

# **Drivers of CO<sub>2</sub> emissions from electricity generation in the European Union 2000-2015 - Appendix**

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# Appendix

## Appendix A: Procedure for five renewables

In structural decomposition analysis (SDA) there are  $n!$  permutations (Dietzenbacher and Los, 1998). De Boer (2009) showed that the (unweighted) arithmetic average, by collecting duplicates, can be reduced to the computation of the weighted average of  $2^{n-1}$  combinations. De Boer and Rodrigues (2019) discussed multiplicative and additive decomposition methods and applied them to the decomposition of carbon dioxide emissions in the Netherlands into five driving factors. The so-called Bennet SDA method is comparable to the procedure we used in this paper to assess the contribution of each of the five renewables.

In this appendix we show that the unweighted arithmetic average of all  $5! = 120$  permutations (equation 8) is equivalent to the weighted average of  $2^4 = 16$  combinations (see the Table below.) The five renewables are denoted by the letters A, B, C, D, E and the fossil factor by the letter F. The common logarithmic mean of all permutations is denoted by LMEAN (see equation 5). Without loss of generality we consider renewable A. In a particular permutation we have in the denominator at position 1 always F and A at respectively, the positions 2 up to and including 6.

**A at position 2.** For positions 3,..., 6 there are  $4! = 24$  permutations of B through E.

In comparison and base period we have:

$$F^1/(F^1+A^1) \text{ and } F^0/(F^0+A^0) \text{ and } LMEAN * \log \{F^1/(F^1+A^1) / F^0/(F^0+A^0)\}.$$

It is an additive decomposition so that only this term matters. The weight of this term in the average is  $24/120 = 1/5$ .

In the sequel we use the superscript  $t$  which is 1 in comparison and 0 in base period. Moreover, we omit the pertinent definition of LMEAN.

**A at position 3.** There are four possibilities for position 2: B, C, D or E. For the positions 4- 6 there are  $3! = 6$  permutations for the three remaining letters. We have:

$$(F^t+B^t)/(F^t+B^t+A^t) \quad (F^t+C^t)/(F^t+C^t+A^t) \quad (F^t+D^t)/(F^t+D^t+A^t) \quad (F^t+E^t)/(F^t+E^t+A^t)$$

Each one has weight  $6/120 = 1/20$ .

**A position 4.** For positions 2 and 3 we have 6 possibilities:

$$(F^t+B^t+C^t)/(F^t+B^t+C^t+A^t) \quad (F^t+B^t+D^t)/(F^t+B^t+D^t+A^t) \quad (F^t+B^t+E^t)/(F^t+B^t+E^t+A^t)$$

$$(F^t+C^t+D^t)/(F^t+C^t+D^t+A^t) \quad (F^t+C^t+E^t)/(F^t+C^t+E^t+A^t) \quad (F^t+D^t+E^t)/(F^t+D^t+E^t+A^t)$$

Each one occurs 4 times, so that each weight is  $1/30$ .

From equation 7 we derive for the final two positions:

**A at position 6** as the mirror of ‘A at position 2’, i.e.

$$(TOT^t-A^t)/TOT^t \text{ with weight } 1/5, \text{ and}$$

**A at position 5** as the mirror of ‘A at position 3’:

$$(TOT^t-B^t-A^t)/TOT^t \quad (TOT^t-C^t-A^t)/TOT^t \quad (TOT^t-D^t-A^t)/TOT^t \quad (TOT^t-E^t-A^t)/TOT^t$$

For each one the weight is  $1/20$ .

Summarizing Table with position of  $A^t$  ( $t = 0, 1$ ) and corresponding weight

Position					Weight
2	$F^t/(F^t+A^t)$				$1/5$
3	$(F^t+B^t)/$ $(F^t+B^t+A^t)$	$(F^t+C^t)/(F^t+C^t+A^t)$	$(F^t+D^t)/$ $(F^t+D^t+A^t)$	$(F^t+E^t)/$ $(F^t+E^t+A^t)$	$1/20$
4	$(F^t+B^t+C^t)/$ $(F^t+B^t+C^t+A^t)$	$(F^t+B^t+D^t)/$ $(F^t+B^t+D^t+A^t)$	$(F^t+B^t+E^t)/$ $(F^t+B^t+E^t+A^t)$		$1/30$
4	$(F^t+C^t+D^t)/$ $(F^t+C^t+D^t+A^t)$	$(F^t+C^t+E^t)/$ $(F^t+C^t+E^t+A^t)$	$(F^t+D^t+E^t)/$ $(F^t+D^t+E^t+A^t)$		$1/30$
5	$(TOT^t-B^t-A^t)/$ $TOT^t$	$(TOT^t-C^t-A^t)/$ $TOT^t$	$(TOT^t-D^t-A^t)/$ $TOT^t$	$(TOT^t-E^t-A^t)/$ $TOT^t$	$1/20$

