**11-3-1** [OYC] A gas mixture contains 5 kg of  $N_2$  and 10 kg of  $O_2$ . Determine (a) the average molar mass and (b) gas constant. (c) What-if Scenario: What would the average molar mass be if the gas mixture contained 10 kg of  $N_2$  and 5 kg of  $O_2$ ?

## **SOLUTION:**

The total mass of the mixture are:

$$m = m_{N_2} + m_{O_2} = 5 + 10 = 15 \text{ kg}$$

The mole of each component is

$$n_{N_2} = \frac{m_{N_2}}{\overline{M}_{N_2}} = \frac{5}{28} = 0.179 \text{ kmol}$$

$$n_{O_2} = \frac{m_{O_2}}{\overline{M}_{O_2}} = \frac{10}{32} = 0.3125 \text{ kmol}$$

$$n = n_{N_2} + n_{O_2} = 0.179 + 0.3125 = 0.4915$$
 kmol

The average molar mass and gas constant of the mixture can now be found.

$$\overline{M} = \frac{m}{n} = \frac{15}{0.4915} = 30.52 \frac{\text{kg}}{\text{kmol}}$$

$$R = \frac{\overline{R}}{\overline{M}} = \frac{8.314}{30.52} = 0.2724 \frac{\text{kJ}}{\text{kg.K}}$$

**TEST Solution and What-if Scenario** Use the n-PG system-state TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

11-3-2 [OYQ] A tank contains 3 kmol  $N_2$  and 7 kmol of  $CO_2$  gases at 25°C, 10 MPa. Based on ideal gas equation of state, determine (a) the average molar mass and (b) volume () of the tank.

Unknown

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# **SOLUTION:**

The total mass and mole of the mixture are:

$$n_{N_2} = 3 \text{ kmol}$$

$$\Rightarrow m_{N_2} = n_{N_2} \overline{M}_{N_2} = (3)(28) = 84 \text{ kg}$$

$$n_{CO_2} = 7 \text{ kmol}$$

$$\Rightarrow m_{CO_2} = n_{CO_2} \overline{M}_{CO_2} = (7)(44) = 308 \text{ kg}$$

$$n = n_{N_2} + n_{CO_2} = 3 + 7 = 10$$
 kmol

$$\Rightarrow m = m_{N_2} + m_{CO_2} = 84 + 308 = 392 \text{ kg}$$

The average molar mass and gas constant of the mixture is given as

$$\overline{M} = \frac{m}{n} = \frac{392}{10} = 39.2 \frac{\text{kg}}{\text{kmol}}$$

The volume of the tank is

$$V = \frac{n\overline{R}T}{p} = \frac{(10)(8.314)(298)}{10,000} = 2.48 \text{ m}$$

11-3-3 [OYT] A gas mixture consists of 9 kmol  $H_2$  and 2 kmol of  $N_2$ . Determine (a) the mass of each gas and (b) the apparent gas constant of the mixture. (c) What-if Scenario: What would the answer in (b) be if hydrogen were replaced by oxygen?

## **SOLUTION:**

The total mass and mole of the mixture are

$$n_{H_2} = 9 \text{ kmol}$$
  
 $\Rightarrow m_{H_2} = n_{H_2} \overline{M}_{H_2} = (9)(2.02) = 18.18 \text{ kg}$ 

$$n_{N_2} = 2 \text{ kmol}$$
  
 $\Rightarrow m_{N_2} = n_{N_2} \overline{M}_{N_2} = (2)(28) = 56 \text{ kg}$ 

$$n = n_{H_2} + n_{N_2} = 9 + 2 = 11 \text{ kmol}$$
  
 $m = m_{H_2} + m_{N_2} = 18.18 + 56 = 74.18 \text{ kg}$ 

The average molar mass and gas constant of the mixture can be found now.

$$\overline{M} = \frac{m}{n} = \frac{74.18}{11} = 6.74 \frac{\text{kg}}{\text{kmol}}$$

$$R = \frac{\overline{R}}{\overline{M}} = \frac{8.314}{6.74} = 1.23 \frac{\text{kJ}}{\text{kg.K}}$$

**TEST Solution and What-if Scenario** Use the n-PG system-state TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

11-3-4 [OYY] A rigid tank contains 4 kmol O<sub>2</sub> and 5 kmol of CO<sub>2</sub> gases at 18°C, 100 kPa. Determine (a) the volume Q of the tank.

Unknown

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# **SOLUTION:**

The total mole of the mixture is:  $n = n_{O_2} + n_{CO_2} = 4 + 5 = 9$  kmol

The volume of the tank is

$$V = \left(\frac{n\overline{R}T}{p}\right) = \frac{(9)(8.314)(273+18)}{100} = 0.1377 \text{ m}^3$$



11-3-5 [OYF] A mixture of CO<sub>2</sub> and water vapor is at 100 kPa, 200°C. As the mixture is cooled at a constant pressure, water vapor begins to condense when the temperature reaches 70°C. Determine (a) the mole fraction and (b) the mass fraction of CO<sub>2</sub> in the mixture.

### **SOLUTION:**

During cooling, the total pressure remains constant.

$$p = p_v + p_{CO_2} = c$$

When water vapor begins to condense, the mixture reaches saturation. For the water vapor to be in equilibrium with the water droplet, the partial pressure of water vapor must be equal to the saturation pressure at the temperature of the mixture. From the temperature saturation table, Table B-2, or the PC system state TESTcalc, obtain:

$$p_{v,\text{sat}} = p_{\text{sat}@70^{\circ}\text{C}} = 31.19 \text{ kPa}$$

With total pressure remaining unchanged

$$p_{CO_2,\text{sat}} = p - p_{\nu,\text{sat}} = 100 - 31.19 = 68.79 \text{ kPa}$$

$$y_{co_2} = \frac{n_{co_2}}{n} = \frac{p_{co_2}}{p} = \frac{68.8}{100} = 0.68$$

$$x_{co_2} = \frac{y_{co_2} \overline{M}_{co_2}}{\sum_k y_k \overline{M}_k} = \frac{(0.68)(44)}{(0.32)(18) + (0.68)(44)} = 0.84$$

$$p = p_v + p_{co_2} = c$$

**TEST Solution** Use the n-PG system-state TESTcalc to verify the solution. Compose the mixture by adding the components according to their mole fraction.

11-3-6 [OYM] A  $0.4 \text{ m}^3$  rigid tank contains  $0.4 \text{ kg N}_2$  and  $0.7 \text{ kg of O}_2$  gases at 350 K. Determine (a) the partial pressure of each gas and (b) the total pressure (p) of the mixture. (c) What-if Scenario: What would the total pressure be if the volume of the rigid tank were  $0.6 \text{ m}^3$ ?

## **SOLUTION:**

The partial pressure of each gas is given as

$$p_{N_2} = \frac{m_{N_2} R_{N_2} T}{V} = \frac{m_{N_2} \overline{R} T}{V \overline{M}_{N_2}} = \frac{(0.4)(8.314)(350)}{(0.4)(28)} = 103.88 \text{ kPa}$$

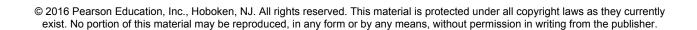
$$p_{O_2} = \frac{m_{O_2} R_{O_2} T}{V} = \frac{m_{O_2} \overline{R} T}{V \overline{M}_{O_2}} = \frac{(0.7)(8.314)(350)}{(0.4)(32)} = 159.128 \text{ kPa}$$

$$p_{O_2} = \frac{m_{O_2} R_{O_2} T}{V} = \frac{m_{O_2} R T}{V M_{O_2}} = \frac{(0.7)(8.314)(350)}{(0.4)(32)} = 159.128 \text{ kPa}$$

The total pressure of the mixture is

$$p = p_{N_2} + p_{O_2} = 103.88 + 159.13 = 263.01 \text{ kPa}$$

TEST Solution and What-if Scenario Use the n-PG system-state TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.



11-3-7 [OYW] A 1 kmol mixture of  $CO_2$  and  $C_2H_6$  (ethane) occupies a volume of 0.2 m<sup>3</sup> at a temperature of 410 K. The mole fraction of  $CO_2$  is 0.3. Using the RG model (Kay's rule), determine (a) the mixture pressure. (b) What-if Scenario: What would the mixture pressure be if IG model were used?

### **SOLUTION:**

The critical temperature and pressure for carbon-di-oxide and ethane is

$$T_{cr,CO_2} = 304 \text{ K}; p_{cr,CO_2} = 7.39 \text{ MPa}$$
  
 $T_{cr,C_2H_6} = 305.5 \text{ K}; p_{cr,C_2H_6} = 4.48 \text{ MPa}$ 

Using Nelson-Obert charts

$$T_{r,CO_{2}} = \frac{T_{m}}{T_{cr,CO_{2}}} = \frac{410}{304} = 1.35$$

$$v_{r,CO_{2}} = \frac{v_{CO_{2}}}{R_{u}T_{cr,CO_{2}} / p_{cr,CO_{2}}}; \Rightarrow v_{r,CO_{2}} = \frac{\cancel{L} \cdot m / N_{CO_{2}}}{R_{u}T_{cr,CO_{2}} / p_{cr,CO_{2}}}$$

$$\Rightarrow v_{r,CO_{2}} = \frac{(0.2)/(0.3)}{(8.314)(304)/(7390)}; \Rightarrow v_{r,CO_{2}} = 1.94$$

$$T_{r,C_{2}H_{6}} = \frac{T_{m}}{T_{cr,C_{2}H_{6}}}; \Rightarrow T_{r,C_{2}H_{6}} = \frac{410}{305.5}; \Rightarrow T_{r,C_{2}H_{6}} = 1.34$$

$$v_{r,C_{2}H_{6}} = \frac{v_{C_{2}H_{6}}}{R_{u}T_{cr,C_{2}H_{6}} / p_{cr,C_{2}H_{6}}}; \Rightarrow v_{r,C_{2}H_{6}} = \frac{\cancel{L} \cdot m / N_{C_{2}H_{6}}}{R_{u}T_{cr,C_{2}H_{6}} / p_{cr,C_{2}H_{6}}}$$

$$\Rightarrow v_{r,C_{2}H_{6}} = \frac{(0.2)/(0.7)}{(8.314)(305.5)/(4480)}; \Rightarrow v_{r,C_{2}H_{6}} = 0.504$$

$$p_{CO_{2}} = Z_{CO_{2}}p_{cr,CO_{2}}; \Rightarrow p_{CO_{2}} = (0.915)(7.39); \Rightarrow p_{CO_{2}} = 6.8 \text{ MPa}$$

$$p_{C_{2}H_{6}} = Z_{C_{2}H_{6}}p_{cr,C_{2}H_{6}}; \Rightarrow p_{C_{2}H_{6}} = (0.74)(4.48); \Rightarrow p_{C_{2}H_{6}} = 3.3 \text{ MPa}$$

$$p_{m} = p_{CO_{2}} + p_{C_{2}H_{6}}; \Rightarrow p_{m} = 10.1 \text{ MPa}$$

**TEST Solution and What-if Scenario** Use the RG-RG system-state TESTcalc to verify the solution and the IG-IG system state TESTcalc to conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

11-3-8 [OYD] A mixture consisting of 0.18 kmol of methane and 0.274 kmol of butane occupies a volume of 0.3 m<sup>3</sup> at a temperature of 240°C. Using the IG model, determine (a) the pressure exerted by the mixture. (b) What-if Scenario: What would the answer in (a) be if RG model (Kay's rule) were used?

### **SOLUTION:**

Given: 
$$n_{\text{CH}_4} = 0.18 \text{ kmol}$$
;  $n_{\text{C}_4\text{H}_{10}} = 0.274 \text{ kmol}$ ;  $\mathcal{L} = 0.3 \text{ m}^3$ ;  $T = 240 ^{\circ}\text{C}$ 

$$pV = n\overline{R}T;$$
  

$$\Rightarrow p = \frac{n\overline{R}T}{V} = \frac{(0.18 + 0.274) \cdot (8.314) \cdot (240 + 273)}{0.3} = 6456 \text{ kPa} = 6.4 \text{ MPa}$$

**TEST Solution and What-if Scenario** Use the n-PG system-state TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.



11-3-9 [OYJ] Determine (a) the mass of 1 m<sup>3</sup> air (N<sub>2</sub>: 79% and O<sub>2</sub>: 21% by volume) at 10 MPa and 160 K, assuming air as a perfect gas mixture. (b) What-if Scenario: What would the answer in (a) be if real gas mixture model were used?

## **SOLUTION:**

The mole and volume fraction being equal

$$y_{N_2} = 0.79; \ y_{O_2} = 0.21;$$

The molar mass being the mass of unit mole of mixture,

$$\bar{M} = y_{N_2} \bar{M}_{N_2} + y_{O_2} \bar{M}_{O_2} = 28.84 \frac{\text{kg}}{\text{kmol}}$$

$$R = \frac{\overline{R}}{\overline{M}} = \frac{8.314}{30.52} = 0.288 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

The mass of the mixture can now be found:

$$p \mathcal{V} = n \overline{R} T = \frac{m}{\overline{M}} \overline{R} T = mRT;$$

$$\Rightarrow m = \frac{pV}{RT} = \frac{(10000)(1)}{(0.288)(160)} = 6456 \text{ kPa} = 216.8 \text{ kg}$$

**TEST Solution and What-if Scenario** Use the n-PG system-state TESTcalc to verify the solution. For the what-if study, use the RG-RG system-state TESTcalc.. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

11-3-10 [OFB] An insulated rigid tank is divided into two compartments by a partition. One compartment contains 8 kg of oxygen gas at 42°C and 100 kPa, and the other compartment contains 4 kg of nitrogen gas at 20°C and 180 kPa. The partition is then removed and the two gases are allowed to mix. Determine (a) the mixture temperature and (b) the mixture pressure after equilibrium has been reached.

