}_{\text{out}} = \dot{m} c_p (T_4 - T_1) = (9.35)(1.005)(675.04 - 298) = 3542.95 \text{ kW}$

The net power is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$$

$$\Rightarrow \dot{W}_{\text{net}} = 6558.56 - 2895.23;$$

$$\Rightarrow \dot{W}_{\text{net}} = 3663.33 \text{ kW}$$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{\dot{W}_{\text{comp}}}{\dot{W}_{\text{turb}}};$$

$$\Rightarrow \text{BWR} = \frac{2895.23}{6558.56};$$

$$\Rightarrow \text{BWR} = 44.1\%$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$

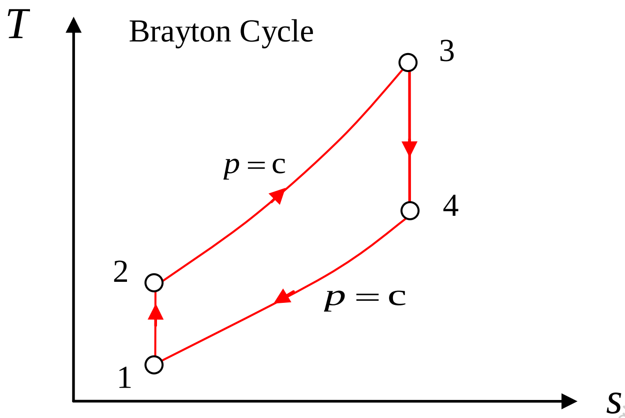
$$\Rightarrow \eta_{\text{th}} = \frac{6558.56}{7206.27};$$

$$\Rightarrow \eta_{\text{th}} = 50.8\ \%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-2 [OZH] A stationary power plant operating on an ideal Brayton cycle has a pressure ratio of 7. The gas temperature is 25°C at the compressor inlet and 1000°C at the turbine inlet. Utilizing the air standard assumptions, determine (a) the gas temperature (T) at the exits of the compressor, (b) back work ratio and (c) the thermal efficiency ($\eta_{\text{th,Brayton}}$). Use the PG model.

SOLUTION



Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(298)}{100} = 0.8553 \frac{\text{m}^3}{\text{kg}}$$

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (298)(7)^{(1.4-1)/1.4} = 519.6 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = \frac{T_3}{r_p^{(k-1)/k}} = \frac{1273}{7^{(1.4-1)/1.4}} = 730 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p\left(T_2 - T_1\right) = (1.005)(519.6 - 298) = 221.6\,\frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = c_p\left(T_3 - T_2\right) = (1.005)(1273 - 519.6) = 757.2\,\frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_T = c_p\left(T_3 - T_4\right) = (1.005)(1273 - 730) = 545.7\,\frac{\text{kJ}}{\text{kg}}$

Device-D (4-1): $q_{\text{out}} = c_p\left(T_4 - T_1\right) = (1.005)(730 - 298) = 434.2\,\frac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$w_{\text{net}} = w_T - w_C;$

$\Rightarrow w_{\text{net}} = 545.7 - 221.6;$

$\Rightarrow w_{\text{net}} = 324.1\,\frac{\text{kJ}}{\text{kg}}$

The thermal efficiency and the back work ratio are

$\text{BWR} = \frac{w_C}{w_T};$

$\Rightarrow \text{BWR} = \frac{221.6}{545.7};$

$\Rightarrow \text{BWR} = 40.6\%$

$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$

$\Rightarrow \eta_{\text{th}} = \frac{324.1}{757.2};$

$\Rightarrow \eta_{\text{th}} = 42.8\%$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-3 [OZN] In an air standard Brayton cycle the air enters the compressor at 0.1 MPa and 20^{o}C. The pressure leaving the compressor is 1 MPa, and the maximum temperature in the cycle is 1225^{o}C. Determine (a) the rate of compressor work per unit mass (w_C), (b) rate of turbine work per unit mass (w_T), and (c) the cycle efficiency ($\eta_{\text{th,Brayton}}$). Use the PG model. (d) **What-if Scenario:** What would the answers be if the compressor exit pressure were 1.5 MPa?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $p_2, s_2 = s_1$):

$$r_p = \frac{p_2}{p_1} = \frac{1}{0.1} = 10$$

$$T_2 = T_1 r_p^{(k-1)/k} = (293)(10)^{(1.4-1)/1.4} = 565.69 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = \frac{T_3}{r_p^{(k-1)/k}} = \frac{1498}{10^{(1.4-1)/1.4}} = 775.89 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p(T_2 - T_1) = (1.005)(565.69 - 293) = 274.05 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = c_p(T_3 - T_2) = (1.005)(1498 - 565.69) = 936.97 \frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_T = c_p(T_3 - T_4) = (1.005)(1498 - 775.89) = 725.72 \frac{\text{kJ}}{\text{kg}}$

Device-D (4-1): $q_{\text{out}} = c_p(T_4 - T_1) = (1.005)(775.89 - 293) = 485.30 \frac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$$w_{\text{net}} = w_T - w_C;$$
$$\Rightarrow w_{\text{net}} = 725.72 - 274.05;$$
$$\Rightarrow w_{\text{net}} = 451.67 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$
$$\Rightarrow \eta_{\text{th}} = \frac{451.67}{936.97};$$
$$\Rightarrow \eta_{\text{th}} = 48.2\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-4 [OZE] Air (use the PG model) enters the compressor of an ideal air standard Brayton cycle at 100 kPa and 305 K with a volumetric flow rate of 5 m^3/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1000 K. Determine (a) the thermal efficiency ($\eta_{\text{th,Brayton}}$), (b) net power output and (c) back work ratio. Use the PG model for air.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given $p_1, T_1, \dot{V}_1$):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(305)}{100} = 0.8754 \frac{\text{m}^3}{\text{kg}}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{5}{0.8754} = 5.712 \frac{\text{kg}}{\text{s}}$$

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (305)(10)^{(1.4-1)/1.4} = 588.86 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = \frac{T_3}{r_p^{(k-1)/k}} = \frac{1000}{10^{(1.4-1)/1.4}} = 517.95 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $\dot{W}_C = \dot{m}c_p(T_2 - T_1) = (5.712)(1.005)(588.86 - 305) = 1629.52 \text{ kW}$

Device-B (2-3): $\dot{Q}_{\text{in}} = \dot{m}c_p(T_3 - T_2) = (5.712)(1.005)(1000 - 588.86) = 2360.17 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}c_p(T_3 - T_4) = (5.712)(1.005)(1000 - 517.95) = 2767.24 \text{ kW}$

Device-D (4-1): $\dot{Q}_{\text{out}} = \dot{m}c_p(T_4 - T_1) = (5.712)(1.005)(517.95 - 305) = 1222.45 \text{ kW}$

The net power is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$$
$$\Rightarrow \dot{W}_{\text{net}} = 2767.24 - 1629.52;$$
$$\Rightarrow \dot{W}_{\text{net}} = 1137.72 \text{ kW}$$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{\dot{W}_C}{\dot{W}_T};$$
$$\Rightarrow \text{BWR} = \frac{1629.52}{2767.24};$$
$$\Rightarrow \text{BWR} = 58.9\%$$
$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$
$$\Rightarrow \eta_{\text{th}} = \frac{1137.72}{2360.17};$$
$$\Rightarrow \eta_{\text{th}} = 48.2\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-5 [OZI] A gas turbine power plant operates on a simple Brayton cycle with air (use the PG model) as the working fluid. The air enters the turbine at 1 MPa and 1000 K and leaves at 125 kPa, 610 K. Heat is rejected to the surroundings at a rate of 8000 kW and air flow rate is 25 kg/s. Assuming a compressor efficiency of 80%, determine (a) the net power output. Use the PG model for air. (b)**What-if Scenario:** What would the net power output be if the compressor efficiency dropped to 75%?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given $p_1 = p_5, \dot{m}, \dot{Q}_{\text{out}}$):

$$T_1 = T_5 - \frac{\dot{Q}_{\text{out}}}{\dot{m}c_p} = 610 - \frac{8000}{(25)(1.005)} = 291.59 \text{ K}$$

State-2 (given $p_2 = p_4, s_2 = s_1$):

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (291.59)\left(\frac{1000}{125}\right)^{(1.4-1)/1.4} = 528.20 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 291.59 + \frac{528.20 - 291.59}{0.80} = 587.35 \text{ K}$$

State-4 (given p_4, T_4)

State-5 (given p_5, T_5)

An energy analysis is carried out for each device as follows

Device-A (1-3): $\dot{W}_C = \dot{m}c_p(T_3 - T_1) = (25)(1.005)(587.35 - 291.59) = 7430.97 \text{ kW}$

Device-B (3-4): $\dot{Q}_{\text{in}} = \dot{m}c_p(T_4 - T_3) = (25)(1.005)(1000 - 587.35) = 10367.83 \text{ kW}$

Device-C (4-5): $\dot{W}_T = \dot{m}c_p(T_4 - T_5) = (25)(1.005)(1000 - 610) = 9798.75 \text{ kW}$

Device-D (5-1): $\dot{Q}_{\text{out}} = 8000 \text{ kW}$

The net power is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$$
$$\Rightarrow \dot{W}_{\text{net}} = 9798.75 - 7430.97;$$
$$\Rightarrow \dot{W}_{\text{net}} = 2367.78 \text{ kW}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-6 [OZG] Repeat problem 8-1-4 [OZE] assuming a compressor efficiency of 80% and a turbine efficiency of 90%, and determine (a) back work ratio, (b) the thermal efficiency ($\eta_{\text{th,Brayton}}$) and (c) the turbine exit temperature. Use the PG model.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given $p_1, T_1, \dot{V}_1$):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(305)}{100} = 0.8754 \frac{\text{m}^3}{\text{kg}}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{5}{0.8754} = 5.712 \frac{\text{kg}}{\text{s}}$$

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (305)(10)^{(1.4-1)/1.4} = 588.86 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 305 + \frac{588.86 - 305}{0.80} = 659.83 \text{ K}$$

State-4 (given $p_4 = p_2, T_4$)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = \frac{T_4}{r_p^{(k-1)/k}} = \frac{1000}{10^{(1.4-1)/1.4}} = 517.95 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 1000 - (0.90)(1000 - 517.95) = 566.16 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-3): $\dot{W}_C = \dot{m} c_p (T_3 - T_1) = (5.712)(1.005)(659.83 - 305) = 2036.92 \text{ kW}$

Device-B (3-4): $\dot{Q}_{\text{in}} = \dot{m} c_p (T_4 - T_3) = (5.712)(1.005)(1000 - 659.83) = 1952.77 \text{ kW}$

Device-C (4-6): $\dot{W}_T = \dot{m} c_p (T_4 - T_6) = (5.712)(1.005)(1000 - 566.16) = 2490.48 \text{ kW}$

Device-D (6-1): $\dot{Q}_{\text{out}} = \dot{m}c_p\left(T_6 - T_1\right) = (5.712)(1.005)(566.16 - 305) = 1499.20\text{ kW}$

The net power is

$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$

$\Rightarrow \dot{W}_{\text{net}} = 2490.48 - 2036.92;$

$\Rightarrow \dot{W}_{\text{net}} = 453.56\text{ kW}$

The thermal efficiency and the back work ratio are

$\text{BWR} = \dfrac{\dot{W}_C}{\dot{W}_T};$

$\Rightarrow \text{BWR} = \dfrac{2036.92}{2490.48};$

$\Rightarrow \text{BWR} = 81.8\%$

$\eta_{\text{th}} = \dfrac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$

$\Rightarrow \eta_{\text{th}} = \dfrac{453.56}{1952.77};$

$\Rightarrow \eta_{\text{th}} = 23.2\%$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-7 [OZZ] In a air standard Brayton cycle the air enters the compressor at 0.1 MPa, 20^{o}C. The pressure leaving the compressor is 1 MPa, and the maximum temperature in the cycle is 1225^{o}C. Assume a compressor efficiency of 80%, a turbine efficiency of 85% and a pressure drop between the compressor and turbine of 25 kPa. Determine (a) the compressor work per unit mass (w_{comp}), (b) the turbine work per unit mass (w_{turb}) and (c) the cycle efficiency ($\eta_{th,Brayton}$). Use the PG model.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(293)}{100} = 0.8409 \frac{\text{m}^3}{\text{kg}}$$

State-2 (given $p_2, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (293)(10)^{(1.4-1)/1.4} = 565.69 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$)

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 293 + \frac{565.69 - 293}{0.80} = 633.86 \text{ K}$$

State-4 (given $p_4 = p_2 - 25, T_4$)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = T_4 \left(\frac{p_5}{p_4}\right)^{(k-1)/k} = (1498)\left(\frac{100}{975}\right)^{(1.4-1)/1.4} = 781.52 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 1498 - (0.85)(1498 - 781.52) = 888.99 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-3): $w_C = c_p (T_3 - T_1) = (1.005)(633.86 - 293) = 342.56 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in}} = c_p\left(T_4 - T_3\right) = (1.005)(1498 - 633.86) = 868.46\,\dfrac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_T = c_p\left(T_4 - T_6\right) = (1.005)(1498 - 888.99) = 612.06\,\dfrac{\text{kJ}}{\text{kg}}$

Device-D (6-1): $q_{\text{out}} = c_p\left(T_6 - T_1\right) = (1.005)(888.99 - 293) = 598.97\,\dfrac{\text{kJ}}{\text{kg}}$

The net power is

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 612.06 - 342.56;$$

$$\Rightarrow w_{\text{net}} = 269.50\,\frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{269.50}{868.46};$$

$$\Rightarrow \eta_{\text{th}} = 31.0\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-8 [OZL] Air enters the compressor of an ideal air standard Brayton cycle at 195 kPa, 290 K, with a volumetric flow rate of 6 m^3/s. The compressor pressure ratio is 9. The turbine inlet temperature is 1400 K. The compressor has an efficiency of 90% and the turbine has an efficiency of 75%. Determine (a) the thermal efficiency ($\eta_{\text{th,Brayton}}$), (b) net power output ($\dot{W}_{\text{net}}$) and (c) back work ratio. Use the PG model for air.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given $p_1, T_1, \dot{V}_1$):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(290)}{195} = 0.4268 \frac{\text{m}^3}{\text{kg}}$$

$$\dot{m} = \frac{\dot{V}}{v_1} = \frac{6}{0.4268} = 14.1 \frac{\text{kg}}{\text{s}}$$

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (290)(9)^{(1.4-1)/1.4} = 543.30 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 290 + \frac{543.30 - 290}{0.90} = 571.44 \text{ K}$$

State-4 (given $p_4 = p_2, T_4$)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = \frac{T_4}{r_p^{(k-1)/k}} = \frac{1400}{9^{(1.4-1)/1.4}} = 747.23 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$)

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 1400 - (0.75)(1400 - 747.23) = 910.42 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-3): $\dot{W}_C = \dot{m} c_p (T_3 - T_1) = (14.1)(1.005)(571.44 - 290) = 3988.15 \text{ kW}$

Device-B (3-4): $\dot{Q}_{\text{in}} = \dot{m}c_p\left(T_4 - T_3\right) = (14.1)(1.005)(1400 - 571.44) = 11741.11\text{ kW}$

Device-C (4-6): $\dot{W}_T = \dot{m}c_p\left(T_4 - T_6\right) = (14.1)(1.005)(1400 - 910.42) = 6937.59\text{ kW}$

Device-D (6-1): $\dot{Q}_{\text{out}} = \dot{m}c_p\left(T_6 - T_1\right) = (14.1)(1.005)(910.42 - 290) = 8791.66\text{ kW}$

