2-1-1 [HD] Mass enters an open system with one inlet and one exit at a constant rate of 50 kg/min. At the exit, the mass flow rate is 60 kg/min. If the system initially contains 1000 kg of working fluid, determine (a) d*m*/d*t* treating the tank as a system and (b) the time when the system mass becomes 500 kg.

SOLUTION

(a) The mass balance equation will yield the rate of change in mass of this open system as it changes from state 1 (m = 1000 kg) to state 2 (m = 500 kg).

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \frac{dm}{dt} = 50 - 60; \quad \Rightarrow \frac{dm}{dt} = -10 \ \frac{\text{kg}}{\text{min}}$$

(b) The change in mass is given by

$$\Delta m = m_2 - m_1; \quad \Rightarrow \Delta m = 500 - 1000; \quad \Rightarrow \Delta m = -500 \text{ kg};$$

Since $\frac{dm}{dt}$ is found to be constant, Δm can be rewritten as

$$\Delta m = \frac{dm}{dt}(t); \quad \Rightarrow t = \frac{\Delta m}{\left(\frac{dm}{dt}\right)}; \quad \Rightarrow t = \frac{-500}{-10}; \quad \Rightarrow t = 50 \ \text{min}$$

2-1-2 [HM] Steam enters an insulated tank through a valve. At a given instant, the mass of steam in the tank is found to be 10 kg, and the conditions at the inlet are measured as follows: A = 50 cm^2, V = 31 m/s, and ρ = 0.6454 kg/m^3. Determine (a) dm/dt treating the tank as a system. (b) Assuming the inlet conditions to remain unchanged, determine the mass of steam in the tank after 10 s.

SOLUTION

(a) $\dfrac{dm}{dt} = \dot{m}_1; \quad \Rightarrow \dfrac{dm}{dt} = \rho_1 A_1 V_1; \quad \Rightarrow \dfrac{dm}{dt} = (0.6454)(50\times 10^{-4})(31);$

$\Rightarrow \dfrac{dm}{dt} = 0.1\ \dfrac{\text{kg}}{\text{s}}$

(b) $\Delta m = \dfrac{dm}{dt}(t); \quad \Rightarrow m_2 - m_1 = \dfrac{dm}{dt}(t); \quad \Rightarrow m_2 = (0.1)(10) + 10;$

$\Rightarrow m_2 = 1 + 10; \quad \Rightarrow m_2 = 11\ \text{kg}$

2-1-3 [HJ] Air is introduced into a piston-cylinder device through a 7 cm-diameter flexible duct. The velocity and specific volume at the inlet at a given instant are measured as 22 m/s and 0.1722 m^3/kg respectively. At the same time air jets out through a 1 mm-diameter leak with a density of 1.742 kg/m^3, the piston rises with a velocity of 10 cm/s, and the mass of the air in the device increases at a rate of 0.415 kg/s. Determine (a) the mass flow rate (*m*) of air at the inlet, and (b) the jet velocity. (c) If the jet pressure is 100 kPa, use the IG flow state daemon to determine the temperature of the jet at the exit.

SOLUTION

(a) $\dot{m}_1 = \rho_1 A_1 V_1 = \dfrac{A_1 V_1}{v_1} = \left(\dfrac{\pi D_1^2}{4}\right)\dfrac{V_1}{v_1} = 0.49167\ \dfrac{\text{kg}}{\text{s}}$

(b) $\dfrac{dm}{dt} = \dot{m}_1 - \dot{m}_2;$

$$\Rightarrow \dot{m}_2 = \dot{m}_1 - \frac{dm}{dt}; \qquad \Rightarrow \dot{m}_2 = 0.0767\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_2 = \frac{A_2 V_2}{v_2};$$

$$\Rightarrow V_2 = \frac{\dot{m}_2 v_2}{A_2}; \qquad \Rightarrow V_2 = 56040\ \frac{\text{m}}{\text{s}}$$

(c) Launch the IG flow-state TESTcalc. Enter the known properties at the inlet and exit states (State-1 and State-2 respectively). T2 is calculated as part of State-2 as 200 K. The TEST-code can be found in the professional TEST site (www.thermofluids.net).

2-1-4 [HW] Air enters an open system with a velocity of 1 m/s and density of 1 kg/m^3 at 500 K through a pipe with a cross-sectional area of 10 cm^2. The mass of air in the tank at a given instant is given by the expression $m = 5p/T$ where p is in kPa and T is in K. If the temperature in the tank remains constant at 500 K due to heat transfer, (a) determine the rate of increase of pressure in the tank. (b) What is the sign of heat transfer, positive (1) or negative (-1)?

