15-2-1 [BHO] Air at 1 MPa and 550°C enters a converging nozzle of throat area 50 cm^2 with a velocity of 100 m/s. Determine the mass flow rate for a back pressure of (a) 0.7 MPa, (b) 0.4 MPa and (c) 0.2 MPa.

SOLUTION:

Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain: $R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet state (Given: $p = 1.0$ MPa; $T = 823$ K; $V = 100\ \dfrac{\text{m}}{\text{s}}$):

$$M = \frac{V}{\sqrt{1000kRT}} = \frac{100}{\sqrt{(1000)(1.4)(0.287)(823.15)}} = 0.17388;$$

$$\frac{p}{p_t} = 0.97912; \quad \frac{T}{T_t} = 0.99399;$$

$$\Rightarrow p_t = \frac{1000}{0.97912} = 1021.33 \text{ kPa}; \quad T_t = \frac{823.15}{0.99399} = 828.13 \text{ K}$$

For the chocked exit,

Exit state (Given: $M = 1$; $T_{te} = T_{ti}$; ; $p_t = p_{ti}$; $A = 50$ cm^2):

$$\frac{p}{p_t} = 0.5283; \quad \frac{T}{T_t} = 0.833;$$

$$\Rightarrow p = (1021.33)(0.5283) = 539.57 \text{ kPa}; \quad T = (828.13)(0.833) = 689.83 \text{ K}$$

For back pressures less than 539.57 kPa, that is, for 400 kPa, and 200 kPa the flow is choked. The mass flow rate for the choked flow can be calculated as,

$$\dot{m} = \rho AV = \frac{p}{RT} AM\sqrt{1000kRT} = \frac{p}{\sqrt{RT}} AM\sqrt{1000k} = \frac{(539.57)(0.005)\sqrt{(1000)(1.4)}}{\sqrt{(0.287)(689.83)}} = 7.17\ \frac{\text{kg}}{\text{s}}$$

For a back pressure of 700 kPa, the flow is subsonic and the back pressure must be equal to the exit pressure.

For isentropic flow,

Exit state (Given: $p = 700$ kPa; $p_{te} = p_{ti}$; $T_{te} = T_{ti}$; $A = 50$ cm^2):

For p/p_t =700/1021.33=0.68538 (using Table H-1 or the gas dynamics TESTcalc):

$$M = 0.75495; \quad \frac{T}{T_t} = 0.89774$$

$$\Rightarrow T = (0.89774)(828.13) = 743.45 \text{ K}$$

$$\dot{m} = \rho AV = \frac{p}{\sqrt{RT}} AM\sqrt{1000k} = \frac{(700)(0.005)(0.75495)\sqrt{(1000)(1.4)}}{\sqrt{(0.287)(743.45)}} = 6.77 \ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-2 [BHA] Air enters a converging-diverging nozzle at 700 K and 1000 kPa with negligible velocity. The exit Mach number is 2 and the throat area is 20 cm^2. Assuming steady isentropic flow, determine (a) the throat velocity, (b) mass flow rate (m) and (c) the exit area.

SOLUTION:

Working fluid: air; From Table C-1 (or gas dynamics TEScalc),

obtain $R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet state (Given: $p, T,$ and $V = 0$):

$$T_t = T = 700\text{ K};\ p_t = p = 1000\text{ kPa}$$

For the supersonic flow in the diverging section, the throat Mach number must be 1.
Inlet state (Given: $M = 1,\ p_t = p_{ti}, T_t = T_{ti}$):

$$\frac{T_t}{T} = 1.2;\quad T = \frac{T_t}{1.2} = \frac{700}{1.2} = 583.3\text{ K}$$

$$V = M\sqrt{1000kRT} = (1)\sqrt{1000(1.4)(0.287)(583.3)} = 484\ \frac{\text{m}}{\text{s}}$$

$$\dot{m} = Ap_t\sqrt{\frac{1000\cdot k}{RT_t}}\left(\frac{k+1}{2}\right)^{\frac{k+1}{2-2k}} = (0.002)(1000)\sqrt{\frac{(1000)(1.4)}{(0.287)(700)}}\left(\frac{1.4+1}{2}\right)^{\frac{1.4+1}{2-2k}} = 3.05\ \frac{\text{kg}}{\text{s}}$$

Exit state (Given: $M = 2,\ p_t = p_{ti}, T_t = T_{ti}$):

$$\left(\frac{A}{A_*}\right)_{M=2} = 1.68 \text{ (Table H-1)}$$

$$A = 1.68A_* = 33.75\text{ cm}^2$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-3 [BHB] Stationary nitrogen at 0.9 MPa and 450 K is accelerated isentropically to a Mach number of 0.6. Determine (a) the temperature (*T*) and the pressure (*p*) of nitrogen after acceleration.

SOLUTION:
Working fluid: nitrogen; From Table C-1(or the gas dynamics TESTcalc), obtain:
$R = 0.297\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet state (Given: p_t, T_t):
$p_t = 0.9\ \text{MPa};\ T_t = 450\ \text{K}$

Local state (Given: $M = 0.6,\ p_t = p_{ti}, T_t = T_{ti}$):

$$\frac{T_t}{T} = 1 + \frac{k-1}{2}M^2; \quad \Rightarrow T = T_t\left(1 + \frac{k-1}{2}M^2\right)^{-1} = 419.776 = 420\ \text{K}$$

$$\frac{p_t}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}; \quad \Rightarrow p = p_t\left(\left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}\right)^{-1} = 0.705604 = 0.7\ \text{MPa}$$

TEST Solution:
Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-4 [BHS] Air is expanded in an isentropic nozzle from 1.0 MPa and 800 K to an exhaust pressure of 100 kPa. If air enters the nozzle with a velocity of 80 m/s, determine the exhaust velocity.

SOLUTION:

Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain $R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet state (Given: $p = 1.0$ MPa; $T = 800$ K; $V = 80\ \frac{\text{m}}{\text{s}}$):

$$c = \sqrt{1000kRT} = \sqrt{1000(1.4)(0.287)(800)} = 567\ \frac{\text{m}}{\text{s}};$$

$$\Rightarrow M = \frac{V}{c} = 0.14$$

From Table H-1 (or the gas dynamics TESTcalc)

$$\frac{p}{p_t} = 0.986; \quad \frac{T}{T_t} = 0.996;$$

$$\Rightarrow p_t = 1014 \text{ kPa}; \quad T_t = 803 \text{ K};$$

Exit state (Given: $p_{te} = p_{ti}$; $T_{te} = T_{ti}$; $p_e = 100$ kPa;):

$$\frac{p}{p_t} = 0.0986;$$

For this pressure ratio, from Table H-1 (or gas dynamics TESTcalc) we obtain:

$$M = 2.166; \quad \frac{T}{T_t} = 0.516;$$

$$\Rightarrow T = 414.5 \text{ K};$$

$$V = Mc = M\sqrt{1000kRT} = (2.166)\sqrt{(1000)(1.4)(0.287)(414.5)} = 883.6\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-5 [BHH] Helium enters a converging-diverging nozzle at a pressure of 800 kPa, 700 K and 100 m/s. Determine (a) the lowest temperature and (b) the lowest pressure that can be obtained at the throat of the nozzle. (c) What-if Scenario: What would the lowest pressure in (b) be if the working gas were air instead?