### **SOLUTION:**

State-1 (pure  $O_2$ ):

$$p_{1} = 100 \text{ kPa}, T_{1} = 42^{\circ}\text{C} = 315 \text{ K}$$

$$m_{1} = 8 \text{ kg}, R_{1} = 0.26 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, c_{\nu 1} = 0.644 \frac{\text{kJ}}{\text{kg} \cdot \text{K}};$$

$$\Rightarrow n_{1} = \frac{m_{1}}{\overline{M}_{1}} = 0.25 \text{ kmol}$$
State-2 (pure  $N_{2}$ ):

$$p_{2} = 180 \text{ kPa}, \ T_{2} = 20^{\circ}\text{C} = 293 \text{ K}$$

$$m_{2} = 4 \text{ kg}, \ R_{2} = 0.297 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, \ c_{v2} = 0.733 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\Rightarrow n_{2} = \frac{m_{2}}{\overline{M}_{2}} = 0.14 \text{ kmol}$$
State-3 (mixture)
$$m_{3} = m_{1} + m_{2}; \ \mathcal{V}_{3} = \mathcal{V}_{1} + \mathcal{V}_{2},$$

$$n_{3} = n_{1} + n_{2} = 0.39 \text{ kmol}$$

$$y_{O_{2,3}} = \frac{n_{O_{2,3}}}{n} = \frac{n_{1}}{n_{3}} = \frac{0.25}{0.39} = 0.64; \ y_{N_{2,3}} = \frac{n_{N_{2,3}}}{n} = \frac{n_{2}}{n_{3}} = \frac{0.14}{0.39} = 0.36$$

$$\Rightarrow \overline{M}_{3} = \sum y_{k,3} \overline{M}_{k} = (0.64 \times 32) + (0.36 \times 28) = 30.56 \frac{\text{kg}}{\text{kmol}}$$

$$\Rightarrow R_{3} = \frac{R}{\overline{M}_{3}} = \frac{8.314}{30.56} = 0.272 \frac{\text{kJ}}{\text{kg} \times \text{K}}$$

(a) The energy equation for this closed process (for the combined system) simplifies to:

$$\Delta U = \cancel{Q}^{0} - \cancel{W_{\text{ext}}}^{0}$$

$$\Rightarrow U_{3} - (U_{1} + U_{2}) = 0$$

$$\Rightarrow (m_{O_{2,3}} u_{O_{2,3}} + m_{N_{2,3}} u_{N_{2,3}}) - (m_{1} u_{1} + m_{2} u_{2}) = 0$$

$$\Rightarrow (m_{1} u_{O_{2,3}} + m_{2} u_{N_{2,3}}) - (m_{1} u_{O_{2,1}} + m_{2} u_{N_{2,2}}) = 0$$

$$\Rightarrow m_{1} (u_{O_{2,3}} - u_{O_{2,1}}) = m_{2} (u_{N_{2,2}} - u_{N_{2,3}}) = 0$$

$$\Rightarrow m_{1} c_{v,O_{2}} (T_{3} - T_{1}) = m_{2} c_{v,N_{2}} (T_{2} - T_{3})$$

$$\Rightarrow (8)(0.644)(T_{3} - 315) = (4)(0.733)(293 - T_{3})$$

$$\Rightarrow (T_{3} - 315) = (0.57) \times (293 - T_{3})$$

$$\Rightarrow T_{3} = 306.845 \text{ K} = 33.84^{\circ}\text{C}$$

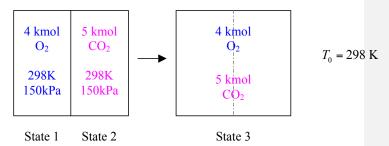
(b)  

$$p_3 \frac{V_3}{V_3} = m_3 R_3 T_3$$
  
 $\Rightarrow p_3 = \frac{m_3 R_3 T_3}{\frac{V_3}{V_3}} = \frac{12 \times 0.272 \times 306.845}{8.48} = 118.13 \text{ kPa}$ 

**TEST Solution** Use the n-PG system-state TESTcalc to verify the solution. Compose the mixture by adding the components according to their mole fraction.

11-3-11 [OFR] An insulated rigid tank is divided into two compartments by a partition. One compartment contains 4 kmol of  $O_2$ , and the other compartment contains 5 kmol of  $CO_2$ . Both gases are initially at 25°C and 150 kPa. The partition is then removed and the two gases are allowed to mix. Assuming the surroundings are at 25°C and both gases behave as ideal gases, determine (a) the entropy change ( $\Delta S$ ) and (b) the exergy destruction (*I*) associated with this process. (c) What-if Scenario: What would the entropy change be if there were 2 kmol of  $O_2$  instead?

### **SOLUTION:**



(a) The energy equation for the combined system going through the closed process simplifies as:

$$\Delta U = \cancel{Q}^{0} - \cancel{W_{\text{ext}}}^{0}$$

$$\Rightarrow U_{3} - (U_{1} + U_{2}) = 0$$

$$\Rightarrow (m_{1}u_{O_{2,3}} + m_{2}u_{CO_{2,3}}) = (m_{1}u_{O_{2,1}} + m_{2}u_{CO_{2,2}})$$

$$\Rightarrow m_{1}(u_{O_{2,3}} - u_{O_{2,1}}) = -m_{2}(u_{CO_{2,3}} - u_{CO_{2,2}})$$

Because u = u(T) is a monotonically increasing function of temperature for the IG model, any rise in temperature will make the left hand side of the equation positive and the right hand side negative. The opposite is true for any fall of temperature. Therefore, the only solution to the energy equation is  $T_3 = T_1 = T_2 = 298 \text{ K}$ 

$$\Delta S = S_3 - (S_2 - S_1) = (m_{O_2,3} s_{O_2,3} + m_{CO_2,3} s_{CO_2,3}) - (m_1 s_1 + m_2 s_2)$$

$$\Rightarrow \Delta S = \left(m_{1} s_{O_{2},3} + m_{2} s_{CO_{2},3}\right) - \left(m_{1} s_{O_{2},1} + m_{2} s_{CO_{2},2}\right)$$

$$\Rightarrow \Delta S = m_{1} \left(s_{O_{2},3} - s_{O_{2},1}\right) + m_{2} \left(s_{CO_{2},3} - s_{CO_{2},2}\right)$$

$$\Rightarrow \Delta S = m_{1} \left(s_{O_{2},3}^{\circ} - s_{O_{2},1}^{\circ} - R_{O_{2}} \ln \frac{p_{O_{2},3}}{p_{1}}\right) - m_{2} \left(s_{CO_{2},3}^{\circ} - s_{CO_{2},2}^{\circ} - R_{CO_{2}} \ln \frac{p_{CO_{2},3}}{p_{2}}\right)$$

$$\Rightarrow \Delta S = -m_{1} R_{O_{2}} \ln \frac{y_{O_{2},3} p_{3}}{p_{1}} - m_{2} R_{CO_{2}} \ln \frac{y_{CO_{2},3} p_{3}}{p_{2}}$$

$$\Rightarrow \Delta S = -\overline{R} \left(n_{O_{2}} \ln y_{O_{2},3} + n_{CO_{2}} \ln y_{CO_{2},3}\right) = -\left(8.314\right) \left(4 \ln \left(\frac{4}{9}\right) + 5 \ln \left(\frac{5}{9}\right)\right) = 51.4 \frac{\text{kJ}}{\text{K}}$$

(b) The entropy equation for the combined system going through the closed process simplifies as:

$$\Delta S = S_f - S_i = \frac{\cancel{Q}^0}{T_B} + S_{\text{gen,univ}}$$

$$\Rightarrow S_{\text{gen,univ}} = \Delta S = 51.4 \frac{\text{kJ}}{\text{K}}$$

Therefore,  $I = T_0 S_{\text{gen,univ}} = 15317.2 \text{ kJ}$ 

**TEST Solution and What-if Scenario** Use the IG-IG mixing closed-process TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

11-3-12 [OFO] An insulated rigid tank is divided into two compartments by a partition. One compartment contains 0.2 kmol of  $CO_2$  at  $26^{\circ}C$ , 180 kPa and the other compartment contains 3 kmol of  $H_2$  gas at  $37^{\circ}C$ , 340 kPa. The partition is then removed and the two gases are allowed to mix. Determine (a) the mixture temperature and (b) the mixture pressure after equilibrium has been established.

### **SOLUTION:**

The molar mass and specific heats of  $CO_2$  and  $H_2$  are 44.01 kg/kmol, 2.016 kg/kmol, 0.657 kJ/kg.K, 10.183 kJ/kg.K respectively

The volume of each gas is

$$\mathcal{V}_{CO_2} = \left(\frac{n\overline{R}T}{p}\right)_{CO_2} = \frac{(0.2)(8.314)(299)}{180} = 2.76 \text{ m}^3$$

$$\mathcal{V}_{H_2} = \left(\frac{n\overline{R}T}{p}\right)_{H_2} = \frac{(3)(8.314)(310)}{340} = 22.74 \text{ m}^3$$

$$\Rightarrow \mathcal{V} = \mathcal{V}_{CO_2} + \mathcal{V}_{H_2} = 25.5 \text{ m}^3$$

(a) The energy equation for the combined system going through a closed process can be simplified as:

$$\Delta U = \mathcal{Q}^{0} - \mathcal{W}_{\text{ext}}^{0}$$

$$\Rightarrow U_{f} - (U_{b,CO_{2}} + U_{b,H_{2}}) = 0$$

$$\Rightarrow (m_{CO_{2}}u_{f,CO_{2}} + m_{H_{2}}u_{f,H_{2}}) - (m_{CO_{2}}u_{b,CO_{2}} + m_{H_{2}}u_{b,H_{2}}) = 0$$

$$\Rightarrow [m(u_{f} - u_{b})]_{CO_{2}} + [m(u_{f} - u_{b})]_{H_{2}} = 0$$

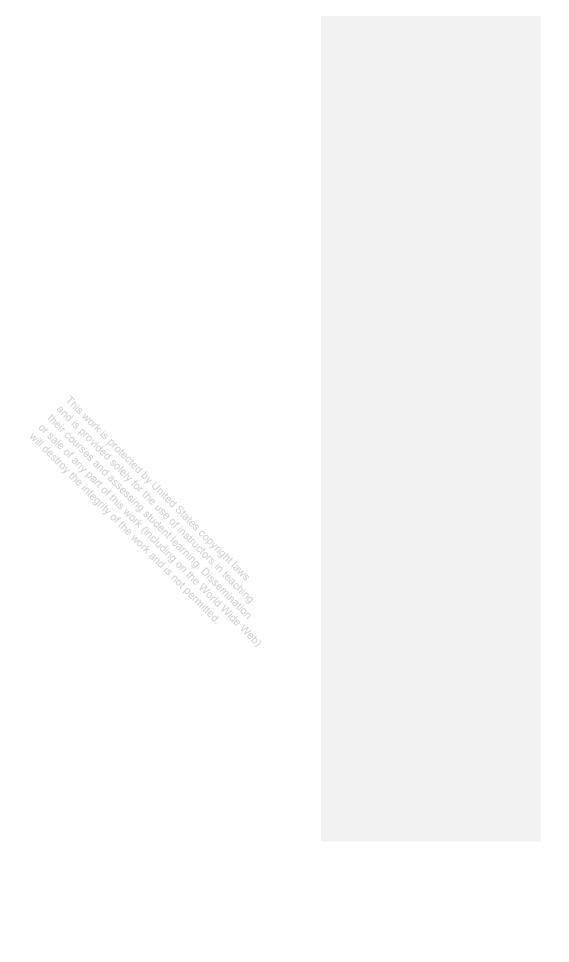
$$\Rightarrow [mc_{v}(T_{f} - T_{b})]_{CO_{2}} + [mc_{v}(T_{f} - T_{b})]_{H_{2}} = 0$$

$$\Rightarrow 0 = (0.2)(44.01)(0.657)(T - 299) + (3)(2.016)(10.183)(T - 310)$$

$$\Rightarrow T = 309.05 \text{ K} = 36.05^{\circ}\text{C}$$

(b) The mixture pressure is

$$p = \frac{n\overline{R}T}{V} = \frac{(3.2)(8.314)(309.05)}{25.5} = 322.44 \text{ kPa}$$



11-3-13 [OFS] A rigid insulated tank is divided into two compartments by a membrane. One compartment contains 0.3 kmol of  $CO_2$  at  $25^{\circ}C$  and 100 kPa, and the other compartment contains 4 kmol of  $H_2$  gas at  $40^{\circ}C$  and 300 kPa. The membrane is then punctured and the two gases are allowed to mix. Determine (a) the mixture temperature and (b) the mixture pressure after equilibrium has been reached. (c) What-if Scenario: What would the mixture temperature be if there were 2 kmol of  $H_2$  instead?

