15-3-1 [BNH] An airstream with a velocity of 600 m/s, a pressure of 50 kPa and a temperature of 250 K undergoes a normal shock. Determine (a) the velocity (*V*) and (b) the pressure (*p*) at the exit. (c) Also determine the loss of total pressure due to the shock. (d) What-if Scenario: What would answers be if the inlet velocity were 1200 m/s instead?

SOLUTION:

Working fluid: air; From Table C-1, obtain: $R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

Inlet state (Given: $p = 50$ kPa; $T = 250$ K; $V = 600 \frac{\text{m}}{\text{s}}$):

$$M_i = \frac{V_i}{\sqrt{1000kRT_i}} = \frac{600}{\sqrt{(1000)(1.4)(0.287)(250)}} = 1.89;$$

$$\frac{p_i}{p_{ti}} = 0.15079; \Rightarrow p_{ti} = \frac{p_i}{0.15079} = \frac{50}{0.15079} = 331.59 \text{ kPa}$$

Exit state (Given: $T_{te} = T_{ti}$, $M_i = 1.89$):

$$\frac{V_e}{V_i} = 0.4; \Rightarrow V_e = (V_i)(0.4) = (600)(0.4) = 239.43 \frac{\text{m}}{\text{s}}$$

$$\frac{p_e}{p_i} = 4.015; \Rightarrow p_e = (p_i)(4.01534) = (50)(4.01534) = 200.77 \text{ kPa}$$

$$\frac{p_{te}}{p_{ti}} = 0.77; \Rightarrow p_{te} = (p_{ti})(0.77) = (331.59)(0.77) = 255.45 \text{ kPa}$$

$$p_{ti} - p_{te} = 331.59 - 255.45 = 76.14 \text{ kPa}$$

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TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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15-3-2 [BNN] Air enters a normal shock at 30 kPa, 220 K and 700 m/s. Determine (a) the total pressure and Mach number upstream of the shock, (b) pressure, temperature, (c) velocity, Mach number and (d) total pressure downstream of the shock.

$p_i = 30 kPa$, $T_i = 220 K$ and $V_i = 700 m/s$

SOLUTION:

Working fluid: air; From Table C-1, obtain: $R = 0.287 \ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

Inlet state (Given: $p = 30$ kPa; $T = 220$ K; $V = 700 \ \dfrac{\text{m}}{\text{s}}$):

$$M_i = \frac{V_i}{\sqrt{1000kRT_i}} = 2.354;$$

$$\Rightarrow \frac{p_i}{p_{ti}} = 0.07341; \quad p_{ti} = \frac{p_{ti}}{p_i} p_i = \left(\frac{p_i}{p_{ti}}\right)^{-1} p_i = 408.7 \text{ kPa}$$

Exit state (Given: $T_{te} = T_{ti}$, $M_i = 2.354$):

$$\frac{p_e}{p_i} = 6.30157, \frac{T_e}{T_i} = 1.99682$$

$$\Rightarrow p_e = \frac{p_e}{p_i} p_i = 189.0 \text{ kPa}$$

$$\Rightarrow T_e = \frac{T_e}{T_i} T_i = 439.3 \text{ K}$$

$M_e =$ **0.52803**

$$\Rightarrow V_e = M_e c_e = M_e \sqrt{(1000)kRT_e} = 221.8 \ \frac{\text{m}}{\text{s}}$$

$$\Rightarrow \frac{p_{te}}{p_{ti}} = 0.55936$$

$$p_{te} = \frac{p_{te}}{p_{ti}} p_{ti} = \frac{p_{te}}{p_{ti}} \left(\frac{p_i}{p_{ti}}\right)^{-1} p_i = 228.6 \text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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15-3-3 [BIS] A meteorite is entering the earth's atmosphere at a Mach number of 25 at the outer atmosphere where the pressure and temperature are 1 kPa and 200 K respectively. Determine (a) the velocity (V) of the meteorite with respect to ground, (b) the relative velocity of the meteorite with respect to the immediate surroundings, (c) the temperature (T) to which the meteorite is subjected.

SOLUTION:

With respect to the meteor, the air approaches it with a Mach number of 25.

Working fluid: air; From Table C-1, obtain: $R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

Inlet state (Given: $p = 1$ kPa; $T = 200$ K; $M = 25$):

$$V_i = M_i c_i = M_i\sqrt{1000kRT_i} = 7086 \text{ m/s};$$

Exit state (Given: $T_{te} = T_{ti}$, $M_i = 25$):

$$\frac{V_e}{V_i} = 0.168; \Rightarrow V_e = 1190 \frac{\text{m}}{\text{s}}$$

$$\frac{T_e}{T_i} = 122.4; \Rightarrow T_e = 24,478 \text{ K}$$

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TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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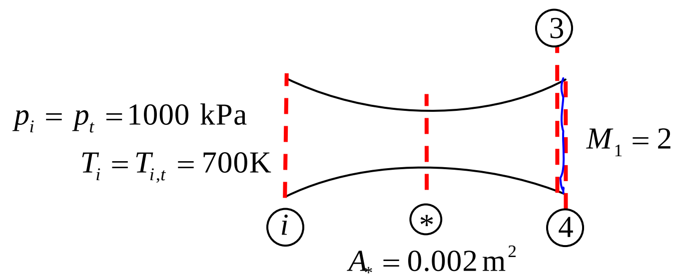
15-3-4 [BNE] Air enters a converging-diverging nozzle at 700 K and 1000 kPa with negligible velocity. The exit Mach number is designed to be 2 and the throat area is 20 cm^2. Now suppose the air experiences a normal shock at the exit plane. Determine (a) the total pressure (p) after the shock, (b) mass flow rate ($\dot{m}$) and (c) the exit velocity.

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

Inlet state (Given: $p = 1000$ kPa; $T = 700$ K; $V = 0 \dfrac{\text{m}}{\text{s}}$):

$\Rightarrow T_{ti} = T_i = 700$ K; $p_{ti} = p_i = 1000$ kPa

Let State-3 and State-4 represent states right before and after the shock at the exit.