The net power is

$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$

$\Rightarrow \dot{W}_{\text{net}} = 6937.59 - 3988.15;$

$\Rightarrow \dot{W}_{\text{net}} = 2949.44\text{ kW}$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{\dot{W}_C}{\dot{W}_T};$$

$$\Rightarrow \text{BWR} = \frac{3988.15}{6937.59};$$

$$\Rightarrow \text{BWR} = 57.5\%$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{2949.44}{11741.11};$$

$$\Rightarrow \eta_{\text{th}} = 25.1\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-9 [OZK] A gas turbine power plant operates on a simple Brayton cycle with air as the working fluid. Air enters the turbine at 800 kPa and 1200 K, and it leaves the turbine at 100 kPa and 750 K. Heat is rejected to the surroundings at a rate of 6800 kW, and air flows through the cycle at a rate of 20 kg/s. Assuming a compressor efficiency of 80%, determine (a) net power output ($\dot{W}_{net}$) of the plant. Use the PG model.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given $p_1 = p_5, \dot{m}, \dot{Q}_{\text{out}}$):

$$T_1 = T_6 - \frac{\dot{Q}_{\text{out}}}{\dot{m}c_p} = 750 - \frac{6800}{(20)(1.005)} = 411.69 \text{ K}$$

State-2 (given $p_2 = p_4, s_2 = s_1$):

$$T_2 = T_1 r_p^{(k-1)/k} = (411.69)(8)^{(1.4-1)/1.4} = 745.75 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 411.69 + \frac{745.75 - 411.69}{0.80} = 829.27 \text{ K}$$

State-4 (given p_4, T_4)

State-5 (given $p_5, s_5 = s_4$):

$$r_p = \frac{p_4}{p_5} = \frac{800}{100} = 8$$

State-6 (given $p_6 = p_5, T_6$)

An energy analysis is carried out for each device as follows

Device-A (1-3): $\dot{W}_C = \dot{m}c_p (T_3 - T_1) = (20)(1.005)(829.27 - 411.69) = 8393.36 \text{ kW}$

Device-B (3-4): $\dot{Q}_{\text{in}} = \dot{m}c_p (T_4 - T_3) = (20)(1.005)(1200 - 829.27) = 7451.67 \text{ kW}$

Device-C (4-6): $\dot{W}_T = \dot{m}c_p (T_4 - T_6) = (20)(1.005)(1200 - 750) = 9045 \text{ kW}$

Device-D (6-1): $\dot{Q}_{\text{out}} = \dot{m}c_p (T_6 - T_1) = (20)(1.005)(750 - 411.69) = 6800.03 \text{ kW}$

The net power is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$$
$$\Rightarrow \dot{W}_{\text{net}} = 9045 - 8393.36;$$
$$\Rightarrow \dot{W}_{\text{net}} = 651.64 \text{ kW}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-10 [OZP] Air is used as the working fluid in a simple ideal Brayton cycle that has a pressure ratio of 12, a compressor inlet temperature of 310 K, and a turbine inlet temperature of 900 K. Determine (a) the required mass flow rate of air for a net power of 25 MW, assuming both the compressor and the turbine have an isentropic efficiency of 90%. Use the PG model. (b) **What-if Scenario:** What would the mass flow rate be if both compressor and the turbine had 100% efficiency?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given T_1)

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (310)(12)^{(1.4-1)/1.4} = 630.52 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 310 + \frac{630.52 - 310}{0.90} = 666.13 \text{ K}$$

State-4 (given T_4)

State-5 (given $s_5 = s_4$):

$$T_5 = \frac{T_4}{r_p^{(k-1)/k}} = \frac{900}{12^{(1.4-1)/1.4}} = 442.49 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 900 - (0.90)(900 - 442.49) = 488.24 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-3): $w_C = c_p (T_3 - T_1) = (1.005)(666.13 - 310) = 357.91 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in}} = c_p (T_4 - T_3) = (1.005)(900 - 666.13) = 235.04 \frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_T = c_p (T_4 - T_6) = (1.005)(900 - 488.24) = 413.82 \frac{\text{kJ}}{\text{kg}}$

Device-D (6-1): $q_{\text{out}} = c_p\left(T_6 - T_1\right) = (1.005)(488.24 - 310) = 179.13\dfrac{\text{kJ}}{\text{kg}}$

The mass flow can now be obtained since the net power is given.

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 413.82 - 357.91;$$

$$\Rightarrow w_{\text{net}} = 55.91\frac{\text{kJ}}{\text{kg}}$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}};$$

$$\Rightarrow \dot{m} = \frac{25000}{55.91};$$

$$\Rightarrow \dot{m} = 447.15\frac{\text{kg}}{\text{s}}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-11 [OZU] A gas turbine power plant operates on the simple Brayton cycle with air as the working fluid and delivers 10 MW of power. The minimum and maximum temperatures in the cycle are 300 K and 1100 K, and the pressure of air at the compressor exit is 9 times the value at the compressor inlet. Assuming an adiabatic efficiency of 80% for the compressor and 90% for the turbine, determine (a) the mass flow rate of air through the cycle. Use the PG model. (b) **What-if Scenario:** What would the mass flow rate be if the IG model were used?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (300)(9)^{(1.4-1)/1.4} = 562.03 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 300 + \frac{562.03 - 300}{0.80} = 627.54 \text{ K}$$

State-4 (given T_4)

State-5 (given $s_5 = s_4$):

$$T_5 = \frac{T_4}{r_p^{(k-1)/k}} = \frac{1100}{9^{(1.4-1)/1.4}} = 587.15 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 1100 - (0.90)(1100 - 587.15) = 638.44 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-3): $w_C = c_p (T_3 - T_1) = (1.005)(627.54 - 310) = 329.18 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in}} = c_p (T_4 - T_3) = (1.005)(1100 - 627.54) = 474.82 \frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_T = c_p (T_4 - T_6) = (1.005)(1100 - 638.44) = 463.87 \frac{\text{kJ}}{\text{kg}}$

Device-D (6-1): $q_{\text{out}} = c_p\left(T_6 - T_1\right) = (1.005)(638.44 - 300) = 340.13\dfrac{\text{kJ}}{\text{kg}}$

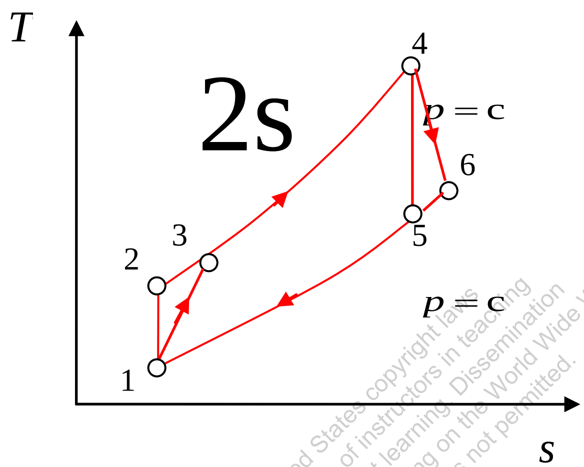
The mass flow can now be obtained since the net power is given.

$$w_{\text{net}} = w_T - w_C;$$
$$\Rightarrow w_{\text{net}} = 463.87 - 329.18;$$
$$\Rightarrow w_{\text{net}} = 134.69\frac{\text{kJ}}{\text{kg}}$$
$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}};$$
$$\Rightarrow \dot{m} = \frac{10000}{134.69};$$
$$\Rightarrow \dot{m} = 74.24\frac{\text{kg}}{\text{s}}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-12 [OZX] Air enters the compressor of a simple gas turbine at 100 kPa, 25°C, with a volumetric flow rate of 6 m^3/s. The compressor pressure ratio is 10 and its isentropic efficiency is 80%. The turbine inlet the pressure is 100 kPa temperature is 1000°C. The turbine has an isentropic efficiency of 88% and the exit pressure is 100 kPa. On the basis of air standard analysis using the PG model, determine (a) the thermal efficiency (η_{th}) of the cycle, (b) net power developed, in kW. (c) **What-if Scenario:** What would the answers be if the inlet temperature increased to 50°C?

SOLUTION



Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given $p_1, T_1, \dot{V}_1$):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(298)}{100} = 0.8553 \frac{\text{m}^3}{\text{kg}}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{6}{0.8553} = 7.015 \frac{\text{kg}}{\text{s}}$$

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (298)(10)^{(1.4-1)/1.4} = 575.35 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 298 + \frac{575.35 - 298}{0.80} = 644.69 \text{ K}$$

State-4 (given p_4, T_4)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = \frac{T_4}{r_p^{(k-1)/k}} = \frac{1273}{10^{(1.4-1)/1.4}} = 659.35 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 1273 - (0.88)(1273 - 659.35) = 732.99 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-3): $\dot{W}_C = \dot{m}c_p(T_3 - T_1) = (7.015)(1.005)(644.69 - 298) = 2444.19 \text{ kW}$

Device-B (3-4): $\dot{Q}_{\text{in}} = \dot{m}c_p(T_4 - T_3) = (7.015)(1.005)(1273 - 644.69) = 4429.63 \text{ kW}$

Device-C (4-6): $\dot{W}_T = \dot{m}c_p(T_4 - T_6) = (7.015)(1.005)(1273 - 732.99) = 3807.11 \text{ kW}$

Device-D (6-1): $\dot{Q}_{\text{out}} = \dot{m}c_p(T_6 - T_1) = (7.015)(1.005)(732.99 - 298) = 3066.71 \text{ kW}$

The net power is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$$

$$\Rightarrow \dot{W}_{\text{net}} = 3807.11 - 2444.19;$$

$$\Rightarrow \dot{W}_{\text{net}} = 1362.92 \text{ kW}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{1362.92}{4429.63};$$

$$\Rightarrow \eta_{\text{th}} = 30.8\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-13 [OZC] Air enters the compressor of a simple gas turbine at 95 kPa, 310 K, where it is compressed to 800 kPa. Heat is transferred to air in the amount of 1000 kJ/kg before it enters the turbine. For a turbine efficiency of 90%, determine (a) the thermal efficiency (η_{th}) of the cycle, and (b) the fraction of turbine work output used to drive the compressor. Use the PG model for air.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $p_2, s_2 = s_1$):

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (310)\left(\frac{800}{95}\right)^{(1.4-1)/1.4} = 569.84 \text{ K}$$

State-3 (given $p_3 = p_2, q_{in}$):

$$T_3 = T_2 + \frac{q_{in}}{c_p} = 569.84 + \frac{1000}{1.005} = 1564.86 \text{ K}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{(k-1)/k} = (1564.86)\left(\frac{95}{800}\right)^{(1.4-1)/1.4} = 851.30 \text{ K}$$

State-5 (given $p_5 = p_4, \eta_T$):

$$T_5 = T_3 - \eta_T\left(T_3 - T_4\right) = 1564.86 - (0.90)(1564.86 - 851.30) = 922.66 \text{ K}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p\left(T_2 - T_1\right) = (1.005)(569.84 - 310) = 261.14 \dfrac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{in} = 1000 \dfrac{\text{kJ}}{\text{kg}}$

Device-C (3-5): $w_T = c_p\left(T_3 - T_5\right) = (1.005)(1564.86 - 922.66) = 645.41 \dfrac{\text{kJ}}{\text{kg}}$

Device-D (5-1): $q_{out} = c_p\left(T_5 - T_1\right) = (1.005)(922.66 - 310) = 615.72 \dfrac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 645.41 - 261.14;$$

$$\Rightarrow w_{\text{net}} = 384.27 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{w_C}{w_T};$$

$$\Rightarrow \text{BWR} = \frac{261.14}{645.41};$$

$$\Rightarrow \text{BWR} = 40.5\%$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{384.27}{1000};$$

$$\Rightarrow \eta_{\text{th}} = 38.4\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-14 [OZF] Air enters the compressor of a simple gas turbine at 0.1 MPa, 300 K. The pressure ratio is 9 and the maximum temperature is 1000 K. The turbine process is divided into two stages each with a pressure ratio of 3, with intermediate reheating to 1000 K. Determine (a) the cycle efficiency (η_{th}) and (b) the net work output per unit mass (w_{net}). Use the PG model. (c) **What-if Scenario:** What would the answers be if the reheat were eliminated?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (300)(9)^{(1.4-1)/1.4} = 562.03 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_3 / 3, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{(k-1)/k} = (1000)\left(\frac{300}{900}\right)^{(1.4-1)/1.4} = 730.60 \text{ K}$$

State-5 (given $p_5 = p_4, T_5$)

State-6 (given $p_6 = p_1$ and $s_6 = s_5$):

$$T_6 = T_5 \left(\frac{p_6}{p_5}\right)^{(k-1)/k} = (1000)\left(\frac{100}{300}\right)^{(1.4-1)/1.4} = 730.60 \text{ K}$$

An energy analysis is carried out for each device as follows.

Device-A (1-2): $w_C = c_p (T_2 - T_1) = (1.005)(562.03 - 300) = 263.34 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = c_p (T_3 - T_2) = (1.005)(1000 - 562.03) = 440.16 \frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_T = c_p (T_3 - T_4) = (1.005)(1000 - 730.60) = 270.75 \frac{\text{kJ}}{\text{kg}}$

Device-D (4-5): $q_{\text{in}} = c_p (T_5 - T_4) = (1.005)(1000 - 730.60) = 270.75 \frac{\text{kJ}}{\text{kg}}$

Device-E (5-6): $w_T = c_p\left(T_5 - T_6\right) = (1.005)(1000 - 730.60) = 270.75\dfrac{\text{kJ}}{\text{kg}}$

Device-F (6-1): $q_{\text{out}} = c_p\left(T_6 - T_1\right) = (1.005)(730.60 - 300) = 432.75\dfrac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$w_{\text{net}} = w_T - w_C;$

$\Rightarrow w_{\text{net}} = (270.75 + 270.75) - 263.34;$

$\Rightarrow w_{\text{net}} = 278.16\dfrac{\text{kJ}}{\text{kg}}$

The thermal efficiency is

$\eta_{\text{th}} = \dfrac{w_{\text{net}}}{q_{\text{in}}};$

$\Rightarrow \eta_{\text{th}} = \dfrac{278.16}{440.16 + 270.75};$

$\Rightarrow \eta_{\text{th}} = 39.1\%$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-15 [OZV] Repeat problem 8-1-14 [OZF] for the net output per kg of air, assuming the pressure ratio of the first stage turbine before reheat to be (a) 7, (b) 5, (c) 3, (d) 2. (e) Use a *T*-*s* diagram to explain why the output increases and then decreases.

SOLUTION

On TEST, find the TEST-code posted for problem 8-1-14 [OZF]. After loading in the TEST-code, vary the pressure ratio that is used in the field for p4, so that it reads '=p3/*x*' where *x* is the pressure ratio of interest, then click Super-Calculate.

