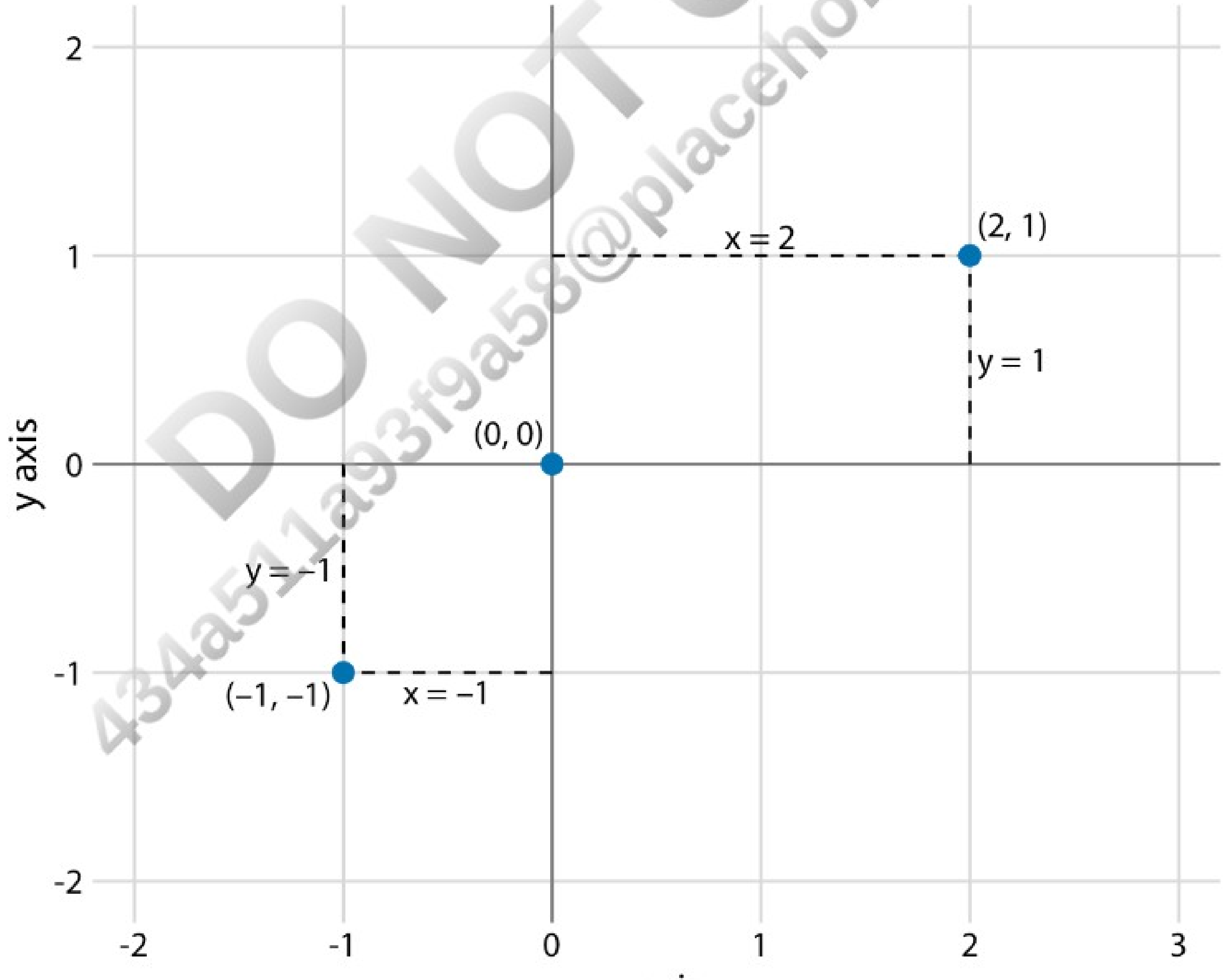
Chapter 3. Coordinate Systems and Axes

To make any sort of data visualization, we need to define position scales, which determine where in a graphic different data values are located. We cannot visualize data without placing different data points at different locations, even if we just arrange them next to each other along a line. For regular 2D visualizations, two numbers are required to uniquely specify a point, and therefore we need two position scales. These two scales are usually but not necessarily the *x* and *y* axes of the plot. We also have to specify the relative geometric arrangement of these scales. Conventionally, the *x* axis runs horizontally and the *y* axis vertically, but we could choose other arrangements. For example, we could have the *y* axis run at an acute angle relative to the *x* axis, or we could have one axis run in a circle and the other run radially. The combination of a set of position scales and their relative geometric arrangement is called a *coordinate system*.

Cartesian Coordinates

The most widely used coordinate system for data visualization is the 2D *Cartesian coordinate system*, where each location is uniquely specified by an *x* and a *y* value. The *x* and *y* axes run orthogonally to each other, and data values are placed in an even spacing along both axes (Figure 3-1). The two axes are continuous position scales, and they can represent both positive and negative real numbers. To fully specify the coordinate system, we need to specify the range of numbers each axis covers. In Figure 3-1, the *x* axis runs from –2.2 to 3.2 and the *y* axis runs from –2.2 to 2.2. Any data values between these axis limits are placed at the appropriate respective location in the plot. Any data values outside the axis limits are discarded.



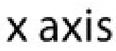


Figure 3-1. Standard Cartesian coordinate system. The horizontal axis is conventionally called x and the vertical axis y. The two axes form a grid with equidistant spacing. Here, both the x and the y grid lines are separated by units of one. The point (2, 1) is located two x units to the right and one y unit above the origin (0, 0). The point (–1, –1) is located one x unit to the left and one y unit below the origin.

Data values usually aren't just numbers, however. They come with units. For example, if we're measuring temperature, the values may be measured in degrees Celsius or Fahrenheit. Similarly, if we're measuring distance, the values may be measured in kilometers or miles, and if we're measuring duration, the values may be measured in minutes, hours, or days. In a Cartesian coordinate system, the spacing between grid lines along an axis corresponds to discrete steps in these data units. In a temperature scale, for example, we may have a grid line every 10 degrees Fahrenheit, and in a distance scale, we may have a grid line every 5 kilometers.

A Cartesian coordinate system can have two axes representing two different units. This situation arises quite commonly whenever we're mapping two different types of variables to *x* and *y*. For example, in Figure 2-3, we plotted temperature versus days of the year. The *y* axis of Figure 2-3 is measured in degrees Fahrenheit, with a grid line every at 20 degrees, and the *x* axis is measured in months, with a grid line at the first of every third month. Whenever the two axes are measured in different units, we can stretch or compress one relative to the other and maintain a valid visualization of the data (Figure 3-2). Which version is preferable may depend on the story we want to convey. A tall and narrow figure emphasizes change along the *y* axis and a short and wide figure does the opposite. Ideally, we want to choose an aspect ratio that ensures that any important differences in position are noticeable.