### **SOLUTION:**

The molar mass and specific heats of  $CO_2$  and  $H_2$  are 44.01 kg/kmol, 2.016 kg/kmol, 0.657 kJ/kg.K, 10.183 kJ/kg.K respectively

The volume of each gas is

$$\mathcal{V}_{CO_2} = \left(\frac{n\overline{R}T}{p}\right)_{CO_2} = \frac{(0.3)(8.314)(298)}{100} = 7.43 \text{ m}^3$$

$$\mathcal{V}_{H_2} = \left(\frac{n\overline{R}T}{p}\right)_{H_2} = \frac{(4)(8.314)(313)}{300} = 34.69 \text{ m}^3$$

$$\mathcal{V} = \mathcal{V}_{CO_2} + \mathcal{V}_{H_2} = 42.12 \text{ m}^3$$

(a) The energy equation for the combined system going through a closed process can be simplified as:

$$\Delta U = \mathcal{Q}^{0} - \mathcal{W}_{\text{ext}}^{0}$$

$$\Rightarrow U_{f} - (U_{b,CO_{2}} + U_{b,H_{2}}) = 0$$

$$\Rightarrow (m_{CO_{2}} u_{f,CO_{2}} + m_{H_{2}} u_{f,H_{2}}) - (m_{CO_{2}} u_{b,CO_{2}} + m_{H_{2}} u_{b,H_{2}}) = 0$$

$$\Rightarrow [m(u_{f} - u_{b})]_{CO_{2}} + [m(u_{f} - u_{b})]_{H_{2}} = 0$$

$$\Rightarrow [mc_{v} (T_{f} - T_{b})]_{CO_{2}} + [mc_{v} (T_{f} - T_{b})]_{H_{2}} = 0$$

$$\Rightarrow 0 = (0.3)(44.01)(0.657)(T_{f} - 298) + (4)(2.016)(10.183)(T_{f} - 313) = 0$$

$$\Rightarrow T_{f} = 311.72 \text{ K} = 38.72^{\circ}\text{C}$$

(b) The mixture pressure is

$$p = \frac{n\overline{R}T}{V} = \frac{(4.3)(8.314)(311.72)}{42.12} = 264.58 \text{ kPa}$$

**TEST Solution and What-if Scenario** Use the PG-PG mixing closed-process TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net. Mill Sole Collis Ovide Dole Clear to the line of the state of the land of the Marion o

11-3-14 [OFA] Two rigid, insulated tanks are interconnected by a valve. Initially 0.79 kmol of nitrogen at 200 kPa and 255 K fills one tank. The other tank contains 0.21 kmol of oxygen at 100 kPa and 300 K. The valve is opened and the gases are allowed to mix until a final equilibrium state is attained. During this process, there are no heat or work interactions between the tank contents and the surroundings. Determine (a) the final temperature of the mixture ( $T_{\rm m}$ ), (b) the final pressure of the mixture ( $p_{\rm m}$ ) and (c) the amount of entropy produced ( $S_{\rm gen}$ ) in the mixing process.

### **SOLUTION:**

The molar mass and specific heats of  $N_2$  and  $O_2$  are 28 kg/kmol, 32 kg/kmol, 0.743 kJ/kg.K, 0.658 kJ/kg.K respectively

The volume of each gas is

$$\mathcal{V}_{N_{2}} = \left(\frac{n\overline{R}T}{p}\right)_{N_{2}}; \quad \Rightarrow \mathcal{V}_{N_{2}} = \frac{(0.79)(8.314)(255)}{200}; \quad \Rightarrow \mathcal{V}_{N_{2}} = 8.37 \text{ m}^{3}$$

$$\mathcal{V}_{O_{2}} = \left(\frac{n\overline{R}T}{p}\right)_{O_{2}}; \quad \Rightarrow \mathcal{V}_{O_{2}} = \frac{(0.21)(8.314)(300)}{100}; \quad \Rightarrow \mathcal{V}_{O_{2}} = 5.24 \text{ m}^{3}$$

$$\mathcal{V}_{M} = \mathcal{V}_{N_{2}} + \mathcal{V}_{O_{2}}; \quad \Rightarrow \mathcal{V}_{M} = 13.6 \text{ m}^{3}$$

(a) The energy equation for the combined system going through a closed process can be simplified as:

$$\Delta U = \emptyset^{0} - W_{\text{ext}}^{0}$$

$$\Rightarrow U_{f} - (U_{b,N_{2}} + U_{b,O_{2}}) = 0$$

$$\Rightarrow (m_{N_{2}}u_{f,N_{2}} + m_{O_{2}}u_{f,O_{2}}) - (m_{N_{2}}u_{b,N_{2}} + m_{O_{2}}u_{b,O_{2}}) = 0$$

$$\Rightarrow [m(u_{f} - u_{b})]_{N_{2}} + [m(u_{f} - u_{b})]_{O_{2}} = 0$$

$$\Rightarrow [mc_{v}(T_{f} - T_{b})]_{N_{2}} + [mc_{v}(T_{f} - T_{b})]_{O_{2}} = 0$$

$$\Rightarrow 0 = (0.79)(28)(0.743)(T_{f} - 255) + (0.21)(32)(0.658)(T_{f} - 300)$$

$$\Rightarrow T_{f} = 264.5 \text{ K}$$

(b) The mixture pressure is

$$p = \frac{n\overline{R}T}{V} = \frac{(1)(8.314)(264.5)}{13.6} = 161.7 \text{ kPa}$$

(c) The change in entropy for the process can be calculated as:

$$\begin{split} &\Rightarrow \Delta S = S_3 - \left(S_2 - S_1\right) = \left(m_1 s_{O_2,3} + m_2 s_{N_2,3}\right) - \left(m_1 s_{O_2,1} + m_2 s_{N_2,2}\right) \\ &\Rightarrow \Delta S = m_1 \left(s_{O_2,3} - s_{O_2,1}\right) + m_2 \left(s_{N_2,3} - s_{N_2,2}\right) \\ &\Rightarrow \Delta S = m_1 \left(c_{p,O_2} \ln \frac{T_3}{T_1} - R_{O_2} \ln \frac{p_{O_2,3}}{p_1}\right) - m_2 \left(c_{p,N_2} \ln \frac{T_3}{T_2} - R_{N_2} \ln \frac{p_{N_2,3}}{p_2}\right) \\ &\Rightarrow \Delta S = m_1 \left(c_{p,O_2} \ln \frac{T_3}{T_1} - R_{O_2} \ln \frac{y_{O_2,3} p_3}{p_1}\right) - m_2 \left(c_{p,N_2} \ln \frac{T_3}{T_2} - R_{N_2} \ln \frac{y_{N_2,3} p_3}{p_2}\right) \\ &\Rightarrow \Delta S = 3.452 \frac{\mathrm{kJ}}{\mathrm{K}} \end{split}$$

The entropy equation for the closed process simplifies as:

$$\Delta S = S_f - S_i = \frac{\cancel{Q}^0}{T_B} + S_{\text{gen,univ}}$$

$$\Rightarrow S_{\text{gen,univ}} = \Delta S = 3.452 \frac{\text{kJ}}{\text{K}}$$

11-3-15 [OFH] A 1.1 m³ rigid tank is divided into two equal compartments by a partition. One compartment contains Ne at 22°C and 120 kPa, and the other compartment contains Ar at 50°C and 200 kPa. The partition is then removed, and the two gases are allowed to mix. Heat is lost to the surrounding air at 20°C during this process in the amount of 18 kJ. Determine (a) the final mixture temperature ( $T_{\rm m}$ ) and (b) the final mixture pressure ( $p_{\rm m}$ ). (c) What-if Scenario: What would the final mixture temperature be if heat lost were in the amount of 20 kJ?

# **SOLUTION:**

(a) Because noble gases are perfect gases; the PG model is ideally suited for this problem.

$$n_{Ne} = \left(\frac{pV}{RT}\right)_{Ne} = \frac{(120)(0.55)}{(8.314)(295)} = 0.0269 \text{ kmol}$$

$$\Rightarrow m_{Ne} = n_{Ne}\overline{M}_{Ne} = 0.543 \text{ kg}$$

$$n_{Ar} = \left(\frac{pV}{RT}\right)_{Ar} = \frac{(200)(0.55)}{(8.314)(323)} = 0.0410 \text{ kmol}$$

$$\Rightarrow m_{Ar} = n_{Ar}\overline{M}_{Ar} = 1.637 \text{ kg}$$

$$y_{Ne} = \frac{n_{1Ne}}{n_2}; \implies y_{Ne} = 0.39635$$
  $y_{Ar} = \frac{n_{1Ar}}{n_2}; \implies y_{Ar} = 0.00278$ 

$$\overline{M}_2 = y_{Ne} \overline{M}_{Ne} + y_{Ar} \overline{M}_{Ar}; \Rightarrow \overline{M}_2 = (0.39635)(20.18) + (0.00278)(39.95); \Rightarrow \overline{M}_2 = 8.1095$$

$$R_2 = \frac{\overline{R}}{\overline{M}_2}; \implies R_2 = 1.025$$

The energy equation for the closed process yields:

$$\Delta U = Q - W_{\text{ext}}^{0} \sum$$

$$\Rightarrow U_{3} - (U_{1} + U_{2}) = Q$$

$$\Rightarrow (m_{3,Ne}u_{3,Ne} + m_{3,Ar}u_{3,Ar}) - (m_{1,Ne}u_{1,Ne} + m_{2,Ar}u_{2,Ar}) = Q$$

$$\Rightarrow (m_{Ne}u_{3,Ne} + m_{Ar}u_{3,Ar}) - (m_{Ne}u_{1,Ne} + m_{Ar}u_{2,Ar}) = Q$$

$$\Rightarrow m_{Ne}(u_{3,Ne} - u_{1,Ne}) + m_{Ar}(u_{3,Ar} - u_{2,Ar}) = Q$$

$$\Rightarrow m_{Ne}(x_{3,Ne} - u_{1,Ne}) + m_{Ar}(x_{3,Ar} - u_{2,Ar}) = Q$$

$$\Rightarrow m_{Ne}c_{v,Ne}(T_{3} - T_{1}) + m_{Ar}c_{v,Ar}(T_{3} - T_{2}) = Q$$

$$\Rightarrow T_{3}(m_{Ne}c_{v,Ne} + m_{Ar}c_{v,Ar}) - T_{1}m_{Ne}c_{v,Ne} - T_{2}m_{Ar}c_{v,Ar} = Q$$

$$\Rightarrow T_{2} = \frac{Q + T_{1}m_{Ne}c_{v,Ne} + T_{2}m_{Ar}c_{v,Ar}}{(m_{Ne}c_{v,Ne} + m_{Ar}c_{v,Ar})}$$

$$\Rightarrow T_{2} = \frac{-18 + (295)(0.543)(0.6179) + (323)(1.637)(0.3121)}{(0.543)(0.6179) + (1.637)(0.3121)} = 290.6K;$$

$$\Rightarrow T_{2} = 17.6^{\circ}C$$
(b)
$$p_{3} = \frac{n_{3}\overline{R}_{3}T_{3}}{V_{3}} = \frac{(0.06789)(8.314)(290.6)}{(1.1)} = 149.1 \text{ kPa}$$

**TEST Solution and What-if Scenario** Use the PG-PG mixing closed-process TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-pro site at www.thermofluids.net.

**11-3-16** [OFN] A rigid tank that contains 3 kg of  $N_2$  at 25°C and 250 kPa is connected to another rigid tank that contains 2 kg of  $O_2$  at 25°C and 450 kPa. The valve connecting the two tanks is opened, and the two gases are allowed to mix. If the final mixture temperature is 25°C, determine (a) the volume Q of each tank and (b) the final mixture pressure  $(p_m)$ . Use the IG Model.

Unknown **Deleted:** V

### **SOLUTION:**

(a) The molar mass and specific heats of  $N_2$  and  $O_2$  are 28 kg/kmol, 32 kg/kmol, 0.743 kJ/kg.K, 0.658 kJ/kg.K respectively

$$n_{N_2} = \frac{m_{N_2}}{\overline{M}_{N_2}} = \frac{3}{28} = 0.107 \text{ kmol}$$

$$\Rightarrow V_{N_2} = \left(\frac{n\overline{R}T}{p}\right)_{N_2} = \frac{(0.107)(8.314)(298)}{250} = 1.06 \text{ m}^3$$

$$n_{O_2} = \frac{m_{O_2}}{\overline{M}_{O_2}} = \frac{2}{32} = 0.0625 \text{ kmol}$$

$$\Rightarrow V_{O_2} = \left(\frac{n\overline{R}T}{p}\right)_{O_2} = \frac{(0.0625)(8.314)(298)}{450} = 0.344 \text{ m}^3$$
For the mixture,
$$V = V_{N_2} + V_{O_2} = 1.404 \text{ m}^3$$

$$n = n_{N_2} + n_{O_2} = 0.1695 \text{ kmol}$$

 $\Rightarrow p = \frac{n\overline{R}T}{V} = \frac{(0.1695)(8.314)(298)}{1.404} = \frac{299 \text{ kPa}}{1.404}$ 

**TEST Solution and What-if Scenario** Use the IG-IG mixing closed-process TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

11-3-17 [OFE] An insulated rigid tank that contains 1 kg of CO<sub>2</sub> at 100 kPa and 25°C and is connected to another insulated rigid tank that contains 1 kg of H<sub>2</sub> at 200 kPa and 500°C. The valve connecting the two tanks is opened, and the two gases are allowed to mix. Determine (a) the final mixture pressure  $(p_m)$ , (b) temperature  $(T_m)$ , (c) entropy generated  $(S_{gen})$  during mixing, and (d) reversible work  $(W_{rev})$  of mixing if the outside temperature is 25°C. Use the IG mixture model. (e) What-if Scenario: (e) How would the answer in part (d) change if both gases were CO<sub>2</sub>?