State-3: $(M = 2)$

$$M_3 = 2$$

$$\Rightarrow \frac{A_3}{A_{th}} = \frac{A_3}{A_*} = 1.68773, \frac{p_3}{p_t} = 0.12781, \frac{T_3}{T_t} = 0.55574$$

$$A_3 = (20)(1.68773) = 33.75\ \text{cm}^2$$

$$p_3 = (1000)(0.12781) = 127.8\ \text{kPa}$$

$$T_3 = (700)(0.55574) = 389.02\ \text{K}$$

$$V_3 = M\sqrt{1000kRT} = (2)\sqrt{1000(1.4)(0.28699)(389)} = 790.6\ \text{m/s}$$

$$\dot{m} = \rho AV = \frac{p}{RT}AV = \left(\frac{127.8}{(0.287)(389)}\right)(0.003375)(790.6) = 3.05\ \frac{\text{kg}}{\text{s}}$$

State-4: $(M_i = 2)$

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$$M_4 = 0.577; \;\; \Rightarrow \frac{p_{t4}}{p_{t3}} = 0.7208; \;\; \frac{V_4}{V_3} = 0.37492$$

$$\Rightarrow p_{t4} = 1000 \times 0.72082 = 720.8 \text{ kPa}$$

$$\Rightarrow V_4 = 790.6 \times 0.375 = 296.4 \text{ m/s}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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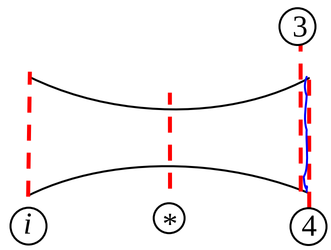
15-3-5 [BNI] Air enters a converging-diverging nozzle of a supersonic wind tunnel at 400 K and 1 MPa with a low velocity. If a normal shock wave occurs at the exit plane of the nozzle at Mach equal to 2, determine (a) the pressure (p), the temperature (T), (b) the Mach number, (c) the velocity (V) and (d) the total pressure after the shock wave. (e) What-if Scenario: What would the Mach number be if a normal shock wave were to occur at the exit plane of the nozzle at Mach equal to 1.5?

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

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Inlet state (Given: $p = 1000$ kPa; $T = 400$ K; $V = 0 \dfrac{\text{m}}{\text{s}}$):

$\Rightarrow T_{ti} = T_i = 400$ K; $p_{ti} = p_i = 1000$ kPa

Let State-3 and State-4 represent states right before and after the shock at the exit.

State-3: ($M = 2$)

$M_3 = 2;\ T_{t3} = T_{ti} = 400$ K; $p_{t3} = p_{ti} = 1000$ kPa

$$\Rightarrow \frac{p_3}{p_t} = 0.12781, \frac{T_3}{T_t} = 0.55574$$

$p_3 = (1000)(0.12781) = 127.8$ kPa

$T_3 = (0.55574)(400) = 222.3$ K

$V_3 = AM\sqrt{1000kRT} = (2)\sqrt{1000(1.4)(0.287)(222.3)} = 597.6$ m/s

State-4: ($M_i = 2$)

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$$M_4 = 0.577; \quad \Rightarrow \frac{p_4}{p_3} = 4.5; \; \frac{p_{t4}}{p_{t3}} = 0.7208; \; \frac{T_4}{T_3} = 1.687; \; \frac{V_4}{V_3} = 0.375$$

$$\Rightarrow p_4 = 127.8 \times 4.5 = 575.1 \text{ kPa}$$

$$\Rightarrow T_4 = 222.3 \times 41.687 = 375.0 \text{ kPa}$$

$$\Rightarrow V_4 = 597.6 \times 0.375 = 224.1 \text{ m/s}$$

$$\Rightarrow p_{t4} = 1000 \times 0.72082 = 720.8 \text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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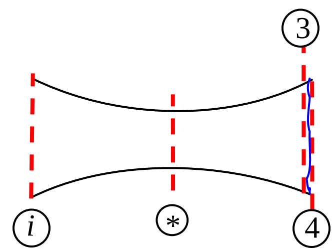
15-3-6 [BNL] Air enters a converging-diverging nozzle with a low velocity at 1.5 MPa and 120^{o}C. If the exit area of the nozzle is 3 times the throat area, determine (a) the back pressure to produce a normal shock at the exit plane of the nozzle. (b) What-if Scenario: What would the back pressure be for a normal shock to occur at a location wherethe cross-sectional area were twice the throat area?

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

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Inlet state (Given: $p = 1500$ kPa; $T = 393$ K; $V = 0 \ \dfrac{\text{m}}{\text{s}}$):

$\Rightarrow T_{ti} = T_i = 393$ K; $\ p_{ti} = p_i = 1500$ kPa

Let State-3 and State-4 represent states right before and after the shock at the exit.

State-3:

$$p_{t3} = p_{ti} = 1500 \ \text{kPa}$$

$$\frac{A_3}{A_{\text{th}}} = \frac{A_3}{A_*} = 3; \quad \Rightarrow M_3 = 2.64; \quad \Rightarrow \frac{p_3}{p_{t3}} = 0.047$$

$$\Rightarrow p_3 = 71 \ \text{kPa}$$

State-4: $(M_i = 2.64)$

$$M_4 = 0.5; \quad \Rightarrow \frac{p_4}{p_3} = 7.945;$$

$$\Rightarrow p_4 = 71 \times 7.945 = 564 \ \text{kPa}$$

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TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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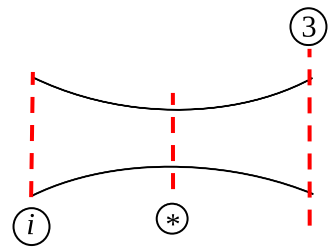
15-3-7 [BHQ] Carbon dioxide enters converging-diverging nozzle at 900 kPa and 900 K with a negligible velocity. The flow is steady and isentropic. For a exit Mach number of 2.5 and a throat area of 25 cm^2, determine (a) the exit area, (b) mass flow rate ($\dot{m}$) and (c) the exit velocity.

SOLUTION:



Working fluid: carbon dioxide; From Table C-1, obtain: $R = 0.189\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.29$

Use the gas dynamics TESTcalc (table panel) to obtain isentropic and shock properties.

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Inlet state (Given: $p = 900$ kPa; $T = 900$ K; $V = 0\ \dfrac{\text{m}}{\text{s}}$):

$\Rightarrow T_{ti} = T_i = 900$ K; $p_{ti} = p_i = 900$ kPa

Let State-3 represent the exit state.