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-16 [OZY] Air enters the compressor of an ideal air standard Brayton cycle at 100 kPa and 290 K with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1500 K. If a regenerator with an effectiveness of 70% is incorporated in the cycle, determine (a) the thermal efficiency ($\eta_{\text{th,Brayton}}$) of the cycle. Use the PG model for air. (b) **What-if Scenario:** What would the thermal efficiency be if the regenerator effectiveness increased to 90% ?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given $p_1, T_1, \dot{m}$)

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (290)(10)^{(1.4-1)/1.4} = 559.90 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_1$):

$$T_4 = \frac{T_3}{r_p^{(k-1)/k}} = \frac{1500}{10^{(1.4-1)/1.4}} = 776.92 \text{ K}$$

State-5 (given $p_5 = p_2, \varepsilon_{\text{reg}}$):

$$T_5 = T_2 + \varepsilon_{\text{reg}}(T_4 - T_2) = 559.90 + (0.70)(776.92 - 559.90) = 711.81 \text{ K}$$

State-6 (given $p_6 = p_1$):

$$T_6 = T_4 - (T_5 - T_2) = 776.92 - (711.81 - 559.90) = 625.01 \text{ K}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $\dot{W}_C = \dot{m}c_p(T_2 - T_1) = (6)(1.005)(559.90 - 290) = 1627.50 \text{ kW}$

Device-B (2-3): $\dot{Q}_{\text{in}} = \dot{m}c_p(T_3 - T_2) = (6)(1.005)(1500 - 559.90) = 5668.80 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}c_p(T_3 - T_4) = (6)(1.005)(1500 - 776.92) = 4360.17 \text{ kW}$

Device-D (4-1): $\dot{Q}_{\text{out}} = \dot{m}c_p(T_4 - T_1) = (6)(1.005)(776.92 - 290) = 2936.13 \text{ kW}$

Now looking at the regenerator

$$\dot{Q}_{\text{reg}} = \dot{m}c_p(T_5 - T_2) = (6)(1.005)(711.81 - 559.90) = 916.02 \text{ kW}$$

Applying the adjustments to heat transfer due to the regenerator

$$\dot{Q}_{\text{in}} = \dot{Q}_{23} - \dot{Q}_{\text{reg}} = 5668.80 - 916.02 = 4752.78\ \text{kW}$$

$$\dot{Q}_{\text{out}} = \dot{Q}_{14} - \dot{Q}_{\text{reg}} = 2936.13 - 916.02 = 2020.11\ \text{kW}$$

The net power is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$$

$$\Rightarrow \dot{W}_{\text{net}} = 4360.17 - 1627.50;$$

$$\Rightarrow \dot{W}_{\text{net}} = 2732.67\ \text{kW}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{2732.67}{4752.78};$$

$$\Rightarrow \eta_{\text{th}} = 57.5\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-17 [OZQ] Repeat problem 8-1-16 [OZY] using the IG model for air.

SOLUTION

State-1 (given $p_1, T_1, \dot{m}$)

$p_{r1} = 1.2311$

$h_1 = 290.16 \frac{\text{kJ}}{\text{kg}}$

State-2 (given $s_2 = s_1, r_p$):

$p_{r2} = r_p p_{r1} = (10)(1.2311) = 12.311$

$T_2 = 554.4 \text{ K}; \; h_2 = 559.9 \frac{\text{kJ}}{\text{kg}}$

State-3 (given $p_3 = p_2, T_3$):

$p_{r3} = 601.9$

$h_3 = 1635.97 \frac{\text{kJ}}{\text{kg}}$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$p_{r4} = \frac{p_4}{p_3} p_{r3} = \left(\frac{1}{10}\right)(601.9) = 60.19$

$T_4 = 849.44 \text{ K}; \; h_4 = 876.55 \frac{\text{kJ}}{\text{kg}}$

State-5 (given $p_5 = p_2, \varepsilon_{\text{reg}}$):

$h_5 = h_2 + \varepsilon_{\text{reg}}(h_4 - h_2) = 559.9 + (0.7)(876.55 - 559.9) = 781.56 \frac{\text{kJ}}{\text{kg}}$

State-6 (given $p_6 = p_1$):

$h_6 = h_4 - (h_5 - h_2) = 876.55 - (781.55 - 559.9) = 654.9 \frac{\text{kJ}}{\text{kg}}$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $\dot{W}_C = \dot{m}(h_2 - h_1) = (6)(559.9 - 290.16) = 1618.44 \text{ kW}$

Device-B (2-3): $\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_2) = (6)(1635.97 - 559.9) = 6456.42 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}(h_3 - h_4) = (6)(1635.97 - 876.55) = 4556.52 \text{ kW}$

Device-D (4-1): $\dot{Q}_{\text{out}} = \dot{m}(h_4 - h_1) = (6)(876.55 - 290.16) = 3518.34 \text{ kW}$

Now looking at the regenerator

$$\dot{Q}_{\text{reg}} = \dot{m}(h_5 - h_2) = (6)(781.56 - 559.9) = 1329.96\ \text{kW}$$

Applying the adjustments to heat transfer due to the regenerator

$$\dot{Q}_{\text{in}} = \dot{Q}_{23} - \dot{Q}_{\text{reg}} = 6456.52 - 1329.96 = 5126.56\ \text{kW}$$

$$\dot{Q}_{\text{out}} = \dot{Q}_{14} - \dot{Q}_{\text{reg}} = 3518.34 - 1329.96 = 2188.38\ \text{kW}$$

The net power is

$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_C;$$

$$\Rightarrow \dot{W}_{\text{net}} = 4556.52 - 1618.44;$$

$$\Rightarrow \dot{W}_{\text{net}} = 2938.08\ \text{kW}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{2938.08}{5126.56};$$

$$\Rightarrow \eta_{\text{th}} = 57.3\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-18 [OZT] A Brayton cycle with regeneration and air at 100 kPa as the working fluid operates on a pressure ratio of 8. The minimum and maximum temperatures of the cycle are 300 and 1200 K. The adiabatic efficiencies of the turbine and the compressor are 80% and 82% respectively. The regenerator effectiveness is 65%. Determine (a) the thermal efficiency ($\eta_{\text{th,Brayton}}$) and (b) net work output per unit mass (w_{net}). Use the PG model.
(c) **What-if Scenario:** What would the thermal efficiency be if the regenerator effectiveness increased to 75%?

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $s_2 = s_1, r_p$):

$$T_2 = T_1 r_p^{(k-1)/k} = (300)(8)^{(1.4-1)/1.4} = 543.43 \text{ K}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$T_3 = T_1 + \frac{T_2 - T_1}{\eta_C} = 300 + \frac{543.43 - 300}{0.82} = 596.87 \text{ K}$$

State-4 (given $p_4 = p_2, T_4$)

State-5 (given $p_5 = p_1, s_5 = s_4$):

$$T_5 = \frac{T_4}{r_p^{(k-1)/k}} = \frac{1200}{8^{(1.4-1)/1.4}} = 662.45 \text{ K}$$

State-6 (given $p_6 = p_5, \eta_T$):

$$T_6 = T_4 - \eta_T (T_4 - T_5) = 1200 - (0.80)(1200 - 662.45) = 769.96 \text{ K}$$

State-7 (given $p_7 = p_2, \varepsilon_{\text{reg}}$):

$$T_7 = T_3 + \varepsilon_{\text{reg}} (T_6 - T_3) = 596.87 + (0.65)(769.96 - 596.87) = 709.38 \text{ K}$$

State-8 (given $p_8 = p_1$)

$$T_8 = T_6 - (T_7 - T_3) = 769.96 - (709.38 - 596.87) = 657.45 \text{ K}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-3): $w_C = c_p\left(T_3 - T_1\right) = \left(1.005\right)\left(596.87 - 300\right) = 298.35\dfrac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{in}} = c_p\left(T_4 - T_3\right) = \left(1.005\right)\left(1200 - 596.87\right) = 606.15\dfrac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_T = c_p\left(T_4 - T_6\right) = \left(1.005\right)\left(1200 - 769.96\right) = 432.19\dfrac{\text{kJ}}{\text{kg}}$

Device-D (6-1): $q_{\text{out}} = c_p\left(T_6 - T_1\right) = \left(1.005\right)\left(769.96 - 300\right) = 472.31\dfrac{\text{kJ}}{\text{kg}}$

Now looking at the regenerator

$$q_{\text{reg}} = c_p\left(T_7 - T_3\right) = \left(1.005\right)\left(709.38 - 596.87\right) = 113.07\frac{\text{kJ}}{\text{kg}}$$

Applying the adjustments to heat transfer due to the regenerator

$$q_{\text{in}} = q_{34} - q_{\text{reg}} = 606.15 - 113.07 = 493.08\frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{out}} = q_{16} - q_{\text{reg}} = 472.31 - 113.07 = 359.24\frac{\text{kJ}}{\text{kg}}$$

The net work per unit mass is given as

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 432.19 - 298.35;$$

$$\Rightarrow w_{\text{net}} = 133.84\frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{133.84}{493.08};$$

$$\Rightarrow \eta_{\text{th}} = 27.1\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-19 [OKS] A 100-hp, regenerative, Brayton-cycle gas turbine operates between a source at 840°C and the reference atmosphere at 21°C. Air enters the compressor at 21°C, 101 kPa. The air is then compressed to 345 kPa and then heated to 840°C. Part of this heating is accomplished in a regenerator whose effectiveness is 90%. Determine (a) the thermal efficiency ($\eta_{\text{th,Brayton}}$) of the cycle, (b) work done by compressor and (c) work done by turbine. Use the PG model.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $p_2, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(k-1)/k} = (294)\left(\frac{345}{101} \right)^{(1.4-1)/1.4} = 417.61 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3} \right)^{(k-1)/k} = (1113)\left(\frac{101}{345} \right)^{(1.4-1)/1.4} = 783.55 \text{ K}$$

State-5 (given $p_5 = p_2, \varepsilon_{\text{reg}}$):

$$T_5 = T_2 + \varepsilon_{\text{reg}} (T_4 - T_2) = 417.61 + (0.90)(783.55 - 417.61) = 746.96 \text{ K}$$

State-6 (given $p_6 = p_1$)

$$T_6 = T_4 - (T_5 - T_2) = 783.55 - (746.96 - 417.61) = 454.20 \text{ K}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p (T_2 - T_1) = (1.005)(417.61 - 294) = 124.23 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = c_p (T_3 - T_2) = (1.005)(1113 - 417.61) = 698.87 \frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_T = c_p (T_3 - T_4) = (1.005)(1113 - 783.55) = 331.10 \frac{\text{kJ}}{\text{kg}}$

Device-D (4-1): $q_{\text{out}} = c_p\left(T_4 - T_1\right) = (1.005)(783.55 - 294) = 492.00\,\dfrac{\text{kJ}}{\text{kg}}$

Now looking at the regenerator

$$q_{\text{reg}} = c_p\left(T_5 - T_2\right) = (1.005)(746.96 - 417.61) = 331.02\,\frac{\text{kJ}}{\text{kg}}$$

Applying the adjustments to heat transfer due to the regenerator

$$q_{\text{in}} = q_{23} - q_{\text{reg}} = 698.87 - 331.02 = 367.85\,\frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{out}} = q_{14} - q_{\text{reg}} = 492.00 - 331.02 = 160.98\,\frac{\text{kJ}}{\text{kg}}$$

The mass flow rate

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 331.10 - 124.23;$$

$$\Rightarrow w_{\text{net}} = 206.87\,\frac{\text{kJ}}{\text{kg}}$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}};$$

$$\Rightarrow \dot{m} = \frac{74.57}{206.87};$$

$$\Rightarrow \dot{m} = 0.36\,\frac{\text{kg}}{\text{s}}$$

The power consumed by the compressor and delivered by the turbine are

$$\dot{W}_C = \dot{m}w_C = (0.36)(124.23) = 44.72\text{ kW}$$

$$\dot{W}_T = \dot{m}w_T = (0.36)(331.10) = 119.20\text{ kW}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{206.87}{367.85};$$

$$\Rightarrow \eta_{\text{th}} = 56.2\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-20 [OZM] Repeat problem 8-1-19 [OKS] assuming the regenerator having an effectiveness of 85%, and determine the thermal efficiency ($\eta_{\text{th,Brayton}}$). Use the IG model.

SOLUTION

State-1 (given p_1, T_1):

$$p_{r1} = 1.2917$$

$$h_1 = 294.17 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $p_2, s_2 = s_1$):

$$p_{r2} = \frac{p_2}{p_1} p_{r1} = \left(\frac{345}{101}\right)(1.2917) = 4.4122$$

$$T_2 = 417.02 \text{ K}; \; h_2 = 418.24 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$)

$$p_{r3} = 175.29$$

$$h_3 = 1176.20 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$p_{r4} = \frac{p_4}{p_3} p_{r3} = \left(\frac{101}{345}\right)(175.29) = 51.3168$$

$$T_4 = 814.74 \text{ K}; \; h_4 = 838.18 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_2, \varepsilon_{\text{reg}}$):

$$h_5 = h_2 + \varepsilon_{\text{reg}}(h_4 - h_2) = 418.24 + (0.85)(838.18 - 418.24) = 775.19 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_1$)

$$h_6 = h_4 - (h_5 - h_2) = 838.18 - (775.19 - 418.24) = 481.23 \frac{\text{kJ}}{\text{kg}}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = h_2 - h_1 = 418.24 - 294.17 = 124.07 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = h_3 - h_2 = 1176.20 - 418.24 = 757.96 \dfrac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_T = h_3 - h_4 = 1176.20 - 838.18 = 338.02 \dfrac{\text{kJ}}{\text{kg}}$

Device-D (4-1): $q_{\text{out}} = h_4 - h_1 = 838.18 - 294.17 = 544.01 \dfrac{\text{kJ}}{\text{kg}}$

Now looking at the regenerator

$$q_{\text{reg}} = h_5 - h_2 = 775.19 - 418.24 = 356.95 \frac{\text{kJ}}{\text{kg}}$$

Applying the adjustments to heat transfer due to the regenerator

$$q_{\text{in}} = q_{23} - q_{\text{reg}} = 757.96 - 356.95 = 401.01 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{out}} = q_{14} - q_{\text{reg}} = 544.01 - 356.95 = 187.06 \frac{\text{kJ}}{\text{kg}}$$

The mass flow rate

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 338.02 - 124.07;$$

$$\Rightarrow w_{\text{net}} = 213.95 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}};$$

$$\Rightarrow \dot{m} = \frac{74.57}{213.95};$$

$$\Rightarrow \dot{m} = 0.35 \frac{\text{kg}}{\text{s}}$$

The power consumed by the compressor and delivered by the turbine are

$$\dot{W}_C = \dot{m} w_C = (0.35)(124.07) = 43.42 \text{ kW}$$

$$\dot{W}_T = \dot{m} w_T = (0.35)(338.02) = 118.31 \text{ kW}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{213.95}{401.01};$$

$$\Rightarrow \eta_{\text{th}} = 53.4\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-21 [OZD] Air enters the compressor of a regenerative gas turbine engine at 100 kPa and 290 K, where it is compressed to 750 kPa and 550 K. The regenerator has an effectiveness of 70%, and the air enters the turbine at 1200 K. For a turbine efficiency of 80%, determine (a) the amount of heat transfer (q) in the regenerator and (b) the thermal efficiency (η_{th}) of the cycle. Assume variable specific heats of air.

