Real Statistics Using Excel
Everything you need to do real statistical analysis using Excel

Grubbs' Test

Grubbs' test can be used to test the presence of one outlier and can be used with data that is normally distributed (except for the outlier) and has at least 7 elements (preferably more).

Here we test the null hypothesis that the data has no outliers vs. the alternative hypothesis that there is one outlier. The ESD test should be used if there is the possibility of more than one outlier.

If you suspect that the maximum value in the data set may be an outlier you can use the test statistic

$$G = \frac{x_{max} - \bar{x}}{s}$$

If you suspect that the minimum value in the data set may be an outlier you can use the test statistic

$$G = \frac{\bar{x} - x_{min}}{s}$$

The critical value for the test is

$$G_{crit} = \frac{(n-1)t_{crit}}{\sqrt{n(n-2+t_{crit}^2)}}$$

where t_{crit} is the critical value of the t distribution $T(n-2)$ and the significance level is α/n. Thus the null hypothesis is rejected if $G > G_{crit}$.

There is also a two-tailed version of the test where G is the larger of the two G values described above and G_{crit} is defined as above except that the significance level for t_{crit} is $\alpha/(2n)$. Alternatively, G can be calculated using the formula

$$G = \frac{max|x_i - \bar{x}|}{s}$$

Example 1: Determine whether the data set {145, 125, 190, 135, 220, 130, 210, 3, 165, 165, 150} has an outlier.

The data set is shown in range A4:A14 of Figure 1. Using the Shapiro-Wilk test, we see that the data is not normally distributed. However, when we remove the data element 3, which seems to be a potential outlier (as shown in range K4:K14), we find that the data is normally distributed, thus allowing us to use Grubbs' test

	A	C	D	K	M	N
3		Shapiro-Wilk Test			Shapiro-Wilk Test	
4	145			145		
5	125		*Group 1*	125		*Group 1*
6	190	W	0.854251	190	W	0.914883
7	135	p-value	0.048484	135	p-value	0.316244
8	220	alpha	0.05	220	alpha	0.05
9	130	normal	no	130	normal	yes
10	210			210		
11	3					
12	165			165		
13	165			165		
14	150			150		

Figure 1 – Testing data for normality

Grubbs' test is implemented in Figure 2.

	A	F	G	I
1	**Grubbs Test**			
2				
3		min	3	=MIN(A4:A14)
4	145	mean	148.9091	=AVERAGE(A4:A14)
5	125	stdev	57.81082	=STDEV.S(A4:A14)
6	190	G	2.523906	=(G4-G3)/G5
7	135			
8	220	alpha	0.05	
9	130	size	11	=COUNT(A4:A14)
10	210	sig value	0.004545	=G8/G9
11	3	df	9	=G9-2
12	165	t-crit	3.309517	=T.INV(1-G10,G11)
13	165	G-crit	2.233908	=(G9-1)*G12/SQRT(G9*(G11+G12^2))
14	150	sig	yes	=IF(G6>G13,"yes","no")

Figure 2 – Grubbs' Test

We see that 3 is a little more than 2.5 standard deviations from the mean (cell G6) and that the test is significant (cell G14), meaning that 3 is an outlier (based on $\alpha = .05$).

Real Statistics Function: The Real Statistics Resource Pack provides the following array function to perform a one-tailed Grubbs' test.

GRUBBS(R1, *lab, alpha*): outputs a 4 × 1 column range with the following entries: potential outlier, G, Gcrit, and test significance

If *lab* = TRUE (default FALSE) then the output is a 4 × 2 matrix with a column of labels added. *alpha* = the significance level (default .05). The potential outlier is either the maximum or minimum value in R1, depending on which is farthest away from the mean of R1. The test significance if "yes" if $G > G_{crit}$ and "no" otherwise.

For Example 1, if you highlight the range P5:Q8, enter the formula =GRUBBS(A4:A14,TRUE) and press **Ctrl-Shft-Enter**, then the output that appears is displayed in Figure 3.