State-3: ($M = 2.5$)

$M_3 = 2.5$

$$\Rightarrow \frac{A_3}{A_{th}} = \frac{A_3}{A_*} = 3,\ \frac{p_3}{p_t} = 0.0566,\ \frac{T_3}{T_t} = 0.527$$

$A_3 = (25)(3) = 75\ \text{cm}^2$

$p_3 = (900)(0.0566) = 51\ \text{kPa}$

$T_3 = (900)(0.527) = 474\ \text{K}$

$V_3 = M\sqrt{1000kRT} = 849\ \text{m/s}$

$$\dot{m} = \rho AV = \frac{p}{RT}AV = \left(\frac{51}{(0.189)(474)}\right)(0.0075)(849) = 3.63\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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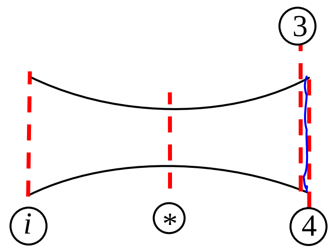
15-3-8 [BNZ] Air enters a converging-diverging nozzle at 700 K and 1000 kPa with a velocity of 75 m/s. The exit Mach number is designed to be 2 and the throat area is 20 cm^2. Suppose the air experiences a normal shock at the exit plane. Determine (a) the mass flow rate (m) and (b) the nozzle efficiency (%). (c) What-if Scenario: What would the mass flow rate and nozzle efficiency be if the inlet kinetic energy were neglected (%)?

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

Inlet state (Given: $p = 1000$ kPa; $T = 700$ K; $V = 75\ \dfrac{\text{m}}{\text{s}}$):

$$M_i = \frac{V_i}{\sqrt{1000kRT_i}} = 0.118;$$

$$\Rightarrow \begin{array}{l} \dfrac{p_i}{p_{ti}} \approx 0.99; \ \Rightarrow p_{ti} = \dfrac{p_{ti}}{p_i} p_i = \left(\dfrac{p_i}{p_{ti}}\right)^{-1} p_i \approx 706.9 \text{ kPa} \\ \dfrac{T_i}{T_{ti}} \approx 0.997; \ \Rightarrow T_{ti} = \dfrac{T_{ti}}{T_i} p_i = \left(\dfrac{T_i}{T_{ti}}\right)^{-1} T_i \approx 1002.8 \text{ K} \end{array};$$

Let State-3 and State-4 represent states right before and after the shock at the exit.

State-3: $(M = 2)$

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$M_3 = 2$

$\Rightarrow \dfrac{A_3}{A_{th}} = \dfrac{A_3}{A_*} = 1.69, \dfrac{p_3}{p_t} = 0.128, \dfrac{T_3}{T_t} = 0.555$

$A_3 = (20)(1.69) = 338 \text{ cm}^2$

$p_3 = (706.9)(0.128) = 89.5 \text{ kPa}$

$T_3 = (1002.8)(0.557) = 555.7 \text{ K}$

$V_3 = M\sqrt{1000kRT} = 845 \text{ m/s}$

$\dot{m} = \rho AV = \dfrac{p}{RT}AV = \left(\dfrac{89.5}{(0.189)(555.7)}\right)(0.00338)(845) = 1.79 \ \dfrac{\text{kg}}{\text{s}}$

State-4: $(M_i = 2)$

$M_4 = 0.577; \ \Rightarrow \ \dfrac{V_4}{V_3} = 0.375$

$\Rightarrow V_4 = 597.6 \times 0.375 = 224.1 \text{ m/s}$

The nozzle efficiency is:

$\eta_{\text{nozzle}} = \dfrac{\text{ke}}{\text{ke}_s} = \dfrac{V_4^2}{V_3^2} = 0.375^2 = 0.14 = 14\%$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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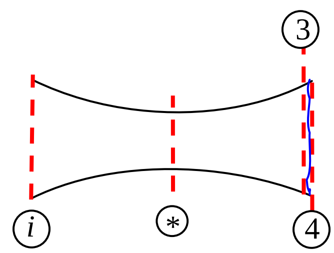
15-3-9 [BNU] Consider the convergent-divergent nozzle of problem 15-2-21 [BHD] in which the diverging section acts as a supersonic nozzle. Assume that a normal shock stands in the exit plane of the nozzle. Determine (a) the static pressure, temperature and (b) total pressure just downstream of the normal shock.

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

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Inlet state (Given: $T_{ti} = 400$ K; $p_{ti} = 1100$ kPa):

Let State-3 and State-4 represent states right before and after the shock at the exit.

State-3:

$$p_{t3} = p_{ti} = 1100 \text{ kPa}$$

$$\frac{A_3}{A_{th}} = \frac{A_3}{A_*} = 1.8; \quad \Rightarrow M_3 = 2.08; \quad \frac{p_3}{p_{t3}} = 0.114; \quad \frac{T_3}{T_{t3}} = 0.537$$

$$\Rightarrow p_3 = 124.9 \text{ kPa}; \quad T_3 = 214.9 \text{ K}$$

State-4: $(M_i = 2.08)$

$$\frac{p_4}{p_3} = 4.86; \quad \frac{T_4}{T_3} = 0.537; \quad \frac{p_{t4}}{p_{t3}} = 0.685;$$

$$\Rightarrow p_4 = 607 \text{ kPa}; \quad T_4 = 376 \text{ K}; \quad p_{t4} = 754 \text{ kPa}$$

TEST Solution:

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Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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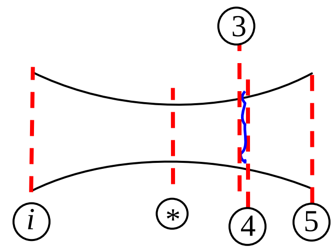
15-3-10 [BNK] Consider the convergent-divergent nozzle of problem 15-3-9 [BNU] . Assume that there is a normal shock wave standing at the point where M = 1.5, determine (a) the exit plane pressure, (b) temperature and (c) Mach number.

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg} \cdot \text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

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Inlet state (Given: $T_{ti} = 400$ K; $p_{ti} = 1100$ kPa):

Let State-3 and State-4 represent states right before and after the shock and State-5 the exit.