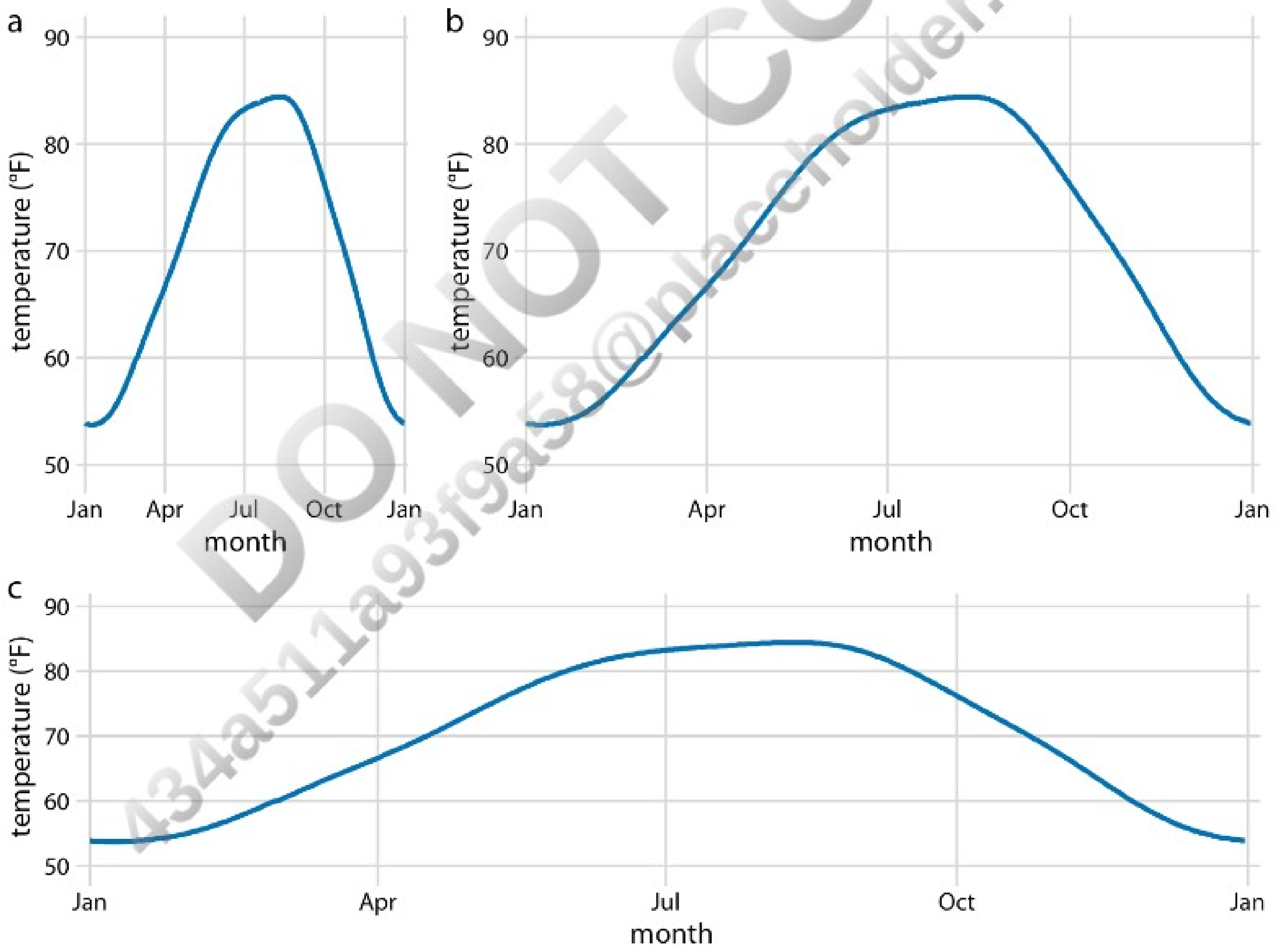


Figure 3-2. Daily temperature normals for Houston, TX. Temperature is mapped to the y axis and day of the year to the x axis. Parts (a), (b), and (c) show the same figure in different aspect ratios. All three parts are valid visualizations of the temperature data. Data source: NOAA.

On the other hand, if the *x* and *y* axes are measured in the same units, then the grid spacings for the two axes should be equal, such that the same distance along the *x* or *y* axis corresponds to the same number of data units. As an example, we can plot the temperature in Houston, TX, against the temperature in San Diego, CA, for every day of the year (Figure 3-3a). Since the same quantity is plotted along both axes, we need to make sure that the grid lines form perfect squares, as is the case in Figure 3-3a.