SOLUTION

State-1 (given p_1, T_1):

$$p_{r1} = 1.2311$$

$$h_1 = 290.16 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given p_2, T_2):

$$p_{r2} = 11.86$$

$$h_2 = 555.74 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$):

$$p_{r3} = 238$$

$$h_3 = 1277.79 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_1, s_4 = s_3$):

$$p_{r5} = \frac{p_5}{p_4} p_{r4} = \left(\frac{100}{750}\right)(238) = 31.73$$

$$T_4 = 718.05 \text{ K}; \; h_4 = 732.72 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, \eta_T$):

$$h_5 = h_3 - \eta_T \left(h_3 - h_4\right) = 1277.79 - (0.80)(1277.79 - 732.72) = 841.73 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_2, \varepsilon_{\text{reg}}$):

$$h_6 = h_2 + \varepsilon_{\text{reg}} \left(h_5 - h_2\right) = 555.74 + (0.70)(841.73 - 555.74) = 755.93 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_1$):

$$h_7 = h_5 - (h_6 - h_2) = 841.73 - (755.93 - 555.74) = 641.54 \frac{\text{kJ}}{\text{kg}}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = h_2 - h_1 = 555.74 - 290.16 = 265.58 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{in}} = h_3 - h_2 = 1277.79 - 555.74 = 722.05 \frac{\text{kJ}}{\text{kg}}$

Device-C (3-5): $w_T = h_3 - h_5 = 1277.79 - 841.73 = 436.06 \frac{\text{kJ}}{\text{kg}}$

Device-D (5-1): $q_{\text{out}} = h_5 - h_1 = 841.73 - 290.16 = 551.57 \frac{\text{kJ}}{\text{kg}}$

Now looking at the regenerator

$$q_{\text{reg}} = h_6 - h_2 = 755.93 - 555.74 = 200.19 \frac{\text{kJ}}{\text{kg}}$$

Applying the adjustments to heat transfer due to the regenerator

$$q_{\text{in}} = q_{23} - q_{\text{reg}} = 722.05 - 200.19 = 521.86 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{out}} = q_{15} - q_{\text{reg}} = 551.57 - 200.19 = 351.38 \frac{\text{kJ}}{\text{kg}}$$

The net work per unit mass is given as

$$w_{\text{net}} = w_T - w_C;$$
$$\Rightarrow w_{\text{net}} = 436.06 - 265.58;$$
$$\Rightarrow w_{\text{net}} = 170.48 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$
$$\Rightarrow \eta_{\text{th}} = \frac{170.48}{521.86};$$
$$\Rightarrow \eta_{\text{th}} = 32.7\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-22 [OKR] Air is compressed from 100 kPa and 310 K to 1000 kPa in a two stage compressor with intercooling between stages. The intercooler pressure is 350 kPa. The air is cooled back to 310 K in the intercooler before entering the second compressor stage. Each compressor stage is isentropic. Determine (a) the temperature (T) at the exit of the second compressor stage and (b) the total compressor work in kJ/kg. Use the PG model.

SOLUTION

Given:

$$c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$k = 1.4$$

State-1 (given p_1, T_1)

State-2 (given $p_2, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (310)\left(\frac{350}{100}\right)^{(1.4-1)/1.4} = 443.41 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{(k-1)/k} = (310)\left(\frac{1000}{350}\right)^{(1.4-1)/1.4} = 418.43 \text{ K}$$

An energy analysis is carried out for compressors as

Device-A (1-2): $w_C = c_p (T_2 - T_1) = (1.005)(443.41 - 310) = 134.08 \dfrac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $w_C = c_p (T_4 - T_3) = (1.005)(418.43 - 310) = 108.97 \dfrac{\text{kJ}}{\text{kg}}$

The total compressor work is

$$w_{C,\text{total}} = 134.08 + 108.97 = 243.05 \frac{\text{kJ}}{\text{kg}}$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-23 [OZJ] Air enters the compressor of an ideal air standard Brayton cycle at 100 kPa, 25°C with a volumetric flow rate of 8 m^3/s and is compressed to 1000 kPa. The temperature at the inlet to the first turbine stage is 1000°C. The expansion takes place isentropically in two stages, with reheat to 1000°C between the stages at a constant pressure of 300 kPa. If a regenerator having an effectiveness of 100% is incorporated in the cycle, determine (a) the thermal efficiency of the cycle ($\eta_{\mathrm{th,Brayton}}$). Use the PG model.

SOLUTION

Given:

$$c_p = 1.005 \frac{\mathrm{kJ}}{\mathrm{kg \cdot K}}$$

$$R = 0.287 \frac{\mathrm{kJ}}{\mathrm{kg \cdot K}}$$

$$k = 1.4$$

State-1 (given $p_1, T_1, \dot{V}_1$):

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287)(298)}{100} = 0.8553 \frac{\mathrm{m^3}}{\mathrm{kg}}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{8}{0.8553} = 9.35 \frac{\mathrm{kg}}{\mathrm{s}}$$

State-2 (given $p_2, s_2 = s_1$):

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(k-1)/k} = (298)\left(\frac{1000}{100} \right)^{(1.4-1)/1.4} = 575.35 \text{ K}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4, s_4 = s_3$):

$$T_4 = T_3 \left(\frac{p_4}{p_3} \right)^{(k-1)/k} = (1273)\left(\frac{300}{1000} \right)^{(1.4-1)/1.4} = 902.47 \text{ K}$$

State-5 (given $p_5 = p_4, T_5$)

State-6 (given $p_6 = p_1, s_6 = s_5$):

$$T_6 = T_5 \left(\frac{p_6}{p_5} \right)^{(k-1)/k} = (1273)\left(\frac{100}{300} \right)^{(1.4-1)/1.4} = 930.05 \text{ K}$$

State-7 (given $p_7 = p_2, \varepsilon_{\mathrm{reg}}$):

$$T_7 = T_2 + \varepsilon_{\text{reg}}(T_6 - T_2) = 575.35 + (1)(930.05 - 575.35) = 930.05 \text{ K}$$

State-8 (given $p_8 = p_1$):

$$T_8 = T_6 - (T_7 - T_2) = 930.05 - (930.05 - 575.35) = 575.35 \text{ K}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $\dot{W}_C = \dot{m}c_p(T_2 - T_1) = (9.35)(1.005)(575.35 - 298) = 2606.19 \text{ kW}$

Device-B (2-3): $\dot{Q}_{\text{in}} = \dot{m}c_p(T_3 - T_2) = (9.35)(1.005)(1273 - 575.35) = 6555.64 \text{ kW}$

Device-C (3-4): $\dot{W}_T = \dot{m}c_p(T_3 - T_4) = (9.35)(1.005)(1273 - 902.47) = 3481.78 \text{ kW}$

Device-D (4-5): $\dot{Q}_{\text{in}} = \dot{m}c_p(T_5 - T_4) = (9.35)(1.005)(1273 - 902.47) = 3481.78 \text{ kW}$

Device-E (5-6): $\dot{W}_T = \dot{m}c_p(T_5 - T_6) = (9.35)(1.005)(1273 - 930.05) = 3222.62 \text{ kW}$

Device-F (6-1): $\dot{Q}_{\text{out}} = \dot{m}c_p(T_6 - T_1) = (9.35)(1.005)(930.05 - 298) = 5939.22 \text{ kW}$

Now looking at the regenerator

$$\dot{Q}_{\text{reg}} = \dot{m}c_p(T_7 - T_2) = (9.35)(1.005)(930.05 - 575.35) = 3333.03 \text{ kW}$$

Applying the adjustments to heat transfer due to the regenerator

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{in,total}} - \dot{Q}_{\text{reg}} = (6555.64 + 3481.78) - 3333.03 = 6704.39 \text{ kW}$$

$$\dot{Q}_{\text{out}} = \dot{Q}_{16} - \dot{Q}_{\text{reg}} = 5939.22 - 3333.03 = 2606.19 \text{ kW}$$

The power is given as

$$\dot{W}_{\text{net}} = \dot{W}_{T,\text{total}} - \dot{W}_C;$$

$$\Rightarrow \dot{W}_{\text{net}} = (3481.78 + 3222.62) - 2606.19;$$

$$\Rightarrow \dot{W}_{\text{net}} = 4098.21 \text{ kW}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$

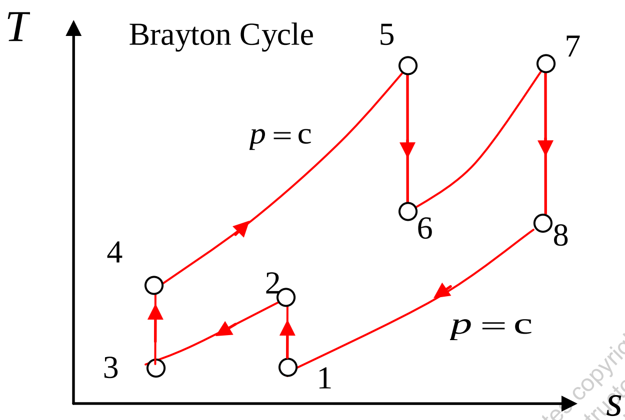
$$\Rightarrow \eta_{\text{th}} = \frac{4098.21}{6704.39};$$

$$\Rightarrow \eta_{\text{th}} = 61.1\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-24 [OZW] Consider an ideal gas turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each stage of the compressor and the turbine is 2. Air (use the IG model) enters each stage of the compressor at 310 K and each stage of the turbine at 1100 K. Determine (a) the thermal efficiency (η_{th}) of the cycle and (b) back work ratio. Use the IG model. (c) **What-if Scenario:** What would the thermal efficiency and BWR be if the pressure ratio across each stage of the compressor and the turbine were 4?

SOLUTION



State-1 (given p_1, T_1):

$$p_{r1} = 1.5546$$

$$h_1 = 310.24 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, r_p$):

$$p_{r2} = r_p p_{r1} = (2)(1.5546) = 3.11$$

$$T_2 = 377.59 \text{ K}; \; h_2 = 378.34 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 310.24 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4, s_4 = s_3$):

$$h_4 = 378.34 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, T_5$):

$$p_{r5} = 167.1$$

$$h_5 = 1161.07 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $s_6 = s_5, r_p$):

$$p_{r6} = \frac{1}{r_p} p_{r5} = \left(\frac{1}{2}\right)(167.1) = 83.55$$

$$T_6 = 924.1 \text{ K}; \; h_6 = 960.05 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6, T_7$):

$$p_{r7} = 167.1$$

$$h_7 = 1161.07 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_1, s_8 = s_7$):

$$T_8 = 924.1 \text{ K}; \; h_8 = 960.05 \frac{\text{kJ}}{\text{kg}}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = h_2 - h_1 = 378.34 - 310.24 = 68.10 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{out}} = h_2 - h_3 = 378.34 - 310.24 = 68.10 \frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_C = h_4 - h_3 = 378.34 - 310.24 = 68.10 \frac{\text{kJ}}{\text{kg}}$

Device-D (4-5): $q_{\text{in}} = h_5 - h_4 = 1161.07 - 378.34 = 782.73 \frac{\text{kJ}}{\text{kg}}$

Device-E (5-6): $w_T = h_5 - h_6 = 1161.07 - 960.05 = 201.02 \frac{\text{kJ}}{\text{kg}}$

Device-F (6-7): $q_{\text{in}} = h_7 - h_6 = 1161.07 - 960.05 = 201.02 \frac{\text{kJ}}{\text{kg}}$

Device-G (7-8): $w_T = h_7 - h_8 = 1161.07 - 960.05 = 201.02 \frac{\text{kJ}}{\text{kg}}$

Device-H (8-1): $q_{\text{out}} = h_8 - h_1 = 960.05 - 310.24 = 649.81 \frac{\text{kJ}}{\text{kg}}$

The net work per unit mass is given as

$$w_{\text{net}} = w_{T,\text{total}} - w_{C,\text{total}};$$
$$\Rightarrow w_{\text{net}} = (201.02 + 201.02) - (68.10 + 68.10);$$
$$\Rightarrow w_{\text{net}} = 265.84 \frac{\text{kJ}}{\text{kg}}$$

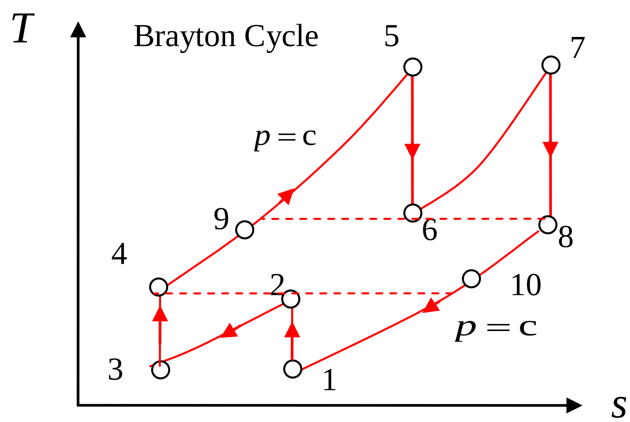
The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{w_{C,\text{total}}}{w_{T,\text{total}}};$$
$$\Rightarrow \text{BWR} = \frac{68.10 + 68.10}{201.02 + 201.02};$$
$$\Rightarrow \text{BWR} = 33.9\%$$
$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in,total}}};$$
$$\Rightarrow \eta_{\text{th}} = \frac{265.84}{782.73 + 201.02};$$
$$\Rightarrow \eta_{\text{th}} = 27.0\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-25 [OKO] Repeat problem 8-1-24 [OZW] assuming a regenerator with 80% effectiveness is added at the end of the last compressor. Determine (a) the thermal efficiency (η_{th}) of the cycle and (b) back work ratio.

SOLUTION



State-1 (given p_1, T_1):

$$p_{r1} = 1.5546$$

$$h_1 = 310.24 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, r_p$):

$$p_{r2} = r_p p_{r1} = (2)(1.5546) = 3.11$$

$$T_2 = 377.59 \text{ K}; \; h_2 = 378.34 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$):

$$h_3 = 310.24 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4, s_4 = s_3$):

$$h_4 = 378.34 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = p_4, T_5$):

$$p_{r5} = 167.1$$

$$h_5 = 1161.07 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $s_6 = s_5, r_p$):

$$p_{r6} = \frac{1}{r_p} p_{r5} = \left(\frac{1}{2}\right)(167.1) = 83.55$$

$$T_6 = 924.1 \text{ K};\ h_6 = 960.05 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6, T_7$):

$$p_{r7} = 167.1$$

$$h_7 = 1161.07 \frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8 = p_1, s_8 = s_7$):

$$T_8 = 924.1 \text{ K};\ h_8 = 960.05 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_4, \varepsilon_{\text{reg}}$):

$$h_9 = h_4 + \varepsilon_{\text{reg}}\left(h_8 - h_4\right) = 378.34 + (0.80)(960.05 - 378.34) = 843.71 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_1$):

$$h_{10} = h_8 - \left(h_9 - h_4\right) = 960.05 - (843.71 - 378.34) = 494.68 \frac{\text{kJ}}{\text{kg}}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = h_2 - h_1 = 378.34 - 310.24 = 68.10 \frac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{out}} = h_2 - h_3 = 378.34 - 310.24 = 68.10 \frac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_C = h_4 - h_3 = 378.34 - 310.24 = 68.10 \frac{\text{kJ}}{\text{kg}}$

Device-D (4-5): $q_{\text{in}} = h_5 - h_4 = 1161.07 - 378.34 = 782.73 \frac{\text{kJ}}{\text{kg}}$

Device-E (5-6): $w_T = h_5 - h_6 = 1161.07 - 960.05 = 201.02 \frac{\text{kJ}}{\text{kg}}$

Device-F (6-7): $q_{\text{in}} = h_7 - h_6 = 1161.07 - 960.05 = 201.02 \frac{\text{kJ}}{\text{kg}}$

Device-G (7-8): $w_T = h_7 - h_8 = 1161.07 - 960.05 = 201.02 \frac{\text{kJ}}{\text{kg}}$

Device-H (8-1): $q_{\text{out}} = h_8 - h_1 = 960.05 - 310.24 = 649.81 \dfrac{\text{kJ}}{\text{kg}}$

Now looking at the regenerator

$$q_{\text{reg}} = h_9 - h_4 = 843.71 - 378.34 = 465.37 \frac{\text{kJ}}{\text{kg}}$$

Applying the adjustments to heat transfer due to the regenerator

$$q_{\text{in}} = q_{\text{in,total}} - q_{\text{reg}} = (782.73 + 201.02) - 465.37 = 518.38 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{out}} = q_{\text{out,total}} - q_{\text{reg}} = (68.10 + 649.81) - 465.37 = 252.54 \frac{\text{kJ}}{\text{kg}}$$

The net work per unit mass is given as

$$w_{\text{net}} = w_{T,\text{total}} - w_{C,\text{total}};$$

$$\Rightarrow w_{\text{net}} = (201.02 + 201.02) - (68.10 + 68.10);$$

$$\Rightarrow w_{\text{net}} = 265.84 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{w_{C,\text{total}}}{w_{T,\text{total}}};$$

$$\Rightarrow \text{BWR} = \frac{68.10 + 68.10}{201.02 + 201.02};$$

$$\Rightarrow \text{BWR} = 33.9\%$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{265.84}{518.38};$$

$$\Rightarrow \eta_{\text{th}} = 51.3\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-26 [OKB] Consider a regenerative gas turbine power plant with two stages of compression and two stages of expansion. The overall pressure ratio of the cycle is 9. Air enters each stage of compressor at 290 K and each stage of turbine at 1400 K, the regenerator has an effectiveness of 100%. Determine (a) the minimum mass flow rate of air needed to develop a net power output of 50 MW. Use the IG model. (b) **What-if Scenario:** What would the mass flow rate be if argon were used as the working fluid ?

