The Complexity of Gradient Descent: CLS = PPAD ∩ PLS Proof Report

This is the proof report generated by the automated prover. The document contains, in order, the following

- A list of the templates the were given as input to the prover.
- For each square in each template, a report giving the output of the proof for that particular square.
- For each square on the boundary of the template "X-Full Boundary", a report giving the output of the boundary proof for that square. Squares that are on multiple boundaries will have one report for each side that they touch.

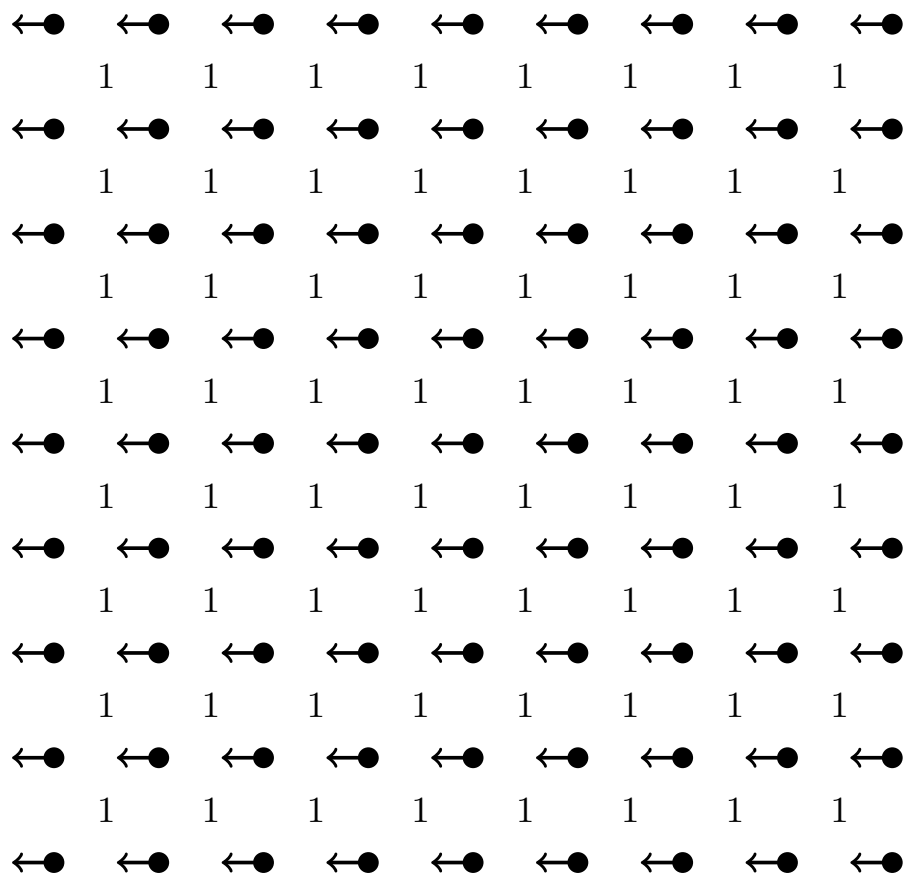
Each square that was checked was assigned a number, and for easy referencing, in the drawing for each template the number of each square is displayed. If a square's number is black, then this indicates that the automated proof verified that the lemma holds for particular square, whereas if a square's number is red, then this indicates that the automated proof falsified the lemma for that particular square.

It can be verified that the red squares appear only at the starts and ends of PPAD lines, as well as at solutions in the PLS labyrinth, which is exactly where we expect solutions of the instances to be.

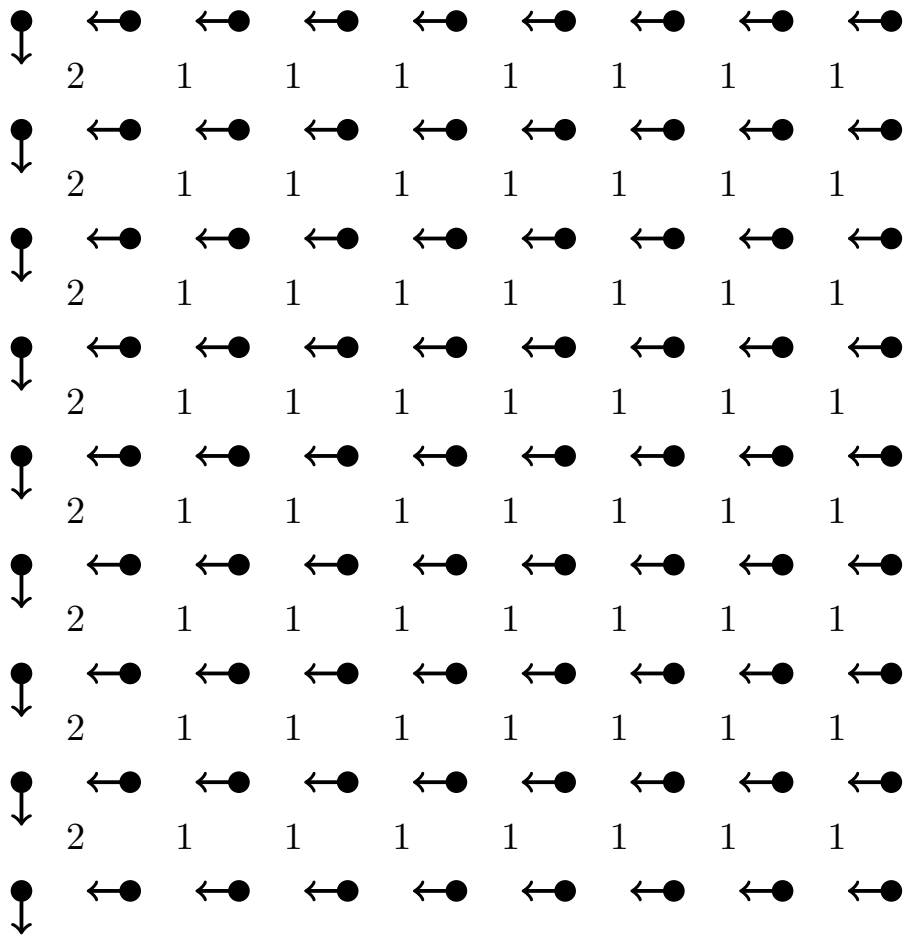
Parameters. For this report, the following parameters were used.

