

14-2-1 [BRF] What is the maximum number of phases that can stay in equilibrium when the system has (a) one component, (b) two components, (c) four components.

SOLUTION

For all three cases start with Gibbs' Phase Rule:

$$q = C - P + 2;$$

$$\Rightarrow P = C - q + 2$$

Since the system is in equilibrium, $q = 0$, therefore

$$P = C + 2$$

When the system has one component, $C = 1$

$$p = 1 + 2 = 3$$

When the system has two components, $C = 2$

$$p = 2 + 2 = 4$$

When the system has four components, $C = 4$

$$p = 4 + 2 = 6$$

14-2-2 [BOO] Consider a liquid-vapor mixture of ammonia and water in equilibrium at 30°C. If the molar composition of the liquid phase is 55% NH₃ and 45% H₂O, determine the composition of the vapor phase of this mixture.

SOLUTION

Given:

$$T = 30^{\circ}\text{C}$$

$$y'_{\text{NH}_3} = 0.55$$

$$y'_{\text{H}_2\text{O}} = 0.45$$

From Table B-12

$$p_{\text{NH}_3, \text{sat}@30^{\circ}\text{C}} = 1167 \text{ kPa}$$

From Table B-2

$$p_{\text{H}_2\text{O}, \text{sat}@30^{\circ}\text{C}} = 4.25 \text{ kPa}$$

Using Raoult's Model

$$p_{\text{NH}_3} = y'_{\text{NH}_3} p_{\text{NH}_3, \text{sat}@30^{\circ}\text{C}} = (0.55)(1167) = 641.85 \text{ kPa}$$

$$p_{\text{H}_2\text{O}} = y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O}, \text{sat}@30^{\circ}\text{C}} = (0.45)(4.25) = 1.91 \text{ kPa}$$

Therefore, the mixture pressure

$$p = p_{\text{NH}_3} + p_{\text{H}_2\text{O}} = 641.85 + 1.91 = 643.76 \text{ kPa}$$

The molar composition of the vapor

$$y_{\text{NH}_3} = \frac{p_{\text{NH}_3}}{p} = \frac{641.85}{643.76} = 0.9970$$

$$y_{\text{H}_2\text{O}} = \frac{p_{\text{H}_2\text{O}}}{p} = \frac{1.91}{643.76} = 0.0030$$

14-2-3 [BRD] Determine the vapor pressure adjacent to the surface of a lake at 15°C (a) assuming air does not affect the equilibrium between water vapor and liquid water, (b) taking into account the presence of air, which affects the chemical potential (Gibbs function).

SOLUTION

If air does not affect the equilibrium between the water vapor and the liquid water, then the vapor pressure of the water is equal to the saturation pressure of the water at $T = 15^\circ\text{C}$. Therefore,

$$p_{v,\text{H}_2\text{O}} = p_{\text{H}_2\text{O},\text{sat}@15^\circ\text{C}} = \mathbf{1.71 \text{ kPa}}$$

The total pressure is known as $p = 100 \text{ kPa}$. When air does affect the equilibrium between water vapor and liquid, the equilibrium between their specific Gibbs functions remains

$$\begin{aligned}\bar{g}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) &= \bar{g}'_{\text{H}_2\text{O}}(p, T); \\ \Rightarrow \bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - T\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) &= \bar{h}'_{\text{H}_2\text{O}}(p, T) - T\bar{s}'_{\text{H}_2\text{O}}(p, T); \\ \Rightarrow \bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - \bar{h}'_{\text{H}_2\text{O}}(p, T) &= T[\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - \bar{s}'_{\text{H}_2\text{O}}(p, T)]\end{aligned}$$

We know that for the vapor phase

$$\bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) \approx \bar{h}_{g,\text{H}_2\text{O}@T}$$

Using a pressure correction for the vapor phase

$$\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) \approx \bar{s}(p_{\text{sat}@T}, T) - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} = \bar{s}_{g,\text{H}_2\text{O}@T} - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}}$$

We know that for the liquid phase

$$\bar{h}'_{\text{H}_2\text{O}}(p, T) \approx \bar{h}_{f,\text{H}_2\text{O}@T} + \bar{v}_{f,\text{H}_2\text{O}@T}(p - p_{\text{sat}@T})$$

$$\bar{s}'_{\text{H}_2\text{O}}(p, T) \approx \bar{s}_{f,\text{H}_2\text{O}@T}$$

Combining all the terms into the equating of the specific Gibbs function yields

$$\bar{h}_{g,\text{H}_2\text{O}@T} - [\bar{h}_{f,\text{H}_2\text{O}@T} + \bar{v}_{f,\text{H}_2\text{O}@T}(p - p_{\text{sat}@T})] = T \left[\left(\bar{s}_{g,\text{H}_2\text{O}@T} - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} \right) - \bar{s}_{f,\text{H}_2\text{O}@T} \right]$$

It can be noticed that

$$\bar{h}_{g,\text{H}_2\text{O}@T} - \bar{h}_{f,\text{H}_2\text{O}@T} = T(\bar{s}_{g,\text{H}_2\text{O}@T} - \bar{s}_{f,\text{H}_2\text{O}@T})$$

Therefore

$$\begin{aligned}
-\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T}) &= -\bar{R}T \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}}; \\
\Rightarrow \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} &= \frac{\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T}; \\
\Rightarrow p_{\text{H}_2\text{O}} &= p_{\text{sat}@T} \exp \left[\frac{\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T} \right]; \\
\Rightarrow p_{\text{H}_2\text{O}} &= p_{\text{sat}@T} \exp \left[\frac{\bar{M}_{\text{H}_2\text{O}} v_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T} \right] = (1.71) \exp \left[\frac{(18)(0.001)(100 - 1.71)}{(8.314)(288)} \right] = \mathbf{1.71 \text{ kPa}}
\end{aligned}$$

As can be seen, taking into consideration the presence of the air had practically no bearing on the equilibrium pressure of the water vapor.

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14-2-4 [BRM] For a mixture of saturated vapor and saturated liquid of water at 200°C, use tabulated properties or the PC system state TESTcalc to show that the specific Gibbs functions of the two phases are equal.

SOLUTION

State-1 (given T_1, x_1):

$$p_1 = p_{\text{sat}@200^\circ\text{C}} = 1.55 \text{ MPa}$$

$$h_1 = h_{f@200^\circ\text{C}} = 852.45 \frac{\text{kJ}}{\text{kg}}; \quad s_1 = s_{f@200^\circ\text{C}} = 2.3309 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given T_2, x_2):

$$p_2 = p_{\text{sat}@200^\circ\text{C}} = 1.55 \text{ MPa}$$

$$h_2 = h_{g@200^\circ\text{C}} = 2793.20 \frac{\text{kJ}}{\text{kg}}; \quad s_2 = s_{g@200^\circ\text{C}} = 6.4323 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

In order for the saturated liquid and saturated vapor to be in equilibrium, their chemical potentials must be equal.

$$\bar{g}'(p, T) = \bar{g}(p, T)$$

$$\Rightarrow \bar{g}_f(p, T) = \bar{g}_g(p, T);$$

$$\Rightarrow g_f(p, T) = g_g(p, T);$$

$$\Rightarrow h_1 - T_1 s_1 = h_2 - T_2 s_2;$$

$$\Rightarrow 852.45 - (473)(2.3309) = 2793.20 - (473)(6.4323);$$

$$\Rightarrow -250.07 \frac{\text{kJ}}{\text{kg}} \approx -249.28 \frac{\text{kJ}}{\text{kg}}$$

14-2-5 [BRJ] For a mixture of saturated vapor and saturated liquid of R-134a at 100 kPa, use tabulated properties or the PC system state TESTcalc to show that the chemical potentials of the two phases are equal.