The impact of renewable A on emissions from fossil sources (F) is equal to:

$$LMEAN \times [(1/5) \times A_{2,6} + (1/20) \times A_{3,5} + (1/30) \times A_4]$$

with:

$$A_{2,6} = \log\{(F^1/(F^1+A^1)/F^0/(F^0+A^0))\} + \log\{(TOT^1-A^1)/TOT^1/(TOT^0-A^0)/TOT^0\}$$

$$A_{3,5} = \log\{(F^1+B^1)/(F^1+B^1+A^1)/ (F^0+B^0)/(F^0+B^0+A^0)\} + \dots +$$

$$\log\{(F^1+E^1)/(F^1+E^1+A^1)/ (F^0+E^0)/(F^0+E^0+A^0)\} +$$

$$\log\{(TOT^1-B^1-A^1)/TOT^1 / (TOT^0-B^0-A^0)/TOT^0\} + \dots +$$

$$\log\{(TOT^1-E^1-A^1)/TOT^1\} / (TOT^0-E^0-A^0)/TOT^0\}$$

$$A_4 = \log\{(F^1+B^1+C^1)/(F^1+B^1+C^1+A^1)\} / (F^0+B^0+C^0)/(F^0+B^0+C^0+A^0)\} + \dots +$$

$$\log\{(F^1+D^1+E^1)/(F^1+D^1+E^1+A^1)\} / (F^0+D^0+E^0)/(F^0+D^0+E^0+A^0)\}$$

## References

Dietzenbacher, E., Los, B., (1998). Structural decomposition techniques: Sense and sensitivity. *Economic Systems Research*, 10, 307-323.

De Boer, P. (2009). Generalized Fisher index or Siegel-Shapley decomposition? *Energy Economics*, 31, 810-814.

De Boer, P., Rodrigues, J.F.D., 2019. Decomposition analysis: When to use which method? *Economic Systems Research* 32, 1-28. DOI: <https://doi.org/10.1080/09535314.2019>

## Appendix B: Proof of the international trade share formula

The starting data are the reported international trade data without reallocation of reexports, matrix  $\mathbf{Z}$ , and final use of electricity, vector  $\mathbf{y}$ . From these, total output of electricity is obtained as

$$\mathbf{x} = \mathbf{Z} \mathbf{i} + \mathbf{y} + \mathbf{exp} \quad \text{Eq. B1}$$

and domestic production of electricity is calculated as:

$$\mathbf{v}' = \mathbf{x}' - \mathbf{i}' \mathbf{Z} - \mathbf{imp}' \quad \text{Eq. B2}$$

Our assumption is that each unit of electricity delivered by a country (whether for exports or for final use) is generated using the same mix of domestically produced electricity and imports. That is, for country  $j$  to deliver one unit of output of electricity it imports from country  $i$  a flow of electricity  $A_{ij} = Z_{ij} / x_j$ , from its domestic production  $b_j = v_j / x_j$ , and from imports  $c_j = imp_j / x_j$ . Using the definition of  $A_{ij}$  Eq. B1 can be rewritten as:

$$\mathbf{Z} + \mathbf{y} + \mathbf{exp} = \mathbf{x} \Leftrightarrow \mathbf{A} \mathbf{x} + \mathbf{y} + \mathbf{exp} = \mathbf{x} \Leftrightarrow \mathbf{y} + \mathbf{exp} = \mathbf{x} - \mathbf{A} \mathbf{x} \Leftrightarrow \mathbf{y} + \mathbf{exp} = (\mathbf{I} - \mathbf{A}) \mathbf{x} \Leftrightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} (\mathbf{y} + \mathbf{exp}) \quad \text{Eq. B3}$$

We can now separate the total electricity production into a domestic and export component (appended with superscripts *dom* and *exp*):

$$\mathbf{x}^{dom} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{y} \text{ and } \mathbf{x}^{exp} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{exp} \quad \text{Eq. B4}$$