SOLUTION

State-1 (given p_1, T_1):

$p_{r1} = 1.2311$

$h_1 = 290.16 \dfrac{\text{kJ}}{\text{kg}}$

State-2 (given $s_2 = s_1, r_p = \sqrt{r_{p,\text{overall}}}$):

$p_{r2} = r_p p_{r1} = (3)(1.2311) = 3.69$

$T_2 = 396.43\ \text{K};\ h_2 = 397.38 \dfrac{\text{kJ}}{\text{kg}}$

State-3 (given $p_3 = p_2, T_3$):

$h_3 = 290.16\ \dfrac{\text{kJ}}{\text{kg}}$

State-4 (given $p_4, s_4 = s_3$):

$T_4 = 396.43\ \text{K};\ h_4 = 397.38 \dfrac{\text{kJ}}{\text{kg}}$

State-5 (given $p_5 = p_4, T_5$):

$p_{r5} = 450.5$

$h_5 = 1515.42 \dfrac{\text{kJ}}{\text{kg}}$

State-6 (given $s_6 = s_5, r_p$):

$$p_{r6} = \frac{1}{r_p} p_{r5} = \left(\frac{1}{3}\right)(450.5) = 150.16$$

$T_6 = 1071.1\ \text{K};\ h_6 = 1127.62 \dfrac{\text{kJ}}{\text{kg}}$

State-7 (given $p_7 = p_6, T_7$):

$p_{r7} = 450.5$

$h_7 = 1515.42 \dfrac{\text{kJ}}{\text{kg}}$

State-8 (given $p_8 = p_1, s_8 = s_7$):

$T_8 = 1071.1 \text{ K}; \; h_8 = 1127.62 \dfrac{\text{kJ}}{\text{kg}}$

State-9 (given $p_9 = p_4, \varepsilon_{\text{reg}}$):

$$h_9 = h_4 + \varepsilon_{\text{reg}}\left(h_8 - h_4\right) = 397.38 + (1)(1127.62 - 397.38) = 1127.62 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_1$):

$$h_{10} = h_8 - \left(h_9 - h_4\right) = 1127.62 - (1127.62 - 397.38) = 397.38 \ \frac{\text{kJ}}{\text{kg}}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = h_2 - h_1 = 397.38 - 290.16 = 107.22 \dfrac{\text{kJ}}{\text{kg}}$

Device-B (2-3): $q_{\text{out}} = h_2 - h_3 = 397.38 - 290.16 = 107.22 \dfrac{\text{kJ}}{\text{kg}}$

Device-C (3-4): $w_C = h_4 - h_3 = 397.38 - 290.16 = 107.22 \dfrac{\text{kJ}}{\text{kg}}$

Device-D (4-5): $q_{\text{in}} = h_5 - h_4 = 1515.42 - 397.38 = 1118.04 \dfrac{\text{kJ}}{\text{kg}}$

Device-E (5-6): $w_T = h_5 - h_6 = 1515.42 - 1127.62 = 387.80 \dfrac{\text{kJ}}{\text{kg}}$

Device-F (6-7): $q_{\text{in}} = h_7 - h_6 = 1515.42 - 1127.62 = 387.80 \dfrac{\text{kJ}}{\text{kg}}$

Device-G (7-8): $w_T = h_7 - h_8 = 1515.42 - 1127.62 = 387.80 \dfrac{\text{kJ}}{\text{kg}}$

Device-H (8-1): $q_{\text{out}} = h_8 - h_1 = 1127.62 - 290.16 = 837.46 \dfrac{\text{kJ}}{\text{kg}}$

Now looking at the regenerator

$q_{\text{reg}} = h_9 - h_4 = 1127.62 - 397.38 = 730.24 \dfrac{\text{kJ}}{\text{kg}}$

Applying the adjustments to heat transfer due to the regenerator

$$q_{\text{in}} = q_{\text{in,total}} - q_{\text{reg}} = (1118.04 + 387.80) - 730.24 = 775.60\,\frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{out}} = q_{\text{out,total}} - q_{\text{reg}} = (107.22 + 837.46) - 730.24 = 214.22\,\frac{\text{kJ}}{\text{kg}}$$

The net work per unit mass is given as

$$w_{\text{net}} = w_{T,\text{total}} - w_{C,\text{total}};$$

$$\Rightarrow w_{\text{net}} = (387.80 + 387.80) - (107.22 + 107.22);$$

$$\Rightarrow w_{\text{net}} = 561.16\,\frac{\text{kJ}}{\text{kg}}$$

The mass flow can now be obtained since the net power is given.

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}};$$

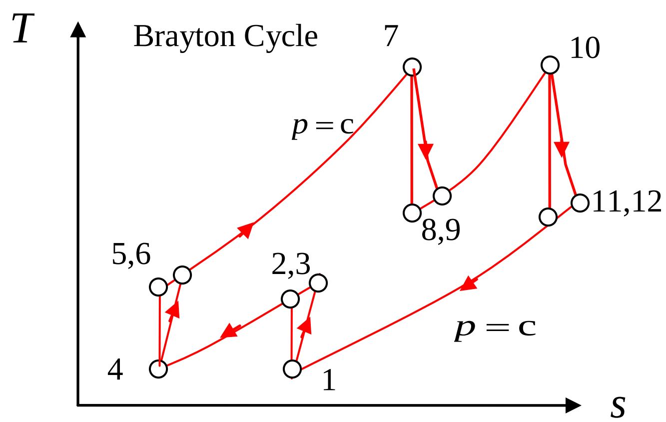
$$\Rightarrow \dot{m} = \frac{50000}{561.16};$$

$$\Rightarrow \dot{m} = 89.10\,\frac{\text{kg}}{\text{s}}$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-27 [OKA] Repeat 8-1-24 [OZW] assuming an efficiency of 80% for each compressor stage and an efficiency of 85% for each turbine stage. Determine (a) the thermal efficiency (η_{th}) of the cycle and (b) back work ratio.

SOLUTION



State-1 (given p_1, T_1):

$p_{r1} = 1.5546$

$h_1 = 310.24 \dfrac{\text{kJ}}{\text{kg}}$

State-2 (given $s_2 = s_1, r_p$):

$p_{r2} = r_p p_{r1} = (2)(1.5546) = 3.11$

$T_2 = 377.59 \text{ K}; \; h_2 = 378.34 \dfrac{\text{kJ}}{\text{kg}}$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 310.24 + \frac{378.34 - 310.24}{0.80} = 395.37 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$)

$p_{r4} = 1.5546$

$h_4 = 310.24 \dfrac{\text{kJ}}{\text{kg}}$

State-5 (given $p_5, s_5 = s_4$):

$$p_{r5} = r_p p_{r4} = (2)(1.5546) = 3.11$$

$$h_5 = 378.34 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_C$):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_C} = 310.24 + \frac{378.34 - 310.24}{0.80} = 395.37 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6, T_7$):

$$p_{r7} = 167.1$$

$$h_7 = 1161.07 \ \frac{\text{kJ}}{\text{kg}}$$

State-8 (given $s_8 = s_7, r_p$):

$$p_{r8} = \frac{1}{r_p} p_{r5} = \left(\frac{1}{2}\right)(167.1) = 83.55$$

$$T_8 = 924.1 \text{ K}; \ h_8 = 960.05 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_8, \eta_T$):

$$h_9 = h_7 - \eta_T (h_7 - h_8) = 1161.07 - (0.85)(1161.07 - 960.05) = 990.20 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_8, T_{10}$):

$$p_{r10} = 167.1$$

$$h_{10} = 1161.07 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):

$$p_{r11} = \frac{1}{r_p} p_{r10} = \left(\frac{1}{2}\right)(167.1) = 83.55$$

$$h_{11} = 960.05 \frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_{11}, \eta_T$):

$$h_{12} = h_{10} - \eta_T (h_{10} - h_{11}) = 1161.07 - (0.85)(1161.07 - 960.05) = 990.20 \frac{\text{kJ}}{\text{kg}}$$

An energy analysis is carried out for each device as follows

Device-A (1-3): $w_C = h_3 - h_1 = 395.37 - 310.24 = 85.13 \dfrac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{out}} = h_3 - h_4 = 395.37 - 310.24 = 85.13 \dfrac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_C = h_6 - h_4 = 395.37 - 310.24 = 85.13 \dfrac{\text{kJ}}{\text{kg}}$

Device-D (6-7): $q_{\text{in}} = h_7 - h_6 = 1161.07 - 395.37 = 765.70 \dfrac{\text{kJ}}{\text{kg}}$

Device-E (7-9): $w_T = h_7 - h_9 = 1161.07 - 990.20 = 170.87 \dfrac{\text{kJ}}{\text{kg}}$

Device-F (9-10): $q_{\text{in}} = h_{10} - h_9 = 1161.07 - 990.20 = 170.87 \dfrac{\text{kJ}}{\text{kg}}$

Device-G (10-12): $w_T = h_{10} - h_{12} = 1161.07 - 990.20 = 170.87 \dfrac{\text{kJ}}{\text{kg}}$

Device-H (12-1): $q_{\text{out}} = h_{12} - h_1 = 990.20 - 310.24 = 679.96 \dfrac{\text{kJ}}{\text{kg}}$

The net work per unit mass is given as

$$w_{\text{net}} = w_{T,\text{total}} - w_{C,\text{total}};$$

$$\Rightarrow w_{\text{net}} = (170.87 + 170.87) - (85.13 + 85.13);$$

$$\Rightarrow w_{\text{net}} = 171.48 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{w_{C,\text{total}}}{w_{T,\text{total}}};$$

$$\Rightarrow \text{BWR} = \frac{85.13 + 85.13}{170.87 + 170.87};$$

$$\Rightarrow \text{BWR} = 49.8\%$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in,total}}};$$

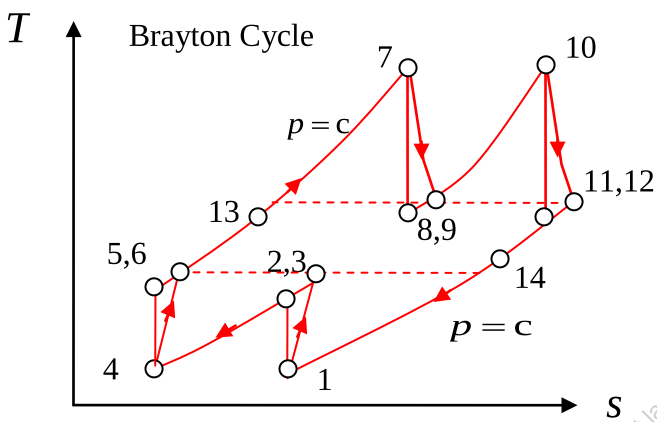
$$\Rightarrow \eta_{\text{th}} = \frac{171.48}{765.70 + 170.87};$$

$$\Rightarrow \eta_{\text{th}} = 18.3\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-28 [OKI] Repeat problem 8-1-24 [OZW] assuming an efficiency of 80% for each compressor stage and an efficiency of 85% for each turbine stage, and a regenerator with 80% effectiveness. Determine (a) the thermal efficiency (η_{th}) of the cycle and (b) the back work ratio.

SOLUTION



State-1 (given p_1, T_1):

$$p_{r1} = 1.5546$$

$$h_1 = 310.24 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $s_2 = s_1, r_p$):

$$p_{r2} = r_p p_{r1} = (2)(1.5546) = 3.11$$

$$T_2 = 377.59 \text{ K}; \; h_2 = 378.34 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 310.24 + \frac{378.34 - 310.24}{0.80} = 395.37 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$)

$$p_{r4} = 1.5546$$

$$h_4 = 310.24 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5, s_5 = s_4$):

$$p_{r5} = r_p p_{r4} = (2)(1.5546) = 3.11$$

$$h_5 = 378.34 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_C$):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_C} = 310.24 + \frac{378.34 - 310.24}{0.80} = 395.37 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_6, T_7$):

$$p_{r7} = 167.1$$

$$h_7 = 1161.07 \ \frac{\text{kJ}}{\text{kg}}$$

State-8 (given $s_8 = s_7, r_p$):

$$p_{r8} = \frac{1}{r_p} p_{r5} = \left(\frac{1}{2}\right)(167.1) = 83.55$$

$$T_8 = 924.1 \text{ K};\ h_8 = 960.05 \frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_8, \eta_T$):

$$h_9 = h_7 - \eta_T (h_7 - h_8) = 1161.07 - (0.85)(1161.07 - 960.05) = 990.20 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_8, T_{10}$):

$$p_{r10} = 167.1$$

$$h_{10} = 1161.07 \frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):

$$p_{r11} = \frac{1}{r_p} p_{r10} = \left(\frac{1}{2}\right)(167.1) = 83.55$$

$$h_{11} = 960.05 \frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_{11}, \eta_T$):

$$h_{12} = h_{10} - \eta_T (h_{10} - h_{11}) = 1161.07 - (0.85)(1161.07 - 960.05) = 990.20 \frac{\text{kJ}}{\text{kg}}$$

State-13 (given $p_{13} = p_6, \varepsilon_{\text{reg}}$):

$$h_{13} = h_6 + \varepsilon_{\text{reg}}\left(h_{12} - h_6\right) = 395.37 + (0.80)(990.20 - 395.37) = 871.23\frac{\text{kJ}}{\text{kg}}$$

State-14 (given $p_{14} = p_1$):

$$h_{14} = h_{12} - \left(h_{13} - h_6\right) = 990.20 - (871.23 - 395.37) = 514.34\frac{\text{kJ}}{\text{kg}}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-3): $w_C = h_3 - h_1 = 395.37 - 310.24 = 85.13\frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $q_{\text{out}} = h_3 - h_4 = 395.37 - 310.24 = 85.13\frac{\text{kJ}}{\text{kg}}$

Device-C (4-6): $w_C = h_6 - h_4 = 395.37 - 310.24 = 85.13\frac{\text{kJ}}{\text{kg}}$

