**11-2-1** [OTQ] Determine the change in (a) enthalpy ( $\Delta h$ ) and (b) entropy ( $\Delta s$ ) of nitrogen as it undergoes a change of state from 200 K and 6 MPa to 300 K and 10 MPa by treating nitrogen as a perfect gas. What-if Scenario: What would the change in enthalpy be if nitrogen were modeled using (c) the ideal gas, or (d) real gas model?

## **SOLUTION:**

From Table C-1 or the PG system-state TESTcalc, obtain the necessary material properties of

$$N_2$$
:  $c_p = 1.039 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ ,  $c_v = 0.743 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ , R=0.2968  $\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ 

Given:  $p_1 = 6 \text{ MPa}$ ;  $T_1 = 200 \text{ K}$ 

Given:  $p_2 = 10 \text{ MPa}$ ;  $T_2 = 300 \text{ K}$ 

## PG Model:

$$\begin{split} \Delta h &= h_2 - h_1 \; ; \quad \Rightarrow \Delta h = c_p \; (T_2 - T_1) \; ; \quad \Rightarrow \Delta h = \left(1.039\right) (300 - 200) \; ; \\ \Rightarrow \Delta h = 103.9 \; \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= u_2 - u_1 \; ; \quad \Rightarrow \Delta u = c_v (T_2 - T_1) \; ; \quad \Rightarrow \Delta u = \left(0.743\right) (300 - 200) \; ; \\ \Rightarrow \Delta u = 74.3 \; \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= s_2 - s_1 \; ; \quad \Rightarrow \Delta s = c_p \; \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \; ; \quad \Rightarrow \Delta s = 1.039 \ln \left(\frac{300}{200}\right) - 0.2968 \ln \left(\frac{10}{6}\right) \; ; \\ \Rightarrow \Delta s &= 0.2697 \; \frac{\text{kJ}}{\text{kg} \; \text{K}} \\ \Delta v &= v_2 - v_1 \; ; \quad \Rightarrow \Delta v = \frac{RT_2}{p_2} - \frac{RT_1}{p_1} \; ; \quad \Rightarrow \Delta v = \frac{(0.2968)(300)}{10000} - \frac{(0.2968)(200)}{6000} \; ; \\ \Rightarrow \Delta v &= -0.000989 \; \frac{\text{m}^3}{\text{kg}} \end{split}$$

## IG Model:

Using the manual approach described in Chapter 3 or the IG system-state TESTcalc, obtain:

$$\Delta h = h_2 - h_1; \quad \Rightarrow \Delta h = 1.523 + 101.586; \quad \Rightarrow \Delta h = 103.109 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1; \quad \Rightarrow \Delta u = -87.556 + 160.971; \quad \Rightarrow \Delta u = 73.415 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = s_2 - s_1; \quad \Rightarrow \Delta s = 5.484 - 5.218; \quad \Rightarrow \Delta s = 0.266 \frac{\text{kJ}}{\text{kg}.\text{K}}$$

$$\Delta v = v_2 - v_1; \quad \Rightarrow \Delta v = 0.00891 - 0.0099; \quad \Rightarrow \Delta v = -0.00099 \frac{\text{m}^3}{\text{kg}}$$

# RG Model (L-K):

From Table E-1, obtain the necessary material properties of  $N_2$ :

$$p_{cr} = 3.39 \text{ MPa}, T_{cr} = 126.2 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}}; \implies p_{r1} = \frac{6}{3.39}; \implies p_{r1} = 1.76991,$$

$$p_{r2} = \frac{p_2}{p_{cr}}; \implies p_{r2} = \frac{10}{3.39}; \implies p_{r2} = 2.950$$

$$T_{r1} = \frac{T_1}{T_{cr}}; \implies T_{r1} = \frac{200}{126.2}; \implies T_{r1} = 1.585,$$

$$T_{r2} = \frac{T_2}{T_{cr}}; \implies T_{r2} = \frac{300}{126.2}; \implies T_{r2} = 2.377$$

From the Lee-Kesler compressibility charts, Table E, we obtain  $Z_1^{\text{L-K}} = 0.88$ ,  $Z_2^{\text{L-K}} = 0.98$ 

$$\Delta h = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1});$$

$$\Rightarrow \Delta h = (1.5228 + 101.586) - 0.2968(126.2)(0.549 - 0.810) = 112.88 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = (h_2 - h_1)^{IG} - R(Z_2^{\text{L-K}}T_2 - Z_1^{\text{L-K}}T_1);$$

$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1});$$

 $\Rightarrow \Delta u = (1.52286 + 101.586) - 0.2968 [(0.98)(300) - (0.876)(200)]; \Rightarrow \Delta u = 67.849 \frac{\text{kJ}}{\text{kg}}$ 

$$\Delta s = (s_2 - s_1) - K(Z_{s,2} - Z_{s,1}),$$

$$\Rightarrow \Delta s = (5.48426 - 5.2179) - 0.2968(0.23 - 0.38) = 0.311 \frac{kJ}{kg \cdot K}$$

$$\Delta v = v_2 - v_1 \; ; \quad \Rightarrow \Delta v = \frac{Z_2^{\text{L-K}} R T_2}{p_2} - \frac{Z_1^{\text{L-K}} R T_1}{p_1} \; ;$$

$$\Rightarrow \Delta v = \frac{(0.98)(0.2968)(300)}{10000} - \frac{(0.876)(0.2968)(200)}{6000} \; ; \quad \Rightarrow \Delta v = 6.2 \times 10^{-5} \; \frac{\text{m}^3}{\text{kg}}$$

## RG Model (N-O):

From the Nelson-Obert compressibility chart, we obtain  $Z_1^{\text{N-O}} = 0.92$ ,  $Z_2^{\text{N-O}} = 0.99$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{1G} - RT_{cr}(Z_{h,2} - Z_{h,1}); \\ \Rightarrow \Delta h &= (1.5228 + 101586) - 0.2968(126.2)(0.49 - 0.567); \quad \Rightarrow \Delta h = 105.993 \ \frac{\text{kJ}}{\text{kg}} \end{split}$$

$$\Delta u = (h_2 - h_1)^{IG} - R(Z_2^{\text{N-O}} T_2 - Z_1^{\text{N-O}} T_1);$$

$$\Rightarrow \Delta u = (1.52286 + 101.586) - 0.2968 [(0.99)(300) - (0.92)(200)]; \Rightarrow \Delta u = 69.571 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1});$$

$$\Rightarrow \Delta s = (5.48426 - 5.2179) - 0.2968(0.25 - 0.375); \Rightarrow \Delta s = 0.3035 \frac{\text{kJ}}{\text{kg.K}}$$

$$\Delta v = v_2 - v_1; \implies \Delta v = \frac{Z_2^{\text{N-O}} R T_2}{p_2} - \frac{Z_1^{\text{N-O}} R T_1}{p_1};$$

$$\Rightarrow \Delta v = \frac{(0.99)(0.2968)(300)}{10000} - \frac{(0.92)(0.2968)(200)}{6000}; \implies \Delta v = -0.000287 \frac{\text{m}^3}{\text{kg}}$$

# **Summary of Results:**

					RG Model		RG N	
	DC	DC	IC M - 1-1	IC	`	[anual]	(TEST)	
	PG	PG	IG Model	IG	Lee-	Nelson-	Lee-	Nelson-
	Model	Model	(Manual)	Model	Kesler	Obert	Kesler	Obert
	(Manual)	(TEST)		(TEST)				
$\left(\frac{kJ}{J}\right)$	103.9	103.11	103.109	103.10 8	112.88	105.993	114.530	114.224
(kg)								
$\Delta u$								
$\left(\frac{\mathrm{kJ}}{\mathrm{kg}}\right)$	74.3	73.417	73.415	73.416	67.849	69.571	78.013	77.697
$\frac{\Delta s}{\left(\frac{kJ}{kg.K}\right)}$	0.2697	0.2664	0.266	0.2664	0.311	0.3035	0.318	0.3214
$\left(\frac{\Delta v}{kg}\right)$	-9.89E-4	-9.90E- 4	-9.9E-4	-9.90E- 4	6.2E-5	-2.87E-4	1.67E-4	1.79E-4

What-if Scenario: Launch the PG system-state TESTcalc and select N2. Evaluate the initial and final states from the known conditions and evaluate the property differences in the I/O panel. Generate the TEST-code using the Super-Calculate button. Launch the RG system-state TESTcalc in a separate tab, paste the TEST-code in the I/O panel, and click the Load button to calculate the states. Evaluate the property differences for the RG model in the I/O panel. The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

**11-2-2** [OTY] Calculate  $\Delta v$ ,  $\Delta u$ ,  $\Delta h$ , and  $\Delta s$  for the following change of state of superheated steam: State-1:  $p_1$ = 2 MPa, saturated vapor; State-2:  $p_2$ = 33 kPa, 400°C. Compare the following models: (a) PC model, (b) PG model, (c) IG model, (d) RG model (L-K) and (e) RG model (N-O). Your answers should be in a tabular form - one table for manual results and another for corresponding TEST results.

### **SOLUTION:**

(a) **PC model**: Using the manual approach (Chapter 3) or the PC system-state TESTcalc obtain:

<b>State-1</b> ( $x_1$ = <b>1</b> )	State-2
$p_1 = 2 \text{ MPa}$	$p_2 = 33 \text{ kPa}$
$T_1 = 212.42 \text{ C}$	$T_2 = 400 \text{ C}$
$v_1 = 0.009961 \text{ m}^3/\text{kg}$	$v_2 = 9.4099 \text{ m}^3/\text{kg}$
$u_1 = 2600.26 \text{ kJ/kg}$	$u_2 = 2968.61 \text{ kJ/kg}$
$h_1 = 2799.47 \text{ kJ/kg}$	$h_2 = 3279.13 \text{ kJ/kg}$
$s_1 = 6.3409 \text{ kJ/kg.K}$	$s_2 = 9.0560 \text{ kJ/kg.K}$

 $\Delta v$ ,  $\Delta u$ ,  $\Delta h$ , and  $\Delta s$  are shown in the table below.

(b) **PG model**, pv = RT;  $c_v = \text{constant}$ . Perfect Gas values for steam are:

$$c_v = 1.4108 \text{ kJ/kg.K}$$
  
 $c_p = 1.8723 \text{ kJ/kg.K}$   
 $R = 0.4615 \text{ kJ/kg.K}$ 

Also,

$$\Delta u = c_v \left( T_2 - T_1 \right)$$

$$\Delta h = c_p \left( T_2 - T_1 \right)$$

$$\Delta s = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right)$$

State properties are found as:

$$v_1 = \frac{RT_1}{p_1}; \implies v_1 = \frac{(0.4615)(485.4 \text{ K})}{(2000 \text{ kPa})}; \implies v_1 = 0.1120 \text{ m}^3/\text{kg}$$

$$v_{2} = \frac{RT_{1}}{p_{1}}; \Rightarrow v_{2} = \frac{(0.4615)(673 \text{ K})}{(33 \text{ kPa})}; \Rightarrow v_{2} = 9.4118 \text{ m}^{3}/\text{kg}$$

$$\Delta u = c_{v} (T_{2} - T_{1}); \Rightarrow \Delta u = (1.4108)(400 - 212.42); \Rightarrow \Delta u = 264.638 \text{ kJ/kg}$$

$$\Delta h = c_{p} (T_{2} - T_{1}); \Rightarrow \Delta h = (1.8723)(400 - 212.42); \Rightarrow \Delta h = 351.206 \text{ kJ/kg}$$

$$\Delta s = c_{p} \ln \left(\frac{T_{2}}{T_{1}}\right) - R \ln \left(\frac{p_{2}}{p_{1}}\right); \Rightarrow \Delta s = (1.8723) \ln \left(\frac{673}{485.42}\right) - (0.4615) \ln \left(\frac{33}{2000}\right);$$

$$\Rightarrow \Delta s = 2.506 \text{ kJ/kg.K}$$

(c) **IG model**, pv = RT;  $c_v = c_v(T)$ .  $v_1$  and  $v_2$  are found as in PG model,

$$v_1 = \frac{RT_1}{p_1}; \implies v_1 = \frac{(0.4615)(485.4 \text{ K})}{(2000 \text{ kPa})}; \implies v_1 = 0.1120 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{RT_1}{p_1}; \implies v_2 = \frac{(0.4615)(673 \text{ K})}{(33 \text{ kPa})}; \implies v_2 = 9.4118 \text{ m}^3/\text{kg}$$

u = u(T), h = h(T), and s = s(T), found by lookup in IG tables:

	20. 70. 00.
State-1 $(x_i = 1)$	State-2
$T_1 = 212.42 \mathrm{C}$	$T_2 = 400 \text{ C}$
$u_1 = 681.044 \text{ kJ/kg}$	$u_2 = 970.619 \text{ kJ/kg}$
$h_1 = 904.885 \text{ kJ/kg}$	$h_2 = 1281.21 \text{ kJ/kg}$
$s_1 = 11.399 \text{ kJ/kg.K}$	$s_2 = 12.0534 \text{ kJ/kg.K}$

 $\Delta v$ ,  $\Delta u$ ,  $\Delta h$ , and  $\Delta s$  are shown in the table below.