SOLUTION

State-1 (given p_1, x_1):

$$T_1 = T_{\text{sat}@100\text{kPa}} = 246.56 \text{ K}$$

$$h_1 = h_{f@100\text{kPa}} = 16.46 \frac{\text{kJ}}{\text{kg}}; \quad s_1 = s_{f@100\text{kPa}} = 0.0684 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given p_2, x_2):

$$T_2 = T_{\text{sat}@100\text{kPa}} = 246.56 \text{ K}$$

$$h_2 = h_{g@100\text{kPa}} = 233.00 \frac{\text{kJ}}{\text{kg}}; \quad s_2 = s_{g@100\text{kPa}} = 0.9465 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

In order for the saturated liquid and saturated vapor to be in equilibrium, their chemical potentials must be equal.

$$\bar{g}'(p, T) = \bar{g}(p, T)$$

$$\Rightarrow \bar{g}_f(p, T) = \bar{g}_g(p, T);$$

$$\Rightarrow g_f(p, T) = g_g(p, T);$$

$$\Rightarrow h_1 - T_1 s_1 = h_2 - T_2 s_2;$$

$$\Rightarrow 16.46 - (246.56)(0.0684) = 233.00 - (246.56)(0.9465);$$

$$\Rightarrow -0.40 \frac{\text{kJ}}{\text{kg}} \approx -0.30 \frac{\text{kJ}}{\text{kg}}$$

14-2-6 [BRW] Show that an equilibrium mixture of saturated vapor and saturated liquid of water at 100°C satisfies the criterion for phase equilibrium.

SOLUTION

State-1 (given T_1, x_1):

$$p_1 = p_{\text{sat}@100^\circ\text{C}} = 101.40 \text{ kPa}$$

$$h_1 = h_{f@100^\circ\text{C}} = 419.04 \frac{\text{kJ}}{\text{kg}}; \quad s_1 = s_{f@100^\circ\text{C}} = 1.3069 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given T_2, x_2):

$$p_2 = p_{\text{sat}@100^\circ\text{C}} = 101.40 \text{ kPa}$$

$$h_2 = h_{g@100^\circ\text{C}} = 2676.10 \frac{\text{kJ}}{\text{kg}}; \quad s_2 = s_{g@100^\circ\text{C}} = 7.3549 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

In order for the saturated liquid and saturated vapor to be in equilibrium, their chemical potentials must be equal.

$$\bar{g}'(p, T) = \bar{g}(p, T)$$

$$\Rightarrow \bar{g}_f(p, T) = \bar{g}_g(p, T);$$

$$\Rightarrow g_f(p, T) = g_g(p, T);$$

$$\Rightarrow h_1 - T_1 s_1 = h_2 - T_2 s_2;$$

$$\Rightarrow 419.04 - (373)(1.3069) = 2676.10 - (373)(7.3549);$$

$$\Rightarrow -68.43 \frac{\text{kJ}}{\text{kg}} \approx -67.28 \frac{\text{kJ}}{\text{kg}}$$

14-2-7 [BOR] Consider a liquid-vapor mixture of ammonia and water in equilibrium at 15°C. If the molar composition of the liquid phase is 50% NH₃ and 50% H₂O, determine the composition of the vapor phase of this mixture.

SOLUTION

Given:

$$T = 15^{\circ}\text{C}$$

$$y'_{\text{NH}_3} = 0.50$$

$$y'_{\text{H}_2\text{O}} = 0.50$$

From Table B-12

$$p_{\text{NH}_3, \text{sat}@15^{\circ}\text{C}} = 728.60 \text{ kPa}$$

From Table B-2

$$p_{\text{H}_2\text{O}, \text{sat}@15^{\circ}\text{C}} = 1.71 \text{ kPa}$$

Using Raoult's Model

$$p_{\text{NH}_3} = y'_{\text{NH}_3} p_{\text{NH}_3, \text{sat}@15^{\circ}\text{C}} = (0.50)(728.60) = 364.30 \text{ kPa}$$

$$p_{\text{H}_2\text{O}} = y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O}, \text{sat}@15^{\circ}\text{C}} = (0.50)(1.71) = 0.86 \text{ kPa}$$

Therefore, the mixture pressure

$$p = p_{\text{NH}_3} + p_{\text{H}_2\text{O}} = 364.30 + 0.86 = 365.16 \text{ kPa}$$

The molar composition of the vapor

$$y_{\text{NH}_3} = \frac{p_{\text{NH}_3}}{p} = \frac{364.30}{365.16} = 0.9976$$

$$y_{\text{H}_2\text{O}} = \frac{p_{\text{H}_2\text{O}}}{p} = \frac{0.86}{365.16} = 0.0024$$

14-2-8 [BOB] A two-phase mixture of ammonia and water is in equilibrium at 40°C. If the molar composition of the vapor phase is 98% NH₃ and 2% H₂O, determine the composition of the liquid phase of this mixture.

SOLUTION

Given:

$$T = 40^{\circ}\text{C}$$

$$y_{\text{NH}_3} = 0.98$$

$$y_{\text{H}_2\text{O}} = 0.02$$

From Table B-12

$$p_{\text{NH}_3, \text{sat}@40^{\circ}\text{C}} = 1554.90 \text{ kPa}$$

From Table B-2

$$p_{\text{H}_2\text{O}, \text{sat}@40^{\circ}\text{C}} = 7.38 \text{ kPa}$$

From the knowledge of the vapor phase

$$p_{\text{NH}_3} = y_{\text{NH}_3} p$$

$$p_{\text{H}_2\text{O}} = y_{\text{H}_2\text{O}} p$$

Using Raoult's Model

$$y'_{\text{NH}_3} = \frac{p_{\text{NH}_3}}{p_{\text{NH}_3, \text{sat}@40^{\circ}\text{C}}} = \frac{y_{\text{NH}_3} p}{p_{\text{NH}_3, \text{sat}@40^{\circ}\text{C}}};$$

$$\Rightarrow p = \frac{y'_{\text{NH}_3} p_{\text{NH}_3, \text{sat}@40^{\circ}\text{C}}}{y_{\text{NH}_3}}$$

$$y'_{\text{H}_2\text{O}} = \frac{p_{\text{H}_2\text{O}}}{p_{\text{H}_2\text{O}, \text{sat}@40^{\circ}\text{C}}} = \frac{y_{\text{H}_2\text{O}} p}{p_{\text{H}_2\text{O}, \text{sat}@40^{\circ}\text{C}}};$$

$$\Rightarrow p = \frac{y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O}, \text{sat}@40^{\circ}\text{C}}}{y_{\text{H}_2\text{O}}}$$