SOLUTION:

Working fluid: helium; From Table C-1, obtain: $R = 2.0770\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.67$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet state (Given: $p = 800$ kPa; $T = 700$ K; $V = 100\ \frac{\text{m}}{\text{s}}$):

$$M = \frac{V}{\sqrt{1000(kRT)}} = \frac{100}{\sqrt{1000(1.67)(2.0770)(700)}} = 0.0642$$

Using formulas (or the gas dynamics TESTcalc)

$$\frac{p}{p_t} = 0.99657,\ \frac{T}{T_t} = 0.99863,\ \frac{A}{A_*} = 8.78508$$

$$\Rightarrow p_t = \frac{800}{0.99657} = 802.75 \text{ kPa};\ T_t = \frac{700}{0.99863} = 700.96 \text{ K}$$

The lowest pressure occurs at throat when the Mach number is the highest, that is, 1.

Throat state (Given: $p_t = p_{ti};\ T_t = T_{ti};\ M = 1$):

At $M_2 = 1.00$, using isentropic formulas or gas dynamics TESTcalc, we get $\frac{p}{p_t} = 0.4871$, $\frac{T}{T_t} = 0.7500$

$$\Rightarrow p = p_t(0.4871) = 391 \text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-6 [BHN] Nitrogen flows steadily through a variable-area duct with a mass flow rate of 3 kg/s. It enters the duct at 1200 kPa and 250^{o}C with a low velocity and expands to a pressure of 300 kPa. The duct is designed so that the flow can be approximated as isentropic. Determine (a) the Mach number, (b) flow area and (c) the velocity at the exit and the throat.

SOLUTION:

Working fluid: nitrogen; From Table C-1, obtain: $R = 0.297\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$

Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet state (Given: $p, T, \dot{m}$, and $V = 0$):

For the supersonic flow in the diverging section, the throat Mach number must be 1.
Throat state (Given: $M = 1,\ p_t = p_{ti}, T_t = T_{ti}$):

$$\frac{T}{T_2} = 0.833; \quad \Rightarrow T = 436\ \text{K}$$

$$V = M\sqrt{1000kRT} = (1)\sqrt{1000(1.4)(0.297)(436)} = 426\ \frac{\text{m}}{\text{s}}$$

$$\dot{m} = Ap_t\sqrt{\frac{1000\cdot k}{RT_t}}\left(\frac{k+1}{2}\right)^{\frac{k+1}{2-2k}}$$

$$\Rightarrow A = 0.00144\ \text{m}^2 = 14.4\ \text{cm}^2$$

Exit state (Given: $p, p_t = p_{ti}, T_t = T_{ti}$):

$$\frac{p}{p_t} = 0.25; \quad \Rightarrow M = 1.56;$$

$$\Rightarrow \frac{T}{T_t} = 0.673; \quad \Rightarrow T = 352\ \text{K};$$

$$V = M\sqrt{1000kRT} = (1.56)\sqrt{1000(1.4)(0.297)(352)} = 596.3\ \frac{\text{m}}{\text{s}}$$

$$\frac{A}{A_*} = \frac{A}{A_{th}} = 1.218; \quad \Rightarrow A = 0.00175\ \text{m}^2 = 17.5\ \text{cm}^2$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-7 [BHE] A convergent nozzle has an exit area of 4 cm^2. Air enters the nozzle with a total pressure of 1200 kPa, and a total temperature of 400 K. Assuming isentropic flow, determine the mass flow rate (m) for back pressure of (a) 900 kPa, (b) 634 kPa and (c) 400 kPa.

SOLUTION:

Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain:

$R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

For chocked exit,

Exit state (Given: $M = 1;\ T_{te} = T_t;\ \ p_t = p_{ti};\ A = 4\ \text{cm}^2$):

$$\frac{p}{p_t} = 0.5283;\quad \frac{T}{T_t} = 0.833;$$

$$\Rightarrow p = (1200)(0.5283) = 634.0\ \text{kPa};\ \ T = (400)(0.833) = 333.4\ \text{K}$$

For back pressures less than or equal to 634 kPa, that is, for 634 kPa and 400 kPa, the flow is choked. The mass flow rate for the choked flow can be calculated as,

$$\dot{m} = \rho AV = \frac{p}{RT} AM\sqrt{1000kRT} = \frac{p}{\sqrt{RT}} AM\sqrt{1000k} = \frac{(634)(0.0004)\sqrt{(1000)(1.4)}}{\sqrt{(0.287)(333.4)}} = 0.97\ \frac{\text{kg}}{\text{s}}$$

For a back pressure of 900 kPa, the flow is subsonic and the back must be equal to the exit pressure.

For isentropic flow,

Exit state (Given: $p = 900$ kPa; $p_{te} = p_t;\ T_{te} = T_t;\ A = 4\ \text{cm}^2$):

For $p/p_t = 900/1200 = 0.75$ (from Table H-1 or gas dynamics TESTcalc) :

$$M = 0.655;\quad \frac{T}{T_t} = 0.92$$

$$\Rightarrow T = 368.5\ \text{K}$$

$$\dot{m} = \rho AV = \frac{p}{RT} AM\sqrt{1000kRT} = \frac{p}{\sqrt{RT}} AM\sqrt{1000k} = \frac{(900)(0.0004)(0.655)\sqrt{(1000)(1.4)}}{\sqrt{(0.287)(368.5)}} = 0.857\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-8 [BHI] Air enters a nozzle at 3000 kPa, 400 K and a velocity of 180 m/s. Assuming isentropic flow, determine (a) the temperature (T), (b) the pressure (p) of the air at a location where the air velocity equals to the velocity of sound, and (c) the ratio of area at this location to the entrance area. (d) What-if Scenario: What would the temperature in (a) be if air entered the nozzle at 200 m/s?

SOLUTION:

Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain:
$R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet state (Given: $p = 3000$ kPa; $T = 400$ K; $V = 180\ \dfrac{\text{m}}{\text{s}}$):

$$M = \frac{V}{\sqrt{1000kRT}} = 0.45;$$

$$\frac{p}{p_t} = 0.87; \quad \frac{T}{T_t} = 0.96;$$

$$\Rightarrow p_t = 3445 \text{ kPa}; \quad T_t = 416 \text{ K}$$

For the chocked state (Given: $M = 1$; $T_t = T_{ti}$; $p_t = p_{ti}$;):

$$\frac{p}{p_t} = 0.5283; \quad \frac{T}{T_t} = 0.833;$$

$$\Rightarrow p = (3000)(0.5283) = 1820 \text{ kPa}; \quad T = (400)(0.833) = 346 \text{ K}$$

$$\frac{A}{A_i} = \frac{A_*}{A_i} = \left(\frac{A_i}{A_*}\right)^{-1} = \left(\left[\frac{A_i}{A_*}\right]_{M=1}\right)^{-1} = (1.451)^{-1} = 0.689$$

TEST Solution:
Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-9 [BHL] An ideal gas with k = 1.5 is flowing through a nozzle such that the Mach number is 3 where the flow area is 30 cm^2. Assuming the flow to be isentropic, determine (a) the flow area at the location where the Mach number is 1.4.