- eps = 0.01 :: SReal
- delta = 0.5 :: SReal
- color offset = 4.0 :: SReal

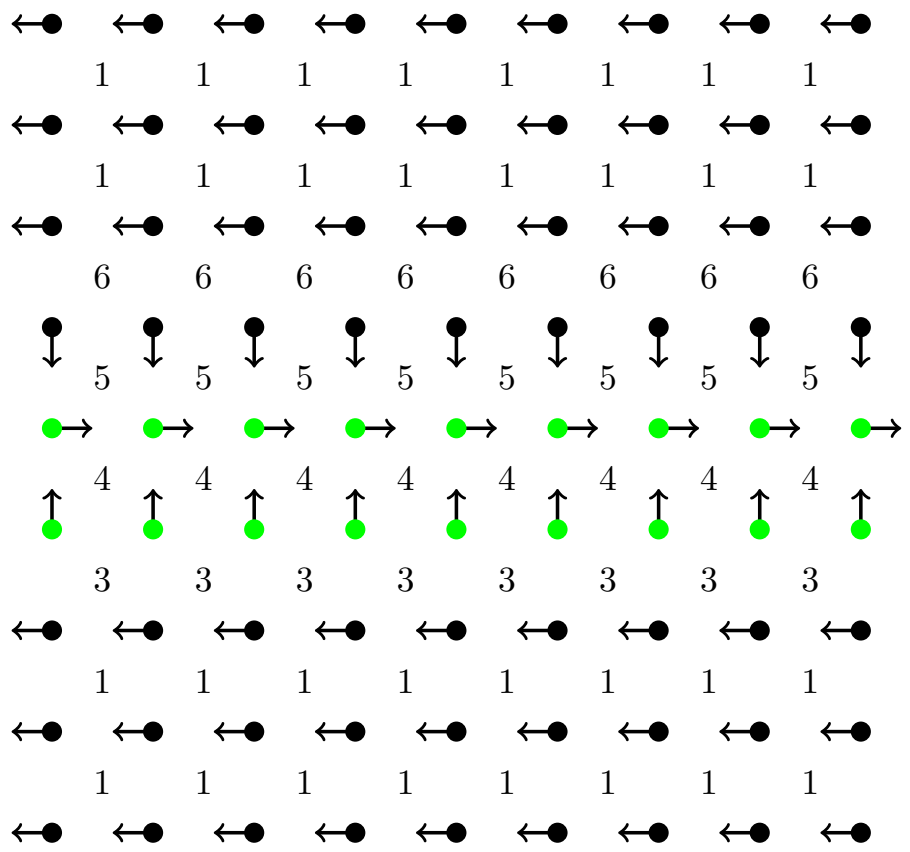
E1-Standard Environment



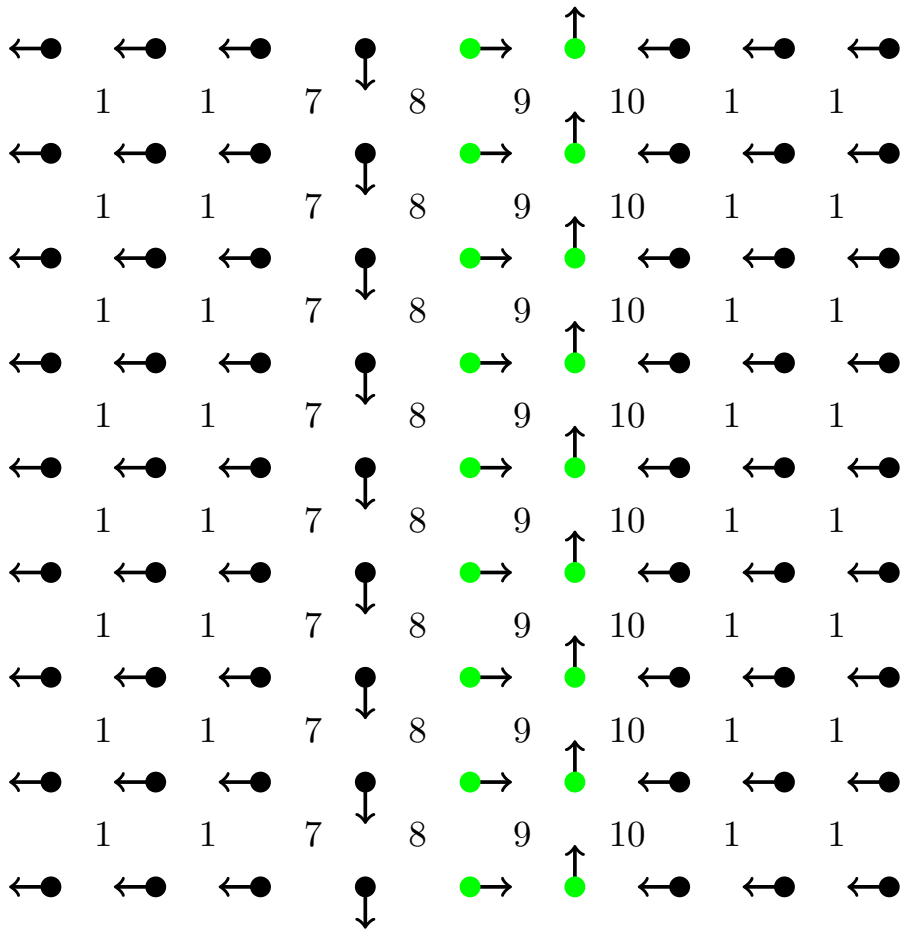
E2-Left Boundary Environment



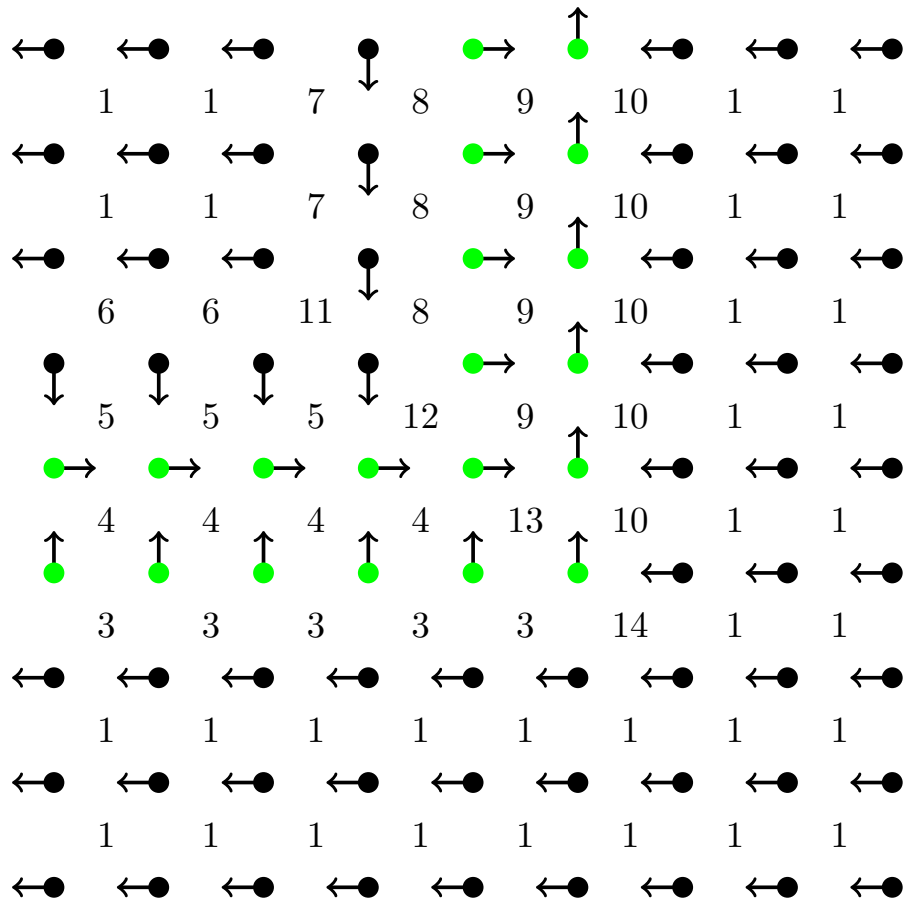
G1-Horizontal Green Path



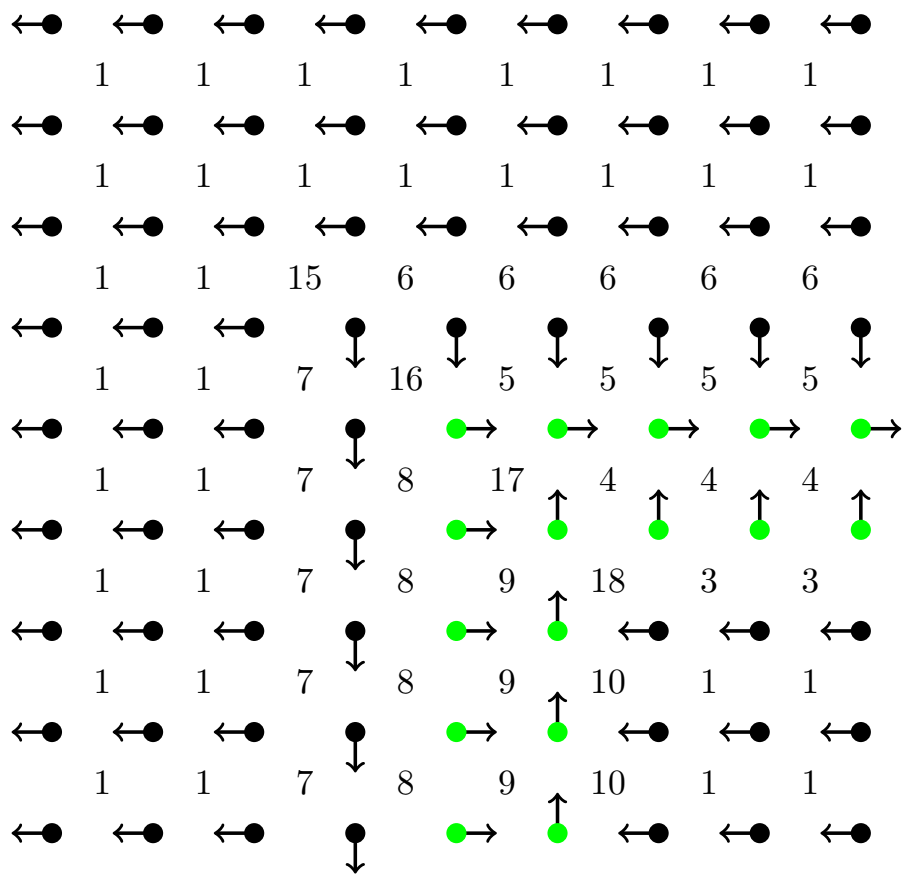
G2-Vertical Green Path



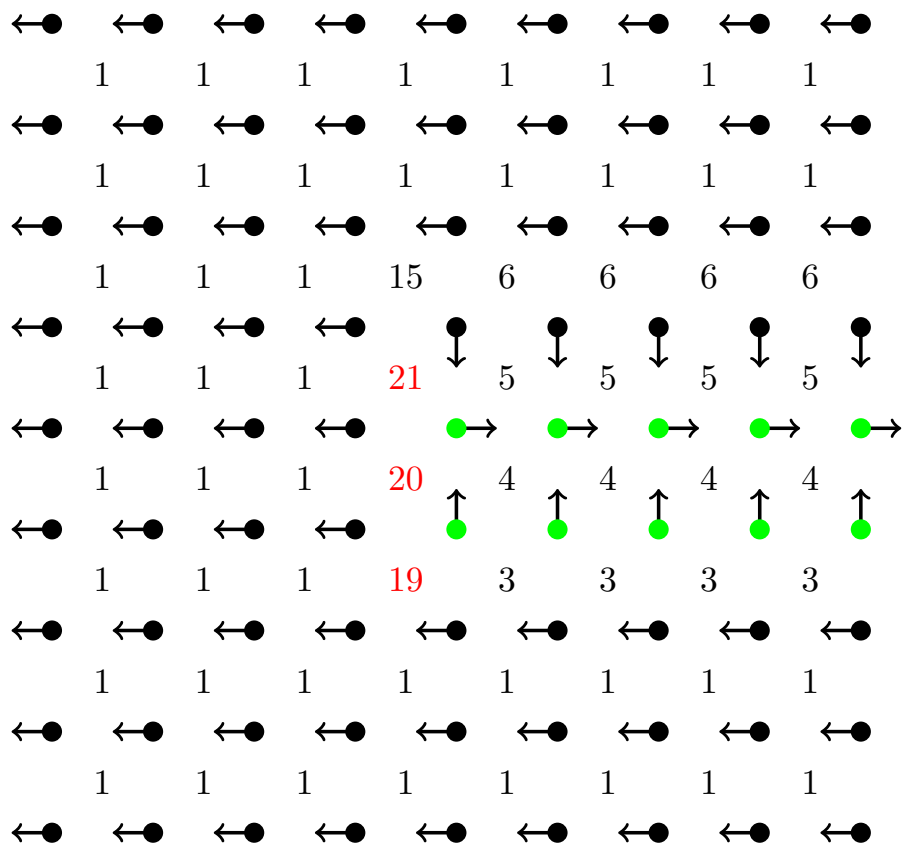
G3-Green Path Turn Up



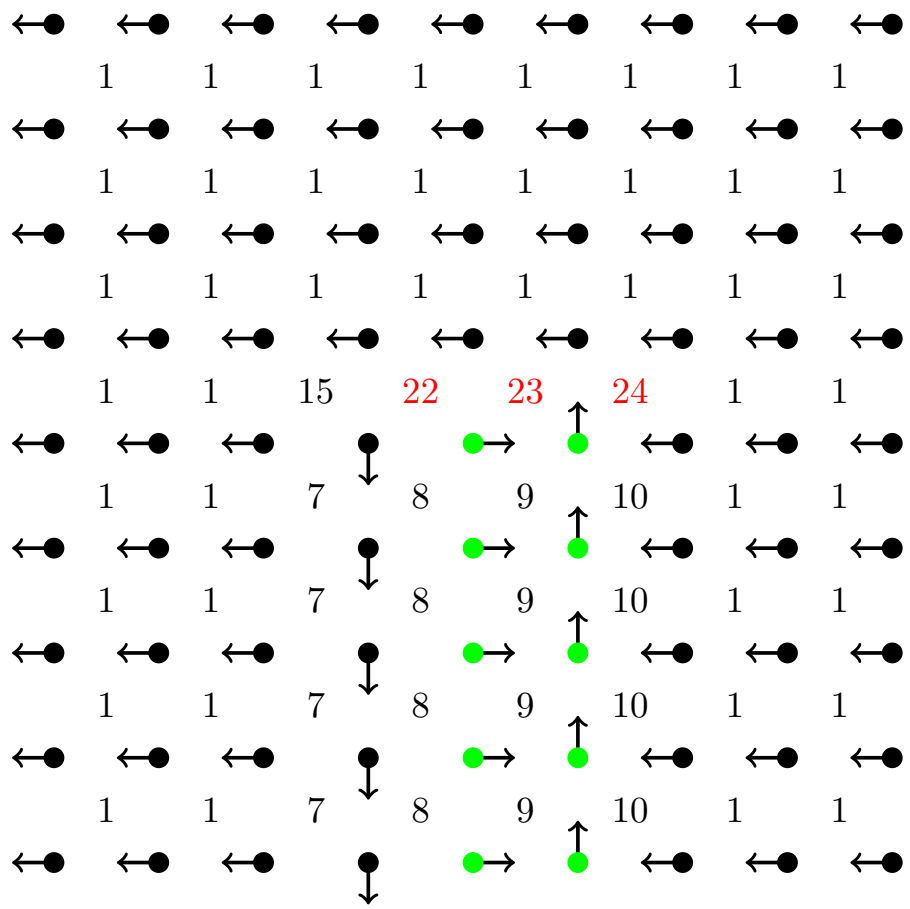
G4-Green Path Turn Right



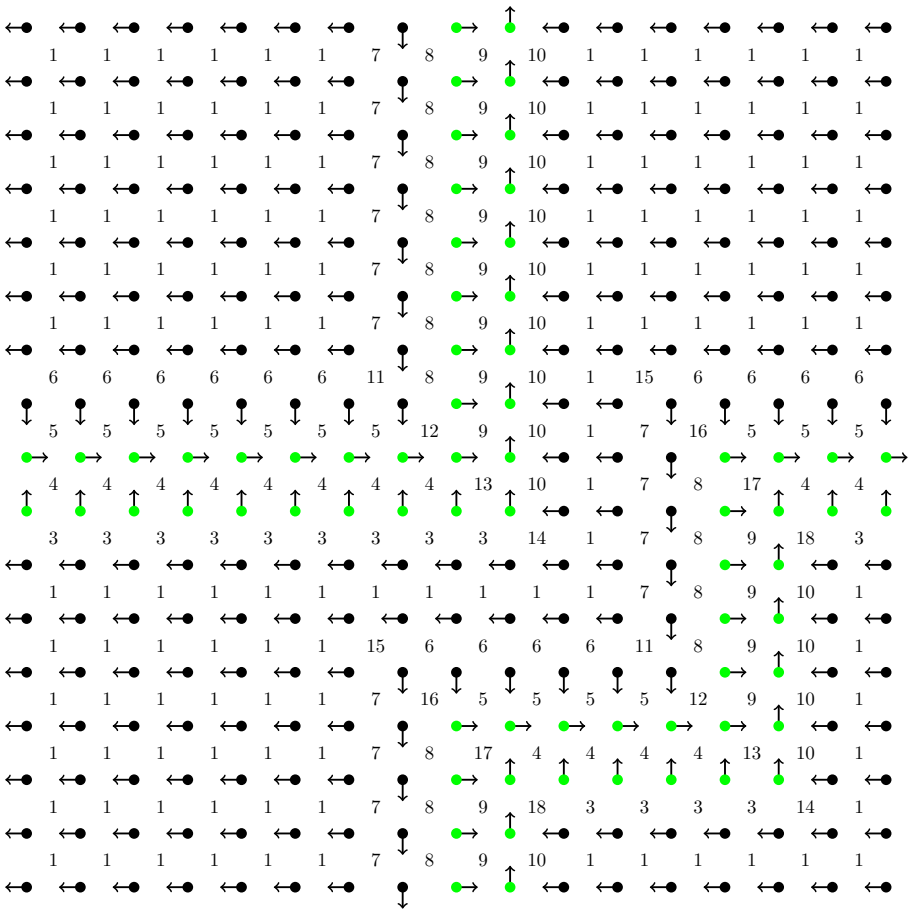
G5-Green Path Source