Then equating the two yields

$$\frac{y'_{\text{NH}_3} p_{\text{NH}_3, \text{sat}@40^{\circ}\text{C}}}{y_{\text{NH}_3}} = \frac{y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O}, \text{sat}@40^{\circ}\text{C}}}{y_{\text{H}_2\text{O}}};$$

$$\Rightarrow y'_{\text{NH}_3} = \frac{y'_{\text{H}_2\text{O}} y_{\text{NH}_3} p_{\text{H}_2\text{O}, \text{sat}@40^{\circ}\text{C}}}{y_{\text{H}_2\text{O}} p_{\text{NH}_3, \text{sat}@40^{\circ}\text{C}}}$$

It is known that

$$y'_{\text{H}_2\text{O}} = 1 - y'_{\text{NH}_3}$$

Substituting into the previously found equation

$$\begin{aligned}
 y'_{\text{NH}_3} &= \frac{(1 - y'_{\text{NH}_3}) y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}}; \\
 \Rightarrow y'_{\text{NH}_3} &= \frac{y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}} - y'_{\text{NH}_3} y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}}; \\
 \Rightarrow y'_{\text{NH}_3} + \frac{y'_{\text{NH}_3} y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}} &= \frac{y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}}; \\
 \Rightarrow y'_{\text{NH}_3} \left(1 + \frac{y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}} \right) &= \frac{y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}}; \\
 \Rightarrow y'_{\text{NH}_3} \left(\frac{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}} + y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}} \right) &= \frac{y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}}; \\
 \Rightarrow y'_{\text{NH}_3} &= \left(\frac{y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}} \right) \left(\frac{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}}}{y_{\text{H}_2\text{O}} P_{\text{NH}_3, \text{sat}@40^\circ\text{C}} + y_{\text{NH}_3} P_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}}} \right); \\
 \Rightarrow y'_{\text{NH}_3} &= \left(\frac{(0.98)(7.38)}{(0.02)(1554.90)} \right) \left(\frac{(0.02)(1554.90)}{(0.02)(1554.90) + (0.98)(7.38)} \right) = 0.1887
 \end{aligned}$$

Having found the molar composition of ammonia in the liquid phase,

$$y'_{\text{H}_2\text{O}} = 1 - y'_{\text{NH}_3} = 1 - 0.1887 = 0.8113$$

14-2-9 [BOA] Repeat 14-1-9 [BOS] for water at 100°C.

SOLUTION

State-1 (given T_1, x_1):

$$p_1 = p_{\text{sat}@100^\circ\text{C}} = 101.40 \text{ kPa}$$

$$h_1 = h_{f@100^\circ\text{C}} = 419.04 \frac{\text{kJ}}{\text{kg}}; \quad s_1 = s_{f@100^\circ\text{C}} = 1.3069 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

State-2 (given T_2, x_2):

$$p_2 = p_{\text{sat}@100^\circ\text{C}} = 101.40 \text{ kPa}$$

$$h_2 = h_{g@100^\circ\text{C}} = 2676.10 \frac{\text{kJ}}{\text{kg}}; \quad s_2 = s_{g@100^\circ\text{C}} = 7.3549 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

In order for the saturated liquid and saturated vapor to be in equilibrium, their chemical potentials must be equal. Also working with a pure substance,

$$\bar{g}_f(p, T) = \bar{g}_g(p, T);$$

$$\Rightarrow g_f(p, T) = g_g(p, T);$$

$$\Rightarrow h_1 - T_1 s_1 = h_2 - T_2 s_2;$$

$$\Rightarrow 419.04 - (373.15)(1.3069) = 2676.10 - (373.15)(7.3549);$$

$$\Rightarrow -68.63 \frac{\text{kJ}}{\text{kg}} \approx -68.38 \frac{\text{kJ}}{\text{kg}}$$

14-2-10 [BOH] Consider a glass of water in a room at 20°C and 100 kPa. If the relative humidity in the room is 100%, and the water and air are in thermal and phase equilibrium, determine (a) the mole fraction of the water vapor in the air and (b) the mole fraction of air in the water.

SOLUTION

Given:

$$T = 20^{\circ}\text{C}$$

$$p = 100 \text{ kPa}$$

From Table B-2

$$p_{\text{H}_2\text{O},\text{sat}@20^{\circ}\text{C}} = 2.34 \text{ kPa}$$

Since the air is saturated,

$$p_{\text{H}_2\text{O}} = p_{\text{H}_2\text{O},\text{sat}@20^{\circ}\text{C}} = 2.34 \text{ kPa}$$

Therefore, the partial pressure of dry air

$$p_{\text{d.a.}} = p - p_{\text{H}_2\text{O}} = 100 - 2.34 = 97.66 \text{ kPa}$$

The mole fraction of water vapor in the air

$$y_{\text{H}_2\text{O}} = \frac{p_{\text{H}_2\text{O}}}{p} = \frac{2.34}{100} = 0.0234$$

Assuming that air is weakly soluble in water, use Henry's Law to find the amount of air dissolved into the liquid water. Henry's constant for air in water at 20°C is

$$H = 65600 \text{ bar}$$

Therefore, using Henry's Law, the mole fraction of air in water

$$y'_{\text{d.a.}} = \frac{p_{\text{d.a.}}}{H} = \frac{0.9766}{65600} = 1.49 \times 10^{-5}$$

14-2-11 [BON] Water is sprayed into air at 75°F and 14.3 psia, and the falling water droplets are collected in a container on the floor. Determine (a) the mass and (b) mole fractions of air dissolved in the water.

SOLUTION

Given:

$$T = 75^\circ\text{F}$$

$$p = 14.3 \text{ psia}$$

$$\bar{M}_{\text{H}_2\text{O}} = 18 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{d.a.}} = 29 \frac{\text{kg}}{\text{kmol}}$$

From Table B-2

$$p_{\text{H}_2\text{O}, \text{sat}@75^\circ\text{F}} = 0.43 \text{ psia}$$

Since the air is saturated,

$$p_{\text{H}_2\text{O}} = p_{\text{H}_2\text{O}, \text{sat}@75^\circ\text{F}} = 0.43 \text{ psia}$$

Therefore, the partial pressure of dry air

$$p_{\text{d.a.}} = p - p_{\text{H}_2\text{O}} = 14.30 - 0.43 = 13.87 \text{ psia}$$

Use Henry's Law to find the amount of air dissolved into the liquid water. Henry's constant for air in water at 75°F is

$$H = 70400 \text{ bar}$$

Therefore, using Henry's Law, the mole fraction of air in water

$$y'_{\text{d.a.}} = \frac{p_{\text{d.a.}}}{H} = \frac{0.9563}{70400} = 1.36 \times 10^{-5}$$

The mass and mole fractions of a mixture are related as follows

$$x_k = y_k \frac{\bar{M}_k}{\bar{M}}$$

The apparent molar mass of the mixture

$$\bar{M} = y'_{\text{H}_2\text{O}} \bar{M}_{\text{H}_2\text{O}} + y'_{\text{d.a.}} \bar{M}_{\text{d.a.}} = (1)(18) + (0)(29) = 18 \frac{\text{kg}}{\text{kmol}}$$

Therefore, the mass fraction of the mixture

$$x'_{\text{d.a.}} = y'_{\text{d.a.}} \frac{\bar{M}_{\text{d.a.}}}{\bar{M}} = (1.36 \times 10^{-5}) \left(\frac{29}{18} \right) = 2.19 \times 10^{-5}$$

14-2-12 [BOE] A mixture of 1 kmol of H_2 and 1 kmol of Ar is heated in a reaction chamber at a constant pressure of 1 atm until 15% of H_2 dissociates into monatomic hydrogen (H). Determine the final temperature (T_2) of the mixture.