SOLUTION

(a) $\frac{dm}{dt} = \dot{m}_1 - \cancel{\dot{m}_2}^{\,0}; \qquad \Rightarrow \frac{dm}{dt} = \dot{m}_1;$

$$\Rightarrow \frac{d}{dt}\left(\frac{5p}{T}\right) = \rho_1 A_1 V_1;$$

$$\Rightarrow \frac{dp}{dt} = \left(\frac{T}{5}\right)\rho_1 A_1 V_1; \qquad \Rightarrow \frac{dp}{dt} = \frac{500}{5}(1)(0.001)(1); \qquad \Rightarrow \frac{dp}{dt} = 0.1\ \frac{\text{kPa}}{\text{s}}$$

(b) The flow work involved in pushing the mass into the system would transfer energy into the system in addition to the stored energy transported by the mass. To maintain a constant temperature (internal energy), heat must be transferred out of the system. Hence the sign of heat transfer must be negative by the WinHip sign convention.

2-1-5 [NR] A propane tank is being filled at a charging station (see figure in problem 2-1-2 [HM]). At a given instant the mass of the tank is found to increase at a rate of 0.4 kg/s. Propane from the supply line at state-1 has the following conditions: D = 5 cm, T = 25°C, and ρ = 490 kg/m^3. Determine the velocity of propane in the supply line.

SOLUTION

$$\frac{dm}{dt} = \dot{m}_1; \qquad \Rightarrow \dot{m}_1 = 0.4 \ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_1 = \rho_1 A_1 V_1;$$

$$\Rightarrow V_1 = \frac{\dot{m}_1}{\rho_1 A_1}; \qquad \Rightarrow V_1 = \frac{\dot{m}_1}{\rho_1}\left(\frac{4}{\pi D_1^2}\right); \qquad \Rightarrow V_1 = 0.416 \ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PC flow-state TESTcalc. Enter the known properties and press Calculate to obtain the velocity along with other properties of the flow state. TEST-code can be found in the TEST professional site at www.thermofluids.net.

2-1-6 [NO] Mass leaves an open system with a mass flow rate of $c*m$, where c is a constant and m is the system mass. If the mass of the system at $t = 0$ is m_0, derive an expression for the mass of the system at time t.

SOLUTION

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \frac{dm}{dt} = 0 - cm; \quad \Rightarrow \frac{dm}{dt} = -cm;$$

$$\Rightarrow \int_{m_0}^{m} \frac{1}{m} dm = -c\int_0^t dt;$$

$$\Rightarrow \ln(m)\Big|_{m_0}^{m} = -ct\Big|_0^t; \quad \Rightarrow \ln(m) - \ln(m_0) = -ct; \quad \Rightarrow \ln\left(\frac{m}{m_0}\right) = -ct;$$

$$\Rightarrow \frac{m}{m_0} = e^{-ct}; \quad \Rightarrow m = m_0 e^{-ct}$$

$$\frac{m}{m_0} = e^{-ct}; \quad \Rightarrow \frac{m}{m_0} = 0.5;$$

$$\Rightarrow t = \frac{\ln(0.5)}{(-c)}; \quad \Rightarrow t = 69.3 \text{ s}$$

2-1-7 [NB] Water enters a vertical cylindrical tank of cross-sectional area 0.01 m^2 at a constant mass flow rate of 5 kg/s. It leaves the tank through an exit near the base with a mass flow rate given by the formula 0.2*h* kg/s, where *h* is the instantaneous height in m. If the tank is initially empty, (a) develop an expression for the liquid height *h* as a function of time *t*. (b) How long does it take for the water to reach a height of 10 m? Assume density of water to remain constant at 1000 kg/m^3.

SOLUTION

(a) $\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \frac{dm}{dt} = 5 - \frac{h}{5};$

$$\frac{d\cancel{V}}{dt} = \frac{\left(\frac{dm}{dt}\right)}{\rho_{water}}; \quad \Rightarrow \frac{d\cancel{V}}{dt} = \frac{5}{1000} - \frac{h}{5000}; \quad \text{and} \quad \frac{dh}{dt} = \frac{\left(\frac{d\cancel{V}}{dt}\right)}{A}; \quad \Rightarrow \frac{dh}{dt} = 100\frac{d\cancel{V}}{dt};$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{2} - \frac{h}{50};$$

$$\Rightarrow \int_0^t dt = t = \int_0^h \frac{1}{\frac{1}{2} - \frac{h}{50}}\, dh;$$