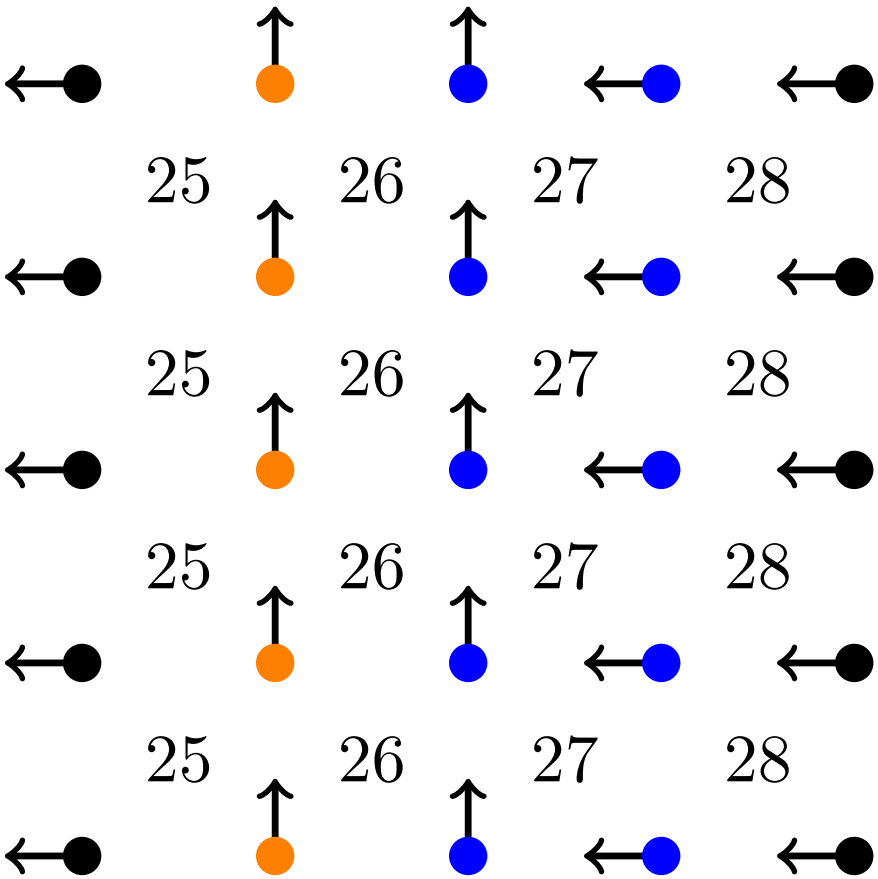
G6-Green Path Sink



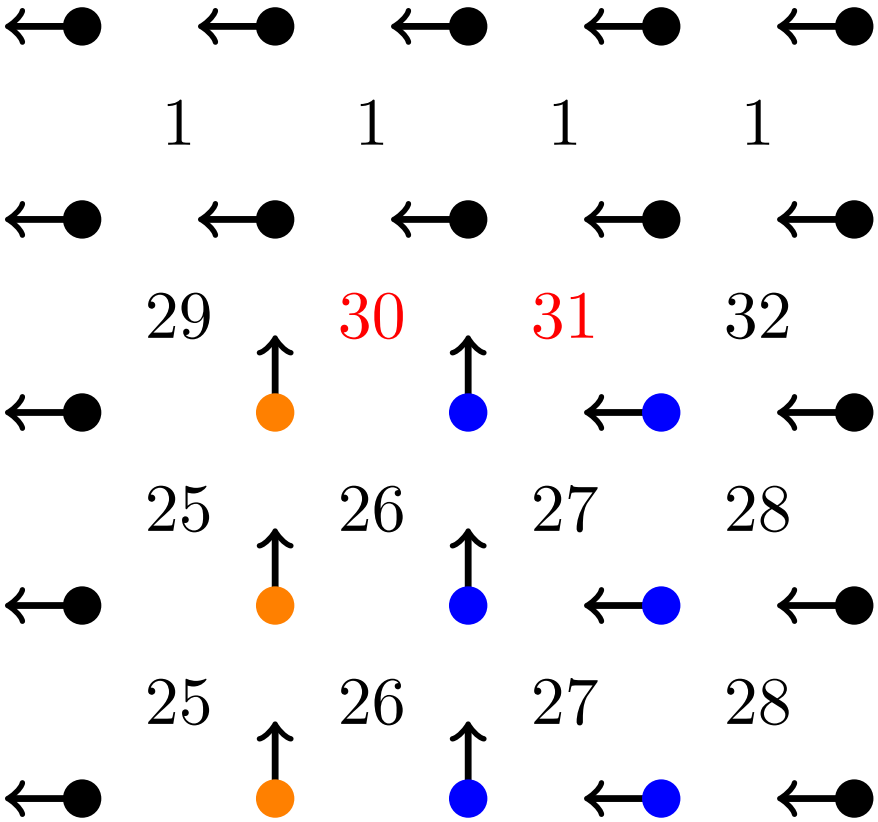
G7-Green Path Crossing



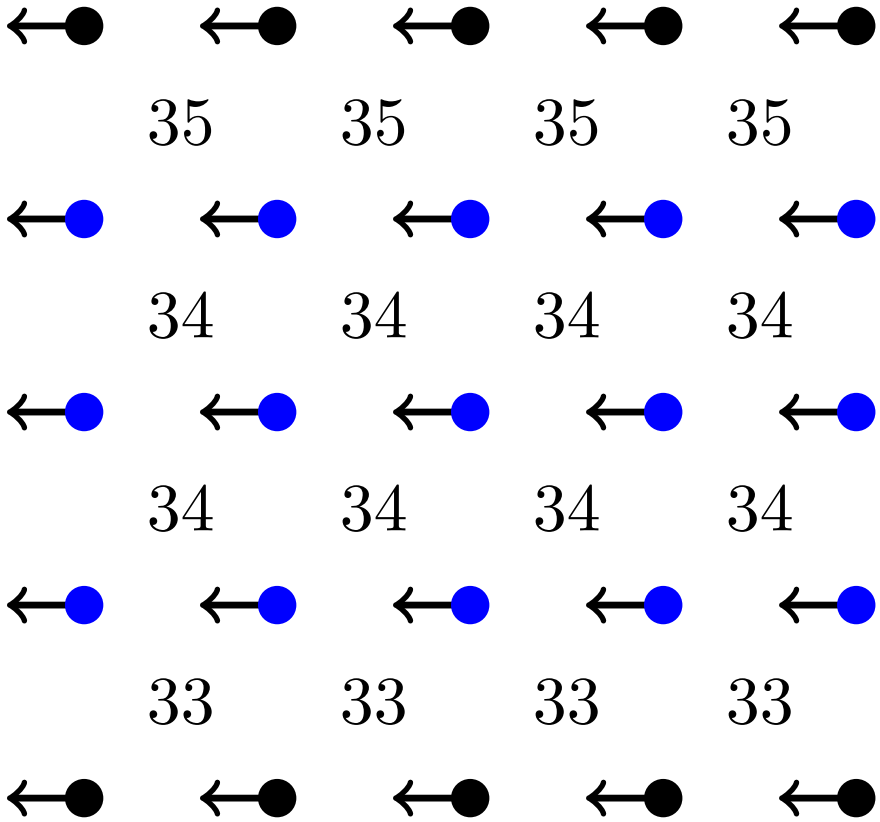
LA1-Orange-Blue Path



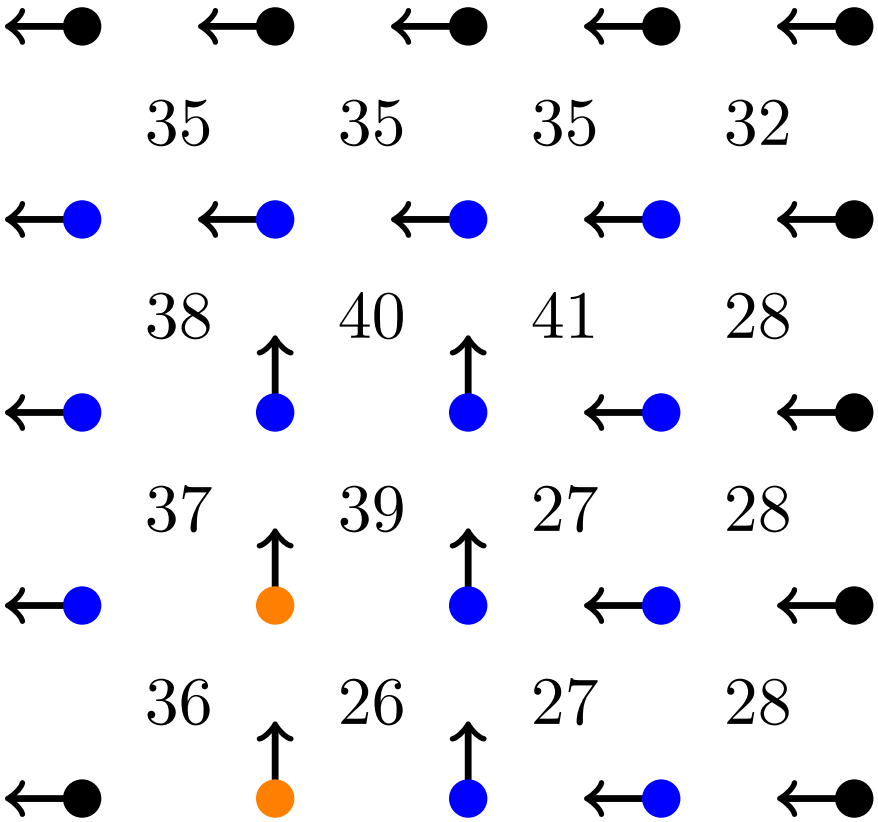
LA2-Orange-Blue Path Sink



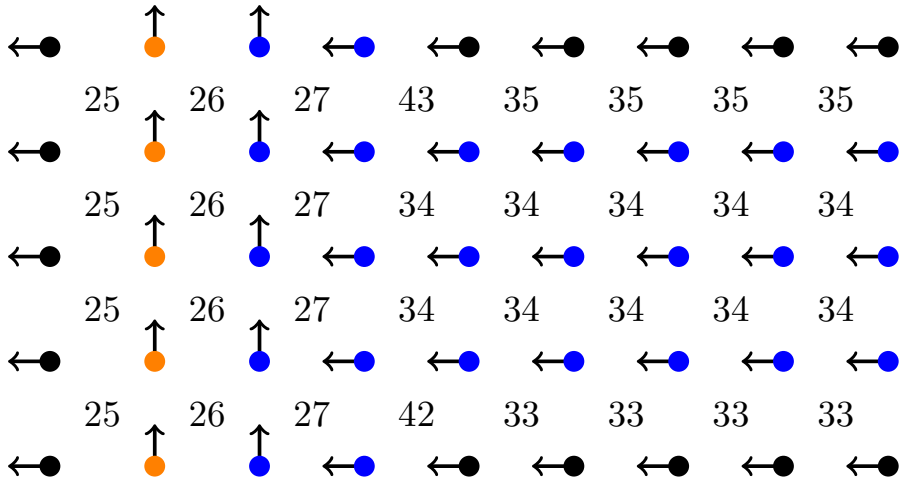
LA3-Blue Path



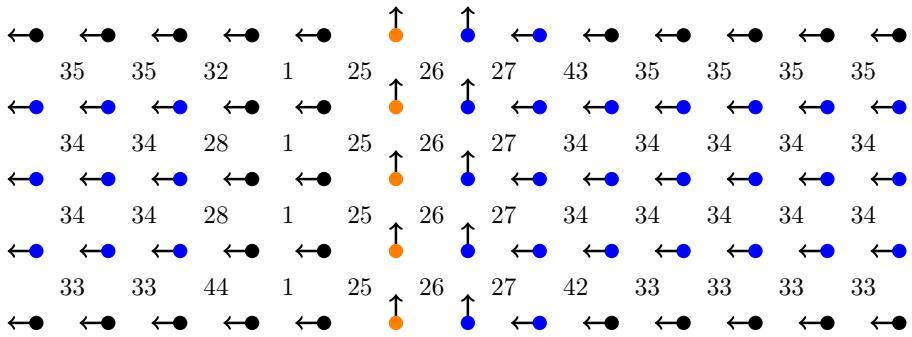
LA4-Orange-Blue Path Turn



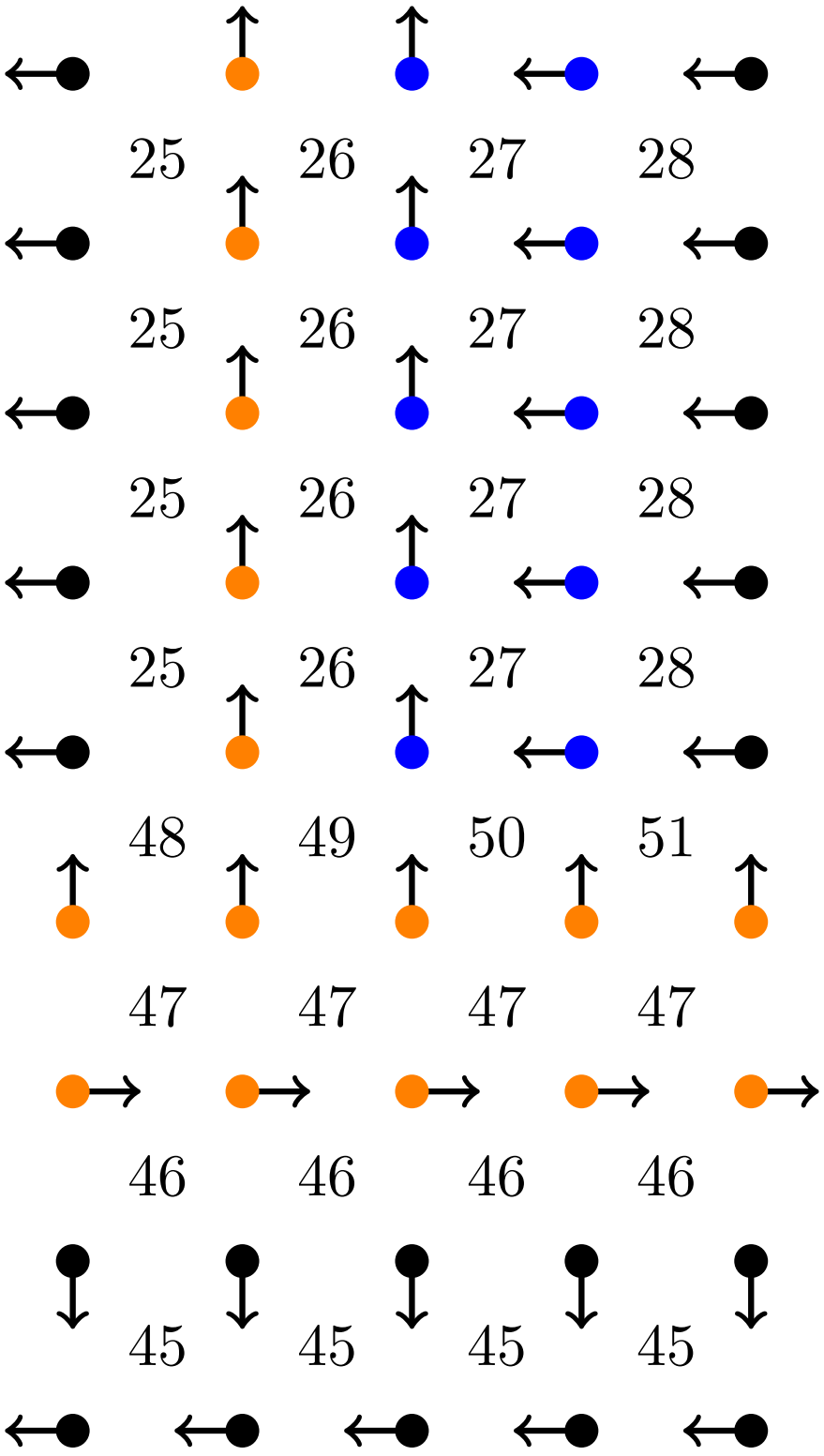
LA5-Blue Path Merging Into Orange-Blue Path



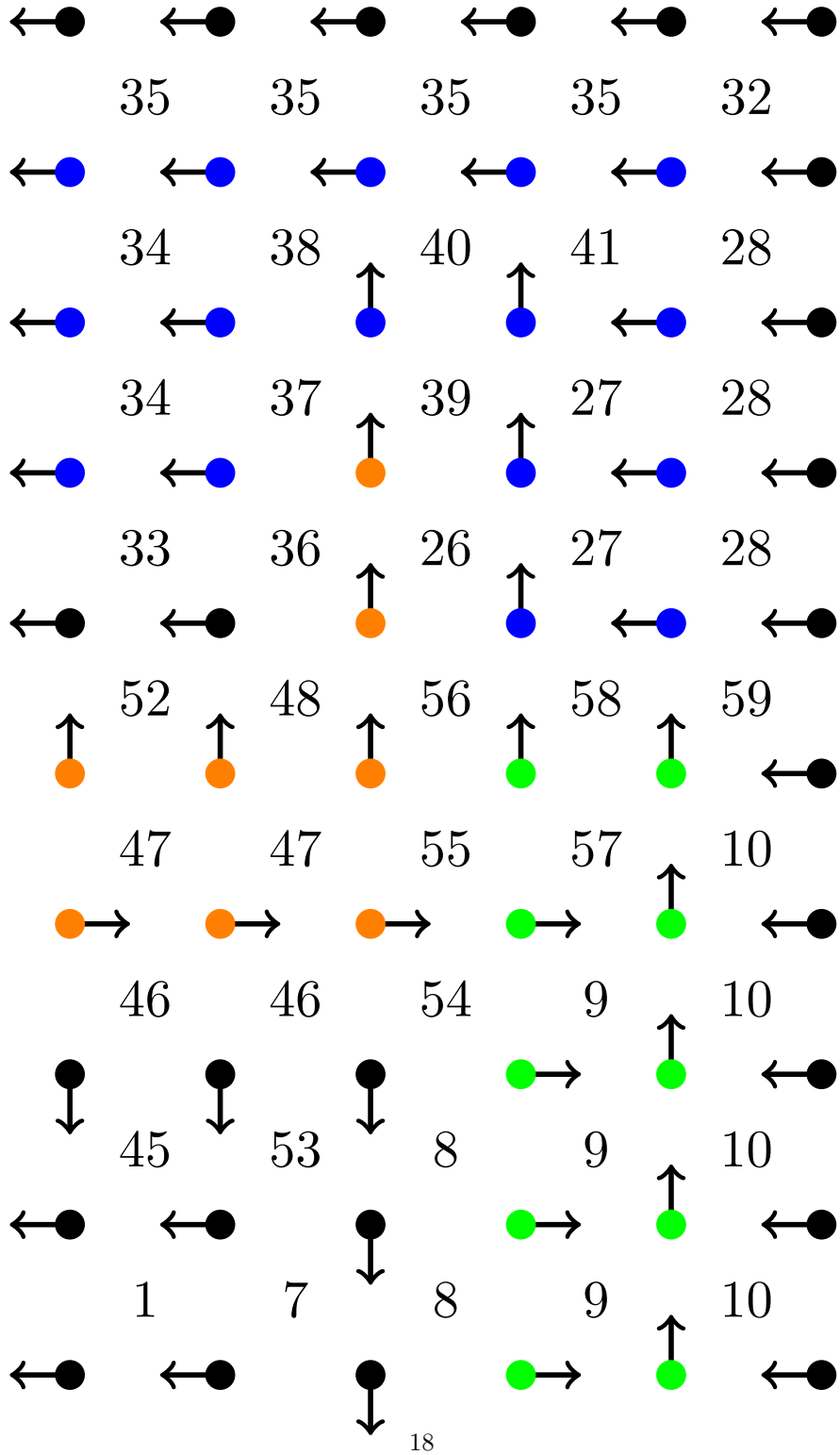
LA6-Blue Path Crossing Over Orange-Blue Path