(d) **RG model (L-K),** pv = ZRT; where

$$p_{cr} = 22.09 \text{ MPa}$$
 $T_{cr} = 647.3 \text{ K}$ 

Where  $p_r = \frac{p}{p_{cr}}$  and  $T_r = \frac{T}{T_{cr}}$ , Thus from the compressibility charts,

Real Gas (Lee-Kesler)						
State-1 State-2						
$p_{r1} = 0.0905$	$p_{r2} = 0.0015$					
$T_{r1} = \text{sat vap}$	$T_{r2} = 1.0397$					
$Z_1 = 0.90$	$Z_2 = 0.99$					
$Z_{h1} = 0.20$	$Z_{h2} = 0.01$					
$Z_{s1} = 0.20$	$Z_{s2} = 0.01$					

$$v_{1} = \frac{Z_{1}RT_{1}}{p_{1}}; \Rightarrow v_{1} = \frac{(0.90)(0.4615)(485.4 \text{ K})}{(2000 \text{ kPa})}; \Rightarrow v_{1} = 0.1007 \text{ m}^{3}/\text{kg}$$

$$v_{2} = \frac{Z_{2}RT_{1}}{p_{1}}; \Rightarrow v_{2} = \frac{(0.99)(0.4615)(673 \text{ K})}{(33 \text{ kPa})}; \Rightarrow v_{2} = 9.318 \text{ m}^{3}/\text{kg}$$

$$\Delta h = \Delta h^{IG} - RT_{cr}(Z_{h2} - Z_{h1});$$

$$\Rightarrow \Delta h = (376.325) - (0.4615)(647.3)(0.01 - 0.20); \Rightarrow \Delta h = 433.084 \text{ kJ/kg}$$

$$\Delta u = \Delta h - R(Z_{2}T_{2} - Z_{1}T_{1});$$

$$\Rightarrow \Delta u = (433.084) - (0.4615)[(0.01)(673) - (0.20)(485)]; \Rightarrow \Delta u = 474.743 \text{ kJ/kg}$$

$$\Delta s = \Delta s^{IG} - R(Z_{s2} - Z_{s1});$$

$$\Rightarrow \Delta s = (0.6549) - (0.4615)(0.01 - 0.20); \Rightarrow \Delta s = 0.74259 \text{ kJ/kg.K}$$

# (e) **RG model (N-O)**, pv = ZRT; where

$$p_{cr} = 22.09 \text{ MPa}$$
 $T_{cr} = 647.3 \text{ K}$ 

Where  $p_r = \frac{p}{p_{cr}}$  and  $T_r = \frac{T}{T_{cr}}$ , Thus from the compressibility charts,

Real Gas (Nelson-Obert)						
State-1	State-2					
$p_{r1} = 0.0905$	$p_{r2} = 0.0015$					
$T_{r1} = \text{sat vap}$	$T_{r2} = 1.0397$					
	(supercritical)					
$Z_1 = 0.913$	$Z_2 = 0.99$					
$Z_{h1} = 0.21$	$Z_{h2} = 0.01$					
$Z_{s1}=0.25$	$Z_{s2} = 0.01$					

$$v_{1} = \frac{Z_{1}RT_{1}}{p_{1}}; \Rightarrow v_{1} = \frac{(0.913)(0.4615)(485.4 \text{ K})}{(2000 \text{ kPa})}; \Rightarrow v_{1} = 0.1022 \text{ m}^{3}/\text{kg}$$

$$v_{2} = \frac{Z_{2}RT_{1}}{p_{1}}; \Rightarrow v_{2} = \frac{(0.99)(0.4615)(673 \text{ K})}{(33 \text{ kPa})}; \Rightarrow v_{2} = 9.318 \text{ m}^{3}/\text{kg}$$

$$\Delta h = \Delta h^{IG} - RT_{cr}(Z_{h2} - Z_{h1});$$

$$\Rightarrow \Delta h = (376.325) - (0.4615)(647.3)(0.01 - 0.21); \Rightarrow \Delta h = 436.071 \text{ kJ/kg}$$

$$\Delta u = \Delta h - R(Z_{2}T_{2} - Z_{1}T_{1});$$

$$\Rightarrow \Delta u = (436.071) - 0.4615[(0.01)(673) - (0.21)(485)]; \Rightarrow \Delta u = 479.969 \text{ kJ/kg}$$

$$\Delta s = \Delta s^{IG} - R(Z_{s2} - Z_{s1});$$

$$\Rightarrow \Delta s = (0.6549) - (0.4615)(0.01 - 0.21); \Rightarrow \Delta s = 0.76566 \text{ kJ/kg.K}$$

## **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

Manual Solutions								
	PC	PG	IG	RG (L-K)	RG (N-O)			
$\Delta v \ (\text{m}^3/\text{kg})$	9.3999	9.4006	9.4006	9.213	9.2158			
$\Delta u$ (kJ/kg)	368.35	264.638	289.575	474.743	479.969			
$\Delta h$ (kJ/kg)	479.66	351.206	376.325	433.084	436.071			
Δs (kJ/kg.K)	2.7151	2.506	0.6549	0.74259	0.76566			
		TEST	Solutions					
	PC	PG	IG	RG (L-K)	RG (N-O)			
$\Delta v \ (\text{m}^3/\text{kg})$	9.3103	9.2994	9.2994	9.314	Could not			
$\Delta u$ (kJ/kg)	368.34	263.8	290.37	321.36	compute;			
$\Delta h$ (kJ/kg)	479.66	349.96	376.91	424.74	Z out of			
$\Delta s$ (kJ/kg.K)	2.715	2.504	2.549	2.619	range			

# **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

**11-2-3** [OTD] Methane is isothermally and reversibly compressed by a piston-cylinder device from 1 MPa,  $100^{\circ}$ C to 4 MPa. Using the Lee-Kesler RG model, calculate (a) the work done ( $w_B$ ) and (b) heat transfer per unit mass (q). (c) What-if Scenario: What would the work done be if the process were isentropic?

## **SOLUTION:**

From Table C-1 or the PG system-state TESTcalc, obtain the necessary material properties of

$$CH_4$$
:  $c_p = 2.254 \frac{\text{kJ}}{\text{kg.K}}$ ,  $c_v = 1.736 \frac{\text{kJ}}{\text{kg.K}}$ ,  $R = 0.5182 \frac{\text{kJ}}{\text{kg.K}}$ 

Given:  $p_1 = 1 \text{ MPa}$ ;  $T_1 = 100 \, ^{\circ}\text{C}$ 

Given:  $p_2 = 4$  MPa;  $T_2 = T_1$ 

# Perfect Gas Model:

$$\Delta h = h_2 - h_1; \quad \Rightarrow \Delta h = C_p \left( T_2 - T_1 \right)^0; \quad \Rightarrow \Delta h = 0 \frac{kJ}{kg}$$

$$\Delta u = u_2 - u_1; \quad \Rightarrow \Delta u = C_v \left( T_2 - T_1 \right)^0; \quad \Rightarrow \Delta u = 0 \frac{kJ}{kg}$$

$$\Delta s = s_2 - s_1; \quad \Rightarrow \Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}; \quad \Rightarrow \Delta s = -0.5182 \ln \frac{4}{1};$$

$$\Rightarrow \Delta s = -0.718 \frac{kJ}{kg K}$$

$$\Delta v = v_2 - v_1; \quad \Rightarrow \Delta v = \frac{RT_2}{p_2} - \frac{RT_1}{p_1}; \quad \Rightarrow \Delta v = \frac{(0.5182)(373.15)}{4000} - \frac{(0.5182)(373.15)}{1000};$$

$$\Rightarrow \Delta v = -0.145 \frac{m^3}{kg}$$

The entropy equation produces

$$\Delta s = \frac{q}{T_B} + S_{gen}^{0};$$

$$q = (\Delta s)(T_B); \quad \Rightarrow q = (-0.718)(298); \quad \Rightarrow q = -214 \frac{\text{kJ}}{\text{kg}}$$

The energy equation yields

$$\Delta u + \Delta k e^{0} + \Delta p e^{0} = q - \left( w_{B} + w_{O}^{0} \right)$$

$$\Rightarrow w_{B} = q - \Delta u; \quad \Rightarrow w_{B} = -214 - 0; \quad \Rightarrow w_{B} = -124 \frac{kJ}{kg}$$

# Ideal Gas Model:

The Ideal gas model utilizes the TEST for finding the properties at the given state.

$$\Delta h = h_2 - h_1; \quad \Rightarrow \Delta h = 174.11 - 174.11; \quad \Rightarrow \Delta h = 0 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1; \quad \Rightarrow \Delta u = -19.30 + 19.30; \quad \Rightarrow \Delta u = 0 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = s_2 - s_1; \quad \Rightarrow \Delta s = 10.226 - 10.945; \quad \Rightarrow \Delta s = -0.719 \frac{\text{kJ}}{\text{kg.K}}$$

$$\Delta v = v_2 - v_1; \quad \Rightarrow \Delta v = 0.04835 - 0.1934; \quad \Rightarrow \Delta v = -0.145 \frac{\text{m}^3}{\text{kg}}$$

The entropy equation produces

$$\Delta s = \frac{q}{T_B} + S_{gen}^{0};$$

$$q = (\Delta s)(T_B); \quad \Rightarrow q = (-0.719)(298); \quad \Rightarrow q = -214 \frac{kJ}{kg}$$

The energy equation yields

$$\Delta u + \Delta k e^{0} + \Delta p e^{0} = q - \left( w_{B} + w_{O}^{0} \right)$$

$$\Rightarrow w_{B} = q - \Delta u; \quad \Rightarrow w_{B} = -214 - 0; \quad \Rightarrow w_{B} = -124 \frac{kJ}{kg}$$

## Real Gas Model (Lee Kesler):

From Table E-1, obtain the necessary material properties of  $CH_4$ :  $p_{cr} = 4.64$  MPa,  $T_{cr} = 191.1$  K

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}}; \quad \Rightarrow \quad p_{r1} = \frac{1}{4.64}; \quad \Rightarrow \quad p_{r1} = 0.216,$$

$$p_{r2} = \frac{p_2}{p_{cr}}; \quad \Rightarrow \quad p_{r2} = \frac{4}{4.64}; \quad \Rightarrow \quad p_{r2} = 0.862$$

$$T_{r1} = \frac{T_1}{T_{cr}}; \quad \Rightarrow \quad T_{r1} = \frac{373.15}{191.1}; \quad \Rightarrow \quad T_{r1} = 1.953,$$

$$T_{r2} = \frac{T_2}{T}; \quad \Rightarrow \quad T_{r2} = \frac{373.15}{191.1}; \quad \Rightarrow \quad T_{r2} = 1.953$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{\text{L-K}} = 0.993$ ,  $Z_2^{\text{L-K}} = 0.975$ 

$$\Delta h = (h_1 - h_1)^{1/3} - RT_{cr}(Z_{h,2} - Z_{h,1}); \quad \Rightarrow \Delta h = -0.5182(191.1)(0.249 - 0.0598)$$

$$\Rightarrow \Delta h = -18.7 \frac{kJ}{kg}$$

$$\Delta u = (h_2 - h_1) - R(Z_2^{L-K}T_2 - Z_1^{L-K}T_1); \quad \Rightarrow = -18.7 - 0.5182[(0.975)(373.15) - (0.993)(373.15)]$$

$$\Rightarrow \Delta u = -15.2 \frac{kJ}{kg}$$

$$\Delta s = (s_2 - s_1)^{1/3} - R(Z_{s,2} - Z_{s,1}); \quad \Rightarrow = -0.719 - 0.5182(0.1056 - 0.0284)$$

$$\Rightarrow \Delta s = -0.76 \frac{kJ}{kg.K}$$

$$\Delta v = v_2 - v_1; \quad \Rightarrow \Delta v = \frac{Z_2^{L-K}RT_2}{P_2} - \frac{Z_1^{L-K}RT_1}{P_1};$$

$$\Rightarrow \Delta v = \frac{(0.975)(0.5182)(373)}{4000} + \frac{(0.993)(0.5182)(373)}{1000}; \quad \Rightarrow \Delta v = -0.145 \frac{m^3}{kg}$$

The entropy equation produces

$$\Delta s = \frac{q}{T_B} + S_{gen}^{0};$$

$$q = (\Delta s)(T_B); \quad \Rightarrow q = (-0.76)(298); \quad \Rightarrow q = -226.5 \frac{kJ}{kg}$$

The energy equation yields

$$\Delta u + \Delta k e^{0} + \Delta p e^{0} = q - \left(w_{B} + w_{O}^{0}\right)$$

$$w_{B} = q - \Delta u; \quad \Rightarrow w_{B} = -226.5 + 15.2; \quad \Rightarrow w_{B} = -211.3 \frac{kJ}{kg}$$

# Real Gas Model (Nelson-Obert)

From the Nelson-Obert compressibility chart, we obtain  $Z_1^{\text{N-O}} = 0.996$ ,  $Z_2^{\text{N-O}} = 0.98$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{G^0} - RT_{cr}(Z_{h,2} - Z_{h,1}); \quad \Rightarrow \Delta h = -0.5182(191.1)(0.1694 - 0.0426) \\ &\Rightarrow \Delta h = -12.56 \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= (h_2 - h_1) - R(Z_2^{\text{N-O}}T_2 - Z_1^{\text{N-O}}T_1); \quad \Rightarrow \Delta u = -12.56 - 0.5182 \big[ (0.98)(373.15) - (0.996)(373.15) \big] \\ &\Rightarrow \Delta u = -9.46 \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}); \quad \Rightarrow \Delta s = -0.719 - 0.5182(0.0923 - 0.0228) \\ &\Rightarrow \Delta s = -0.76 \frac{\text{kJ}}{\text{kg.K}} \\ \Delta v &= v_2 - v_1; \quad \Rightarrow \Delta v = \frac{Z_2^{\text{N-O}}RT_2}{p_2} - \frac{Z_1^{\text{N-O}}RT_1}{p_1}; \\ &\Rightarrow \Delta v = \frac{(0.98)(0.5182)(373)}{4000} - \frac{(0.996)(0.5182)(373)}{1000}; \quad \Rightarrow \Delta v = -0.145 \frac{\text{m}^3}{\text{kg}} \end{split}$$

The entropy equation produces

$$\Delta s = \frac{q}{T_B} + S_{gen}^{0};$$

$$q = (\Delta s)(T_B); \quad \Rightarrow q = (-0.76)(298); \quad \Rightarrow q = -226.5 \frac{\text{kJ}}{\text{kg}}$$

The energy equation yields

$$\Delta u + \Delta k e^{0} + \Delta p e^{0} = q - \left( w_{B} + w_{O}^{0} \right)$$

$$\Rightarrow w_{B} = q - \Delta u; \quad \Rightarrow w_{B} = -226.5 + 9.46; \quad \Rightarrow w_{B} = -217 \frac{kJ}{kg}$$

					RG N		RG Model	
					(Manual)		(TE	ST)
	PG	PG	IG Model	IG	Lee-	Nelson-	Lee-	Nelson-
	Model	Model	(Manual)	Model	Kesler	Obert	Kesler	Obert
	(Manual)	(TEST)		(TEST)				
$\Delta h$								
(kJ)	0	0	0	0	-18.7	-12.56	-18.7	-12.54
$\left({\mathrm{kg}}\right)$								
$\Delta u$								
(kJ)	0	0	0	0	-15.2	-9.46	-15.24	-9.36
$\left(\frac{\mathrm{kJ}}{\mathrm{kg}}\right)$								
$\Delta s$								
( kJ )	-0.718	-0.7185	-0.719	-0.7185	-0.76	-0.75	-0.75	-0.76
$\left(\overline{\text{kg.K}}\right)$								
$\Delta v$				3	is hind for Med	\		
$\left( m^{3}\right)$	-0.145	-0.145	-0.145	-0.145	-0.145	-0.145	-0.145	-0.145
$\left({\mathrm{kg}}\right)$				es controls	is Noneitied			
			Inited	ise derilling	not P			

What-if Scenario: Use TEST solution to verify these answers and pursue what-if scenarios. The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

**11-2-4** [OTT] A cylindrical tank contains 4.0 kg of carbon monoxide at -45°C has an inner diameter of 0.2 m and a length of 1 m. Using the RG model (L-K charts), determine (a) the pressure exerted by the gas. (b) What-if Scenario: What would the pressure exerted by the gas be if the IG model were used instead?

