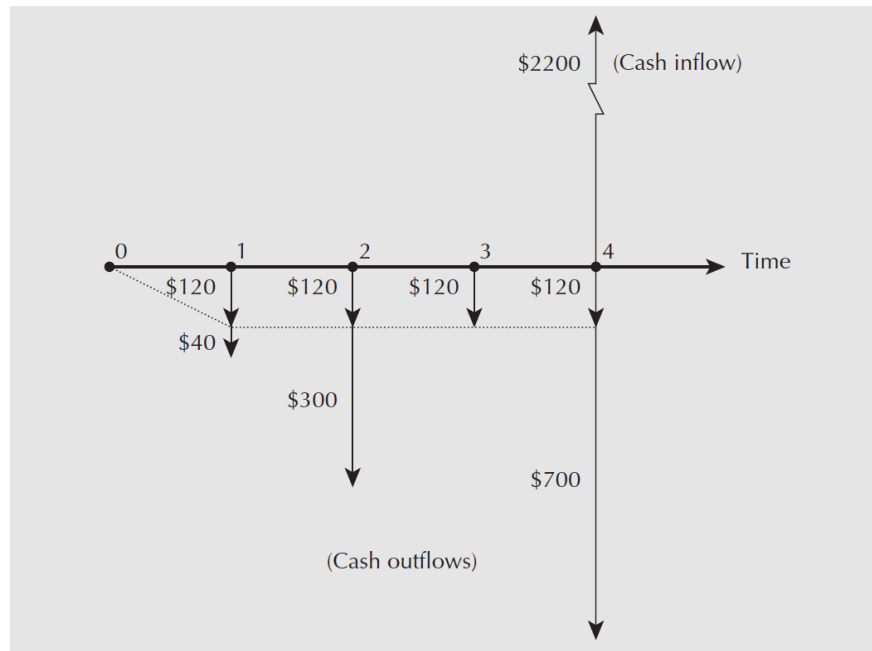


## MSCI 261 Midterm Review (Chpt. 2-5)

### Cash Flow Diagrams



- Cash inflows and outflows are represented by arrows
- Each “year” point represents the beginning of that year

### Interest

- **Compound interest:**  $F = P(1 + i)^N$ 
  - $F$  = future value (value at the end of year N)
  - $P$  = present value (value at the beginning of year 0)
  - $i$  = interest rate (per period)
  - $N$  = number of compounding periods
- **Simple interest:**  $F = PN(1 + i)$
- **Nominal interest rate:**  $i_s$ 
  - “Normal” way of stating interest rate
  - If annual nominal rate = 12%/year, then monthly nominal rate = 1%/month
- **Effective interest rate:**  $i_e$ 
  - “Actual” interest rate
- **Converting from smaller period to large period:**
  - Suppose  $i_s$  is stated over a small period

- Then  $i_e$  over a large period, which consists of  $m$  small periods, is

$$i_e = (1 + i_s)^m - 1$$

- i.e. effective interest is the rate such that  $P(1 + i_s)^m = P(1 + i_e)$

- **Converting from large period to small period:**

- If  $i_s$  is given over a large period =  $m$  small periods, then interest for small period is simply

$$i = i_s/m$$

- **Converting nominal annual to effective annual rate:**

- i.e. converting  $i_s$  to  $i_e$  for the same large period, which consists of  $m$  small compounding periods

$$i_e = \left(1 + \frac{i_s}{m}\right)^m - 1 \quad \text{where } m = \# \text{ compounding periods in a year}$$

- Continuous compounding – compounding period is infinitesimally small

$$\begin{aligned} i_e &= \lim_{m \rightarrow \infty} \left(1 + \frac{i_s}{m}\right)^m - 1 \\ &= e^{i_s} - 1 \end{aligned}$$

## Compound Interest Factors

- Compound interest factors are just notations to represent formulas used to calculate  $F$  (future value),  $P$  (present value), or  $A$  (annuity).
- e.g.  $(F/P, i, N) \rightarrow$  returns  $F$ , given  $P$ ,  $i$ , and  $N$
- **Compound amount factor** =  $(F/P, i, N) = (1 + i)^N$ 
  - Given how much a payment is worth now, how much is it worth in  $N$  years?