SOLUTION:

$$\left(\frac{A}{A_*}\right)_{M=3} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}M^2\right)\right]^{\frac{k+1}{2k-2}}$$

$$= \frac{1}{3}\left[\left(\frac{2}{1.5+1}\right)\left(1+\frac{1.5-1}{2}3^2\right)\right]^{\frac{1.5+1}{2\cdot 1.5-2}} = 3.63$$

$$A_* = \frac{A}{3.63} = \frac{30}{3.63} = 8.26\,\mathrm{cm}^2$$

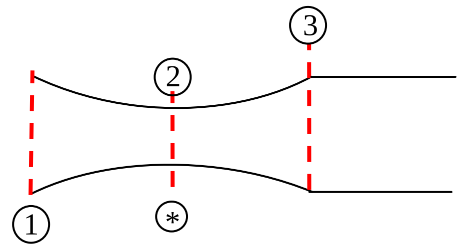
$$A_{M=1.4} = \left(\frac{A}{A_*}\right)_{M=1.4} A_* = \left\{\frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}M^2\right)\right]^{\frac{k+1}{2k-2}}\right\}_{M=1.4} A_* = (1.11)\cdot(8.26) = 9.168\ \mathrm{cm}^2$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. Use custom fluid. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-10 [BHG] Air enters a converging-diverging nozzle of a supersonic wind tunnel at 1000 kPa and 35°C with a low velocity. The flow area of the test section is equal to the exit area of the nozzle, which is 0.5 m^2. Determine (a) the pressure (p), temperature (T), (b) velocity (V) and (c) the mass flow rate (m) in the test section where M = 2.

SOLUTION:



Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain:

$R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Given: $p, T,$ and $V = 0$):

$T_t = T = 35$ °C; $p_t = p = 1000$ kPa

State-3

$M = 2,\ p_t = p_{t1}, T_t = T_{t1}$

$\Rightarrow \dfrac{T}{T_t} = 0.555$ and $\dfrac{p}{p_t} = 0.127$

$p = 0.127 p_t = 0.127 \times 1000 = 127$ kPa

$T = 0.555 T_t = 0.555 \times 308 = 171.2$ K $= -101.95$°C

$$V = Mc = M\sqrt{1000kRT} = (2)\sqrt{1000(1.4)(0.287)(171.2)} = 524.7\ \frac{\text{m}}{\text{s}}$$

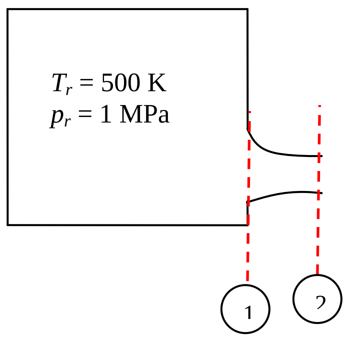
$$\dot{m} = \rho AV = \left(\frac{p}{RT}\right)AV \Rightarrow \left(\frac{127.8}{(0.287)(171.2)}\right)(0.5)(524.7) = 682.2\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-11 [BHZ] Compressed air is discharged through a converging nozzle as shown in the accompanying figure. The conditions in the tank are 1 MPa and 500 K while the outside pressure is 100 kPa. The inlet area of the nozzle is 100 cm^2 and the exit area is 35 cm^2. Determine (a) the exit velocity, (b) the exit temperature and (c) the force of the air on the nozzle. Assume the conditions inside to remain unchanged during the discharge.

SOLUTION:



Working fluid: air; From Table C-1 (or gas dynamics TESTcalc), obtain:

$R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Let us first find the condition for the flow to be chocked.

State-1:

$p_t = p_r = 1000$ kPa; $T_t = T_r = 500$ K;

State-2 (exit):

$M = 1$; $p_t = p_{t,2}$; $T_t = T_{t,2}$

$$\Rightarrow \frac{p}{p_t} = 0.528;\ \ p = (0.5283)(1000) = 528.3 \text{ kPa}$$

$$\Rightarrow \frac{T}{T_t} = 0.833;\ \ T = (0.833)(500) = 416.65 \text{ K}$$

For, $p_b \le 528.3$ kPa the flow is choked at the nozzle exit. Since $p_{atm} = 100$ kPa is the back pressure, the flow must be choked.

$$V = Mc = M\sqrt{1000kRT} = \sqrt{1000(1.4)(287)(416.65)} = 409.16 \text{ m/s}.$$

$$\dot{m} = \rho AV = \frac{p}{RT}AV = 6.33\ \frac{\text{kg}}{\text{s}}$$

State-1:

$p_t = p_r = 1000 \text{ kPa}; \; T_t = T_r = 500 \text{ K};$

$$\frac{A}{A_*} = \frac{A_1}{A_2} = \frac{100}{35} = 2.857,$$

$$\Rightarrow M = 0.208$$

$$\frac{p}{p_t} = 0.970; \; p = (0.970)(1000) = 970 \text{ kPa}$$

$$\frac{T}{T_t} = 0.991; \; T = (0.991)(500) = 495.7 \text{ K}. \text{ Thus,}$$

$$V = M\sqrt{(1000)kRT} = (0.208)\sqrt{(1000)(1.4)(287)(495.7)} = 92.83 \frac{\text{m}}{\text{s}}$$

For a cylindroid nozzle with a control volume drawn around the edges of the nozzle, any radial forces sum to zero, so the net force on the nozzle by the air can be assumed to be in the x-direction only. Thus, a momentum balance gives,

$$\cancel{\frac{dM_x}{dt}} = \dot{m}_i V_{x,i} - \dot{m}_e V_{x,e} + \sum F_x, \text{ or } 0 = \frac{\dot{m}(V_i - V_e)}{1000} + T + p_i A_i - p_e A_e - p_b (A_i - A_e).$$

$$\Rightarrow T = \frac{\dot{m}(V_e - V_i)}{1000} - p_i A_i + p_e A_e + p_b (A_i - A_e)$$

$$= \frac{\dot{m}(V_e - V_i)}{1000} + (p_e - p_b) A_e - (p_i - p_b) A_i$$

$$= \frac{(6.33)(409.16 - 92.83)}{1000} + (528.3 - 100)(0.0035) - (970 - 100)(0.01) = -5.2 \text{ kN}$$

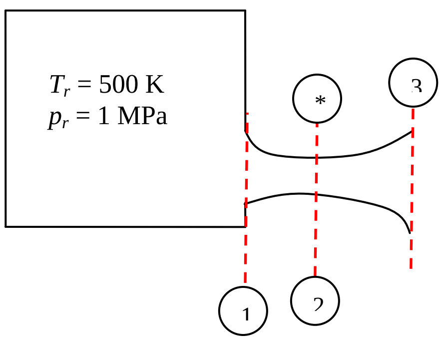
The negative sign means the nozzle will exert a force on the tank in the opposite direction of the velocity.

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-12 [BHK] For the converging-diverging nozzle shown in the accompanying figure, (a) find the maximum back pressure below which the flow is choked and (b) the mass flow rate for a choked nozzle. The throat area is 10 cm^2 and the exit area is 40 cm^2.