### **SOLUTION:**

State-1 (pure  $CO_2$ ):

$$p_1 = 100 \text{ kPa}, T_1 = 298 \text{ K}, m_1 = 8 \text{ kg};$$

$$\Rightarrow n_1 = \frac{m_1}{\overline{M}_1} = 0.0227 \text{ kmol}; \ \ \mathcal{V}_1 = \left(\frac{n\overline{R}T}{p}\right)_1 = \frac{(0.0227)(8.314)(298)}{100} = 0.5632 \text{ m}^3$$

State-2 (pure  $H_2$ ):

$$p_2 = 200 \text{ kPa}, T_2 = 773 \text{ K}, m_2 = 1 \text{ kg};$$

$$\Rightarrow n_2 = \frac{m_2}{\overline{M}_2} = 0.4961 \text{ kmol}; \ \frac{V_2}{V_2} = \left(\frac{n\overline{R}T}{p}\right)_2 = \frac{(0.4961)(8.314)(773)}{200} = 15.944 \text{ m}^3$$

State-3 (mixture)

$$m_3 = m_1 + m_2$$
;  $V_3 = V_1 + V_2 = 16.507 \text{ m}^3$ ;  $n_3 = n_1 + n_2 = 0.5188 \text{ kmol}$ 

$$\Rightarrow y_{CO_2,3} = \frac{n_1}{n_3} = 0.0438; y_{H_2,3} = 1 - y_{CO_2,3} = 0.0956;$$

(a) The energy equation for this closed process (for the combined system) simplifies to:

$$\Delta U = \cancel{Q}^0 - \cancel{W_{\rm ext}}^0$$

$$\Rightarrow U_3 - (U_1 + U_2) = 0$$

$$\Rightarrow \left( m_1 u_{CO_{2,3}} + m_2 u_{H_{2,3}} \right) - \left( m_1 u_{CO_{2,1}} + m_2 u_{H_{2,2}} \right) = 0$$

$$\Rightarrow m_1 \left( u_{CO_{2,3}} - u_{CO_{2,1}} \right) = m_2 \left( u_{H_{2,2}} - u_{H_{2,3}} \right)$$

Iterating over  $T_3$ , the equation is satisfied when

$$\Rightarrow T_3 = 306.845 \text{ K} = 738.6 \text{ K} = 465.4^{\circ}\text{C}$$

(b) 
$$p_3 \frac{V_3}{V_3} = m_3 R_3 T_3$$

$$\Rightarrow p_3 = \frac{m_3 R_3 T_3}{V_3} = 193 \text{ kPa}$$

$$\begin{split} &\Delta S = S_3 - \left(S_2 - S_1\right) \\ &\Rightarrow \Delta S = \left(m_1 s_{CO_2,3} + m_2 s_{H_2,3}\right) - \left(m_1 s_{CO_2,1} + m_2 s_{H_2,2}\right) \\ &\Rightarrow \Delta S = m_1 \left(s_{CO_2,3} - s_{CO_2,1}\right) + m_2 \left(s_{H_2,3} - s_{H_2,2}\right) \\ &\Rightarrow \Delta S = m_1 \left(s_{CO_2,3}^\circ - s_{CO_2,1}^\circ - R_{CO_2} \ln \frac{p_{CO_2,3}}{p_1}\right) - m_2 \left(s_{H_2,3}^\circ - s_{H_2,2}^\circ - R_{H_2} \ln \frac{p_{H_2,3}}{p_2}\right) \\ &\Rightarrow \Delta S = m_1 \left(s_{CO_2,3}^\circ - s_{CO_2,1}^\circ - R_{CO_2} \ln \frac{p_{CO_2,3}}{p_1}\right) - m_2 \left(s_{H_2,3}^\circ - s_{H_2,2}^\circ - R_{H_2} \ln \frac{p_{H_2,3}}{p_2}\right) \\ &\Rightarrow \Delta S = 1.029 \frac{kJ}{K} \end{split}$$

The entropy equation for the combined system going through the closed process simplifies as:

$$\Delta S = S_f - S_i = \frac{\cancel{Q}^0}{T_B} + S_{\text{gen,univ}}$$

$$\Rightarrow S_{\text{gen,univ}} = \Delta S = 1.029 \frac{\text{kJ}}{\text{K}}$$
(d)
$$W_{\text{rev}} = \cancel{W_{\text{ext}}}^0 + I = I = T_0 S_{\text{gen,univ}} = (298)(1.029) = 306.64 \text{ kJ}$$

**TEST Solution** Use the n-PG system-state TEST calc to verify the solution. Compose the mixture by adding the components according to their mole fraction.

11-3-18 [OFI]  $N_2$  at 100 kPa, 30°C with a flow rate of 100 m<sup>3</sup>/min is mixed with CO<sub>2</sub> at 200°C, 100 kPa with a flow rate of 50 m<sup>3</sup>/min. Determine (a) the final temperature ( $T_2$ ) and (b) rate of generation of entropy ( $S_{gen}$ ). Assume the gases to behave as a perfect gas mixture. (c) What-if Scenario: What would the final temperature be if the ideal gas mixture model were used?

### **SOLUTION:**

The molar mass and specific heats of  $N_2$  and  $CO_2$  are 28 kg/kmol, 44 kg/kmol, 1.039 kJ/kg.K, 0.846 kJ/kg.K respectively

Given: 
$$N_2$$
:  $p_1 = 100 \text{ kPa}$ ;  $T_1 = 30^{\circ}\text{ C}$ ;  $T_1 = 100 \frac{\text{m}^3}{\text{min}}$   
 $R_2 = \frac{RTT_1^{\otimes}}{p_1}$ ;  $\Rightarrow R_3 = \frac{(0.297)(303)(1.67)}{100}$ ;  $\Rightarrow R_4 = 1.5 \frac{\text{kg}}{\text{s}}$ 

Given: 
$$CO_2$$
:  $p_2 = p_1$  kPa;  $T_2 = 200^{\circ}$  C;  $\frac{18}{2} = 50 \frac{\text{m}^3}{\text{min}}$   
 $RB_2 = \frac{RT^{\frac{18}{2}}}{p_2}$ ;  $\Rightarrow RB_2 = \frac{(0.1889)(473)(0.83)}{100}$ ;  $\Rightarrow RB_2 = 0.742 \frac{\text{kg}}{\text{s}}$ 

For the mixture: 
$$p_3 = p_1 = p_2$$
;  $n_{3} = n_{3} + n_{3}$ ;  $\Rightarrow n_{3} = 2.24 \frac{\text{kg}}{\text{s}}$ 

The energy balance equation for this mixing open-steady system simplifies as:

$$\frac{dE_{0}^{0}}{dt}^{0} = nB_{1}j_{1} + nB_{2}j_{2} - nB_{3}j_{3} + \mathcal{D}^{0} - \mathcal{D}^{0}_{ext}^{0} \cong nB_{1}h_{1} + nB_{2}h_{2} - nB_{3}h_{3}$$

$$\Rightarrow nB_{1}h_{N_{2},3} + nB_{2}h_{CO_{2},3} = nB_{1}h_{N_{2},1} + nB_{2}h_{CO_{2},2}$$

$$\Rightarrow nB_{1}\left(h_{N_{2},3} - h_{N_{2},1}\right) + nB_{2}\left(h_{CO_{2},3} - h_{CO_{2},2}\right) = 0$$

$$\Rightarrow \left[nB_{1}C_{p}\left(T_{3} - T_{1}\right)\right]_{N_{2}} + \left[nB_{1}C_{p}\left(T_{3} - T_{2}\right)\right]_{CO_{2}} = 0$$

$$\Rightarrow \left(1.5\right)\left(1.039\right)\left(T_{3} - 303\right) + \left(0.742\right)\left(0.846\right)\left(T_{3} - 473\right) = 0$$

$$\Rightarrow T_{3} = 351.47 \text{ K}$$

The molar flow rate of the two gases in the mixture is

$$\frac{R_{N_2}}{R_{N_2}} = \left(\frac{R_R}{\overline{M}}\right)_{N_2} = \frac{1.5}{28} = 0.054 \frac{\text{kmol}}{\text{s}}$$

$$\frac{R_{N_2}}{R_{N_2}} = \left(\frac{R_R}{\overline{M}}\right)_{CO_2} = \frac{0.742}{44} = 0.0169 \frac{\text{kmol}}{\text{s}}$$

The mole fraction of each gas becomes

$$y_{N_2} = \frac{0.054}{0.054 + 0.0169} = 0.762$$
$$y_{CO_2} = \frac{0.0169}{0.054 + 0.0169} = 0.238$$

The entropy balance equation simplifies as:

$$\frac{dS'}{dt}^{0} = nR_{1}S_{1} + nR_{2}S_{2} - nR_{3}S_{3} + \frac{g^{0}}{T_{B}} + g^{0}_{gen}$$

$$\Rightarrow S_{gen}^{L} = nR_{3}S_{3} - nR_{4}S_{1} - nR_{2}S_{2}$$

$$\Rightarrow S_{gen}^{L} = \left[nR_{2}(S_{3} - S_{1})\right]_{N_{2}} + \left[nR_{2}(S_{3} - S_{2})\right]_{CO_{2}}$$

$$\Rightarrow S_{gen}^{L} = \left[nR_{2}(C_{p} \ln \frac{T_{3}}{T_{1}} - R \ln y)\right]_{N_{2}} + \left[nR_{2}(C_{p} \ln \frac{T_{3}}{T_{2}} - R \ln y)\right]_{CO_{2}}$$

$$\Rightarrow S_{gen}^{L} = \left[(1.5)\left(1.039 \ln \frac{351.47}{303} - 0.297 \ln 0.762\right)\right] + \left[(0.742)\left(0.846 \ln \frac{351.47}{473} - 0.1889 \ln 0.238\right)\right]$$

$$\Rightarrow S_{gen}^{L} = 0.37 \frac{kW}{K}$$

**TEST Solution and What-if Scenario** Use the PG-PG mixing open-steady TESTcalc to verify the solution and conduct the what-if study. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

11-3-19 [OFL] An equimolar mixture of oxygen and nitrogen enters a compressor operating at steady state at 10 bar, 220 K with a mass flow rate (m) of 1 kg/s. The mixture exits the compressor at 60 bar, 400 K with no significant change in KE or PE. The heat transfer from the compressor can be neglected. Using the RG model (Kay's rule), determine (a) the external power  $(W_{\rm ext})$  and (b) the rate of entropy production  $(S_{\rm gen})$ . (c) What-if Scenario: What would the required power be if the ideal gas mixture model were used?

### **SOLUTION::**

Given: 
$$p_1 = 1$$
 MPa;  $T_1 = 220$  K;  $n_{\text{K}} = 1$   $\frac{\text{kg}}{\text{s}}$ 

Given: 
$$p_2 = 6 \text{ MPa}$$
;  $T_2 = 400 \text{ K}$ ;  $n = 200 \text{ K}$ 

From Table E-1, obtain the necessary material properties of  $N_2$  and  $O_2$ :

$$p_{cr,N_2} = 3.39 \text{ MPa}, T_{cr,N_2} = 126.2 \text{ K}$$

$$p_{cr,O_2} = 5.08 \text{ MPa}, T_{cr,O_2} = 154.8 \text{ K}$$

The mixture molar mass of the mixture

$$\bar{M} = y_{N_2} \bar{M}_{N_2} + y_{O_2} \bar{M}_{O_2}$$

$$\Rightarrow \bar{M} = (0.5)(28) + (0.5)(32) = 30 \frac{\text{kg}}{\text{kmol}}$$

Using Kay's rule, the gas mixture can be treated as a pseudo substance whose critical temperature and pressure are

$$T_{cr} = y_{N_2} T_{cr,N_2} + y_{O_2} T_{cr,O_2} = (0.5)(126.2) + (0.5)(154.8) = 140.5 \text{ K}$$
  
 $p_{cr} = y_{N_2} p_{cr,N_2} + y_{O_2} p_{cr,O_2} = (0.5)(3.39) + (0.5)(5.08) = 4.235 \text{ MPa}$ 

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = \frac{1}{4.235} = 0.236; \ p_{r2} = \frac{p_2}{p_{cr}} = \frac{6}{4.235} = 1.417$$

$$T_{r1} = \frac{T_1}{T_{cr}} = \frac{220}{140.5} = 1.57; \ T_{r2} = \frac{T_2}{T_{cr}} = \frac{400}{140.5} = 2.85$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{\text{L-K}} = 0.982$ ,  $Z_2^{\text{L-K}} = 1.006$ Assuming ideal gas behavior, the inlet and exit enthalpies of  $N_2$  and  $O_2$ 

$$h_{1,N_2} = 6391 \frac{\text{kJ}}{\text{kmol}}; \ h_{2,N_2} = 11640 \frac{\text{kJ}}{\text{kmol}}$$

$$h_{1,O_2} = 6404 \frac{\text{kJ}}{\text{kmol}}; \ h_{2,O_2} = 11711 \frac{\text{kJ}}{\text{kmol}}$$

Thus, for the mixture:

$$\Delta \bar{h}^{IG} = y_{N_2} (\bar{h}_2 - \bar{h}_1)_{N_2} + y_{O_2} (\bar{h}_2 - \bar{h}_1)_{O_2}$$

$$\Rightarrow \Delta \bar{h}^{IG} = (0.5) (11640 - 6391) + (0.5) (11711 - 6404) = 5278 \frac{\text{kJ}}{\text{kmol}}$$

$$\Rightarrow \Delta h^{IG} = \frac{\Delta \bar{h}^{IG}}{\bar{M}} = 175 \frac{\text{kJ}}{\text{kg}}$$

Therefore,

$$\Delta h = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}) = 175 - 0.277(140.5)(1.006 - 0.982) = 174 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}) = (5.96 - 5.87) - 0.277(1.006 - 0.982) = 0.083 \frac{\text{kJ}}{\text{kg, K}}$$

The energy equation and entropy equations yields

$$V_{ext}^{\&} = -n\&(h_2 - h_1) + \&^0 = -174 \text{ kW}$$

$$\&_{gen}^{\&} = n\&(s_2 - s_1) - \&^0 = 0.083 \text{ kW}$$

**TEST Solution and What-if Scenario** Use the RG-RG single-flow open-steady TESTcalc to verify the solution. For the what-if study, use the corresponding IG-IG TESTcalc. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

**11-3-20** [OFZ] A mixture of 0.5 kg of carbon dioxide and 0.3 kg of nitrogen is compressed from 100 kPa, 300 K to 300 kPa in a polytropic process for which n = 1.25. Determine (a) the final temperature  $(T_2)$ , (b) the work (W), (c) the heat transfer (Q) and (d) the change in entropy  $(\Delta S)$  of the mixture.