## **SOLUTION:**

From Table, obtain the necessary material properties of CO:  $\overline{M} = 28 \text{ kg/kmol}$ ,  $p_{cr} = 3.5 \text{ MPa}$ ,  $T_{cr} = 133 \text{ K}$ . Calculate R = 8.314/28 = 0.297 kJ/kg·K.

State-1:

Given:  $m_1 = 4 \text{ kg}$ ;  $T_1 = 228 \text{ K}$ ;

$$V_{1} = \left(\pi \frac{d^{2}}{4}\right) l; \quad \Rightarrow V_{1} = \left(\pi \frac{\left(0.2\text{m}\right)^{2}}{4}\right) 1.0 \text{ m}; \quad \Rightarrow V_{1} = 0.0314 \text{ m}^{3}$$

$$n^{\text{IG}} = {}^{m_{1}RT_{1}} : \quad \Rightarrow n^{\text{IG}} = {}^{4\text{x}0.297\text{x}228} : \quad \Rightarrow n^{\text{IG}} = 8.63 \text{ MPa}$$

$$p_1^{\text{IG}} = \frac{m_1 R T_1}{V_1}; \quad \Rightarrow p_1^{\text{IG}} = \frac{4 \times 0.297 \times 228}{0.0314}; \quad \Rightarrow p_1^{\text{IG}} = 8.63 \text{ MPa}$$

The reduced pressure and temperature for this state are

$$p_{r1} = 8.63/3.5 = 2.46$$
 and  $T_{r1} = 228/133 = 1.7$ 

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{L-K} = 0.9$ 

Hence,

$$p_1^{\text{RG,L-K}} = Z_1^{\text{L-K}} p_1; \implies p_1^{\text{RG,L-K}} = 0.9 \times 8.63; \implies p_1^{\text{RG,L-K}} = 7.76 \text{ MPa}$$

**What-if Scenario:** Use TEST solution to verify these answers and pursue what-if scenarios. The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

11-2-5 [OTF] Methane is adiabatically compressed by a piston-cylinder device from 1 MPa,  $100^{\circ}$ C to 4 MPa. Calculate (a) the work done per unit mass ( $w_B$ ). Assume the adiabatic efficiency to be 90%. Use the real gas model. (b) What-if Scenario: What would the work done per unit mass be if the compressed gas were ethane instead?

## **SOLUTION:**

# Real Gas Model (Lee Kesler):

From Table E-1 or the RG closed-process TESTCalc, obtain the necessary material properties of  $CH_{4}$ :

$$p_{cr} = 4.64 \text{ MPa}, T_{cr} = 191.1 \text{ K}$$

State-1 (given 
$$p_1 = 1 \text{ MPa}$$
;  $T_1 = 373 \text{ K}$ ;  $m = 1 \text{ kg}$ )

$$p_{r1} = \frac{p_1}{p_{cr}} = 0.216, \ T_{r1} = \frac{T_1}{T_{cr}} = 1.96$$

$$\Rightarrow Z_1^{\text{L-K}} = 0.996, Z_{h1}^{\text{L-K}} = -2.52, Z_{s1}^{\text{L-K}} = -15.77;$$

$$\Rightarrow h_1 = 171.6 \frac{\text{kJ}}{\text{kg}}, s_1 = 10.96 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, u_1 = -21.6 \frac{\text{kJ}}{\text{kg}}$$

State-2 (given 
$$p_2 = 4 \text{ MPa}$$
;  $s_2 = s_1$ ;  $m = 1 \text{ kg}$ )

$$p_{r2} = \frac{p_1}{p_{cr}} = 0.87, \ s_2 = 10.96 \ \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\Rightarrow Z_{s2}^{L-K} = -16.15$$
 (through iteration or using TEST);

$$\Rightarrow T_{r2} = 2.6, Z_{h2}^{L-K} = -2.52, h_2 = 480.7 \frac{kJ}{kg}$$

State-3 (given 
$$p_3 = 4$$
 MPa;  $h_3 = h_1 + (h_2 - h_1)/0.9 = 515 \frac{kJ}{kg}$ ;  $m = 1 \text{ kg}$ )

$$p_{r3} = 0.87, \ h_3 = 515 \ \frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow Z_{h3}^{L-K} = -7.98$$
 (through iteration or using TEST);

$$\Rightarrow T_{r3} = 2.66, T_3 = 505.5 \frac{\text{kJ}}{\text{kg}}, u_1 = 252.2 \frac{\text{kJ}}{\text{kg}}$$

The energy equation for the closed-process produces:

$$\Delta U = \cancel{Q}^{0} - W_{\text{ext}}$$

$$\Rightarrow w_{\text{ext}} = \frac{W_{\text{ext}}}{m} = -(u_{2} - u_{1}) = 273.8 \frac{\text{kJ}}{\text{kg}}$$

# **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.



11-2-6 [OTM] Propane is compressed isothermally in an internally reversible manner by a piston-cylinder device from 1.5 MPa,  $90^{\circ}$ C to 4 MPa. Using the Nelson-Obert charts, determine (include sign) (a) the work done ( $w_{\rm B}$ ) and (b) the heat transfer per unit mass of propane (q). (c) What-if Scenario: What would be the work done if the PC model were used?

## **SOLUTION:**

Given:  $p_1 = 1.5 \text{ MPa}$ ;  $T_1 = 90 \, ^{\circ}\text{C}$ ; and  $p_2 = 4 \, \text{MPa}$ ;  $T_2 = T_1 \, ^{\circ}$ 

# Real Gas Model (Nelson-Obert)

From Table E-1, obtain the necessary material properties of  $C_3H_8$ :  $p_{cr} = 4.26$  MPa,  $T_{cr} = 370$  K

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}}; \quad \Rightarrow \quad p_{r1} = \frac{1.5}{4.26}; \quad \Rightarrow p_{r1} = 0.352,$$

$$p_{r2} = \frac{p_2}{p_{cr}}; \quad \Rightarrow p_{r2} = \frac{4}{4.26}; \quad \Rightarrow p_{r2} = 0.94$$

$$T_{r1} = \frac{T_1}{T_{cr}}; \quad \Rightarrow T_{r1} = \frac{363.15}{370}; \quad \Rightarrow T_{r1} = 0.981,$$

$$T_{r2} = \frac{T_2}{T_{cr}}; \quad \Rightarrow T_{r2} = \frac{363.15}{370}; \quad \Rightarrow T_{r2} = 0.981$$

From the Nelson-Obert compressibility chart, we obtain

$$\Delta h = (h_2 - h_1)^{IG^{-0}} - RT_{cr}(Z_{h,2} - Z_{h,1}); \Rightarrow \Delta h = -0.1885(370)(1.995 - 0.503)$$

$$\Rightarrow \Delta h = -104.1 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = (h_2 - h_1) - R(Z_2^{\text{N-O}}T_2 - Z_1^{\text{N-O}}T_1); \Rightarrow \Delta u = -104.1 - 0.1885 [(0.385)(363.15) - (0.856)(363.15)]$$

$$\Rightarrow \Delta u = -71.85 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}); \Rightarrow \Delta s = -0.185 - 0.1885(2.84 - 0.39)$$

$$\Rightarrow \Delta s = -0.646 \frac{\text{kJ}}{\text{kg}.\text{K}}$$

$$\Delta v = v_2 - v_1; \Rightarrow \Delta v = \frac{Z_2^{\text{N-O}}RT_2}{n_2} - \frac{Z_1^{\text{N-O}}RT_1}{n_2};$$

The entropy equation produces

$$\Delta s = \frac{q}{T_B} + S_{gen}^{0};$$

$$q = (\Delta s)(T_B); \quad \Rightarrow q = (-0.646)(273+90); \quad \Rightarrow q = -234.5 \frac{kJ}{kg}$$

The energy equation yields

$$\Delta u + \Delta k e^{0} + \Delta p e^{0} = q - \left(w_{B} + w_{O}^{0}\right)$$

$$\Rightarrow w_{B} = q - \Delta u; \quad \Rightarrow w_{B} = -234.5 + 71.85; \quad \Rightarrow w_{B} = -162.65 \frac{kJ}{kg}$$

### **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Repeating the solution with the PC closed-process TESTcalc, the answers can be obtained from the analysis panel as -266 kJ and -90 kJ respectively. The LK model can be shown to produce results that are much closer to the PC model solution.

11-2-7 [OTJ] Methane is isothermally compressed in an internally reversible manner by a piston-cylinder device from 1 MPa,  $100^{\circ}$ C to 4 MPa. Calculate (a) the entropy generation ( $s_{gen}$ ) and (b) the irreversibility associated with the process if the ambient temperature is  $25^{\circ}$ C. Use the real gas model.

## **SOLUTION:**

From Table C-1, obtain the necessary material properties of  $CH_4$ :

$$c_p = 2.254 \frac{\text{kJ}}{\text{kg.K}}, c_v = 1.736 \frac{\text{kJ}}{\text{kg.K}}, R = 0.5182 \frac{\text{kJ}}{\text{kg.K}}$$

Given:  $p_1 = 1 \text{ MPa}$ ;  $T_1 = 100 \, ^{\circ}\text{C}$ 

Given:  $p_2 = 4$  MPa;  $T_2 = T_1$ 

# Ideal Gas Model:

The Ideal gas model utilizes the TEST for finding the properties at the given state.

$$\Delta h = h_2 - h_1; \implies \Delta h = 174.11 - 174.11; \implies \Delta h = 0 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1$$
;  $\Rightarrow \Delta u = -19.30 + 19.30$ ;  $\Rightarrow \Delta u = 0$   $\frac{kJ}{kg}$ 

$$\Delta s = s_2 - s_1;$$
  $\Rightarrow$   $\Delta s = 10.226 - 10.945;$   $\Rightarrow$   $\Delta s = -0.719 \frac{\text{kJ}}{\text{kg.K}}$ 

$$\Delta v = v_2 - v_1; \implies \Delta v = 0.04835 - 0.1934; \implies \Delta v = -0.145 \frac{\text{m}^3}{\text{kg}}$$

From Table E-1, obtain the necessary material properties of  $CH_4$ :

$$p_{cr} = 4.64 \text{ MPa}, T_{cr} = 191.1 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}}; \quad \Rightarrow \quad p_{r1} = \frac{1}{4.64}; \quad \Rightarrow \quad p_{r1} = 0.216,$$

$$p_{r2} = \frac{p_2}{p_{cr}}; \quad \Rightarrow \quad p_{r2} = \frac{4}{4.64}; \quad \Rightarrow \quad p_{r2} = 0.862$$

$$T_{r1} = \frac{T_1}{T_{cr}}; \quad \Rightarrow \quad T_{r1} = \frac{373.15}{191.1}; \quad \Rightarrow \quad T_{r1} = 1.953,$$

$$T_{r2} = \frac{T_2}{T_{cr}}; \quad \Rightarrow \quad T_{r2} = \frac{373.15}{191.1}; \quad \Rightarrow \quad T_{r2} = 1.953$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{\text{L-K}} = 0.993$ ,  $Z_2^{\text{L-K}} = 0.975$ 

$$\begin{split} \Delta h &= \underbrace{(h_2 - h_1)^{1G^{-0}}}_{l_1} - RT_{cr}(Z_{h,2} - Z_{h,1}); \quad \Rightarrow \Delta h = -0.5182(191.1)(0.249 - 0.0598) \\ \Rightarrow \Delta h = -18.7 \frac{\mathrm{kJ}}{\mathrm{kg}} \\ \Delta u &= (h_2 - h_1) - R(Z_2^{\mathrm{L-K}}T_2 - Z_1^{\mathrm{L-K}}T_1); \quad \Rightarrow = -18.7 - 0.5182 \big[ (0.975)(373.15) - (0.993)(373.15) \big] \\ \Rightarrow \Delta u = -15.2 \frac{\mathrm{kJ}}{\mathrm{kg}} \\ \Delta s &= (s_2 - s_1)^{1G} - R(Z_{s,2} - Z_{s,1}); \quad \Rightarrow = -0.719 - 0.5182(0.1056 - 0.0284) \\ \Rightarrow \Delta s &= -0.76 \frac{\mathrm{kJ}}{\mathrm{kg.K}} \\ \Delta v &= v_2 - v_1; \quad \Rightarrow \Delta v = \frac{Z_2^{\mathrm{L-K}}RT_2}{P_2} - \frac{Z_1^{\mathrm{L-K}}RT_1}{P_1}; \\ \Rightarrow \Delta v &= \frac{(0.975)(0.5182)(373)}{4000} - \frac{(0.993)(0.5182)(373)}{1000}; \quad \Rightarrow \Delta v = -0.145 \frac{\mathrm{m}^3}{\mathrm{kg}} \end{split}$$

The entropy equation for the internal system produces

$$\Delta s = \frac{q}{T_B} + S_{gen}^{0};$$

$$q = (\Delta s)(T_B); \implies q = (-0.76)(273+100); \implies q = -283.5 \frac{kJ}{kg}$$

The entropy equation for the system's universe produces

$$\Delta s = \frac{q}{T_B} + s_{\text{gen,univ}};$$

$$s_{\text{gen,univ}} = \Delta s - \frac{q}{T_B} = -0.76 - \frac{-283.5}{(273 + 25)}; \quad \Rightarrow s_{\text{gen,univ}} = 0.191 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

The irreversibility per unit mass can be calculated as

$$i = T_0 s_{\text{gen,univ}} = (298)(0.191) = 56.92 \frac{\text{kJ}}{\text{kg}}$$

# **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Note that the heat transfer in the analysis panel is entered as an expression '=T1\*(s2-s1)'.



