Big-O Notation Practice

This is an ungraded class activity and you may not have time to answer all of the questions. You have until approximately 2:35-2:40 to work on this.

Review of limits - evaluate the following:

$\lim_{n\to\infty} \frac{n^{10}-1000n^4+12}{3n^{10}-n^9} = \lim_{n\to\infty} \frac{n^{10}}{3n^{10}} = \frac{1}{3}$

$\lim_{n\to\infty} \frac{1}{e^n} = 0$

$\lim_{n\to\infty} \frac{n!}{n^n} = 0$ (n^n grows much faster than $n!$)

$\lim_{n\to\infty} \frac{n^4+n^3-36}{n+8} = \infty$

Instructions: Answer the remainder of the questions on this worksheet **twice** - once using the limit definition of big-O notation, and once using the inequality definition.

An algorithm takes $f(n) = n^7 + 12$ operations to execute. How should you describe this run-time using big-O notation? Prove that your answer is correct using each definition.

Should describe it as $O(n^7)$

limit def:

$\lim_{n\to\infty} \frac{n^7+12}{n^7} = 1$

Since 1 is finite then n^7+12 is $O(n^7)$

Inequality def: $f(n) = n^7+12$
$g(n) = n^7$

$n^7 + 12 \le C \cdot n^7$

$1 + \frac{12}{n^7} \le C$ if $n_0 = 1$ and $C = 13$ then

this inequality is true for any $n \ge n_0$

since $1 + \frac{12}{n^7} \le 13$ whenever $n \ge 1$.

Thus $f(n)$ is $O(n^7)$

Prove that the function from the last question is $O(8n^{11})$. Even though this fits the definition of big-O, what is wrong with using $8n^{11}$ to describe the run-time of your algorithm?

$$\lim_{n\to\infty} \frac{n^7+12}{8n^{11}} = 0$$

0 is $< \infty$ so $f(n)$ is $O(8n^{11})$. However, this is not a good way to describe the run-time of the algorithm; we want to use $O(n^7)$ because it is closer to the actual number of operations

$$n^7 + 12 \le C \cdot 8n^{11}$$

$$\Rightarrow \frac{1}{8n^4} + \frac{12}{8n^{11}} \le C$$

using $n_0 = 1$ we have $C = \frac{1}{8} + \frac{12}{8} = \frac{13}{8}$,

i.e. $$\frac{1}{8n^4} + \frac{12}{8n^{11}} \le \frac{13}{8}$$

Since this inequality is true whenever $n \ge 1$, we know that n^7+12 is $O(8n^{11})$ (but it's still better to describe it as $O(n^7)$)

Prove that the function f is not $O(n+\sqrt{n})$.

first, using limits:

$$\lim_{n\to\infty} \frac{n^7+12}{n+\sqrt{n}} = \infty$$

Since the limit is <u>not</u> finite, n^7+12 is not $O(n+\sqrt{n})$

using inequality def:

is $n^7 + 12 \le c(n+\sqrt{n})$? divide both sides by $n+\sqrt{n}$

Is $\frac{n^7+12}{n+\sqrt{n}} \le c$ for some number c?

NO! because $\frac{n^7+12}{n+\sqrt{n}}$ will grow (arbitrarily large) as n grows. So no matter what value we pick for c the inequality will not be true for all values of $n \ge n_0$, so $f(n)$ is <u>not</u> $O(n+\sqrt{n})$.