SOLUTION:



Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain:

$R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1:

$p_t = p_r = 1000$ kPa; $T_t = T_r = 500$ K;

State-2:

$M = 1$; $A = A_* = 10 \text{ cm}^2$;

State-3 (subsonic):

$T_{t,3} = T_t$; $p_{t,3} = p_t$

$$\frac{A}{A_*} = \frac{40}{10} = 4 \Rightarrow M = 0.146 \text{ (subsonic solution)};$$

$$\Rightarrow \frac{T}{T_t} = 0.995; \quad \frac{p}{p_t} = 0.985$$

$$\Rightarrow T = 497.5 \text{ K}, \quad p = 985 \text{ kPa}$$

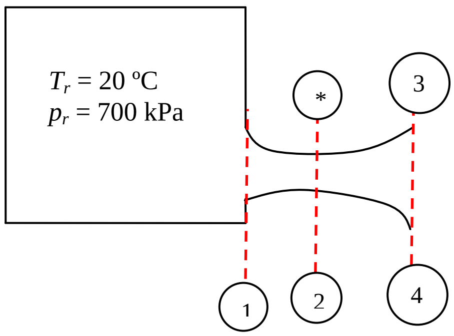
$$\dot{m} = \rho AV = \frac{p}{RT} AM\sqrt{1000kRT} = \frac{p}{\sqrt{RT}} AM\sqrt{1000k} = \frac{(985)(0.004)(0.146)\sqrt{(1000)(1.4)}}{\sqrt{(0.287)(497.5)}} = 1.807 \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-13 [BHP] A converging-diverging nozzle with an exit area of 35 cm^2 and a throat area of 10 cm^2, is attached to a reservoir which contains air at 700 kPa and 20^{o}C absolute. Determine (a) the two exit pressures that result in M = 1 at the throat for an isentropic flow, (b) the associated exit temperatures and (c) velocities.

SOLUTION:



Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain:

$R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1:

$p_r = p_t = 700$ kPa, $T_r = T_t = 20\ ^oC = 293$ K

State-2:

$M = 1$; $A = A_* = 10$ cm^2;

State-3 (subsonic):

$T_{t,3} = T_t$; $p_{t,3} = p_t$

$$\frac{A}{A_*} = \frac{35}{10} = 3.5 \Rightarrow M = 0.168,\ \frac{p}{p_t} = 0.98,\ \frac{T}{T_t} = 0.99$$

$\Rightarrow p = 686$ kPa and $T = 290.2$ K

$$V = \sqrt{2000c_p(T_r - T)} = \sqrt{2000(1.005)(293 - 290.2)} = 57.54\ \frac{\text{m}}{\text{s}}$$

State-3 (supersonic):

$$\frac{A}{A_*} = \frac{35}{10} = 3.5 \Rightarrow M = 2.799, \ \frac{p}{p_r} = 0.036, \ \frac{T}{T_r} = 0.389$$

$$\frac{p}{700} = 0.036 \Rightarrow p = 25.808 \text{ kPa}$$

$$\frac{T}{20 + 273.15} = 0.389 \Rightarrow T = 114.2 \text{ K}$$

$$V = \sqrt{2000 c_p (T_r - T)} = \sqrt{2000(1.005)(293 - 114.2)} = 599.68 \ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-14 [BHU] A rocket motor is fired on a test stand. Hot exhaust gases leave the exit with a velocity of 700 m/s at a mass flow rate of 10 kg/s. The exit area is 0.01 m^2 and the exit pressure is 50 kPa. For an ambient pressure of 100 kPa, determine the rocket motor thrust that is transmitted to the stand. Assume steady state and one-dimensional flow.

SOLUTION:

The momentum equation produces;

$$0=\frac{\dot{m}\left(0-V_e\right)}{1000}+T+p_0A-p_eA_e-p_0\left(A-A_e\right)$$

$$T=\frac{\dot{m}V_e}{1000_e}+A_e\left(p_e-p_0\right)$$

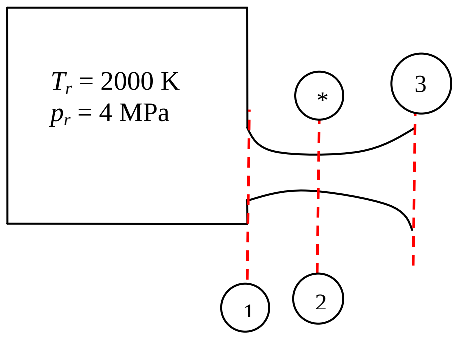
$$=10(700)\frac{1}{1000}+0.01(50-100)$$

$$=6.5\text{ kN}$$

Use I/O panel to solve it.

15-2-15 [BHC] A rocket nozzle has an exit-to-throat area ratio of 4.0 and a throat area of 100 cm^2. The exhaust gases are generated in a combustion chamber with stagnation pressure equal to 4 MPa and stagnation temperature equal to 2000 K. Assume the working fluid to behave as a perfect gas with k = 1.3 and molar mass = 20 kg/kmol. Determine (a) the rocket exhaust velocity and (b) the mass flow rate. Assume isentropic steady flow.

SOLUTION:



Working fluid is a perfect gas with $k = 1.3, \; M = 20$ kg/kmol. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$T_r = T_t = 2000 \text{ K}, \; p_r = p_t = 4 \text{ MPa}$

State-2 (Throat);

$M = 1; \; A = A_* = 100 \text{ cm}^2;$

State-3 (Exit);

$$\frac{A}{A_*} = 4 \Rightarrow M = 2.774, \; \frac{p}{p_r} = 0.036, \; \frac{T}{T_r} = 0.464$$

$$\frac{p}{4000} = 0.036 \Rightarrow p = 143.7 \text{ kPa}$$

$$\frac{T}{2000} = 0.464 \Rightarrow T = 928.25 \text{ K}$$

$$V = \sqrt{1000kRT_3} = \sqrt{1000(1.3)(0.41)928.25} = 1965 \; \frac{\text{m}}{\text{s}}$$

$$\dot{m} = \frac{p}{RT} AM \sqrt{1000kRT}$$

$$\dot{m} = \frac{143.7}{1.3 \times 928.25} (4 \times 0.01)(2.774) \sqrt{1000 \times 1.3 \times 0.41 \times 928.25}$$

$$\dot{m} = (0.3724)(0.04)(2.774)(708.3)$$

$$\dot{m} = 29.27 \, \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-16 [BHV] In problem 15-2-15 [BHC] determine the thrust if the outside pressure is 0 kPa.

SOLUTION:

$$T = \frac{\dot{m}V_e}{1000_e} + A_e\left(p_e - p_0\right)$$

$$T = 29.27\left(1965\right)\frac{1}{1000} + 0.04\left(143.7 - 0\right)$$

$$T = 63.25 \text{ kN}$$

Use I/O panel to solve it.

15-2-17 [BHX] On a test stand, a nozzle operates isentropically with a chamber pressure of 2 MPa and chamber temperature of 2500 K. If the products of combustion are assumed to behave as a perfect gas with k = 1.3 and molar mass = 20 kg/kmol, determine the rocket motor thrust that is transmitted to the stand. Assume the nozzle exit area to be 0.01 m^2 and the back pressure to be 50 kPa.