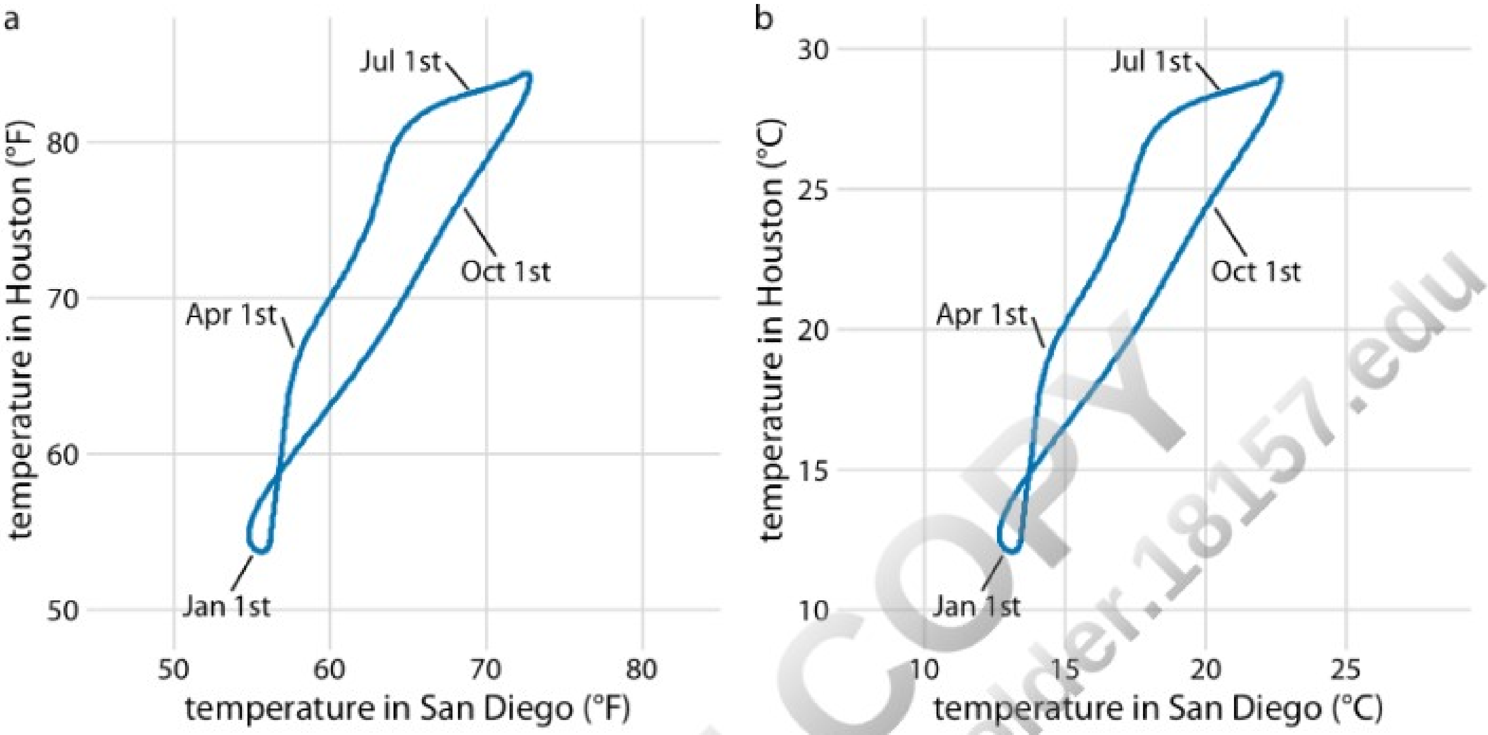


Figure 3-3. Daily temperature normals for Houston, TX, plotted versus the respective temperature normals of San Diego, CA. The first days of the months January, April, July, and October are highlighted to provide a temporal reference. (a) Temperatures are shown in degrees Fahrenheit. (b) Temperatures are shown in degrees Celsius. Data source: NOAA.

You may wonder what happens if you change the units of your data. After all, units are arbitrary, and your preferences might be different from somebody else's. A change in units is a linear transformation, where we add or subtract a number to or from all data values and/or multiply all data values with another number. Fortunately, Cartesian coordinate systems are invariant under such linear transformations. Therefore, you can change the units of your data and the resulting figure will not change as long as you change the axes accordingly. As an example, compare Figures 3-3a and 3-3

3-3b. Both show the same data, but in part (a) the temperature units are degrees Fahrenheit and in part (b) they are degrees Celsius. Even though the grid lines are in different locations and the numbers along the axes are different, the two data visualizations look exactly the same.

Nonlinear Axes

In a Cartesian coordinate system, the grid lines along an axis are spaced evenly both in data units and in the resulting visualization. We refer to the position scales in these coordinate systems as *linear*. While linear scales generally provide an accurate representation of the data, there are scenarios where nonlinear scales are preferred. In a nonlinear scale, even spacing in data units corresponds to uneven spacing in the visualization, or conversely even spacing in the visualization corresponds to uneven spacing in data units.

The most commonly used nonlinear scale is the *logarithmic scale*, or *log scale* for short. Log scales are linear in multiplication, such that a unit step on the scale corresponds to multiplication with a fixed value. To create a log scale, we need to log-transform the data values while exponentiating the numbers that are shown along the axis grid lines. This process is demonstrated in Figure 3-4, which shows the numbers 1, 3.16, 10, 31.6, and 100 placed on linear and log scales. The numbers 3.16 and 31.6 may seem like strange choices, but they were selected because they are exactly halfway between 1 and 10 and between 10 and 100 on a log scale. We can see this by observing that $10^{0.5} = \sqrt{10} \approx 3.16$, and equivalently $3.16 \times 3.16 \approx 10$. Similarly, $10^{1.5} = 10 \times 10^{0.5} \approx 31.6$.

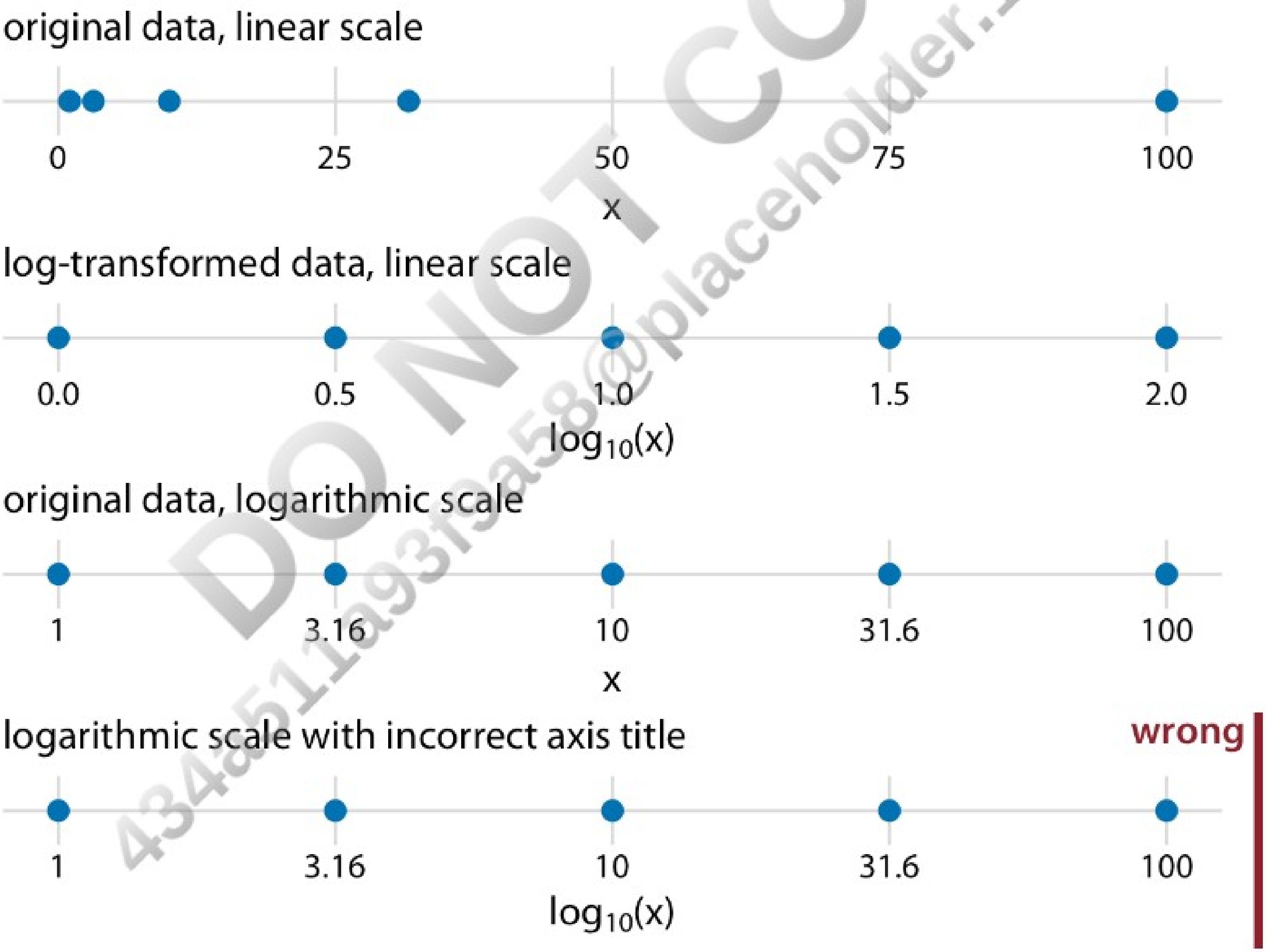


Figure 3-4. Relationship between linear and logarithmic scales. The dots correspond to the data values 1, 3.16, 10, 31.6, and 100, which are evenly spaced numbers on a logarithmic scale. We can display these data points on a linear scale, we can log-transform them and then show them on a linear scale, or we can show them on a logarithmic scale. Importantly, the correct axis title for a logarithmic scale is the name of the variable shown, not the logarithm of that variable.