State-3:

$p_{t3} = p_{ti} = 1100$ kPa; $T_{t3} = T_{ti} = 400$ K; $A_{*3} = A_{\text{th}} = 5 \text{ cm}^2$;

$M_3 = 1.5$;

State-4: ($M_i = 1.5$)

$$T_{t4} = T_{t3} = 400 \text{ K}; \; p_{t4} = \frac{p_{t4}}{p_{t3}} p_{t3} = (0.93) p_{t3} = 1023 \text{ kPa}; \; A_{*4} = \frac{A_{*4}}{A_{*3}} A_{*3} = (1.076) A_{*3} = 5.38 \text{ cm}^2;$$

State-5: (Given: $A_5 = (1.8) A_{\text{th}} = 9 \text{ cm}^2$)

$T_{t5} = T_{t4} = 400$ K; $p_{t5} = p_{t4} = 1023$ kPa; $A_{*5} = A_{*4} = 5.38 \text{ cm}^2$;

$$\frac{A_5}{A_{5*}} = \frac{9}{5.38} = 1.67; \;\; \Rightarrow M_5 = 0.377$$

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$$\frac{p_5}{p_{t5}} = 0.907; \ \frac{T_5}{T_{t5}} = 0.972;$$

$$\Rightarrow p_5 = 927 \text{ kPa}; \quad T_5 = 389 \text{ K};$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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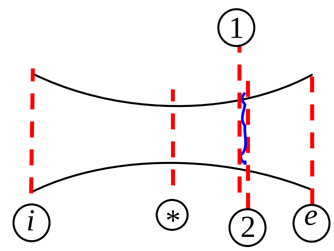
15-3-11 [BNP] Air flowing steadily in a nozzle experiences a normal shock at a Mach number of 2.5. If the pressure and temperature of air are 60 kPa and 273 K, respectively, upstream of the shock, determine (a) the pressure (*p*), temperature (*T*), (b) velocity (*V*), (c) Mach number and (d) the total pressure downstream of the shock. (e) What-if Scenario: What would the velocity be for helium undergoing a normal shock under the same conditions?

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties. Let States 1 and 2 represent the states just before and after the shock.

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Given $M_1 = 2.5$;

$\Rightarrow M_2 = 0.513$

Also,

$$\frac{p_2}{p_1} = 7.125 \Rightarrow \quad p_2 = 7.125(60\text{kPa}) = 427.5 \text{ kPa}$$

$$\frac{T_2}{T_1} = 2.1375 \Rightarrow \quad T_2 = 2.1375(273\text{K}) = 583.5 \text{ K}$$

Therefore, $V_2 = M_2\sqrt{(1000)kRT_2} = (0.513)\sqrt{(1000)(1.40)(0.287)(583.5)} = 248.3 \ \frac{\text{m}}{\text{s}}$

And, $p_{t,2} = \frac{p_{t,2}}{p_{t,1}} p_{t,1} = \frac{p_{t,2}}{p_{t,1}} \frac{p_{t,1}}{p_1} p_1 = (0.499)\frac{1}{(0.0585)}(60) = 511.5 \text{ kPa}$

TEST Solution:

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Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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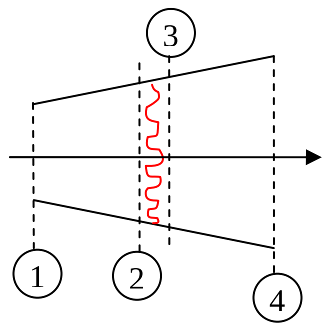
15-3-12 [BNX] An airstream at Mach 2.0, with pressure of 100 kPa and temperature of 300 K, enters a diverging channel with a area ratio of 3 between the exit and inlet. Determine the back pressure necessary to produce a normal shock in the channel at an area equal to twice the inlet area.

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties. Let States 2 and 3 represent the states just before and after the shock.

State-1 (Given: p, T, M):

$$\frac{p_1}{p_{t1}} = 0.128;\ \frac{T_1}{T_{t1}} = 0.556;\ \frac{A_1}{A_{*1}} = 1.688$$
$$\Rightarrow p_{t1} = 782.4 \text{ kPa};\ \Rightarrow T_{t1} = 539.8 \text{ K}$$

State-2 (Given: $T_{t2} = T_{t1},\ p_{t2} = p_{t1},\ A_{*2} = A_{*1},\ \dfrac{A_2}{A_1} = 2$):

$$\frac{A_2}{A_{2*}} = \frac{A_2}{A_1}\frac{A_1}{A_{2*}} = \frac{A_2}{A_1}\frac{A_1}{A_{1*}} = 3.374$$
$$\Rightarrow M_2 = 2.76$$

State-3 (Given: $T_{t3} = T_{t2},\ \dfrac{A_3}{A_1} = 2$):

$$\Rightarrow M_3 = 0.491$$

$$\frac{A_{3*}}{A_{2*}} = 2.485;\ \frac{p_{t3}}{p_{t2}} = 0.402$$

$$\Rightarrow \frac{A_3}{A_{3*}} = \frac{A_{2*}}{A_{3*}}\frac{A_2}{A_{2*}} = \frac{3.374}{2.485} = 1.357;$$

$$\Rightarrow p_{t3} = (0.402)\, p_{t2} = 50.96 \text{ kPa}$$

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State-4 (Given: $T_{t4} = T_{t1}$, $p_{t4} = p_{t3}$, $A_{*4} = A_{*3}$, $\dfrac{A_4}{A_1} = 3$):

$$\frac{A_4}{A_{4*}} = \frac{A_4}{A_1}\frac{A_1}{A_{4*}} = \frac{A_4}{A_1}\frac{A_1}{A_{3*}} = \frac{A_4}{A_1}\frac{A_3}{A_{3*}}\frac{A_1}{A_3} = (3)(1.357)\frac{1}{2} = 2.036$$

$$\frac{p_4}{p_{t4}} = 0.94;$$

$$\Rightarrow p_4 = (0.94)\,p_{t4} = (0.94)\,p_{t3} = 47.9 \text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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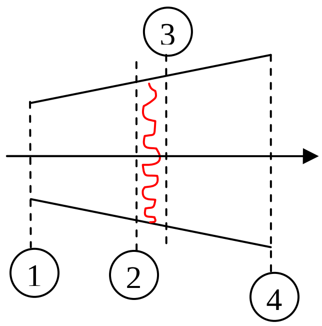
15-3-13 [BNC] A supersonic flow of air at M = 3.0 is to be slowed down via a normal shock in a diverging channel (see Anim. 15-3-12) with an exit to inlet area ratio of 2. At the exit M = 0.5. Find the ratio of exit to inlet pressure.

