13-3-1 [OWJ] A rigid tank contains a mixture of 1 lbm of methane gas and 5 lbm of O_2 at 77°F and 25 psia. Upon ignition, the contents of the tank burns completely. If the final temperature is 1500°R, determine (a) the final pressure in the tank and (b) the amount of heat transfer during the process.

SOLUTION

The reaction of methane and pure oxygen is described by:

$$\mathrm{CH_4} + 2\mathrm{O_2} \rightarrow \mathrm{CO_2} + 2\mathrm{H_2O}$$

a) The final pressure is:

$$\frac{p_f}{p_b} = \frac{T_f \Sigma \nu_p}{T_b \Sigma \nu_r} \Rightarrow p_f = \frac{T_f \Sigma \nu_p}{T_b \Sigma \nu_r} p_b = \frac{(1500)(3)}{(537)(3)}(25) \Rightarrow p_f = 69.83 \text{ psia}$$

b) The heat transfer is;

$$n_{\mathrm{CH_4}} = \frac{m}{\bar{M}} = \frac{1}{16.043} = 0.06233 \text{ lbmol CH}_4$$

$$\frac{Q}{n_F} = \bar{h}_p - \bar{h}_r - \bar{R}\left[T_f \sum_p \nu_p - T_b \sum_r \nu_r\right] \Rightarrow Q = \left(\bar{h}_p - \bar{h}_r - \bar{R}\left[T_f \sum_p \nu_p - T_b \sum_r \nu_r\right]\right) n_F$$

$$Q = \left(-350245 - (-17626) - 1.9858\left[1500(3) - 537(3)\right]\right)(0.06233)$$

$$Q = -21089.7 \text{ Btu}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-2 [OWW] A 10 m^3 insulated rigid tank contains a mixture of 1 kmol of octane (liquid) and the theoretical amount of air at 25°C. The contents are ignited and the mixture burns completely. Determine the final temperature (T_2) and pressure (p_2). Assume the products to be (a) a perfect gas mixture with a c_p of 1.005 kJ/kg K and (b) an ideal gas mixture.

SOLUTION

The reaction of octane and theoretical amount of air is described by:

$$C_8H_{18}(l) + 12.5(O_2 + 3.76N_2) \to 8CO_2 + 9H_2O + 47N_2$$

$$p_f V_f = n_f RT \Rightarrow p_f = 15724.38 \text{ kPa}$$

(a) For PG mixture:

Applying energy equation;

$$\frac{Q}{n_F} = \bar{h}_p - \bar{h}_r - \bar{R}\left[T_f \sum_p \nu_p - T_b \sum_r \nu_r\right]$$

$$0 = \bar{h}_P - \bar{h}_R - \bar{R}\left[T_f \sum_p \nu_p - T_b \sum_r \nu_r\right]$$

$$0 = 1660288.52 - (-250238.9) - 8.314\left[64T_f - 298(60.5)\right]$$

$$\Rightarrow T_f = 3866.32 \text{ K}$$

To find the final pressure:

$$\frac{p_f}{p_b} = \frac{T_f \Sigma \nu_p}{T_b \Sigma \nu_r} \Rightarrow p_f = \frac{T_f \Sigma \nu_p}{T_b \Sigma \nu_r} p_b = \frac{(3866.52)(64)}{298(60.5)}(15724.38) \Rightarrow p_f = 205801.84 \text{ kPa} = 205.8 \text{ MPa}$$

(b) For IG mixture:

$$0 = \bar{h}_P - \bar{h}_R - \bar{R}\left[T_f \sum_p \nu_p - T_b \sum_r \nu_r\right]$$

$$0 = 1151909 - (-250207.25) - 8.314\left[64T_f - 298(60.5)\right]$$

$$\Rightarrow T_f = 2911.18 \text{ K}$$

From the IG law:

$$\frac{p_f}{p_b} = \frac{T_f \Sigma \nu_p}{T_b \Sigma \nu_r} \Rightarrow p_f = \frac{T_f \Sigma \nu_p}{T_b \Sigma \nu_r} p_b = \frac{(2911.18)(64)}{298(60.5)}(15724.38) \Rightarrow p_f = 154.9 \text{ MPa}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG and n-PG Model to verify the results.

