Lab 4

The *binomial theorem* gives a way to expand out an expression of the form $(a+b)^n$:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.$$

For example, if we wanted to expand the polynomial $(x+2)^3$ we could apply the binomial theorem and find that

$$\begin{aligned}(x+2)^3 &= \binom{3}{0} x^3 2^0 + \binom{3}{1} x^2 2^1 + \binom{3}{2} x^1 2^2 + \binom{3}{3} x^0 2^3 \\ &= 1x^3 2^0 + 3x^2 2^1 + 3x^1 2^2 + 1x^0 2^3 \\ &= x^3 + 6x^2 + 12x + 8.\end{aligned}$$

Use the binomial theorem to answer the following questions. This lab is broken into a group portion (worth 4 points) and an individual portion (worth 6 points).

Group Questions: Hand the solutions in for these questions by the end of the lab period.

1. (1/2 point) Expand $(x+y)^5$. What is the degree of this polynomial?

2. (1/2 point) What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x-3y)^{200}$?

3. (1 point) Give a formula for the coefficient of x^k in the expansion of $(x+1/x)^{100}$, where k is an integer.

4. (2 points) Prove the binomial theorem using mathematical induction.

Individual Questions: On back. Due in two weeks.

Individual Questions:

1. (2 points) A polynomial in any number of variables is called *homogeneous* if the sums of the exponents on the variables in each term add up to the same number. For example, $x^2 + 2xy + y^2$ is homogeneous because the exponents on x and y in each term add to 2. For which values of k is $(x+y)^k$ homogeneous? Prove your answer (using full sentences!).

2. (1 point) For what values of k, if any, is $(x+y+z)^k$ a homogeneous polynomial? When the polynomial is homogeneous, what is the degree of the polynomial? You do not have to prove your answer, just state it.

3. (3 points) What is the coefficient of w^4x^2yz in the expansion of $(w + 2x + y - z)^8$? You do not need to write a proof, but please explain your answer well.