SOLUTION:

To calculate the force of the rocket on the stand, use the momentum equation for forces in the x-direction;

$$\frac{dM_x}{dt} = \frac{\dot{m}_i V_{x,i}}{1000} - \frac{\dot{m}_e V_{x,e}}{1000} + F_x - (p_e - p_a) A_e \;;$$

With steady state the equation reduces to:

$$\frac{dM_x}{dt} = 0 \;\Rightarrow \sum F_x = \frac{\dot{m} V_{x,2}}{1000} - \cancelto{0}{\frac{\dot{m} V_{x,1}}{1000}} + (p_e - p_a) A_e \;\Rightarrow\; F_x = \frac{\dot{m} V_{x,2}}{1000} + (p_e - p_a) A_e$$

Assuming isentropic flow, the energy and entropy equations produce:

$$j_1 = j_2; \quad \Rightarrow h_1 + \cancelto{0}{\mathrm{ke}_1} = h_2 + \mathrm{ke}_2 \Rightarrow h_1 = h_2 + \frac{V_2^2}{2000} \Rightarrow V_2 = \sqrt{2000 c_p (T_1 - T_2)} \;;$$

$$k = \frac{c_p}{c_v} = \frac{c_p}{(c_p - R)} \Rightarrow c_p = \frac{k}{(k-1)} R; \quad c_p = \frac{1.3}{(1.3-1)}(0.4157) \Rightarrow c_p = 1.80137 \; \frac{\text{kJ}}{\text{kg K}} \;;$$

$$s_2 = s_1; \quad \Rightarrow \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \Rightarrow T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 2500 \left(\frac{50}{2000}\right)^{(1.3-1)/1.3} \Rightarrow T_2 = 1067.17 \text{ K} \;;$$

$$V_2 = \sqrt{2000 c_p (T_1 - T_2)} = \sqrt{2000(1.80137)(2500 - 1067.17)} \Rightarrow V_2 = 2272.03 \; \frac{\text{m}}{\text{s}}$$

Now all the values necessary to calculate the mass flow are available:

$$\dot{m} = \rho_e A_e V_e = \frac{p_e}{R T_e} A_e V_e \Rightarrow \left(\frac{50}{(0.4157)(1067.17)}\right)(0.01)(2272.03)$$

$$\Rightarrow \dot{m} = 2.56 \; \frac{\text{kg}}{\text{s}}$$

Substituting $\dot{m}$ and V_e back into the force equation gives the value of the force transmitted to the nozzle stand:

$$\Rightarrow F_x = \frac{(2.56)(2272.03)}{1000} + (50 - 50)A_e \Rightarrow F_x = 5.82 \text{ kN}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-18 [BHT] Nitrogen enters a duct with varying flow area at 500 K, 100 kPa and Mach number 0.4. Assuming a steady isentropic flow, determine (a) the Mach number, (b) pressure (p) and (c) the temperature (T) at a location where the flow area has been reduced by 30%. (d) What-if Scenario: What would the pressure be for a location where the flow area has been reduced by 20%?

SOLUTION:

Working fluid: nitrogen; From Table C-1 (or gas dynamics TESTcalc), obtain:
$R = 0.297 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet):

$$M = 0.4,\ p = 100\text{ kPa},\ T = 500\text{ K}$$
$$\Rightarrow \frac{p}{p_t} = 0.895,\ \frac{T}{T_t} = 0.968 \text{ and } \frac{A}{A_*} = 1.59$$
$$\Rightarrow A_* = 0.6288A$$

State-2 (pinched location):

For $A_2 = 0.7A$ and $A_2 = 0.8A$, $A_2 > A_*$. Therefore the flow is isentropic.

$$\frac{A_2}{A_*} = \frac{0.70A}{0.6288A} = 1.113$$
$$\Rightarrow M = 0.675$$

$$\frac{p}{p_t} = 0.736 \quad \Rightarrow p = 0.736(11.65) \Rightarrow p = 82.2\text{ kPa}$$

$$\frac{T}{T_t} = 0.916 \quad \Rightarrow T = 0.916(516) \Rightarrow T = 472.6\text{ K}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-19 [BHY] Products of combustion enters the nozzle of a gas turbine at the design conditions of 420 kPa, 1200 K and 200 m/s, and they exit at a pressure of 290 kPa at a rate of 3 kg/s. Take k = 1.34 and c_p = 1.16 kJ/kg-k for the combustion products. Assuming an isentropic flow, determine (a) whether the nozzle is converging or converging-diverging: (1 if converging, 2 if converging-diverging), (b) the exit velocity and (c) the exit area.

SOLUTION:

To determine the type of nozzle, compare the exit state to a state where $M=1$ to see if $M_e > 1$ for a converging-diverging nozzle.

State-1 (Inlet);

$$p_r = p_t = 420 \text{ kPa};\ T_r = T_t = 1200 \text{ K}$$

$$T_t = T + \frac{V^2}{2000c_p} = 1200 + \frac{200^2}{2000(1.16)} \Rightarrow T_t = 1217.2 \text{ K}$$

$$\left(\frac{p_t}{p}\right) = \left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} \Rightarrow p_t = p\left(\frac{T_t}{T}\right)^{\frac{k}{k-1}} = (420)\left(\frac{1217.2}{1200}\right)^{\frac{1.34}{1.34-1}} \Rightarrow p_t = 444.3 \text{ kPa}$$

State-2 (Throat);

$$M = 1;\ T_t = T_{t,2};\ p_t = p_{t,2}$$

State-3 (Exit);

$$\frac{p}{p_t} = \frac{290}{444.3} = 0.653 \Rightarrow M = 0.82;\ \frac{T}{T_t} = 0.897 \Rightarrow T = 1091.8 \text{ K}$$

$M = 0.82 < 1$ so the exit is subsonic, and the nozzle is converging.

$$R = c_p - \frac{c_p}{k} = 1.16 - \frac{1.16}{1.34} = 0.294 \text{ kJ/kg}\cdot\text{K}$$

$$M = \frac{V}{c} = \frac{V}{\sqrt{1000kRT}} = \frac{V}{\sqrt{1000(1.34)(0.294)(1091.8)}} = 0.82$$

$$\Rightarrow V = 538.2\ \frac{\text{m}}{\text{s}}$$

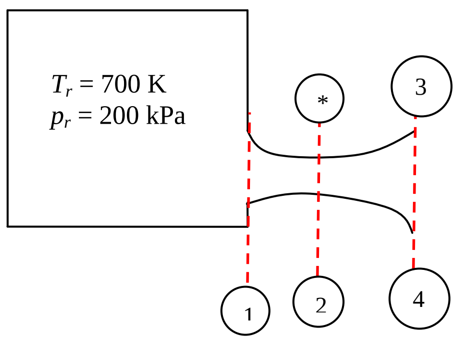
$$M = 0.82 \Rightarrow \frac{A}{A_*} = 1.031 \Rightarrow A = 61.8 \text{ cm}^2$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-20 [BHF] Combustion products enter a nozzle with total temperature of 700 K and total pressure of 200 kPa. For a back pressure of 60 kPa and a nozzle efficiency of 90%, determine the exit velocity. Assume the combustion gases to have the properties of air with $k = 1.33$.