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties. Let States 2 and 3 represent the states just before and after the shock.

State-1:

$M_1 = 3;$

$\Rightarrow \dfrac{A_1}{A_{*1}} = 4.237;\ \ \dfrac{p_1}{p_{t1}} = 0.0272;$

State-4 (Given $A_4 = 2A_1$):

$M_4 = 0.5;$

$\Rightarrow \dfrac{A_4}{A_{*4}} = 1.34;\ \ \dfrac{p_4}{p_{t4}} = 0.8431;$

State-2:

$$\frac{A_{*3}}{A_{*2}} = \frac{A_{*4}}{A_{*1}} = \frac{A_{*4}}{A_4}\frac{A_4}{A_{*1}} = \frac{A_{*4}}{A_4}\frac{A_4}{A_1}\frac{A_1}{A_{*1}} = \frac{1}{1.34}(2)(4.237) = 6.32$$

$$\Rightarrow M_2 = 3.845;\ \ \frac{p_{t3}}{p_{t2}} = 0.158;$$

$$\frac{p_4}{p_1} = \frac{p_4}{p_{t4}}\frac{p_{t4}}{p_1} = \frac{p_4}{p_{t4}}\frac{p_{t3}}{p_{t2}}\frac{p_{t2}}{p_1} = \frac{p_4}{p_{t4}}\frac{p_{t3}}{p_{t2}}\frac{p_{t1}}{p_1} = (0.8431)(0.1587)\left(\frac{1}{0.0272}\right) = 4.91$$

TEST Solution:

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Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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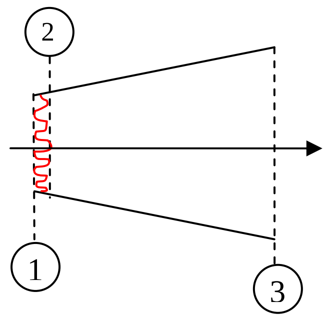
15-3-14 [BNF] Air approaches a diffuser with a pressure of 20 kPa at a Mach number of 2. A normal shock occurs at the inlet of the channel as shown in the accompanying figure. For an exit to inlet area ratio of 3, find (a) the loss of total pressure and (b) the Mach number at the exit. (c) What-if Scenario: What would the answers be if the shock occurred at the exit?

SOLUTION:



$p_i = 20\,\text{kPa}$ and $M_i = 2$ produces $p_{t1} = 156.5$ kPa (from isentropic table)

$A_e = 3A_i$

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Working fluid: air; From Table C-1, obtain: $R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties. Let States 1 and 2 represent the states just before and after the shock.

State-1:

$M_1 = 2;$

$\Rightarrow \dfrac{p_1}{p_{t1}} = 0.1278;\ \Rightarrow p_{t1} = \dfrac{20}{0.1278} = 156.47\text{ kPa};$

State-2:

$M_1 = 2;$

$\Rightarrow M_2 = 0.577;\ \dfrac{p_{t2}}{p_{t1}} = 0.72;\ \dfrac{A_2}{A_{2*}} = 1.21654$

$\Rightarrow p_{t2} = (0.72)156.47 = 112.8\text{ kPa};$

$\Rightarrow p_{t1} - p_{t2} = 43.68\text{ kPa};$

State-3:

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$$\frac{A_3}{A_{3*}} = \frac{A_3}{A_{2*}} = \frac{3A_1}{A_{2*}} = 3\frac{A_2}{A_{2*}} = 3.65$$

$$\Rightarrow M_3 = 0.16;$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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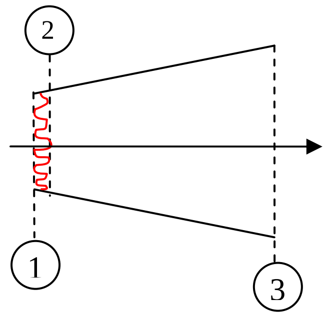
15-3-15 [BNV] In problem 15-3-14 [BNF] determine the diffuser efficiency if the shock is (a) at the inlet and (b) at the exit (%).

SOLUTION:



Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties. Let States 1 and 2 represent the states just before and after the shock, State-3 the exit state, and State-4 is a fictitious inlet state (with same static properties as State-1), but with a lower ke so that the total pressure at State-4 is the same as that at State-3, the diffuser exit.

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State-1:

$M_1 = 2;\ T_1 = 250 \text{ K};$

$\Rightarrow \dfrac{A_1}{A_{1*}} = 1.688$

$\Rightarrow \dfrac{p_1}{p_{t1}} = 0.1278;\ \Rightarrow p_{t1} = 156.47 \text{ kPa};$

$\Rightarrow \dfrac{T_1}{T_{t1}} = 0.555;\ \Rightarrow T_{t1} = 449.8 \text{ K};$

$\Rightarrow V_1 = M_1\sqrt{(1000)kRT_1} = 633.8 \text{ m/s};$

State-2:

$M_1 = 2;$

$\Rightarrow M_2 = 0.577;\ T_{t2} = T_{t1};\ \dfrac{p_{t2}}{p_{t1}} = 0.72;\ \dfrac{A_2}{A_{2*}} = 1.21654$

$\Rightarrow p_{t2} = (0.72)156.47 = 112.8 \text{ kPa};$

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State-3:

$$\frac{A_3}{A_{3*}} = \frac{A_3}{A_{2*}} = \frac{3A_1}{A_{2*}} = 3\frac{A_2}{A_{2*}} = 3.65$$

$$\Rightarrow M_3 = 0.16; \quad \frac{T_3}{T_{t3}} = 0.995; \quad \Rightarrow T_3 = 447.5 \text{ K};$$

State-4:

$$p_4 = p_1 = 20 \text{ kPa}; \quad T_4 = T_1 = 250 \text{ K}; \quad p_{t4} = p_{t3} = p_{t2} = 112.8 \text{ kPa};$$

$$\frac{p_4}{p_{t4}} = \frac{20}{112.8} = 0.177$$

$$\Rightarrow M_4 = 1.787;$$

$$\Rightarrow V_4 = M_4\sqrt{(1000)kRT_4} = M_4\sqrt{(1000)kRT_1} = 566.6 \text{ m/s};$$

Therefore, the diffuser efficiency is:

$$\eta_{\text{diffuser}} = \frac{\text{ke}_4}{\text{ke}_1} = \frac{V_4^2}{V_1^2} = \frac{566^2}{663.8^2} = 0.375^2 = 0.8 = 80\%$$

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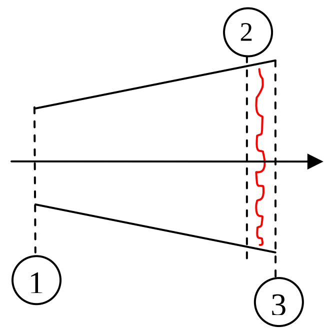
Shock at the exit:



State-1 is unaltered.