### **SOLUTION:**

(a) For the mixture,

$$\begin{split} \overline{M} &= \frac{m}{n} = \frac{\left(0.5 + 0.3\right)}{0.5 / \overline{M}_{CO_2} + 0.3 / \overline{M}_{N_2}} = 36.24 \frac{\text{kg}}{\text{kmol}} \\ &\stackrel{\rightleftharpoons}{\rightleftharpoons} \Re \sum_{k} \frac{\overline{R}}{\sqrt{n}} c_{\overline{p},k} \frac{8.314}{36.00.5} \frac{0.5}{0.5} \frac{229}{0.5} c_{\overline{p},k} \frac{\text{kJ}}{\sqrt{n}} + \frac{0.3}{\sqrt{n}} \frac{0.3}{\sqrt{n}} \left(0.5 + 0.3\right) c_{p,N_2} = \left(0.625\right) \left(0.844\right) + \left(0.375\right) \left(1.031\right) = 0.914 \frac{\text{kJ}}{\text{kg}} \\ c_v &= c_p - R = 0.6845 \frac{\text{kJ}}{\text{kg}} \end{split}$$

Treating the mixture as a perfect gas, the polytropic relation (Eq. 5.17), we obtain:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = \left(\frac{300}{100}\right)^{(1.25-1)/1.25}$$

$$\Rightarrow T_2 = (300)3^{0.2} = 373.3 \text{ K}$$

(b) Using Eq. 5.18,

$$W_{B,\text{polytropic}} = \frac{mR(T_2 - T_1)}{1 - n} = \frac{(0.8)(0.229)(373.3 - 300)}{1 - 1.25} = -54.12 \text{ kJ}$$

(c) For this closed process, the energy equation simplifies as:

$$\Delta U = Q - W_{\text{ext}};$$

$$\Rightarrow Q = mc_v (T_2 - T_1) + W_{B,\text{polytropic}} = (0.8)(0.6845)(373.3 - 300) + (-54.12) = -14.0 \text{ kJ}$$

(c) The change in entropy can be calculated as:

$$\Delta S = m\Delta S = m(s_2 - s_1) = m\left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}\right) = -0.0417 \frac{\text{kJ}}{\text{K}}$$

**TEST Solution** Use the PG-PG system-state TESTcalc to verify the solution. Compose the mixture by adding the components according to their mole fraction. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.

**11-3-21** [OFP] Helium at 200 kPa, 20°C is heated by mixing it with argon at 200 kPa, 500°C in an adiabatic chamber. Helium enters the chamber at 2 kg/s and argon at 0.5 kg/s. If the mixture leaves at 200 kPa, determine (a) the temperature ( $T_2$ ) at the exit and (b) the rate of entropy generation ( $S_{gen}$ ) due to mixing.

### **SOLUTION:**

(a) The molar mass and specific heats of He and Ar are 4 kg/kmol, 40 kg/kmol, 5.193 kJ/kg.K, 0.5203 kJ/kg.K respectively

State-1: He: 
$$p_1 = 200 \text{ kPa}$$
;  $T_1 = 20^{\circ}\text{C}$ ;  $n_{\text{K}} = 2 \frac{\text{kg}}{\text{s}}$ 

State-2: Ar: 
$$p_2 = p_1 \text{ kPa}$$
;  $T_2 = 500^{\circ}\text{ C}$ ;  $n = 0.5 \frac{\text{kg}}{\text{s}}$ 

State-3: Mixture: 
$$p_3 = p_1 \text{ kPa}$$
;  $n k = n k + n k = 2.5 \frac{\text{kg}}{\text{s}}$ 

The energy balance equation for this mixing open-steady system simplifies as:

$$\begin{split} \frac{dE^{\prime 0}}{dt} &= n \Re_1 j_1 + n \Re_2 j_2 - n \Re_3 j_3 + \Re_2^{0} - N \Re_3^{0} = n \Re_1 h_1 + n \Re_2 h_2 - n \Re_3 h_3 \\ &\Rightarrow n \Re_1 h_{He,3} + n \Re_2 h_{Ar,3} = n \Re_1 h_{He,1} + n \Re_2 h_{Ar,2} \\ &\Rightarrow n \Re_1 \left( h_{He,3} - h_{He,1} \right) + n \Re_2 \left( h_{Ar,3} - h_{Ar,2} \right) = 0 \\ &\Rightarrow \left[ n \Re_2 \left( T_3 - T_1 \right) \right]_{He} + \left[ n \Re_2 \left( T_3 - T_2 \right) \right]_{Ar} = 0 \\ &\Rightarrow (2)(5.193)(T_3 - 293) + (0.5)(0.5203)(T_3 - 773) = 0 \\ &\Rightarrow T_3 = 304.9 \text{ K} \end{split}$$

(b) The molar flow rate of the two gases in the mixture is

$$R_{He} = \left(\frac{R_{K}}{\overline{M}}\right)_{He} = \frac{2}{4} = 0.5 \frac{\text{kmol}}{\text{s}}; \quad R_{Ar} = \left(\frac{R_{K}}{\overline{M}}\right)_{Ar} = \frac{0.5}{40} = 0.0125 \frac{\text{kmol}}{\text{s}}$$

The mole fraction of each gas becomes

$$y_{He} = \frac{0.5}{0.5 + 0.0125} = 0.976; \ y_{Ar} = \frac{0.0125}{0.5 + 0.0125} = 0.0244$$

The entropy balance equation simplifies as:

$$\frac{dS'^{0}}{dt} = nR_{t}S_{1} + nR_{2}S_{2} - nR_{2}S_{3} + \frac{g^{0}}{T_{B}} + g^{0}_{gen}$$

$$\Rightarrow S_{gen}^{2} = nR_{3}S_{3} - nR_{5}S_{1} - nR_{2}S_{2}$$

$$\Rightarrow S_{gen}^{2} = \left[nR_{2}(s_{3} - s_{1})\right]_{He} + \left[nR_{2}(s_{3} - s_{2})\right]_{Ar}$$

$$\Rightarrow S_{gen}^{2} = \left[nR_{2}(c_{p} \ln \frac{T_{3}}{T_{1}} - R \ln \frac{y p_{3}}{p_{3}})\right]_{He} + \left[nR_{2}(c_{p} \ln \frac{T_{3}}{T_{2}} - R \ln \frac{y p_{3}}{p_{3}})\right]_{Ar}$$

$$\Rightarrow S_{gen}^{2} = \left[nR_{2}(c_{p} \ln \frac{T_{3}}{T_{1}} - R \ln y)\right]_{He} + \left[nR_{2}(c_{p} \ln \frac{T_{3}}{T_{2}} - R \ln y)\right]_{Ar}$$

$$\Rightarrow S_{gen}^{2} = \left[(2)\left(5.193 \ln \frac{304.9}{293} - 2.0769 \ln 0.976\right)\right] + \left[(0.5)\left(0.5203 \ln \frac{304.9}{773} - 0.2081 \ln 0.02448\right)\right]$$

$$\Rightarrow S_{gen}^{2} = (2)(0.257) + (0.5)(0.289) = 0.66 \frac{kW}{K}$$



**11-3-22** [OFG] Repeat problem 11-3-21 [OFP] with argon entering the chamber at the same temperature as helium (all other conditions remaining the same). (c) Explain the change in entropy generation rate ( $S_{gen}$ ).

#### **SOLUTION:**

(a) With temperature of the incoming gases being the same, that is,  $T_2 = T_1$ , the energy equation produces:

$$\begin{split} \left[ \mathbf{nBC}_{p} \left( T_{3} - T_{1} \right) \right]_{He} + \left[ \mathbf{nBC}_{p} \left( T_{3} - T_{1} \right) \right]_{Ar} &= 0 \\ \Rightarrow \left( T_{3} - T_{1} \right) \left[ \left( \mathbf{nBC}_{p} \right)_{He} + \left( \mathbf{nBC}_{p} \right)_{Ar} \right] &= 0 \\ \Rightarrow T_{3} &= T_{1} &= 293 \text{ K} \end{split}$$

(b) The entropy balance equation simplifies as:

$$S_{gen}^{\mathcal{R}} = [R_{\mathbf{x}}(s_3 - s_1)]_{He} + [R_{\mathbf{x}}(s_3 - s_2)]_{Ar}$$

$$\Rightarrow S_{gen}^{\mathcal{R}} = \left[R_{\mathbf{x}}(c_p \ln \frac{T_e^{-1}}{f_1^r} - R \ln y)\right]_{He} + \left[R_{\mathbf{x}}(c_p \ln \frac{T_e^{-1}}{f_2^r} - R \ln y)\right]_{Ar}$$

$$\Rightarrow S_{gen}^{\mathcal{R}} = (2)(0.0505) + (0.5)(0.772)$$

$$\Rightarrow S_{gen}^{\mathcal{R}} = 0.487 \frac{kW}{K}$$

(c) With temperature of the two streams being the same, there is less difference to destroy. Hence the rate of entropy generation is lower.

**11-3-23** [OFK] Repeat problem 11-3-22 [OFG] with argon replaced by neon, entering the chamber at the same temperature as helium (all other conditions remaining the same). (c) Explain the change in entropy generation rate ( $S_{gen}$ ).

## **SOLUTION:**

With argon replaced by neon, the only change in the solution comes from properties of

neon: 
$$\overline{M}_{Ne} = 20.183 \frac{\text{kg}}{\text{kmol}}$$
,  $c_{p,Ne} = 1.03 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ 

(a) With temperature of the incoming gases being the same, that is,  $T_2 = T_1$ , the energy equation produces:

$$\left[ n \mathcal{B} \boldsymbol{c}_{P} \left( T_{3} - T_{1} \right) \right]_{He} + \left[ n \mathcal{B} \boldsymbol{c}_{P} \left( T_{3} - T_{1} \right) \right]_{Ne} = 0$$

$$\Rightarrow \left( T_{3} - T_{1} \right) \left[ \left( n \mathcal{B} \boldsymbol{c}_{P} \right)_{He} + \left( n \mathcal{B} \boldsymbol{c}_{P} \right)_{Ne} \right] = 0$$

$$\Rightarrow T_{3} = T_{1} = 293 \text{ K}$$

(b) The molar flow rate of the two gases in the mixture is

$$R_{He} = \left(\frac{nR}{\overline{M}}\right)_{He} = \frac{2}{4} = 0.5 \frac{\text{kmol}}{\text{s}}; \quad R_{Ar} = \left(\frac{nR}{\overline{M}}\right)_{Ne} = \frac{0.5}{20.183} = 0.025 \frac{\text{kmol}}{\text{s}}$$

The mole fraction of each gas becomes

$$y_{He} = \frac{0.5}{0.5 + 0.025} = 0.952; \ y_{Ar} = \frac{0.0125}{0.5 + 0.025} = 0.048$$

The entropy balance equation simplifies as:

$$\mathcal{S}_{gen}^{\mathbf{k}} = \left[ n \mathbf{R} (s_3 - s_1) \right]_{He} + \left[ n \mathbf{R} (s_3 - s_2) \right]_{Ne}$$

$$\Rightarrow \mathcal{S}_{gen}^{\mathbf{k}} = \left[ n \mathbf{R} \left( c_p \ln \frac{T_f^{\prime}}{T_1} - R \ln y \right) \right]_{He} + \left[ n \mathbf{R} \left( c_p \ln \frac{T_f^{\prime}}{T_2} - R \ln y \right) \right]_{Ne}$$

$$\Rightarrow \mathcal{S}_{gen}^{\mathbf{k}} = (2)(0.1022) + (0.5)(1.275)$$

$$\Rightarrow \mathcal{S}_{gen}^{\mathbf{k}} = 0.842 \frac{\mathbf{kW}}{\mathbf{k}}$$

(c) Because of lower molar mass of neon (compared to argon), the difference in mole fractions of the components can be seen to be higher in the mixture. The mixing friction increases as a result explaining a higher rate of entropy generation.

**11-3-24** [OFU] Repeat problem 11-3-21 [OFP] with the hot gas argon replaced by neon (all other conditions remaining the same). (c) Explain the change in entropy generation rate ( $S_{gen}$ ).