Using the substitution method where

$$U = \frac{1}{2} - \frac{h}{50}; \quad \text{and} \quad dU = -\frac{1}{50}dh; \quad \Rightarrow dh = -50dU;$$

$$\Rightarrow t = -50\int_{\frac{1}{2}}^{\frac{1}{2}-\frac{h}{2}} \frac{1}{U}dU; \quad \Rightarrow t = -50\left[\ln\left(\frac{1}{2} - \frac{h}{50}\right) - \ln\left(\frac{1}{2}\right)\right];$$

$$\Rightarrow t = -50\ln\left(\frac{\frac{1}{2} - \frac{h}{50}}{\frac{1}{2}}\right); \quad \Rightarrow t = -50\ln\left(1 - \frac{h}{25}\right);$$

$$\Rightarrow h = 25\left(1 - e^{-0.02t}\right)$$

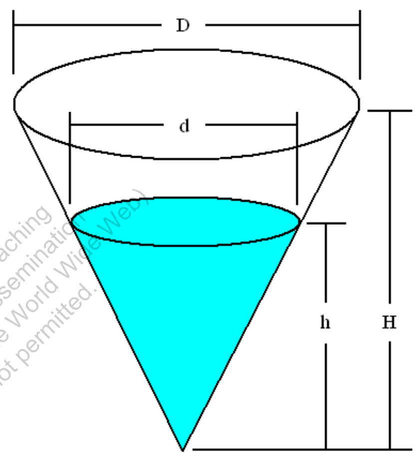
(b) $10 = 25\left(1 - e^{-0.02t}\right); \quad \Rightarrow e^{-0.02t} = 1 - \frac{10}{25};$

$$\Rightarrow -0.02t = \ln\left(\frac{15}{25}\right); \quad \Rightarrow t = 25.5 \text{ s}$$

2-1-8 [NS] A conical tank of base diameter *D* and height *H* is suspended in an inverted position to hold water. A leak at the apex of the cone causes water to leave with a mass flow rate of *c*sqrt(h)*, where *c* is a constant and *h* is the height of the water level from the leak at the bottom.(a) Determine the rate of change of height *h*. (b) Express *h* as a function of time *t* and other known constants, ρ (constant density of water), *D*, *H*, and *c* if the tank were completely full at $t = 0$. (c) If D= 1 m, H=1 m, $\rho = 1000$ kg/m3, and c= 1 kg/(s.m$^{1/2}$), how long does it take for the tank to empty?

SOLUTION

$$V = \frac{1}{12}\pi d^2 h;$$



Using similar triangles to express d in terms of D, H, and h

$$\frac{D}{H} = \frac{d}{h}; \quad \Rightarrow d = \frac{Dh}{H}; \quad \text{and} \quad V = \frac{\pi D^2 h^3}{12H^2};$$

$$m = V\rho; \quad \Rightarrow m = \frac{\pi D^2 h^3 \rho}{12H^2};$$

The derivative of the mass equation produces $\frac{dh}{dt}$

$$\frac{dm}{dt} = \left(\frac{\pi D^2 h^2 \rho}{4H^2}\right)\frac{dh}{dt};$$

From the mass balance equation

$$\dot{m}_e = -\frac{dm}{dt}; \quad \Rightarrow \dot{m}_e = -\left(\frac{\pi D^2 h^2 \rho}{4H^2}\right)\frac{dh}{dt};$$

(a) To solve for the rate of change in height, $\frac{dh}{dt}$, replace $\dot{m}_e$ with $c\sqrt{h}$

$$\frac{dh}{dt} = -\frac{4cH^2}{\pi D^2 h^{3/2} \rho}$$

(b) It is necessary to separate the variables in order to solve for the height h

$$\frac{dh}{dt} = -\frac{4cH^2}{\pi D^2 h^{3/2} \rho}; \quad \Rightarrow h^{3/2} dh = -\frac{4cH^2}{\pi D^2 \rho} dt;$$

$$\Rightarrow \int_H^h h^{3/2} dh = -\frac{4cH^2}{\pi D^2 \rho}\int_0^t dt; \quad \Rightarrow \frac{2}{5} h^{5/2}\Big|_H^h = -\frac{4cH^2}{\pi D^2 \rho}(t)\Big|_0^t;$$

$$\Rightarrow \frac{2}{5}h^{5/2} - \frac{2}{5}H^{5/2} = -\left(\frac{4cH^2}{\pi D^2 \rho}\right)t;$$

$$h = \left[H^{5/2} - \left(\frac{10cH^2}{\pi D^2 \rho}\right)t\right]^{2/5}$$

(c) $0 = h; \qquad \Rightarrow 0 = \left[H^{5/2} - \left(\frac{10cH^2}{\pi D^2 \rho}\right)t\right]^{2/5};$

$$\Rightarrow H^{5/2} = \left(\frac{10cH^2}{\pi D^2 \rho}\right)t;$$

$$\Rightarrow t = \frac{\pi D^2 \rho \sqrt{H}}{10c}; \qquad \Rightarrow t = \frac{\pi(1)(1000)\sqrt{1}}{10(1)}; \qquad \Rightarrow t = 314 \text{ s}$$

2-1-9 [NA] Steam enters a mixing chamber at 100 kPa, 20 m/s and a specific volume of 0.4 m^3/kg. Liquid water at 100 kPa and 25^{o}C enters the chamber through a separate duct with a flow rate of 50 kg/s and a velocity of 5 m/s. If liquid water leaves the chamber at 100 kPa, 43^{o}C, 5.58 m/s and a volumetric flow rate of 3.357 m^3/min, determine the port areas at (a) the inlets and (b) the exit. Assume liquid water density to be 1000 kg/m^3 and steady state operation.