13-3-3 [BRR] A mixture of 1 kmol of gaseous methane and 2 kmol of oxygen initially at 298 K and 100 kPa burns completely in a closed, rigid container. Heat transfer occurs until the products are cooled to 1000 K. If the reactants and the products each form ideal gaseous mixtures, determine (a) the amount of heat transfer (Q) and (b) the final pressure (P_2). (c) What-if Scenario: What would the final pressure be if the heat transfer occurred until the products were cooled to 800 K?

SOLUTION

The reaction of methane and oxygen is described by:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

(a) The amount of heat transfer is:

$$\bar{h}_R = \sum_r \nu_r \bar{h}_r = -74923.32 \text{ kJ/kmol}$$

$$\bar{h}_P = \sum_p \nu_p \bar{h}_p = -791703.88 \text{ kJ/kmol}$$

$$\frac{Q}{\eta_f} = \bar{h}_P - \bar{h}_R - \bar{R}(T_f \sum_p \nu_p - T_b \sum_r \nu_r) = -791703.88 - (-74923.32) - 8.314(1000(3) - 298(3))$$

$$Q = (-716913 - 8.314(3000 - 894)) = -734289.8 \text{ kJ}$$

(b) The final pressure is:

$$P_b = 100 \text{ kPa} \qquad T_b = 298 \text{ K} \qquad T_f = 1000 \text{ K}$$

From the IG law;

$$\frac{P_f V_f}{P_b V_b} = \frac{\eta_f \bar{R} T_f}{\eta_b \bar{R} T_b} \Rightarrow P_f = P_b \left(\frac{\sum_p \nu_p}{\sum_r \nu_r} \right) \left(\frac{T_f}{T_b} \right) = 100 \left(\frac{3}{3} \right) \left(\frac{1000}{298} \right) = 335.5 \text{ kPa}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-4 [BRS] A constant-volume tank contains 1 kmol of methane (CH_4) gas and 3 kmol of O_2 at 25°C and 100 kPa. The contents of the tank are ignited, and the methane gas burns completely. If the final temperature is 700°C, determine (a) the final pressure (P_2) in the tank and (b) the heat transfer during this process.

SOLUTION

The balanced reaction equation for complete combustion is,

$$CH_4(g) + 4O_2 \rightarrow CO_2 + 2H_2O + 2O_2$$

(a) Final pressure in the tank is;

Ideal Gas relations:

$$P_P = P_R \left(\frac{\eta_P}{\eta_R}\right)\left(\frac{T_P}{T_R}\right)$$

$$P_P = (100)\left(\frac{973.15}{298.15}\right) = 326.4 \text{ kPa}$$

(b) Heat transfer during process:

$$\frac{Q}{\eta_f} = \bar{h}_P - \bar{h}_R - \bar{R}(T_f \sum_p \nu_p - T_b \sum_r \nu_r) = -751775.84 - (-74969.93) - 8.314(973.15(5) - 298(5))$$

$$= -704871.9 \text{ kJ/kmol}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-5 [BRO] Ethane (C_2H_6) is burned with 200% theoretical air at 500 kPa. Assuming complete combustion at constant pressure, determine (a) the air-fuel ratio and (b) the dew point temperature of the products.

SOLUTION

The combustion equation can be written as follows,

$$C_2H_6 + 7(O_2 + 3.76\,N_2) \rightarrow +\ 2\,CO_2 + 3H_2O + 3.5O_2 + 26.33N_2$$

(a) Air-fuel ration determined by taking the ratio of the mass of air to the mass of the fuel:

$$\text{AF} = \frac{\text{mass of air}}{\text{mass of fuel}} = \frac{(7)(4.76)(29)}{2(12)+3(2)} = 32.21\ \text{kg air/kg fuel}$$

(b) The dew point temperature for the products by first determining the partial pressure (P_v) of the products.

$$P_\text{v} = \left(\frac{\eta_\text{v}}{\eta_P}\right)(P_P) = \left(\frac{3}{34.83}\right)500 = 43.08\ \text{kPa}$$

From (Table B-2)

$$T_\text{dp} = T_{\text{sat@43.08 kPa}} = 77.7^\circ\text{C}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-6 [BRB] A closed combustion chamber is designed so that it maintains a constant pressure (p) of 120 kPa during the combustion process. The combustion chamber has an initial volume of 0.6 m^3 and contains a stoichiometric mixture of octane (C_8H_{18}) gas and air at 25°C. The mixture is now ignited, and the product gases are observed to be at 900 K at the end of the combustion process. Assuming complete combustion, and treating both the reactants and the products as ideal gases, determine (a) the heat transfer (Q) from the combustion chamber during this process. (b) What-if Scenario: What would the heat transfer be if the initial volume were 0.8 m^3?

