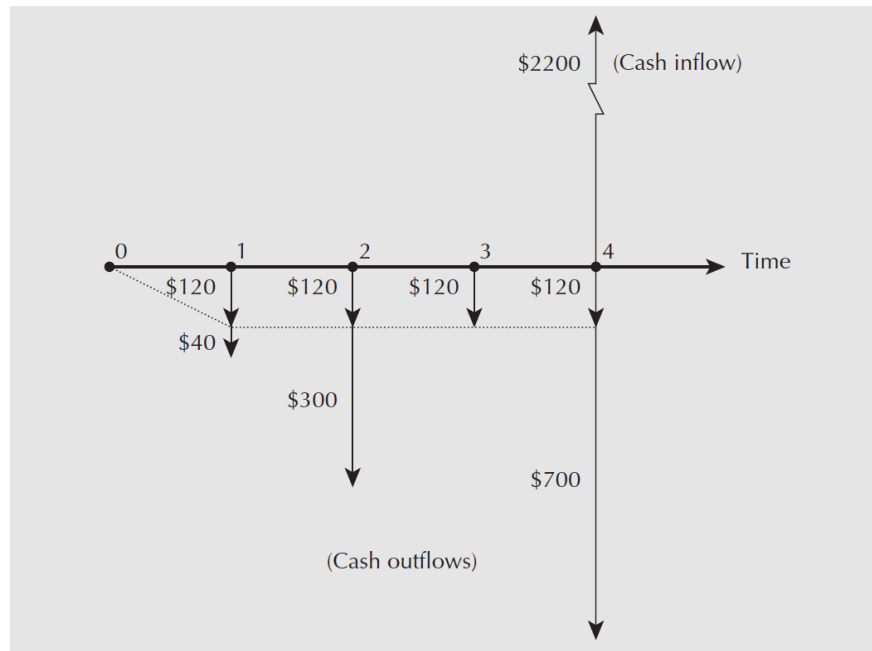


MSCI 261 Midterm Review (Chpt. 2-5)

Cash Flow Diagrams



- Cash inflows and outflows are represented by arrows
- Each “year” point represents the beginning of that year

Interest

- **Compound interest:** $F = P(1 + i)^N$
 - F = future value (value at the end of year N)
 - P = present value (value at the beginning of year 0)
 - i = interest rate (per period)
 - N = number of compounding periods
- **Simple interest:** $F = PN(1 + i)$
- **Nominal interest rate:** i_s
 - “Normal” way of stating interest rate
 - If annual nominal rate = 12%/year, then monthly nominal rate = 1%/month
- **Effective interest rate:** i_e
 - “Actual” interest rate
- **Converting from smaller period to large period:**
 - Suppose i_s is stated over a small period

- Then i_e over a large period, which consists of m small periods, is

$$i_e = (1 + i_s)^m - 1$$

- i.e. effective interest is the rate such that $P(1 + i_s)^m = P(1 + i_e)$

- **Converting from large period to small period:**

- If i_s is given over a large period = m small periods, then interest for small period is simply

$$i = i_s/m$$

- **Converting nominal annual to effective annual rate:**

- i.e. converting i_s to i_e for the same large period, which consists of m small compounding periods

$$i_e = \left(1 + \frac{i_s}{m}\right)^m - 1 \quad \text{where } m = \# \text{ compounding periods in a year}$$

- Continuous compounding – compounding period is infinitesimally small

$$\begin{aligned} i_e &= \lim_{m \rightarrow \infty} \left(1 + \frac{i_s}{m}\right)^m - 1 \\ &= e^{i_s} - 1 \end{aligned}$$

Compound Interest Factors

- Compound interest factors are just notations to represent formulas used to calculate F (future value), P (present value), or A (annuity).
- e.g. $(F/P, i, N) \rightarrow$ returns F , given P , i , and N
- **Compound amount factor** = $(F/P, i, N) = (1 + i)^N$
 - Given how much a payment is worth now, how much is it worth in N years?

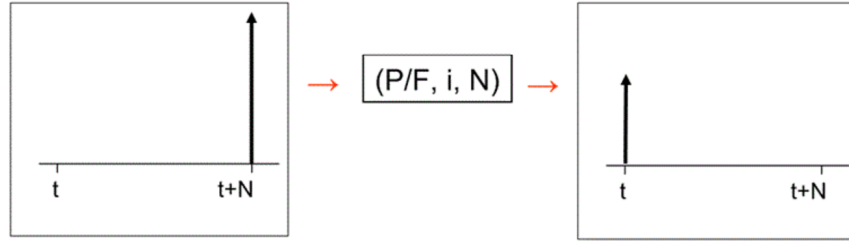
$$F = P(F/P, i, N)$$

- **Present worth factor** = $(P/F, i, N) = \frac{1}{(1 + i)^N}$
 - Given how much a payment will be worth in N years, how much is it worth now?

$$P = F(P/F, i, N)$$

- **Sinking fund factor** = $(A/F, i, N) = \frac{i}{(1 + i)^N - 1}$
 - Given how much an amount should be worth in N years, how much should I deposit/pay each year (i.e. annuity) ?