## **SOLUTION:**

(a) Material properties of helium and neon (Table C-1) are:

$$\overline{M}_{He} = 4 \text{ kg/kmol}, \ \overline{M}_{Ne} = 20.18 \text{ kg/kmol}$$

$$c_{p,He} = 5.193 \,\text{kJ/kg} \cdot \text{K}, \ c_{p,Ne} = 1.03 \,\text{kJ/kg} \cdot \text{K}$$

State-1: He: 
$$p_1 = 200 \text{ kPa}$$
;  $T_1 = 20^{\circ}\text{C}$ ;  $nR_1 = 2 \frac{\text{kg}}{\text{s}}$ 

State-2: Ne: 
$$p_2 = p_1$$
 kPa;  $T_2 = 500^{\circ}$  C;  $n_2 = 0.5 \frac{\text{kg}}{\text{s}}$ 

State-3: Mixture: 
$$p_3 = p_1 \text{ kPa}$$
;  $m_3 = m_1 + m_2 = 2.5 \frac{\text{kg}}{\text{s}}$ 

The energy balance equation for this mixing open-steady system simplifies as:

$$\frac{dE^{0}}{dt}^{0} = n k_{1} j_{1} + n k_{2} j_{2} - n k_{3} j_{3} + \sum_{i=0}^{\infty} (1 - i k_{1})^{0} = n k_{1} k_{1} + n k_{2} k_{2} - n k_{3} k_{3}$$

$$\Rightarrow n k_1 h_{He,3} + n k_2 h_{Ne,3} = n k_1 h_{He,1} + n k_2 h_{Ne,2}$$

$$\Rightarrow \left[ n \mathcal{B} \boldsymbol{c}_{P} \left( T_{3} - T_{1} \right) \right]_{He} + \left[ n \mathcal{B} \boldsymbol{c}_{P} \left( T_{3} - T_{1} \right) \right]_{Ne} = 0$$

$$\Rightarrow (T_3 - T_1) \left[ (n \mathcal{B} \mathcal{C}_p)_{He} + (n \mathcal{B} \mathcal{C}_p)_{Ne} \right] = 0$$

$$\Rightarrow (2)(5.193)(T_3 - 293) + (0.5)(1.03)(T_3 - 773) = 0$$

$$\Rightarrow T_3 = 315.7 \text{ K}$$

(b) The molar flow rate of the two gases in the mixture is

$$R_{He} = \left(\frac{m_{N}^{2}}{\overline{M}}\right)_{He} = \frac{2}{4} = 0.5 \frac{\text{kmol}}{\text{s}}; \quad R_{Ar} = \left(\frac{m_{N}^{2}}{\overline{M}}\right)_{Ne} = \frac{0.5}{20.183} = 0.025 \frac{\text{kmol}}{\text{s}}$$

The mole fraction of each gas becomes

$$y_{He} = \frac{0.5}{0.5 + 0.025} = 0.952; \ y_{Ar} = \frac{0.0125}{0.5 + 0.025} = 0.048$$

The entropy balance equation simplifies as:

$$\frac{dS}{dt}^{0} = iR_{1}S_{1} + iR_{2}S_{2} - iR_{3}S_{3} + \underbrace{\frac{g^{0}}{T_{B}}}^{0} + g^{0}_{gen}$$

$$\Rightarrow g^{0}_{gen} = iR_{3}S_{3} - iR_{4}S_{1} - iR_{2}S_{2}$$

$$\Rightarrow g^{0}_{gen} = \left[iR_{2}(S_{3} - S_{1})\right]_{He} + \left[iR_{2}(S_{3} - S_{2})\right]_{Ne}$$

$$\Rightarrow g^{0}_{gen} = \left[iR_{2}(C_{p} \ln \frac{T_{3}}{T_{1}} - R \ln \frac{y p_{3}}{p_{3}})\right]_{He} + \left[iR_{2}(C_{p} \ln \frac{T_{3}}{T_{2}} - R \ln \frac{y p_{3}}{p_{3}})\right]_{Ne}$$

$$\Rightarrow g^{0}_{gen} = \left[iR_{2}(C_{p} \ln \frac{T_{3}}{T_{1}} - R \ln y)\right]_{He} + \left[iR_{2}(C_{p} \ln \frac{T_{3}}{T_{2}} - R \ln y)\right]_{Ne}$$

$$\Rightarrow g^{0}_{gen} = (2)\left((5.1965)\ln\left(\frac{315.3}{293.15}\right) - (2.0785)\ln(0.953)\right)$$

$$+ (0.5)\left((1.03)\ln\left(\frac{315.3K}{773.15K}\right) - (0.412)\ln(0.047)\right)$$

$$\Rightarrow g^{0}_{gen} = 1.141kW/K$$

c) The entropy generation rate changes from problem 11-3-20 because argon has a different molar mass from neon, so the mole fraction of the final mixture is different for the two problems. It is the increase in the difference between the component mole fraction that increases the mixing friction and, hence, the rate of entropy generation.

11-3-25 [OFX] Hydrogen at 100 kPa, 25°C is mixed with oxygen in an adiabatic mixing chamber. The flow rate of hydrogen is 2 kmol/s and that of oxygen is 1 kmol/s. If the mixture leaves the chamber at 100 kPa, determine (a) the temperature at the exit ( $T_2$ ) and (b) the rate of entropy generation ( $S_{gen}$ ) in the device. Assume no chemical reaction and use the PG mixture model.

### **SOLUTION:**

(a) The molar mass and gas constant of hydrogen and oxygen are 2 kg/kmol, 32 kg/kmol, 4.124 kJ/kg.K, 0.2598 kJ/kg.K respectively

State-1:: 
$$p_1 = 200 \text{ kPa}$$
;  $T_1 = 25^{\circ}\text{C}$ ;  $R_2 = 2 \frac{\text{kmol}}{\text{s}}$ ;  $R_3 = R_4 \overline{M}_{H_2} = 4 \frac{\text{kg}}{\text{s}}$ 

State-2: Ar: 
$$p_2 = p_1 \text{ kPa}$$
;  $T_2 = 25^{\circ}\text{C}$ ;  $R_2 = 1 \frac{\text{kmol}}{\text{s}}$ ;  $R_2 = R_2 \overline{M}_{Q_2} = 32 \frac{\text{kg}}{\text{s}}$ 

State-3: Mixture: 
$$p_3 = p_1$$
 kPa;  $\frac{1}{8} = \frac{1}{8} + \frac{1}{8} = 1.5 + \frac{1}{$ 

The energy balance equation for this mixing open-steady system simplifies as:

$$\begin{split} \frac{dE^{\prime}}{dt}^{0} &= n B_{1} j_{1} + n B_{2} j_{2} - n B_{3} j_{3} + O^{\prime} - V^{\prime}_{\text{ext}}^{0} \cong n B_{1} h_{1} + n B_{2} h_{2} - n B_{3} h_{3} \\ \Rightarrow n B_{1} h_{H_{2},3} + n B_{2} h_{O_{2},3} = n B_{1} h_{H_{2},1} + n B_{2} h_{O_{2},2} \\ \Rightarrow n B_{1} \left( h_{H_{2},3} - h_{H_{2},1} \right) + n B_{2} \left( h_{O_{2},3} - h_{O_{2},2} \right) = 0 \\ \Rightarrow \left[ n B_{1} \mathcal{C}_{p} \left( T_{3} - T_{1} \right) \right]_{H_{2}} + \left[ n B_{1} \mathcal{C}_{p} \left( T_{3} - T_{2} \right) \right]_{O_{2}} = 0 \\ \Rightarrow \left[ n B_{1} \mathcal{C}_{p} \left( T_{3} - T_{1} \right) \right]_{H_{2}} + \left[ n B_{1} \mathcal{C}_{p} \left( T_{3} - T_{1} \right) \right]_{O_{2}} = 0 \\ \Rightarrow \left( T_{3} - T_{1} \right) \left[ \left( n B_{1} \mathcal{C}_{p} \right)_{H_{2}} + \left( n B_{2} \mathcal{C}_{p} \right)_{O_{2}} \right] = 0 \\ \Rightarrow \left( 2 \right) \left( 5.193 \right) \left( T_{3} - 293 \right) + \left( 0.5 \right) \left( 0.5203 \right) \left( T_{3} - 773 \right) = 0 \\ \Rightarrow T_{3} = T_{1} = 298 \text{ K} \end{split}$$

(b)

The mole fraction of each component in the mixture:

$$y_{H_2} = \frac{18_{H_2}}{182} = \frac{2}{3} = 0.667; \quad y_{O_2} = \frac{18_{O_2}}{182} = \frac{1}{3} = 0.333$$

The entropy balance equation simplifies as:

$$\frac{dS}{dt}^{0} = n\mathbf{S}_{t}s_{1} + n\mathbf{S}_{t}s_{2} - n\mathbf{S}_{t}s_{3} + \frac{\mathbf{S}_{t}^{0}}{T_{B}} + \mathbf{S}_{gen}^{0}$$

$$\Rightarrow \mathbf{S}_{gen}^{0} = n\mathbf{S}_{t}s_{3} - n\mathbf{S}_{t}s_{1} - n\mathbf{S}_{t}s_{2}$$

$$\Rightarrow \mathbf{S}_{gen}^{0} = \left[n\mathbf{S}_{t}(s_{3} - s_{1})\right]_{H_{2}} + \left[n\mathbf{S}_{t}(s_{3} - s_{2})\right]_{O_{2}}$$

$$\Rightarrow \mathbf{S}_{gen}^{0} = \left[n\mathbf{S}_{t}(s_{3} - s_{1})\right]_{H_{2}} + \left[n\mathbf{S}_{t}(s_{3} - s_{2})\right]_{O_{2}}$$

$$\Rightarrow \mathbf{S}_{gen}^{0} = \left[n\mathbf{S}_{t}(s_{1} - s_{1})\right]_{H_{2}} + \left[n\mathbf{S}_{t}(s_{1} - s_{2})\right]_{O_{2}}$$

$$\Rightarrow \mathbf{S}_{gen}^{0} = \left[n\mathbf{S}_{t}(s_{1} - s_{2})\right]_{H_{2}} + \left[n\mathbf{S}_{t}(s_{1} - s_{2})\right]_{O_{2}}$$

$$\Rightarrow \mathbf{S}_{gen}^{0} = \left[n\mathbf{S}_{t}(s_{1} - s_{2})\right]_{H_{2}} + \left[n\mathbf{S}_{t}(s_{1} - s_{2})\right]_{O_{2}}$$

**TEST Solution** Use the PG-PG mixing open-steady TESTcalc to verify the solution. The TEST-code for this problem can be found in the TEST-Pro site at www.thermofluids.net.



11-3-26 [OFC] An insulated rigid tank that contains 1 kg of  $H_2$  at 25°C and 100 kPa is connected to another insulated rigid tank that contains 1 kg of He at 25°C and 100 kPa. The valve connecting the two tanks is opened, and the two gases are allowed to mix adiabatically. Determine (a) the entropy generated ( $s_{gen}$ ) during mixing per unit mass of the mixture. (b) Now replace the content of the second tank with different gases of same mass and for each gas determine the entropy generated per unit mass of the total mixture (2 kg). Plot a bar chart of entropy generation per unit mass against different gases on the x-axis. Use IG mixture model.

## **SOLUTION:**

State-1 (pure  $H_2$ ):

$$p_1 = 100 \text{ kPa}, \ T_1 = 298 \text{ K}, \ m_1 = 1 \text{ kg};$$
  
 $\Rightarrow n_1 = \frac{m_1}{\overline{M}_1} = 0.496 \text{ kmol}; \ \frac{V_1}{V_1} = \left(\frac{n\overline{R}T}{p}\right)_1 = 12.3 \text{ m}^3$   
State-2 (pure  $He$ ):

$$p_2 = 100 \text{ kPa}, \ T_2 = 298 \text{ K}, \ m_2 = 1 \text{ kg};$$
  
 $\Rightarrow n_2 = \frac{m_2}{\overline{M}_2} = 0.25 \text{ kmol}; \ \ \frac{1}{2} = \left(\frac{n\overline{R}T}{p}\right)_2 = 6.2 \text{ m}^3$ 

State-3 (mixture)

$$m_3 = m_1 + m_2 = 2 \text{ kg}; \ \frac{V_3}{V_3} = \frac{V_1}{V_1} + \frac{V_2}{V_2} = 18.5 \text{ m}^3; \ n_3 = n_1 + n_2 = 0.746 \text{ kmol}$$

$$\Rightarrow y_{H_2,3} = \frac{n_1}{n_3} = 0.665; y_{He,3} = 1 - y_{H_2,3} = 0.335;$$

$$\overline{M}_3 = \sum_k y_k \overline{M}_k = 2.681 \frac{\text{kg}}{\text{kmol}}; R = \frac{8.314}{2.681} = 3.101 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

The energy equation for this closed process (for the combined system) simplifies to:

$$\begin{split} &\Delta U = \cancel{\mathcal{Q}}^0 - \cancel{\mathcal{W}_{\text{ext}}}^0 \\ &\Rightarrow U_3 - \left(U_1 + U_2\right) = 0 \\ &\Rightarrow \left(m_1 u_{H_{2,3}} + m_2 u_{He,3}\right) - \left(m_1 u_{H_{2,1}} + m_2 u_{He,2}\right) = 0 \\ &\Rightarrow m_1 \left(u_{H_{2,3}} - u_{H_{2,1}}\right) = m_2 \left(u_{He,2} - u_{He,3}\right) \\ &\Rightarrow m_1 \left(u_{H_{2,3}} - u_{H_{2,1}}\right) = m_1 \left(u_{He,1} - u_{He,3}\right) \\ &\Rightarrow \int_1^3 c_{v,H_2} dT = \int_3^1 c_{v,He} dT \\ &\Rightarrow \int_1^3 \left(c_{v,H_2} + c_{v,He}\right) dT = 0 \end{split}$$

