

# CS 240 Midterm Review (Module 1–7)

## Asymptotic Analysis

- Problem instance (I) – *input* for the specified problem
- Problem solution – *output* for the specified problem instance
- Problem size –  $\text{Size}(I)$  = size of instance I
- Algorithm - a step-by-step process for carrying out a series of computations
  - An algorithm A solves a problem P if, for every instance I of P, A computes a valid solution for I in finite time
- RAM model
  - Assume any memory access & primitive operation is constant time
  - Assume infinite amount of memory
  - Sequential operation
  - Running time is determined by the # of memory accesses & primitive operations
- Order notations
  - $f(n) \in O(g(n))$  if  $\exists c > 0$  and  $n_0 > 0$  such that  $0 \leq f(n) \leq cg(n) \forall n \geq n_0$ 
    - $f$  “grows no faster than”  $g$
    - $f$  is “upper-bounded” by  $g$
  - $f(n) \in \Omega(g(n))$  if  $\exists c > 0$  and  $n_0 > 0$  such that  $0 \leq cg(n) \leq f(n) \forall n \geq n_0$ 
    - $f$  “grows no slower than”  $g$
    - $f$  is “lower-bounded” by  $g$
  - $f(n) \in \Theta(g(n))$  if  $\exists c_1, c_2 > 0$  and  $n_0 > 0$  such that  $0 \leq c_1g(n) \leq f(n) \leq c_2g(n) \forall n \geq n_0$ 
    - $f$  and  $g$  grow at the same rate
  - $f(n) \in o(g(n))$  if  $\forall c > 0, \exists n_0 > 0$  such that  $0 \leq f(n) < cg(n) \forall n \geq n_0$ 
    - $f$  *always* grows slower than  $g$ , given a  $n_0$
  - $f(n) \in \omega(g(n))$  if  $\forall c > 0, \exists n_0 > 0$  such that  $0 \leq cg(n) < f(n) \forall n \geq n_0$ 
    - $f$  *always* grows faster than  $g$ , given a  $n_0$
  - Suppose  $L = \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$ 
    - If  $L = 0$  then  $f \in o(g)$
    - If  $0 < L < \infty$  then  $f \in \Theta(g)$
    - If  $L = \infty$  then  $f \in \omega(g)$

## Thing

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