State-2:

$$\frac{A_2}{A_{2*}} = \frac{A_2}{A_1}\frac{A_1}{A_{2*}} = \frac{A_2}{A_1}\frac{A_1}{A_{1*}} = 5$$

$$\Rightarrow M_2 = 3.187;$$

State-3:

$$M_2 = 3.187;$$

$$\Rightarrow M_3 = 0.465;\ \ \frac{p_{t3}}{p_{t2}}=0.279;\ \ \Rightarrow p_{t3}=43.69\text{ K};$$

State-4 (isentropic inlet state):

$$p_4 = p_1 = 20\text{ kPa};\ \ T_4 = T_1 = 250\text{ K};\ \ p_{t4} = p_{t3} = 43.69\text{ kPa};$$

$$\frac{p_4}{p_{t4}} = \frac{20}{43.69} = 0.4577$$

$$\Rightarrow M_4 = 1.118;$$

$$\Rightarrow\ V_4 = M_4\sqrt{(1000)kRT_4} = M_4\sqrt{(1000)kRT_1}=354.4\text{ m/s};$$

Therefore, the diffuser efficiency is:

$$\eta_{\text{diffuser}} = \frac{\text{ke}_4}{\text{ke}_1} = \frac{V_4^2}{V_1^2} = \frac{354.4^2}{663.8^2} = 0.313=31.3\%$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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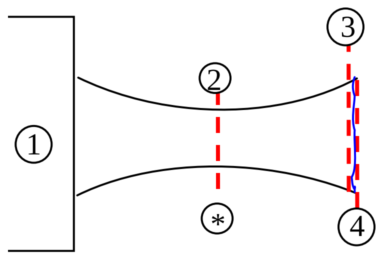
15-3-16 [BNQ] A rocket nozzle has an exit-to-throat area ratio of 4.0. The exhaust gases are generated in a combustion chamber with stagnation pressure equal to 4 MPa and stagnation temperature equal to 2000 K. Assume the working fluid to behave as a perfect gas with k = 1.3 and molar mass = 20 kg/kmol. Determine (a) the rocket exhaust velocity and (b) the mass flow rate. Assume isentropic steady flow except at the nozzle exit where a normal shock is located as shown in the accompanying figure.

SOLUTION:



Working fluid: custom; From given molar mass obtain: $R=\dfrac{\bar{R}}{\bar{M}}=0.416\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k=1.3$

Use the gas dynamics TESTcalc (table panel) to obtain isentropic and shock properties.

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State-1

Given: $T_{t1}=2000$ K; $p_{t1}=4000$ kPa ;

State-2

$M_2=1;\ A_2=A_*=100\ \text{cm}^2;$

$$p_2=\frac{p_2}{p_{2t}}p_{2t}=\frac{p_2}{p_{2t}}p_{1t}=(0.5457)(4000)=2183\ \text{kPa}$$

$$T_2=\frac{T_2}{T_{2t}}T_{2t}=\frac{T_2}{T_{2t}}T_{1t}=(0.869)(2000)=1739\ \text{K}$$:

$$V_2=M\sqrt{1000kRT_2}=(1)\sqrt{(1000)(1.3)(0.4157)(1739)}=969\ \text{m/s}$$

$$\dot{m}=\rho AV=\frac{p}{RT}AV=\left(\frac{2183}{(0.4157)(1739)}\right)(0.01)(969)=29.27\ \frac{\text{kg}}{\text{s}}$$

Let State-3 and State-4 represent states right before and after the shock at the exit.

State-3:

$$p_{t3}=p_{t1}=4000\ \text{kPa}$$

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$$\frac{A_3}{A_{th}} = \frac{A_3}{A_*} = 4; \quad \Rightarrow M_3 = 2.774; \quad \frac{p_3}{p_{t3}} = 0.036; \quad \frac{T_3}{T_{t3}} = 0.464$$

$$\Rightarrow p_3 = 143.8 \text{ kPa}; \; T_3 = 928 \text{ K}$$

State-4:

$$M_3 = 2.774$$

$$\frac{p_4}{p_3} = 8.568; \; \frac{T_4}{T_3} = 2.086;$$

$$\Rightarrow p_4 = 1232 \text{ kPa}; \; T_4 = 1936.5 \text{ K};$$

$$V_4 = M_4\sqrt{1000kRT_4} = (2.774)\sqrt{(1000)(1.3)(0.4157)(1936.5)} = 478.3 \text{ m/s}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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15-3-17 [BNT] In problem 15-3-16 [BNQ] (a) determine the thrust if the outside pressure is 0 kPa. (b) What-if Scenario: What would the thrust be if the normal shock did not exist?

SOLUTION:

With a normal shock at the exit, there will be contribution from both momentum and pressure.

$$T = \frac{\dot{m}}{(1000\ \text{N/kN})}\left(V_4 - \cancel{V_a}^{0}\right) + A_4\left(p_4 - \cancel{p_a}^{0}\right)$$

$$\Rightarrow T = \frac{29.27}{1000}(478.3) + (4)(0.01)(1232) = 14.0 + 49.29 = 63.29\ \text{kN}$$

If the normal shock was absent, State-3 would be the exit state

$$T = \frac{\dot{m}}{(1000\ \text{N/kN})}\left(V_3 - \cancel{V_a}^{0}\right) + A_4\left(p_3 - \cancel{p_a}^{0}\right)$$

$$\Rightarrow T = \frac{29.27}{1000}(969) + (4)(0.01)(143.8) = 57.5 + 5.75 = 63.25\ \text{kN}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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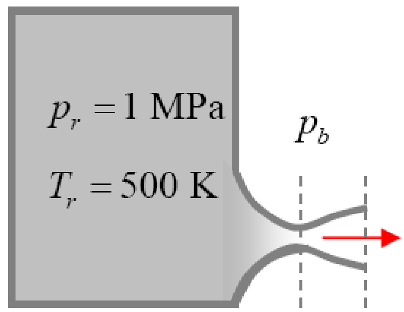
15-3-18 [BNY] For the converging-diverging nozzle shown in the accompanying figure, (a) find the range of back pressure for which a normal shock appears in the diverging section, and (b) the mass flow rate ($\dot{m}$) when the shock is present. The throat area is 10 cm^2 and the exit area is 40 cm^2.