SOLUTION

From the mass balance equation,

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3;$$

$$\text{Where } \dot{m}_2 = 50\ \frac{\text{kg}}{\text{s}}; \quad \text{and} \quad \dot{m}_3 = \rho_3 \dot{\cancel{V}}_3; \quad \Rightarrow \dot{m}_3 = (1000)\left(\frac{3.357}{60}\right); \quad \Rightarrow \dot{m}_3 = 55.95\ \frac{\text{kg}}{\text{s}};$$

$$\text{(a.1)}\ \dot{m}_1 = \dot{m}_3 - \dot{m}_2; \quad \Rightarrow \dot{m}_1 = 55.95 - 50; \quad \Rightarrow \dot{m}_1 = 5.95\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_1 = \rho_1 A_1 \mathrm{V}_1; \quad \Rightarrow \dot{m}_1 = \frac{A_1 \mathrm{V}_1}{v_1};$$

$$A_1 = \frac{\dot{m}_1 v_1}{\mathrm{V}_1}; \quad \Rightarrow A_1 = \frac{(5.95)(0.4)}{20}; \quad \Rightarrow A_1 = 0.119\ \text{m}^2; \quad \Rightarrow A_1 = 1{,}190\ \text{cm}^2$$

$$\text{(a.2)}\ \dot{m}_2 = \rho_2 A_2 \mathrm{V}_2;$$

$$A_2 = \frac{\dot{m}_2}{\rho_2 \mathrm{V}_2}; \quad \Rightarrow A_2 = \frac{50}{(1000)(5)}; \quad \Rightarrow A_2 = 0.01\ \text{m}^2; \quad \Rightarrow A_2 = 100\ \text{cm}^2$$

$$\text{(b)}\ A_3 = \frac{\dot{V}_3}{\mathrm{V}_3}; \quad \Rightarrow A_3 = \frac{\left(\frac{3.357}{60}\right)}{5.58}; \quad \Rightarrow A_3 = 0.01\ \text{m}^2; \quad \Rightarrow A_3 = 100\ \text{cm}^2$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the two inlet states, State-1 and State-2, and the exit state, State-3 from the properties supplied. The areas are calculated as part of the flow states. The TEST-code for this problem can be found in the professional site at www.thermofluids.net.

2-1-10 [NH] The diameter of the ports in the accompanying figures are 10 cm, 5 cm, and 1 cm at port 1, 2, and 3 respectively. At a given instant, water enters the tank at port 1 with a velocity of 2 m/s and leaves through port 2 with a velocity of 1 m/s. Assuming air to be insoluble in water and density of water and air to remain constant at 1000 kg/m^3 and 1 kg/m^3 respectively, determine:

(a) the mass flow rate (m) of water in kg/s at the inlet,

(b) the rate of change of mass of water $(dm/dt)_W$ in the tank

(c) the rate of change of mass of air $(dm/dt)_A$ in the tank,

(d) the velocity of air port 3.

SOLUTION

Since the water and air do not mix and their densities remain constant, we can find the rate of increase of volume of water in the tank which must be equal to the rate at which air is expelled.

(a) $\dot{m}_1 = \rho_w A_1 V_1; \quad \Rightarrow \dot{m}_1 = (1000)\left(\frac{\pi}{4}(0.1)^2\right)(2); \quad \Rightarrow \dot{m}_1 = 15.708 \ \frac{\text{kg}}{\text{s}}$

(b) $\dot{m}_2 = \rho_w A_2 V_2; \quad \Rightarrow \dot{m}_2 = (1000)\left(\frac{\pi}{4}(0.05)^2\right)(1); \quad \Rightarrow \dot{m}_2 = 1.96 \ \frac{\text{kg}}{\text{s}};$

Accumulation rate $(\dot{m}) = \dot{m}_1 - \dot{m}_2; \quad \Rightarrow \dot{m} = 15.708 - 1.96; \quad \Rightarrow \dot{m} = 13.74 \ \frac{\text{kg}}{\text{s}}$

(c) Volume of water accumulating is same as volume of air coming out of port 3.