**11-2-8** [OTW] A piston-cylinder device contains 2 kg of H<sub>2</sub> and 14 kg of O<sub>2</sub> at 150 K and 5000 kPa. Heat is then transferred until the mixture expands at constant pressure (why does the pressure remain constant?) until the temperature rises to 200 K. Determine (a) the heat transfer (*Q*) by treating the mixture as perfect gas. (b) What-if Scenario: What would the conclusion be if the device contained 3 kg of H<sub>2</sub> and 12 kg of O<sub>2</sub> instead?

#### **SOLUTION:**

From Table C-1 or the PG system-state TESTcalc, obtain:

$$\overline{M}_{\rm O_2} = 32 \ \frac{\rm kg}{\rm kmol}; \ \overline{M}_{\rm H_2} = 2 \ \frac{\rm kg}{\rm kmol}; \ c_{\rm p,O_2} = 0.905 \ \frac{\rm kJ}{\rm kg \cdot K}; \ c_{\rm p,H_2} = 14.3 \ \frac{\rm kJ}{\rm kg \cdot K};$$

Given:  $m_{O_2} = 14 \text{ kg}$ ;  $m_{H_2} = 2 \text{ kg}$ ;  $T_1 = 150 \text{ K}$ ;  $T_2 = 200 \text{ K}$ ;

For the constant-pressure process, the energy equation simplifies to:

$$\Delta E = Q - W_{\text{ext}} = Q - W_{B}$$

$$\Rightarrow Q = \Delta U + W_{B} = \Delta H = [m\Delta h]_{O_{2}} + [m\Delta h]_{H_{2}}$$

$$\Rightarrow Q = [mc_{p}(T_{2} - T_{1})]_{O_{2}} + [mc_{p}(T_{2} - T_{1})]_{H_{2}}$$

$$\Rightarrow Q = 2063.05 \text{ kJ}$$

## **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use the PG/PG closed-process TESTcalc to verify the results and pursue the what-if scenario.

**11-2-9** [OYB] A piston-cylinder device contains 1 lbm of  $O_2$  and 9 lbm of  $N_2$  at  $300^{\circ}R$  and 900 psia. The gas mixture is now heated at constant pressure to  $400^{\circ}R$ . Determine (a) the heat transfer (Q) during the expansion process by treating the mixture as a perfect gas mixture. (b) What-if Scenario: What would the heat transfer be if the real gas mixture model (with Kay's rule) were used?

#### **SOLUTION:**

From Table C-1 or the PG system-state TESTcalc, obtain:

$$\overline{M}_{O_2} = 32 \frac{\text{kg}}{\text{kmol}}; \ \overline{M}_{N_2} = 28 \frac{\text{kg}}{\text{kmol}}; \ c_{p,O_2} = 0.905 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ c_{p,N_2} = 1.03 \frac{\text{kJ}}{\text{kg} \cdot \text{K}};$$

Given:  $m_{O_2} = 1 \text{ lbm} = 0.45 \text{ kg}$ ;  $m_{N_2} = 9 \text{ lbm} = 4.08 \text{ kg}$ ;  $T_1 = 150 \text{ K}$ ;  $T_2 = 200 \text{ K}$ ; For the constant-pressure process, the energy equation simplifies to:

$$\Delta E = Q - W_{\text{ext}} = Q - W_{B}$$

$$\Rightarrow Q = \Delta U + W_{B} = \Delta H = [m\Delta h]_{O_{2}} + [m\Delta h]_{N_{2}}$$

$$\Rightarrow Q = [mc_{p}(T_{2} - T_{1})]_{O_{2}} + [mc_{p}(T_{2} - T_{1})]_{N_{2}}$$

$$\Rightarrow Q = 256 \text{ kJ}$$

## **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use the PG/PG closed-process TESTcalc to verify the results and the RG/RG TESTcalc to pursue the what-if scenario. The real gas mixture model produces an answer of 378 kJ.

**11-2-10** [OYR] An insulated piston-cylinder device contains 0.1 kg of  $N_2$  and 0.2 kg of  $CO_2$  at 300 K and 100 kPa. The gas mixture is now compressed isentropically to a pressure of 1000 kPa. Determine (a) the final temperature ( $T_2$ ) by treating the mixture as a perfect gas. (b) What-if Scenario: What would the final temperature be if the ideal gas model were used? **SOLUTION:** 

From Table C-1 or the PG system-state TESTcalc, obtain:

$$\overline{M}_{\text{CO}_2} = 44 \frac{\text{kg}}{\text{kmol}}; \ \overline{M}_{\text{N}_2} = 28 \frac{\text{kg}}{\text{kmol}}; \ c_{p,\text{CO}_2} = 0.844 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \ c_{p,\text{N}_2} = 1.03 \frac{\text{kJ}}{\text{kg} \cdot \text{K}};$$

Given:  $m_{CO_2} = 0.2$  kg;  $m_{N_2} = 0.1$  kg;  $T_1 = 300$  K;  $p_1 = 100$  kPa;  $s_2 = s_1$ ; For the isentropic process, the energy equation simplifies to:

$$\Delta S = 0;$$

$$\Rightarrow \left[ m\Delta s \right]_{CO_2} + \left[ m\Delta s \right]_{N_2} = 0;$$

$$\Rightarrow \left[ mc_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right]_{CO_2} + \left[ mc_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right]_{N_2} = 0;$$

$$\Rightarrow T_2 = 531 \text{ kJ}$$

## **TEST Solution:**

The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use the PG/PG closed-process TESTcalc to verify the results and the IG/IG TESTcalc to pursue the what-if scenario. The ideal gas mixture model produces an answer of 511 K.

**11-2-11** [OYO] A rigid tank contains 3 m<sup>3</sup> of argon at -100°C and 1 MPa. Heat is transferred until the temperature rises to 0°C. Determine (a) the mass of argon, (b) the final pressure  $(p_2)$  and (c) heat transferred (Q). Use the real gas model. (d) What-if Scenario: What would the mass be if the tank contained 1 m<sup>3</sup> of argon?

## **SOLUTION:**

From Table C-1, obtain the necessary material properties of Ar:  $R=0.2081 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ 

Given: 
$$p_1 = 1 \text{ MPa}$$
;  $T_1 = -100 \text{ °C}$ ;  $\frac{V_1}{V_1} = 3 \text{ m}^3$ ,  $m = \frac{p_1 V_1}{RT_1} = 83.3 \text{ kg}$ 

Given: 
$$T_2 = 0$$
 °C;  $V_2 = V_1$ ;  $p_2 = \frac{mRT_2}{V_2} = 1.58$  MPa;

### Ideal Gas Model:

The Ideal gas model utilizes the TEST for finding the properties at the given state.

$$\Delta h = h_2 - h_1; \implies \Delta h = -13.00 + 65.03; \implies \Delta h = 52.03 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1; \implies \Delta u = -69.85 + 101.07; \implies \Delta u = 31.22 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = s_2 - s_1; \quad \Rightarrow \Delta s = 3.259 - 3.116; \quad \Rightarrow \Delta s = 0.14 \frac{\text{kJ}}{\text{kg.K}}$$

$$\Delta v = v_2 - v_1;$$
  $\Rightarrow$   $\Delta v = 0.03603 - 0.03603;$   $\Rightarrow$   $\Delta v = 0$   $\frac{\text{m}^3}{\text{kg}}$ 

The energy equation yields

$$\Delta U + \Delta K E^{0} + \Delta P E^{0} = Q - W_{ext}^{0}$$

$$\Rightarrow Q = \Delta U; \quad \Rightarrow Q = m_{1}(u_{2} - u_{1}); \quad \Rightarrow Q = 2600 \text{ kJ}$$

# Real Gas Model (Lee Kesler):

From Table E-1, obtain the necessary material properties of Ar:

$$p_{cr} = 4.86 \text{ MPa}, T_{cr} = 151 \text{ K}$$

Given: 
$$p_1 = 1 \text{ MPa}$$
;  $T_1 = -100 \text{ °C}$ ;  $\frac{V_1}{I} = 3 \text{ m}^3$ ,  $Z_1^{\text{L-K}} = 0.954$ ,  $m = \frac{p_1 V_1}{Z_1^{\text{L-K}} R T_1}$ ;  $\Rightarrow m = 87.3 \text{ kg}$ 

Given: 
$$T_2 = 0$$
 °C;  $\frac{1}{V_2} = \frac{1}{V_1}$ ;

Since  $p_2$  is used to find  $Z_2$ , we must iterate, first assuming Z is constant.

$$p_2 = \frac{Z_1^{\text{L-K}}RT_2}{v_1}; \quad \Rightarrow p_2 = \frac{\left(0.95\right)\left(0.2081\right)\left(273\right)}{\left(0.0342\right)}; \quad \Rightarrow p_2 = 1.578 \text{ MPa}$$
This yield  $p_{r2} = \frac{1.578}{4.84} = 0.326$ , and  $T_{r2} = \frac{273}{151} = 1.808$ ; which gives  $Z_2^{\text{L-K}} = 0.98$ 

And so 
$$p_2 = \frac{Z_2^{\text{L-K}}RT_2}{v_2}$$
;  $\Rightarrow p_2 = \frac{(0.98)(0.2081)(273)}{(0.0342)}$ ;  $\Rightarrow p_2 = 1.63 \text{ MPa}$  (iteration confirms this).

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}}; \quad \Rightarrow p_{r1} = \frac{1}{4.86}; \quad \Rightarrow p_{r1} = 0.206,$$

$$p_{r2} = \frac{p_2}{p_{cr}}; \quad \Rightarrow p_{r2} = \frac{1.63}{4.86}; \quad \Rightarrow p_{r2} = 0.335$$

$$T_{r1} = \frac{T_1}{T_{cr}}; \quad \Rightarrow T_{r1} = \frac{173}{151}; \quad \Rightarrow T_{r1} = 1.146,$$

$$T_{r2} = \frac{T_2}{T_{cr}}; \quad \Rightarrow T_{r2} = \frac{273}{151}; \quad \Rightarrow T_{r2} = 1.81$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{\text{L-K}} = 0.954$ ,  $Z_2^{\text{L-K}} = 0.985$ 

$$\Delta h = (h_2 - h_1)^{1G} - RT_{cr}(Z_{h,2} - Z_{h,1}); \quad \Rightarrow \Delta h = 52.03 - 0.2081(151)(0.113 - 0.165)$$

$$\Rightarrow \Delta h = 53.66 \frac{kJ}{kg}$$

$$\Delta u = (h_2 - h_1) - R(Z_2^{L-K}T_2 - Z_1^{L-K}T_1); \quad \Rightarrow \Delta u = 53.66 - 0.2081[(0.985)(273) - (0.954)(173)]$$

$$\Rightarrow \Delta u = 32.05 \frac{kJ}{kg}$$

$$\Delta s = (s_2 - s_1)^{1G} - R(Z_{s,2} - Z_{s,1}); \quad \Rightarrow \Delta s = 0.14 - 0.2081(0.0506 - 0.1005)$$

$$\Rightarrow \Delta s = 0.15 \frac{kJ}{kg.K}$$

$$\Delta v = v_2 - v_1; \quad \Rightarrow \Delta v = \frac{Z_2^{L-K}RT_2}{p_2} - \frac{Z_1^{L-K}RT_1}{p_1};$$

$$\Rightarrow \Delta v = \frac{(0.985)(0.2081)(273)}{1630} - \frac{(0.954)(0.2081)(173)}{1000}; \quad \Rightarrow \Delta v = 0 \frac{m^3}{kg}$$

The energy equation yields

$$\Delta U + \Delta K E^{0} + \Delta P E^{0} = Q - W_{ext}^{0}$$

$$\Rightarrow Q = \Delta U; \quad \Rightarrow Q = m_{1}(u_{2} - u_{1}); \quad \Rightarrow Q = 2797.9 \text{ kJ}$$

# Real Gas Model (Nelson-Obert)

Given: 
$$p_1 = 1 \text{ MPa}$$
;  $T_1 = -100 \text{ °C}$ ;  $V_1 = 3 \text{ m}^3$ ,  $Z_1^{\text{N-O}} = 0.952$ ,  $m = \frac{p_1 V_1}{Z_1^{\text{L-K}} R T_1}$ ;  $\Rightarrow m = 87.5 \text{ kg}$ 

Given: 
$$T_2 = 0$$
 °C;  $\frac{1}{12} = \frac{1}{12}$ ;

Since  $p_2$  is used to find  $Z_2^{\text{N-O}}$ , we must iterate, first assuming  $Z^{\text{N-O}}$  is constant.

$$p_2 = \frac{Z_1^{\text{N-O}}RT_2}{v_1}; \implies p_2 = \frac{(0.952)(0.2081)(273)}{(0.0343)}; \implies p_2 = 1.577 \text{ MPa}$$

This yield 
$$p_{r2} = \frac{1.577}{4.84} = 0.326$$
, and  $T_{r2} = \frac{273}{151} = 1.808$ ; which gives  $Z_2^{\text{N-O}} = 0.98$ 

And so 
$$p_2 = \frac{Z_2^{\text{N-O}}RT_2}{v_2}$$
;  $\Rightarrow p_2 = \frac{(0.98)(0.2081)(273)}{(0.0343)}$ ; (iteration confirms this).  
 $\Rightarrow p_2 = 1.62 \text{ MPa}$ 