Device-D (6-7): $q_{\text{in}} = h_7 - h_6 = 1161.07 - 395.37 = 765.70\frac{\text{kJ}}{\text{kg}}$

Device-E (7-9): $w_T = h_7 - h_9 = 1161.07 - 990.20 = 170.87\frac{\text{kJ}}{\text{kg}}$

Device-F (9-10): $q_{\text{in}} = h_{10} - h_9 = 1161.07 - 990.20 = 170.87\frac{\text{kJ}}{\text{kg}}$

Device-G (10-12): $w_T = h_{10} - h_{12} = 1161.07 - 990.20 = 170.87\frac{\text{kJ}}{\text{kg}}$

Device-H (12-1): $q_{\text{out}} = h_{12} - h_1 = 990.20 - 310.24 = 679.96\frac{\text{kJ}}{\text{kg}}$

Now looking at the regenerator

$$q_{\text{reg}} = h_{13} - h_6 = 871.23 - 395.37 = 475.86\frac{\text{kJ}}{\text{kg}}$$

Applying the adjustments to heat transfer due to the regenerator

$$q_{\text{in}} = q_{\text{in,total}} - q_{\text{reg}} = (765.70 + 170.87) - 475.86 = 460.71\frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{out}} = q_{\text{out,total}} - q_{\text{reg}} = (85.13 + 679.96) - 475.86 = 289.23\frac{\text{kJ}}{\text{kg}}$$

The net work per unit mass is given as

$$w_{\text{net}} = w_{T,\text{total}} - w_{C,\text{total}};$$

$$\Rightarrow w_{\text{net}} = (170.87 + 170.87) - (85.13 + 85.13);$$

$$\Rightarrow w_{\text{net}} = 171.48 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{w_{C,\text{total}}}{w_{T,\text{total}}};$$

$$\Rightarrow \text{BWR} = \frac{85.13 + 85.13}{170.87 + 170.87};$$

$$\Rightarrow \text{BWR} = 49.8\%$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{171.48}{460.71};$$

$$\Rightarrow \eta_{\text{th}} = 37.2\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-29 [OKH] A regenerative gas turbine with intercooling and reheat operates at steady state. Air enters the compressor at 100 kPa, 300 K with a mass flow rate ($\dot{m}$) of 8 kg/s. The pressure ratio across the two stage compressor and two stage turbine is 12. The intercooler and reheater each operate at 300 kPa. At the inlets to the turbine stages the temperature is 1500 K. The temperature at the inlet to the second compressor stage is 300 K. The efficiency of each compressor is 85%, and turbine stage is 80%. The regenerator effectiveness is 75%. Determine (a) the thermal efficiency (η_{th}), (b) the back work ratio and (c) the net power developed. Use the IG model.

SOLUTION

State-1 (given $p_1, T_1, \dot{m}$):

$$p_{r1} = 1.3860$$

$$h_1 = 300.19 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $p_2, s_2 = s_1$):

$$p_{r2} = \left(\frac{p_2}{p_1}\right) p_{r1} = (3)(1.3860) = 4.158$$

$$T_2 = 410.14 \text{ K}; \; h_2 = 411.26 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 300.19 + \frac{411.26 - 300.19}{0.85} = 430.86 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$$p_{r4} = 1.3860$$

$$h_4 = 300.19 \frac{\text{kJ}}{\text{kg}}$$

State-5 (given $p_5 = 12 p_1, s_5 = s_4$):

$$p_{r5} = \left(\frac{p_5}{p_4}\right) p_{r4} = \left(\frac{12}{3}\right)(1.3860) = 5.544$$

$$T_5 = 444.79 \text{ K}; \; h_5 = 446.49 \frac{\text{kJ}}{\text{kg}}$$

State-6 (given $p_6 = p_5, \eta_C$):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_C} = 300.19 + \frac{446.49 - 300.19}{0.85} = 472.31 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_5, T_7$):

$$p_{r7} = 601.9$$

$$h_7 = 1636.00\,\frac{\text{kJ}}{\text{kg}}$$

State-8 (given $p_8, s_8 = s_7$):

$$p_{r8} = \left(\frac{p_8}{p_7}\right) p_{r7} = \left(\frac{3}{12}\right)(601.9) = 150.48$$

$$T_8 = 1071.60 \text{ K};\ h_8 = 1128.30\,\frac{\text{kJ}}{\text{kg}}$$

State-9 (given $p_9 = p_8, \eta_T$):

$$h_9 = h_7 - \eta_T\left(h_7 - h_8\right) = 1636.00 - (0.80)(1636.00 - 1128.30) = 1229.84\,\frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_8, T_{10}$):

$$p_{r10} = 601.9$$

$$h_{10} = 1636.00\,\frac{\text{kJ}}{\text{kg}}$$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):

$$p_{r11} = \left(\frac{p_1}{p_{10}}\right) p_{r10} = \left(\frac{1}{3}\right)(601.9) = 200.63$$

$$T_{11} = 1150.70 \text{ K};\ h_{11} = 1220.00\,\frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_{11}, \eta_T$):

$$h_{12} = h_{10} - \eta_T\left(h_{10} - h_{11}\right) = 1636.00 - (0.80)(1636.00 - 1220.00) = 1303.20\,\frac{\text{kJ}}{\text{kg}}$$

State-13 (given $p_{13} = p_6, \varepsilon_{\text{reg}}$):

$$h_{13} = h_6 + \varepsilon_{\text{reg}}\left(h_{12} - h_6\right) = 472.31 + (0.75)(1303.20 - 472.31) = 1095.48\,\frac{\text{kJ}}{\text{kg}}$$

State-14 (given $p_{14} = p_1$):

$$h_{14} = h_{12} - \left(h_{13} - h_6\right) = 1303.20 - (1095.48 - 472.31) = 680.03\,\frac{\text{kJ}}{\text{kg}}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-3): $\dot{W}_C = \dot{m}\left(h_3 - h_1\right) = (8)(430.86 - 300.19) = 1045.36 \text{ kW}$

Device-B (3-4): $\dot{Q}_{\text{out}} = \dot{m}\left(h_3 - h_4\right) = (8)(430.86 - 300.19) = 1045.36 \text{ kW}$

Device-C (4-6): $\dot{W}_C = \dot{m}\left(h_6 - h_4\right) = (8)(472.31 - 300.19) = 1376.96 \text{ kW}$

Device-D (6-7): $\dot{Q}_{\text{in}} = \dot{m}\left(h_7 - h_6\right) = (8)(1636.00 - 472.31) = 9309.52 \text{ kW}$

Device-E (7-9): $\dot{W}_T = \dot{m}\left(h_7 - h_9\right) = (8)(1636.00 - 1229.84) = 3249.28 \text{ kW}$

Device-F (9-10): $\dot{Q}_{\text{in}} = \dot{m}\left(h_{10} - h_9\right) = (8)(1636.00 - 1229.84) = 3249.28 \text{ kW}$

Device-G (10-12): $\dot{W}_T = \dot{m}\left(h_{10} - h_{12}\right) = (8)(1636.00 - 1303.20) = 2662.40 \text{ kW}$

Device-H (12-1): $\dot{Q}_{\text{out}} = \dot{m}\left(h_{12} - h_1\right) = (8)(1303.20 - 300.19) = 8024.08 \text{ kW}$

Now looking at the regenerator

$$\dot{Q}_{\text{reg}} = \dot{m}\left(h_{13} - h_6\right) = (8)(1095.48 - 472.31) = 4985.36 \text{ kW}$$

Applying the adjustments to heat transfer due to the regenerator

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{in,total}} - \dot{Q}_{\text{reg}} = (9309.52 + 3249.28) - 4985.36 = 7573.44 \text{ kW}$$

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{out,total}} - \dot{Q}_{\text{reg}} = (1045.36 + 8024.08) - 4985.36 = 4084.08 \text{ kW}$$

The net power is given as

$$\dot{W}_{\text{net}} = \dot{W}_{T,\text{total}} - \dot{W}_{C,\text{total}};$$

$$\Rightarrow \dot{W}_{\text{net}} = (3249.28 + 2662.40) - (1045.36 + 1376.96);$$

$$\Rightarrow \dot{W}_{\text{net}} = 3489.36 \text{ kW}$$

The thermal efficiency and the back work ratio are

$$\text{BWR} = \frac{\dot{W}_{C,\text{total}}}{\dot{W}_{T,\text{total}}};$$

$$\Rightarrow \text{BWR} = \frac{1045.36 + 1376.96}{3249.28 + 2662.40};$$

$$\Rightarrow \text{BWR} = 41.0\%$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$

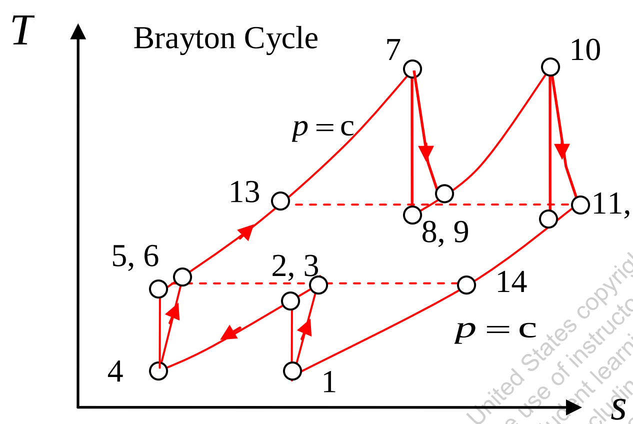
$$\Rightarrow \eta_{\text{th}} = \frac{3489.36}{7573.44};$$

$$\Rightarrow \eta_{\text{th}} = 46.1\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-30 [OKN] Air enters steadily the first compressor of the gas turbine at 100 kPa and 300 K with a mass flow rate of 50 kg/s. The pressure ratio across the two-stage compressor and turbine is 15. The intercooler and reheater each operate at an intermediate pressure given by the square root of the product of the first compressor and turbine inlet pressures. The inlet temperature of each turbine is 1400 K and that of the second compressor is 300 K. The isentropic efficiency of each compressor and turbine is 80% and the regenerator effectiveness is also 80%. Determine (a) the thermal efficiency (η_{th}). Use the IG model for air. (b) **What-if Scenario:** What would the thermal efficiency of the cycle be if the turbine and compressor efficiency increased to 90%?

SOLUTION



State-1 (given $p_1, T_1, \dot{m}$):

$$p_{r1} = 1.3860$$

$$h_1 = 300.19 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given $p_2 = \sqrt{p_1 p_7}, s_2 = s_1$):

$$p_{r2} = \left(\frac{p_2}{p_1}\right) p_{r1} = \left(\frac{\sqrt{(100)(1500)}}{100}\right)(1.3860) = 5.368$$

$$T_2 = 440.81 \text{ K}; \; h_2 = 442.44 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, \eta_C$):

$$h_3 = h_1 + \frac{h_2 - h_1}{\eta_C} = 300.19 + \frac{442.44 - 300.19}{0.80} = 478.00 \frac{\text{kJ}}{\text{kg}}$$

State-4 (given $p_4 = p_2, T_4$):

$p_{r4} = 1.3860$

$h_4 = 300.19 \dfrac{\text{kJ}}{\text{kg}}$

State-5 (given $p_5 = 15p_1, s_5 = s_4$):

$$p_{r5} = \left(\frac{p_5}{p_4}\right) p_{r4} = \left(\frac{1500}{\sqrt{(100)(1500)}}\right)(1.3860) = 5.368$$

$T_5 = 440.81\,\text{K};\; h_5 = 442.44 \dfrac{\text{kJ}}{\text{kg}}$

State-6 (given $p_6 = p_5, \eta_C$):

$$h_6 = h_4 + \frac{h_5 - h_4}{\eta_C} = 300.19 + \frac{442.44 - 300.19}{0.80} = 478.00 \frac{\text{kJ}}{\text{kg}}$$

State-7 (given $p_7 = p_5, T_7$):

$p_{r7} = 450.50$

$h_7 = 1515.40 \dfrac{\text{kJ}}{\text{kg}}$

State-8 (given $p_8 = \sqrt{p_1 p_7}, s_8 = s_7$):

$$p_{r8} = \left(\frac{p_8}{p_7}\right) p_{r7} = \left(\frac{\sqrt{(100)(1500)}}{1500}\right)(450.50) = 116.32$$

$T_8 = 1004.90\,\text{K};\; h_8 = 1051.70 \dfrac{\text{kJ}}{\text{kg}}$

State-9 (given $p_9 = p_8, \eta_T$):

$$h_9 = h_7 - \eta_T\left(h_7 - h_8\right) = 1515.40 - (0.80)(1515.40 - 1051.70) = 1144.44 \frac{\text{kJ}}{\text{kg}}$$

State-10 (given $p_{10} = p_8, T_{10}$):

$p_{r10} = 450.50$

$h_{10} = 1515.40 \dfrac{\text{kJ}}{\text{kg}}$

State-11 (given $p_{11} = p_1, s_{11} = s_{10}$):

$$p_{r11} = \left(\frac{p_1}{p_{10}}\right) p_{r10} = \left(\frac{100}{\sqrt{(100)(1500)}}\right)(450.50) = 116.32$$

$$T_{11} = 1004.90 \text{ K}; \; h_{11} = 1051.70 \frac{\text{kJ}}{\text{kg}}$$

State-12 (given $p_{12} = p_{11}, \eta_T$):

$$h_{12} = h_{10} - \eta_T (h_{10} - h_{11}) = 1515.40 - (0.80)(1515.40 - 1051.70) = 1144.44 \frac{\text{kJ}}{\text{kg}}$$

State-13 (given $p_{13} = p_6, \varepsilon_{\text{reg}}$):

$$h_{13} = h_6 + \varepsilon_{\text{reg}} (h_{12} - h_6) = 478.00 + (0.80)(1144.44 - 478.00) = 1011.15 \frac{\text{kJ}}{\text{kg}}$$

State-14 (given $p_{14} = p_1$):

$$h_{14} = h_{12} - (h_{13} - h_6) = 1144.44 - (1011.15 - 478.00) = 611.29 \frac{\text{kJ}}{\text{kg}}$$

Ignoring the regenerator, an energy analysis is carried out for each device as follows

Device-A (1-3): $\dot{W}_C = \dot{m}(h_3 - h_1) = (50)(478.00 - 300.19) = 8890.50 \text{ kW}$

Device-B (3-4): $\dot{Q}_{\text{out}} = \dot{m}(h_3 - h_4) = (50)(478.00 - 300.19) = 8890.50 \text{ kW}$

Device-C (4-6): $\dot{W}_C = \dot{m}(h_6 - h_4) = (50)(478.00 - 300.19) = 8890.50 \text{ kW}$

Device-D (6-7): $\dot{Q}_{\text{in}} = \dot{m}(h_7 - h_6) = (50)(1515.40 - 478.00) = 51870.00 \text{ kW}$

Device-E (7-9): $\dot{W}_T = \dot{m}(h_7 - h_9) = (50)(1515.40 - 1144.44) = 18548.00 \text{ kW}$

Device-F (9-10): $\dot{Q}_{\text{in}} = \dot{m}(h_{10} - h_9) = (50)(1515.40 - 1144.44) = 18548.00 \text{ kW}$

Device-G (10-12): $\dot{W}_T = \dot{m}(h_{10} - h_{12}) = (50)(1515.40 - 1144.44) = 18548.00 \text{ kW}$

Device-H (12-1): $\dot{Q}_{\text{out}} = \dot{m}(h_{12} - h_1) = (50)(1144.44 - 300.19) = 42212.50 \text{ kW}$

Now looking at the regenerator

$$\dot{Q}_{\text{reg}} = \dot{m}(h_{13} - h_6) = (50)(1011.15 - 478.00) = 26657.50 \text{ kW}$$

Applying the adjustments to heat transfer due to the regenerator

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{in,total}} - \dot{Q}_{\text{reg}} = (51870.00 + 18548.00) - 26657.50 = 43760.50 \text{ kW}$$

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{out,total}} - \dot{Q}_{\text{reg}} = (8890.50 + 42212.50) - 26657.50 = 24445.50 \text{ kW}$$

The net power is given as

$$\dot{W}_{\text{net}} = \dot{W}_{T,\text{total}} - \dot{W}_{C,\text{total}};$$
$$\Rightarrow \dot{W}_{\text{net}} = (18548.00 + 18548.00) - (8890.50 + 8890.50);$$
$$\Rightarrow \dot{W}_{\text{net}} = 19315.00 \text{ kW}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}};$$
$$\Rightarrow \eta_{\text{th}} = \frac{19315.00}{43760.50};$$
$$\Rightarrow \eta_{\text{th}} = 44.1\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-31 [OKE] Consider an ideal Ericsson cycle with air as working fluid executed in a steady-flow system. Air is at 30°C and 115 kPa at the beginning of the isothermal compression process during which 155 kJ/kg of heat is rejected. Heat transfer to air occurs at 1250 K. Determine (a) the maximum pressure in the cycle, (b) the net work output per unit mass (w_{net}) of air and (c) the thermal efficiency (η_{th}) of the cycle. Use the PG model.