SOLUTION:

Working fluid: air; From Table C-1, obtain: $R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

(a) range of back pressure for which a normal shock appears in the diverging section

Let State I and e be the states right before and after the shock when it is at the exit location. p_b is minimum for this case.

$$\frac{A_e}{A_*} = 4; \quad \Rightarrow \frac{p_i}{p_{ti}} = 0.0298; \; p_i = \left(\frac{p_i}{p_{ti}}\right) p_{ti} = (0.0298)(1000) = 29.8 \text{ kPa}$$

$$\frac{p_e}{p_i} = 9.91447; \quad \Rightarrow p_b = p_e = \left(\frac{p_e}{p_i}\right) p_i = (9.91447)(29.8) = 295.5 \text{ kPa}$$

For subsonic flow in the diverging section, we will have maximum p_b

$$\frac{p_i}{p_{ti}} = 0.98511; \quad \Rightarrow p_b = p_i = \left(\frac{p_i}{p_{ti}}\right) p_{ti} = (0.98511)(1000) = 985.11 \text{ kPa}$$

$$295.5 kPa \le p_b \le 985.11 kPa$$

(b) the mass flow rate when the shock is present, the Mach number must be 1 at the throat. Let State-2 represent the throat state.

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$$M_2 = 1;\ p_{t2} = p_r = 1000 \text{ kPa};\ T_{t2} = T_r = 500 \text{ K}$$

$$\frac{p_2}{p_{t2}} = 0.5283;\ \frac{T_2}{T_{t2}} = 0.833$$

$$\Rightarrow p_2 = 528.3 \text{kPa};\ T_2 = 416.7 \text{ K}$$

$$\dot{m} = \frac{p_2}{RT_2} A_2 M_2 \sqrt{kRT_2} = \frac{p_2}{RT_2} A_2 \sqrt{kRT_2} = 1.81 \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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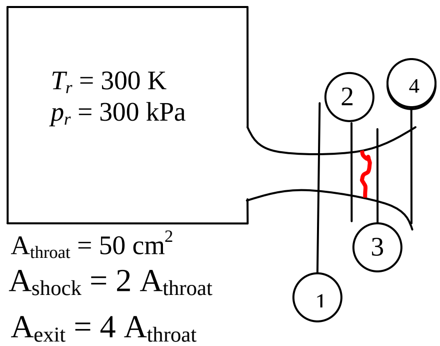
15-3-19 [BNM] For the converging-diverging nozzle shown in the accompanying figure, find the exit Mach number.

SOLUTION:



A_{throat} = 50 cm^2

A_{shock} = 2 A_{throat}

A_{exit} = 4 A_{throat}

Working fluid: air; From Table C-1, obtain: $R = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties. Because the flow is supersonic in the diverging section (after the throat), the throat state is the critical state; that is, Mach number is one and $A_{1*} = A_1 = 50\ \text{cm}^2$.

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State-2:

$$\frac{A_2}{A_{2*}} = \frac{A_2}{A_{1*}} = 2; \quad \Rightarrow M_2 = 2.197.$$

From normal shock table:

$M_2 = 2.197; \quad \Rightarrow M_3 = 0.5474$

$\dfrac{A_3}{A_{*3}} = 1.588; \quad \Rightarrow A_3 = 79.436\ \text{cm}^2$

State-4:

$$A_4 = 4A_1 = 4(50) = 200\ \text{cm}^2$$

$$\frac{A_4}{A_{*4}} = \frac{A_4}{A_{*3}} = \frac{200}{79.436} = 2.517$$

$$\Rightarrow M_4 = 0.2377$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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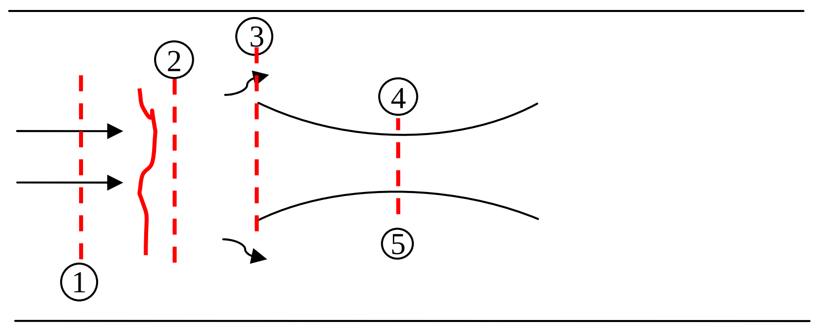
15-3-20 [BIR] A symmetric converging diverging duct with an area ratio of 2 is placed in wind tunnel where it encounters a 700 m/s flow of air at 50 kPa and 300 K. Determine the bypass ratio (diverted flow / incoming flow).

SOLUTION:



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Working fluid: air; From Table C-1, obtain: $R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$; $k = 1.4$

Use the gas dynamics TESTcalc (table panel) or Table H-1 and H-2 to obtain isentropic and shock properties.

Let State-1 represent a flow state upstream of the nozzle where the flow is uniform and parallel. In terms of inlet area A, the incoming mass flow rate can be expressed as follows.