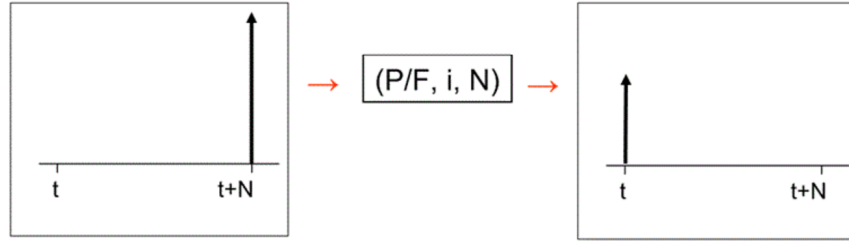
$$F = P(F/P, i, N)$$

- **Present worth factor** =  $(P/F, i, N) = \frac{1}{(1 + i)^N}$ 
  - Given how much a payment will be worth in  $N$  years, how much is it worth now?

$$P = F(P/F, i, N)$$

- **Sinking fund factor** =  $(A/F, i, N) = \frac{i}{(1 + i)^N - 1}$ 
  - Given how much an amount should be worth in  $N$  years, how much should I deposit/pay each year (i.e. annuity) ?

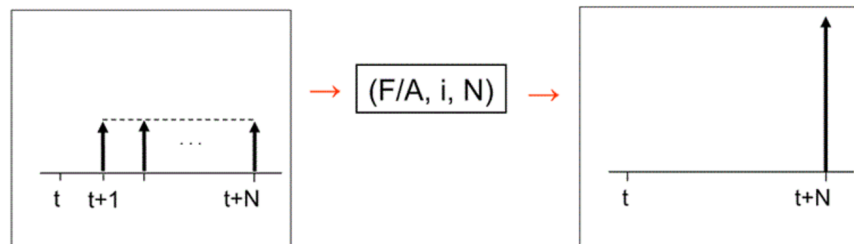
$$A = F(A/F, i, N)$$



- **Uniform series compound amount factor**  $= (F/A, i, N) = \frac{(1+i)^N - 1}{i}$

- If I deposit/pay  $A$  each year, how much will it be worth in  $N$  years?

$$F = A(F/A, i, N)$$



- **Capital recovery factor**  $= (A/P, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$

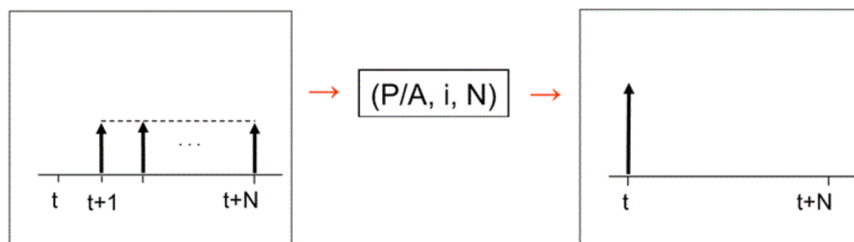
- Given how much a payment is worth now, how much should I deposit/pay each year in order to recover this payment in  $N$  years?

$$A = P(A/P, i, N)$$

- **Series present worth factor**  $= (P/A, i, N) = \frac{(1+i)^N - 1}{i(1+i)^N}$

- If I despoit/pay  $A$  each year for  $N$  years, how much is it all worth today?

$$P = A(P/A, i, N)$$



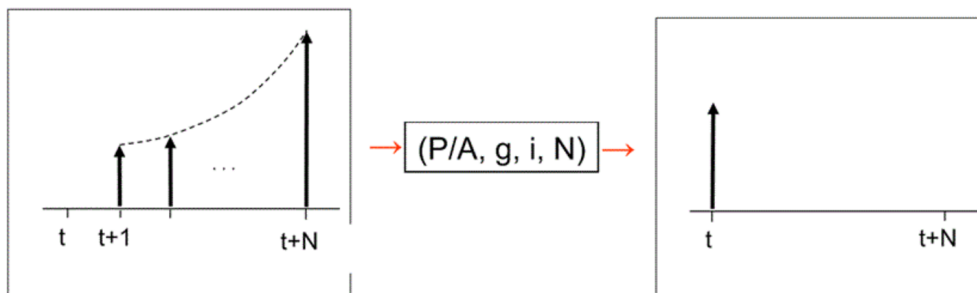
## Conversion Factors

- **Arithmetic gradient to annuity conversion factor**  $= (A/G, i, N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$

