Truth Tables

We can model compound Boolean conditions using truth tables. Take, for example:

```
if (roll1 == 4 || roll2 == 4) {
     //statements to be executed go here
 }
```

The corresponding truth table is:

`roll1 == 4`	`roll2 == 4`	`roll1 == 4 || roll2 == 4`
T	T	T
T	F	T
F	T	T
F	F	F

In other words, if `roll1 == 4` evaluates to `T`, `roll2 == 4` evaluates to `T`, then we can look at the corresponding row of the truth table and see that `(roll1 == 4 || roll2 == 4)` will also evaluate to `T`.

Often, instead of writing out entire statements like `(roll1 == 4 || roll2 == 4)` we will use variables (P,Q,R…), called predicates, when modeling this logic. Here is the general truth table for (inclusive) or:

P	**Q**	**P || Q**
T	T	T
T	F	T
F	T	T
F	F	F

The other basic truth tables:

P	!P
T	
T	

P	Q	P && Q
T	T	
T	F	
F	T	
F	F	

P	Q	P xor Q
T	T	
T	F	
F	T	
F	F	

P	Q	P == Q
T	T	
T	F	
F	T	
F	F	

To do more complicated truth tables, we generally create extra columns to help us through the intermediate steps.

Example: The truth table for !(!P && Q) is

Q	**P**	**!P**	**!P && Q**	**!(!P && Q)**
T	T	F		
T	F	T		
F	T	F		
F	F	T		

Example: The truth table for P || (Q && R) is

P	**Q**	**R**	**Q && R**	**P || (Q && R)**
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Example: The truth table for (P && Q && R) || !P

P	Q	R	P && Q && R	(P && Q && R) || !P

Number of rows:

A truth table with one proposition P has 2 rows and a truth table with two propositions P and Q has 4 rows.

How many rows are there in a truth table with 3 propositions?

4 propositions?

n propositions?

DeMorgan's Laws:

!(P && Q) =

!(P && R) =

Distributive Laws:

P && (Q || R) =

P || (Q && R) =

Tautology and Contradition:

Order of Operations:

1. ( )
2. !
3. == and !=
4. &&
5. ||

Example: Make a truth table for P && Q || P && R. Simplify the expression first using the distributive law.

P	Q	R		

Example: Prove one of DeMorgan's Laws by making two truth tables and showing that both expressions are equivalent.

P	Q	!(P && Q)
T	T	
T	F	
F	T	
F	F	

P	Q	! P || ! Q
T	T	
T	F	
F	T	
F	F	