	P	Q
4		
5	outlier	3
6	G	2.523906
7	G-crit	2.233908
8	sig	yes

Figure 3 – Output from GRUBBS formula

23 Responses to *Grubbs' Test*

Tuba *says:*
February 29, 2020 at 9:47 pm

Hi Charles,

I wanted to get your opinion on the appropriate alpha level to use. I see that you have used alpha=0.05 for Grubbs' test. Several references suggest setting a lower alpha level (ie. 0.01) for outlier tests when using the same data for other statistical tests (ie 0.05). What are your thoughts on this? Thank you.

Tuba

Reply

Charles *says:*
March 1, 2020 at 9:54 am

Hello Tuba,
I haven't thought about this before and so don't have a strong opinion one way or the other. In general, I tend to view such tests as guidelines, and in any case, and don't pay strict attention to the alpha value. If the statistic is low, then I suspect that there is a potential outlier and try to figure whether or not I should be concerned about it. I also tend to use other tests to see whether these give me similar information.
If you have a sample with 100 elements and one of them is 2.5 standard deviations from the mean, then this isn't very surprising. If the sample has 20 elements, then I would take much more seriously that one is 2.5 standard deviations from the mean.
Charles

Reply

Aleksandra Michalska *says:*
December 4, 2019 at 9:35 pm

Hello Charles,

Could you please help me understand what the sig value is and why are you using it for t crit calculation instead of alpha?

Reply

Charles *says:*
December 5, 2019 at 10:18 am

Either approach should give the same result; i.e. for any statistic v, p-value < alpha is equivalent to v < v-crit (if v-crit is the left critical value or v > v-crit if v-crit is the right critical value). This is explained at

Null and Alternative Hypothesis
Charles

Reply

Yahn *says:*
September 4, 2019 at 8:20 am

Thanks for the information on Grubbs' test. I noticed that it's said on Graphpad website: Grubb's paper(1) gives critical values for the one-sided test, but the values in the alpha=0.025 one sided test are also for the alpha=0.05 two sided test.
https://www.graphpad.com/support/faqid/1598/

Do you agree with that the alpha shoud be 0.025 in one sided test?
Thanks

Reply

Charles *says:*
September 4, 2019 at 2:44 pm

For a one-tail test you need to double alpha. alpha = .05 for the two-tailed test is equivalent to alpha = .10 for the one-tailed test.
Charles

Reply

Jill *says:*
September 5, 2018 at 6:09 pm

Hi, Charles. Thank you for this fantastic site. I was given a table for the Grubbs Test Critical Value, and the lowest n listed is 3. Here is the table citation: Frank E. Grubbs and Glenn Beck, "Extension of Sample Sizes and Percentage Points for Significance Tests of Outlying Observations", Technometrics, 14(4), 847-854 (1972).

We are working with datasets that have n=4, using quarterly sampling data. In the past, Grubbs test was used for outlier testing and has been accepted by the regulating agency. Should we change to a different test due to sample size, and if so, what would be more appropriate? Thank you, Jill

Reply

Charles *says:*
September 6, 2018 at 1:22 pm

Jill,
You cab use Real Statistics Descriptive Data and Normality data analysis tool to identity potential outliers. The Box Plots with Outliers and Grubbs test options are both relevant.
Charles

Reply

Hadi *says:*
June 14, 2018 at 1:51 pm

Hi Charles!
First and foremost, would like to thank you for changing my understanding of statistics and providing the RealStatistics tool generously. Thank you so much.

I have a dataset which consists of 4 series of data, consisting of 4 numeric parameters of 34 samples. I ran the 'Descriptive Statistics and Normality' tool and found out that there was a outlier. It turned out that that record was wrongly measured, so I removed it. After running the test again, the d'Agostino-Pearson test tells me that all 4 series are normal now. The Plot box shows no outliers and Grubbs' Test is insignificant which shows that there is no outlier in the data. It is also confirmed by 'Outliers and missing data' in the RealStatistics test results.
The problem is that when I run Shapiro-Wilk Test, one of the series(out of 4) is not recognized as normal with the p-value of 0.04987
How would you interprete that and would you approve that my t-test and ANOVA would make sense considering that the p-value is very close to 0.05?