- Returns an annuity value, **not** the present worth
- Annuity increases/decreases by an amount  $G$  each year
  - Year 1:  $A = A'$
  - Year 2:  $A = A' + G$
  - Year 3:  $A = A' + 2G$
  - Year  $N$ :  $A = A' + (N - 1)G$
- First find  $A_{total} = A' + G(A/G, i, N)$ , then  $P = A_{total}(P/A, i, N)$



- **Geometric gradient to present worth conversion factor**  $= (P/A, g, i, N) = \frac{(P/A, i^o, N)}{1 + g}$ 
  - Annuity grows by a rate  $g$  each year
    - Year 1:  $A = A'$
    - Year 2:  $A = A'(1 + g)$
    - Year 3:  $A = A'(1 + g)^2$
    - Year  $N$ :  $A = A'(1 + g)^{N-1}$
  - **Growth-adjusted interest rate**  $= i^o = \frac{1 + i}{1 + g} - 1$
  - If  $g = i > 0$ , the growth rate cancels the interest rate so  $i^o = 0$ , and  $P = \frac{NA}{1 + g}$



Compound Interest Factor	Excel Function
$P = A(P/A, i, N)$	$P = PV(i, N, -A)$
$P = F(P/F, i, N)$	$P = PV(i, N, 0, -F)$
$F = A(F/A, i, N)$	$F = FV(i, N, -A)$
$F = P(F/P, i, N)$	$F = FV(i, N, 0, -P)$
$A = P(A/P, i, N)$	$A = PMT(i, N, -P)$
$A = F(A/F, i, N)$	$A = PMT(i, N, 0, -F)$

## Calculating Present and Future Worth

- Present worth can also be calculated as = sum of revenue – cost in each year, divided by the discount in that year  $(1 + i)^k$

$$\blacksquare PW = -C_{initial} + \frac{R_1 - C_1}{(1 + i)^1} + \frac{R_2 - C_2}{(1 + i)^2} + \dots + \frac{R_N - C_N}{(1 + i)^N}$$

- **Capital recovery formula:**

- Given initial purchase cost  $P$  (year 0) and final salvage value  $S$  (year  $N$ ), what's the annual saving  $A$  required to justify this purchase?

$$A = P(A/P, i, N) - S(A/F, i, N) = (P - S)(A/P, i, N) + S \cdot i$$

- If payment period  $\neq$  compound period for annuities:
  - Method 1: calculate PV or FV of each annuity individually and sum
    - PV of each year =  $A(P/F, i, N)$
    - FV of each year =  $A(F/P, i, N - \text{current year})$
  - Method 2: convert compounding period  $\rightarrow$  payment period (i.e. find effective interest)
    - $i_e = (1 + i)^m - 1$ , where  $m$  = the # of compounding periods in a payment period
  - Method 3: convert annuity  $\rightarrow$  equivalent annual annuity (can't use this for annuities with gradients)
    - i.e. an annuity payment at the end of  $m$  years is the FV of  $m$  years of equivalent annual annuities
    - $A_{annual} = A(A/F, i, m)$ , then find PV or FV over total # of compounding years
- If  $N \rightarrow \infty$ :
  - Present worth of a project that continues indefinitely, with infinite series of uniform cash flows is called the **capitalized value**

$$P = \lim_{N \rightarrow \infty} A(P/A, i, N) = \frac{A}{i}$$

## Relations Between Projects

- **Independent:** costs and benefits of each project is not affected by whether any other project is chosen; any projects (or none) can be chosen
- **Mutually exclusive:** choosing one project excludes all other projects from being chosen; only one of many projects can be chosen
- **Related but not mutually exclusive:** costs and benefits of a project depends on another project, but both can also be chosen
  - Put projects into mutually exclusive sets
  - $n$  projects can be put into  $2^n$  sets
  - Example: need to pick a number of projects with various costs, given a total budget
    - Evaluate all combinations of projects, and those combinations' present worths and costs
    - Find the combination of projects that has the highest PW, while satisfying the budget

## Comparison Methods

- **Minimum acceptable rate of return (MARR):**
  - Rate of return required for investing in a project to be acceptable
    - “The investment should return *at least* this much”
    - i.e. investing in a project that returns at less than MARR is deemed undesirable, since the money could be invested elsewhere with higher returns
  - When doing comparisons, MARR is used as the interest rate
- **Present/future worth comparison**
  - For independent projects, choose all with  $PW \geq 0$
  - For mutually exclusive projects, choose the one with highest  $PW$
- **Annual worth comparison**
  - Convert all cash flows to annual worth
  - Useful when many cash flows are given as annuities