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}}; \quad \Rightarrow \quad p_{r1} = \frac{1}{4.86}; \quad \Rightarrow \quad p_{r1} = 0.206,$$

$$p_{r2} = \frac{p_2}{p_{cr}}; \quad \Rightarrow \quad p_{r2} = \frac{1.62}{4.86}; \quad \Rightarrow \quad p_{r2} = 0.335$$

$$T_{r1} = \frac{T_1}{T_{cr}}; \quad \Rightarrow \quad T_{r1} = \frac{173}{151}; \quad \Rightarrow \quad T_{r1} = 1.146,$$

$$T_{r2} = \frac{T_2}{T_{cr}}; \quad \Rightarrow \quad T_{r2} = \frac{273}{151}; \quad \Rightarrow \quad T_{r2} = 1.81$$

From the Nelson-Obert compressibility chart, we obtain  $Z_1^{\text{N-O}} = 0.952$ ,  $Z_2^{\text{N-O}} = 0.986$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}); \quad \Rightarrow \Delta h = 52.03 - 0.2081(151)(0.1144 - 0.163) \\ \Delta h &\Rightarrow \Delta h = 53.6 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= (h_2 - h_1) - R(Z_2^{\text{L-K}}T_2 - Z_1^{\text{L-K}}T_1); \quad \Rightarrow \Delta u = 53.6 - 0.2081\big[(0.986)(273) - (0.952)(173)\big] \\ &\Rightarrow \Delta u = 31.9 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}); \quad \Rightarrow \Delta s = 0.14 - 0.2081(0.055 - 0.116) \\ &\Rightarrow \Delta s = 0.15 \ \frac{\text{kJ}}{\text{kg.K}} \\ \Delta v &= v_2 - v_1; \quad \Rightarrow \Delta v = \frac{Z_2^{\text{N-O}}RT_2}{p_2} - \frac{Z_1^{\text{N-O}}RT_1}{p_1}; \\ &\Rightarrow \Delta v = \frac{(0.86)(0.2081)(273)}{1620} - \frac{(0.952)(0.2081)(173)}{1000}; \quad \Rightarrow \Delta v = 0 \ \frac{\text{m}^3}{\text{kg}} \end{split}$$

The energy equation yields 
$$\Delta U + \Delta K E^{0} + \Delta P E^{0} = Q - W_{\text{ext}}^{0}$$
 
$$\Rightarrow Q = \Delta U; \quad \Rightarrow Q = m_{1}(u_{2} - u_{1}); \quad \Rightarrow Q = 2791.3 \text{ kJ}$$

					RG Model (Manual)		RG N (TE	Model
	PG Model (Manual	PG Model (TEST)	IG Model (Manual	IG Model (TEST)	Lee- Kesler	Nelson- Obert	Lee- Kesler	Nelson- Obert
$\frac{\Delta h}{\left(\frac{\text{kJ}}{\text{kg}}\right)}$	52.03	52.03	52.03	52.03	53.66	53.6	53.66	53.6
$\left(\frac{kJ}{kg}\right)$	31.22	31.22	31.22	31.22	32.05	31.9	32.01	31.8
$\frac{\Delta s}{\left(\frac{kJ}{kg.K}\right)}$	0.142	0.143	0.14	0.14	0.15	0.15	0.147	0.147
$\frac{\Delta v}{\left(\frac{m^3}{kg}\right)}$	0	0	0	Original	ESELLISTADE SERVICE THE SERVICE THE	0	0	0

What-if Scenario: The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use it to pursue the what-if scenario.

**11-2-12** [OYS] A 0.5 m<sup>3</sup> well-insulated rigid tank contains oxygen at 200 K and 9 MPa. A paddle wheel placed in the tank is turned on, and the temperature of the oxygen rises to 240 K. Determine (a) the final pressure in the tank and (b) the paddle-wheel work (*W*) done during the process. Use the RG Model (L-K Charts).

# **SOLUTION:**

From Table E-1, obtain the necessary material properties of  $O_2$ :  $\overline{M}$  =32 kg/kmol,  $p_{cr}$  =5.08 MPa,  $T_{cr}$  =154.8 K. Calculate R =8.314/32=0.2598 kJ/kg·K.

State-1 (Given: 
$$p_1 = 9 \text{ MPa}, \frac{V_1}{I} = 0.5 \text{ m}^3, T_1 = 200 \text{ K}$$
)

$$p_{r1} = \frac{9}{5.08} = 1.78, \ T_{r1} = \frac{200}{154.8} = 1.29$$

$$\Rightarrow Z_1^{\text{L-K}} = 0.713, Z_{h1}^{\text{L-K}} = 1.40,$$

$$\Rightarrow v_1 = \frac{Z_1 R T_1}{p_1} = 0.0041 \frac{\text{m}^3}{\text{kg}}; m_1 = \frac{V_1}{v_1} = 121.5 \text{ kg};$$

State-2 (given 
$$v_2 = v_2$$
;  $T_2 = 240 \text{ K}$ )

$$T_{r2} = \frac{T_2}{T_{crit}} = 1.55$$

$$\Rightarrow p_{r2} = 2.5; \Rightarrow p_2 = p_{r2} p_{cr} = 12.6 \text{ MPa};$$

$$\Rightarrow Z_2 = 0.839, Z_{h2} = 1.117, Z_{s2} = 0.555$$

$$\Delta u = \Delta h - \Delta (pv) = \Delta h - \Delta (ZRT) = \Delta h^{IG} + RT_c - \Delta (pv) = \Delta h - \Delta (pv);$$

$$\Rightarrow \Delta u = \Delta h^{1G} - RT_{cr}(Z_{h,2} - Z_{h,1}) - R(Z_2T_2 - Z_1T_1) = 29.7 \frac{kJ}{kg}$$

The energy equation yields

$$\Delta U + \Delta K E^{0} + \Delta P E^{0} = \cancel{Q}^{0} - W_{\text{ext}}$$

$$\Rightarrow W_{\text{ext}} = -\Delta U = -m_1 \Delta u = -3611 \text{ kJ}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

**11-2-13** [OYA] A closed, rigid, insulated vessel having a volume of 0.15 m<sup>3</sup> contains oxygen initially at 10 MPa and 280 K. The oxygen is stirred by a paddle wheel until pressure increases to 15 MPa. Stirring ceases and the gas attains a final equilibrium state. Using the RG model (N-O charts), determine (a) the final temperature ( $T_2$ ), (b) work done (W) during the process and (c) the amount of availability destroyed in the process. Let  $T_0 = 280$  K.

### **SOLUTION:**

From Table E-1, obtain the necessary material properties of  $O_2$ :  $\bar{M}$  =32 kg/kmol,  $p_{cr}$  =5.08 MPa,  $T_{cr}$  =154.8 K. Calculate R =8.314/32=0.2598 kJ/kg·K.

State-1 (Given: 
$$p_1 = 10 \text{ MPa}$$
,  $T_1 = 280 \text{ K}$ ,  $\frac{V_1}{V_1} = 0.15 \text{ m}^3$ )

$$p_{r1} = 1.984, T_{r1} = 1.81$$

$$\Rightarrow Z_1^{\text{N-O}} = 0.922, Z_{b1}^{\text{N-O}} = 0.592, Z_{c1}^{\text{N-O}} = 0.26$$

$$\Rightarrow v_1 = \frac{Z_1 R T_1}{p_1} = 0.0067 \frac{\text{m}^3}{\text{kg}}; m_1 = \frac{V_1}{v_1} = 23.37 \text{ kg};$$

State-2 (given 
$$v_2 = v_2$$
;  $p_2 = 15 \text{ MPa}$ )

$$p_{r2} = \frac{p_2}{p_{cr}} = 2.98$$

$$\Rightarrow T_{r2} = 2.48; \Rightarrow T_2 = T_{r2} T_{cr} = 384 \text{ K};$$

$$\Rightarrow Z_2^{\text{N-O}} = 1.008, Z_{h2}^{\text{N-O}} = 0.459, Z_{s2}^{\text{N-O}} = 0.178$$

$$\Delta u = \Delta h - \Delta (pv) = \Delta h - \Delta (ZRT) = \Delta h^{IG} + RT_c - \Delta (pv) = \Delta h - \Delta (pv);$$

$$\Rightarrow \Delta u = \Delta h^{\text{IG}} - RT_{cr}(Z_{h,2} - Z_{h,1}) - R(Z_2T_2 - Z_1T_1) = 68.1 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = \Delta s^{IG} - R(Z_{s,2} - Z_{s,1}) = 0.21 \frac{kJ}{kg \cdot K}$$

The energy equation yields

$$\Delta U + \Delta K E^{0} + \Delta P E^{0} = Q^{0} - W_{\text{ext}}$$

$$\Rightarrow W_{\text{ext}} = -\Delta U = -m_1 \Delta u = -1523 \text{ kJ}$$

The entropy equation yields

$$\Delta S = \frac{Q^{\prime}}{I_B}^0 + S_{\text{gen}}; \implies S_{\text{gen}} = \Delta S = m_1 \Delta s = 4.63 \frac{\text{kJ}}{\text{K}}$$

Therefore, the loss of availability, which is same as the exergy destroyed during the process is:

$$I = T_0 S_{\text{gen}} = 1295.6 \text{ kJ}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.



**11-2-14** [OYH] Steam is throttled from 10 MPa,  $400^{\circ}$ C to 3 MPa. If the ambient temperature is 25°C, determine (a) the change in temperature ( $\Delta T$ ) and (b) the irreversibility for a flow rate of 1 kg/s. Use the real gas model. (c) What-if Scenario: What would the answer in (b) be if steam were throttled from 10 MPa,  $400^{\circ}$ C to 5 MPa?

# **SOLUTION:**

From Table, obtain the necessary material properties of  $H_2O$ :  $\overline{M} = 18 \text{ kg/kmol}$ ,  $p_{cr} = 22.1 \text{ MPa}$ ,  $T_{cr} = 647.3 \text{ K}$ . Calculate  $R = 0.462 \text{ kJ/kg} \cdot \text{K}$ .

State-1 (given: 
$$p_1 = 10 \text{ MPa}$$
;  $T_1 = 673 \text{ K}$ ;  $\dot{m}_1 = 1 \frac{\text{kg}}{\text{s}}$ )

$$p_{r1} = 0.452, T_{r1} = 1.04$$

$$\Rightarrow Z_1 = 0.846, Z_{h1} = 0.553, Z_{s1} = 0.405$$

$$\Rightarrow v_1 = \frac{Z_1 R T_1}{p_1} = 0.0263 \frac{\text{m}^3}{\text{kg}};$$

State-2 (given: 
$$p_2 = 3$$
 MPa;  $h_2 = h_1$ ;  $\dot{m}_2 = \dot{m}_1$ )

$$p_{r2} = \frac{p_2}{p_{cr}} = 0.136$$

$$\Delta h = 0$$
:

$$\Rightarrow 0 = (h_2 - h_1)^{1G} - RT_{cr}(Z_{h2} - Z_{h1});$$

$$\Rightarrow (h_2 - h_1)^{IG} = RT_{cr}(Z_{h2} - Z_{h1});$$

$$\Rightarrow$$
  $T_2 = 620 \text{ K}$ ; (iterating with guessed values of  $T_2$ )

$$\Rightarrow \Delta T = T_2 - T_1 = -53 \text{ K}$$

$$\Rightarrow T_{r2} = 0.958 \text{ K}; Z_{s2} = 0.169$$

$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s2} - Z_{s1});$$

$$\Rightarrow \Delta s = 0.4975 \frac{kJ}{kg \cdot K}$$

The energy equation and entropy equations yields

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1) + \frac{\dot{\cancel{D}}^0}{T_R} = \dot{m}\Delta s = 0.4975 \frac{kW}{K}$$

$$\dot{I} = T_0 \dot{S}_{gen}; \implies \dot{I} = (298)(0.4975) = 148 \text{ kW}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use to perform the what-if study.



11-2-15 [OYN] Oxygen is throttled from 10 MPa, 400 K to 2 MPa. Using the RG model (L-K charts), determine (a) the change in temperature ( $\Delta T$ ). (b) What-if Scenario: What would the change in temperature be if steam were throttled from 15 MPa?

### **SOLUTION:**

From Table, obtain the necessary material properties of O<sub>2</sub>:  $\overline{M} = 32 \text{ kg/kmol}$ ,  $p_{cr} = 5.04 \text{ MPa}$ ,  $T_{cr} = 154.6 \text{ K}$ . Calculate  $R = 0.26 \text{ kJ/kg} \cdot \text{K}$ .

State-1 (given: 
$$p_1 = 10 \text{ MPa}$$
;  $T_1 = 400 \text{ K}$ )  
 $p_{r1} = 1.984$ ,  $T_{r1} = 2.587$   
 $\Rightarrow Z_{h1} = 0.303$   
State-2 (given:  $p_2 = 2 \text{ MPa}$ ;  $h_2 = h_1$ )  
 $p_{r2} = \frac{p_2}{r_1} = 0.397$ 

$$p_{r2} = \frac{p_2}{p_{cr}} = 0.397$$

$$\Delta h = 0;$$

$$\Rightarrow 0 = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h2} - Z_{h1});$$

$$\Rightarrow (h_2 - h_1)^{IG} = RT_{cr}(Z_{h2} - Z_{h1});$$

$$\Rightarrow T_2 = 391 \text{ K}$$
; (iterating with guessed values of  $T_2$ )

$$\Rightarrow \Delta T = T_2 - T_1 = -9.0 \text{ K}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use it to perform the what-if study.

**11-2-16** [OYE] Nitrogen gas enters a turbine at 7 MPa, 500 K, 100 m/s and leaves at 1 MPa, 300 K, 150 m/s at a flow rate (m) of 2 kg/s. Heat is being lost to the surroundings at 25°C at a rate of 100 kW. Determine (a) the power output  $(W_{\rm ext})$  and (b) irreversibility. Use the real gas model. (c) What-if Scenario: What would the power output be if the mass flow rate were 1 kg/s?

# **SOLUTION:**

From Table C-1 and E-1, obtain the necessary material properties of  $N_2$ :  $\overline{M} = 28$  kg/kmol,  $p_{cr} = 3.39$  MPa,  $T_{cr} = 126.2$  K. Calculate R = 0.2968 kJ/kg·K.