SOLUTION

The theoretical reaction is:

$$C_8H_{18}(g) + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

$$\frac{Q}{n_f} = \bar{h}_P - \bar{h}_R - \bar{R}\left[T_f \sum_p \nu_p - T_b \sum_r \nu_r\right]$$

$$\bar{h}_R = -208707.26 \text{ kJ/kmol}$$

$$\bar{h}_P = -4041065.24 \text{ kJ/kmol}$$

Applying $pV = mRT$;

$$m_F = \frac{120 \times 0.6 \times 1}{0.07278 \times 298 \times 60.5} = 0.0548 \text{ kg}$$

$$n_F = \frac{0.0548}{114} = 0.0004807 \text{ kmol}$$

$$\frac{Q}{n_F} = \bar{h}_P - \bar{h}_R - \bar{R}\left[T_F \sum_P \nu_P - T_b \sum_r \nu_r\right]$$

$$= -4041065.24 + 208707.26 - 8.314\left[900(64) - 298(60.5)\right] = -4161351.3 \text{ kJ/kmol}$$

$$\Rightarrow Q = -2000.37 \text{ kJ}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-7 [BRA] A constant volume tank contains a mixture of 1 kmol of benzene (C_6H_6) gas and 20% excess air at 25°C and 1 atm. The contents of the tank are now ignited, and all hydrogen in the fuel burns to H_2O but only 93% of the carbon burns to CO_2, the remaining 7% forming CO. If the final temperature (T_2) in the tank is 1000 K, determine (a) the heat transfer (Q) from the combustion chamber during this process. Use the IG mixture model. (b) What-if Scenario: What would the heat transfer be if the PG mixture model were used?

SOLUTION

The theoretical reaction is:

$$C_6H_6 + 7.5(O_2 + 3.76N_2) \rightarrow 6CO_2 + 3H_2O + 28.2N_2$$

With 20 % excess air is:

$$C_6H_6 + 9(O_2 + 3.76N_2) \rightarrow 5.58CO_2 + 0.42CO + 3H_2O + 33.84N_2 + 1.71O_2$$

$$\frac{Q}{n_F} = \bar{h}_P - \bar{h}_R - \bar{R}\left(T_f \sum_p \nu_p - T_b \sum_r \nu_r\right)$$

$$Q = -1926113.95 - 82764.55 - 8.314(1000(44.54) - 298(43.84))$$

$$Q = -2270650.5 \text{ kJ/kmol}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-8 [BRH] A constant volume tank contains a mixture of 150 g of methane (CH_4) gas and 750 g air at 25^{o}C, 150 kPa. The contents of the tank are now ignited, and the methane gas burns completely. If the final temperature in the tank is 1100 K, determine (a) the final pressure (p_2) in the tank and (b) the heat transfer (Q) during this process.

SOLUTION

The balanced theoretical reaction is:

$$\mathrm{CH_4} + 2\left(\mathrm{O_2} + 3.76\mathrm{N_2}\right) \rightarrow \mathrm{CO_2} + 2\mathrm{H_2O} + 7.52\mathrm{N_2}$$

(a) Applying $p\cancel{V} = n\bar{R}T$, the IG equation of state, to the reactants and products mixture, the final pressure can be evaluated as follows.

$$\frac{p_f \cancel{V}_f}{p_b \cancel{V}_b} = \frac{n_f \bar{R} T_f}{n_b \bar{R} T_b}; \quad \Rightarrow \ p_f = p_b \frac{\sum_p \nu_p}{\sum_r \nu_r} \frac{T_f}{T_b} = \left(150\text{kPa}\right)\frac{\left(1+2+7.52\right)}{\left(1+2+7.52\right)}\frac{1100\text{K}}{298\text{K}} = 553.7\text{kPa}$$

The balanced reaction is:

$$\mathrm{CH_4} + 0.58\left(\mathrm{O_2} + 4.76\mathrm{N_2}\right) \rightarrow 0.3\mathrm{CO_2} + 0.58\mathrm{H_2O} + 2.18\mathrm{N_2} + 0.7\mathrm{CH_4}$$