Mathematically, there is no difference between plotting the log-transformed data on a linear scale or plotting the original data

Mathematically, there is no difference between plotting the log-transformed data on a linear scale or plotting the original data on a logarithmic scale (Figure 3-4). The only difference lies in the labeling for the individual axis ticks and for the axis as a whole. In most cases, the labeling for a logarithmic scale is preferable, because it places less mental burden on the reader to interpret the numbers shown as the axis tick labels. There is also less of a risk of confusion about the base of the logarithm. When working with log-transformed data, we can get confused about whether the data was transformed using the natural logarithm or the logarithm to base 10. And it's not uncommon for labeling to be ambiguous—e.g., log(*x*), which doesn't specify a base at all. I recommend that you always verify the base when working with log-transformed data. When plotting log-transformed data, always specify the base in the labeling of the axis.

Because multiplication on a log scale looks like addition on a linear scale, log scales are the natural choice for any data that has been obtained by multiplication or division. In particular, ratios should generally be shown on a log scale. As an example, I have taken the number of inhabitants in each county in Texas and divided it by the median number of inhabitants across all Texas counties. The resulting ratio is a number that can be larger or smaller than 1. A ratio of exactly 1 implies that the corresponding county has the median number of inhabitants. When visualizing these ratios on a log scale, we can see that the population numbers in Texas counties are symmetrically distributed around the median, and that the most populous counties have over 100 times more inhabitants than the median while the least populous counties have over 100 times fewer inhabitants (Figure 3-5).

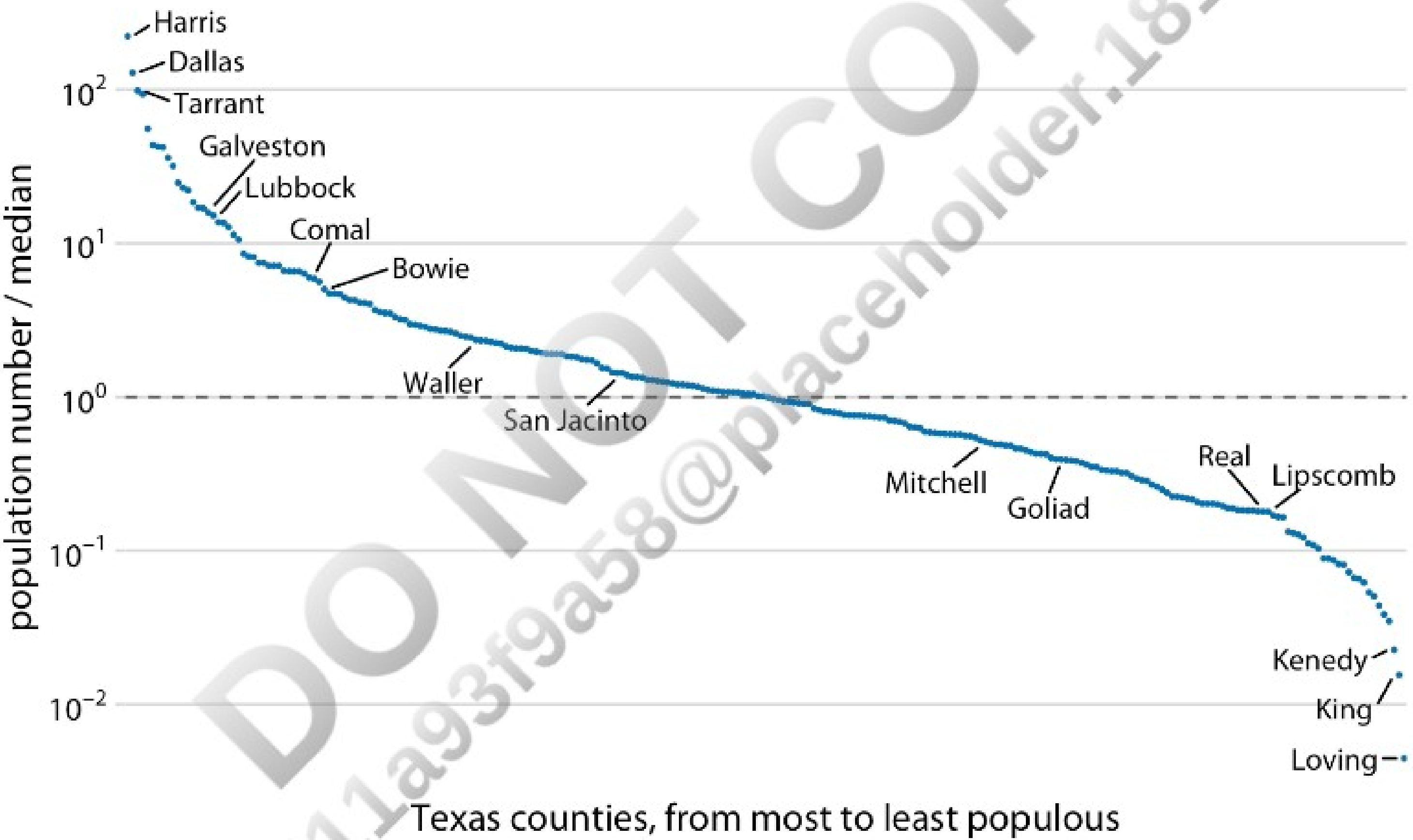


Figure 3-5. Population numbers of Texas counties relative to their median value. Select counties are highlighted by name. The dashed line indicates a ratio of 1, corresponding to a county with median population number. The most populous counties have approximately 100 times more inhabitants than the median county, and the least populous counties have approximately 100 times fewer inhabitants than the median county. Data source: 2010 US Decennial Census.

By contrast, for the same data, a linear scale obscures the differences between a county with median population number and a county with a much smaller population number than median (Figure 3-6).