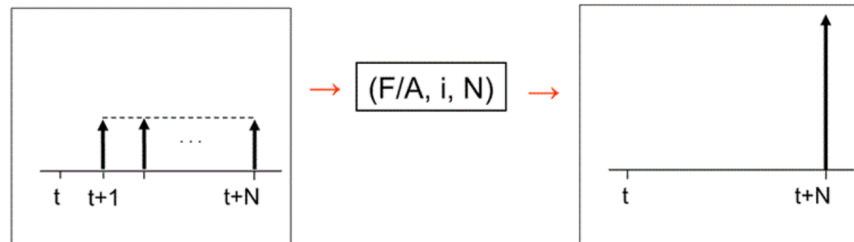
$$A = F(A/F, i, N)$$



- **Uniform series compound amount factor** $= (F/A, i, N) = \frac{(1+i)^N - 1}{i}$

- If I deposit/pay A each year, how much will it be worth in N years?

$$F = A(F/A, i, N)$$



- **Capital recovery factor** $= (A/P, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$

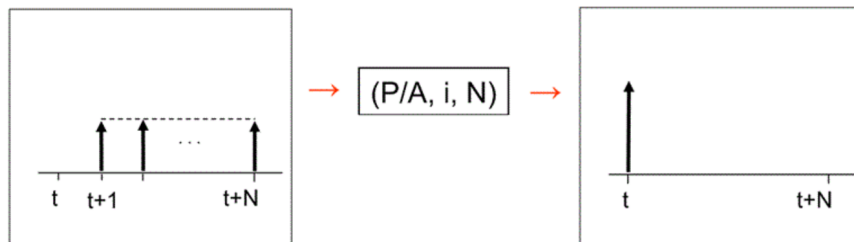
- Given how much a payment is worth now, how much should I deposit/pay each year in order to recover this payment in N years?

$$A = P(A/P, i, N)$$

- **Series present worth factor** $= (P/A, i, N) = \frac{(1+i)^N - 1}{i(1+i)^N}$

- If I despoit/pay A each year for N years, how much is it all worth today?

$$P = A(P/A, i, N)$$



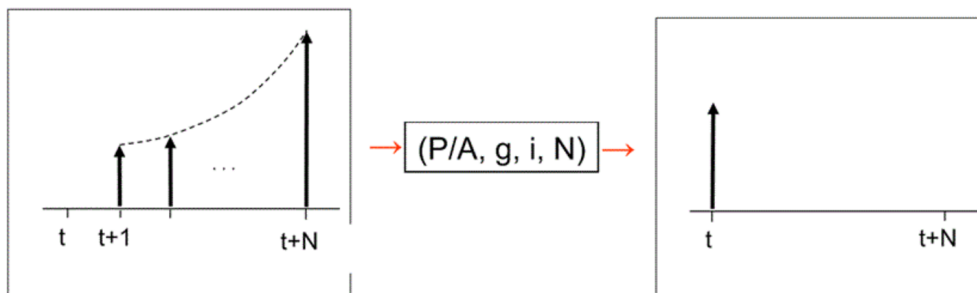
Conversion Factors

- **Arithmetic gradient to annuity conversion factor** $= (A/G, i, N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$

- Returns an annuity value, **not** the present worth
- Annuity increases/decreases by an amount G each year
 - Year 1: $A = A'$
 - Year 2: $A = A' + G$
 - Year 3: $A = A' + 2G$
 - Year N : $A = A' + (N - 1)G$
- First find $A_{total} = A' + G(A/G, i, N)$, then $P = A_{total}(P/A, i, N)$



- **Geometric gradient to present worth conversion factor** $= (P/A, g, i, N) = \frac{(P/A, i^o, N)}{1 + g}$
 - Annuity grows by a rate g each year
 - Year 1: $A = A'$
 - Year 2: $A = A'(1 + g)$
 - Year 3: $A = A'(1 + g)^2$
 - Year N : $A = A'(1 + g)^{N-1}$
 - **Growth-adjusted interest rate** $= i^o = \frac{1 + i}{1 + g} - 1$
 - If $g = i > 0$, the growth rate cancels the interest rate so $i^o = 0$, and $P = \frac{NA}{1 + g}$