SOLUTION

Given:

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-1 (given p_1, T_1)

State-2 (given $T_2 = T_1,\ q_{12}$):

$$\cancel{\frac{ds}{dt}}^{0} = (s_1 - s_2) + \frac{q_{12}}{T_2};\ \Rightarrow s_2 - s_1 = \frac{q_{12}}{T_2};\ \Rightarrow c_p \ln\cancel{\frac{T_2}{T_1}}^{0} - R\ln\frac{p_2}{p_1} = \frac{q_{12}}{T_2};\ \Rightarrow p_2 = p_1 e^{-\frac{q_{12}}{RT_2}};$$

$$\Rightarrow p_2 = (115)e^{-\left(\frac{-155}{(0.287)(303)}\right)} = 683.58 \text{ kPa}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1,\ T_4 = T_3$):

$$\cancel{\frac{ds}{dt}}^{0} = (s_3 - s_4) + \frac{q_{34}}{T_4};\ \Rightarrow s_4 - s_3 = \frac{q_{34}}{T_4};\ \Rightarrow q_{34} = T_4\left(c_p \ln\cancel{\frac{T_4}{T_3}}^{0} - R\ln\frac{p_4}{p_3}\right);$$

$$\Rightarrow q_{34} = -(1250)(0.287)\ln\left(\frac{115}{683.58}\right) = 639.44 \frac{\text{kJ}}{\text{kg}}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p \cancel{(T_2 - T_1)}^{0} + q_{21} = 155.00 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $w_T = c_p \cancel{(T_3 - T_4)}^{0} + q_{34} = 639.44 \frac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 639.44 - 155.00;$$

$$\Rightarrow w_{\text{net}} = 484.44 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{484.44}{639.44};$$

$$\Rightarrow \eta_{\text{th}} = 75.8\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-32 [OKG] Air enters the turbine of an Ericsson cycle at 1000 kPa and 1200 K, with a mass flow rate of 1 kg/s. The temperature and pressure at the inlet to the compressor are 250 K and 100 kPa respectively. Determine (a) the thermal efficiency (η_{th}) of the cycle, (b) the net work output (w_{net}) per unit mass of air. Use the PG model.

SOLUTION

Given:

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-1 (given $p_1, T_1, \dot{m}$)

State-2 (given $p_2 = p_3, T_2 = T_1$):

$$\cancel{\frac{ds}{dt}}^0 = (s_1 - s_2) + \frac{q_{12}}{T_2}; \Rightarrow s_2 - s_1 = \frac{q_{12}}{T_2}; \Rightarrow q_{12} = T_2 \left(c_p \ln \cancel{\frac{T_2}{T_1}}^0 - R \ln \frac{p_2}{p_1} \right);$$

$$\Rightarrow q_{12} = -(250)(0.287) \ln\left(\frac{1000}{100}\right) = -165.21 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given p_3, T_3)

State-4 (given $p_4 = p_1, T_4 = T_3$):

$$\cancel{\frac{ds}{dt}}^0 = (s_3 - s_4) + \frac{q_{34}}{T_4}; \Rightarrow s_4 - s_3 = \frac{q_{34}}{T_4}; \Rightarrow q_{34} = T_4 \left(c_p \ln \cancel{\frac{T_4}{T_3}}^0 - R \ln \frac{p_4}{p_3} \right);$$

$$\Rightarrow q_{34} = -(1200)(0.287) \ln\left(\frac{100}{1000}\right) = 793.01 \frac{\text{kJ}}{\text{kg}}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p \cancel{(T_2 - T_1)}^0 + q_{21} = 165.21 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $w_T = c_p \cancel{(T_3 - T_4)}^0 + q_{34} = 793.01 \frac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 793.01 - 165.21;$$

$$\Rightarrow w_{\text{net}} = 627.80 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{627.80}{793.01};$$

$$\Rightarrow \eta_{\text{th}} = 79.2\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-33 [OKK] An ideal Ericsson cycle with air as the working fluid has a compression ratio of 10. Isothermal expansion takes place at 1000 K. Heat transfer from the compressor occurs at 350 K. Determine (a) the net work (w_{net}) per kg of air in the cycle, and (b) the thermal efficiency (η_{th}).

SOLUTION

Given:

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-1 (given p_1, T_1)

State-2 (given $T_2 = T_1, r_p$):

$$\cancelto{0}{\frac{ds}{dt}} = (s_1 - s_2) + \frac{q_{12}}{T_2}; \Rightarrow s_2 - s_1 = \frac{q_{12}}{T_2}; \Rightarrow q_{12} = T_2\left(c_p \ln\cancelto{0}{\frac{T_2}{T_1}} - R\ln r_p\right);$$

$$\Rightarrow q_{12} = -(350)(0.287)\ln(10) = -231.29 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given $p_3 = p_2, T_3$)

State-4 (given $p_4 = p_1, T_4 = T_3$):

$$\cancelto{0}{\frac{ds}{dt}} = (s_3 - s_4) + \frac{q_{34}}{T_4}; \Rightarrow s_4 - s_3 = \frac{q_{34}}{T_4}; \Rightarrow q_{34} = T_4\left(c_p \ln\cancelto{0}{\frac{T_4}{T_3}} - R\ln\frac{1}{r_p}\right);$$

$$\Rightarrow q_{34} = -(1000)(0.287)\ln\left(\frac{1}{10}\right) = 660.84 \frac{\text{kJ}}{\text{kg}}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p \cancelto{0}{(T_2 - T_1)} + q_{21} = 231.29 \dfrac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $w_T = c_p \cancelto{0}{(T_3 - T_4)} + q_{34} = 660.84 \dfrac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$$w_{\text{net}} = w_T - w_C;$$

$$\Rightarrow w_{\text{net}} = 660.84 - 231.29;$$

$$\Rightarrow w_{\text{net}} = 429.55 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}};$$

$$\Rightarrow \eta_{\text{th}} = \frac{429.55}{660.84};$$

$$\Rightarrow \eta_{\text{th}} = 65.0\%$$

TEST Solution Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

8-1-34 [OKL] Hydrogen enters the turbine of an Ericsson cycle at 1500 kPa and 900 K, with a mass flow rate of 1 kg/s. The temperature and pressure at the inlet to the compressor are 320 K and 150 kPa respectively. Determine (a) the thermal efficiency (η_{th}) of the cycle, (b) the net work (w_{net}) output per unit mass of air. Use the PG model. (b) **What-if Scenario:** How would the answers change if the IG model were used?

SOLUTION

Given:

$$R = 4.124 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State-1 (given p_1, T_1)

State-2 (given $p_2 = p_3, T_2 = T_1$):

$$\cancel{\frac{ds}{dt}}^{0} = (s_1 - s_2) + \frac{q_{12}}{T_2};\ \Rightarrow s_2 - s_1 = \frac{q_{12}}{T_2};\ \Rightarrow q_{12} = T_2\left(c_p \cancel{\ln\frac{T_2}{T_1}}^{0} - R\ln\frac{p_2}{p_1}\right);$$

$$\Rightarrow q_{12} = -(320)(4.124)\ln\left(\frac{1500}{150}\right) = -3038.68 \frac{\text{kJ}}{\text{kg}}$$

State-3 (given p_3, T_3)

State-4 (given $p_4 = p_1, T_4 = T_3$):

$$\cancel{\frac{ds}{dt}}^{0} = (s_3 - s_4) + \frac{q_{34}}{T_4};\ \Rightarrow s_4 - s_3 = \frac{q_{34}}{T_4};\ \Rightarrow q_{34} = T_4\left(c_p \cancel{\ln\frac{T_4}{T_3}}^{0} - R\ln\frac{p_4}{p_3}\right);$$

$$\Rightarrow q_{34} = -(900)(4.124)\ln\left(\frac{150}{1500}\right) = 8546.27 \frac{\text{kJ}}{\text{kg}}$$

An energy analysis is carried out for each device as follows

Device-A (1-2): $w_C = c_p \cancel{(T_2 - T_1)}^{0} + q_{21} = 3038.68 \frac{\text{kJ}}{\text{kg}}$

Device-B (3-4): $w_T = c_p \cancel{(T_3 - T_4)}^{0} + q_{34} = 8546.27 \frac{\text{kJ}}{\text{kg}}$

The net work per unit mass is

$$w_{net} = w_T - w_C;$$

$$\Rightarrow w_{net} = 8546.27 - 3038.68;$$

$$\Rightarrow w_{net} = 5507.59 \frac{\text{kJ}}{\text{kg}}$$

The thermal efficiency is

$$\eta_{th} = \frac{w_{net}}{q_{in}};$$

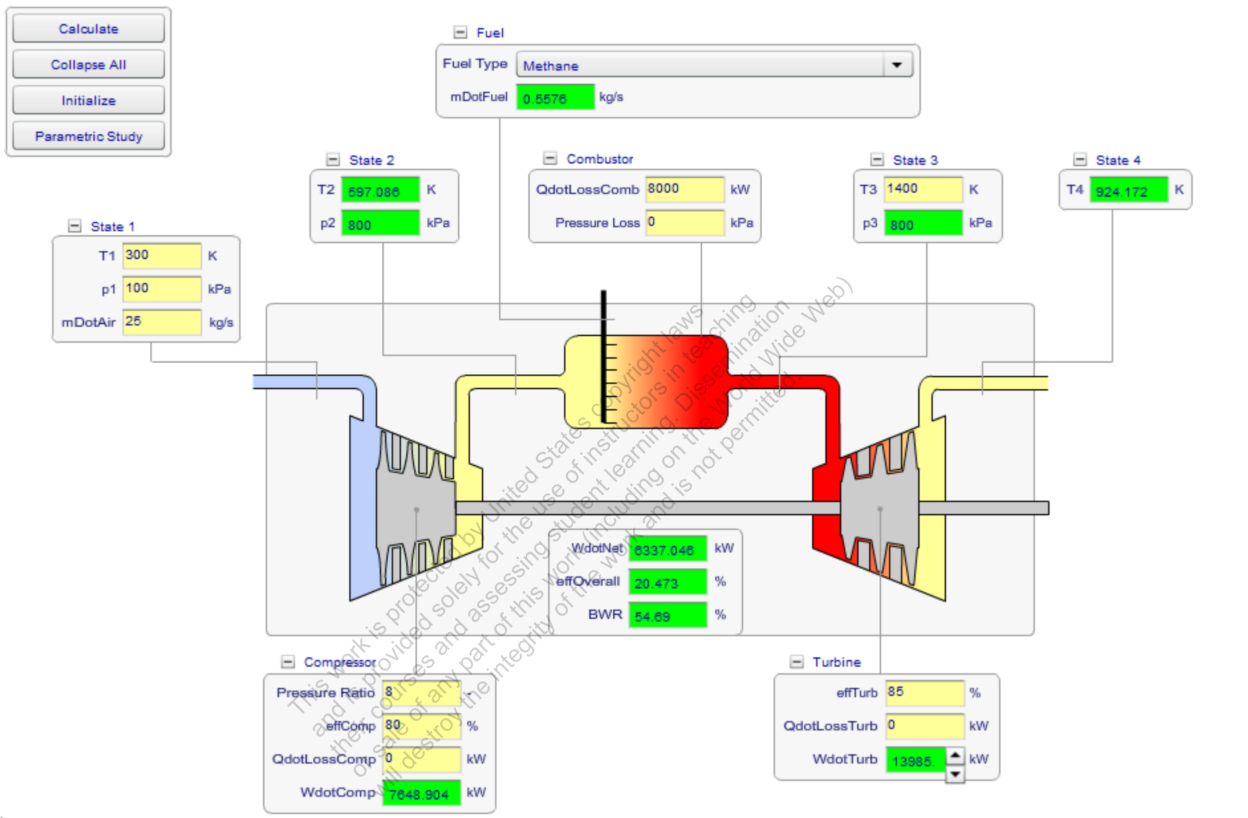
$$\Rightarrow \eta_{th} = \frac{5507.59}{8546.27};$$

$$\Rightarrow \eta_{th} = 64.4\%$$

TEST Solution and What-if Scenario Use the PG (or IG based on problem statement) gas-power cycle TESTcalc to verify the solution and perform the what-if study. The TEST-code for this problem can be found in the problem module of the professional TEST site at www.thermofluids.net.

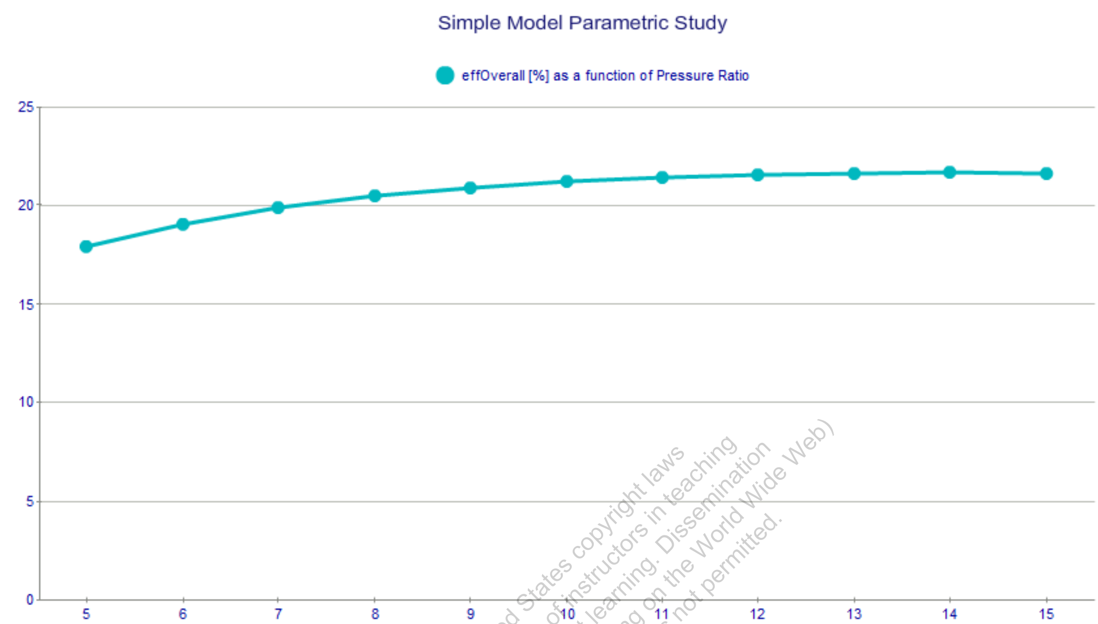
8-1-35 [BCQ] A gas turbine power plant operates on a simple Brayton cycle with air as the working fluid having a pressure ratio of 8. The compressor efficiency is 80% and its inlet conditions are 100 kPa and 300 K with a mass flow rate of 25 kg/s. The turbine efficiency is 85% and its inlet temperature is 1400 K. Heat is rejected to the surroundings at a rate of 8000 kW. Using Gas Turbine Simulator RIA (linked from left margin) determine (a) the net power output, (b) the thermal efficiency (η_{th}), and (c) the back work ratio.