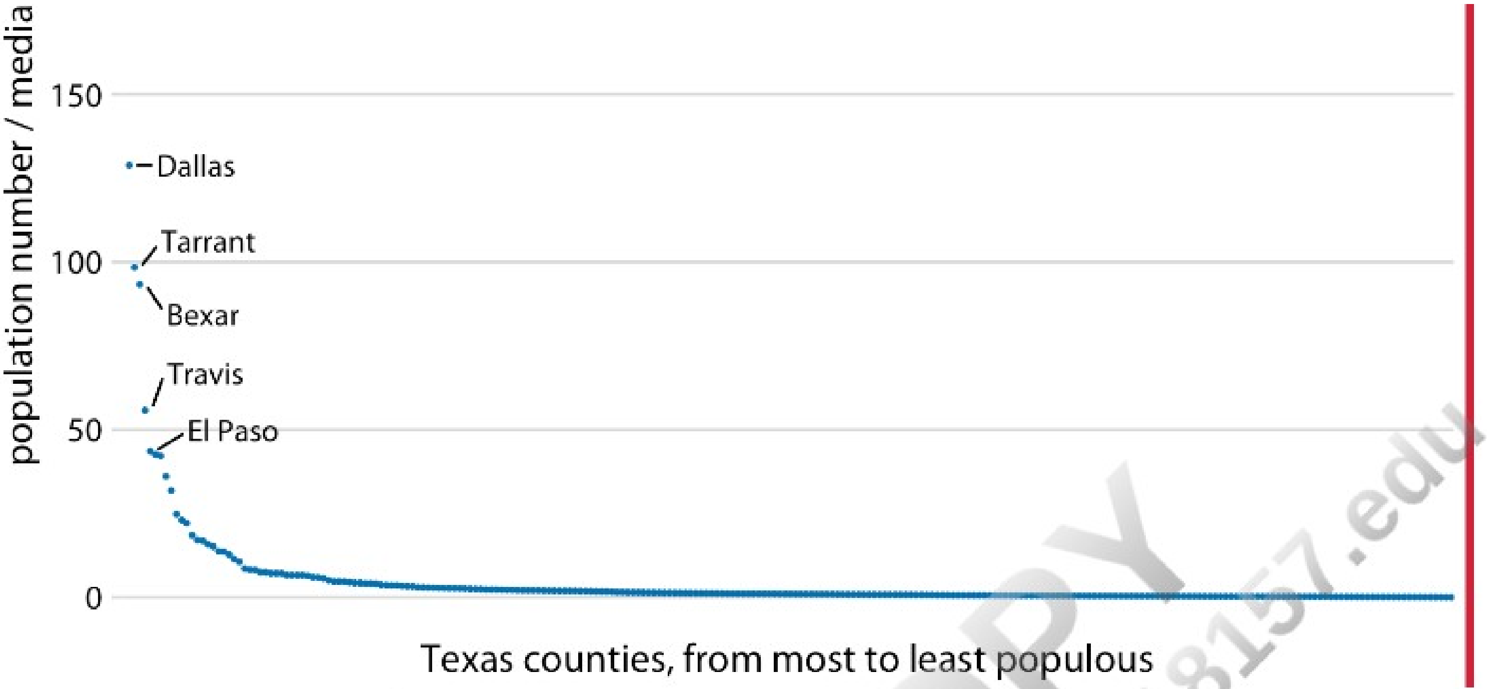
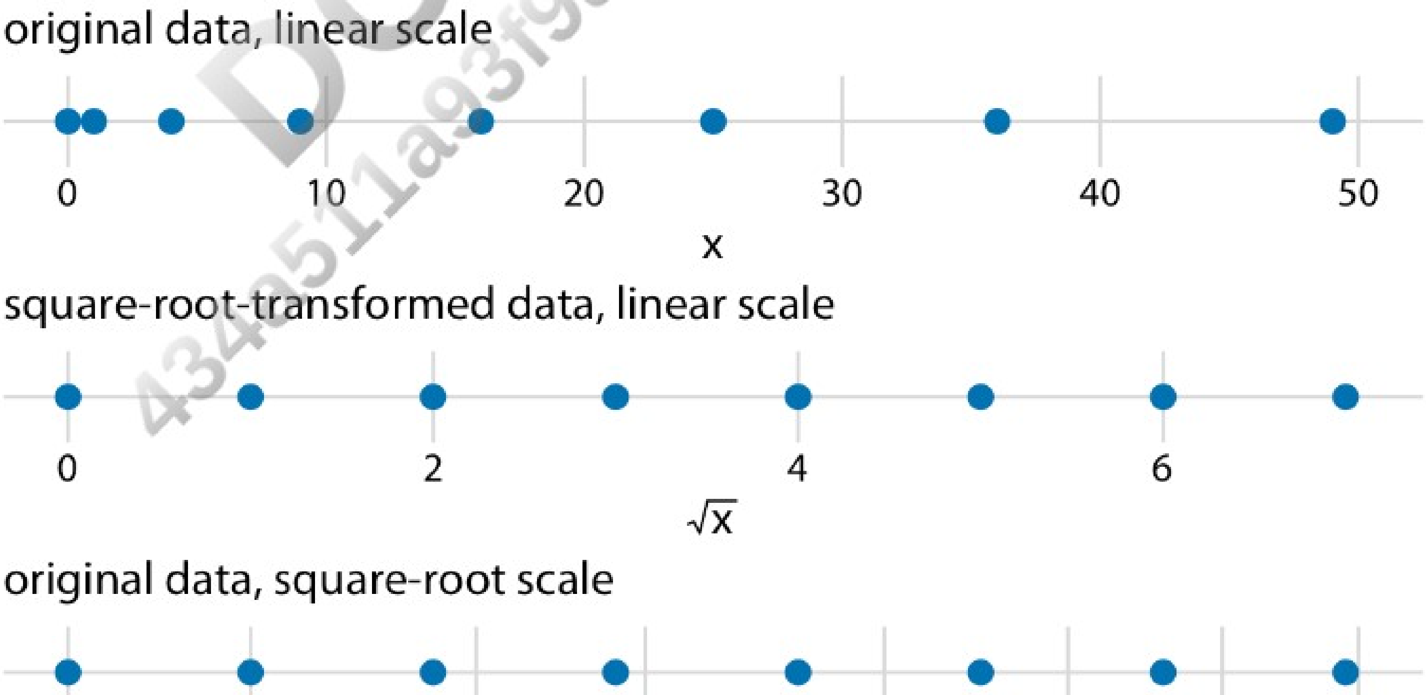


Figure 3-6. Population sizes of Texas counties relative to their median value. By displaying a ratio on a linear scale, we have overemphasized ratios > 1 and have obscured ratios < 1. As a general rule, ratios should not be displayed on a linear scale. Data source: 2010 US Decennial Census.