$\dot{m} = \rho_w \dot{\mathcal{V}}; \quad \Rightarrow \dot{\mathcal{V}} = \frac{\dot{m}}{\rho_w}; \quad \Rightarrow \dot{\mathcal{V}} = \frac{13.74}{1000}; \quad \Rightarrow \dot{\mathcal{V}} = 0.01374 \ \frac{\text{m}^3}{\text{s}};$

$\dot{m}_3 = \rho_a \dot{\mathcal{V}}; \quad \Rightarrow \dot{m}_3 = (1)(0.01374); \quad \Rightarrow \dot{m}_3 = 0.01374 \ \frac{\text{kg}}{\text{s}}$

(d) $\dot{m}_3 = \rho_a A_3 V_3; \quad \Rightarrow V_3 = \frac{\dot{m}_3}{A_3 \rho_a}; \quad \Rightarrow V_3 = \frac{0.01374}{\left(\frac{\pi}{4}(0.01)^2\right)(1)};$

$\Rightarrow V_3 = 174.94 \ \frac{\text{m}}{\text{s}}$

2-1-11 [NN] Air is pumped into and withdrawn from a 10 m^3 rigid tank as shown in the accompanying figure. The inlet and exit conditions are as follow. Inlet: $v_1 = 2$ m^3/kg, $V_1 =$ 10 m/s, $A_1 = 0.01$ m^2; Exit: $v_2 = 5$ m^3/kg, $V_2 = 5$ m/s, $A_2 = 0.015$ m^2. Assuming the tank to be uniform at all time with the specific volume and pressure related through $p*v = 9.0$ (kPa-m^3), determine the rate of change of pressure in the tank.

SOLUTION

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \frac{dm}{dt} = \frac{A_1V_1}{v_1} - \frac{A_2V_2}{v_2}; \quad \Rightarrow \frac{dm}{dt} = \frac{(0.01)10}{2} - \frac{(0.015)5}{5};$$

$$\Rightarrow \frac{dm}{dt} = 0.035 \ \frac{\text{kg}}{\text{s}};$$

$$\frac{d\rho}{dt} = \frac{d}{dt}\left(\frac{m}{\forall}\right); \quad \Rightarrow \frac{d\rho}{dt} = \frac{1}{\forall}\frac{dm}{dt}; \quad \Rightarrow \frac{d\rho}{dt} = \frac{0.035}{10}; \quad \Rightarrow \frac{d\rho}{dt} = 0.0035 \ \frac{\text{kg}}{\text{m}^3\text{s}};$$

$$pv = 9.0; \quad \Rightarrow p = 9.0(\rho);$$

Taking the derivative of the pressure equation gives

$$\frac{dp}{dt} = 9.0\frac{d\rho}{dt}; \quad \Rightarrow \frac{dp}{dt} = (9.0)(0.0035); \quad \Rightarrow \frac{dp}{dt} = 0.0315 \ \frac{\text{kPa}}{\text{s}}$$

2-1-12 [NE] A gas flows steadily through a circular duct of varying cross-section area with a mass flow rate of 10 kg/s. The inlet and exit conditions are as follows. Inlet: V_1 = 400 m/s, A_1 = 179.36 cm^2; Exit: V_2 = 584 m/s, v_2 = 1.1827 m^3/kg. (a) Determine the exit area. (b) Do you find the increase in velocity of the gas accompanied by an increase in flow area counter-intuitive? Why?

SOLUTION

(a) $\cancel{\dfrac{dm}{dt}}^{\,0} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \dot{m}_i = \dot{m}_e = \dot{m} = 10\ \dfrac{\text{kg}}{\text{s}};$

$$\dot{m} = \frac{A_2 V_2}{v_2}; \quad \Rightarrow \dot{m} = 10\ \frac{\text{kg}}{\text{s}};$$

$$\Rightarrow A_2 = \frac{10(v_2)}{V_2}; \quad \Rightarrow A_2 = \frac{10(1.1827)}{584}; \quad \Rightarrow A_2 = 0.0202517\ \text{m}^2;$$

$$\Rightarrow A_2 = 202.52\ \text{cm}^2$$

(b) The increase in velocity and flow area may seem counter intuitive because we are used to flow of constant-density fluids such as water in which case an increase in area must accompany a decrease in velocity as required by the mass balance equation.

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the two states partially after selecting Custom gas. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-13 [NI] A pipe with a diameter of 10 cm carries nitrogen with a velocity of 10 m/s and specific volume 5 m^3/kg into a chamber. Surrounding the pipe, in an annulus of outer diameter 20 cm, is a flow of hydrogen entering the chamber at 20 m/s with a specific volume of 1 m^3/kg . The mixing chamber operates at steady state with a single exit of diameter 5 cm. If the velocity at the exit is 62 m/s, determine: (a) the mass flow rate in kg/s at the exit, (b) the specific volume of the mixture at the exit in m^3/kg, and (c) the apparent molar mass in kg/kmol of the mixture at the exit.