(b) The heat transfer during the process is:

$$\frac{Q}{n_F} = \bar{h}_P - \bar{h}_R - \bar{R}\left[T_f \sum_p \nu_p - T_b \sum_r \nu_r\right]$$

$$\frac{Q}{n_F} = \left(-192617.04\right) - \left(-74887.3\right) - 8.314\left[1100\left(3.76\right) - 298\left(3.76\right)\right]$$

$$\frac{Q}{n_F} = -142856.69 \text{ kJ/kmol}$$

The molar amount of fuel used is $n_F = \dfrac{0.15}{16} = 0.00935$ kmol

$$Q = 0.00935(-142856.69) = -1335.7 \text{ kJ}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-9 [BRN] One kmol of gaseous ethylene (C_2H_4) and 4 kmol of oxygen at 25^oC react in a constant volume bomb. Heat is transferred until the products are cooled to 800 K. Determine (a) the amount of heat transfer (Q) from the system. (b) What-if Scenario: What would the amount of heat transfer be if ethylene reacted with 3 kmol of oxygen?

SOLUTION

The balanced theoretical reaction is:

$$\mathrm{C_2H_4 + 4O_2 \rightarrow 2CO_2 + 2H_2O + O_2}$$

(a) Heat transfer during process:

$$\frac{Q}{\eta_f} = \bar{h}_P - \bar{h}_R - \bar{R}(T_f \sum_p \nu_p - T_b \sum_r \nu_r) = -1173057.3 - (52366.6) - 8.314(800(5) - 298(5))$$
$$= -1246292.1 \ \mathrm{kJ/kmol}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.

13-3-10 [BRE] An adiabatic constant volume tank contains a mixture of 1 kmol of hydrogen (H_2) gas and the stoichiometric amount of air at 25°C and 1 atm. The contents of the tank are now ignited. Assuming complete combustion, determine (a) the final temperature (T_2) and (b) pressure (p_2) in the tank. Use the PG mixture model. (c) What-if Scenario: What would the final temperature and pressure be if the IG mixture model were used?

SOLUTION

The balanced theoretical reaction is:

$$H_2 + 0.5(O_2 + 3.76N_2) \rightarrow H_2O + 1.88N_2$$

(a) The equation that is used to determine the final temperature is:

$$\frac{Q}{n_F} - \frac{W_{ext}}{n_F} = \bar{h}_P - \bar{h}_R - \bar{R}\left(T_f \sum \nu_p - T_b \sum \nu_r\right)$$

For an adiabatic volume with no work exerted, Q=0 and W=0.

$$0 = \bar{h}_P - \bar{h}_R - \bar{R}\left(T_f \sum \nu_p - T_b \sum \nu_r\right)$$
$$0 = 88775.03 - (-14.71) - 8.314\left(2.88T_f - (3.38)298\right)$$

Solving for the final temperature;

$$T_f = 4056.6 \text{ K}$$

(b) From IG law;

$$\frac{P_f V_f}{P_b V_b} = \frac{\eta_f \bar{R} T_f}{\eta_b \bar{R} T_b} \Rightarrow P_f = P_b\left(\frac{\sum_p \nu_p}{\sum_r \nu_r}\right)\left(\frac{T_f}{T_b}\right) = 100\left(\frac{2.88}{3.38}\right)\left(\frac{4056.6}{298}\right) = 1159.9 \text{ kPa}$$

TEST Verification: Use the n-PG premixed open-steady combustion TESTcalc to verify the results.

13-3-11 [BRI] Consider the same combustion process as in 13-2-17 [OWE], but assume that the reactants consists of a mixture at 1 atm, 25°C and that the product also consist of a mixture at 1 atm, 25°C. Determine (a) the work (*w*) that would be done if this combustion process took place reversibly and in pressure and temperature equilibrium with the surroundings.

SOLUTION

The balanced reaction is:

$$C_2H_4 + 12(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 45.12N_2 + 9O_2$$

$$W = \bar{h}_p - \bar{h}_r - \bar{R}\left[T\sum_p \nu_p - T\sum_f \nu_f\right]$$

$$= -1271611.21 - 52241.21 - 8.314\left[298(58.12) - 298(58.12)\right]$$

$$= 1323852.4 \text{ kJ/kmol}$$

$$\Rightarrow \frac{1323852.4}{28} = 47280.4 \ \text{kJ/kg}$$

TEST Verification: Use the closed process premixed combustion TESTcalc: n-IG Model to verify the results.