MATH 213 (Important Results)

Separable ODE

- **Goal**: separate functions and derivatives of x and y to either side of the equation

$$\begin{aligned} f(x) &= g(y)y' \\ \implies \int f(x)dx &= \int g(y)dy \qquad \text{then integrate both sides as normal} \end{aligned}$$

Exact ODE

$$\begin{aligned} M(x,y)dx + N(x,y)dy &= 0 \\ &= du \quad \text{where } u \text{ is some function of } x \text{ and } y \end{aligned}$$

- **Goal**: find function $u(x,y) = C$ (aka. an implicit solution of y)
- The equation is **exact** if & only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

$$\begin{aligned} u &= \int M dx + k(y) \\ \text{then} \quad N &= \frac{\partial u}{\partial y} \\ &= \frac{\partial}{\partial y}\int M dx + \frac{d}{dy}k(y) \quad \text{to solve for } k(y) \end{aligned}$$

First-Order Linear ODE (With Variable Coefficients)

- **Homogeneous**: $y' + p(x)y = 0$

$$y(x) = Ce^{-h}, \quad h = \int p(x)dx$$

- **Nonhomogeneous**: $y' + p(x)y = q(x)$

$$y(x) = e^{-h}(\int e^h q(x)dx + C), \quad h = \int p(x)dx$$