Because  $(c_{v,H_2} + c_{v,He})$  is non-zero, the integral can be zero only when  $\Rightarrow T_3 = T_1$ 

$$p_{3} \frac{Y_{3}}{Y_{3}} = m_{3} R_{3} T_{3}$$

$$\Rightarrow p_{3} = \frac{m_{3} R_{3} T_{3}}{Y_{3}} = 100 \text{ kPa}$$
(a)
$$\Delta S = S_{3} - (S_{2} - S_{1})$$

$$\Rightarrow \Delta S = (m_{1} S_{H_{2},3} + m_{2} S_{He,3}) - (m_{1} S_{H_{2},1} + m_{2} S_{He,2})$$

$$\Rightarrow \Delta S = m_{1} \left(S_{H_{2},3} - S_{H_{2},1}\right) + m_{2} \left(S_{He,3} - S_{He,2}\right)$$

$$\Rightarrow \Delta S = m_{1} \left(S_{H_{2},3}^{\circ} - S_{H_{2},1}^{\circ} - R_{H_{2}} \ln \frac{p_{H_{2},3}}{p_{1}}\right) - m_{2} \left(S_{He,3}^{\circ} - S_{He,2}^{\circ} - R_{He} \ln \frac{p_{He,3}}{p_{2}}\right)$$

$$\Rightarrow \Delta S = m_{1} \left(S_{H_{2},3}^{\circ} - S_{H_{2},1}^{\circ} - R_{H_{2}} \ln \frac{y_{CO_{2},3} p_{3}}{p_{1}}\right) - m_{2} \left(S_{He,3}^{\circ} - S_{He,2}^{\circ} - R_{He} \ln \frac{y_{He,3}}{p_{2}}\right)$$

$$\Rightarrow \Delta S = 3.956 \frac{kJ}{K}$$

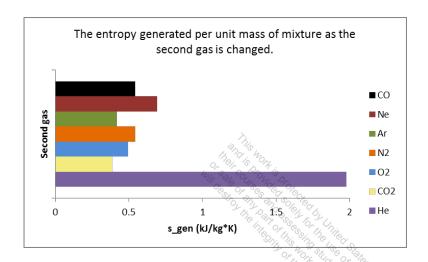
The entropy equation for the combined system going through the closed process simplifies as:

$$\Delta S = S_f - S_i = \frac{\cancel{Q}^0}{T_B} + S_{\text{gen,univ}}$$

$$\Rightarrow S_{\text{gen,univ}} = \Delta S = 3.956 \frac{\text{kJ}}{\text{K}}$$

$$\Rightarrow \frac{S_{\text{gen,univ}}}{m_3} = \frac{3.956}{2} = 1.978 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

**TEST Solution** Use the IG-IG mixing closed-process TESTcalc to verify the solution. The TEST-code for this problem can be found in the problems module of the professional edition of TEST at www.thermofluids.net and can be used for the what-if study.



11-3-27 [OFV] An insulated rigid tank that contains 1 kg of  $O_2$  at 25°C and 500 kPa is connected to another insulated rigid tank that contains 1 kg of  $O_2$  at 25°C and 1000 kPa. The valve connecting the two tanks is opened, and the two gases are allowed to mix adiabatically. Determine (a) the entropy generated ( $s_{\rm gen}$ ) during mixing per unit mass of the mixture. (b) Now repeat the calculations by varying the pressure in the second tank from a range of 10 kPa through 10 MPa and for each pressure determine the entropy generated per unit mass of the total mixture (2 kg). Plot the entropy generation per unit mass (y-axis) against the initial pressure ratio. Use IG mixture model.

## **SOLUTION:**

State-1 (pure O<sub>2</sub>):

$$p_1 = 500 \text{ kPa}, \ T_1 = 298 \text{ K}, \ m_1 = 1 \text{ kg};$$
  
 $\Rightarrow n_1 = \frac{m_1}{\overline{M}_1} = 0.0313 \text{ kmol}; \ \frac{V}{I} = \left(\frac{n\overline{R}T}{p}\right)_1 = 0.1549 \text{ m}^3$   
State-2 (pure O<sub>2</sub>):

$$p_2 = 1000 \text{ kPa}, \ T_2 = 298 \text{ K}, \ m_2 = 1 \text{ kg};$$

$$\Rightarrow n_2 = \frac{m_2}{\overline{M}_2} = 0.0313 \text{ kmol}; \ \mathcal{V}_2 = \left(\frac{n\overline{R}T}{p}\right)_2 = 0.0775 \text{ m}^3$$
State-3 (mixture)
$$m_3 = m_1 + m_2 = 2 \text{ kg}; \ \mathcal{V}_3 = \mathcal{V}_1 + \mathcal{V}_2 = 0.232 \text{ m}^3; \ n_3 = n_1 + n_2 = 0.0625 \text{ kmol}$$

The energy equation for this closed process (for the combined system) simplifies to:

$$\begin{split} \Delta U &= \cancel{\mathcal{Q}}^0 - \cancel{\mathcal{W}_{\text{ext}}}^0 \\ \Rightarrow U_3 - (U_1 + U_2) &= 0 \\ \Rightarrow (m_1 + m_2) u_{O_2, 3} - (m_1 u_{O_2, 1} + m_2 u_{O_2, 2}) &= 0 \\ \Rightarrow (m_1 + m_2) (u_{O_2, 3} - u_{O_2, 1}) &= 0 \\ \Rightarrow (u_{O_2, 3} - u_{O_2, 1}) &= 0 \\ \Rightarrow \int_0^3 c_{v, O_2} dT &= 0 \end{split}$$

Because  $c_{v,\mathcal{O}_2}$  is non-zero, the integral can be zero only when

$$\Rightarrow T_3 = T_1$$

$$\begin{split} p_{3} & \not\vdash_{3} = m_{3} R_{3} T_{3} \\ & \Rightarrow p_{3} = \frac{m_{3} R_{3} T_{3}}{\not\vdash_{3}} = 666.7 \text{ kPa} \\ \text{(a)} \\ & \Delta S = S_{3} - \left(S_{2} - S_{1}\right) \\ & \Rightarrow \Delta S = \left(m_{1} + m_{2}\right) S_{O_{2},3} - \left(m_{1} S_{O_{2},1} + m_{2} S_{O_{2},2}\right) \\ & \Rightarrow \Delta S = m_{1} \left(S_{O_{2},3} - S_{O_{2},1}\right) + m_{2} \left(S_{O_{2},3} - S_{O_{2},2}\right) \\ & \Rightarrow \Delta S = m_{1} \left(S_{O_{2},3}^{\circ} - S_{O_{2},1}^{\circ} - R_{O_{2}} \ln \frac{p_{3}}{p_{1}}\right) + m_{2} \left(S_{O_{2},3}^{\circ} - S_{O_{2},2}^{\circ} - R_{O_{2}} \ln \frac{p_{3}}{p_{2}}\right) \\ & \Rightarrow \Delta S = -m_{1} R_{O_{2}} \ln \frac{p_{3}}{p_{1}} - m_{2} R_{O_{2}} \ln \frac{p_{3}}{p_{2}} \\ & \Rightarrow \Delta S = 0.0306 \frac{\text{kJ}}{\text{K}} \end{split}$$

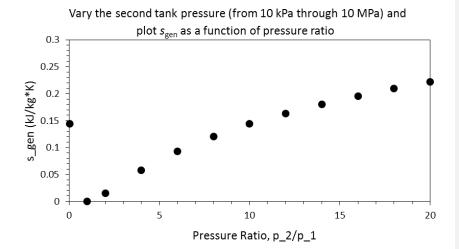
The entropy equation for the combined system going through the closed process simplifies as:

$$\Delta S = S_f - S_i = \frac{\cancel{Q}^0}{T_B} + S_{\text{gen,univ}}$$

$$\Rightarrow S_{\text{gen,univ}} = \Delta S = 0.0306 \frac{\text{kJ}}{\text{K}}$$

$$\Rightarrow S_{\text{gen,univ}} = \frac{S_{\text{gen,univ}}}{m_3} = \frac{0.0306}{2} = 0.0153 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

**TEST Solution** Use the IG mixing closed-process TESTcalc to verify the solution. The TEST-code for this problem can be found in the problems module of the professional edition of TEST at www.thermofluids.net and can be used for the what-if study.





11-3-28 [OFQ] An insulated rigid tank that contains 1 kg of  $CO_2$  at 300 K and 500 kPa is connected to another insulated rigid tank that contains 1 kg of  $CO_2$  at 400 K and 500 kPa. The valve connecting the two tanks is opened, and the two gases are allowed to mix adiabatically. Determine (a) the entropy generated ( $s_{gen}$ ) during mixing per unit mass of the mixture. (b) Now repeat the calculations by varying the temperature in the second tank from a range of 100 K through 2000 K. Plot the entropy generation per unit mass (y-axis) against the initial temperature ratio. Use IG mixture model.

### **SOLUTION:**

State-1 (pure CO<sub>2</sub>):

$$p_1 = 500 \text{ kPa}, T_1 = 300 \text{ K}, m_1 = 1 \text{ kg};$$
  

$$\Rightarrow V_1 = \left(\frac{mRT}{p}\right)_1 = 0.1133 \text{ m}^3$$

State-2 (pure  $CO_2$ ):

$$p_2 = 500 \text{ kPa}, \ T_2 = 400 \text{ K}, \ m_2 = 1 \text{ kg};$$
  

$$\Rightarrow \frac{V_2}{V_2} = \left(\frac{mRT}{p}\right)_2 = 0.1511 \text{ m}^3$$
State-3 (mixture)  
 $m_3 = m_1 + m_2 = 2 \text{ kg}; \ \frac{V_3}{V_3} = \frac{V_1}{V_1} + \frac{V_2}{V_2} = 0.169 \text{ m}^3;$ 

The energy equation for this closed process (for the combined system) simplifies to:

$$\Delta U = \cancel{Q}^{0} - \cancel{W_{\text{ext}}}^{0}$$

$$\Rightarrow U_{3} - (U_{1} + U_{2}) = 0$$

$$\Rightarrow (m_{1} + m_{2}) u_{CO_{2},3} - (m_{1} u_{CO_{2},1} + m_{2} u_{CO_{2},2}) = 0$$

$$\Rightarrow u_{CO_{2},3} = \frac{(u_{CO_{2},3} - u_{CO_{2},1})}{2}$$

$$\Rightarrow T_{3} = 352 \text{ K}$$

$$p_3 \frac{V_3}{V_3} = m_3 R_3 T_3$$
  
 $\Rightarrow p_3 = \frac{m_3 R_3 T_3}{V_3} = 502 \text{ kPa}$   
(a)

$$\begin{split} &\Delta S = S_3 - \left(S_2 - S_1\right) \\ &\Rightarrow \Delta S = \left(m_1 + m_2\right) s_{CO_2,3} - \left(m_1 s_{CO_2,1} + m_2 s_{CO_2,2}\right) \\ &\Rightarrow \Delta S = m_1 \left(s_{CO_2,3} - s_{CO_2,1}\right) + m_2 \left(s_{CO_2,3} - s_{CO_2,2}\right) \\ &\Rightarrow \Delta S = m_1 \left(s_{CO_2,3}^\circ - s_{CO_2,1}^\circ - R_{CO_2} \ln \frac{p_3}{p_1}\right) + m_2 \left(s_{CO_2,3}^\circ - s_{CO_2,2}^\circ - R_{CO_2} \ln \frac{p_3}{p_2}\right) \\ &\Rightarrow \Delta S = 0.01835 \frac{\text{kJ}}{\text{K}} \end{split}$$

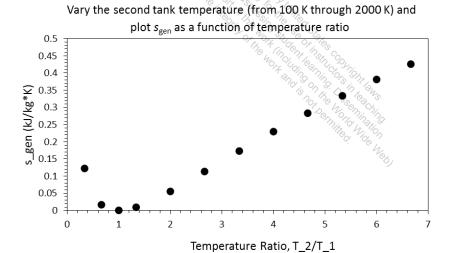
The entropy equation for the combined system going through the closed process simplifies as:

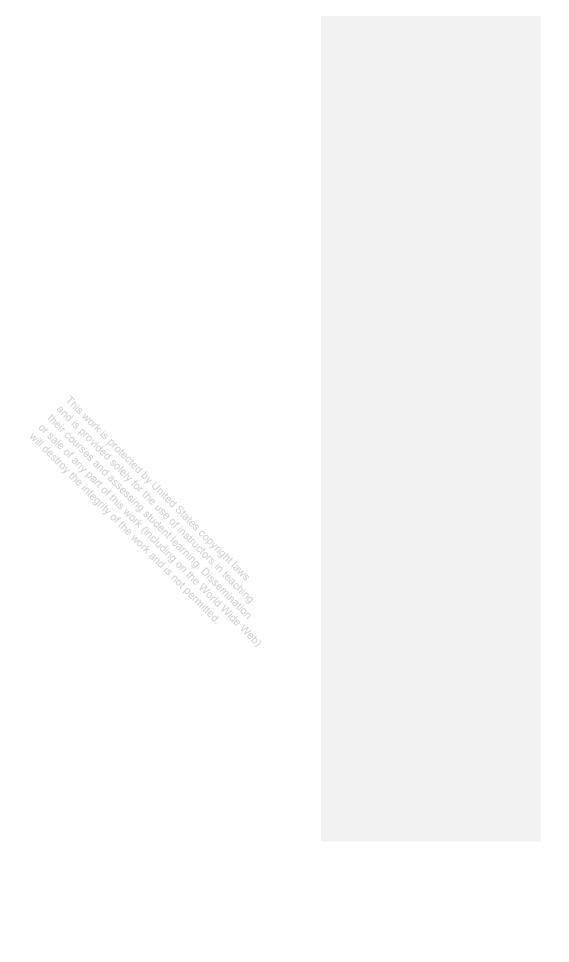
$$\Delta S = S_f - S_i = \frac{\cancel{Q}^0}{T_B} + S_{\text{gen,univ}}$$

$$\Rightarrow S_{\text{gen,univ}} = \Delta S = 0.01835 \frac{\text{kJ}}{\text{K}}$$

$$\Rightarrow S_{\text{gen,univ}} = \frac{S_{\text{gen,univ}}}{m_3} = \frac{0.01835}{2} = 0.0092 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

**TEST Solution** Use the IG mixing closed-process TESTcalc to verify the solution. The TEST-code for this problem can be found in the problems module of the professional edition of TEST at www.thermofluids.net and can be used for the what-if study.