We have identified the total electricity output that is generated by final demand in each member state. However, we are not interested in obtaining a vector  $\mathbf{x}^{dom}$  (which combines the total electricity output of country  $i$  that results from the final demand of all countries), but a matrix  $\mathbf{X}^{dom}$  (which identifies the total electricity output in country  $i$  that results from final demand in country  $j$ ), so we diagonalize the demand vector:

$$\mathbf{X}^{dom} = (\mathbf{I} - \mathbf{A})^{-1} \text{diag}(\mathbf{y}) \quad \text{Eq. B5}$$

Finally, we are interested in obtaining the domestically produced electricity output, which can be obtained by appending the diagonal of vector  $\mathbf{b}$  to  $\mathbf{X}^{dom}$ :

$$\mathbf{R} = \text{diag}(\mathbf{b}) \mathbf{X}^{dom} = \text{diag}(\mathbf{b}) (\mathbf{I} - \mathbf{A})^{-1} \text{diag}(\mathbf{y}) \quad \text{Eq. B6}$$

Entry  $R_{ij}$  indicates the electricity domestically produced in country  $i$  to satisfy final use in country  $j$ . We can also define the vectors of domestic production to satisfy exports,  $\mathbf{r}^{exp}$ , and imports to satisfy domestic consumption,  $\mathbf{r}^{imp}$ :

$$\mathbf{r}^{exp} = \text{diag}(\mathbf{b}) \mathbf{x}^{exp} = \text{diag}(\mathbf{b}) (\mathbf{I} - \mathbf{A})^{-1} \mathbf{exp} \text{ and } (\mathbf{r}^{imp})' = \mathbf{c}' \mathbf{X}^{dom} = \mathbf{c}' (\mathbf{I} - \mathbf{A})^{-1} \text{diag}(\mathbf{y}) \quad \text{Eq. B7}$$

Note that the column sum of  $\mathbf{R}$  plus  $(\mathbf{r}^{imp})'$  is identical to  $\mathbf{y}'$  (the production for final demand in  $j$  over all countries and imports matches total final demand of  $j$ ) and the row sum of  $\mathbf{R}$  plus  $\mathbf{r}^{exp}$  is

identical to  $\mathbf{v}$  (the domestic production in  $i$  is exhaustively split over the final use of all countries and exports).

That is, we first want to prove that  $\mathbf{i}' \mathbf{R} + (\mathbf{r}^{imp})' = \mathbf{y}'$ :

$$\mathbf{i}' \mathbf{R} + (\mathbf{r}^{imp})' = (\mathbf{i}' \text{diag}(\mathbf{b}) + \mathbf{c}')(\mathbf{I} - \mathbf{A})^{-1} \text{diag}(\mathbf{y}) = (\mathbf{b}' + \mathbf{c}')(\mathbf{I} - \mathbf{A})^{-1} \text{diag}(\mathbf{y}) = \mathbf{y}' \text{diag}((\mathbf{b}' + \mathbf{c}') (\mathbf{I} - \mathbf{A})^{-1})$$

So now we need to prove that  $\text{diag}((\mathbf{b}' + \mathbf{c}') (\mathbf{I} - \mathbf{A})^{-1}) = \mathbf{I}$

$$\begin{aligned} \text{diag}((\mathbf{b}' + \mathbf{c}') (\mathbf{I} - \mathbf{A})^{-1}) = \mathbf{I} &\Leftrightarrow (\mathbf{b}' + \mathbf{c}') (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{i}' \Leftrightarrow \mathbf{b}' + \mathbf{c}' = \mathbf{i}' (\mathbf{I} - \mathbf{A}) \Leftrightarrow \mathbf{b}' + \mathbf{c}' + \mathbf{i}' \mathbf{A} = \mathbf{i}' \\ &\Leftrightarrow (\mathbf{v}' + \mathbf{imp}' + \mathbf{i}' \mathbf{Z}) \text{diag}(\mathbf{x})^{-1} = \mathbf{i}' \Leftrightarrow \mathbf{v}' + \mathbf{imp}' + \mathbf{i}' \mathbf{Z} = \mathbf{i}' \text{diag}(\mathbf{x}) \Leftrightarrow \mathbf{x}' = \mathbf{x}' \end{aligned}$$

This completes the proof.