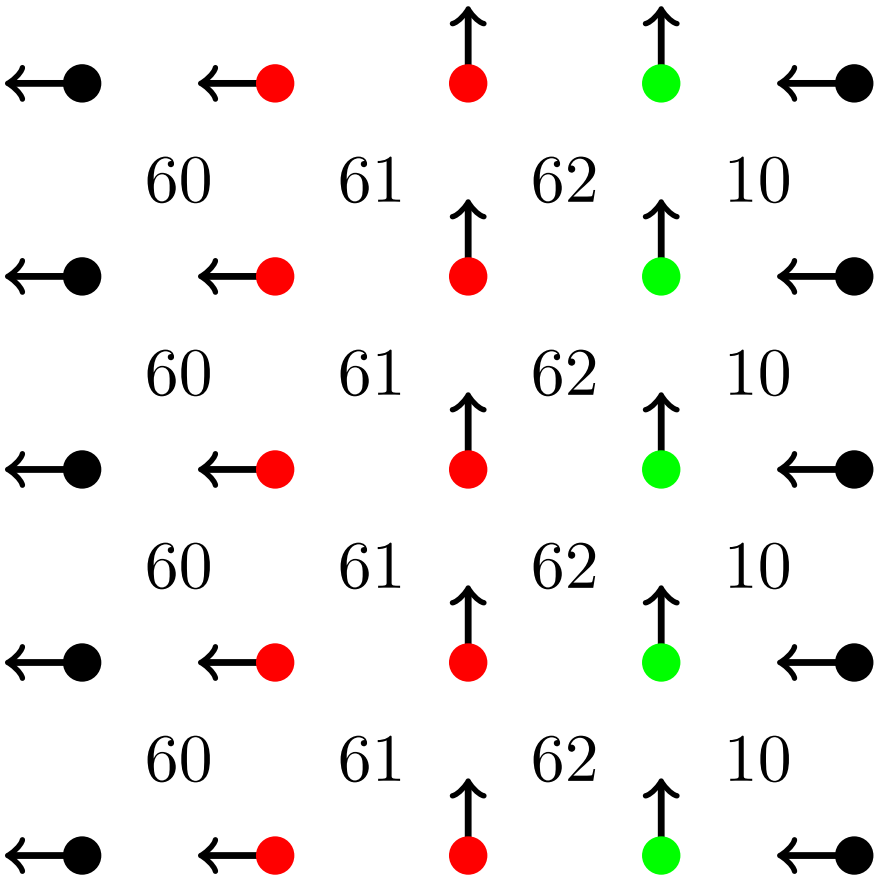
LAX1-Orange-Blue Path Start



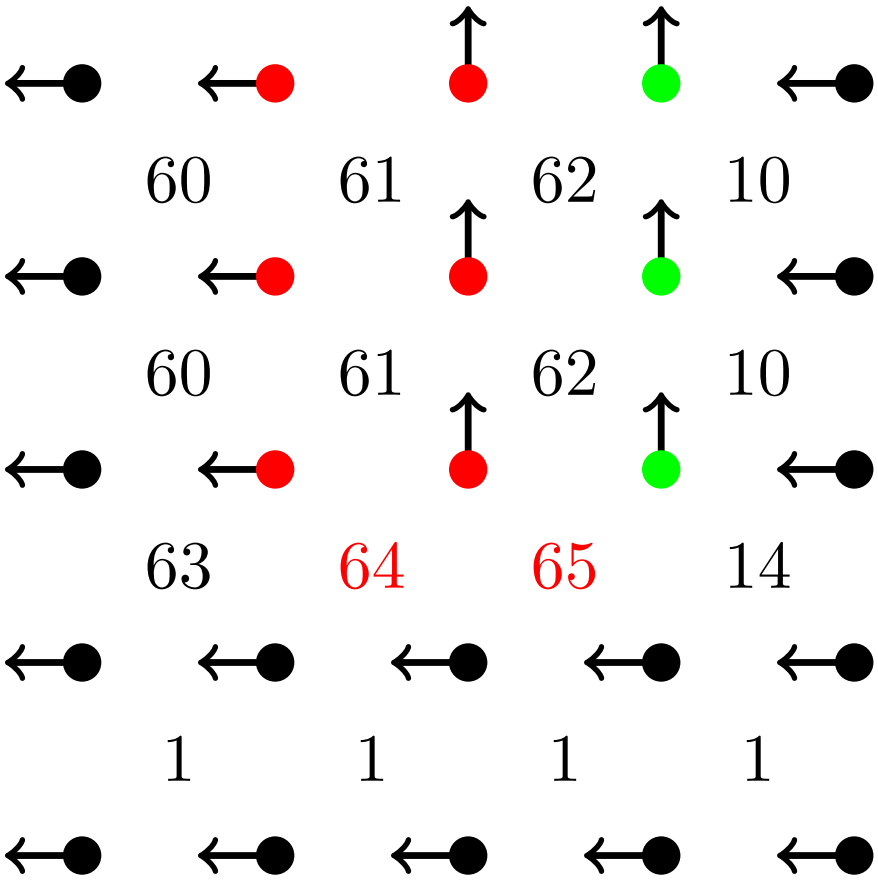
LAX2-PLS-Labyrinth A Origin



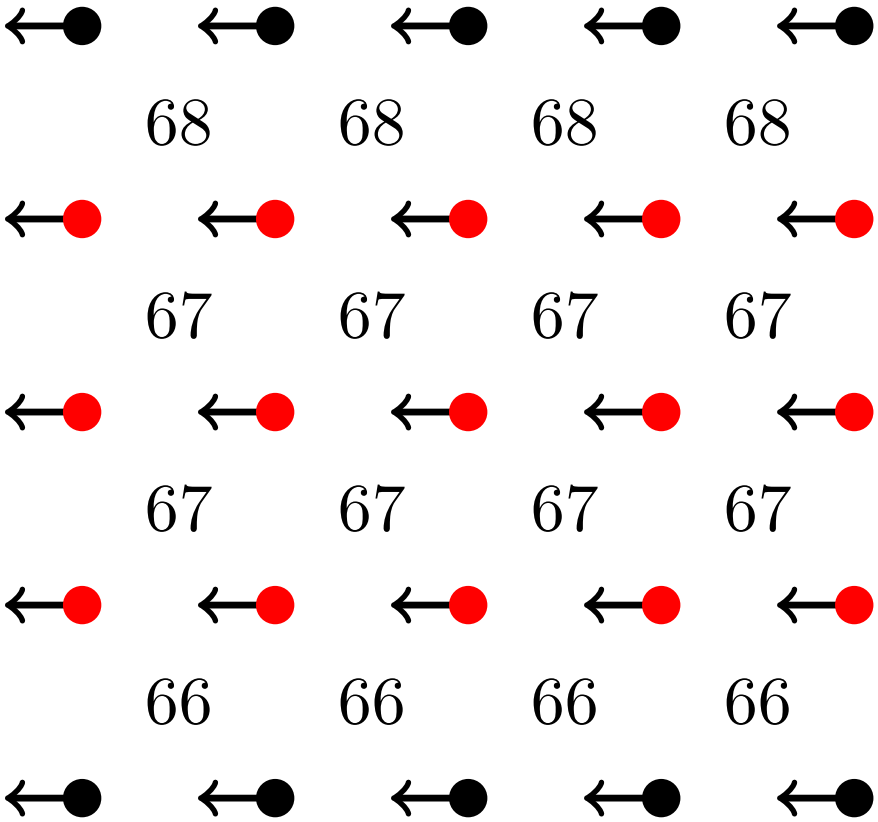
LB1-Red-Green Path



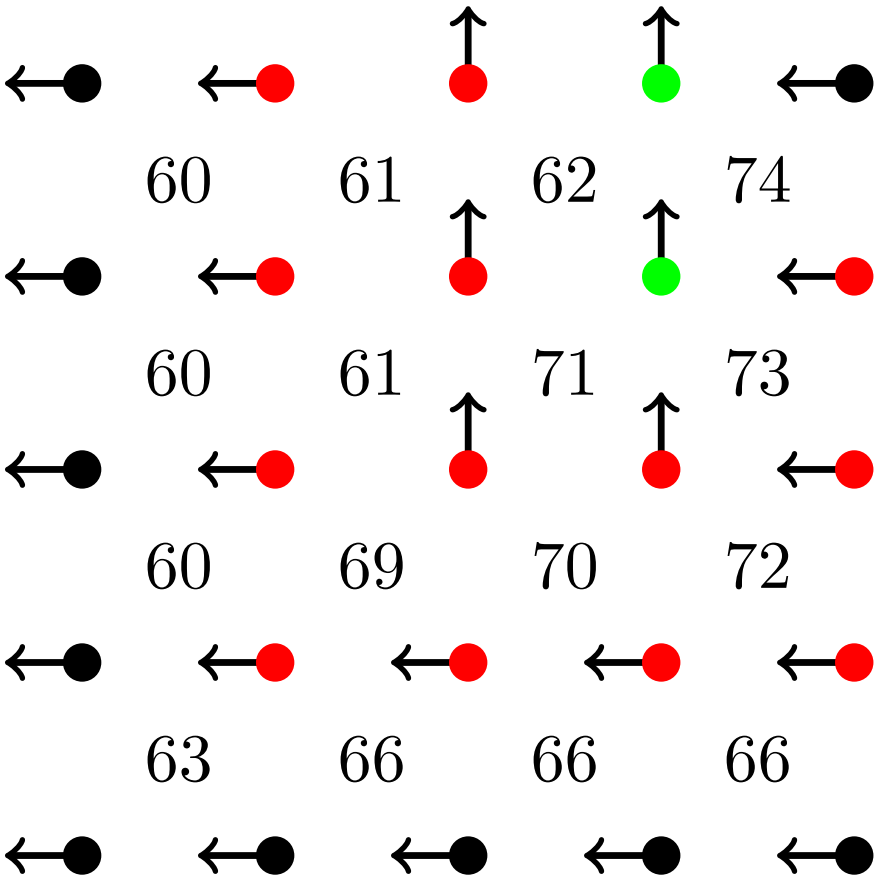
LB2-Red-Green Path Sink



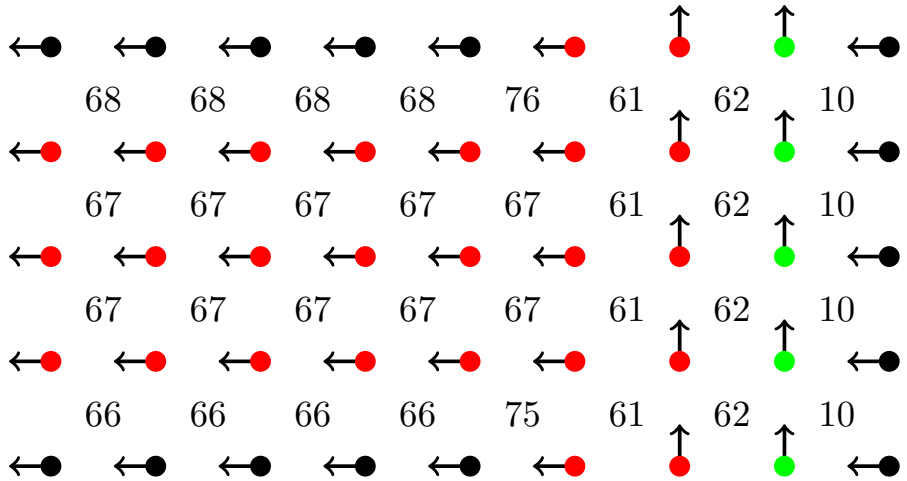
LB3-Red Path



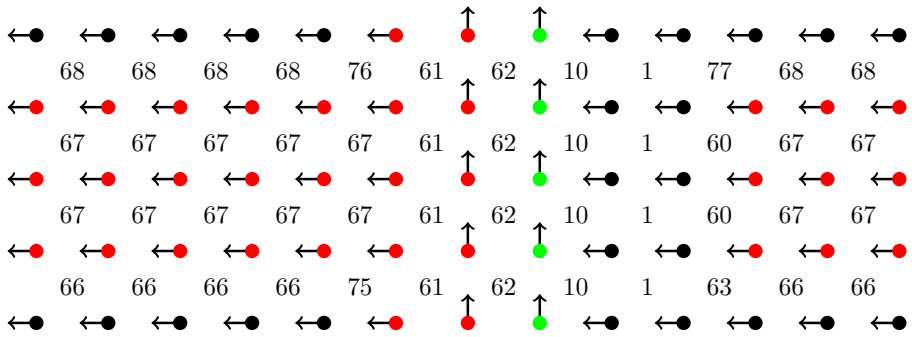
LB4-Red-Green Path Turn



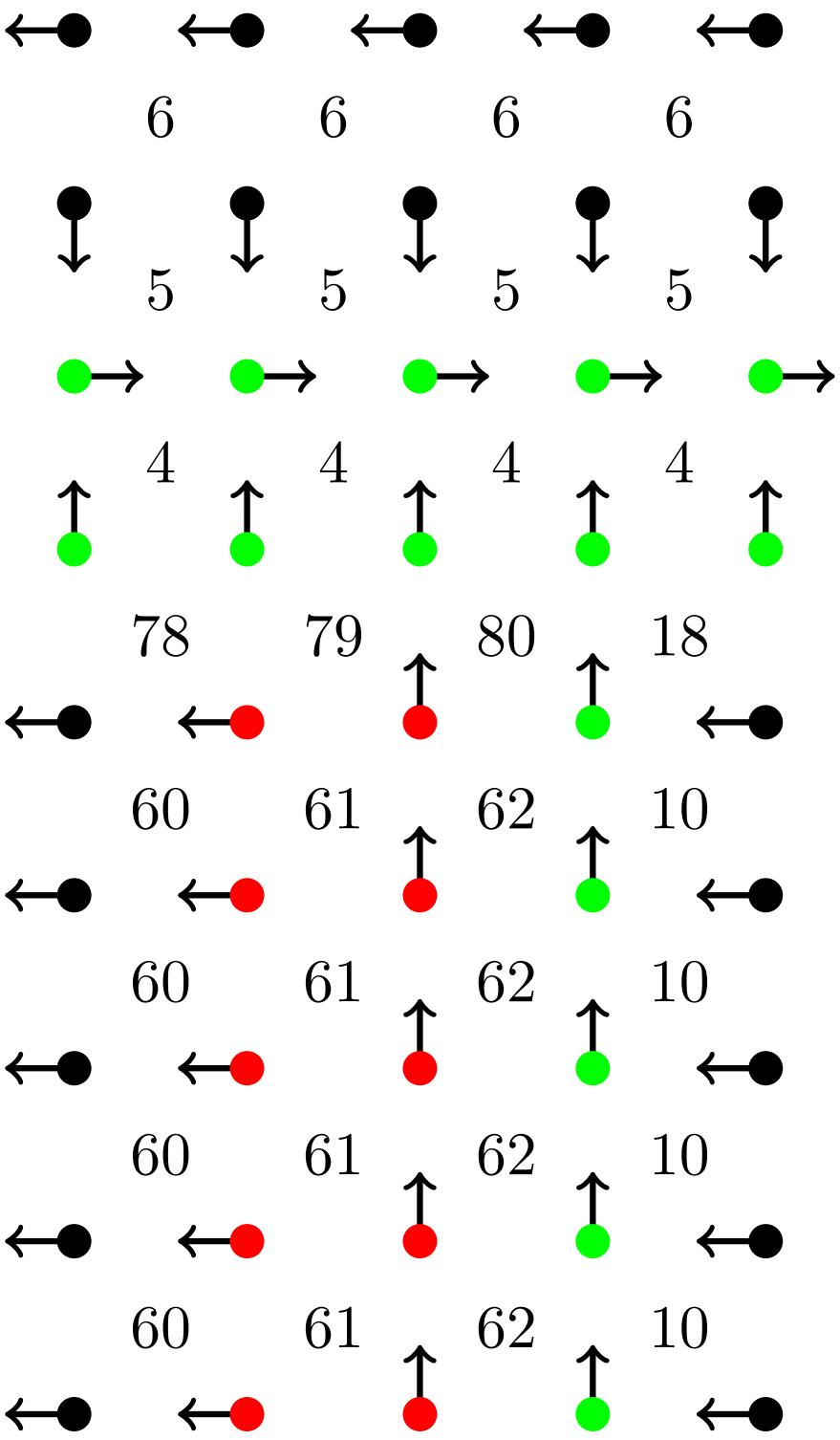
LB5-Red Path Merging Into Red-Green Path



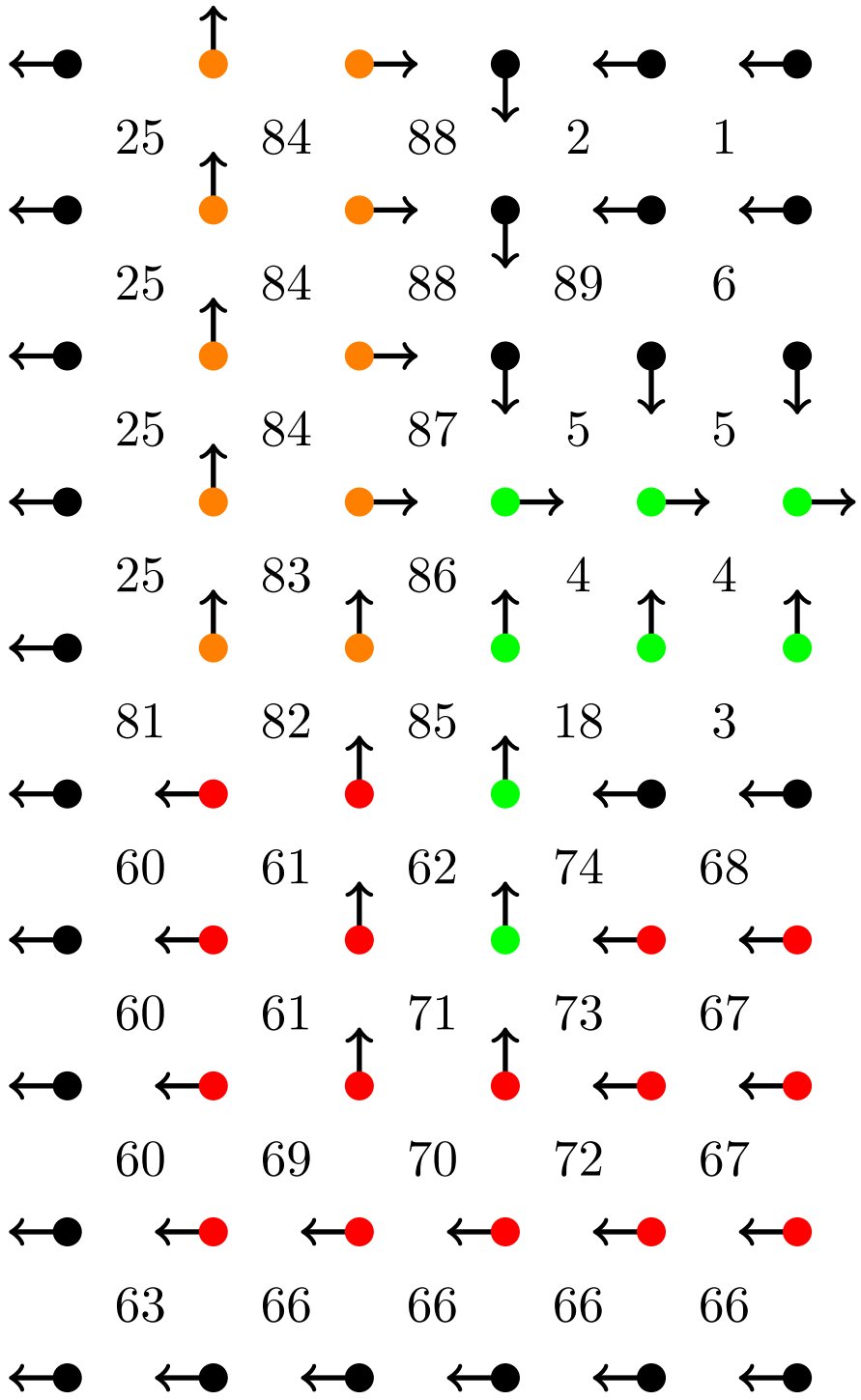
LB6-Red Path Crossing Over Red-Green Path