SOLUTION:



The properties of air with $k = 1.33;\ R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$ are used. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$T_r = T_t = 700\text{ K},\ p_r = p_t = 200\text{ kPa}$

State-2 (Throat);

$M = 1;\ T_t = T_{t,2} = 700\text{ K},\ p_t = p_{t,2} = 200\text{ kPa}$

State-3 (Isentropic exit);

$p = 60\text{ kPa}$

$$\frac{P}{P_t} = \frac{60}{200} = 0.3 \Rightarrow M = 1.45;\quad \frac{T}{T_t} = 0.74 \Rightarrow T_3 = 518\text{ K}$$

State-4 (Actual exit conditions with 90% efficiency);

Using the nozzle efficiency the actual exit temperature can be calculated.

$$\eta_{nozzle} = \frac{ke_{actual}}{ke_{isentropic}} = \frac{T_t - T_4}{T_t - T_3} \Rightarrow 0.9 = \frac{700 - T_4}{700 - 518} \Rightarrow T_4 = 536.2\text{ K}$$

$$\frac{T_4}{T_t} = \frac{536.2}{700} = 0.766 \Rightarrow M = 1.36$$

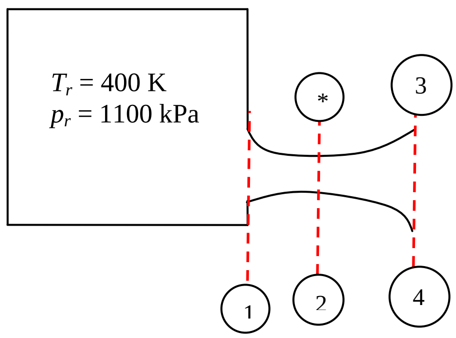
$$M = \frac{V}{c} = \frac{V}{\sqrt{1000kRT}} \Rightarrow 1.36 = \frac{V_4}{\sqrt{1000(1.33)(0.287)(536.2)}} \Rightarrow V = 615.2\,\frac{\mathrm{m}}{\mathrm{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net

15-2-21 [BHD] A converging-diverging nozzle has an exit area to throat area ratio of 1.8. Air enters the nozzle with a total pressure of 1100 kPa and a total temperature of 400 K. The throat area is 5 cm^2. If the velocity at the throat is sonic, and the diverging section acts as a nozzle, determine (a) the mass flow rate, (b) the exit pressure and temperature, (c) the exit Mach number and (d) the exit velocity. (e) What-if Scenario: What would the exit pressure and temperature be if the velocity at the throat was sonic, and the diverging section acted as a diffuser?

SOLUTION:



Working fluid: air; From Table C-1 (or gas dynamics TEScalc), obtain:

$R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$$T_r = T_t = 400 \text{ K},\ p_r = p_t = 1100 \text{ kPa}$$

State-2 (Throat);

$$A_* = 5 \text{ cm}^2$$

$$T_t = T_{t,2} = 400 \text{ K};\ p_t = p_{t,2} = 1100 \text{ kPa}$$

$$M = 1 \Rightarrow \frac{p}{p_t} = 0.528;\ \frac{T}{T_t} = 0.833$$

$$\Rightarrow p = 0.52 p_t \Rightarrow 0.52(1100) \Rightarrow p = 581.1 \text{kPa}$$

$$\Rightarrow T = 0.83 T_t \Rightarrow 0.83(400) \Rightarrow T = 333.3 \text{K}$$

$$\dot{m} = \rho A_* V = \left(\frac{p}{RT}\right) A_* \left(M\sqrt{kRT}\right) \Rightarrow \left(\frac{581.1}{(0.287)(333.3)}\right)(0.0005)\left(1\sqrt{(1.4)(287)(333.3)}\right)$$

$$\Rightarrow \dot{m} = 1.112 \frac{\text{kg}}{\text{s}}$$

State-3 (Exit);

$$\frac{A}{A_*} = 1.8 \Rightarrow M = 2.076; \quad \frac{p}{p_t} = 0.11; \quad \frac{T}{T_t} = 0.53$$

$$\Rightarrow p = 124.88\,\text{kPa} \text{ and } T = 214.9\,\text{K}$$

$$\frac{V^2}{2000c_p} = T_t - T_3 \Rightarrow V = \sqrt{2000c_p\left(T_t - T\right)} = \sqrt{2000(1.005)\left(400 - 214.89\right)}$$

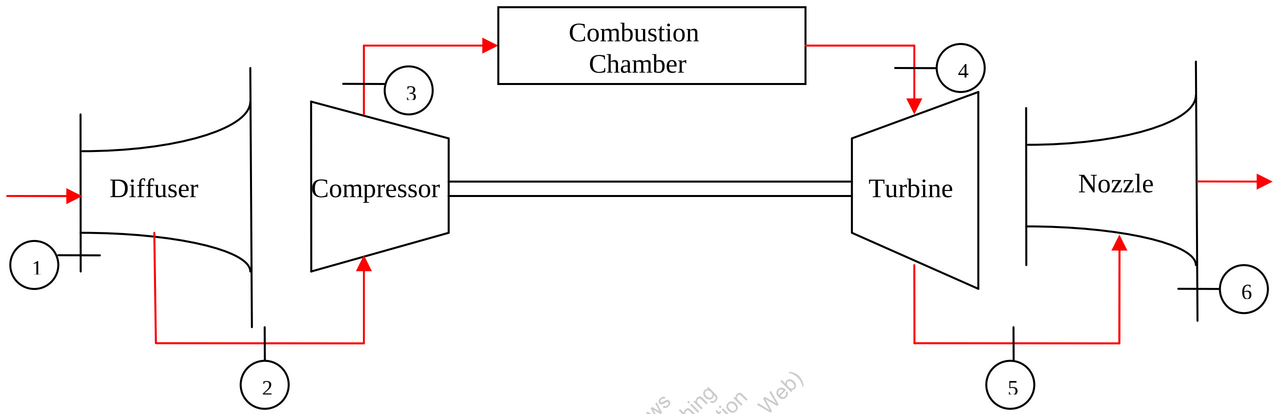
$$\Rightarrow V = 609.9\,\frac{\text{m}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-22 [BHM] A jetliner is flying at 275 m/s at a high altitude where the atmospheric pressure and temperature are 50 kPa and 250 K, respectively. Air is first decelerated in a diffuser, before it is compressed by an isentropic compressor with a compression ratio of 10. Determine (a) the pressure at the compressor inlet and (b) the compressor work per unit mass of air.

SOLUTION:



Working fluid: air; From Table C-1(or gas dynamics TEScalc) obtain

$R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

(a) To determine the exit pressure p_2, the isentropic table relation is used.

$$T_2 = T_1 + \frac{V_1^2}{2000c_p} = 250\frac{275}{2000(1.005)} = 287.6 \text{ K}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \Rightarrow p_2 = p_1 \cdot \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = 50 \cdot \left(\frac{287.6}{205}\right)^{\frac{1.4}{1.4-1}} \Rightarrow p_2 = 81.69 \text{ kPa}$$

(b) The compressor work per unit mass is:

$$\dot{W}_{c,2,3} = \dot{m}(j_2 - j_1) = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1)$$

$$\dot{W}_{c,2,3} = \dot{m}c_pT_2\left[\left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}} - 1\right] = \frac{\dot{m}kRT_2}{k-1}\left[\left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}} - 1\right]$$

$$\dot{W}_{c,2,3} = \frac{1(1.4)(0.287)(287.6)}{1.4-1}\left[\left(\frac{10(81.69)}{81.69}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$\Rightarrow \dot{W}_{c,2,3} = 268.81 \text{ kW}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-23 [BHJ] Carbon dioxide flows steadily through a variable-area duct with a mass flow rate of 2 kg/s. It enters the duct at 1400 kPa and 250°C with negligible velocity and expands to a pressure of 200 kPa isentropically. Determine (a) the exit Mach number, (b) flow area and (c) the critical flow area. (d) What-if Scenario: What would the critical flow area be if the mass flow rate of carbon dioxide were 3 kg/s?