SOLUTION

$$\dot{m}_{\mathrm{N_2}} = \frac{A_{\mathrm{N_2}} V_{\mathrm{N_2}}}{v_{\mathrm{N_2}}}; \quad \Rightarrow \dot{m}_{\mathrm{N_2}} = \frac{\left(\frac{\pi}{4}(0.1)^2\right)(10)}{5}; \quad \Rightarrow \dot{m}_{\mathrm{N_2}} = 0.0157 \ \frac{\mathrm{kg}}{\mathrm{s}};$$

$$A_{\mathrm{H_2}} = \frac{\pi}{4}\left[(0.2)^2 - (0.1)^2\right]; \quad \Rightarrow A_{\mathrm{H_2}} = 0.0236 \ \mathrm{m}^2;$$

$$\dot{m}_{\mathrm{H_2}} = \frac{A_{\mathrm{H_2}} V_{\mathrm{H_2}}}{v_{\mathrm{H_2}}}; \quad \Rightarrow \dot{m}_{\mathrm{H_2}} = \frac{(0.0236)(20)}{1}; \quad \Rightarrow \dot{m}_{\mathrm{H_2}} = 0.4712 \ \frac{\mathrm{kg}}{\mathrm{s}};$$

(a) $\dot{m}_e = \dot{m}_{\mathrm{H_2}} + \dot{m}_{\mathrm{N_2}}; \quad \Rightarrow \dot{m}_e = 0.4712 + 0.0157; \quad \Rightarrow \dot{m}_e = 0.4869 \ \frac{\mathrm{kg}}{\mathrm{s}}$

(b) $v_e = \frac{A_e V_e}{\dot{m}_e}; \quad \Rightarrow v_e = \frac{\left(\frac{\pi}{4}(0.05)^2\right)(62)}{0.4869}; \quad \Rightarrow v_e = 0.25 \ \frac{\mathrm{m}^3}{\mathrm{kg}}$

(c) $\bar{M} = \frac{\dot{m}}{\dot{n}}; \quad \Rightarrow \bar{M} = \frac{\dot{m}_{\mathrm{N_2}} + \dot{m}_{\mathrm{H_2}}}{\dot{n}_{\mathrm{N_2}} + \dot{n}_{\mathrm{H_2}}}; \quad \Rightarrow \bar{M} = \frac{\dot{m}_{\mathrm{N_2}} + \dot{m}_{\mathrm{H_2}}}{\left(\dot{m}_{\mathrm{N_2}} / \bar{M}_{\mathrm{N_2}}\right) + \left(\dot{m}_{\mathrm{H_2}} / \bar{M}_{\mathrm{H_2}}\right)};$

$$\Rightarrow \bar{M} = \frac{0.4869}{(0.0157/28) + (0.4712/2)}; \quad \Rightarrow \bar{M} = 2.06 \ \frac{\mathrm{kg}}{\mathrm{kmol}}$$

TEST Solution:
Launch the PG flow-state TESTcalc. Evaluate the three states partially after selecting Custom gas. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-14 [NL] A pipe with a diameter of 15 cm carries hot air with a velocity 200 m/s and temperature 1000 K into a chamber. Surrounding the pipe, in an annulus of outer diameter 20 cm, is a flow of cooler air entering the chamber at 10 m/s with a temperature of 300 K . The mixing chamber operates at steady state with a single exit of diameter 20 cm. If the air exits at 646 K and the specific volume of air is proportional to the temperature (in K), determine: (a) the exit velocity in m/s and (b) the d*m*/d*t* for the mixing chamber in kg/s.

SOLUTION

$v = cT$ (Given)

$$\dot{m}_h = \frac{A_h V_h}{v_h}; \quad \Rightarrow \dot{m}_h = \frac{A_h V_h}{cT_h}; \quad \Rightarrow \dot{m}_h = \frac{\left(\frac{\pi}{4}(0.15)^2\right)(200)}{c(1000)}; \quad \Rightarrow \dot{m}_h = \frac{0.00353}{c}\ \frac{\text{kg}}{\text{s}};$$

$$A_c = \frac{\pi}{4}\left[(0.2)^2 - (0.15)^2\right]; \quad \Rightarrow A_c = 0.0137\ \text{m}^2;$$

$$\dot{m}_c = \frac{A_h V_h}{v_c}; \quad \Rightarrow \dot{m}_c = \frac{A_c V_c}{cT_c}; \quad \Rightarrow \dot{m}_c = \frac{(0.0137)(10)}{c(300)}; \quad \Rightarrow \dot{m}_c = \frac{0.000458}{c}\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_e = \dot{m}_h + \dot{m}_c; \quad \Rightarrow \dot{m}_e = \frac{0.00353}{c} + \frac{0.000458}{c}; \quad \Rightarrow \dot{m}_e = \frac{0.00399}{c}\ \frac{\text{kg}}{\text{s}};$$

(a) $$\dot{m}_e = \frac{A_e V_e}{v_e}; \quad \Rightarrow V_e = \frac{\dot{m}_e v_e}{A_e}; \quad \Rightarrow V_e = \frac{\left(\frac{0.00399}{\cancel{c}}\right)(\cancel{c})(646)}{0.0314}; \quad \Rightarrow V_e = 82.09\ \frac{\text{m}}{\text{s}}$$

(b) At steady state, the global state of the mixing chamber does not change with time. Therefore, its mass remains constant and

$$\frac{dm}{dt} = 0\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the three states partially after selecting Custom gas. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-15 [NG] Steam enters a turbine through a duct of diameter 0.25 m at 10 MPa, 600°C and 100 m/s. It exits the turbine through a duct of 1 m diameter at 400 kPa and 200°C. For steady state operation, determine (a) the exit velocity and (b) the mass flow rate of steam through the turbine. Use the PC flow-state daemon to obtain the density of steam at the inlet and exit ports. (c) What-if Scenario: What would the exit velocity be if the exit area were equal to the inlet area?