11-3-29 [OFT] Carbon-dioxide at 100 kPa,  $25^{\circ}\text{C}$  enters an adiabatic mixing chamber with a mass flow rate of 1 kg/s is mixed with hydrogen entering at 100 kPa and  $25^{\circ}\text{C}$ . Plot the entropy generated per unit mass of the mixture ( $s_{\text{gen}}$ ) as a function of mass fraction of hydrogen in the mixture (vary x from 0 to 1). (b) Repeat the plot with entropy generation expressed on the basis of unit mole (1 kmol) of the flow and mass fraction replaced with mole fraction.

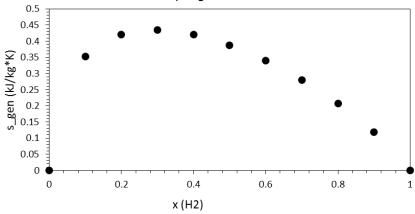
### **TEST Solution:**

Use the IG-IG mixing open-steady TEST calc to for this study. Keep the total mass flow rate constant by using mdot2 = 1 - mdot1.

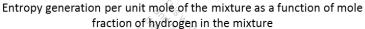
The TEST-code is shown below:

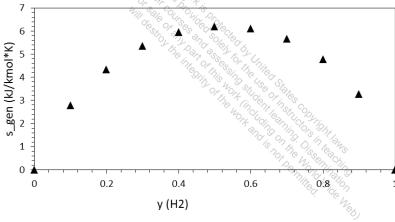
By varying mdot1 from 1 to 0, the mass fraction of hydrogen can be varied from 0 to 1. The entropy generated is displayed in the I/O panel.  $s_{\text{gen}} = S_{\text{gen}}^2 / nR_{\text{g}}$  is plotted agains the mass fraction of hydrogen in the figure below:

# Entropy generated per unit mass of the mixture as a function of mass fraction of hydrogen in the mixture



By converting the mass flow rates to mole flow rate, the entropy generated per unit mole of the total flow can be plotted against the mole fraction of hydrogen:





11-3-30 [OFY] Repeat problem 11-3-29 [OFT] with a completely different pair of gases. Can you come up with a generalized mixing criterion that maximizes entropy generation per unit mass or mole?

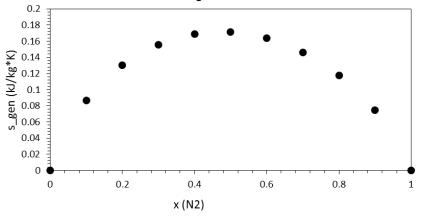
### **TEST Solution:**

Selecting argon and nitrogen as a new pair of gases, the TEST-code is modified as follows:

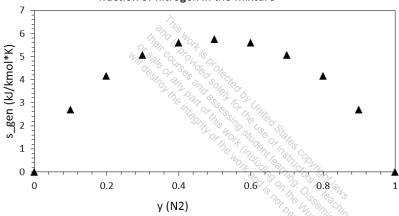
```
States {
                                  State-1: Ar, N2;
                                  Given: { p1=100.0 \text{ kPa}; T1=298.0008 \text{ K}; Vel1=0.0 \text{ m/s}; z1=0.0 \text{ m};
mdot1 = 0.5 \text{ kg/s}; x_A1 = 1.0 \text{ fraction}; 
                                  State-2: Ar, N2;
                                  Given: \{p2=100.0 \text{ kPa}; T2=298.0 \text{ K}; Vel2=0.0 \text{ m/s}; z2=0.0 \text{ m};
mdot2= "1-mdot1" kg/s; x A2= 0.0 fraction; }
                                   State-3: Ar, N2;
                                  Given: \{p3="p1" kPa; Vel3=0.0 m/s; z3=0.0 m; mdot3=
"mdot1+mdot2" kg/s; x_A3= mdot1/(mdot1+mdot2)" fraction; }
     Analysis {
                                  Device-A: i-State = State-1, State-2; e-State = State-3; Mixing: true;
                                  Given: { Qdot= 0.0 kW; Wdot_ext= 0.0 kW; T_B= 298.15 K; }
                                                                                                                                                                                                               To Color of the Co
```

The resulting plots are as follows:

# Entropy generated per unit mass of the mixture as a function of mass fraction of nitrogen in the mixture



Entropy generation per unit mole of the mixture as a function of mole fraction of nitrogen in the mixture



Comparing the plots with the plots generated in the previous problem, it can be concluded that when the mole fraction of the gases are 0.5, the entropy generation is maximized.

**11-3-31** [OFF] Argon at 100 kPa, 600 K enters an adiabatic mixing chamber with a mass flow rate of 1 kg/s is mixed with with nitrogen entering at 100 kPa, 600 K and 1 kg/s. (a) Determine the entropy generated per unit mass of the mixture ( $s_{gen}$ ). (b) Vary the inlet temperature of nitrogen (from 200 K to 2000 K) and plot  $s_{gen}$  as a function of temperature ratio of the incoming streams.

#### **SOLUTION:**

The molar mass and gas constant of argon and nitrogen are 39.95 kg/kmol, 28 kg/kmol, 0.2081 kJ/kg.K, 0.2970 kJ/kg.K respectively

State-1:: 
$$p_1 = 100 \text{ kPa}$$
;  $T_1 = 600 \text{ K}$ ;  $n_1^{8} = n_2^{8} = 1 \frac{\text{kg}}{\text{s}}$ ;  $n_2^{8} = \frac{n_2^{8}}{\overline{M}_{dr}} = 0.025 \frac{\text{kmol}}{\text{s}}$ ;

State-2: Ar: 
$$p_2 = p_1$$
;  $T_2 = T_1$ ;  $nB_2 = 1$   $\frac{kg}{s}$ ;  $nB_2 = \frac{nB_2}{\overline{M}_{N_2}} = 0.0357$   $\frac{kmol}{s}$ ;

State-3: Mixture: 
$$p_3 = p_1$$
;  $k_3 = k_4 + k_2 = 0.0607 \frac{\text{kmol}}{\text{s}}$ ;  $k_3 = k_4 + k_2 = 2 \frac{\text{kg}}{\text{s}}$ 

The energy balance equation for this mixing open-steady system simplifies as:

$$\begin{split} \frac{dE^{0}}{dt} &= n R_{1} j_{1} + n R_{2} j_{2} - n R_{3} j_{3} + O - N R_{2} i_{2} = n R_{1} h_{1} + n R_{2} h_{2} - n R_{3} h_{3} \\ \Rightarrow n R_{1} h_{Ar,3} + n R_{2} h_{N_{2},3} &= n R_{1} h_{Ar,1} + n R_{2} h_{N_{2},2} \\ \Rightarrow n R_{1} \int_{T_{1}}^{T_{3}} c_{p,Ar} dT &= n R_{2} \int_{T_{2}}^{T_{3}} c_{p,N_{2}} dT \\ \Rightarrow n R_{1} \int_{T_{1}}^{T_{3}} c_{p,Ar} dT &= n R_{2} \int_{T_{1}}^{T_{3}} c_{p,N_{2}} dT \\ \Rightarrow \int_{T_{1}}^{T_{3}} \left( n R_{1} c_{p,Ar} - n R_{2} c_{p,N_{2}} \right) dT &= 0 \\ \Rightarrow T_{3} &= T_{1} = 600 \text{ K} \end{split}$$

The mole fraction of each component in the mixture:

$$y_{Ar} = \frac{k g_{H_2}}{k g_2} = 0.412; \quad y_{N_2} = \frac{k g_{O_2}}{k g_2} = 0.588$$

The entropy balance equation simplifies as:

$$\frac{dS^{0}}{dt} = nB_{1}S_{1} + nB_{2}S_{2} - nB_{3}S_{3} + \frac{g^{0}}{T_{B}} + g^{0}_{gen}$$

$$\Rightarrow g^{0}_{gen} = nB_{3}S_{3} - nB_{4}S_{1} - nB_{2}S_{2}$$

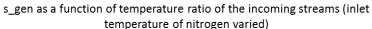
$$\Rightarrow g^{0}_{gen} = \left[nB_{2}(S_{3} - S_{1})\right]_{Ar} + \left[nB_{3}(S_{3} - S_{2})\right]_{N_{2}}$$

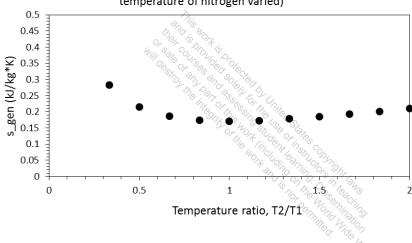
$$\Rightarrow g^{0}_{gen} = \left[nB_{3}(S_{3} - S_{1})\right]_{Ar} + \left[nB_{3}(S_{3} - S_{2})\right]_{N_{2}} + \left[nB_{3}(S_{3} - S_{2}) - R \ln \frac{y p_{3}}{p_{3}}\right]_{N_{2}}$$

$$\Rightarrow g^{0}_{gen} = \left[nB_{3}R \ln y\right]_{Ar} + \left[nB_{3}R \ln y\right]_{N_{2}} = 0.3422 \text{ kW/K}$$

$$\Rightarrow S_{gen} = \frac{g^{0}_{gen}}{nB_{3}} = \left[nB_{3}R \ln y\right]_{Ar} + \left[nB_{3}R \ln y\right]_{N_{2}} = 0.1733 \frac{\text{kW}}{\text{kg·K}}$$

**TEST Solution** Use the IG-IG mixing open-steady TESTcalc to verify the solution. The TEST-code for this problem can be found in the TEST-Pro site at <a href="www.thermofluids.net">www.thermofluids.net</a> and is used to generate the following plot.

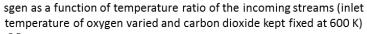


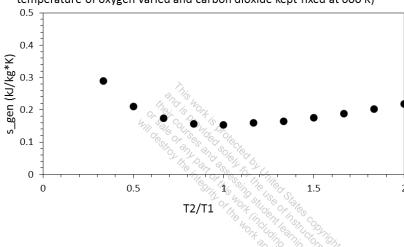


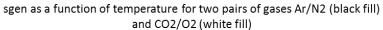
**11-3-32** [OFJ] Repeat problem 11-3-31 [OFF] with a completely different pair of gases. Can you come up with a generalized mixing curve where data from different pairs fall on the same line as far as entropy generation is concerned?

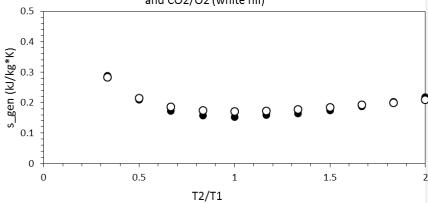
## **TEST Solution:**

Launch the IG-IG mixing open-steady TESTcalc. Paste the TEST-code from the previous problem in the I/O panel and click the Load button. Now, select a new pair of gases, carbon dioxide and oxygen. Change T2 and click Super-Calculate to obtain Sdot\_gen in the I/O panel. Calculate s\_gen = Sdot\_gen/2 and plot it against T2/T1. By changing the gas pairs, it seems that the entropy generation per unit mass of the mixture is primarily driven by the temperature ratio of the incoming gases.











11-3-33 [OFD] Argon at 1000 kPa, 300 K enters an adiabatic mixing chamber with a mass flow rate of 1 kg/s is mixed with with nitrogen entering at 1000 kPa, 300 K and 1 kg/s. (a) Determine the entropy generated per unit mass of the mixture ( $s_{\rm gen}$ ). (b) Vary the mixing pressure (from 10 kPa through 10 MPa) and plot  $s_{\rm gen}$  as a function of chamber pressure.

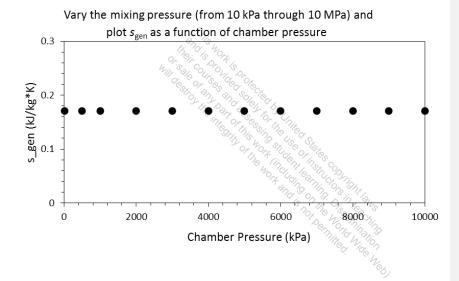
### **SOLUTION:**

The manual solution for the first part is identical to that of problem 11-3-31.

$$s_{\text{gen}} = \frac{\$_{\text{gen}}}{n \$_3} = 0.1733 \frac{\text{kW}}{\text{kg} \cdot \text{K}}$$

#### **TEST Solution:**

For the parametric study, use the TEST-code from 11-3-31 and vary the mixing chamber pressure (p1=p2=p3) from 10 kPa through 10 MPa and plot s\_gen agains chamber pressure. The entropy generation is independent of chamber pressure.



**11-3-34** [OFM] Repeat problem 11-3-33 [OFD] with a completely different pair of gases. Can you come up with a generalized mixing curve where data from different pairs fall on the same line as far as effect of pressure on entropy generation is concerned?

### **TEST Solution:**

Repeat the solution of 11-3-33 with a different pair of gases, CO2 and O2. The resulting plot shows that s\_gen (entropy generated per unit mass of the mixture) is slightly different, 0.1527 kJ/kg.K as opposed to 0.1711 kJ/kg.K with the different pair of gases. However, s\_gen is still independent of the chamber pressure.