Then we want to prove that  $\mathbf{R} \mathbf{i} + \mathbf{r}^{exp} = \mathbf{v}$ :

$$\mathbf{R} \mathbf{i} + \mathbf{r}^{exp} = \text{diag}(\mathbf{b}) (\mathbf{I} - \mathbf{A})^{-1} \text{diag}(\mathbf{y}) \mathbf{i} + \mathbf{r}^{exp} =$$

$$\text{diag}(\mathbf{b}) (\mathbf{I} - \mathbf{A})^{-1} (\mathbf{y} + \mathbf{exp}) = \text{diag}(\mathbf{b}) \mathbf{x} = \text{diag}(\mathbf{v}) \text{diag}(\mathbf{x})^{-1} \mathbf{x} = \text{diag}(\mathbf{x}) \text{diag}(\mathbf{x})^{-1} \mathbf{v} = \mathbf{v}$$

This completes the proof.

Appendix C: Discussion figures

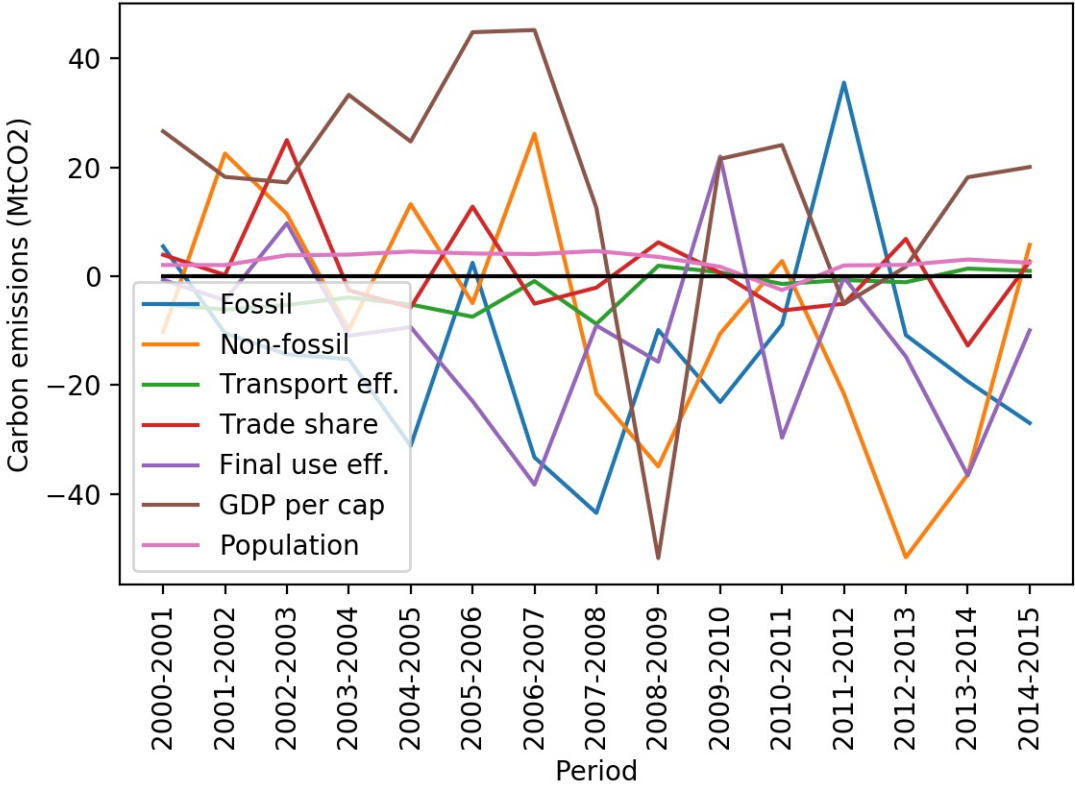


Figure SI.1a) Yearly decomposition by factor.