SOLUTION

$$A_1 = \frac{\pi(0.25)^2}{4}; \quad \Rightarrow A_1 = 0.049 \text{ m}^3; \quad \text{and} \quad A_2 = \frac{\pi(1)^2}{4}; \quad \Rightarrow A_2 = 0.7854 \text{ m}^3;$$

Using the surface state PC model for steam

$$\rho_1 = 26.0624 \ \frac{\text{kg}}{\text{m}^3}; \qquad \rho_2 = 1.87193 \ \frac{\text{kg}}{\text{m}^3};$$

At steady state

$$\frac{dm}{dt} = 0 = m_1 - m_2; \quad \Rightarrow m_1 = m_2;$$

(a) $\rho_1 A_1 \text{v}_1 = \rho_2 A_2 \text{v}_2;$

$$\Rightarrow v_2 = \frac{\rho_1 A_1 \text{v}_1}{\rho_2 A_2}; \quad \Rightarrow v_2 = \frac{(26.0624)(0.049)(100)}{(1.87193)(0.7854)}; \quad \Rightarrow v_2 = 86.9 \ \frac{\text{m}}{\text{s}}$$

(b) $\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho_1 A_1 \text{v}_1;$

$$\dot{m} = (26.0624)(0.049)(100); \quad \Rightarrow \dot{m} = 127.7 \ \frac{\text{kg}}{\text{s}}$$

(c) To find v_2 when $A_2 = A_1$

$$\rho_1 A_1 \text{v}_1 = \rho_2 A_1 \text{v}_2; \quad \Rightarrow \text{v}_2 = \frac{\rho_1 \cancel{A_1} \text{v}_1}{\rho_2 \cancel{A_1}}; \quad \Rightarrow \text{v}_2 = \frac{\rho_1 \text{v}_1}{\rho_2};$$

$$\Rightarrow \text{v}_2 = \frac{(26.0624)(100)}{(1.87193)}; \quad \Rightarrow \text{v}_2 = 1392.27 \ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-16 [NZ] Steam enters a turbine with a mass flow rate of 10 kg/s at 10 MPa, 600°C and 30 m/s. It exits the turbine at 45 kPa, 30 m/s and a quality of 0.9. Assuming steady-state operation, determine (a) the inlet area and (b) the exit area. Use the PC flow-state daemon.

SOLUTION

$$\dot{m}_i = \frac{A_i \mathrm{V}_i}{v_i}; \qquad \Rightarrow A_i = \frac{\dot{m}_i v_i}{\mathrm{V}_i};$$

Using the superheated vapor table, $v_{i(10\text{ MPa, }600^\circ\text{C})} = 0.03837\ \frac{\text{m}^3}{\text{kg}}$;

(a) $A_i = \frac{10(0.03837)}{30}; \qquad \Rightarrow A_i = 0.01279\text{ m}^2$

(b) Using the saturation pressure table for steam,

$$v_{f(45\text{ kPa})} = 0.00103\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{g(45\text{kPa})} = 3.5846\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{e(45\text{ kPa, }0.9)} = v_{f(45\text{ kPa})}(1-x) + v_{g(45\text{ kPa})}(x); \quad \Rightarrow v_{e(45\text{ kPa, }0.9)} = 0.00103(1-0.9) + 3.5846(0.9);$$

$$\Rightarrow v_{e(45\text{ kPa, }0.9)} = 3.226\ \frac{\text{m}^3}{\text{kg}};$$

At steady state

$$\dot{m}_i = \dot{m}_e = 10\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_e = \frac{A_e \mathrm{V}_e}{v_e}; \qquad \Rightarrow A_e = \frac{\dot{m}_e v_e}{\mathrm{V}_e}; \qquad \Rightarrow A_e = \frac{10(3.226)}{30}; \qquad \Rightarrow A_e = 1.075\text{ m}^2$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-17 [NK] Refrigerant R-134 enters a device as saturated liquid at 500 kPa with a velocity of 10 m/s and a mass flow rate of 2 kg/s. At the exit the pressure is 150 kPa and the quality is 0.2. If the exit velocity is 65 m/s, determine the (a) inlet and (b) exit areas. Use the PC flow-state daemon.