Given: 
$$p_1 = 7$$
 MPa;  $T_1 = 500$  K;  $\dot{m}_1 = 2$   $\frac{\text{kg}}{\text{s}}$ ;  $V_1 = 100$   $\frac{\text{m}}{\text{s}}$ 

Given: 
$$p_2 = 1$$
 MPa;  $T_2 = 300$  K;  $\dot{m}_2 = \dot{m}_1$ ;  $V_2 = 150$   $\frac{\text{m}}{\text{s}}$ 

# Real Gas Model (Lee Kesler):

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}}; \quad \Rightarrow \quad p_{r1} = \frac{7}{3.39}; \quad \Rightarrow \quad p_{r1} = 2.065,$$

$$p_{r2} = \frac{p_2}{p_{cr}}; \quad \Rightarrow \quad p_{r2} = \frac{1}{3.39}; \quad \Rightarrow \quad p_{r2} = 0.295$$

$$T_{r1} = \frac{T_1}{T_{cr}}; \quad \Rightarrow \quad T_{r1} = \frac{500}{126.2}; \quad \Rightarrow \quad T_{r1} = 3.96,$$

$$T_{r2} = \frac{T_2}{T}; \quad \Rightarrow \quad T_{r2} = \frac{300}{126.2}; \quad \Rightarrow \quad T_{r2} = 2.38$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{L-K} = 1.026$ ,  $Z_2^{L-K} = 0.997$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}); \\ \Rightarrow \Delta h &= -209.74 - 0.2968(126.2)(0.0589 - 0.0773); \quad \Rightarrow \Delta h = -209.05 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}); \\ \Rightarrow \Delta s &= 0.0429 - 0.2968(0.03925 - 0.04543); \quad \Rightarrow \Delta s = 0.0447 \ \frac{\text{kJ}}{\text{kg.K}} \end{split}$$

The energy equation and entropy equations yield:

$$\frac{d\vec{E}'^{0}}{dt} = \dot{m}(j_{1} - j_{2}) + \dot{Q} - \dot{W}_{\text{ext}} = \dot{m}(h_{1} + \text{ke}_{1} - h_{2} - \text{ke}_{2}) + \dot{Q} - \dot{W}_{\text{ext}}$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{Q} - \dot{m}\left(h_{2} - h_{1} + \frac{V_{2}^{2} - V_{1}^{2}}{2000}\right)$$

$$\Rightarrow \dot{W}_{\text{ext}} = -100 - 2\left(-209.05 + \frac{150^{2} - 100^{2}}{2000}\right) = 305.6 \text{ kW}$$

$$\frac{dS^{0}}{dt} = \dot{m}(s_{1} - s_{2}) + \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}(s_{2} - s_{1}) + \frac{\dot{Q}}{T_{B}} = 2(0.0447) + \frac{100}{298} = 0.425 \frac{\text{kW}}{\text{K}}$$

$$\dot{I} = T_0 \dot{S}_{\text{gen,univ}} = (298)(0.425) = 127 \text{ kW}$$

What-if Scenario: The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use it to perform the what-if study.

11-2-17 [OYI] Nitrogen gas enters a turbine operating at steady state at 10 MPa,  $26^{\circ}$ C with a mass flow rate (m) of 1 kg/s and exits at 4 MPa,  $-28^{\circ}$ C. Using the RG model (N-O charts) and ignoring the heat transfer with the surrounding, determine (a) the work developed ( $W_{\text{ext}}$ ). (b) What-if Scenario: What would the work developed be if the mass flow rate were 0.5 kg/s?

### **SOLUTION:**

From Table C-1, obtain the necessary material properties of  $N_2$ :

$$c_p = 1.039 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, \ c_v = 0.743 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, \ R = 0.2968 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Given: 
$$p_1 = 10 \text{ MPa}$$
;  $T_1 = 26 \, ^{\circ}\text{C}$ ;  $\dot{m}_1 = 1 \, \frac{\text{kg}}{\text{s}}$ 

Given: 
$$p_2 = 4$$
 MPa;  $T_2 = -28$  °C;  $\dot{m}_2 = \dot{m}_1$ 

## Perfect Gas Model:

$$\begin{split} \Delta h &= h_2 - h_1 = c_p \ (T_2 - T_1) = 1.039(245 - 299) = -56.1 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= u_2 - u_1 = c_v (T_2 - T_1) = 0.743(245 - 299) = -40.1 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= s_2 - s_1 = c_p \ \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 1.039 \ln \left( \frac{245}{299} \right) - 0.2968 \ln \left( \frac{4}{10} \right) = 0.065 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta v &= v_2 - v_1 = \frac{RT_2}{p_2} - \frac{RT_1}{p_1} = \frac{(0.2968)(245)}{4000} - \frac{(0.2968)(299)}{10000} = 0.0093 \ \frac{\text{m}^3}{\text{kg}} \end{split}$$

The energy equation yields

$$\frac{d\vec{E}'^{0}}{dt} = \dot{m}(j_1 - j_2) + \not Q^{0} - \dot{W}_{\text{ext}} = \dot{m}(h_1 - h_2) - \dot{W}_{\text{ext}}$$

$$\Rightarrow \dot{W}_{\text{ext}} = -\dot{m}(h_2 - h_1) = 56 \text{ kJ}$$

### Ideal Gas Model:

Using the manual approach (Chapter 3) or the IG system-state TESTcalc, obtain:

$$\Delta h = h_2 - h_1; \quad \Rightarrow \Delta h = -55.03 - 0.646; \quad \Rightarrow \Delta h = -55.7 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1; \quad \Rightarrow \Delta u = -127.82 + 88.17; \quad \Rightarrow \Delta u = 39.6 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = s_2 - s_1; \quad \Rightarrow \Delta s = 5.548 - 5.481; \quad \Rightarrow \Delta s = 0.067 \frac{\text{kJ}}{\text{kg}.K}$$

$$\Delta v = v_2 - v_1; \quad \Rightarrow \Delta v = 0.0182 - 0.0088; \quad \Rightarrow \Delta v = 0.0094 \frac{\text{m}^3}{\text{kg}}$$

The energy equation yields

$$\dot{W}_{\rm ext} = -\dot{m}(h_2 - h_1) = 55.7 \text{ kJ}$$

# Real Gas Model (Lee Kesler):

From Table, obtain the necessary material properties of  $N_2$ :

$$p_{cr} = 3.39 \text{ MPa}, T_{cr} = 126.2 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = \frac{10}{3.39} = 2.95, \ p_{r2} = \frac{p_2}{p_{cr}} = \frac{4}{3.39} = 1.179$$

$$T_{r1} = \frac{T_1}{T_{cr}} = \frac{299}{126.2} = 2.369, \ T_{r2} = \frac{T_2}{T_{cr}} = \frac{245}{126.2} = 1.94$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{L-K} = 0.996$ ,  $Z_2^{L-K} = 0.966$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}); \\ \Rightarrow \Delta h = -55.7 - 0.2968(126.2)(0.3479 - 0.539) = -48.5 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= (h_2 - h_1) - R(Z_2^{\text{L-K}}T_2 - Z_1^{\text{L-K}}T_1); \\ \Rightarrow \Delta u = -48.5 - 0.2968 \big[ (0.966)(245) - (0.996)(299) \big] = -30.4 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}); \\ \Rightarrow \Delta s = 0.067 - 0.2968(0.148 - 0.231) = 0.09 \ \frac{\text{kJ}}{\text{kg.K}} \\ \Delta v &= v_2 - v_1 = \frac{Z_2^{\text{L-K}}RT_2}{p_2} - \frac{Z_1^{\text{L-K}}RT_1}{p_1}; \\ \Rightarrow \Delta v = \frac{(0.966)(0.2968)(245)}{4000} - \frac{(0.996)(0.2968)(299)}{10000} = 0.0087 \ \frac{\text{m}^3}{\text{kg}} \end{split}$$
 The energy equation yields

# Real Gas Model (Nelson-Obert)

 $\dot{W}_{\rm avt} = -\dot{m}(h_2 - h_1) = 48.5 \text{ kJ}$ 

From the Nelson-Obert compressibility chart, we obtain  $Z_1^{\text{N-O}} = 0.994$ ,  $Z_2^{\text{N-O}} = 0.964$ 

$$\Delta h = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1});$$

$$\Rightarrow \Delta h = -55.7 - 0.2968(126.2)(0.314 - 0.505) = -48.5 \frac{kJ}{kg}$$

$$\Delta u = (h_2 - h_1) - R(Z_2^{L-K}T_2 - Z_1^{L-K}T_1);$$

$$\Rightarrow \Delta u = -48.5 - 0.2968[(0.964)(245) - (0.994)(299)] = -30.4 \frac{kJ}{kg}$$

$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1});$$

$$\Rightarrow \Delta s = 0.067 - 0.2968(0.139 - 0.202) = 0.085 \frac{kJ}{kg \cdot K}$$

$$\Delta v = v_2 - v_1 = \frac{Z_2^{N-O}RT_2}{p_2} - \frac{Z_1^{N-O}RT_1}{p_1};$$

$$\Rightarrow \Delta v = \frac{(0.964)(0.2968)(245)}{4000} - \frac{(0.994)(0.2968)(299)}{10000} = 0.0087 \frac{m^3}{kg}$$

The energy equation yields

$$=-\dot{m}(h_2-h_1)=48.5 \text{ kJ}$$

					RG Model (Manual)		RG Model (TEST)	
	PG	PG	IG	IG	Lee-	Nelson-	Lee-	Nelson-
	Model	Model	Model	Model	Kesler	Obert	Kesler	Obert
	(Manual	(TEST)	(Manual	(TEST)				
	)		)					
$\Delta h$								
(kJ)	-56.1	-55.6	-55.7	-55.7	-48.5	-48.5	-49.1	-49.1
$\left(\frac{\overline{kg}}{g}\right)$								
$\Delta u$								
(kJ)	-40.1	-39.7	-39.6	-39.6	-30.4	-30.4	-31.0	-30.96
$\left(\frac{\mathrm{kJ}}{\mathrm{kg}}\right)$					- 00 0 18 <sup>0</sup>	`		
$\Delta s$					0.09			
( kJ )	0.065	0.067	0.067	0.067	0.09	0.085	0.089	0.083
$\left(\overline{\text{kg.K}}\right)$				States tructing;	He John.			
Δν			Jnite	Siendent Inding	)			
$\left(\mathbf{m}^{3}\right)$	0.0093	0.0094	0.0094	0.0094	0.0087	0.0087	0.0087	0.0087
$\left( \overline{\mathrm{kg}} \right)$			15 16 6 50 16 56 5 17	15 Me tille				
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**What-if Scenario:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use it to perform the what-if study.

11-2-18 [OYL] Methane at 9.3 MPa, 300 K enters the turbine operating at steady state at a mass flow rate (m) of 1 kg/s, expands adiabatically through a 6:1 pressure ratio, and exits at 225 K. KE and PE effects are negligible. Using the RG model (L-K charts), determine (a) the power developed ( $W_{\text{ext}}$ ) and (b) the entropy produced ( $S_{\text{gen}}$ ). (c) What-if Scenario: What would the power developed be if PG model were used?

#### **SOLUTION:**

From Table, obtain the necessary material properties of  $CH_4$ :

$$c_p = 2.2537 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, c_v = 1.7354 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, R = 0.5182 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Given: 
$$p_1 = 9.3 \text{ MPa}$$
;  $T_1 = 300 \text{ K}$ ;  $\dot{m}_1 = 1 \frac{\text{kg}}{\text{s}}$ 

Given: 
$$p_2 = \frac{p_1}{6} = 1.55 \text{ MPa}$$
;  $T_2 = 225 \text{ K}$ ;  $\dot{m}_2 = \dot{m}_1$ 

# Perfect Gas Model:

$$\Delta h = h_2 - h_1 = c_p (T_2 - T_1) = 2.2537(225 - 300) = -169.03 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1 = c_v (T_2 - T_1) = 1.7354(225 - 300) = -130.2 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 2.2537 \ln \frac{225}{300} - 0.5182 \ln \frac{1}{6} = 0.280 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\Delta v = v_2 - v_1 = \frac{RT_2}{p_2} - \frac{RT_1}{p_1} = \frac{(0.5182)(225)}{1550} - \frac{(0.5182)(300)}{9300} = 0.056 \frac{\text{m}^3}{\text{kg}}$$

The energy and entropy equations yield

$$\frac{d\vec{E}^{\prime 0}}{dt} = \dot{m}(j_1 - j_2) + \not Q^0 - \dot{W}_{\text{ext}} = \dot{m}(h_1 - h_2) - \dot{W}_{\text{ext}}$$

$$\Rightarrow \dot{W}_{\text{ext}} = -\dot{m}(h_2 - h_1) = 169.03 \text{ kW}$$

$$\frac{dS'}{dt}^{0} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}(s_2 - s_1) + \frac{\dot{Q}}{T_B}^{0} = 0.280 \frac{\text{kW}}{\text{K}}$$

### Ideal Gas Model:

Using the manual approach (Chapter 3) or the IG system-state TESTcalc, obtain:

$$\Delta h = h_2 - h_1; \implies \Delta h = -162.47 - 4.189; \implies \Delta h = -166.6 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1; \implies \Delta u = -279.1 + 151.30; \implies \Delta u = -127.8 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = s_2 - s_1; \quad \Rightarrow \Delta s = 9.572 - 9.283; \quad \Rightarrow \Delta s = 0.289 \frac{\text{kJ}}{\text{kg.K}}$$

$$\Delta v = v_2 - v_1; \implies \Delta v = 0.075 - 0.0167; \implies \Delta v = 0.058 \frac{\text{m}^3}{\text{kg}}$$

The energy equation and entropy equations yields

$$\dot{W}_{\rm ext} = -\dot{m}(h_2 - h_1) = 166.6 \text{ kW}$$

$$\dot{S}_{\text{gen,univ}} = \dot{m}(s_2 - s_1) = 0.289 \frac{\text{kW}}{\text{K}}$$