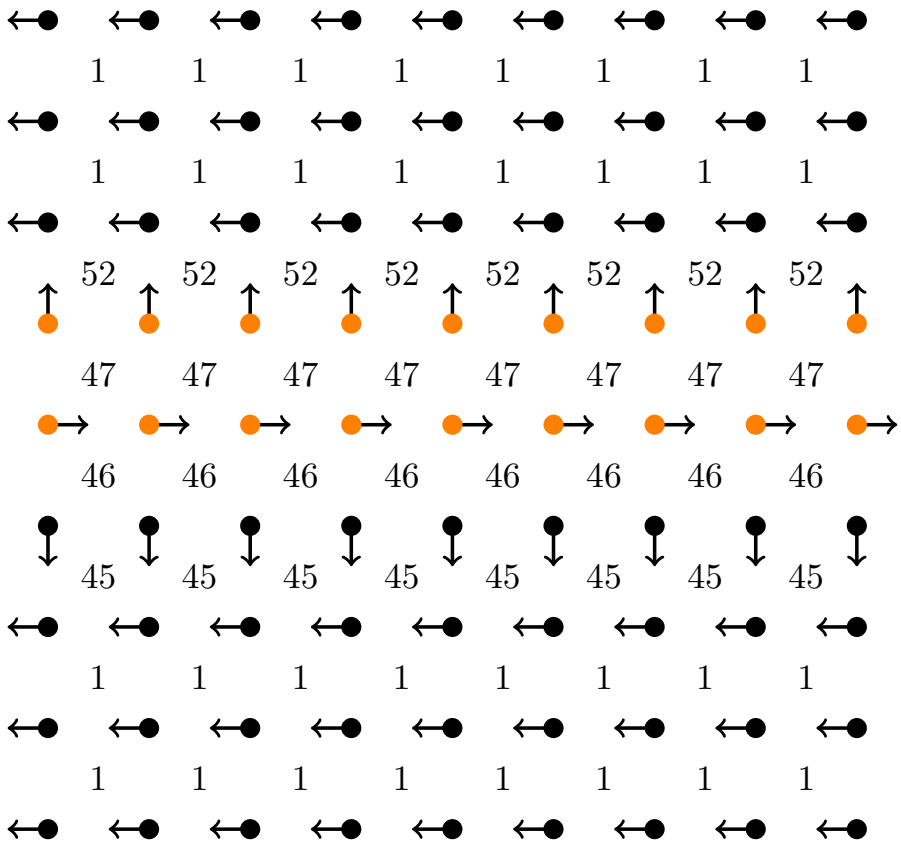
LBX1-Red-Green Path Start



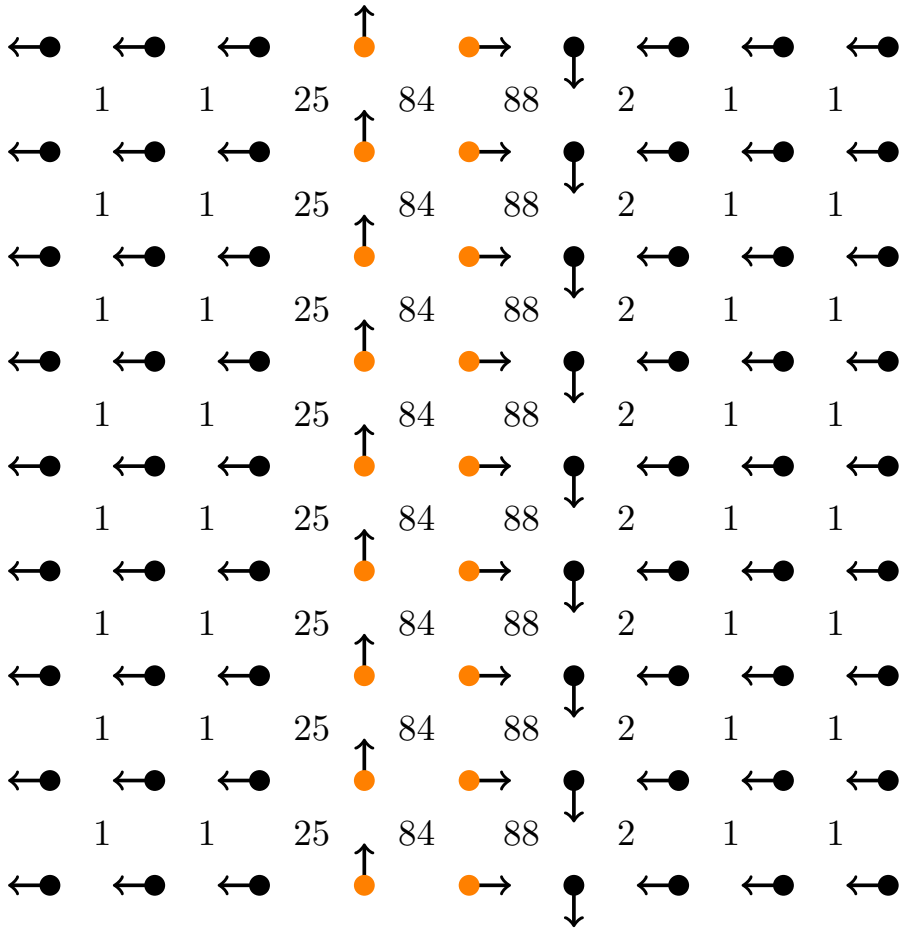
LBX2-PLS-Labyrinth B Origin



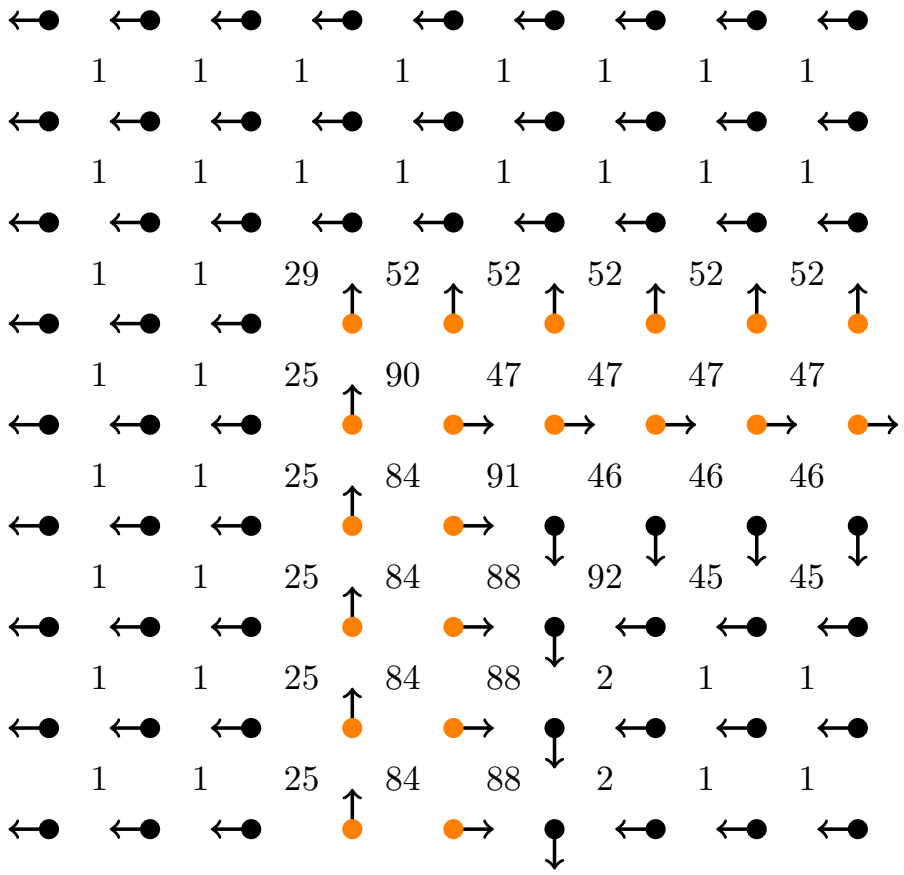
O1-Horizontal Orange Path



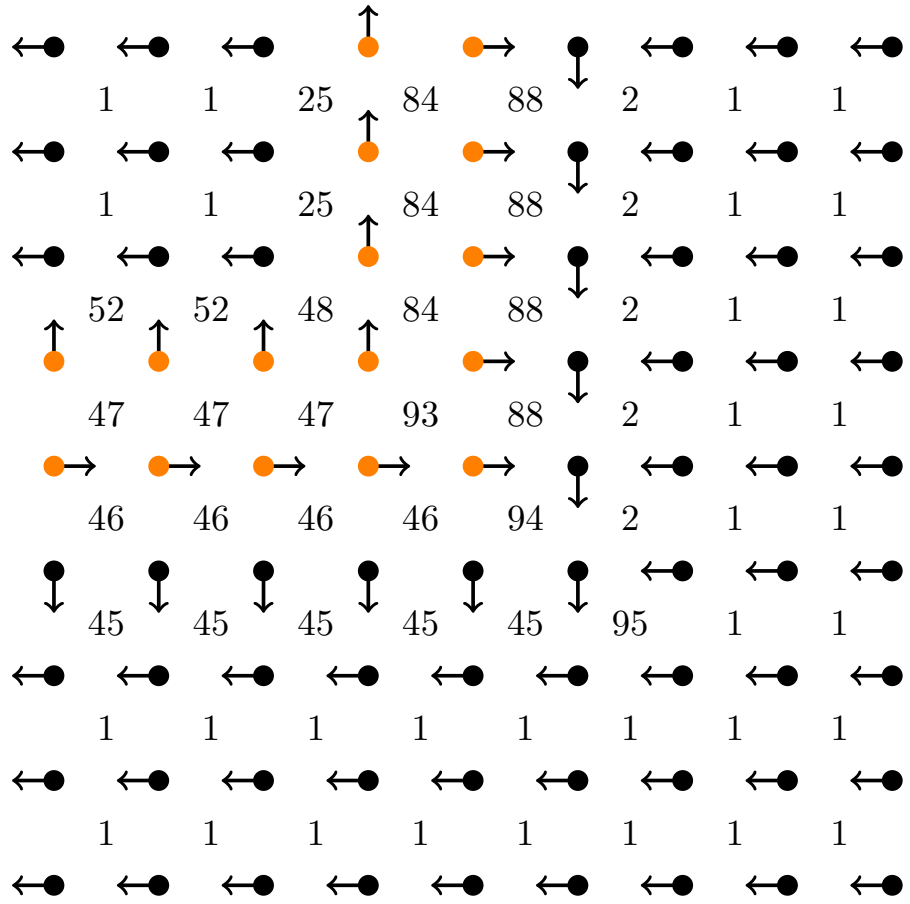
O2-Vertical Orange Path



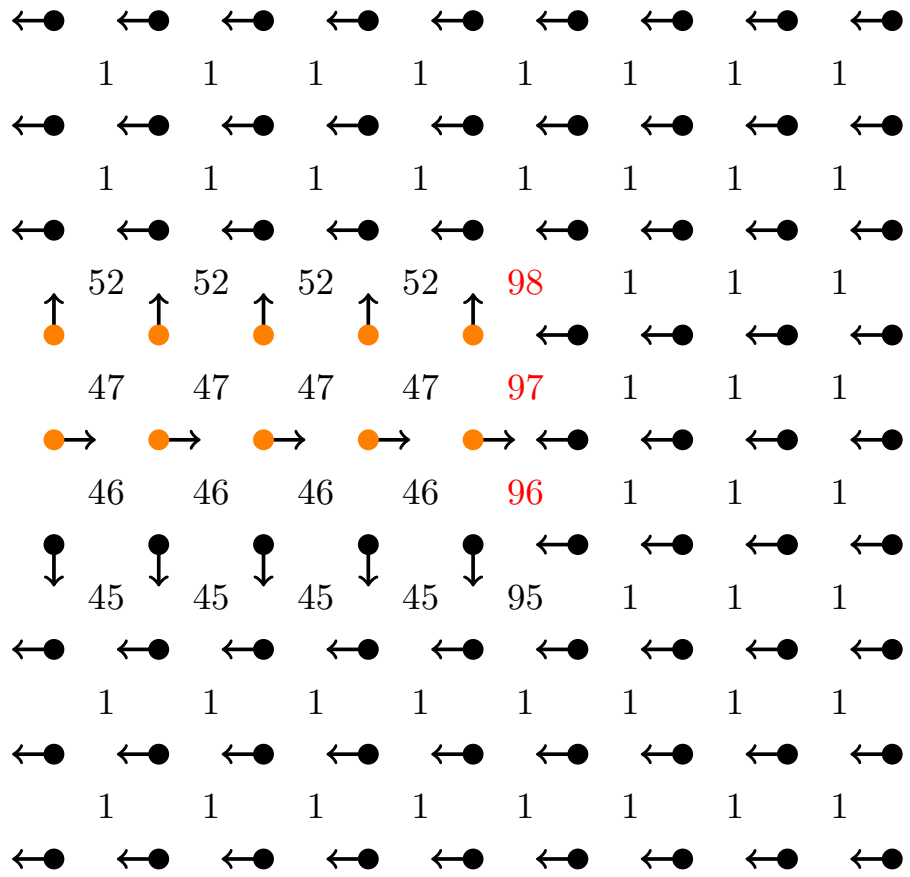
O3-Orange Path Turn Down



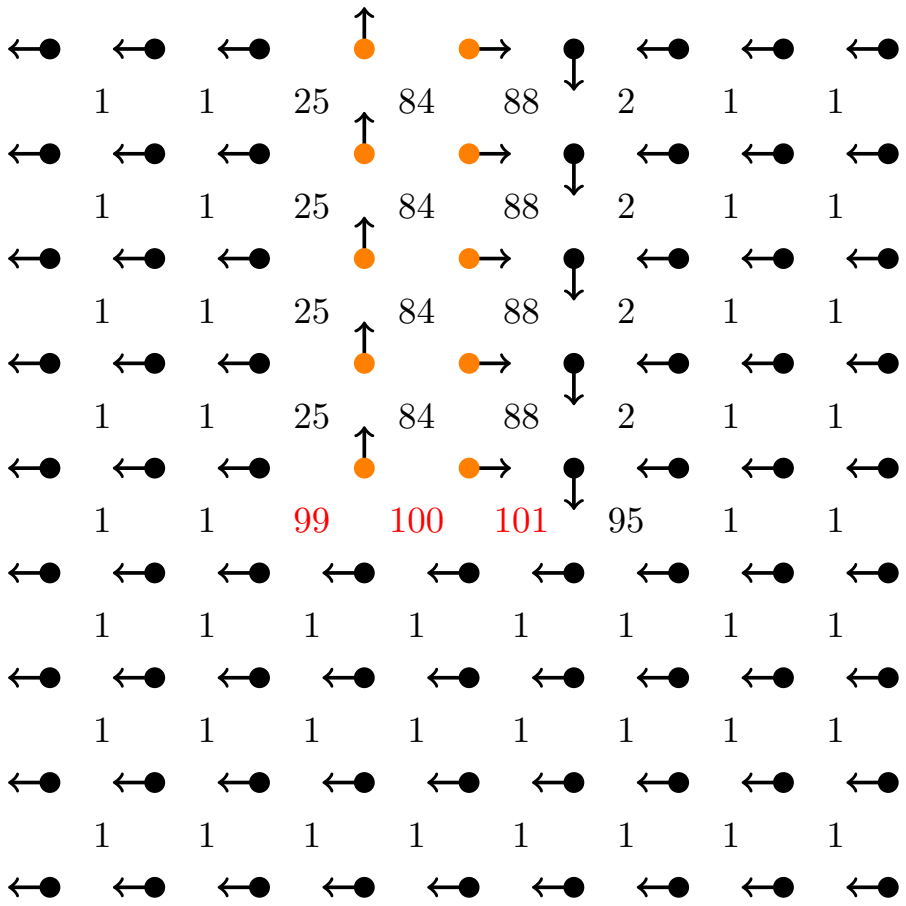
O4-Orange Path Turn Left



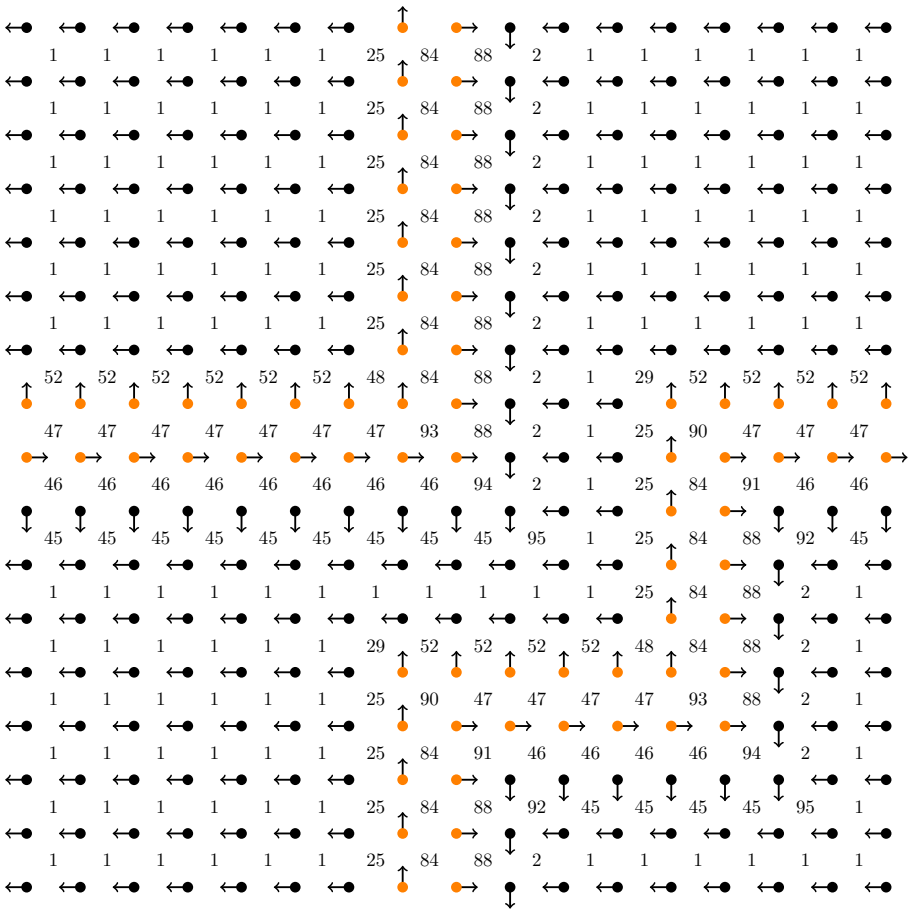
O5-Orange Path Source



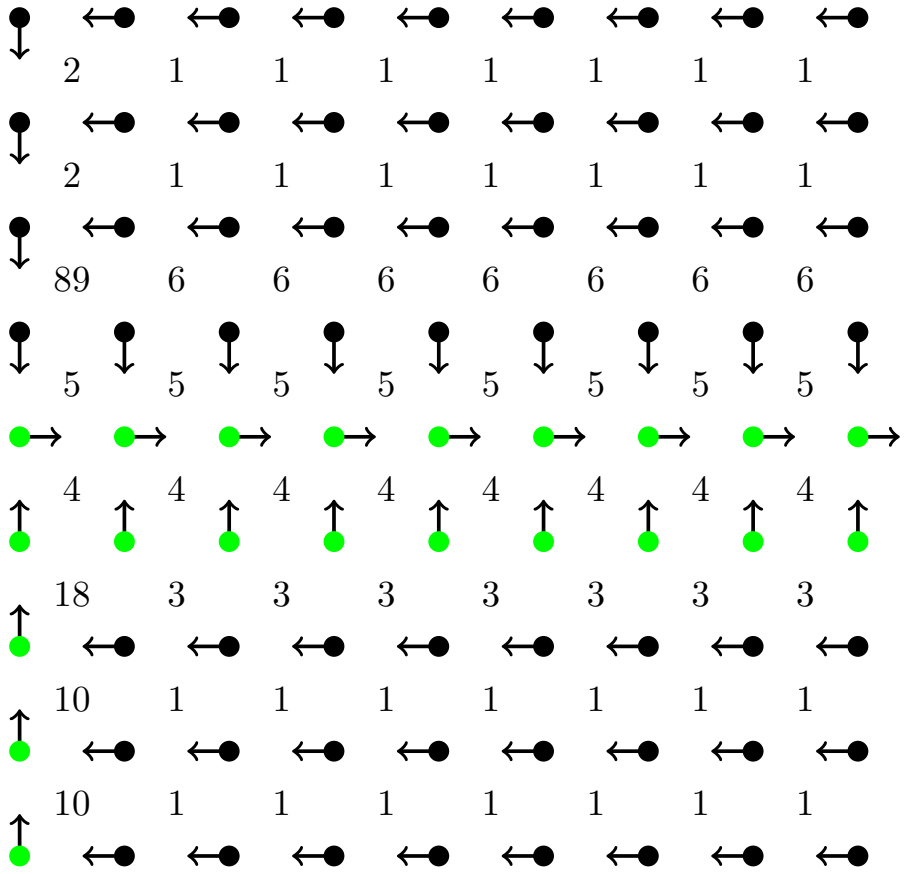
O6-Orange Path Sink



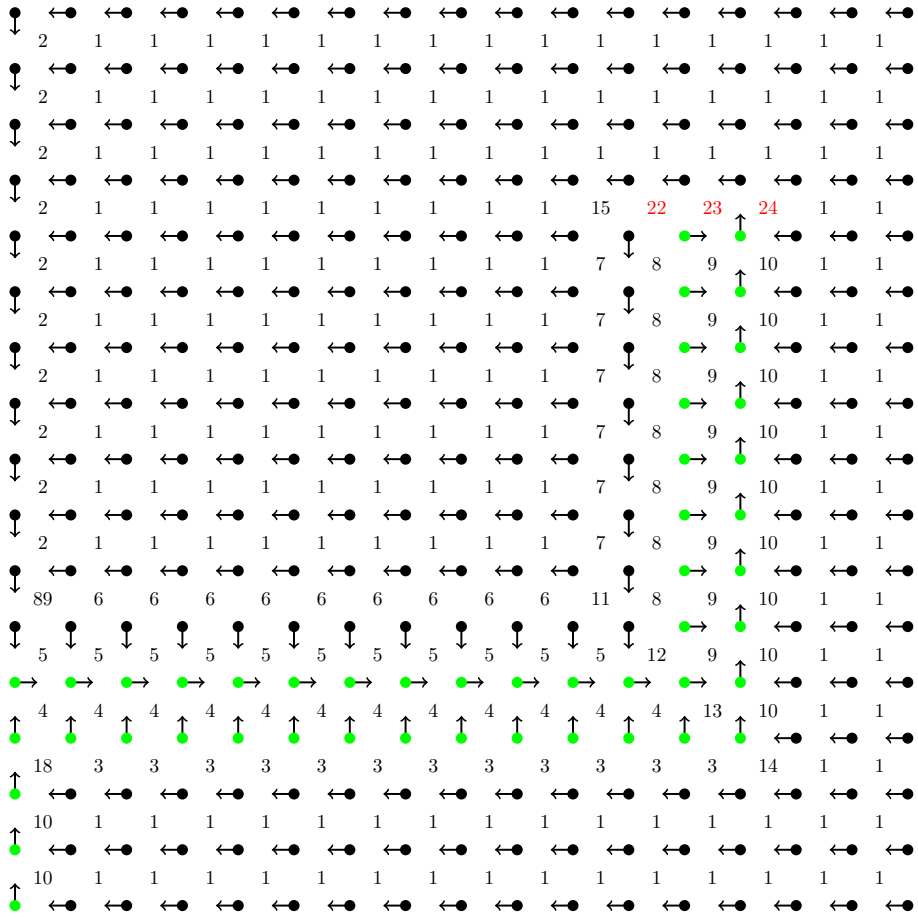
O7-Orange Path Crossing



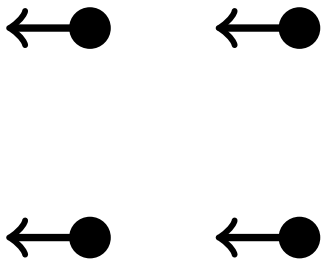
S-Origin Big Square



X-Full Boundary



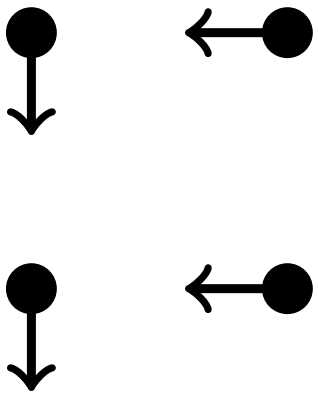
Square 1



Solver reported:

`Q.E.D.`

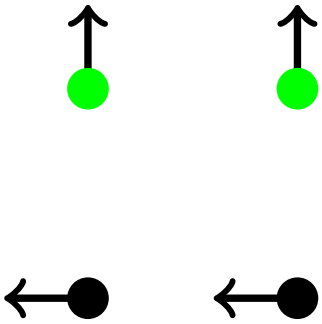
Square 2



Solver reported:

`Q.E.D.`

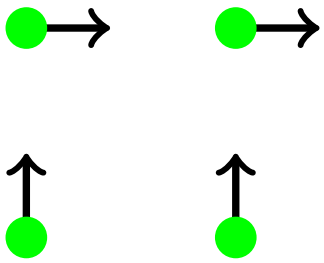
Square 3



Solver reported:

`Q.E.D.`

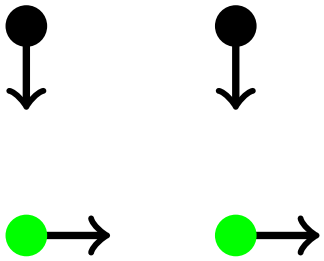
Square 4



Solver reported:

`Q.E.D.`

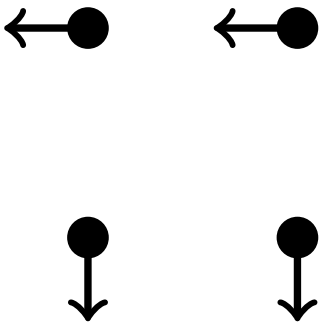
Square 5



Solver reported:

`Q.E.D.`

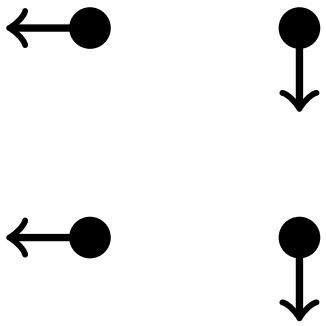
Square 6



Solver reported:

`Q.E.D.`

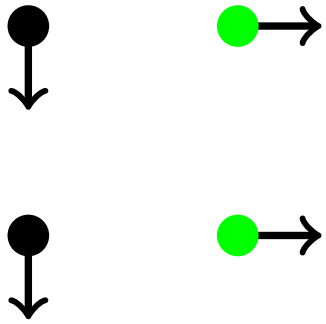
Square 7



Solver reported:

`Q.E.D.`

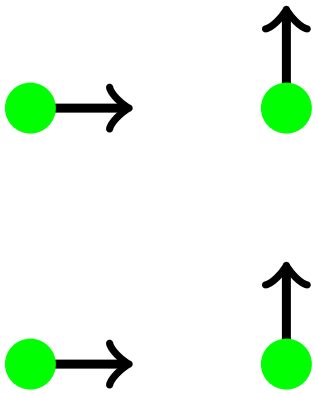
Square 8



Solver reported:

`Q.E.D.`

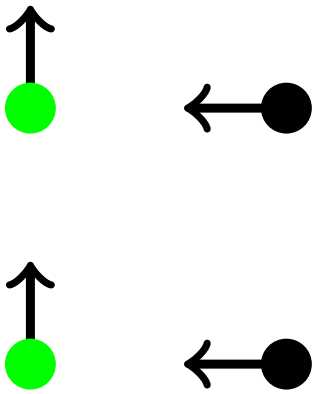
Square 9



Solver reported:

`Q.E.D.`

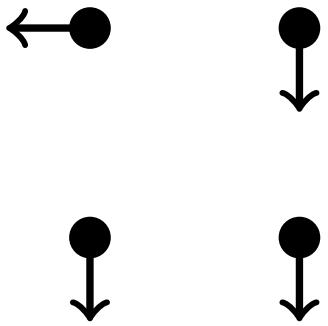
Square 10



Solver reported:

`Q.E.D.`

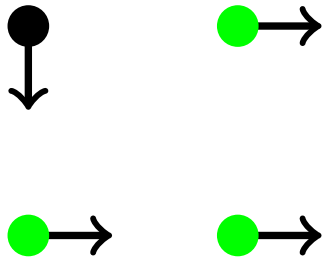
Square 11



Solver reported:

`Q.E.D.`

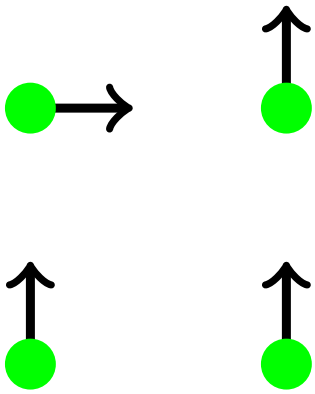
Square 12



Solver reported:

`Q.E.D.`

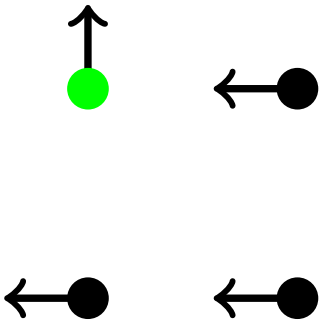
Square 13



Solver reported:

`Q.E.D.`

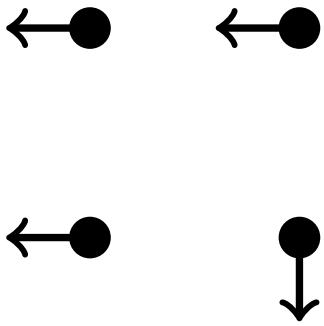
Square 14



Solver reported:

`Q.E.D.`

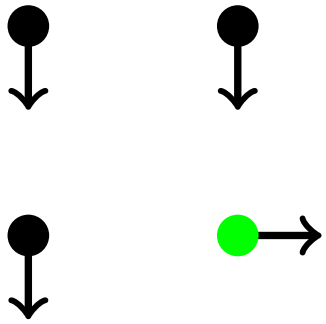
Square 15



Solver reported:

`Q.E.D.`

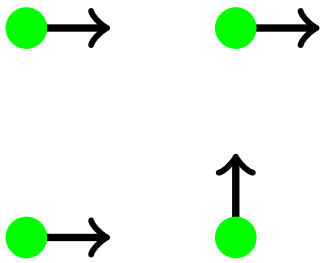
Square 16



Solver reported:

`Q.E.D.`

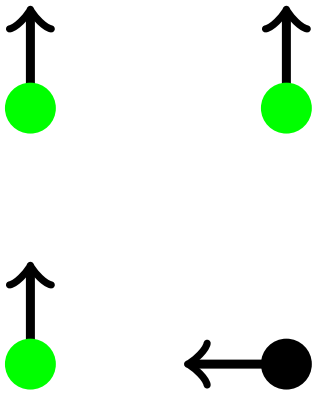
Square 17



Solver reported:

`Q.E.D.`

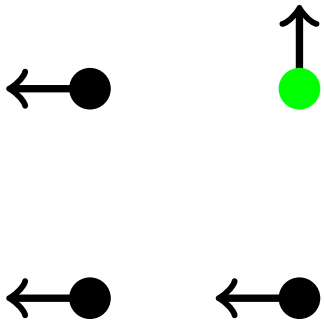
Square 18



Solver reported:

`Q.E.D.`

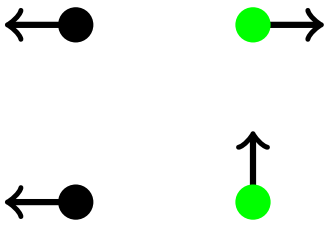
Square 19



Solver reported:

```
Falsifiable. Counter-example:
  red    =    9.875 :: Real
  orange =    4.875 :: Real
  black  =   -0.125 :: Real
  green  =    -4.25 :: Real
  blue   =    -9.25 :: Real
  x      = 0.015625 :: Real
  y      =      1.0 :: Real
```

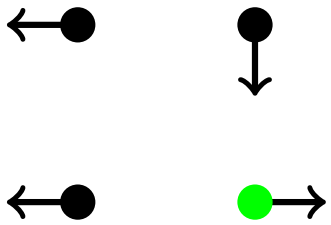
Square 20



Solver reported:

```
Falsifiable. Counter-example:
  red    =          10.5 :: Real
  orange =           5.5 :: Real
  black  =           0.5 :: Real
  green  =          -5.5 :: Real
  blue   =        -10.5 :: Real
  x      =     0.015625 :: Real
  y      = 0.0009765625 :: Real
```

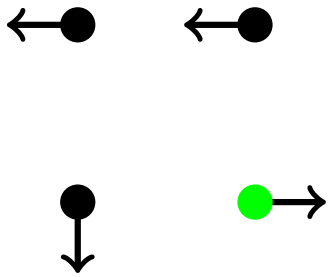
Square 21



Solver reported:

```
Falsifiable. Counter-example:
  red    =          9.0 :: Real
  orange =          4.0 :: Real
  black  =         -1.0 :: Real
  green  =        -7.25 :: Real
  blue   =       -12.25 :: Real
  x      =     0.015625 :: Real
  y      = 0.0009765625 :: Real
```

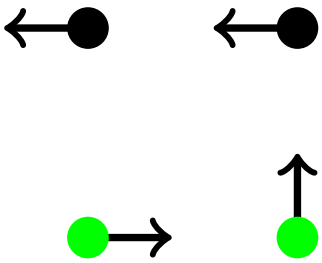
Square 22



Solver reported:

```
Falsifiable. Counter-example:
  red    =                    9.5 :: Real
  orange =                    4.5 :: Real
  black  =                   -0.5 :: Real
  green  =          -7798325284.0 :: Real
  blue   =          -7798325289.0 :: Real
  x      = 0.00000095367431640625 :: Real
  y      =            0.998046875 :: Real
```

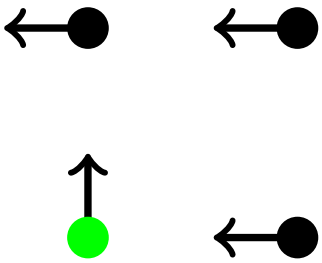
Square 23



Solver reported:

```
Falsifiable. Counter-example:
  red    =      9.5 :: Real
  orange =      4.5 :: Real
  black  =     -0.5 :: Real
  green  =   -4.625 :: Real
  blue   =   -9.625 :: Real
  x      =      1.0 :: Real
  y      = 0.015625 :: Real
```

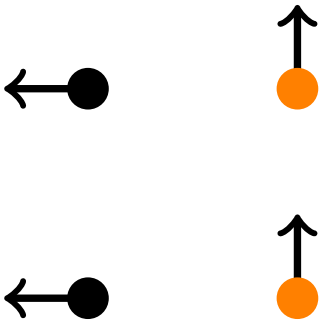
Square 24



Solver reported:

```
Falsifiable. Counter-example:
  red    =           9.5 :: Real
  orange =           4.5 :: Real
  black  =          -0.5 :: Real
  green  =         -4.75 :: Real
  blue   =         -9.75 :: Real
  x      = 0.00048828125 :: Real
  y      =     0.0234375 :: Real
```

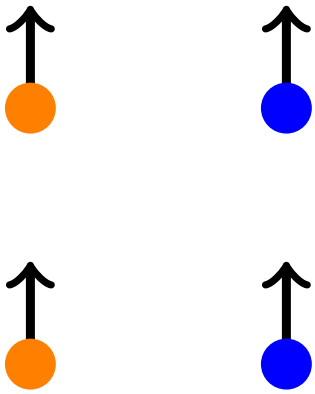
Square 25



Solver reported:

`Q.E.D.`

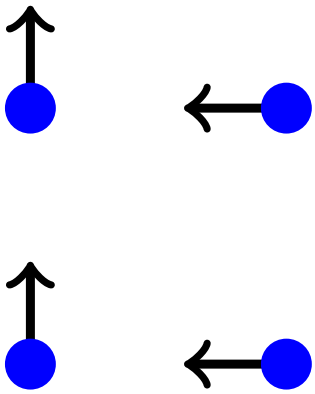
Square 26



Solver reported:

`Q.E.D.`

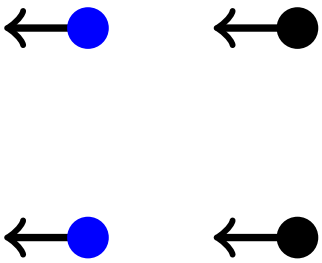
Square 27



Solver reported:

`Q.E.D.`

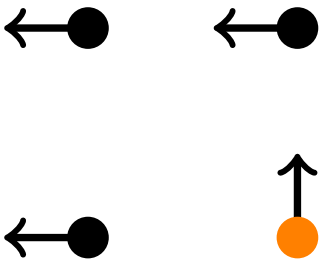
Square 28



Solver reported:

`Q.E.D.`

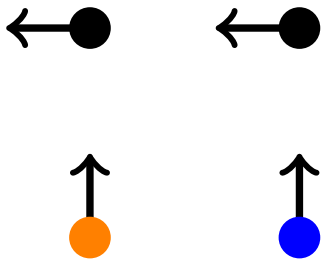
Square 29



Solver reported:

`Q.E.D.`

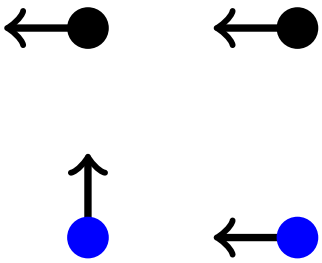
Square 30



Solver reported:

```
Falsifiable. Counter-example:
  red    =      4.5 :: Real
  orange =     -0.5 :: Real
  black  = -9.40625 :: Real
  green  =    -14.0 :: Real
  blue   =    -19.0 :: Real
  x      =      0.5 :: Real
  y      =    0.875 :: Real
```

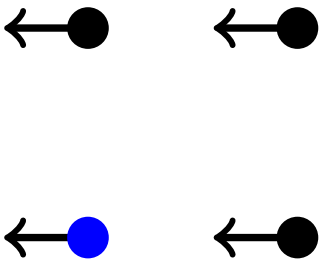
Square 31



Solver reported:

```
Falsifiable. Counter-example:
  red    =       10.125 :: Real
  orange =        5.125 :: Real
  black  =        0.125 :: Real
  green  =        -81.0 :: Real
  blue   =        -86.0 :: Real
  x      =  0.001953125 :: Real
  y      = 0.0009765625 :: Real
```

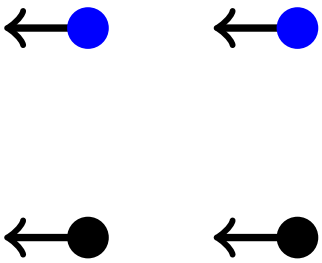
Square 32



Solver reported:

`Q.E.D.`

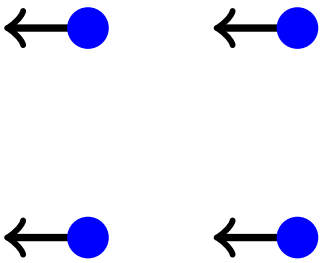
Square 33



Solver reported:

`Q.E.D.`

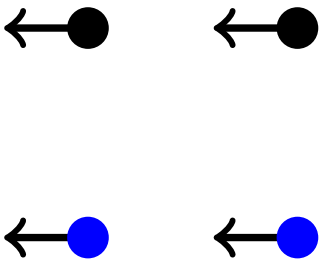
Square 34



Solver reported:

`Q.E.D.`

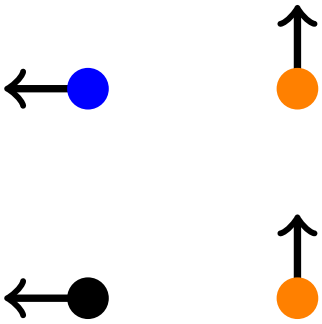
Square 35



Solver reported:

```
Q.E.D.
```

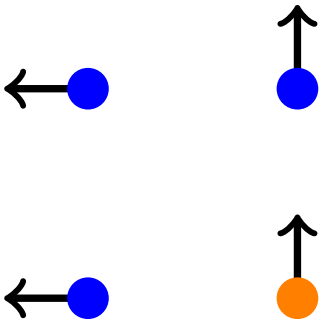
Square 36



Solver reported:

`Q.E.D.`

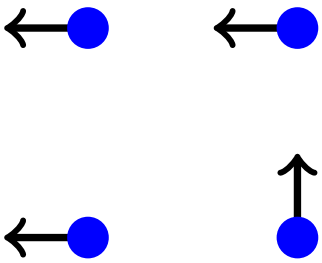
Square 37



Solver reported:

`Q.E.D.`

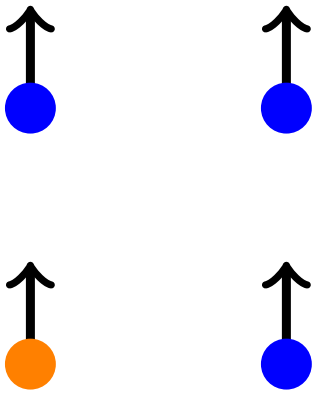
Square 38



Solver reported:

`Q.E.D.`

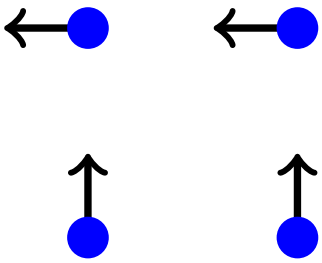
Square 39



Solver reported:

`Q.E.D.`

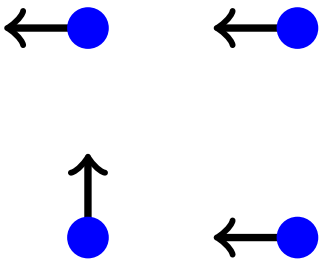
Square 40



Solver reported:

```
Q.E.D.
```

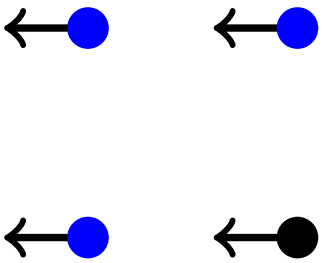
Square 41



Solver reported:

`Q.E.D.`

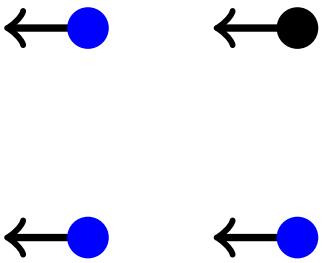
Square 42



Solver reported:

`Q.E.D.`

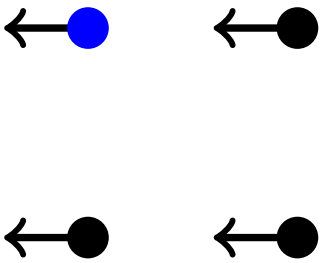
Square 43



Solver reported:

`Q.E.D.`

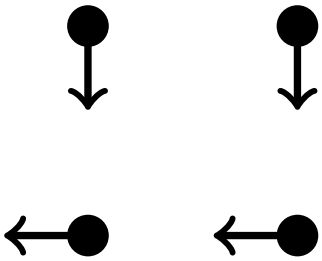
Square 44



Solver reported:

`Q.E.D.`

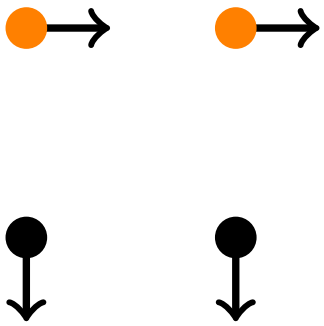
Square 45



Solver reported:

`Q.E.D.`

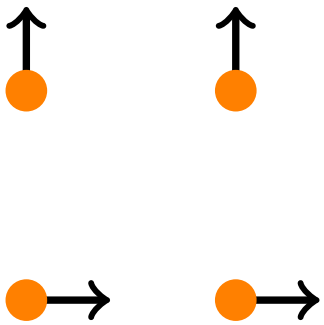
Square 46



Solver reported:

`Q.E.D.`

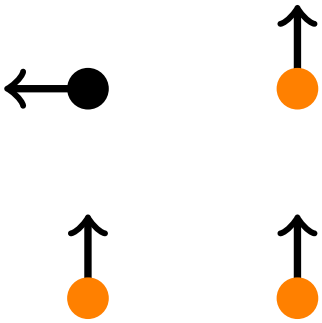
Square 47



Solver reported:

`Q.E.D.`

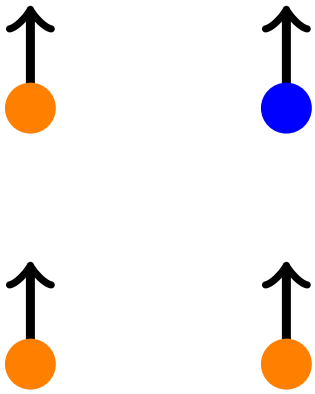
Square 48



Solver reported:

`Q.E.D.`

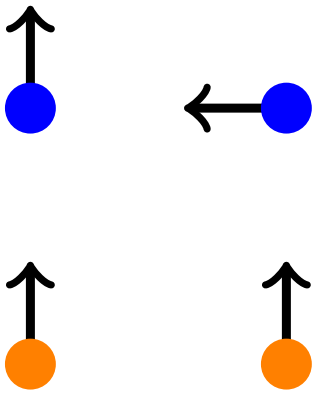
Square 49



Solver reported:

`Q.E.D.`

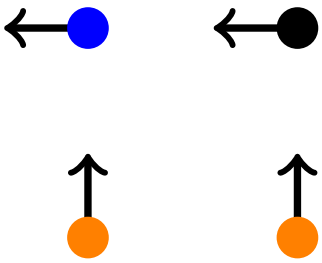
Square 50



Solver reported:

```
Q.E.D.
```

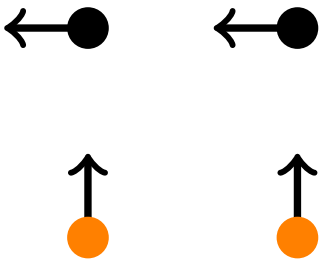
Square 51



Solver reported:

`Q.E.D.`

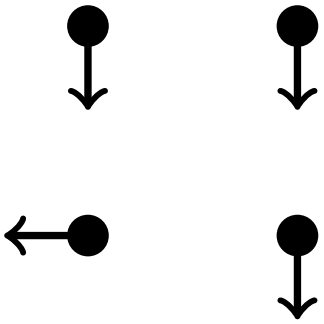
Square 52



Solver reported:

`Q.E.D.`

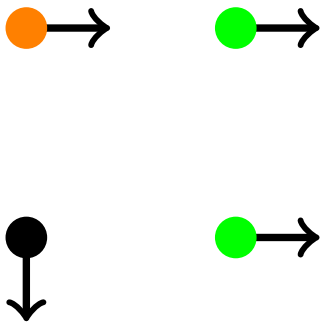
Square 53



Solver reported:

`Q.E.D.`

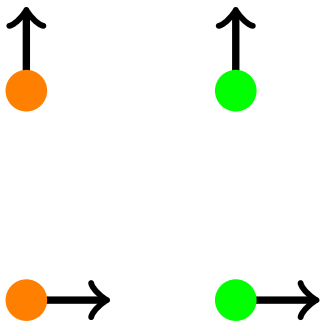
Square 54



Solver reported:

`Q.E.D.`

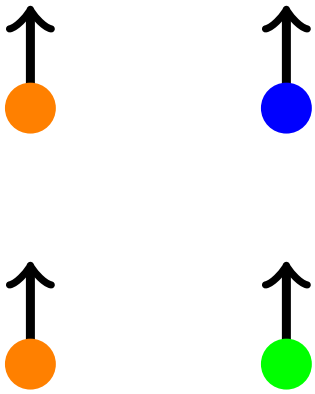
Square 55



Solver reported:

`Q.E.D.`

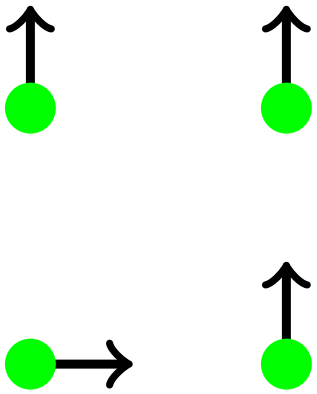
Square 56



Solver reported:

`Q.E.D.`

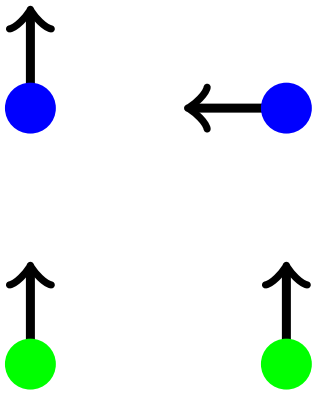
Square 57



Solver reported:

```
Q.E.D.
```

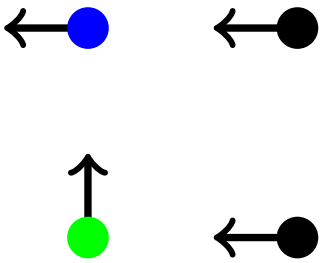
Square 58



Solver reported:

`Q.E.D.`

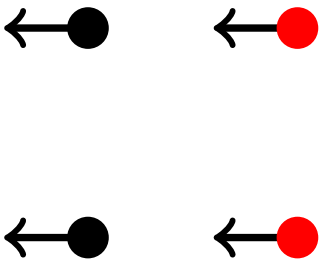
Square 59



Solver reported:

`Q.E.D.`

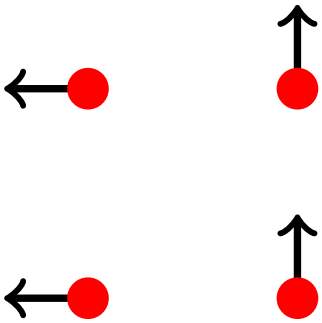
Square 60



Solver reported:

`Q.E.D.`

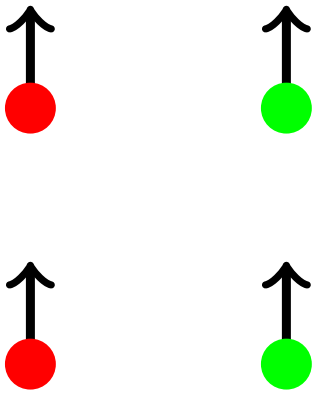
Square 61



Solver reported:

`Q.E.D.`

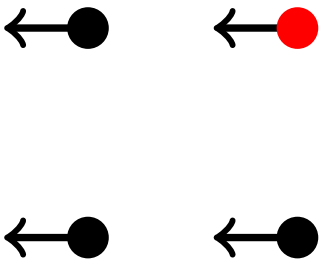
Square 62



Solver reported:

`Q.E.D.`

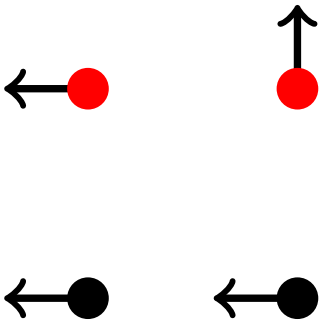
Square 63



Solver reported:

`Q.E.D.`

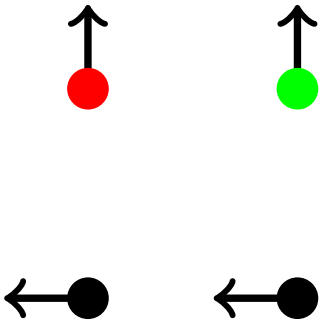
Square 64



Solver reported:

```
Falsifiable. Counter-example:
  red    =       0.125 :: Real
  orange =        -6.0 :: Real
  black  =       -10.5 :: Real
  green  =       -15.5 :: Real
  blue   =       -20.5 :: Real
  x      = 0.998046875 :: Real
  y      =   0.9921875 :: Real
```

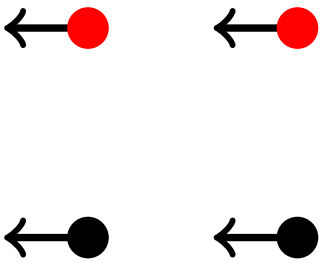
Square 65



Solver reported:

```
Falsifiable. Counter-example:
  red    =      -0.5 :: Real
  orange =      -6.0 :: Real
  black  = -10.09375 :: Real
  green  =     -19.0 :: Real
  blue   =     -24.0 :: Real
  x      =       0.5 :: Real
  y      =     0.125 :: Real
```

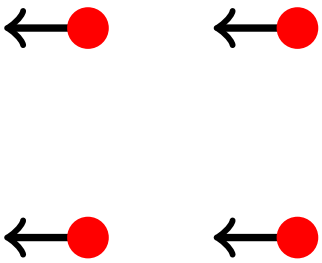
Square 66



Solver reported:

`Q.E.D.`

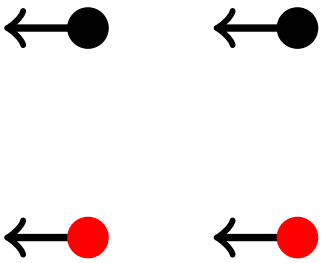
Square 67



Solver reported:

`Q.E.D.`

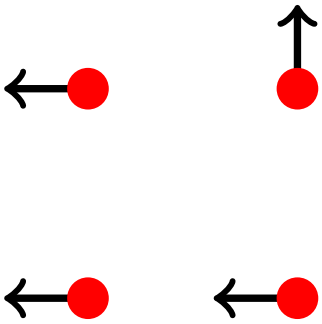
Square 68



Solver reported:

`Q.E.D.`

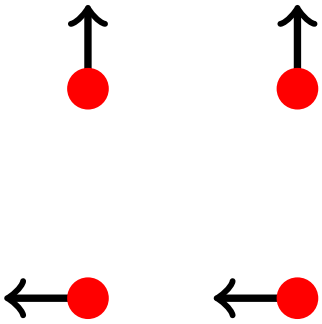
Square 69



Solver reported:

`Q.E.D.`

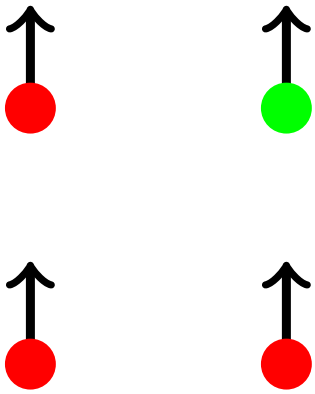
Square 70



Solver reported:

`Q.E.D.`

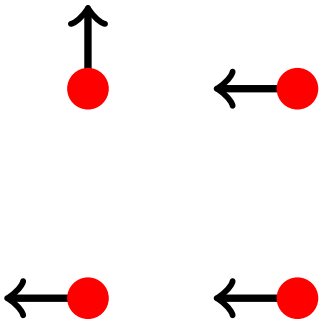
Square 71



Solver reported:

`Q.E.D.`

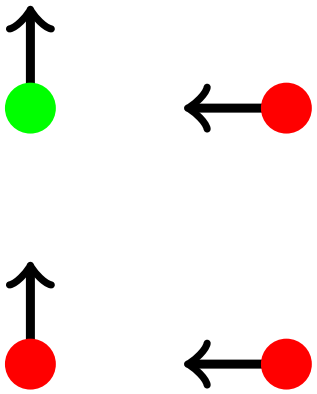
Square 72



Solver reported:

`Q.E.D.`

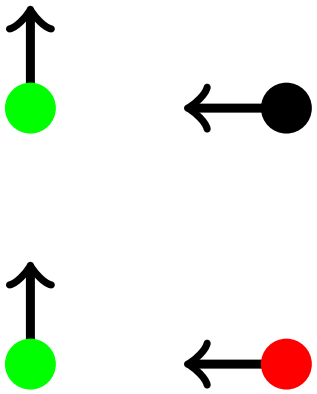
Square 73



Solver reported:

`Q.E.D.`

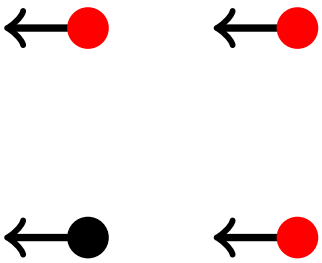
Square 74



Solver reported:

`Q.E.D.`

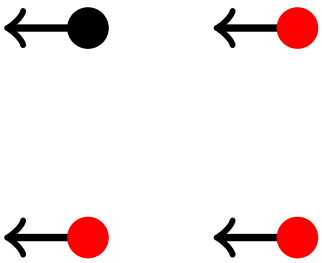
Square 75



Solver reported:

`Q.E.D.`

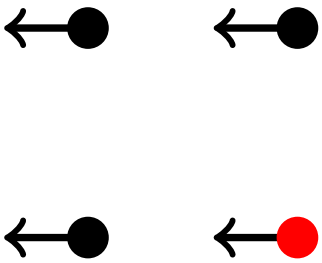
Square 76



Solver reported:

`Q.E.D.`

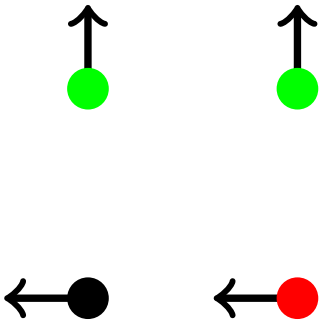
Square 77



Solver reported:

`Q.E.D.`

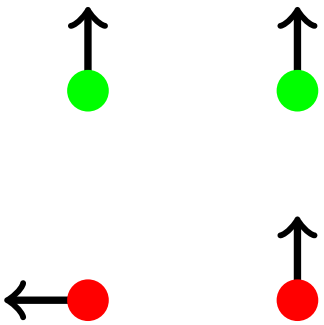
Square 78



Solver reported:

`Q.E.D.`

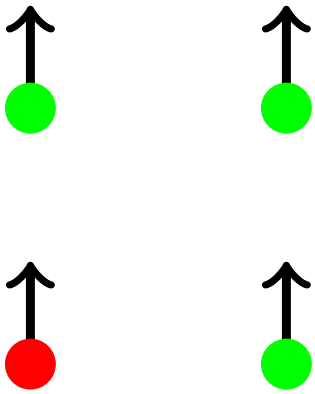
Square 79



Solver reported:

`Q.E.D.`

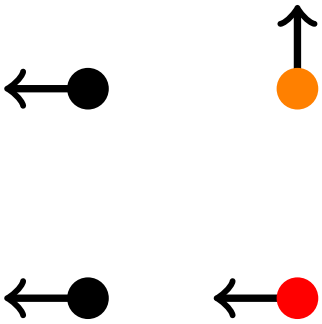
Square 80



Solver reported:

`Q.E.D.`

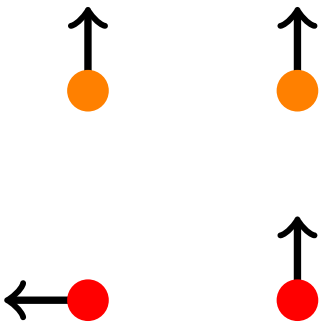
Square 81



Solver reported:

`Q.E.D.`

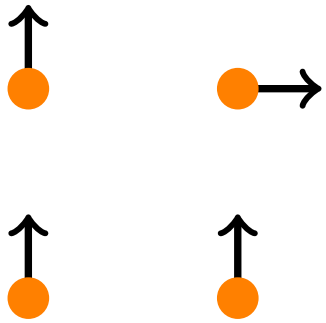
Square 82



Solver reported:

`Q.E.D.`

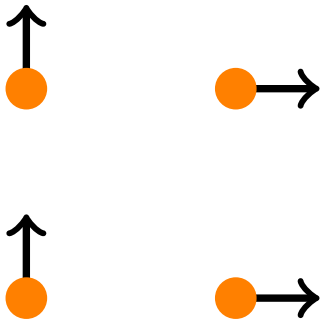
Square 83



Solver reported:

`Q.E.D.`

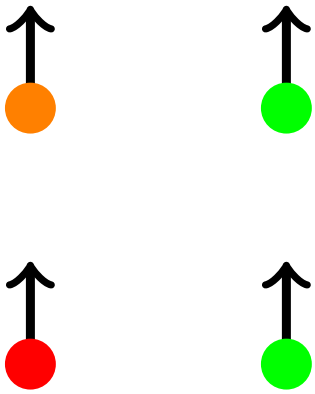
Square 84



Solver reported:

`Q.E.D.`

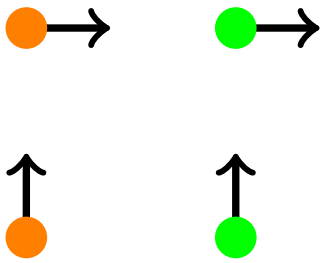
Square 85



Solver reported:

`Q.E.D.`

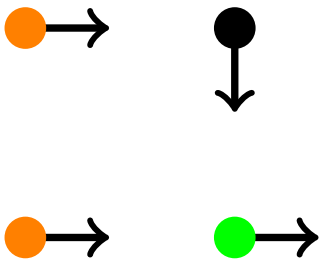
Square 86



Solver reported:

```
Q.E.D.
```

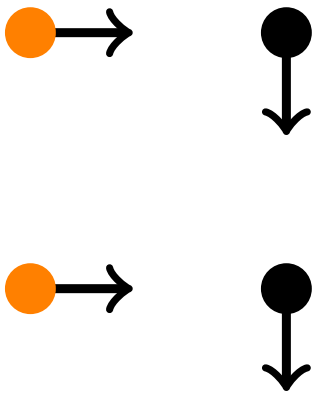
Square 87



Solver reported:

```
Q.E.D.
```

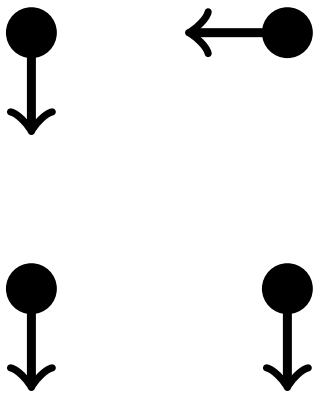
Square 88



Solver reported:

```
Q.E.D.
```

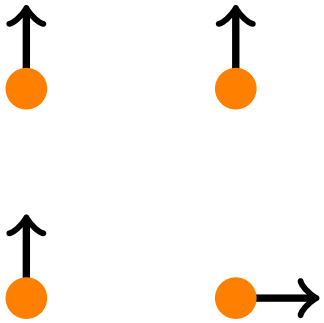
Square 89



Solver reported:

`Q.E.D.`

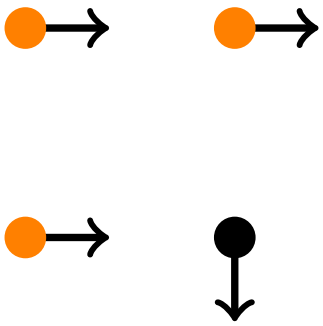
Square 90



Solver reported:

`Q.E.D.`

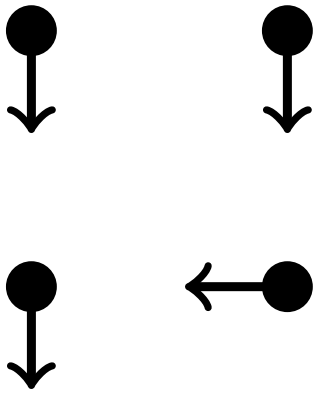
Square 91



Solver reported:

`Q.E.D.`

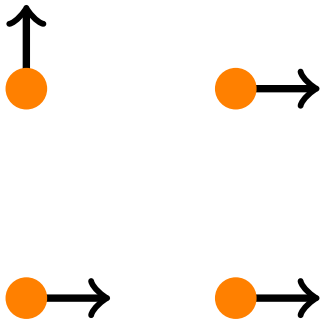
Square 92



Solver reported:

`Q.E.D.`

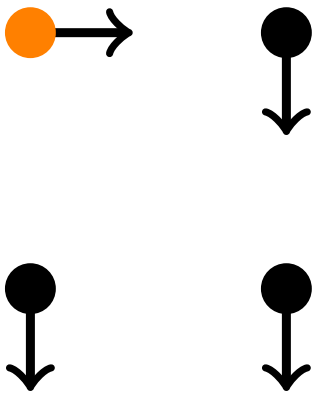
Square 93



Solver reported:

`Q.E.D.`

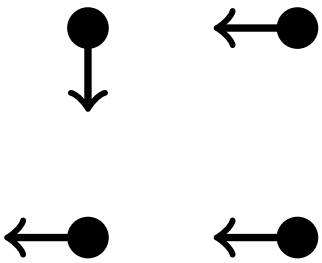
Square 94



Solver reported:

```
Q.E.D.
```

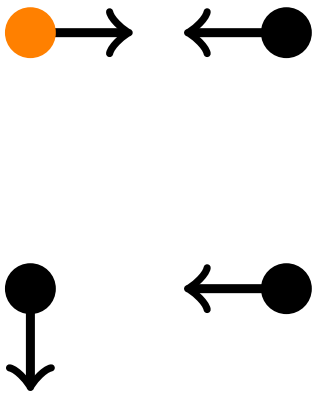
Square 95



Solver reported:

```
Q.E.D.
```

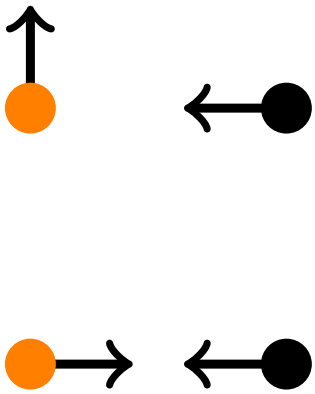
Square 96



Solver reported:

```
Falsifiable. Counter-example:
  red    =          4.0 :: Real
  orange =         -1.0 :: Real
  black  =        -7.25 :: Real
  green  =       -12.25 :: Real
  blue   =       -17.25 :: Real
  x      =     0.984375 :: Real
  y      = 0.9990234375 :: Real
```

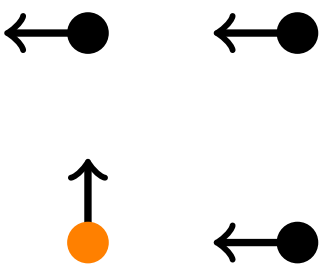
Square 97



Solver reported:

```
Falsifiable. Counter-example:
  red    =          6.0 :: Real
  orange =          1.0 :: Real
  black  =        -3.25 :: Real
  green  =        -8.25 :: Real
  blue   =       -13.25 :: Real
  x      =     0.984375 :: Real
  y      = 0.0009765625 :: Real
```

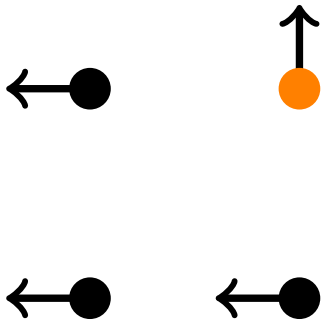
Square 98



Solver reported:

```
Falsifiable. Counter-example:
  red    =       4.0 :: Real
  orange =      -1.0 :: Real
  black  =   -5.3125 :: Real
  green  =  -10.3125 :: Real
  blue   =  -15.3125 :: Real
  x      = 0.7421875 :: Real
  y      =      0.75 :: Real
```

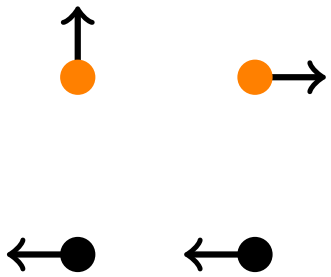
Square 99



Solver reported:

```
Falsifiable. Counter-example:
  red    =     4.75 :: Real
  orange =    -0.25 :: Real
  black  =    -6.25 :: Real
  green  =   -11.25 :: Real
  blue   =   -16.25 :: Real
  x      =      1.0 :: Real
  y      = 0.984375 :: Real
```

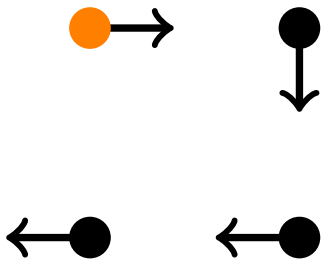
Square 100



Solver reported:

```
Falsifiable. Counter-example:
  red    =      4.5 :: Real
  orange =     -0.5 :: Real
  black  =   -4.625 :: Real
  green  =   -9.625 :: Real
  blue   =  -14.625 :: Real
  x      =      0.0 :: Real
  y      = 0.984375 :: Real
```

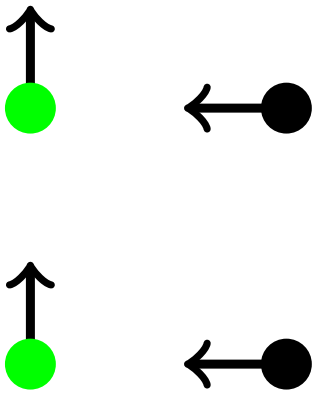
Square 101



Solver reported:

```
Falsifiable. Counter-example:
  red    =                 4.5 :: Real
  orange =                -0.5 :: Real
  black  =       -3896834241.0 :: Real
  green  =       -3896834246.0 :: Real
  blue   =       -3896834251.0 :: Real
  x      = 0.99999237060546875 :: Real
  y      =        0.0009765625 :: Real
```

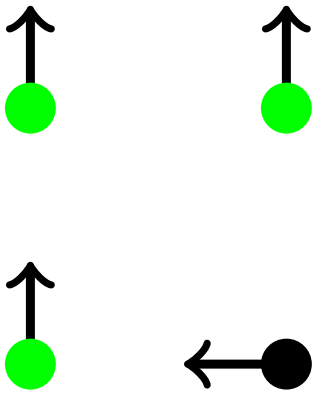
Boundary Square 1



Checked on BoundaryLeft
Solver reported:

```
Q.E.D.
```

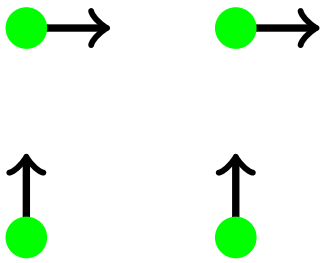
Boundary Square 2



Checked on BoundaryLeft
Solver reported:

```
Q.E.D.
```

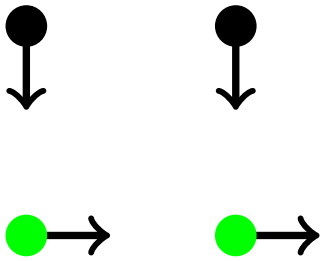
Boundary Square 3



Checked on BoundaryLeft
Solver reported:

```
Q.E.D.
```

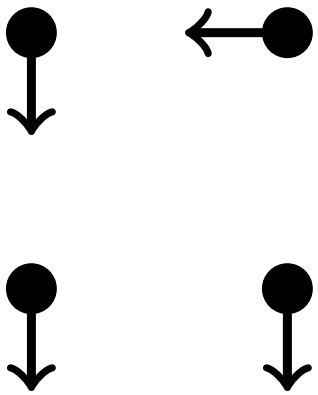
Boundary Square 4



Checked on BoundaryLeft
Solver reported:

```
Q.E.D.
```

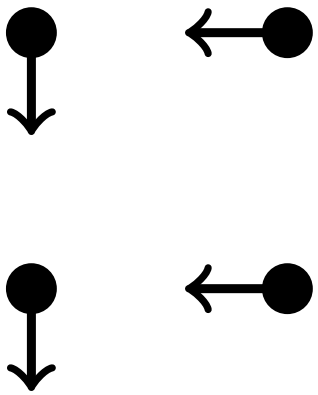
Boundary Square 5



Checked on BoundaryLeft
Solver reported:

```
Q.E.D.
```

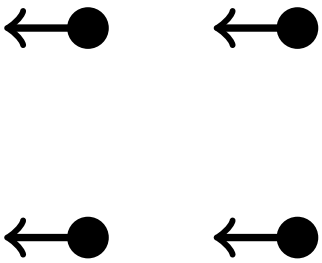
Boundary Square 6



Checked on BoundaryLeft
Solver reported:

```
Q.E.D.
```

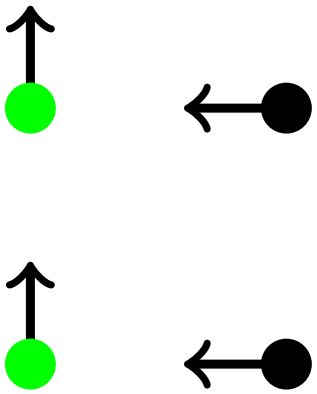
Boundary Square 7



Checked on BoundaryRight
Solver reported:

```
Q.E.D.
```

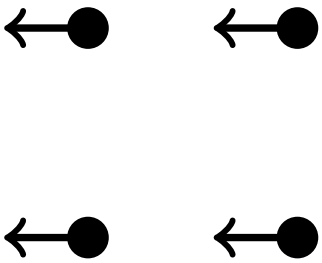
Boundary Square 8



Checked on BoundaryBottom
Solver reported:

```
Q.E.D.
```

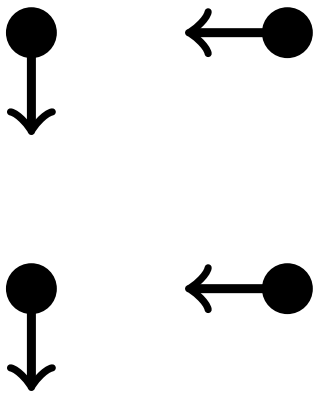
Boundary Square 9



Checked on BoundaryBottom
Solver reported:

```
Q.E.D.
```

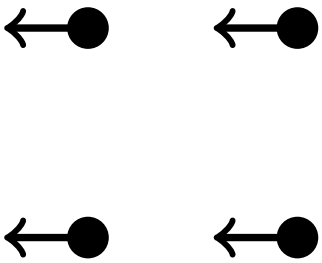
Boundary Square 10



Checked on BoundaryTop
Solver reported:

```
Q.E.D.
```

Boundary Square 11



Checked on BoundaryTop
Solver reported:

```
Q.E.D.
```