SOLUTION



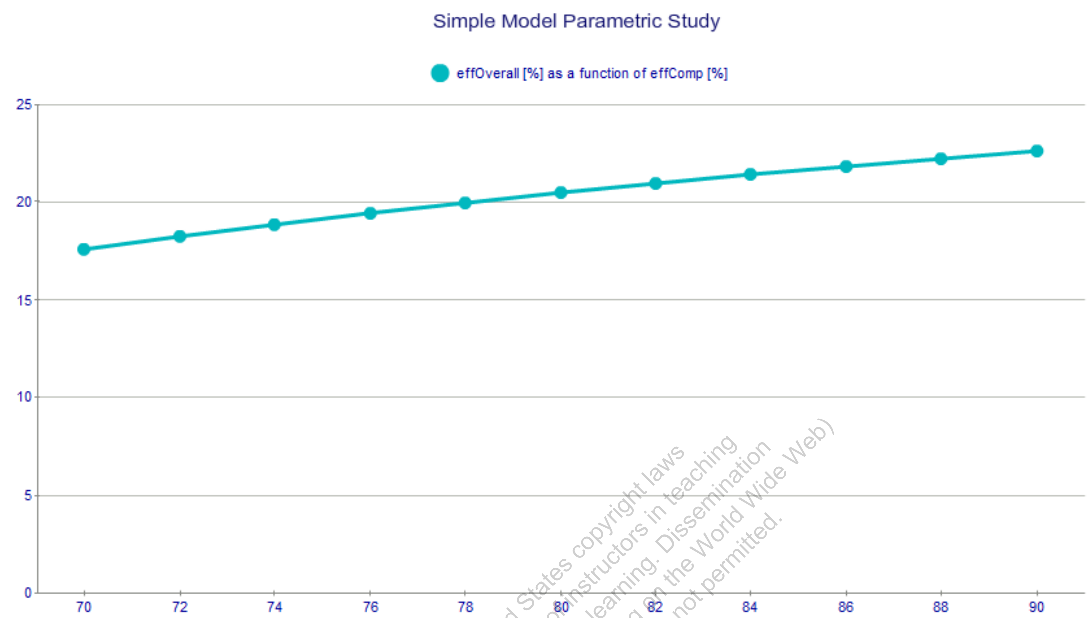
8-1-36 [BCT] Using the gas turbine power plant described in the previous problem, 8-1-35[BCQ], plot how thermal efficiency ($\eta_{th,Brayton}$) varies with pressure ratio varying from 5 to 15, all other input parameters remaining unchanged.

SOLUTION



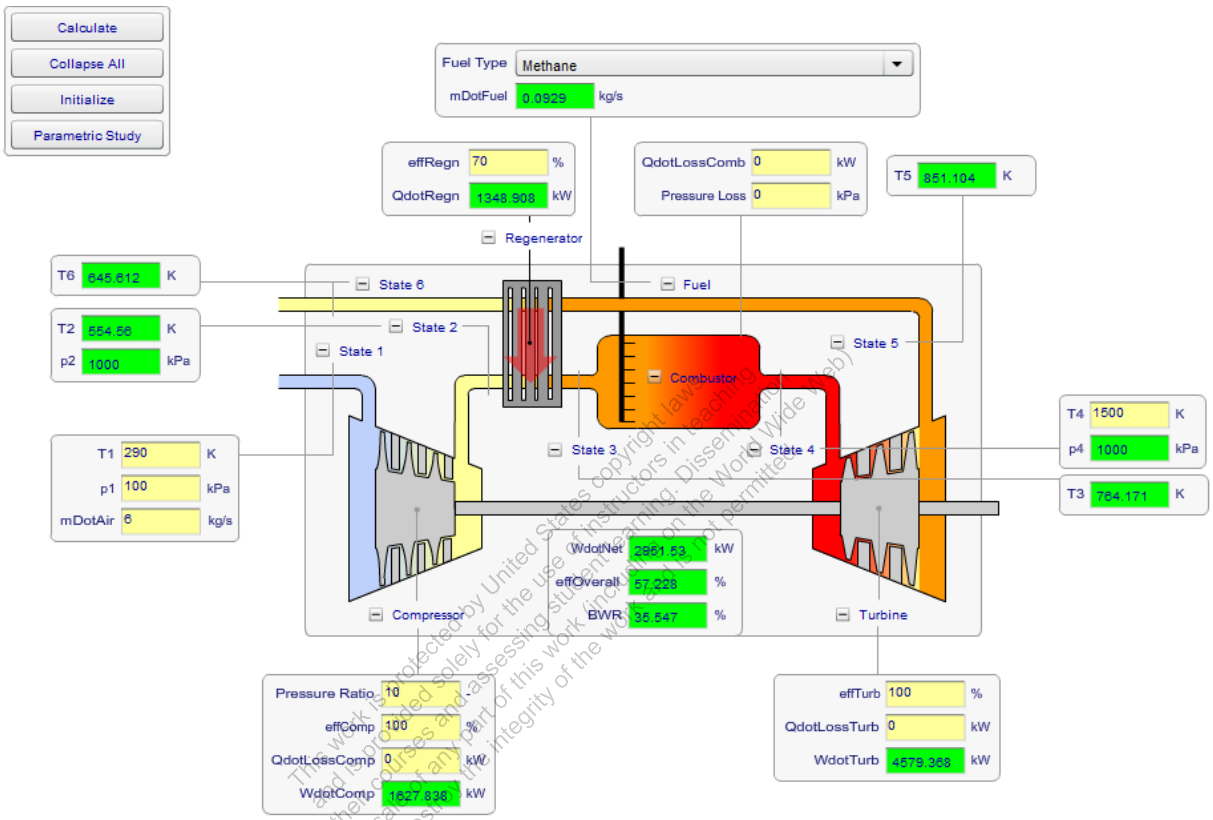
8-1-37 [BCY] Using the gas turbine power plant described in the previous problem, 8-1-35[BCQ], plot how thermal efficiency ($\eta_{th,Brayton}$) varies with compressor efficiency varying from 70% to 90%, all other input parameters remaining unchanged.

SOLUTION



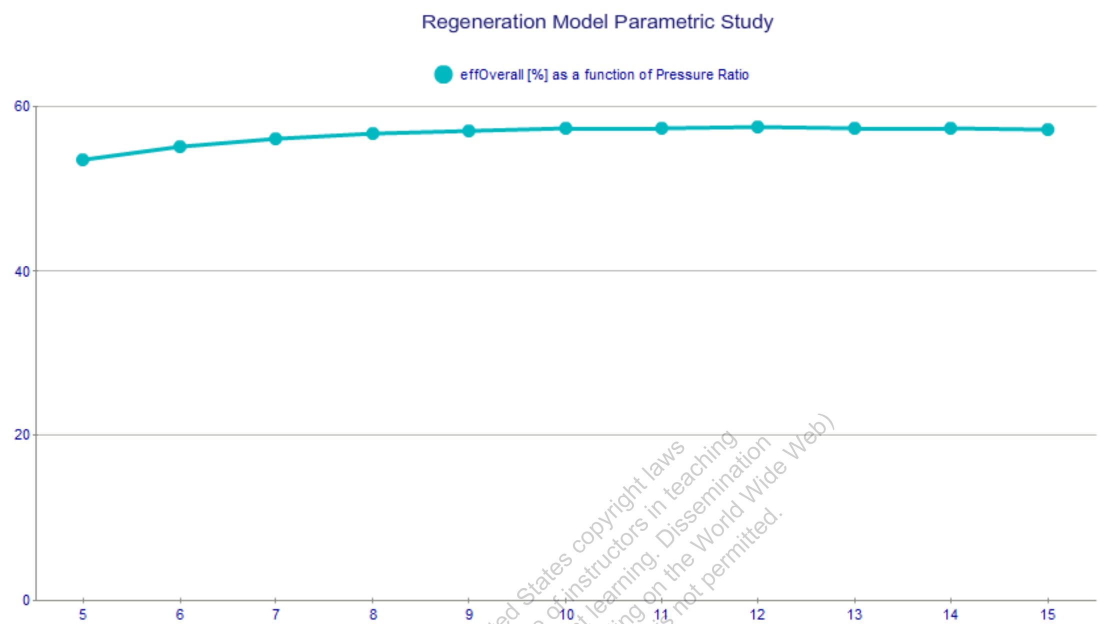
8-1-38 [BCD] Air enters the compressor of an ideal air standard Brayton cycle at 100 kPa, 290 K, with a mass flow rate ($\dot{m}$) of 6 kg/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1500 K. If a regenerator with an effectiveness of 70% is incorporated in the cycle, using Gas Turbine Simulator RIA determine (a) the net power output ($\dot{W}_{net}$), (b) the thermal efficiency ($\eta_{th,Brayton}$) of the cycle.

SOLUTION



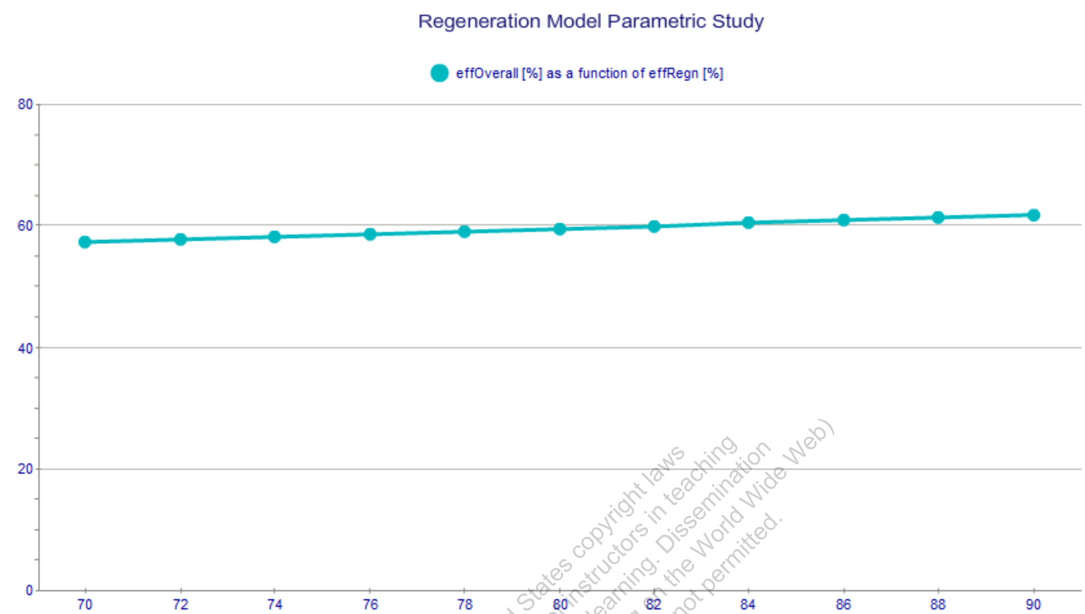
8-1-39 [BCM] Using the air standard Brayton cycle described in the previous problem, 8-1-38[BCD], plot how thermal efficiency varies with pressure ratio varying from 5 to 15, all other input parameters remaining unchanged.

SOLUTION



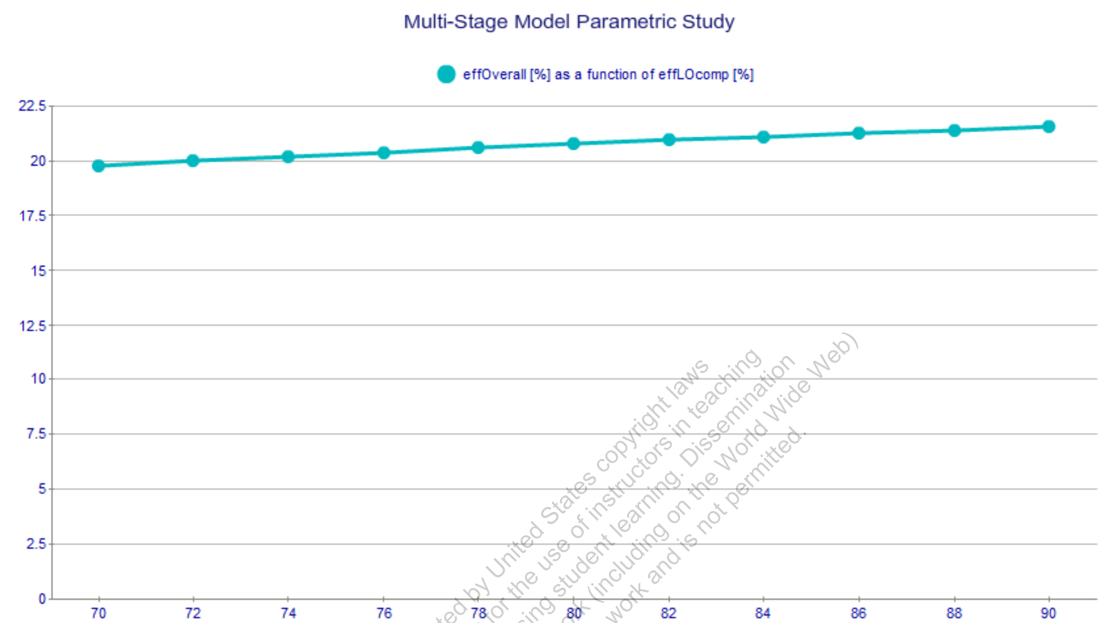
8-1-40 [BCJ] Using the air standard Brayton cycle described in the previous problem, 8-1-38[BCD], plot how thermal efficiency ($\eta_{\text{th,Brayton}}$) varies with regenerative effectiveness varying from 70% to 90%, all other input parameters remaining unchanged.

SOLUTION



8-1-41 [BCW] The gas turbine power plant described in problem, 8-1-35[BCQ], is converted into a multi-stage cycle with two stages of compression and two stages of expansion. The pressure ratio across each stage of the compressor and the turbine is 2. Using the Gas Turbine Simulator RIA, plot how thermal efficiency ($\eta_{th,Brayton}$) varies with first compressor efficiency varying from 70% to 90%.

SOLUTION



8-1-42 [BVR] Repeat the previous problem, 8-1-41[BCW], assuming a regenerator with 80% effectiveness is added at the end of second compressor. Plot how thermal efficiency ($\eta_{\text{th,Brayton}}$) varies with first compressor efficiency varying from 70% to 90%.

SOLUTION