SOLUTION:

Working fluid: CO_2; From Table C-1(or gas dynamics TEScalc) obtain:

$R = 0.188\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.28$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$$p_r = p_t = 1400\ \text{kPa};\ T_r = T_t = 250°\text{C}$$

State-2 (Throat);

$$p_t = p_{t,2};\ T_t = T_{t,2}$$

$$M = 1 \Rightarrow \frac{T}{T_t} = 0.874 \Rightarrow T = 457.23\ \text{K}$$

$$V = M\sqrt{1000kRT} = (1)\sqrt{1000(1.28)(0.189)(457.23)} = 333.58\ \frac{\text{m}}{\text{s}}$$

$$\dot{m} = Ap_t\sqrt{\frac{1000\cdot k}{RT_t}}\left(\frac{k+1}{2}\right)^{\frac{k+1}{2-2k}}$$

$$\Rightarrow A = 6.7\ \text{cm}^2$$

State-3 (Exit);

$$p_t = p_{t,3};\ T_t = T_{t,3}$$

$$\frac{p}{p_t} = 0.142 \Rightarrow M = 1.95;\ \frac{T}{T_t} = 0.646;\ \frac{A}{A_*} = 1.702$$

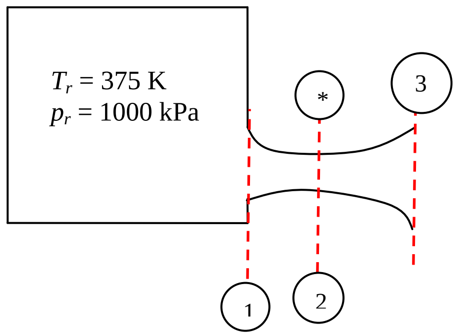
$$\Rightarrow T = 338.3\ \text{K and } A = 11.5\ \text{cm}^2$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-24 [BHW] A converging-diverging nozzle has a throat area of 100 mm^2 and an an exit area of 160 mm^2. The inlet flow is helium at a total pressure of 1 MPa and total temperature of 375 K. Determine the back pressure that will give sonic condition at the throat, but subsonic everywhere else.

SOLUTION:



Working fluid: He; From Table C-1(or gas dynamics TESTcalc) obtain

$R = 2.078 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.66$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$p_r = p_t = 1000 \text{ kPa};\ T_r = T_t = 375 \text{ K}$

State-2 (Throat);

$M = 1;\ p_t = p_{t,2};\ T_t = T_{t,2}$

$A_* = 100 \text{ mm}^2$

State-3 (Exit);

$p_t = p_{t,3};\ T_t = T_{t,3}$

$$\frac{A}{A_*} = \frac{160}{100} = 1.6 \Rightarrow M = 0.387;\ \frac{p}{p_t} = 0.8849;\ \frac{T}{T_t} = 0.952$$

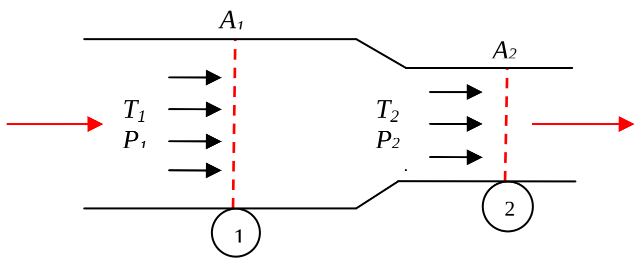
$\Rightarrow p = 884.9 \text{ kPa}$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-25 [BNR] Oxygen flows at Mach 0.5 in a channel with a cross-sectional area of 0.16 m^2. The temperature and pressure are 800 K and 800 kPa, respectively. (a) Calculate the mass flow rate through the channel. The cross-sectional area is now reduced to 0.15 m^2. Determine the (b) Mach number and (c) flow velocity at the reduced area. Assume the flow to be isentropic.

SOLUTION:



Working fluid: O_2; From Table C-1 (or gas dynamics TEScalc)obtain:

$R = 0.26\dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.394$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$$M = 0.5 \Rightarrow \frac{p}{p_t} = 0.843; \quad \frac{T}{T_t} = 0.953$$

$$\Rightarrow p_t = 948.41 \text{ kPa}; \quad \text{T}_t = 839.47 \text{ K}$$

$$M = \frac{V}{\sqrt{1000kRT_1}} \Rightarrow 0.5 = \frac{V}{\sqrt{1000(1.395)(0.26)800}} \Rightarrow V = 269.2\frac{\text{m}}{\text{s}}$$

$$\dot{m}_1 = \rho AV = \left(\frac{p}{RT}\right)AV = \frac{800}{0.26(800)}0.16(269.2) = 165.7\frac{\text{kg}}{\text{s}}$$

State-2 (Reduced area);

$$\dot{m}_1 = \dot{m}_2 = 165.7\frac{\text{kg}}{\text{s}}$$

Since $p_t = p_{t,2}$ and $T_t = T_{t,2}$

$$\dot{m} = \rho AV = \left(\frac{p}{RT}\right)AV \Rightarrow 165.7 = \frac{776.68}{0.26(793.33)}(0.15)\ V \Rightarrow V = 293.3\frac{\text{m}}{\text{s}}$$

$$M = \frac{V}{\sqrt{1000kRT}} = \frac{293.3}{\sqrt{1000(1.394)(0.26)(793.33)}} = 0.54$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-26 [BNO] Air flows at Mach 0.5 in a channel with cross-sectional area of 0.16 m^2. The temperature and pressure are 800 K and 800 kPa, respectively. (a) Calculate the mass flow rate ($\dot{m}$) through the channel. (b) What should the cross-sectional area be reduced to before the mass flow rate is affected? Assume the flow to be isentropic.

SOLUTION:

Working fluid: air; From Table C-1(or gas dynamics TEScalc) obtain

$R = 0.287 \ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

(a) The mass flow rate is:

$$\dot{m} = \frac{p_1}{RT_1} A_1 M_1 \sqrt{kRT_1} = \frac{800}{0.287(800)}(0.16)(0.5)\sqrt{1.4(0.287)(800)} = 158 \ \frac{\text{kg}}{\text{s}}$$

(b) The mass flow rate will be affected if the area reduced less than A_*.

$$M = 0.5 \Rightarrow \frac{A}{A_*} = 1.34$$

$$\Rightarrow A_* = \frac{A_1}{1.34} = \frac{0.16}{1.34} = 0.119 \text{ m}^2$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-27 [BNB] Air enters a diffuser with a velocity of 200 m/s, a static pressure of 80 kPa and a temperature of 268 K. The velocity leaving the diffuser is 50 m/s and the static pressure at the diffuser exit is 90 kPa. Determine (a) the static temperature (T) at the diffuser exit, (b) the total pressure (p) at the the exit and (c) the diffuser efficiency(%).