SOLUTION

(a) From the problem statement, $v_i = v_{f(500\text{ kPa})}$

Using the saturation pressure table for R-134,

$$v_{f(500\text{kPa})} = 0.000806\ \frac{\text{m}^3}{\text{kg}};$$

$$\dot{m}_i = \frac{A_i V_i}{v_i};\quad \Rightarrow A_i = \frac{\dot{m}_i v_i}{V_i};\quad \Rightarrow A_i = \frac{2(0.000806)}{10};\quad \Rightarrow A_i = 0.000161\text{ m}^2;$$

$$\Rightarrow A_i = 1.61\text{ cm}^2$$

(b) Using the saturation pressure table for R134,

$$v_{f(150\text{ kPa})} = 7.4\times10^{-4}\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{g(150\text{ kPa})} = 0.13149\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{e(150\text{ kPa, }0.2)} = v_{f(150\text{ kPa})}(1-x) + v_{g(150\text{ kPa})}(x);$$

$$\Rightarrow v_{e(150\text{ kPa, }0.2)} = (7.4\times10^{-4})(1-0.2) + 0.13149(0.2);\quad \Rightarrow v_{e(150\text{ kPa, }0.2)} = 0.02689\ \frac{\text{m}^3}{\text{kg}};$$

At steady state, $\dot{m}_i = \dot{m}_e = 2\ \frac{\text{kg}}{\text{s}};$

$$\dot{m}_e = \frac{A_e \text{V}_e}{v_e};\quad \Rightarrow A_e = \frac{\dot{m}_e v_e}{\text{V}_e};\quad \Rightarrow A_e = \frac{2(0.02689)}{65};$$

$$\Rightarrow A_e = 0.000827\text{ m}^2;\quad \Rightarrow A_e = 8.27\text{ cm}^2$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-18 [NP] Air enters a 0.5m diameter fan at 25°C, 100 kPa and is discharged at 28°C, 105 kPa and a volume flow rate of 0.8 m^3/s. Determine for steady-state operation, (a) the mass flow rate of air in kg/min and (b) the inlet and (c) exit velocities. Use the PG flow-state daemon.

SOLUTION

From the ideal gas property table for air,

$$\rho_{i(100\text{ kPa},\ 25^{\circ}\text{C})} = 1.169\ \frac{\text{kg}}{\text{m}^3};$$

$$\rho_{e(105\text{ kPa},\ 28^{\circ}\text{C})} = 1.215\ \frac{\text{kg}}{\text{m}^3};$$

$$A_i = A_e;$$

$$A = \frac{\pi(d)^2}{4}; \qquad \Rightarrow A = \frac{\pi(0.5)^2}{4}; \qquad \Rightarrow A = 0.19635\ \text{m}^3;$$

(a) At steady state,

$$\dot{m} = \dot{m}_i = \dot{m}_e = \dot{V}_e \rho_e;$$

$$\dot{m} = (0.8)(1.215); \qquad \Rightarrow \dot{m} = 0.97193\ \frac{\text{kg}}{\text{s}}; \qquad \Rightarrow \dot{m} = 58.3\ \frac{\text{kg}}{\text{min}}$$

(b) $\dot{m} = \rho_i A_i \text{v}_i;$

$$\text{v}_i = \frac{\dot{m}}{\rho_i A_i}; \qquad \Rightarrow \text{v}_i = \frac{0.97193}{(1.1687)(0.19635)}; \qquad \Rightarrow \text{v}_i = 4.23\ \frac{\text{m}}{\text{s}}$$

(c) $$\text{v}_e = \frac{\dot{V}_e}{A}; \qquad \Rightarrow \text{v}_e = \frac{0.8}{0.19635}; \qquad \Rightarrow \text{v}_e = 4.07\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-19 [NU] Air enters a nozzle, which has an inlet area of 0.1 m^2, at 200 kPa, 500°C and 10 m/s. At the exit the conditions are 100 kPa and 443°C. If the exit area is 35 cm^2, determine the steady-state exit velocity. Use the IG flow-state daemon.

SOLUTION

From the ideal gas property table for air,

$$\rho_{i(200\text{ kPa, }500^{o}\text{C})} = 0.9014 \ \frac{\text{kg}}{\text{m}^3};$$

$$\rho_{e(100\text{ kPa, }443^{o}\text{C})} = 0.4866 \ \frac{\text{kg}}{\text{m}^3};$$

At steady state,

$$\dot{m}_i = \dot{m}_e; \qquad \Rightarrow \rho_i A_i \text{v}_i = \rho_e A_e \text{v}_e;$$

$$\text{v}_e = \frac{\rho_i A_i \text{v}_i}{\rho_e A_e}; \qquad \Rightarrow \text{v}_e = \frac{(0.9014)(0.1)(10)}{(0.4866)(0.0035)}; \qquad \Rightarrow \text{v}_e = 529.3 \ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.