SOLUTION

Given:

$$p = 1 \text{ atm}$$

Argon is inert, so the stoichiometric equation is



Therefore,

$$\nu_{H_2} = 1; \nu_H = 2$$

The actual equation



Therefore,

$$n_{H_2} = 0.85; n_H = 0.30; n_{Ar} = 1$$

The mole fractions

$$n_{\text{total}} = n_{H_2} + n_H + n_{Ar} = 0.85 + 0.30 + 1 = 2.15$$

$$y_{H_2} = \frac{0.85}{2.15} = 0.3953$$

$$y_H = \frac{0.30}{2.15} = 0.1395$$

The equilibrium constant K

$$K = \frac{y_H^{\nu_H}}{y_{H_2}^{\nu_{H_2}}} \left(\frac{p}{p_0} \right)^{\nu_H - \nu_{H_2}} = \frac{0.1395^2}{0.3953} (1)^{2-1} = 0.0492$$

Taking the natural logarithm

$$\ln K = \ln 0.0492 = -3.011$$

The corresponding temperature from Table G-3

$$T = 3117 \text{ K}$$

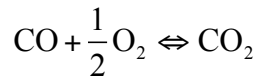
14-2-13 [BOI] Carbon monoxide at 300 K, 1 atm reacts with theoretical amount of air at 300 K and 1 atm in a chamber. An equilibrium mixture of CO₂, CO, O₂ and N₂ exits the chamber at 1 atm. Determine the composition and temperature (*T*) of the exiting mixture.

SOLUTION

Given:

$$p = 1 \text{ atm}$$

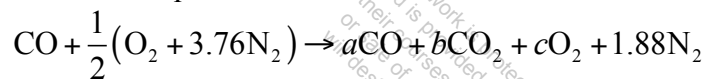
The stoichiometric equation is



Therefore,

$$\nu_{\text{CO}} = 1; \nu_{\text{O}_2} = 0.5; \nu_{\text{CO}_2} = 1$$

The actual equation



Using the m-IGE Model Chemical Equilibrium TESTcalc, the composition and temperature of the exiting mixture



$$T = 2399.30 \text{ K}$$

14-2-14 [BOL] Derive an expression for estimating the pressure (p) at which graphite and diamond exist in equilibrium at 300 K and 100 kPa in terms of the specific Gibbs function.

SOLUTION

The chemical potential for a substance that can be modeled as a solid (SL model) can be expressed as follows:

$$\begin{aligned}\bar{g}_k^{\text{SL}}(p, T) &= \bar{\mu}_k^{\text{SL}}(p, T) \equiv \bar{h}_k^{\text{SL}}(p, T) - T\bar{s}_k^{\text{SL}}(p, T) \\ &= [\bar{h}_{f,k}^{\circ} + \Delta\bar{h}_k^{\text{SL}}(p, T)] - T[\bar{s}_k^{\circ} + \Delta\bar{s}_k^{\text{SL}}(T)] \\ &= \bar{h}_k^{\circ} - T\bar{s}_k^{\circ} + \Delta\bar{h}_k^{\text{SL}}(p, T) - T\Delta\bar{s}_k^{\text{SL}}(T) \\ &= \bar{g}_k^{\circ} + \bar{c}_v(T - T_0) + \bar{v}(p - p_0) - T\bar{c}_v \ln \frac{T}{T_0}; \\ &= \bar{g}_k^{\circ} + \bar{c}_v(T - T_0) - T\bar{c}_v \ln \frac{T}{T_0} + \bar{v}(p - p_0); \\ &= \bar{g}_k^{\text{SL}}(p_0, T) + \bar{v}(p - p_0); \quad \left[\frac{\text{kJ}}{\text{kmol}} \right]\end{aligned}$$

At a given temperature, equating the expressions for diamond and graphite, we obtain:

$$\begin{aligned}\bar{g}_k^{\text{Diamond}}(p_0, T) + \bar{v}^{\text{Diamond}}(p - p_0) &= \bar{g}_k^{\text{Graphite}}(p_0, T) + \bar{v}^{\text{Graphite}}(p - p_0) \\ \Rightarrow (p - p_0)(\bar{v}^{\text{Diamond}} - \bar{v}^{\text{Graphite}}) &= \bar{g}_k^{\text{Graphite}}(p_0, T) - \bar{g}_k^{\text{Diamond}}(p_0, T) \\ \Rightarrow p &= p_0 + \frac{\bar{g}_k^{\text{Graphite}}(p_0, T) - \bar{g}_k^{\text{Diamond}}(p_0, T)}{\bar{v}^{\text{Diamond}} - \bar{v}^{\text{Graphite}}}\end{aligned}$$

14-2-15 [BOG] In a closed chamber at 30°C, 100 kPa, liquid water is in equilibrium with water vapor and dry air. Assuming air does not dissolve in water, determine (a) the partial pressure of water vapor (a) using equality of Gibbs function (chemical potential), and (b) Raoult's law.

SOLUTION

First using the equality of Gibbs functions:

The total pressure is known as $p = 100$ kPa. It is also assumed that air does not dissolve into the liquid water. The following equilibrium between the specific Gibbs functions exists

$$\begin{aligned}\bar{g}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) &= \bar{g}'_{\text{H}_2\text{O}}(p, T); \\ \Rightarrow \bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - T\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) &= \bar{h}'_{\text{H}_2\text{O}}(p, T) - T\bar{s}'_{\text{H}_2\text{O}}(p, T); \\ \Rightarrow \bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - \bar{h}'_{\text{H}_2\text{O}}(p, T) &= T[\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - \bar{s}'_{\text{H}_2\text{O}}(p, T)]\end{aligned}$$

We know that for the vapor phase

$$\bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) \cong \bar{h}_{g, \text{H}_2\text{O}@T}$$

Using a pressure correction for the vapor phase

$$\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) \cong \bar{s}(p_{\text{sat}@T}, T) - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} = \bar{s}_{g, \text{H}_2\text{O}@T} - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}}$$

We know that for the liquid phase

$$\bar{h}'_{\text{H}_2\text{O}}(p, T) \cong \bar{h}_{f, \text{H}_2\text{O}@T} + \bar{v}_{f, \text{H}_2\text{O}@T}(p - p_{\text{sat}@T})$$

$$\bar{s}'_{\text{H}_2\text{O}}(p, T) \cong \bar{s}_{f, \text{H}_2\text{O}@T}$$

Combining all the terms into the equating of the specific Gibbs function yields

$$\bar{h}_{g, \text{H}_2\text{O}@T} - [\bar{h}_{f, \text{H}_2\text{O}@T} + \bar{v}_{f, \text{H}_2\text{O}@T}(p - p_{\text{sat}@T})] = T \left[\left(\bar{s}_{g, \text{H}_2\text{O}@T} - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} \right) - \bar{s}_{f, \text{H}_2\text{O}@T} \right]$$

It can be noticed that

$$\bar{h}_{g, \text{H}_2\text{O}@T} - \bar{h}_{f, \text{H}_2\text{O}@T} = T(\bar{s}_{g, \text{H}_2\text{O}@T} - \bar{s}_{f, \text{H}_2\text{O}@T})$$

