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Problemas Clase 2:

1.5.7.

2) $\langle r| = x^i \hat{e}_i$, $\langle a(r)| = a^i(x,y,z)\hat{e}_i$, $\langle b(r)| = b^i(x,y,z)\hat{e}_i$

$\Phi = \Phi(r) = \Phi(x,y,z)$, $\Psi(r) = \Psi(x,y,z)$

a) $\nabla(\Phi\Psi) = \partial^i(\Phi\Psi) = (\partial^i\Phi)\Psi\hat{e}_i + (\partial^i\Psi)\Phi\hat{e}_i$

$= \Psi\partial^i\Phi(x^i)\hat{e}_i + \Phi\partial^i\Psi(x^i)\hat{e}_i = \Psi\nabla\Phi + \Phi\nabla\Psi$

d) $\nabla\cdot(\nabla\times a) = \partial_i(\varepsilon^{jkm}\partial_k a_m e_j)$

* $\nabla\times(\nabla\cdot a)$ no es posible de realizar, puesto que $\langle\nabla|a\rangle$ resulta en un número, y para realizar el producto vectorial es necesario tener dos vectores.

f) $\nabla\times(\nabla\times a) = \varepsilon^{ijk}\partial_j(\varepsilon_{klm}\partial^l a^m) = \underline{\varepsilon^{ijk}}\,\underline{\varepsilon_{klm}}\partial_j\partial^l a^m$

$= (\delta^i_l\delta^j_m - \delta^i_m\delta^j_l)\partial_j\partial^l a^m \longrightarrow \partial_m(\partial^i a^m) - \underline{\partial_l}(\underline{\partial^l} a^i)$

$= \nabla(\nabla\cdot a) - \nabla^2 a$

1.6.6

2. a) $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)$

$$= Re(\cos(3\alpha) + i\operatorname{sen}(3\alpha)) = Re(e^{i3\alpha})$$

$$= Re((e^{i\alpha})^3) = Re((\cos(\alpha) + i\operatorname{sen}(\alpha))^3)$$

$$= Re(\underbrace{\cos^3\alpha}_{Re} + \underbrace{3i\cos^2\alpha\operatorname{sen}\alpha}_{Im} - \underbrace{3\cos\alpha\operatorname{sen}^2\alpha}_{Re} - \underbrace{i\operatorname{sen}^3\alpha}_{Im})$$

$$= \cos^3\alpha - 3\cos\alpha\operatorname{sen}^3\alpha$$

b) $\operatorname{sen}(3\alpha) = 3\cos^2\alpha\operatorname{sen}\alpha - \operatorname{sen}^3\alpha$

$$= Im(\cos(3\alpha) + i\operatorname{sen}(3\alpha)) = \dots$$

$$= \dots = Im(\underbrace{\cos^3\alpha}_{Re} + \underbrace{3i\cos^2\alpha\operatorname{sen}\alpha}_{Im} - \underbrace{3\cos\alpha\operatorname{sen}^2\alpha}_{Re} - \underbrace{i\operatorname{sen}^3\alpha}_{Im})$$

$$= 3\cos^2\alpha\operatorname{sen}\alpha - \operatorname{sen}^3\alpha$$

5. $\sqrt[n]{z} = z^{1/n} = (|z| \cdot e^{i\theta + 2\pi k})^{1/n} = |z|^{1/n} \cdot e^{\frac{i\theta + 2\pi k}{n}}$, $k \in \mathbb{N}_0$ ó $\mathbb{Z}^+$

a) $\sqrt{2i} = \sqrt{2} \cdot e^{\frac{i\pi/2 + 2\pi k}{2}}$

$|z| = 2$

$\theta = \pi/2$

b) $\sqrt{1-\sqrt{3}i} = \sqrt{2} \cdot e^{\frac{i\frac{5\pi}{3} + 2\pi k}{2}}$

$|z| = 2$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = \frac{5\pi}{3}$

c) $\sqrt[3]{-1} = \sqrt[3]{-1} \cdot e^{\frac{i\pi + 2\pi k}{3}}$

$|z| = -1$ $\quad = -e^{\frac{i\pi + 2\pi k}{3}}$

$\theta = \pi$

d) $\sqrt[6]{8} = \sqrt[6]{8} \cdot e^{\frac{i2\pi k}{6}}$

$|z| = 8$ $\quad = \sqrt{2} \cdot e^{\frac{i2\pi k}{6}}$

$\theta = 0$

e) $\sqrt[4]{-8-8\sqrt{3}\,i} = \sqrt[4]{16} \cdot e^{\frac{i\frac{\pi}{3}+2\pi k}{4}}$

$= \pm 2 \cdot e^{\frac{i\pi/3 + 2\pi k}{4}}$

$|z| = 16$

$\theta = \tan^{-1}\left(\frac{8\sqrt{3}}{8}\right) = \frac{\pi}{3}$ //

6. $z = |z| e^{i\theta} \rightarrow Ln(z) = Ln(|z|) + Ln(e^{i\theta}) = Ln|z| + i\theta$

• para $n \neq 0 \rightarrow Log(z) = Ln|z| + i\theta + 2\pi n$

a) $Log(-ie) = 1 - \pi/2 \, i$

$|z| = e, \; \theta = -\pi/2 \rightarrow Ln(e) + (-\pi/2)\,i = 1 - \pi/2 \, i$ //

b) $Log(1-i) = \frac{1}{2} Ln(2) - \pi/4 \, i$

$|z| = \sqrt{2} = 2^{1/2}$

$\theta = -\pi/4$

$\rightarrow Ln(2^{1/2}) + (-\pi/4)\,i = \frac{1}{2} Ln(2) - \pi/4 \, i$ //

c) $Log(e) = 1 + 2n\pi i$

$|z| = e \rightarrow Ln(e) + \overbrace{2n\pi}^{0}\, i = 1 + 2n\pi i$ // $\quad n \in \mathbb{Z}$

$\theta = 0 = 2n\pi$ ↺

d) $Log(i) = (2n + \frac{1}{2})\pi i$

$|z| = 1$

$\theta = \pi/2 = (4n+1)\frac{\pi}{2}$ ↺

$\rightarrow Ln(1) + (4n+1)\frac{\pi}{2} i = (2n + \frac{1}{2})\pi i$ //