SOLUTION:

Working fluid: air; From Table C-1 (or the gas dynamics TESTcalc)

obtain $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

Inlet State;

$$M = \frac{V}{c} = \frac{V}{\sqrt{1000kRT_1}} = \frac{200}{\sqrt{1000(1.4)(0.287)(268)}} = 0.6$$

$$M = 0.6 \Rightarrow \frac{P}{P_t} = 0.784; \quad \frac{T}{T_t} = 0.932$$

$$\Rightarrow P_t = 102.8 \text{ kPa}; \quad \text{T}_t = 287.9 \text{ K}$$

Exit State;

$$\text{T}_t = \text{T}_{t,e} = 287.9 \text{ K}$$

Based on the energy equation and PG model, the energy balance reduces to:

$$\frac{V^2}{2000} = c_p\left(T_i - T_e\right) \Rightarrow T_e = T_i - \frac{V^2}{2000c_p} = 287.9 - \frac{50^2}{2000\left(1.005\right)} = 286.65 \text{ K}$$

$$M = \frac{V}{c} = \frac{V}{\sqrt{1000kRT}} = \frac{50}{\sqrt{1000(1.4)(0.287)(286.65)}} = 0.14$$

$$M = 0.14 \Rightarrow \left(\frac{p}{p_t}\right) = 0.986 \Rightarrow p_t = 91.37 \text{ kPa}$$

$$\eta_{diffuser} = \frac{ke_{actual}}{ke_{isentropic}} = \frac{\left[1 + M_i^2\dfrac{(k-1)}{2}\right]F_p^{\frac{k-1}{k}} - 1}{M_i^2\dfrac{(k-1)}{2}}; \quad F_p = \frac{p_{t,e}}{p_{t,i}} = \frac{91.3}{102.8} = 0.88$$

$$\Rightarrow \eta_{diffuser} = 0.521 = 52.1\%$$

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15-2-28 [BNS] Helium enters a variable-area duct at 400 K, 100 kPa and M = 0.3. Assuming steady isentropic flow, determine (a) the Mach number at a location where the area is 20% smaller. (b) What-if Scenario: What would the Mach number be if the area were 30% smaller?

SOLUTION:

Working fluid: He; From Table C-1 (or the gas dynamics TESTcalc) obtain $R = 2.078 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.66$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$$M = 0.3,\ p = 100 \text{ kPa},\ T = 400 \text{ K}$$
$$\Rightarrow \frac{p}{p_t} = 0.895,\ \frac{T}{T_t} = 0.928 \text{ and } \frac{A}{A_*} = 1.99$$
$$\Rightarrow A_* = 0.502A$$

State-2 (pinched location):

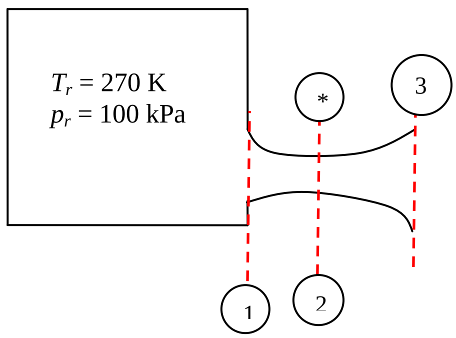
For $A_2 = 0.7A$ and $A_2 = 0.8A$, $A_2 > A_*$. Therefore the flow is isentropic.

$$\frac{A_2}{A_*} = \frac{0.80A}{0.502A} = 1.593 \Rightarrow M = 0.39$$

TEST Solution:
Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-29 [BNA] A converging-diverging nozzle has an area ratio of 3:1 and a throat area of 50 cm^2. The nozzle is supplied from a tank containing helium at 100 kPa and 270 K. Find (a) the maximum mass flow rate possible through the nozzle and (b) the range of back pressures over which the mass flow can be attained. (c) What-if Scenario: What would the answers be if hydrogen were the working fluid?

SOLUTION:



Working fluid: He; From Table C-1 (or the gas dynamics TESTcalc) obtain $R = 2.078\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.66$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Inlet);

$p_r = p_t = 100\ \text{kPa};\ T_r = T_t = 270\ \text{K}$

State-2 (Throat);

$M = 1;\ A_* = 50\ \text{cm}^2$

State-3 (Exit);

$\frac{A}{A_*} = 3 \Rightarrow M = 0.192;\ \frac{p}{p_t} = 0.969 \Rightarrow p_t = 96.98\ \text{kPa}$

$p_t < 96.98\ \text{kPa}$

$$\dot{m}_{max} = A_* p_t \sqrt{\frac{(1000)k}{RT_t}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} = (0.005)(100)\sqrt{\frac{1000(1.66)}{2.078(270)}} \left(\frac{2}{1.66+1}\right)^{\frac{1.66+1}{2(1.66-1)}} = 0.484\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

15-2-30 [BIO] A symmetric converging diverging duct with an area ratio of 2 is placed in a wind tunnel where it encounters a 300 m/s flow of air at 50 kPa, 300 K. Determine the bypass ratio (diverted flow / incoming flow).

SOLUTION:

Working fluid: Air; From Table C-1(or the gas dynamics TESTcalc) obtain $R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$. Use Table H-1 (or the gas dynamics TESTcalc) to obtain isentropic properties.

State-1 (Upstream of the inlet);

$$p = 50\text{ kPa};\ T = 300\text{ K};\ V = 300\ \text{m/s}$$

$$\dot{m} = \frac{AVp}{RT} = \frac{1(300)50}{0.287\cdot 300} = 174.2\frac{\text{kg}}{\text{s}}$$

$$M = \frac{V}{\sqrt{1000kRT}} = \frac{300}{\sqrt{1000(1.4)(0.287)(300)}} = 0.864$$

$$M = 0.864 \Rightarrow \frac{p}{p_{t,1}} = 0.614 \text{ and } \frac{T}{T_t} = 0.981$$

$$\Rightarrow p_t = 81.38\text{ kPa};\ \text{T}_t = 344.77\text{ K}$$

State-2 (Throat);

$$M = 1;\ p_t = p_{t2};\ \text{T}_t = T_{t2}$$

$$\dot{m}_2 = A_* p_t\sqrt{\frac{1000k}{RT_t}}\cdot\left(\frac{2}{k+1}\right)^{\frac{k+1}{2k-2}} \Rightarrow \dot{m}_2 = 88.57\frac{\text{kg}}{\text{s}}$$

For an isentropic flow through the nozzle, $\dot{m}_2$ is the maximum flow rate the nozzle can handle. Some of the incoming flow, therefore, must be bypassed.

$$\text{Bypass ratio }\% = \frac{\dot{m}_1 - \dot{m}_3}{\dot{m}_1}\cdot 100 = \frac{174.2 - 88.57}{174.2}\cdot 100 = 49\%$$

TEST Solution:

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15-2-31 [BIB] In problem 15-2-30[BIO], determine the air speed below which flow bypass is not necessary.

SOLUTION:

$$M_3 = \frac{V_3}{\sqrt{1000kRT_3}} \Rightarrow 0.306 = \frac{V_3}{\sqrt{1000(1.4)(0.287)(338.44)}} \Rightarrow V_3 = 112.8\,\frac{\mathrm{m}}{\mathrm{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.