# Real Gas Model (Lee Kesler):

From Table, obtain the necessary material properties of  $CH_4$ :

$$p_{cr} = 4.64 \text{ MPa}, T_{cr} = 191.1 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = \frac{9.3}{4.64} = 2.004, \ p_{r2} = \frac{p_2}{p_{cr}} = \frac{1.55}{4.64} = 0.334$$

$$T_{r1} = \frac{T_1}{T_{cr}} = \frac{300}{191.1} = 1.57, \ T_{r2} = \frac{T_2}{T_{cr}} = \frac{225}{191.1} = 1.18$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{L-K} = 0.863$ ,  $Z_2^{L-K} = 0.929$ 

$$\Delta h = (h_2 - h_1)^{1G} - RT_{cr}(Z_{h,2} - Z_{h,1}) = -166.6 - 0.5182(191.1)(0.264 - 0.941) = -99.5 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = (h_2 - h_1) - R(Z_2^{\text{L-K}} T_2 - Z_1^{\text{L-K}} T_1);$$

$$\Rightarrow \Delta u = -99.5 - 0.5182 [(0.929)(225) - (0.863)(300)] = -73.65 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1});$$
  
 $\Rightarrow \Delta s = 0.289 - 0.5182(0.1627 - 0.452) = 0.438 \frac{\text{kJ}}{\text{kg.K}}$ 

$$\Delta v = v_2 - v_1 = \frac{Z_2^{L-K} R T_2}{p_2} - \frac{Z_1^{L-K} R T_1}{p_1};$$

$$\Rightarrow \Delta v = \frac{(0.929)(0.5182)(225)}{1550} - \frac{(0.863)(0.2968)(300)}{9300} = 0.0554 \frac{\text{m}^3}{\text{kg}}$$

The energy equation and entropy equations yields

$$\dot{W}_{\text{ext}} = -\dot{m}(h_2 - h_1) = 99.5 \text{ kW}$$

$$\dot{S}_{\text{gen,univ}} = \dot{m}(s_2 - s_1) = 0.438 \frac{\text{kW}}{\text{K}}$$

# Real Gas Model (Nelson-Obert)

From the Nelson-Obert compressibility chart, we obtain  $Z_1^{\text{N-O}} = 0.865$ ,  $Z_2^{\text{N-O}} = 0.928$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}) = -166.6 - 0.5182(191.1)(0.279 - 0.879) = -107.18 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= (h_2 - h_1) - R(Z_2^{\text{L-K}}T_2 - Z_1^{\text{L-K}}T_1); \\ \Rightarrow \Delta u &= -107.18 - 0.5182 \big[ (0.928)(225) - (0.865)(300) \big] = -80.9 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}); \\ \Rightarrow \Delta s &= 0.289 - 0.5182(0.189 - 0.425) = 0.411 \ \frac{\text{kJ}}{\text{kg.K}} \\ \Delta v &= v_2 - v_1 = \frac{Z_2^{\text{N-O}}RT_2}{p_2} - \frac{Z_1^{\text{N-O}}RT_1}{p_1}; \\ \Rightarrow \Delta v &= \frac{(0.928)(0.5182)(225)}{1550} - \frac{(0.865)(0.5182)(300)}{9300} = 0.055 \ \frac{\text{m}^3}{\text{kg}} \end{split}$$

The energy equation and entropy equations yield

$$\dot{W}_{\text{ext}} = -\dot{m}(h_2 - h_1) = 107.18 \text{ kW}$$

$$\dot{S}_{\text{gen,univ}} = \dot{m}(s_2 - s_1) = 0.411 \frac{\text{kW}}{\text{K}}$$

					RG Model (Manual)		RG Model (TEST)	
	PG	PG	IG	IG	Lee-	Nelson-	Lee-	Nelson-
	Model	Model	Model	Model	Kesler	Obert	Kesler	Obert
	(Manual	(TEST)	(Manual	(TEST)				
	)		)					
$\Delta h$								
(kJ)	-169.03	-166.7	-166.7	-166.66	-99.5	-107.18	-102.5	-110.15
$\left({\mathrm{kg}}\right)$								
$\Delta u$								
(kJ)	-130.2	-127.8	-127.8	-127.8	-73.65	-80.9	-76.6	-83.8
$\left({\mathrm{kg}}\right)$								
$\Delta s$								
( kJ )	0.280	0.289	0.289	0.289	0.438	0.411	0.432	0.403
$\left({\mathrm{kg.K}}\right)$					3	\		
$\frac{\Delta v}{\Delta v}$				18	12 CHILD HOLL MEN	*		
				idht.	es dill die			
$\left(\frac{\mathbf{m}^3}{}\right)$	0.056	0.058	0.058	0.058	0.055	0.055	0.055	0.055
(kg)				State instraining	Wir bei			

What-if Scenario: The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use it to perform the what-if study.

**11-2-19** [OYG] Argon gas enters a turbine operating at steady state at 10 MPa,  $51^{\circ}$ C with a mass flow rate (m) of 1 kg/s and expands adiabatically to 4 MPa,  $-35^{\circ}$ C with no change in KE or PE. Using the RG model (N-O charts), determine (a) the work developed ( $W_{\text{ext}}$ ) and (b) the entropy generated ( $S_{\text{gen}}$ ). (c) What-if Scenario: What would the work developed be if IG model were used?

### **SOLUTION:**

From Table, obtain the necessary material properties of Ar:

$$c_p = 0.5203 \frac{\text{kJ}}{\text{kg.K}}, c_v = 0.3122 \frac{\text{kJ}}{\text{kg.K}}, R = 0.2081 \frac{\text{kJ}}{\text{kg.K}}$$

Given: 
$$p_1 = 10 \text{ MPa}$$
;  $T_1 = 51 \, ^{\circ}\text{C}$ ;  $\dot{m}_1 = 1 \, \frac{\text{kg}}{\text{s}}$ 

Given: 
$$p_2 = 4$$
 MPa;  $T_2 = -35$  °C;  $\dot{m}_2 = \dot{m}_1$ 

# Real Gas Model (Lee Kesler):

From Table, obtain the necessary material properties of Ar :  $p_{cr} = 4.86$  MPa,  $T_{cr} = 151$  K

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = \frac{10}{4.86} = 2.058, \ p_{r2} = \frac{p_2}{p_{cr}} = \frac{4}{4.86} = 0.823$$

$$T_{r1} = \frac{T_1}{T_1} = \frac{324}{4.86} = 2.15, \ T_{r2} = \frac{T_2}{T_2} = \frac{238}{4.86} = 1.58$$

$$T_{r1} = \frac{T_1}{T_{cr}} = \frac{324}{151} = 2.15, \ T_{r2} = \frac{T_2}{T_{cr}} = \frac{238}{151} = 1.58$$

From the Lee-Kesler compressibility chart, we obtain  $Z_1^{L-K} = 0.974$ ,  $Z_2^{L-K} = 0.939$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}) \,; \\ \Rightarrow \Delta h &= -44.75 - 0.2081(151)(0.3669 - 0.488) = -40.9 \, \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= (h_2 - h_1) - R(Z_2^{\text{L-K}}T_2 - Z_1^{\text{L-K}}T_1) \,; \\ \Rightarrow \Delta u &= -40.9 - 0.2081 \big[ (0.939)(238) - (0.974)(324) \big] = -21.74 \, \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}) = 0.03 - 0.2081(0.175 - 0.2108) \, = 0.037 \, \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ \Delta v &= v_2 - v_1 = \frac{Z_2^{\text{L-K}}RT_2}{p_2} - \frac{Z_1^{\text{L-K}}RT_1}{p_1} \,; \\ \Rightarrow \Delta v &= \frac{(0.939)(0.2081)(238)}{4000} - \frac{(0.974)(0.2081)(324)}{10000} \,; \quad \Rightarrow \Delta v = 0.0051 \, \frac{\text{m}^3}{\text{kg}} \end{split}$$

The energy and entropy equations yield

$$\frac{d\vec{E}'^{0}}{dt} = \dot{m}(j_{1} - j_{2}) + \dot{\cancel{Q}}^{0} - \dot{W}_{\text{ext}} = \dot{m}(h_{1} - h_{2}) - \dot{W}_{\text{ext}}$$

$$\Rightarrow \dot{W}_{\text{ext}} = -\dot{m}(h_{2} - h_{1}) = 40.9 \text{ kW}$$

$$\frac{dS'^{0}}{dt} = \dot{m}(s_{1} - s_{2}) + \frac{\dot{Q}}{T_{B}} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}(s_{2} - s_{1}) + \frac{\dot{Q}^{0}}{T_{B}} = 0.037 \frac{\text{kW}}{\text{K}}$$

### Real Gas Model (Nelson-Obert)

From the Nelson-Obert compressibility chart, we obtain  $Z_1^{\text{N-O}} = 0.965$ ,  $Z_2^{\text{N-O}} = 0.943$ 

$$\begin{split} \Delta h &= (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}) = -44.75 - 0.2081(151)(0.355 - 0.436) = -42.2 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta u &= (h_2 - h_1) - R(Z_2^{\text{L-K}}T_2 - Z_1^{\text{L-K}}T_1); \\ \Rightarrow \Delta u &= -42.2 - 0.2081 \big[ (0.943)(238) - (0.965)(324) \big] = -23.84 \ \frac{\text{kJ}}{\text{kg}} \\ \Delta s &= (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}); \\ \Rightarrow \Delta s &= 0.03 - 0.2081(0.143 - 0.179) = 0.037 \ \frac{\text{kJ}}{\text{kg.K}} \\ \Delta v &= v_2 - v_1 = \frac{Z_2^{\text{N-O}}RT_2}{p_2} - \frac{Z_1^{\text{N-O}}RT_1}{p_1}; \\ \Rightarrow \Delta v &= \frac{(0.943)(0.2081)(238)}{4000} - \frac{(0.965)(0.2081)(324)}{1000} = 0.0052 \ \frac{\text{m}^3}{\text{kg}} \end{split}$$

The energy equation and entropy equations yield

$$\dot{W}_{\text{ext}} = -\dot{m}(h_2 - h_1) = 42.2 \text{ kW}$$

$$\dot{S}_{\text{gen,univ}} = \dot{m}(s_2 - s_1) = 0.037 \frac{\text{kW}}{\text{K}}$$

			RG M (Mai	· . ·	RG Model (TEST)	
	PG	IG S	Lee-	Nelson-	Lee-	Nelson-
	Model	Model	Kesler	Obert	Kesler	Obert
	(TEST)	(TEST)	K Sid des			
$\left(\frac{\mathrm{kJ}}{\mathrm{kg}}\right)$	-44.74	-44.74	-40.9	-42.2	-40.9	-42.2
$\frac{\Delta u}{\left(\frac{\mathrm{kJ}}{\mathrm{kg}}\right)}$	-26.9	-26.9	-21.74	-23.84	-21.8	-23.8
$\left(\frac{kJ}{kg.K}\right)$	0.03	0.03	0.037	0.037	0.038	0.038
$\left(\frac{\Delta v}{kg}\right)$	0.0056	0.0056	0.0051	0.0052	0.005	0.0052

**What-if Scenario:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. Use it to perform the what-if study.



**11-2-20** [OYZ] Air (79%  $N_2$  and 21%  $O_2$  by volume) is compressed isothermally at 500 K from 4 MPa to 8 MPa in a steady-flow compressor at a rate of 5 kg/s. Assuming no irreversibilities, determine the power input to the compressor. Treat air as a mixture of (a) perfect gases, (b) ideal gases and (c) real gases.

### **SOLUTION:**

(a) PG Model

Obtain mixture material properties as:

$$\overline{M} = \sum_{k} y_{k} \overline{M}_{k} = 28.84 \frac{\text{kg}}{\text{kmol}}; R = \frac{\overline{R}}{\overline{M}} = 0.288 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; c_{p} = \sum_{k} y_{k} c_{p,k} = 1.001 \frac{\text{kJ}}{\text{kg} \cdot \text{K}};$$

The energy and entropy equations yield

$$\frac{d\vec{E}^{0}}{dt} = \dot{m}(j_{1} - j_{2}) + \dot{Q} - \dot{W}_{\text{ext}} = \dot{m}(h_{1} - h_{2}) + \dot{Q} - \dot{W}_{\text{ext}} = \dot{m}c_{p}(T_{1} - T_{2})^{0} + \dot{Q} - \dot{W}_{\text{ext}}$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{Q}$$

$$\frac{dS^{\prime 0}}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}^{0}$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{Q} = \dot{m}(s_2 - s_1)T_B = \dot{m}\left(c_p \ln \frac{T_A^{\prime 0}}{T_1} - R \ln \frac{p_2}{p_1}\right)T_B = -500 \text{ kW}$$

(a) IG Model

Because the temperature remains constant, there is no temperature contribution to entropy change and ideal gas model produces the same result as the PG model.

$$\dot{W}_{\text{ext}} = \dot{Q} = \dot{m}(s_2 - s_1)T_B = \dot{m}\left[ (s_2^0 - s_1)^0 - R \ln \frac{p_2}{p_1} \right] T_B = -500 \text{ kW}$$

(a) RG Model

Using Kay's rule obtain the critical properties of the mixture:

$$p_{cr} = \sum_{k} y_k p_{k,cr} = 3.74 \text{ MPa}, T_{cr} = \sum_{k} y_k T_{k,cr} = 132.2 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = 1.07, \ p_{r2} = \frac{p_2}{p_{cr}} = 2.14$$

$$T_{r1} = \frac{T_1}{T_{cr}} = 3.783, \ T_{r2} = \frac{T_2}{T_{cr}} \ 3.783$$

Using the Lee-Kesler compressibility chart

$$\Delta h = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}) = -1.7 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta s = (s_2 - s_1)^{1G} - R(Z_{s,2} - Z_{s,1}) = -0.2069 \frac{kJ}{kg \cdot K}$$

The energy and entropy equations yield

$$\frac{dS^{\prime 0}}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}^{0}$$

$$\Rightarrow \dot{Q} = \dot{m}(s_2 - s_1)T_B = -517.22 \text{ kW}$$

$$\frac{dE^{\prime 0}}{dt} = \dot{m}(j_1 - j_2) + \dot{Q} - \dot{W}_{\text{ext}} = \dot{m}(h_1 - h_2) + \dot{Q} - \dot{W}_{\text{ext}} = \dot{m}c_p \left(T_1 - T_2\right)^0 + \dot{Q} - \dot{W}_{\text{ext}}$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{Q} + \dot{m}(h_1 - h_2) = -508.7 \text{ kW}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

**11-2-21** [OYK] Methane is compressed adiabatically by a steady flow compressor from 3 MPa and -15°C to 10 MPa and 100°C at a rate of 0.9 kg/s. Determine the power input  $(W_{\text{ext}})$  to the compressor. Use the real gas model.