Therefore

$$\begin{aligned}
 -\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T}) &= -\bar{R}T \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}}; \\
 \Rightarrow \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} &= \frac{\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T}; \\
 \Rightarrow p_{\text{H}_2\text{O}} &= p_{\text{sat}@T} \exp \left[\frac{\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T} \right]; \\
 \Rightarrow p_{\text{H}_2\text{O}} &= p_{\text{sat}@T} \exp \left[\frac{\bar{M}_{\text{H}_2\text{O}} v_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T} \right] = (4.246) \exp \left[\frac{(18)(0.001)(100 - 4.246)}{(8.314)(303)} \right] = 4.25 \text{ kPa}
 \end{aligned}$$

The Raoult's Law approach:

Given:

$$p = 100 \text{ kPa}$$

$$T = 30^\circ\text{C}$$

$$y'_{\text{H}_2\text{O}} = 1$$

From Table B-2

$$p_{\text{H}_2\text{O},\text{sat}@30^\circ\text{C}} = 4.25 \text{ kPa}$$

Using Raoult's Model

$$p_{\text{H}_2\text{O}} = y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O},\text{sat}@30^\circ\text{C}} = (1)(4.25) = 4.25 \text{ kPa}$$

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14-2-16 [BOZ] Consider a two-phase, liquid-vapor $\text{NH}_3\text{-H}_2\text{O}$ system in equilibrium at 30°C . The mole fraction of ammonia in the liquid phase is 80%. Determine the pressure (p) in kPa and the mole fraction of ammonia in the vapor phase. Use Raoult's law.

SOLUTION

Given:

$$T = 30^\circ\text{C}$$

$$y'_{\text{NH}_3} = 0.80$$

$$y'_{\text{H}_2\text{O}} = 1 - y'_{\text{NH}_3} = 0.20$$

From Table B-12

$$p_{\text{NH}_3, \text{sat}@30^\circ\text{C}} = 1167.00 \text{ kPa}$$

From Table B-2

$$p_{\text{H}_2\text{O}, \text{sat}@30^\circ\text{C}} = 4.25 \text{ kPa}$$

Using Raoult's Model

$$p_{\text{NH}_3} = y'_{\text{NH}_3} p_{\text{NH}_3, \text{sat}@30^\circ\text{C}} = (0.80)(1167.00) = 933.60 \text{ kPa}$$

$$p_{\text{H}_2\text{O}} = y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O}, \text{sat}@30^\circ\text{C}} = (0.20)(4.25) = 0.85 \text{ kPa}$$

Therefore, the mixture pressure

$$p = p_{\text{NH}_3} + p_{\text{H}_2\text{O}} = 933.60 + 0.85 = 934.45 \text{ kPa}$$

The molar composition of the vapor

$$y_{\text{NH}_3} = \frac{p_{\text{NH}_3}}{p} = \frac{933.60}{934.45} = 0.9991$$

14-2-17 [BOP] Consider a two-phase, liquid-vapor $\text{NH}_3\text{-H}_2\text{O}$ system in equilibrium at 40°C , 150 kPa. Determine the mole fractions of ammonia in the liquid and vapor phases. Use Raoult's law.

SOLUTION

Given:

$$p = 150 \text{ kPa}$$

$$T = 40^\circ\text{C}$$

From Table B-12

$$p_{\text{NH}_3, \text{sat}@40^\circ\text{C}} = 1554.90 \text{ kPa}$$

From Table B-2

$$p_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}} = 7.38 \text{ kPa}$$

Using Raoult's Law and other known relations, a system with 6 equations and 6 unknown is created

$$y'_{\text{NH}_3} p_{\text{NH}_3, \text{sat}@40^\circ\text{C}} = p_{\text{NH}_3}$$

$$y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}} = p_{\text{H}_2\text{O}}$$

$$\frac{p_{\text{NH}_3}}{y_{\text{NH}_3}} = \frac{p_{\text{H}_2\text{O}}}{y_{\text{H}_2\text{O}}} = p$$

$$y'_{\text{NH}_3} + y'_{\text{H}_2\text{O}} = 1$$

$$y_{\text{NH}_3} + y_{\text{H}_2\text{O}} = 1$$

First find the relation between the mole fractions of ammonia in the liquid phase to that in the vapor phase

$$p_{\text{NH}_3} = y'_{\text{NH}_3} p_{\text{NH}_3, \text{sat}@40^\circ\text{C}} = 1554.90 y'_{\text{NH}_3}$$

$$\frac{p_{\text{NH}_3}}{y_{\text{NH}_3}} = \frac{1554.90 y'_{\text{NH}_3}}{y_{\text{NH}_3}} = 150;$$

$$\Rightarrow y'_{\text{NH}_3} = 0.09647 y_{\text{NH}_3}$$

Repeating the process for water

$$p_{\text{H}_2\text{O}} = y'_{\text{H}_2\text{O}} p_{\text{H}_2\text{O}, \text{sat}@40^\circ\text{C}} = 7.38 y'_{\text{H}_2\text{O}}$$

$$\frac{p_{\text{H}_2\text{O}}}{y_{\text{H}_2\text{O}}} = \frac{7.38 y'_{\text{H}_2\text{O}}}{y_{\text{H}_2\text{O}}} = 150;$$

$$\Rightarrow y'_{\text{H}_2\text{O}} = 20.32520 y_{\text{H}_2\text{O}}$$

Substituting into the relation stating that the sum of the mole fractions in the liquid phase must sum to 1

$$y'_{\text{NH}_3} + y'_{\text{H}_2\text{O}} = 0.09647y_{\text{NH}_3} + 20.32520y_{\text{H}_2\text{O}} = 1$$

Rearranging the equation for the sum of the vapor phase mole fractions

$$y_{\text{NH}_3} + y_{\text{H}_2\text{O}} = 1;$$

$$\Rightarrow y_{\text{H}_2\text{O}} = 1 - y_{\text{NH}_3}$$

Substituting this in and solving for the mole fraction of ammonia in the vapor phase

$$0.09647y_{\text{NH}_3} + 20.32520(1 - y_{\text{NH}_3}) = 1;$$

$$\Rightarrow 0.09647y_{\text{NH}_3} + 20.32520 - 20.32520y_{\text{NH}_3} = 1;$$

$$\Rightarrow -20.22873y_{\text{NH}_3} = -19.32520;$$

$$\Rightarrow y_{\text{NH}_3} = 0.9553$$

Going back to the previous relation between the mole fractions of ammonia in both phases

$$y'_{\text{NH}_3} = 0.09647y_{\text{NH}_3} = (0.09647)(0.9553) = 0.0922$$

14-2-18 [BOV] In a closed chamber at 10°C and 100 kPa, liquid water is in equilibrium with water vapor and dry air. Assuming air does not dissolve in water, determine (a) the partial pressure of water vapor using equality of Gibbs function (chemical potential). (b) Compare your result with saturation vapor pressure. (c) What-if Scenario: What would be the answer in part a if the pressure in the chamber were 1 MPa?