Compound Interest Factor	Excel Function
$P = A(P/A, i, N)$	$P = PV(i, N, -A)$
$P = F(P/F, i, N)$	$P = PV(i, N, 0, -F)$
$F = A(F/A, i, N)$	$F = FV(i, N, -A)$
$F = P(F/P, i, N)$	$F = FV(i, N, 0, -P)$
$A = P(A/P, i, N)$	$A = PMT(i, N, -P)$
$A = F(A/F, i, N)$	$A = PMT(i, N, 0, -F)$

Calculating Present and Future Worth

- Present worth can also be calculated as = sum of revenue – cost in each year, divided by the discount in that year $(1 + i)^k$

$$\blacksquare PW = -C_{initial} + \frac{R_1 - C_1}{(1 + i)^1} + \frac{R_2 - C_2}{(1 + i)^2} + \dots + \frac{R_N - C_N}{(1 + i)^N}$$

- **Capital recovery formula:**

- Given initial purchase cost P (year 0) and final salvage value S (year N), what's the annual saving A required to justify this purchase?

$$A = P(A/P, i, N) - S(A/F, i, N) = (P - S)(A/P, i, N) + S \cdot i$$

- If payment period \neq compound period for annuities:
 - Method 1: calculate PV or FV of each annuity individually and sum
 - PV of each year = $A(P/F, i, N)$
 - FV of each year = $A(F/P, i, N - \text{current year})$
 - Method 2: convert compounding period \rightarrow payment period (i.e. find effective interest)
 - $i_e = (1 + i)^m - 1$, where m = the # of compounding periods in a payment period
 - Method 3: convert annuity \rightarrow equivalent annual annuity (can't use this for annuities with gradients)
 - i.e. an annuity payment at the end of m years is the FV of m years of equivalent annual annuities
 - $A_{annual} = A(A/F, i, m)$, then find PV or FV over total # of compounding years
- If $N \rightarrow \infty$:
 - Present worth of a project that continues indefinitely, with infinite series of uniform cash flows is called the **capitalized value**

$$P = \lim_{N \rightarrow \infty} A(P/A, i, N) = \frac{A}{i}$$

Relations Between Projects

- **Independent:** costs and benefits of each project is not affected by whether any other project is chosen; any projects (or none) can be chosen
- **Mutually exclusive:** choosing one project excludes all other projects from being chosen; only one of many projects can be chosen
- **Related but not mutually exclusive:** costs and benefits of a project depends on another project, but both can also be chosen
 - Put projects into mutually exclusive sets
 - n projects can be put into 2^n sets
 - Example: need to pick a number of projects with various costs, given a total budget
 - Evaluate all combinations of projects, and those combinations' present worths and costs
 - Find the combination of projects that has the highest PW, while satisfying the budget

Comparison Methods

- **Minimum acceptable rate of return (MARR):**
 - Rate of return required for investing in a project to be acceptable
 - “The investment should return *at least* this much”
 - i.e. investing in a project that returns at less than MARR is deemed undesirable, since the money could be invested elsewhere with higher returns
 - When doing comparisons, MARR is used as the interest rate
- **Present/future worth comparison**
 - Calculate PW or FW for all cash flows for each project, and compare
 - For independent projects, choose all with $PW \geq 0$
 - For mutually exclusive projects, choose the one with highest PW
- **Annual worth comparison**
 - Convert all cash flows to annual worth
 - Useful when many cash flows are given as annuities
- Comparing projects with different timespans
 - **Note:** no special techniques are required when comparing using the AW method
 - **Repeated lives:** repeat the “service life” of each alternative so that the timespans under consideration are equal
 - i.e. find the least common multiple of the service lives
 - **Study period:** choose a period over which the alternatives' PWs will be compared
 - If the period $<$ the lifespan of an alternative, its salvage value at that time must

be assumed

- **Payback period**

- $\frac{\text{First cost}}{\text{Annual return}} = \text{the \# of years for an investment to be recovered, assuming } i = 0$
- For non-constant annual returns, deduct each year's savings one year at a time until the first cost has been returned completely
- Shorter payback period is preferred

- **Internal rate of return**

- IRR = interest rate at which the project breaks even
- i.e. IRR is i^* such that

$$\sum_{t=0}^N \frac{R_t - D_t}{(1 + i^*)^t} = 0 \quad \text{where } R_t = \text{receipts in year } t \text{ and } D_t = \text{disbursements in year } t$$

- i^* can be solved by setting $PW(\text{receipts}) = PW(\text{disbursements})$, or similarly using FW or AW
 - Without spreadsheets, i^* can be estimated by trial and error, then determined through linear interpolation
- For independent projects, choose all projects with $IRR \geq \text{MARR}$

- **External rate of return**

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