### **SOLUTION:**

From Table E-1, obtain the necessary material properties of CH<sub>4</sub>:

$$p_{cr} = 4.64 \text{ MPa}, T_{cr} = 191.1 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = \frac{3}{4.64} = 0.652, \ p_{r2} = \frac{p_2}{p_{cr}} = \frac{10}{4.64} = 2.17$$

$$T_{r1} = \frac{T_1}{T} = \frac{258.15}{191.1} = 1.356, \ T_{r2} = \frac{T_2}{T} = \frac{373.15}{191.1} = 1.953$$

Use IG system-state TESTcalc or Table D to obtain:

$$\Delta h^{IG} = (h_2 - h_1)^{IG} = 214.9 \text{ kg}$$

Using the Lee-Kesler compressibility chart, we obtain:

$$\Delta h = (h_2 - h_1)^{1G} - RT_{cr}(Z_{h,2} - Z_{h,1}) = 245.4 \frac{\text{kJ}}{\text{kg}}$$

The energy and entropy equations yield

$$\frac{d\vec{k}'^{0}}{dt} = \dot{m}(j_{1} - j_{2}) + \dot{\cancel{Q}}^{0} - \dot{W}_{\text{ext}} = \dot{m}(h_{1} - h_{2}) - \dot{W}_{\text{ext}}$$

$$\Rightarrow \dot{W}_{\text{ext}} = -\dot{m}(h_{2} - h_{1}) = -(0.9)(245.4) = -220.9 \text{ kW}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

11-2-22 [OYU] Carbon dioxide enters an adiabatic nozzle at 10 MPa, 450 K with a low velocity and leaves at 3 MPa, 350 K. Using the RG model (N-O charts), determine (a) the exit velocity  $(V_2)$ .

#### **SOLUTION:**

Given:  $p_1 = 10 \text{ MPa}$ ;  $T_1 = 450 \text{ K}$ 

Given:  $p_2 = 3 \text{ MPa}$ ;  $T_2 = 350 \text{ MPa}$ 

# Ideal Gas Model:

Use IG system-state TESTcalc or Table D to obtain:

$$\Delta h = h_2 - h_1 = -8896.60 + 8802.76 = -93.84 \frac{\text{kJ}}{\text{kg}}$$

# Real Gas Model (Nelson-Obert)

From Table E-1, obtain the necessary material properties of  $C_3H_8$ :

$$p_{cr} = 7.39 \text{ MPa}, T_{cr} = 304.2 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = \frac{10}{7.39} = 1.35, \ p_{r2} = \frac{p_2}{p_{cr}} = \frac{3}{7.39} = 0.41$$

$$T_{r1} = \frac{T_1}{T_{cr}} = \frac{450}{304.2} = 1.479, \ T_{r2} = \frac{T_2}{T_{cr}} = \frac{350}{304.2} = 1.15$$

Using the Nelson-Obert compressibility chart, we obtain:

$$\Delta h = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}); \quad \Rightarrow \Delta h = -93.84 - 0.1889(304.2)(0.3784 - 0.7337)$$

$$\Rightarrow \Delta h = -73.45 \frac{\text{kJ}}{\text{kg}}$$

For an adiabetic nozzle, the energy equation simplifies as follows:

$$j_{1} = j_{2};$$

$$\Rightarrow h_{1} + ke_{1}^{0} + pe_{1}^{0} = h_{2} + ke_{2} + pe_{2}^{0}$$

$$\Rightarrow ke_{2} = h_{1} - h_{2}$$

$$\Rightarrow \frac{V_{2}^{2}}{2000} = 73.45$$

$$\Rightarrow V_{2} = \sqrt{146900} \Rightarrow V = 383.02 \text{ m/s}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.



11-2-23 [OYP] An adiabatic 1 m<sup>3</sup> rigid tank is initially evacuated. It is filled to a pressure of 10 MPa from a supply line that carries nitrogen at 275 K and 10 MPa. Determine (a) the final temperature  $(T_2)$  and (b) the mass in the tank. Use the real gas model. (c) Whatif Scenario: What would the mass in the tank be if the tank temperature were maintained at 300 K, the surrounding's temperature?

#### **SOLUTION:**

(a) Let state-1 and 2 represent the initial and final states and State-3 the supply line state. The energy equations for the open process with  $m_1 = 0$  reduces to:

$$\Delta E = m_3 j_3; \quad \Rightarrow \quad \Delta U + \Delta K E^0 + \Delta P E^0 = m_3 h_3 + m_3 k e_3^0 + m_3 p e_3^0;$$
  

$$\Rightarrow \quad m_2 u_2 - m_1^0 u_1 = m_3 h_3; \quad \Rightarrow \quad u_2 = h_3;$$

State-3 can be evaluated from  $p_3$  and  $T_3$  and State-2 from  $p_2 = p_3$  and  $u_2 = h_3$ . For the PG model, the task becomes simpler as  $u_2 = h_3$  can be manipulated as follows:

$$u_{2} = h_{3};$$
  
 $\Rightarrow h_{2} - p_{2}v_{2} = h_{3}; \Rightarrow h_{2} - ZRT_{2} = h_{3};$   
 $\Rightarrow h_{2} - h_{3} = Z_{2}RT_{2};$   
 $\Rightarrow (h_{2} - h_{3})^{IG} - RT_{cr}(Z_{h,2} - Z_{h,3}) = Z_{2}RT_{2};$ 

From Table E-1, obtain the necessary material properties of N<sub>2</sub>:

$$p_{cr} = 3.39 \text{ MPa}, T_{cr} = 126.2 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = 0, \quad p_{r2} = \frac{p_2}{p_{cr}} = 2.95, \quad p_{r3} = \frac{p_2}{p_{cr}} = 2.95$$

$$T_{r1} = \frac{T_1}{T_{cr}} = 0, \quad T_{r2} = \frac{T_2}{T_{cr}} = ?, \quad T_{r3} = \frac{T_3}{T_{cr}} = 2.18$$

Iterating over possible values of  $T_2$ , we obtain:

$$T_{r2} = 2.94$$
  
 $\Rightarrow S_{\text{gen,univ}} = m_2 s_2 - p f_1^0 s_1 - m_3 s_3 = m_2 (s_2 - s_3)$   
 $\Rightarrow T_2 = 371 \text{ K}$ 

(b) The real gas equation of state yields:

$$m_2 = \frac{p_2 V_2}{Z_2 R T_2} = \frac{(10000)(1)}{(1.026)(0.297)(371)} = 88.5 \text{ kg}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. While iterating for T2, evaluate Qdot by clicking the Super-Calculate button after every new guess for T2. When Qdot approaches zero, the final temperature matches the adiabatic condition.



**11-2-24** [OYX] Oxygen enters a nozzle operating at steady state at 6 MPa, 300 K, 1 m/s and expands isentropically to 3 MPa. Using the RG model (L-K charts), determine (a) the exit temperature ( $T_2$ ) and (b) the exit velocity ( $V_2$ ).

### **SOLUTION:**

Given:  $p_1 = 6$  MPa;  $T_1 = 300$  K,  $V_1 = 1$  m/s

Given:  $p_2 = 3$  MPa;  $s_2 = s_1$ 

Ideal Gas Model:

Use IG system-state TESTcalc or Table D. obtain:

$$\Delta h = h_2 - h_1 = -49 \frac{\text{kJ}}{\text{kg}}$$

### Real Gas Model (L-K model)

From Table E-1, obtain the necessary material properties of O<sub>2</sub>:

$$p_{cr} = 5.03 \text{ MPa}, T_{cr} = 154.6 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = 1.19, \ p_{r2} = \frac{p_2}{p_{cr}} = 0.60$$

$$T_{r1} = \frac{T_1}{T_{...}} = 1.94$$

(a) Using the Lee-Kesler compressibility chart and iterating over guessed value of  $T_{r2}$ , we obtain:

For 
$$T_{r2} = \frac{T_2}{T_{r2}} = 1.58$$
, or  $T_2 = 245 \text{ K}$ ,  $\Delta s = (s_2 - s_1)^{1G} - R(Z_{s,2} - Z_{s,1}) = 0$ ;

$$\Delta h = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}); \implies \Delta h = -49.22 \frac{\text{kJ}}{\text{kg}}$$

(b) For an adiabatic nozzle, the energy equation simplifies as follows:

$$j_{1} = j_{2};$$

$$\Rightarrow h_{1} + ke_{1}^{0} + pe_{1}^{0} = h_{2} + ke_{2} + pe_{2}^{0}$$

$$\Rightarrow ke_{2} = h_{1} - h_{2}$$

$$\Rightarrow \frac{V_{2}^{2}}{2000} = 49.22$$

$$\Rightarrow V_{2} = 307.3 \text{ m/s}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.



11-2-25 [OYV] One kmol of argon at 320 K is initially confined to one side of a rigid, insulated container divided into equal volumes of  $0.2 \text{ m}^3$  by a partition. The other side is initially evacuated. The partition is removed and argon expands to fill the entire container. Using the RG model (L-K charts), determine (a) the final temperature ( $T_2$ ) of argon. (b) What-if Scenario: What would the final temperature be if PG model were used?

# **SOLUTION:**

$$T_{cr} = 150.8 \text{ K}; \ p_{cr} = 4.865 \text{ MPa};$$

State-1 (given  $T_1, \frac{1}{1}, n_1$ )

$$v_1 = \frac{m_1}{V_1} = \frac{n_1 \overline{M}}{V_1} = 0.005 \frac{\text{m}^3}{\text{kg}}; \quad T_{r1} = \frac{T_1}{T_{cr}} = 2.122;$$
  
 $\Rightarrow p_r = 2.65; \quad p_1 = 12.917 \text{ kPa}$ 

State-2 (given 
$$T_2 = 0$$
,  $m_2 = 0$ )

State-3 (given 
$$V_3 = 2V_1$$
,  $m_3 = m_1$ ):

From the energy balance equation

$$\Delta U + \Delta K E^{0} + \Delta P E^{0} = \cancel{Q}^{0} - \cancel{W_{\text{ext}}}^{0}$$

$$\Rightarrow m_3 u_3 - m_1 u_1 - m_2^{0} u_2 = 0;$$

$$\Rightarrow m(u_3 - u_1) = 0;$$

$$\Rightarrow h_3 - p_3 v_3 - h_1 + p_1 v_1 = 0;$$

$$\Rightarrow h_3 - h_1 - R(Z_3T_3 - Z_1T_1) = 0;$$

$$\Rightarrow (h_3 - h_1)^{1G} - RT_{cr}(Z_{h,3} - Z_{h,1}) - R(Z_3T_3 - Z_1T_1) = 0;$$

Iterating over guessed value of  $T_3$ 

$$T_3 = 320 \text{ K};$$

What-if Scenario: Use TEST solution to verify this answer and for carrying out the what-if scenario. The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net.

**11-2-26** [BPU] Ex: 11-9 Nitrogen at 10 MPa and 150 K flows steadily through a tube with a mass flow rate (m) of 1 kg/s, receiving heat from the surroundings at 300 K. At the end of the tube it enters an expansion valve and leaves at 1 MPa and 125 K. Using the real gas model determine (a) the rate of heat transfer (Q), and (b) entropy generation rate  $(S_{\text{gen,univ}})$  in the thermodynamic universe. What-if Scenario: What would the answers be if (c) the ideal gas, or (d) the phase-change model were used instead?

# **SOLUTION:**

Given:  $p_1 = 10 \text{ MPa}$ ;  $T_1 = 150 \text{ K}$ ,  $\dot{m}_1 = 1 \text{ kg/s}$ 

Given:  $p_2 = 1 \text{ MPa}$ ;  $T_2 = 125 \text{ K}$ ,  $\dot{m}_2 = \dot{m}_1$ 

Ideal Gas Model:

Use IG system-state TESTcalc or Table D. obtain:

$$\Delta h = h_2 - h_1 = -25.78 \frac{\text{kJ}}{\text{kg}}; \ \Delta s = s_2 - s_1 = 0.4957 \frac{\text{kJ}}{\text{kg} \cdot \text{K}};$$

Real Gas Model (L-K model)

From Table E-1, obtain the necessary material properties of N<sub>2</sub>:

$$p_{cr} = 3.39 \text{ MPa}, T_{cr} = 126.2 \text{ K}$$

The reduced pressure and temperature for the given states are

$$p_{r1} = \frac{p_1}{p_{cr}} = 2.95, \ p_{r2} = \frac{p_2}{p_{cr}} = 0.295$$

$$T_{r1} = \frac{T_1}{T_{cr}} = 1.19, \ T_{r2} = \frac{T_2}{T_{cr}} = 0.99,$$

(a) Using the Lee-Kesler compressibility chart, we obtain:

$$\Delta h = (h_2 - h_1)^{IG} - RT_{cr}(Z_{h,2} - Z_{h,1}) = 68 \frac{\text{kJ}}{\text{kg}}$$
  
$$\Delta s = (s_2 - s_1)^{IG} - R(Z_{s,2} - Z_{s,1}) = 0.946 \frac{\text{kJ}}{\text{kg} \cdot \text{K}};$$

The energy equation simplifies as follows:

$$\frac{d\vec{E}^{0}}{dt} = \dot{m}(j_1 - j_2) + \dot{Q} - \dot{W}_{\text{ext}}^{0} = \dot{m}(h_1 - h_2) + \dot{Q}$$

$$\Rightarrow \dot{Q} = \dot{m}(h_2 - h_1) = 68 \text{ kW}$$

The entropy equation simplifies to:

$$\frac{dS^{'0}}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}(s_2 - s_1) - \frac{\dot{Q}}{T_B} = (1)(0.9455) - \frac{68}{300} = 0.718 \frac{\text{kW}}{\text{K}}$$

**TEST Solution:** The TEST-code for this problem can be found in the problems module of the TEST-pro site at www.thermofluids.net. It can be used to perform the what-if studies.