SOLUTION

The total pressure is known as $p = 100$ kPa. It is also assumed that air does not dissolve into the liquid water. The following equilibrium between the specific Gibbs functions exists

$$\begin{aligned}\bar{g}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) &= \bar{g}'_{\text{H}_2\text{O}}(p, T); \\ \Rightarrow \bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - T\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) &= \bar{h}'_{\text{H}_2\text{O}}(p, T) - T\bar{s}'_{\text{H}_2\text{O}}(p, T); \\ \Rightarrow \bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - \bar{h}'_{\text{H}_2\text{O}}(p, T) &= T[\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) - \bar{s}'_{\text{H}_2\text{O}}(p, T)]\end{aligned}$$

We know that for the vapor phase

$$\bar{h}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) \approx \bar{h}_{g, \text{H}_2\text{O}@T}$$

Using a pressure correction for the vapor phase

$$\bar{s}_{\text{H}_2\text{O}}(p_{\text{H}_2\text{O}}, T) \approx \bar{s}(p_{\text{sat}@T}, T) - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} = \bar{s}_{g, \text{H}_2\text{O}@T} - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}}$$

We know that for the liquid phase

$$\bar{h}'_{\text{H}_2\text{O}}(p, T) \approx \bar{h}_{f, \text{H}_2\text{O}@T} + \bar{v}_{f, \text{H}_2\text{O}@T}(p - p_{\text{sat}@T})$$

$$\bar{s}'_{\text{H}_2\text{O}}(p, T) \approx \bar{s}_{f, \text{H}_2\text{O}@T}$$

Combining all the terms into the equating of the specific Gibbs function yields

$$\bar{h}_{g, \text{H}_2\text{O}@T} - [\bar{h}_{f, \text{H}_2\text{O}@T} + \bar{v}_{f, \text{H}_2\text{O}@T}(p - p_{\text{sat}@T})] = T \left[\left(\bar{s}_{g, \text{H}_2\text{O}@T} - \bar{R} \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} \right) - \bar{s}_{f, \text{H}_2\text{O}@T} \right]$$

It can be noticed that

$$\bar{h}_{g, \text{H}_2\text{O}@T} - \bar{h}_{f, \text{H}_2\text{O}@T} = T(\bar{s}_{g, \text{H}_2\text{O}@T} - \bar{s}_{f, \text{H}_2\text{O}@T})$$

Therefore

$$\begin{aligned}
-\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T}) &= -\bar{R}T \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}}; \\
\Rightarrow \ln \frac{p_{\text{H}_2\text{O}}}{p_{\text{sat}@T}} &= \frac{\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T}; \\
\Rightarrow p_{\text{H}_2\text{O}} &= p_{\text{sat}@T} \exp \left[\frac{\bar{v}_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T} \right]; \\
\Rightarrow p_{\text{H}_2\text{O}} &= p_{\text{sat}@T} \exp \left[\frac{\bar{M}_{\text{H}_2\text{O}} v_{f,\text{H}_2\text{O}@T} (p - p_{\text{sat}@T})}{\bar{R}T} \right] = (1.228) \exp \left[\frac{(18)(0.001)(100 - 1.228)}{(8.314)(283)} \right] = 1.229 \text{ kPa}
\end{aligned}$$

Finding the percent difference,

$$\frac{p_{\text{H}_2\text{O}} - p_{\text{sat}@T}}{p_{\text{sat}@T}} = \frac{1.229 - 1.228}{1.228} = 0.0008 = 0.08\%$$

Comparing the resultant vapor pressure and the saturation pressure shows a negligible difference of 0.08%.

14-2-19 [BOK] Fresh water is to be extracted at a rate of 100 L/s from brackish water at 15°C with a salinity of 0.05% (by mass). Determine (a) the mole fraction of water in the brackish water, (b) the minimum power (in kW) required, and (c) the minimum pressure to which the brackish water must be pumped if the fresh water is to be obtained by reverse osmosis using semipermeable membrane.

SOLUTION

Given:

$$T = 15^\circ\text{C}$$

$$\dot{V} = 100 \frac{\text{L}}{\text{s}} = 0.1 \frac{\text{m}^3}{\text{s}}$$

$$\frac{m_{\text{NaCl}}}{m_{\text{NaCl}} + m_{\text{H}_2\text{O}}} = 0.0005$$

$$\bar{M}_{\text{H}_2\text{O}} = 18 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{NaCl}} = 58.4 \frac{\text{kg}}{\text{kmol}}; \rho_{\text{NaCl}} = 2170 \frac{\text{kg}}{\text{m}^3}$$

First find the mass of salt with respect to the mass of water

$$\frac{m_{\text{NaCl}}}{m_{\text{NaCl}} + m_{\text{H}_2\text{O}}} = 0.0005;$$

$$\Rightarrow m_{\text{NaCl}} = 0.0005(m_{\text{NaCl}} + m_{\text{H}_2\text{O}});$$

$$\Rightarrow 0.9995m_{\text{NaCl}} = 0.0005m_{\text{H}_2\text{O}};$$

$$\Rightarrow m_{\text{NaCl}} = 0.0005m_{\text{H}_2\text{O}}$$

From the PC Model at 1 atm and 15°C

$$\rho_{\text{H}_2\text{O}} = 999.00 \frac{\text{kg}}{\text{m}^3}$$

With the volume flow rate known

$$m_{\text{H}_2\text{O}} = \rho_{\text{H}_2\text{O}} \dot{V} = (999.00)(0.1) = 99.90 \frac{\text{kg}}{\text{s}}$$

Substituting to find the mass flow rate of salt

$$m_{\text{NaCl}} = 0.0005m_{\text{H}_2\text{O}} = (0.0005)(99.9) = 0.05 \frac{\text{kg}}{\text{s}}$$

The molar fraction of water in the brackish water

$$y'_{\text{H}_2\text{O}} = \frac{\frac{\dot{m}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}}{\frac{\dot{m}_{\text{NaCl}}}{\bar{M}_{\text{NaCl}}} + \frac{\dot{m}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}} = \frac{\frac{99.90}{18}}{\frac{0.05}{58.4} + \frac{99.90}{18}} = 0.9998$$

The minimum power required

$$\dot{W}_{\text{rev}} = - \frac{(\dot{m}_{\text{H}_2\text{O}} + \dot{m}_{\text{NaCl}}) \bar{R} T}{\bar{M}_{\text{H}_2\text{O}}} \ln \frac{1}{y'_{\text{H}_2\text{O}}} = - \frac{(99.90 + 0.05)(8.314)(288)}{18} \ln \frac{1}{0.9998} = -2.66 \text{ kW}$$

$$\dot{W}_p = 2.66 \text{ kW}$$

Using Eq. 14.36

$$\Delta p = \frac{(\rho_{\text{H}_2\text{O}} + \rho_{\text{NaCl}}) \bar{R} T}{\bar{M}_{\text{H}_2\text{O}}} \ln \frac{1}{y'_{\text{H}_2\text{O}}} = \frac{(999.00 + 2170)(8.314)(288)}{18} \ln \frac{1}{0.9998} = 84.32 \text{ kPa}$$

Therefore, the minimum pressure that the brackish water must be raised to for reverse osmosis to occur is

$$p = p_{\text{atm}} + \Delta p = (101.325 + 84.32) = 185.65 \text{ kPa}$$

14-2-20 [BOU] Fresh water is to be extracted from brackish water at 15°C with a salinity of 0.1% (by mass). Determine (a) the minimum work required to separate 1 kg of brackish water completely into pure water and pure salts, and (b) the minimum work required to obtain 1 kg of fresh water.

