NUMERICAL ANALYSIS PROGRAMING PROJECT
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0. Introduction

Tom the Cat is chasing Jerry the Mouse, with an initial gap between them of 100m. Tom and Jerry's velocities are given as $v_c = 4 - at$ ms^{-1} and $v_m = v_{max} - ks = 3 - 0.02s$ ms^{-1}, respectively, with $0 < a$. The velocity of the change in the gap between Tom and Jerry, s, is given by $\frac{ds}{dt} = v_m - v_c = -1 - 0.02s + at$ ms^{-1}.

1. Problem

Find the true solution for when Tom will catch Jerry by plotting the gap distance.

First, we need to solve $\frac{ds}{dt}$. Noting that our equation is a linear first-order ODE, we need to put it into standard form:

$$\frac{ds}{dt} + 0.02s = at - 1$$

Next, we find the integration factor. Observing that in the second additive term on the left hand side we are multiplying by t^0, we see the integration factor is $e^{0.02t}$. This gives us the form:

$$\frac{d}{dt}s \cdot e^{0.02t} = (at - 1) \cdot e^{0.02t}$$

Taking the antiderivative of both sides gives:

$$\int \frac{d}{dt}s \cdot e^{0.02t}dt = a \cdot \int t \cdot e^{0.02t}dt - \int e^{0.02t}dt$$

$$s \cdot e^{0.02t} = 50at \cdot e^{0.02t} = 2500a \cdot e^{0.02t} - 50e^{0.02t} + c$$

Then, canceling $e^{0.02t}$ gives:

$$s = 50a(t - 50) - 50 + c \cdot e^{-0.02t}$$

Solving for c at our initial value of $s(0) = 100$ m will yield an equation we can use software to plot. Since $t = 0$, we have:

$$100 = -2500a - 50 + c \cdot e^{-0.02t}$$

$$c = 2500a + 150$$

So, our final equaiton we want to plot is:

$$s(a, t) = 50a(t - 50 + 50 \cdot e^{-0.02t}) + 150 \cdot e^{-0.02t} - 50$$

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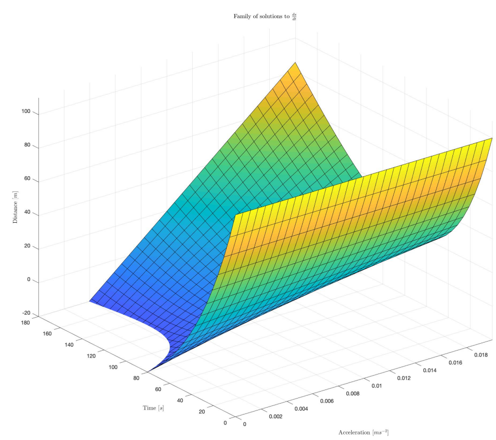

FIGURE 1. Plot of solutions to $\frac{ds}{dt}$

The exact solutions to when Tom catches Jerry are the points on the surface in Figure 1 that intersect with the plane at $s = 0$ with minimal t value .

2. PROBLEM

For $a = 0.01$ ms^{-2}, use the fourth-order Runge-Kutta method to compute when Tom will catch Jerry. Use an appropriate step size to ensure an accurate result.

3. PROBLEM

Use the Adams-Bashforth forth-order predictor-corrector to compute when Tom will catch Jerry using the results form Runge-Kutta, above, for the initial values of Adams-Bashforth.

4. PROBLEM

Suppose Tom's acceleration is unknown. If Tom does not catch Jerry in 120s, is it possible that Tom will catch Jerry?

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