On a log scale, the value 1 is the natural midpoint, similar to the value 0 on a linear scale. We can think of values greater than 1 as representing multiplications and values less than 1 divisions. For example, we can write 10 = 1 × 10 and 0.1 = 1/10. The value 0, on the other hand, can never appear on a log scale. It lies infinitely far from 1. One way to see this is to consider that log(0) = –∞. Or, alternatively, consider that to go from 1 to 0, it takes either an infinite number of divisions by a finite value (e.g., 1/10/10/10/10/10/10· · · = 0) or one division by infinity (i.e., 1/∞ = 0).

Log scales are frequently used when the dataset contains numbers of very different magnitudes. For the Texas counties shown in Figures 3-5 and 3-6, the most populous one (Harris) had 4,092,459 inhabitants in the 2010 US Census while the least populous one (Loving) had 82. So, a log scale would be appropriate even if we hadn't divided the population numbers by their median to turn them into ratios. But what would we do if there was a county with 0 inhabitants? This county could not be shown on the logarithmic scale, because it would lie at minus infinity. In this situation, the recommendation is sometimes to use a *square-root scale*, which uses a square-root transformation instead of a log transformation (Figure 3-7). Just like a log scale, a square-root scale compresses larger numbers into a smaller range, but unlike a log scale, it allows for the presence of 0.



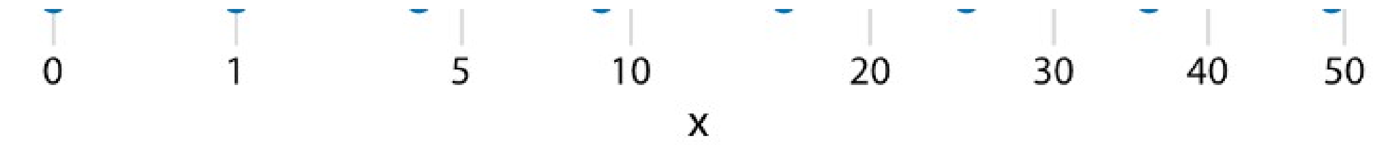


Figure 3-7. Relationship between linear and square-root scales. The dots correspond to the data values 0, 1, 4, 9, 16, 25, 36, and 49, which are evenly spaced numbers on a square-root scale, since they are the squares of the integers from 0 to 7. We can display these data points on a linear scale, we can square-root-transform them and then show them on a linear scale, or we can show them on a square-root scale.

I see two problems with square-root scales. First, while on a linear scale one unit step corresponds to addition or subtraction of a constant value, and on a log scale it corresponds to multiplication with or division by a constant value, no such rule exists for a square-root scale. The meaning of a unit step on a square-root scale depends on the scale value at which we're starting. Second, it is unclear how to best place axis ticks on a square-root scale. To obtain evenly spaced ticks, we would have to place them at squares, but axis ticks at, for example, positions 0, 4, 25, 49, and 81 (every second square) would be unintuitive. Alternatively, we could place them at linear intervals (10, 20, 30, etc.), but this would result in either too few axis ticks near the low end of the scale or too many near the high end. In Figure 3-7, I have placed the axis ticks at positions 0, 1, 5, 10, 20, 30, 40, and 50 on the square-root scale. These values are arbitrary but provide a reasonable covering of the data range.

Despite these problems with square-root scales, they are valid position scales and I do not discount the possibility that they have appropriate applications. For example, just like a log scale is the natural scale for ratios, one could argue that the square-root scale is the natural scale for data that comes in squares. One scenario in which data is naturally squares is in the context of geographic regions. If we show the areas of geographic regions on a square-root scale, we are highlighting the regions' linear extent from east to west or north to south. These extents could be relevant, for example, if we were wondering how long it might take to drive across a region. Figure 3-8 shows the areas of states in the US Northeast on both a linear and a square-root scale. Even though the areas of these states are quite different (Figure 3-8a), the relative time it will take to drive across each state is more accurately represented by the figure on the square-root scale (Figure 3-8b) than the figure on the linear scale (Figure 3-8a).

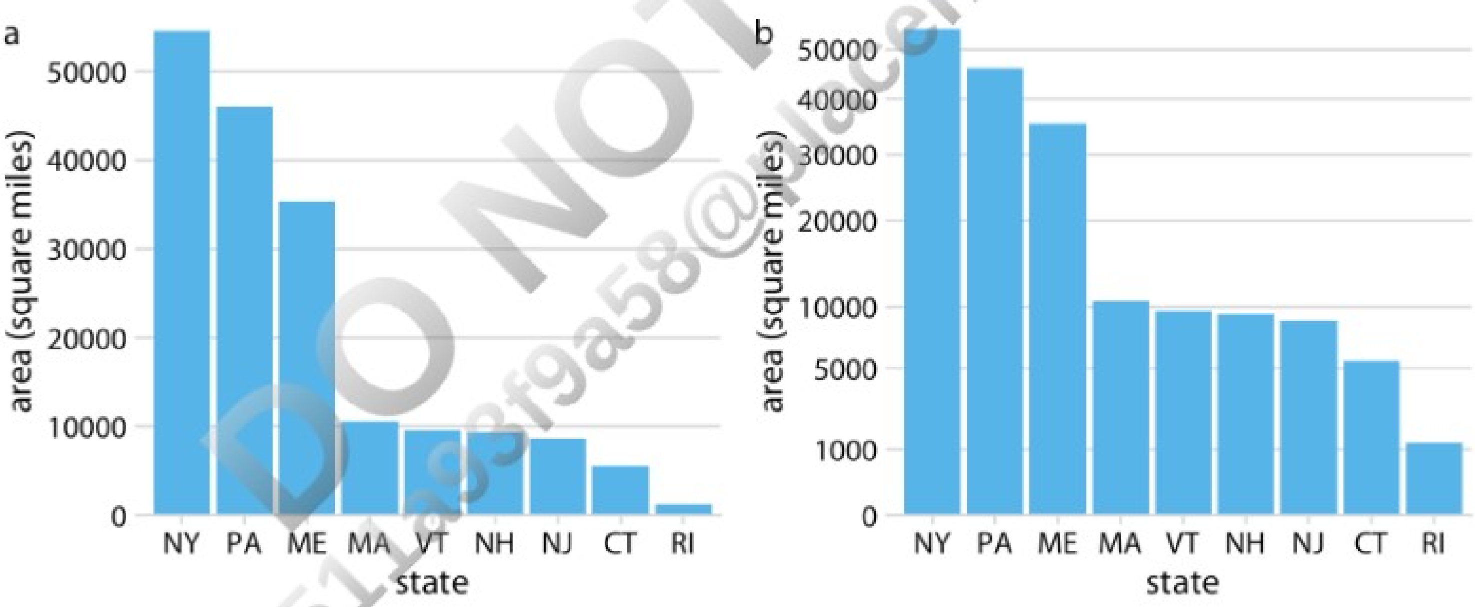


Figure 3-8. Areas of northeastern US states. (a) Areas shown on a linear scale. (b) Areas shown on a square-root scale. Data source: Google.