SOLUTION

Given:

$$T = 15^\circ\text{C}$$

$$\frac{m_{\text{NaCl}}}{m_{\text{NaCl}} + m_{\text{H}_2\text{O}}} = 0.001$$

$$\bar{M}_{\text{H}_2\text{O}} = 18 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{NaCl}} = 58.4 \frac{\text{kg}}{\text{kmol}}$$

First find the mass of salt with respect to the mass of water

$$\frac{m_{\text{NaCl}}}{m_{\text{NaCl}} + m_{\text{H}_2\text{O}}} = 0.001;$$

$$\Rightarrow m_{\text{NaCl}} = 0.001(m_{\text{NaCl}} + m_{\text{H}_2\text{O}});$$

$$\Rightarrow 0.999m_{\text{NaCl}} = 0.001m_{\text{H}_2\text{O}};$$

$$\Rightarrow m_{\text{NaCl}} = 0.001m_{\text{H}_2\text{O}}$$

From the PC Model at 1 atm and 15°C

$$\rho_{\text{H}_2\text{O}} = 999.00 \frac{\text{kg}}{\text{m}^3}$$

In the first scenario, the total mass of brackish water is known

$$m_{\text{NaCl}} + m_{\text{H}_2\text{O}} = 1 \text{ kg};$$

$$\Rightarrow 0.001m_{\text{H}_2\text{O}} + m_{\text{H}_2\text{O}} = 1;$$

$$\Rightarrow m_{\text{H}_2\text{O}} = 0.999 \text{ kg} \therefore m_{\text{NaCl}} = 0.001 \text{ kg}$$

Now the molar fraction of water in the brackish water

$$y'_{\text{H}_2\text{O}} = \frac{\frac{m_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}}{\frac{m_{\text{NaCl}}}{\bar{M}_{\text{NaCl}}} + \frac{m_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}} = \frac{\frac{0.999}{18}}{\frac{0.001}{58.4} + \frac{0.999}{18}} = 0.9997$$

The minimum work required to separate

$$W_{\text{rev}} = - \frac{(m_{\text{H}_2\text{O}} + m_{\text{NaCl}}) \bar{R}T}{\bar{M}_{\text{H}_2\text{O}}} \ln \frac{1}{y'_{\text{H}_2\text{O}}} = - \frac{(0.999 + 0.001)(8.314)(288)}{18} \ln \frac{1}{0.9997} = -0.04 \text{ kW}$$

$$W_p = 0.04 \text{ kW}$$

In the second scenario, the desired amount of fresh water is known

$$m_{\text{H}_2\text{O}} = 1 \text{ kg}$$

$$m_{\text{NaCl}} = 0.001 m_{\text{H}_2\text{O}} = 0.001(1) = 0.001 \text{ kg}$$

The molar fraction of water in the brackish water

$$y'_{\text{H}_2\text{O}} = \frac{\frac{m_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}}{\frac{m_{\text{NaCl}}}{\bar{M}_{\text{NaCl}}} + \frac{m_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}} = \frac{\frac{1}{18}}{\frac{0.001}{58.4} + \frac{1}{18}} = 0.9997$$

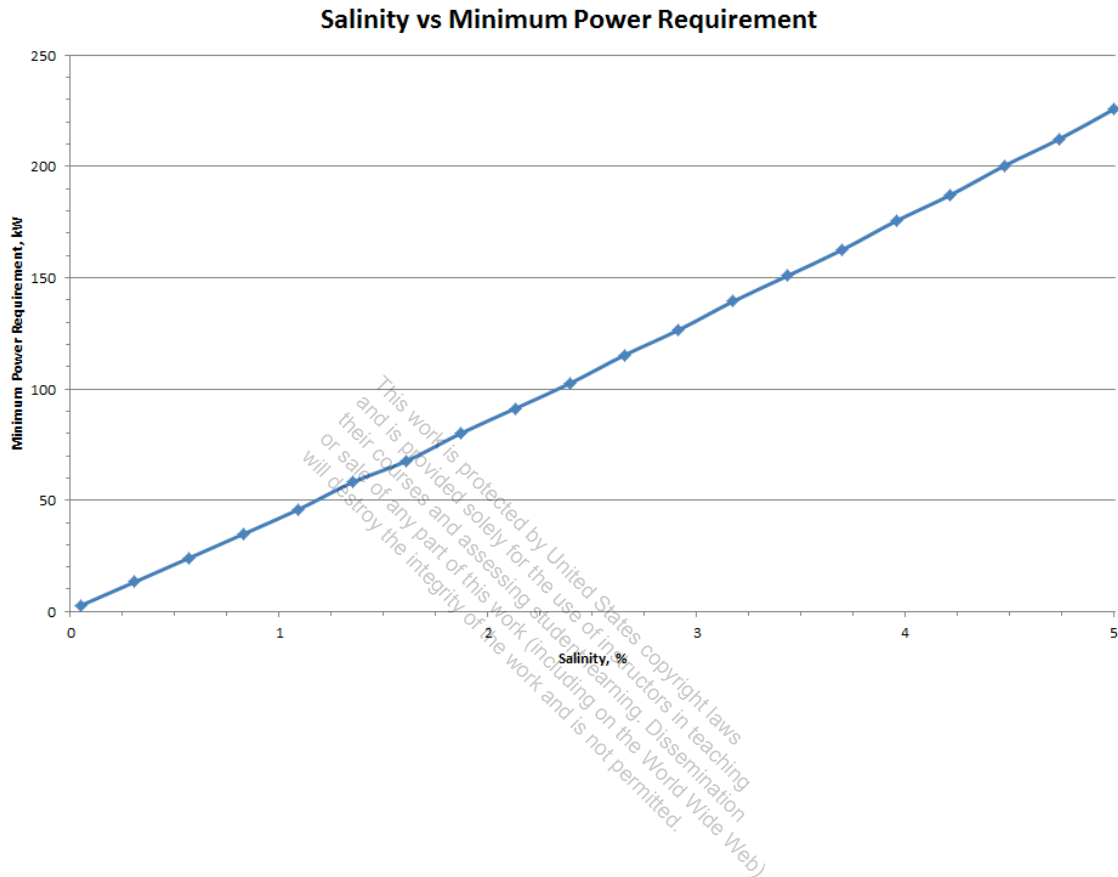
The minimum work required to obtain one kg of fresh water

$$W_{\text{rev}} = - \frac{(m_{\text{H}_2\text{O}} + m_{\text{NaCl}}) \bar{R}T}{\bar{M}_{\text{H}_2\text{O}}} \ln \frac{1}{y'_{\text{H}_2\text{O}}} = - \frac{(1 + 0.001)(8.314)(288)}{18} \ln \frac{1}{0.9997} = -0.04 \text{ kW}$$

$$W_p = 0.04 \text{ kW}$$

14-2-21 [BOQ] In problem 14-2-19 [BOK] change the salinity from 0.05% through 5% and plot how the minimum power requirement varies with the salinity of the brackish water.

SOLUTION



14-2-22 [BOX] A desalination plant produces fresh water from seawater at 10°C with a salinity of 3.1% (on a mass basis) through reverse osmosis at a rate of 1.5 m³/s while consuming 9 MW of power. The amount of fresh water is negligible compared to the seawater used. Determine the exergetic efficiency of the plant.