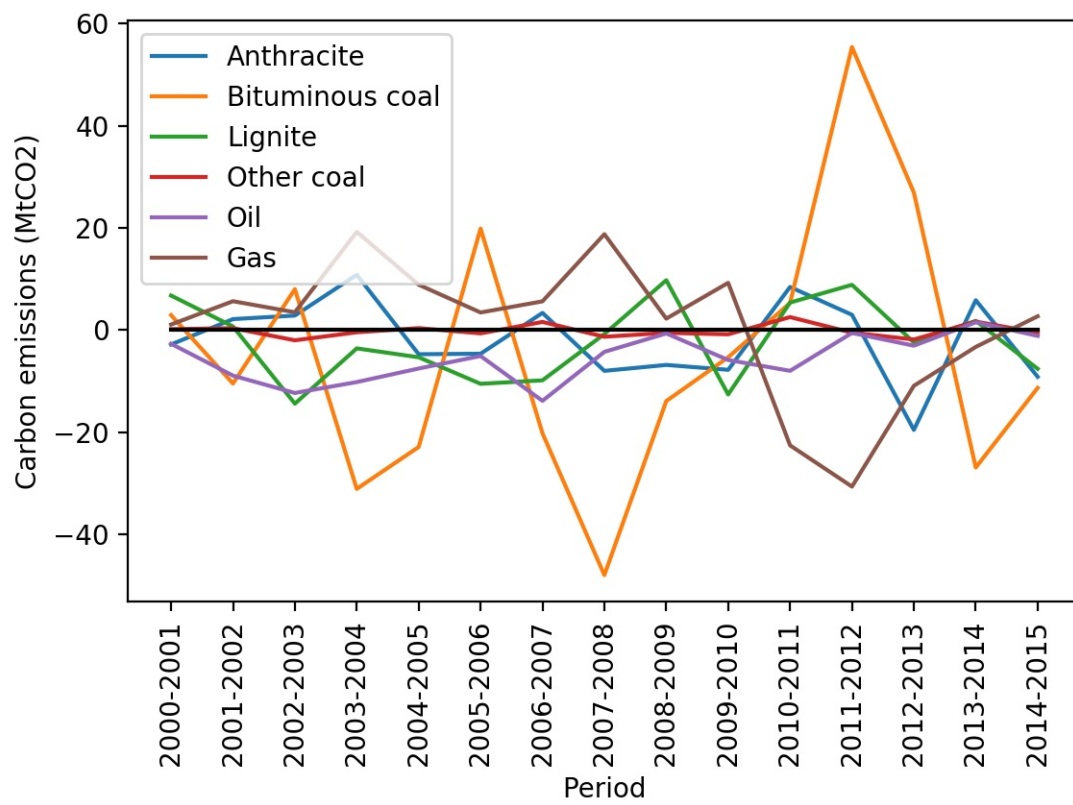


Figure SI.1b) Yearly decomposition by fossil fuel carrier.