State-1 (Given: $p = 50$ kPa; $T = 300$ K; $V = 700\ \frac{\text{m}}{\text{s}}$):

$$M_1 = \frac{V_1}{\sqrt{1000kRT_1}} = \frac{700}{\sqrt{(1000)(1.4)(0.287)(300)}} = 2.016;$$

$$\frac{p_1}{p_{t1}} = 0.12467; \quad \Rightarrow p_{t1} = 401.35\ \text{kPa}$$

$$\frac{T_1}{T_{t1}} = 0.5518; \quad \Rightarrow T_{t1} = 543.8\ \text{K}$$

$$\dot{m}_1 = \rho_1 A V_1 = \frac{p_1}{RT_1} A V_1 = \left(\frac{50}{(0.287)(300)}\right) A(700) = 406.5A\ \frac{\text{kg}}{\text{s}}$$

State-5

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If the flow was isentropic through the nozzle, it can be shown that the maximum flow through the nozzle would occur when the Mach number is one at the throat.

$$M_5 = 1;\ A_5 = \frac{A}{2};\ p_{t5} = p_{t1};\ T_{t5} = T_{t1};$$

$$\frac{p_5}{p_{t5}} = 0.5283;\ \Rightarrow p_5 = 212\ \text{kPa}$$

$$\frac{T_5}{T_{t5}} = 0.833;\ \Rightarrow T_5 = 453\ \text{K}$$

$$\dot{m}_5 = \rho_5 A_5 V_5 = \frac{p_5}{RT_5}\frac{A}{2}V_5 = 347.8A\ \frac{\text{kg}}{\text{s}}$$

Obviously, the nozzle cannot handle the incoming flow of 406.5A kg/s and some flow must be diverted. However, a supersonic flow is unaware of the existence of the nozzle, so a shock must be formed upstream of the nozzle inlet as shown in the figure.

State-2:

$$M_1 = 2.016;$$

$$\Rightarrow M_2 = 0.5746;\ \frac{p_{t2}}{p_{t1}} = 0.7133;\ \Rightarrow p_{t2} = 401.35\ \text{kPa};\ T_{t2} = T_{t1};$$

The subsonic flow at State-2 will now negotiate the nozzle and an appropriate amount will be bypassed. Assuming isentropic flow through the nozzle, the maximum amount of flow that can be handled by the nozzle occurs when the throat is choked at State-4.

State-4

$$M_4 = 1;\ A_4 = \frac{A}{2};\ p_{t4} = p_{t2};\ T_{t4} = T_{t1};$$

$$\frac{p_4}{p_{t4}} = 0.5283;\ \Rightarrow p_4 = 151\ \text{kPa}$$

$$\frac{T_4}{T_{t4}} = 0.833;\ \Rightarrow T_4 = 453\ \text{K}$$

$$\dot{m}_4 = \rho_4 A_4 V_4 = \frac{p_4}{RT_4}\frac{A}{2}V_4 = 248A\ \frac{\text{kg}}{\text{s}}$$

Therefore, the bypass ratio is: $\dfrac{\text{Diverted flow}}{\text{Incoming flow}} = \dfrac{\dot{m}_1 - \dot{m}_1}{\dot{m}_1} = \dfrac{406.5A - 248A}{406.5A} = 0.39 = 39\%$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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15-3-21 [BND] Supersonic air at Mach 3 and 75 kPa impinges on a two dimensional wedge of half angle 10^o. Determine (a) the two possible oblique shock angles that could be formed by this wedge, (b) the pressure downstream of the oblique shock for each case and (c) the Mach number downstream of the oblique shock for each case.

SOLUTION:

Working fluid: air; From Table C-1, obtain: $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$

Use the gas dynamics TESTcalc (Delta-Theta tab on the table panel) to obtain theta from delta and tables H-1 and H-2 (or the table panel of the gas dyanmiccs TESTcalc) to obtain isentropic and normal-shock properties.

State-1:

Given

$$M_1 = 3,\ p_1 = 75 \text{ kPa}$$

$$\Rightarrow \frac{p_1}{p_{t1}} = 0.02722;\ p_{t1} = 2755.6 \text{ kPa};$$

The flow has to be turned by $\delta = 10^o$

State-2 (after the oblique shock):

Weak-shock:

$$M_1 = 3;\ \delta = 10^o;$$

$$\Rightarrow \theta_{\text{weak}} = 27.4^o;\ M_{2,\text{weak}} = 2.51;$$

Using the normal component of the shock:

$$M_{2n} = M_2 \sin\theta = (2.51)\sin 27.4 = 1.155$$

$$p_2 = \left(\frac{p_2}{p_1}\right) p_1 = (1.39)(75) = 104.3 \text{ kPa}$$

Strong-shock:

$$M_1 = 3;\ \delta = 10^o;$$

$$\theta_{\text{strong}} = 86.4^o;\ M_{2,\text{strong}} = 2.51$$

Using the normal component of the shock:

$$M_{2n} = M_2 \sin\theta = (2.51)\sin 86.4 = 2.505$$

$$p_2 = \left(\frac{p_2}{p_1}\right) p_1 = (7.154)(75) = 536.4 \text{ kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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15-3-22 [BNJ] An airflow with velocity, temperature and pressure of 850 m/s, 25°C and 50 kPa absolute, respectively, is turned with an oblique shock emanating from the wall, which makes an abrupt 20° corner. For a weak shock, determine (a) the downstream Mach number and (b) the pressure (*p*). (c) What-if Scenario: What would the downstream Mach number be for a strong shock?

SOLUTION:

Working fluid: air; From Table C-1, obtain: $R = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};\ k = 1.4$

Use the gas dynamics TESTcalc (Delta-Theta tab on the table panel) to obtain theta from delta and tables H-1 and H-2 (or the table panel of the gas dyanmiccs TESTcalc) to obtain isentropic and normal-shock properties.

State-1:

Given $V_1 = 850\ \text{m/s},\ T_1 = 25^{\circ}\text{C},\ p_1 = 50\ \text{kPa}$

The flow has to be turned by $\delta = 20^{\circ}$

$$M_1 = \frac{V_1}{\sqrt{1000kRT_1}} = \frac{850}{\sqrt{1000(1.4)(0.28699)(298)}} = 2.456$$

State-2 (after the week oblique shock):

$$M_1 = 2.456;\ \delta = 20^{\circ};$$
$$\Rightarrow \theta_{\text{weak}} = 43.5^{\circ};\ M_{2,\text{weak}} = 1.61$$

Using the normal component of the shock:

$$M_{2n} = M_1 \sin\theta = (2.456)\sin 43.5 = 1.69$$

$$p_2 = \left(\frac{p_2}{p_1}\right)p_1 = (3.165)(50\text{kPa}) = 158.3\ \text{kPa}$$

TEST Solution:

Use the gas dynamics TESTcalc to verify the solution and perform the what-if study. The TEST-code for this solution can be found in the professional site of TEST at www.thermofluids.net.

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