SOLUTION

Given:

$$T = 10^{\circ}\text{C}$$

$$\dot{V} = 1.5 \frac{\text{m}^3}{\text{s}}$$

$$\dot{W}_{\text{net}} = 9 \text{ MW}$$

$$\frac{\dot{m}_{\text{NaCl}}}{\dot{m}_{\text{NaCl}} + \dot{m}_{\text{H}_2\text{O}}} = 0.031$$

$$\bar{M}_{\text{H}_2\text{O}} = 18 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{NaCl}} = 58.4 \frac{\text{kg}}{\text{kmol}}$$

First find the mass of salt with respect to the mass of water

$$\frac{\dot{m}_{\text{NaCl}}}{\dot{m}_{\text{NaCl}} + \dot{m}_{\text{H}_2\text{O}}} = 0.031;$$

$$\Rightarrow \dot{m}_{\text{NaCl}} = 0.031(\dot{m}_{\text{NaCl}} + \dot{m}_{\text{H}_2\text{O}});$$

$$\Rightarrow 0.969\dot{m}_{\text{NaCl}} = 0.031\dot{m}_{\text{H}_2\text{O}};$$

$$\Rightarrow \dot{m}_{\text{NaCl}} = 0.0320\dot{m}_{\text{H}_2\text{O}}$$

From the PC Model at 1 atm and 10°C

$$\rho_{\text{H}_2\text{O}} = 999.50 \frac{\text{kg}}{\text{m}^3}$$

With the volume flow rate known

$$\dot{m}_{\text{H}_2\text{O}} = \rho_{\text{H}_2\text{O}} \dot{V} = (999.50)(1.5) = 1499.25 \frac{\text{kg}}{\text{s}}$$

Substituting to find the mass flow rate of salt

$$\dot{m}_{\text{NaCl}} = 0.032\dot{m}_{\text{H}_2\text{O}} = (0.032)(1499.25) = 47.98 \frac{\text{kg}}{\text{s}}$$

The molar fraction of water in the brackish water

$$y'_{\text{H}_2\text{O}} = \frac{\frac{\dot{m}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}}{\frac{\dot{m}_{\text{NaCl}}}{\bar{M}_{\text{NaCl}}} + \frac{\dot{m}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}} = \frac{\frac{1499.25}{18}}{\frac{47.98}{58.4} + \frac{1499.25}{18}} = 0.9902$$

The minimum power required

$$\dot{W}_{\text{rev}} = - \frac{(\dot{m}_{\text{H}_2\text{O}} + \dot{m}_{\text{NaCl}}) \bar{R} T}{\bar{M}_{\text{H}_2\text{O}}} \ln \frac{1}{y'_{\text{H}_2\text{O}}} = - \frac{(1499.25 + 47.98)(8.314)(283)}{18} \ln \frac{1}{0.9902} = -1991.78 \text{ kW}$$

$$\dot{W}_p = 1991.78 \text{ kW} = 1.99 \text{ MW}$$

The exergetic efficiency

$$\eta_{\text{II}} = \frac{\dot{W}_{\text{rev}}}{\dot{W}_{\text{net}}} = \frac{1.99}{9} = 0.2211 = 22.11\%$$

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14-2-23 [BOC] A river discharges fresh water at 20°C at a rate of 20,000 m³/s into an ocean at the same temperature with a salinity of 3.5% (by mass). Determine the amount of power that can be generated if the river water mixes with the ocean water in a reversible manner.

SOLUTION

Given:

$$T = 20^{\circ}\text{C}$$

$$\dot{V} = 20000 \frac{\text{m}^3}{\text{s}}$$

$$\frac{m_{\text{NaCl}}}{m_{\text{NaCl}} + m_{\text{H}_2\text{O}}} = 0.035$$

$$\bar{M}_{\text{H}_2\text{O}} = 18 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{NaCl}} = 58.4 \frac{\text{kg}}{\text{kmol}}$$

The amount of power that can be generated is the negative of the minimum pumping requirement to separate through reverse osmosis.

First find the mass of salt with respect to the mass of water

$$\frac{m_{\text{NaCl}}}{m_{\text{NaCl}} + m_{\text{H}_2\text{O}}} = 0.035;$$

$$\Rightarrow m_{\text{NaCl}} = 0.035(m_{\text{NaCl}} + m_{\text{H}_2\text{O}});$$

$$\Rightarrow 0.965 m_{\text{NaCl}} = 0.035 m_{\text{H}_2\text{O}};$$

$$\Rightarrow m_{\text{NaCl}} = 0.0363 m_{\text{H}_2\text{O}}$$

From the PC Model at 1 atm and 20°C

$$\rho_{\text{H}_2\text{O}} = 998.00 \frac{\text{kg}}{\text{m}^3}$$

With the volume flow rate known

$$m_{\text{H}_2\text{O}} = \rho_{\text{H}_2\text{O}} \dot{V} = (998.00)(20000) = 2.00 \times 10^7 \frac{\text{kg}}{\text{s}}$$

Substituting to find the mass flow rate of salt

$$m_{\text{NaCl}} = 0.035 m_{\text{H}_2\text{O}} = (0.035)(2.00 \times 10^7) = 7.00 \times 10^5 \frac{\text{kg}}{\text{s}}$$

The molar fraction of water in the brackish water

$$y'_{\text{H}_2\text{O}} = \frac{\frac{\dot{m}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}}{\frac{\dot{m}_{\text{NaCl}}}{\bar{M}_{\text{NaCl}}} + \frac{\dot{m}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}}} = \frac{\frac{2.00 \times 10^7}{18}}{\frac{7.00 \times 10^5}{58.4} + \frac{2.00 \times 10^7}{18}} = 0.9893$$

The amount of power that can be generated

$$\dot{W}_{\text{rev}} = \frac{(\dot{m}_{\text{H}_2\text{O}} + \dot{m}_{\text{NaCl}}) \bar{R} T}{\bar{M}_{\text{H}_2\text{O}}} \ln \frac{1}{y'_{\text{H}_2\text{O}}} = \frac{(2.00 \times 10^7 + 7.00 \times 10^5)(8.314)(293)}{18} \ln \frac{1}{0.9893} = 3.01 \times 10^7 \text{ kW}$$

$$\dot{W}_{\text{rev}} = \mathbf{30100 \text{ MW}}$$

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14-2-24 [BOT] A desalination plant produces fresh water through reverse osmosis at a rate of $1\text{ m}^3/\text{s}$, consuming 6 MW of power. The plant has an exergetic efficiency of 20%. Determine the power (W) that can be produced from reversible mixing of the produced fresh water with seawater.

SOLUTION

Only a fraction of the power consumption goes into separation to produce fresh water, and in reversible mixing that fraction can be recovered. From the definition of exergetic efficiency

$$\eta_{II} = \frac{\dot{W}_{rev}}{\dot{W}_{net}};$$

$$\Rightarrow \dot{W}_{rev} = \eta_{II} \dot{W}_{net} = (0.20)(6) = 1.2 \text{ MW}$$

The reversible mixing is equal to the required reversible power required for the pumps for reverse osmosis, so **1.2 MW** could be produced.

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