Coordinate Systems with Curved Axes

All the coordinate systems we have encountered so far have used two straight axes positioned at a right angle to each other, even if the axes themselves established a nonlinear mapping from data values to positions. There are other coordinate systems, however, where the axes themselves are curved. In particular, in the *polar* coordinate system, we specify positions via an angle and a radial distance from the origin, and therefore the angle axis is circular (Figure 3-9).

Polar coordinates can be useful for data of a periodic nature, such that data values at one end of the scale can be logically joined to data values at the other end. For example, consider the days in a year. December 31st is the last day of the year, but it is also one day before the first day of the year. If we want to show how some quantity varies over the year, it can be appropriate to use polar coordinates with the angle coordinate specifying each day. Let’s apply this concept to the temperature normals of Figure 2-3. Because temperature normals are average temperatures that are not tied to any specific year, Dec. 31st can be thought of as 366 days later than Jan. 1st (temperature normals include Feb. 29th) and also 1 day earlier.

By plotting the temperature normals in a polar coordinate system, we emphasize this cyclical property they have (Figure 3-10). In comparison to Figure 2-3, the polar version highlights how similar the temperatures are in Death Valley, Houston, and San Diego from late fall to early spring. In the Cartesian coordinate system, this fact is obscured because the temperature values in late December and in early January are shown in opposite parts of the figure and therefore don’t form a single visual unit.

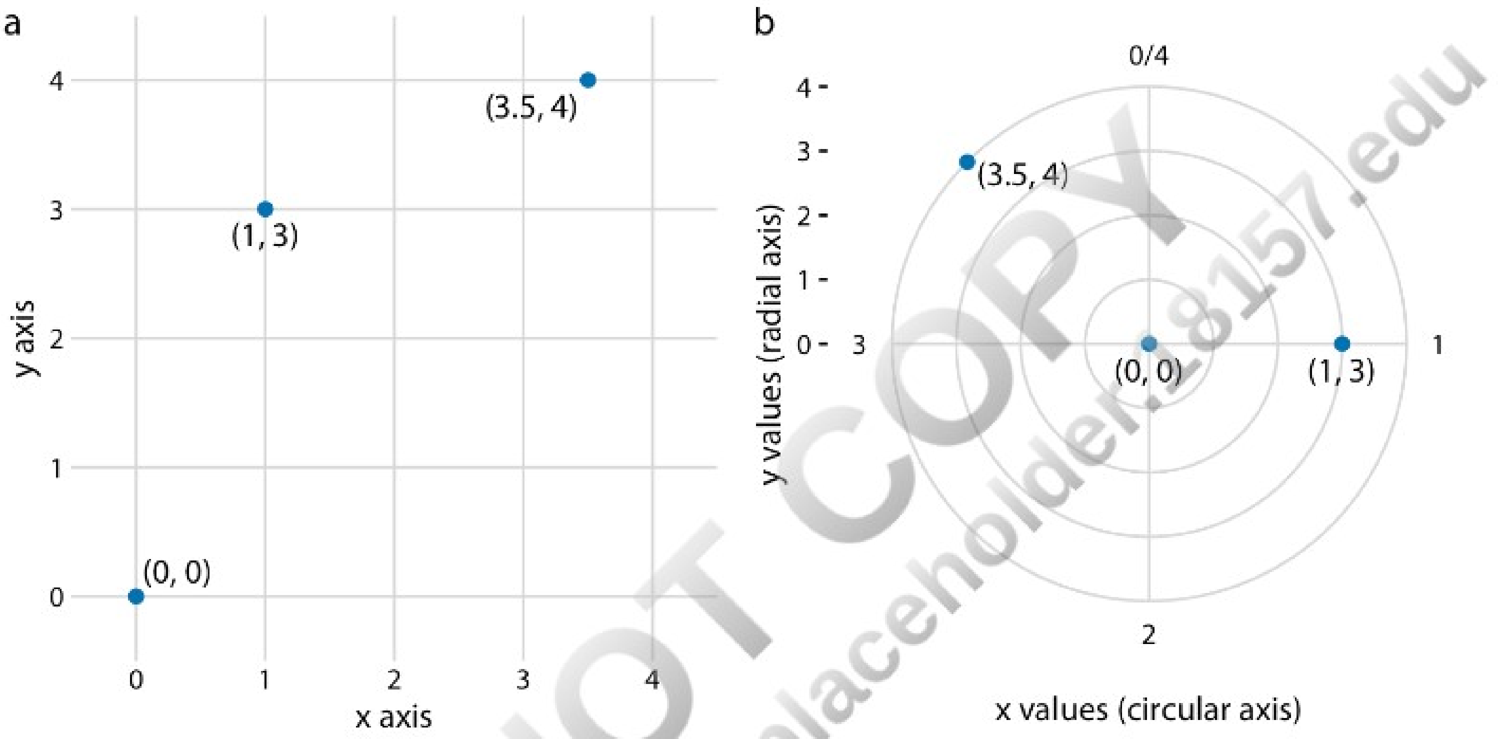
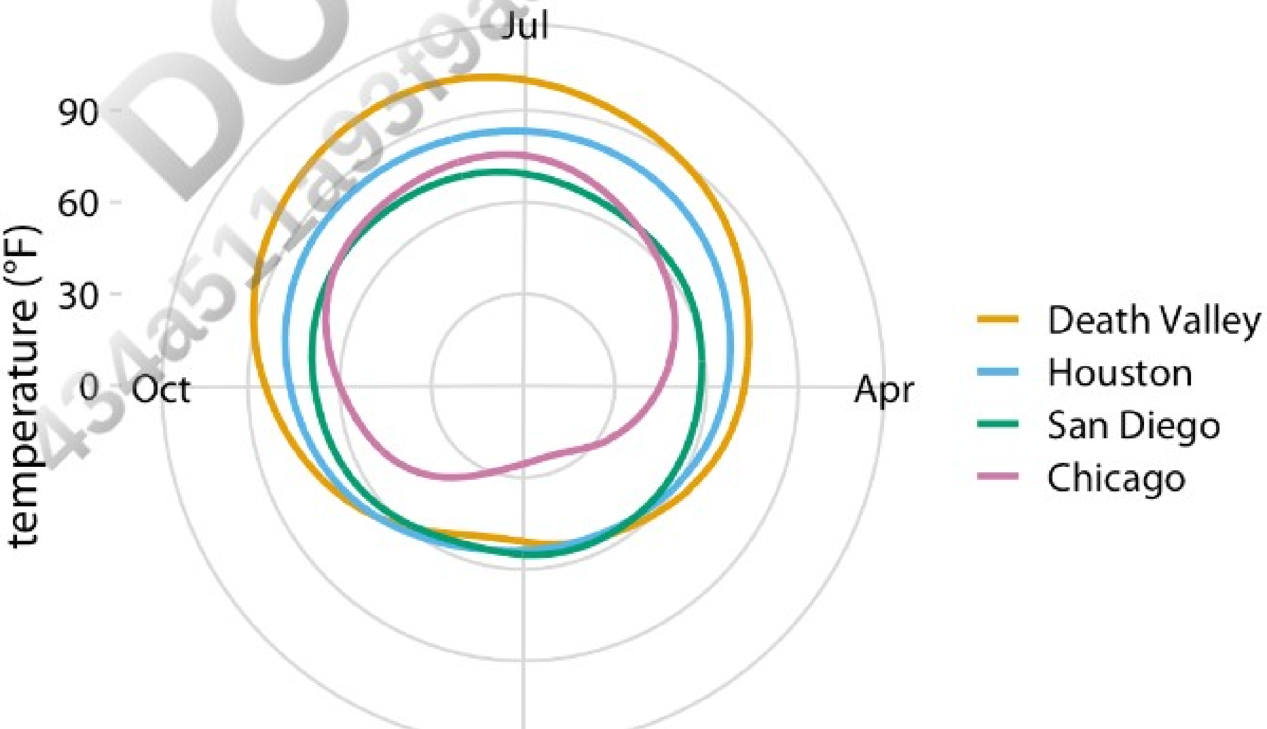