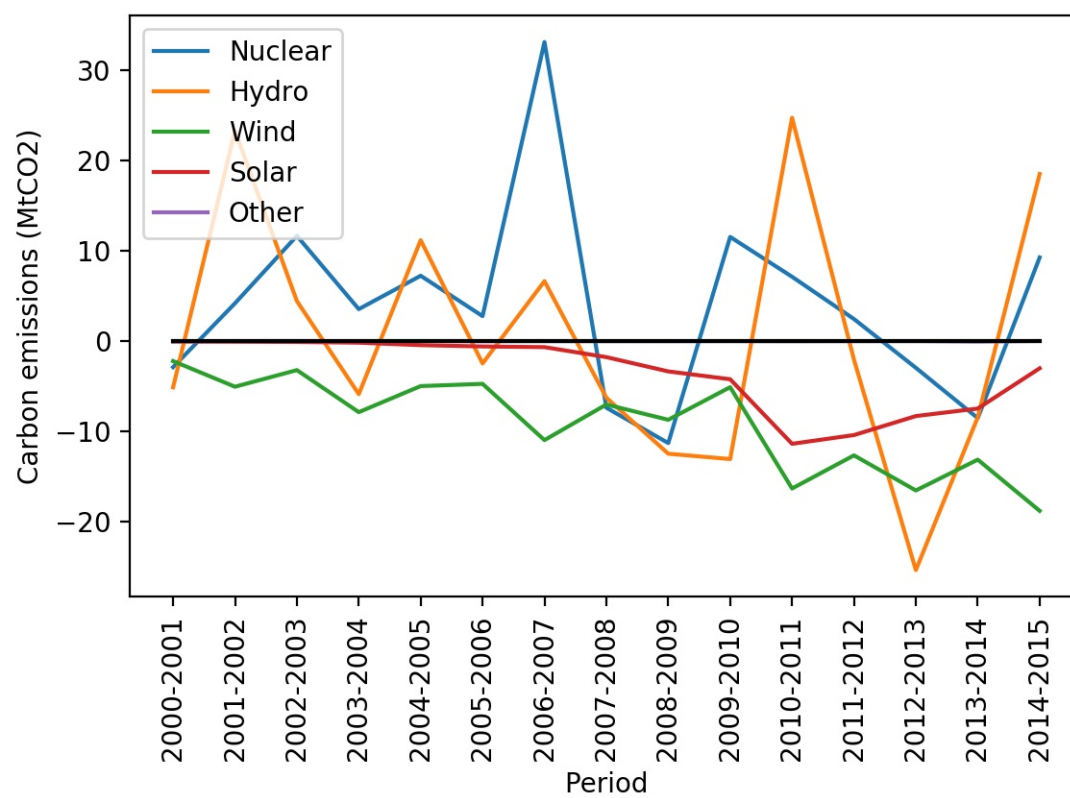


Figure SI.1c) Yearly decomposition by non-fossil electricity type.





Figure SI.1d) Yearly decomposition by final use of electricity.

Figure SI1) The results of yearly index decomposition of carbon emissions from electricity generation in the EU25. Decomposition by factor type, fossil fuel carriers, non-fossil electricity types and final use of electricity. NS = non-specified.

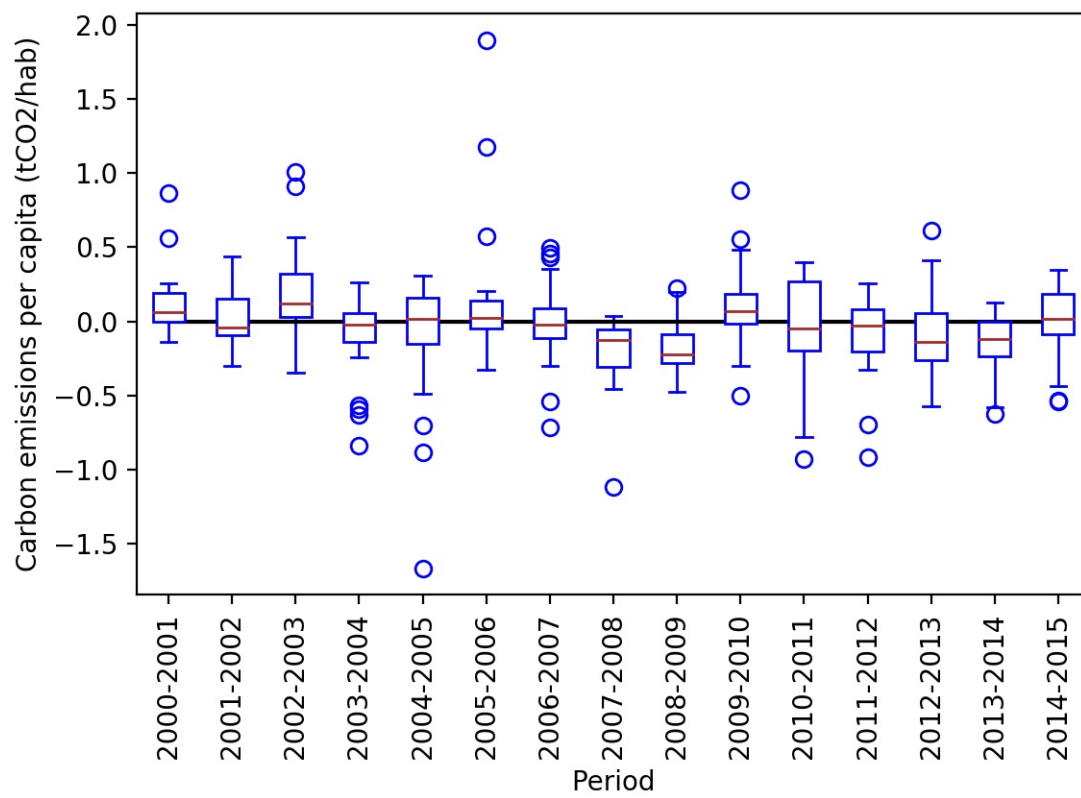


Figure SI2) Boxplot of the per capita yearly decomposition by country. Brown line is the median, the box is the first and third quartile and the whisker is 1.5 times the interquartile distance, with circles being outliers, i.e., countries whose carbon emissions per capita lie outside the whiskers.