Thank you in advance
Hadi

Reply

Charles *says:*
June 14, 2018 at 11:46 pm

Hadi,
The t test and ANOVA are pretty robust to violations of normality, and so based on the results that you have found, I would proceed with using a t test. I would also use ANOVA provided the homogeneity of variance assumption is met.
Charles

Reply

MARCOS FERREIRA *says:*
January 24, 2018 at 10:04 pm

Hi,

Thanks for your post.

Reply

Joe Young *says:*
July 14, 2017 at 1:28 pm

I am trying to find out if the G critical value table uses the N sample size or the Degrees of Freedom. Calculating the values uses the DF but the table could be constructed to reference N as the sample size. I am fairly sure the N is sample size but want to check with out bothering the calculate the G critical from the formula.
I have also read that the Grubbs test should not be used on sample sizes of less than 6. Grubbs test is not reliable with these small sample sizes and leads to rejecting the Ho at an alpha where it should be accepted. You don't state this but is that your understanding?

Reply

Charles *says:*
July 18, 2017 at 9:14 am

Joe,
There are tables of critical values for Grubbs' test, but I don't use them since we can use the critical values of the t distribution instead.
You are correct that the test is not reliable for small samples (as are many other statistical test). I have now updated the webpage with this fact.
Charles

Reply

Pedro Perez *says:*
February 27, 2017 at 2:08 am

Hello Mr. Zaiontz
I got a data set of n=365 and need to find out what values are outliers. Does Grubb's test consider the max and min values only, or it can examine all of the values one by one?

Once the outlier(s) is(are) found.. Should it(they) be removed from the data set and start all over?

Reply

Charles *says:*
February 27, 2017 at 9:53 am

Pedro,
Grubbs test only finds one outlier (which or course will be the max or min, although it needs to consider all the values in determining whether one of these is an outlier). You can use the ESD extension to Grubbs test to identify more than one potential outlier.
Whether or not you remove an outlier from the data set depends on the type of test (e.g. various nonparametric tests are robust to outliers) and the reason that the data element is an outlier (i.e. if it is a measurement error or typo then it can be removed). Often you will need to run your test twice, once with the outlier included and onec without, with both results reported.
Charles

Reply

AKROUR Rabah *says:*
August 17, 2016 at 11:03 am

You said "data that is normally distributed (except for the outlier)."
how to know that our data is normally distributed if we don't know the outliers yet, you said except for outliers??????????
Or is it a supposition that we do and at last we re-verify it ?

Reply

Charles *says:*
August 18, 2016 at 5:37 pm

Akrour,
Yes, you can verify this assumption after you identify the outlier.
Charles

Reply

Silva *says:*
June 16, 2016 at 2:35 pm

Hi Charles,
when I click on cell G12 of Grubbs' Test, t-crit = T.INV(2*G10;G11) so cell I12 is a typo.
But if 2*G10 = 2*G8/G9 = 2*alpha/size is correct then I didn't understand where you write ...G-crit is defined as above except that the significance level for t-crit is α/(2n). Is this another typo?
Thank you in advance for your reply and for your helpful website.

Reply

Charles *says:*
June 16, 2016 at 6:40 pm

Silva,
TINV(2*G10;G11) = T.INV(1-G10;G11). Note that the expression on the left is TINV and not T.INV.
Charles

Reply

Silva *says:*
June 17, 2016 at 8:18 am

Hi Charles,
I realized what the problem is. I have Excel 2010 localized in Italian so I have to pay attention to the functions names.
English localization
TINV = two-tailed inverse
T.INV = left-tailed inverse
Italian localization
INVT = left-tailed inverse
INV.T = two-tailed inverse
To me it seemed more logical T.INV = INV.T but it's wrong.

Reply

Charles *says:*
June 17, 2016 at 8:44 am

Silva,
I understand the problem very well since I too have the Italian version of Excel on one of my computers.
Charles

Reply

david *says:*
April 16, 2016 at 4:06 am

Interesante lo aplicaré en mi expo.

Reply

Willie Delport *says:*
December 14, 2015 at 8:26 am

Thanks for the information on Grubbs, and in addition I learned to evaluate data for their normal distribution characteristics
.
This most kind of you.
Willie

Reply