Figure 3-9. Relationship between Cartesian and polar coordinates. (a) Three data points shown in a Cartesian coordinate system. (b) The same three data points shown in a polar coordinate system. We have taken the x coordinates from part (a) and used them as angular coordinates and the y coordinates from part (a) and used them as radial coordinates. The circular axis runs from 0 to 4 in this example, and therefore x = 0 and x = 4 are the same locations in this coordinate system.



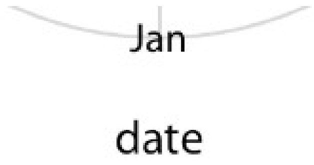


Figure 3-10. Daily temperature normals for four selected locations in the US, shown in polar coordinates. The radial distance from the center point indicates the daily temperature in Fahrenheit, and the days of the year are arranged counterclockwise starting with Jan. 1st at the 6:00 position. Data source: NOAA.

A second setting in which we encounter curved axes is in the context of geospatial data, i.e., maps. Locations on the globe are specified by their longitude and latitude. But because the earth is a sphere, drawing latitude and longitude as Cartesian axes is misleading and not recommended (Figure 3-11

Figure 3-11). Instead, we use various types of nonlinear projections that attempt to minimize artifacts and that strike different balances between conserving areas or angles relative to the true shape lines on the globe (Figure 3-11).

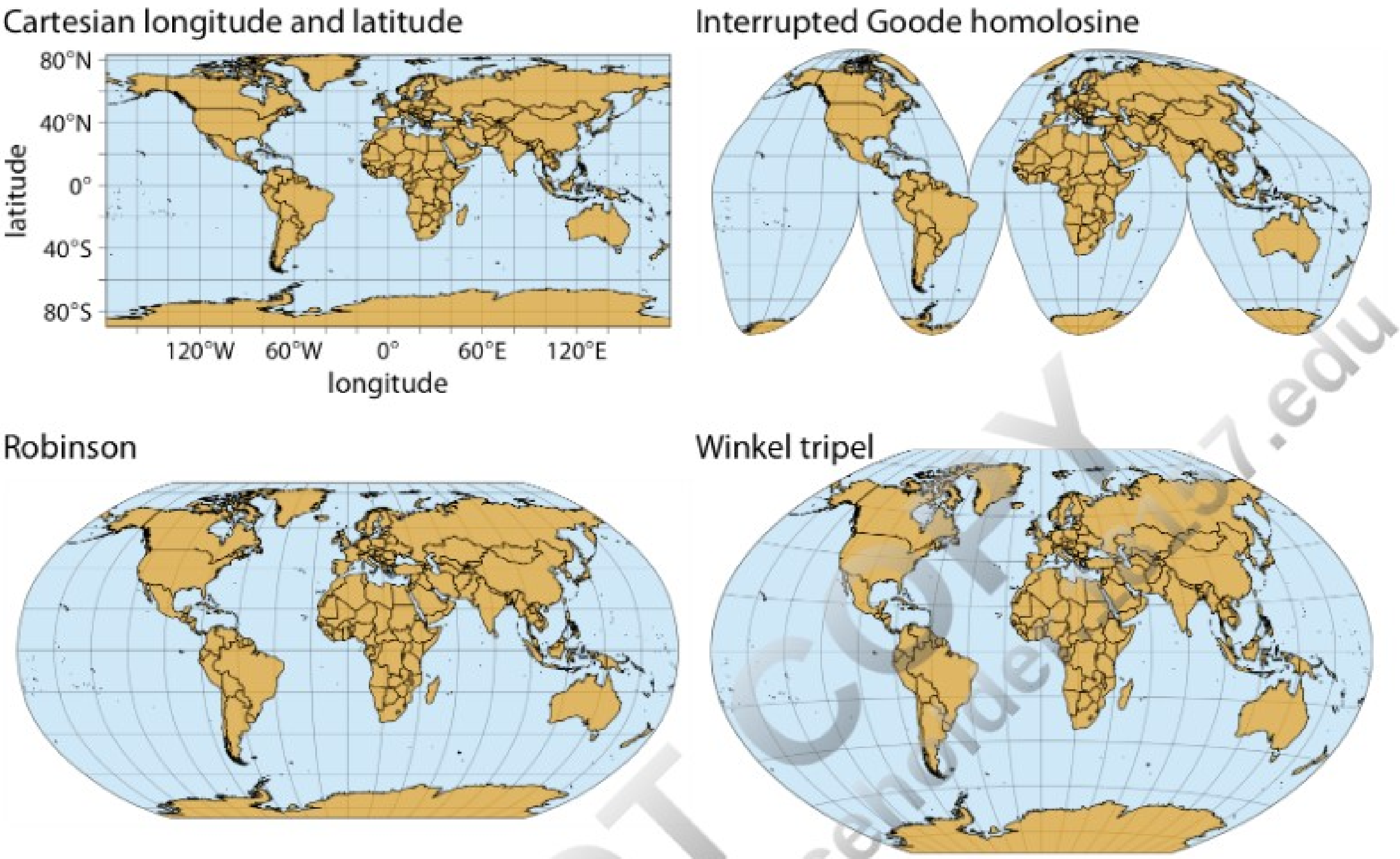


Figure 3-11. Map of the world, shown in four different projections. The Cartesian longitude and latitude system maps the longitude and latitude of each location onto a regular Cartesian coordinate system. This mapping causes substantial distortions in both areas and angles relative to their true values on the 3D globe. The interrupted Goode homolosine projection perfectly represents true surface areas, at the cost of dividing some land masses into separate pieces, most notably Greenland and Antarctica. The Robinson projection and the Winkel tripel projection both strike a balance between angular and area distortions, and they are commonly used for maps of the entire globe.