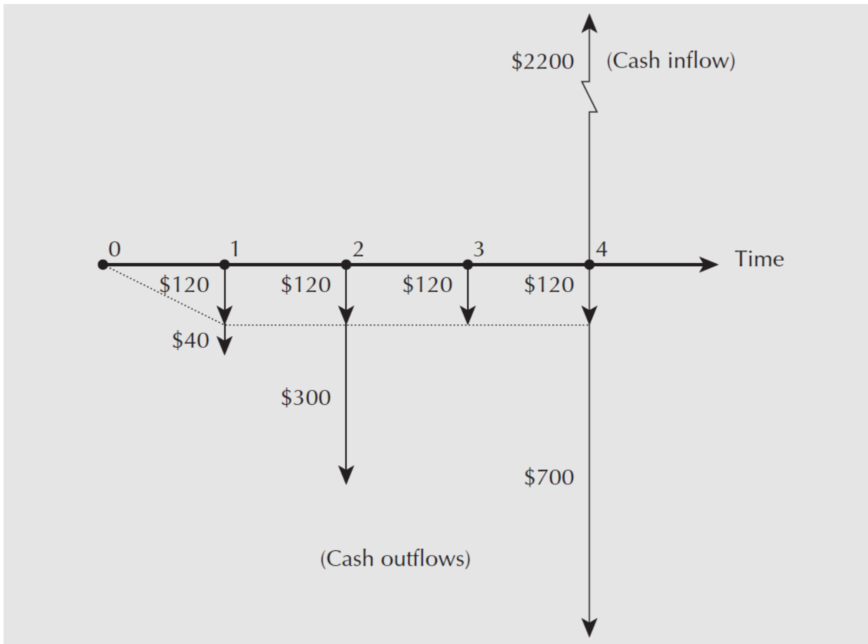
MSCI 261 Midterm Review (Chpt. 2-5)

Cash Flow Diagrams



- Cash inflows and outflows are represented by arrows
- Each "year" point represents the *beginning* of that year

Interest

- **Compound interest**: $F = P(1+i)^N$
  - F = future value (value at the end of year N)
  - P = present value (value at the beginning of year 0)
  - i = interest rate (per period)
  - N = number of compounding periods
- **Simple interest**: $F = PN(1+i)$
- **Nominal interest rate**: i_s
  - "Normal" way of stating interest rate
  - If annual nominal rate = 12%/year, then monthly nominal rate = 1%/month
- **Effective interest rate**: i_e
  - "Actual" interest rate
- **Converting from smaller period to large period**:
  - Suppose i_s is stated over a small period

- Then i_e over a large period, which consists of m small periods, is

$$i_e = (1 + i_s)^m - 1$$

- i.e. effective interest is the rate such that $P(1 + i_s)^m = P(1 + i_e)$

- **Converting from large period to small period:**
  - If i_s is given over a large period $= m$ small periods, then interest for small period is simply

$$i = i_s/m$$

- **Converting nominal annual to effective annual rate**:
  - i.e. converting i_s to i_e for the same large period, which consists of m small compounding periods

$$i_e = (1 + \frac{i_s}{m})^m - 1 \qquad \text{where } m = \#\text{ compounding periods in a year}$$

- Continuous compounding – compounding period is infinitesimally small

$$\begin{aligned} i_e &= \lim_{m \to \infty} \left(1 + \frac{i_s}{m}\right)^m - 1 \\ &= e^{i_s} - 1 \end{aligned}$$

Compound Interest Factors

- Compound interest factors are just notations to represent formulas used to calculate F (future value), P (present value), or A (annuity).
- e.g. $(F/P, i, N) \rightarrow$ returns F, given P, i, and N
- **Compound amount factor** $= (F/P, i, N) = (1 + i)^N$
  - Given how much a payment is worth now, how much is it worth in N years?

$$F = P(F/P, i, N)$$

- **Present worth factor** $= (P/F, i, N) = \dfrac{1}{(1 + i)^N}$
  - Given how much a payment will be worth in N years, how much is it worth now?

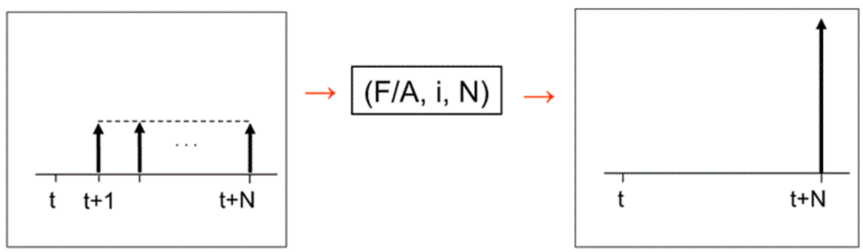
$$P = F(P/F, i, N)$$

- **Sinking fund factor** $= (A/F, i, N) = \dfrac{i}{(1 + i)^N - 1}$
  - Given how much an amount should be worth in N years, how much should I deposit/pay each year (i.e. <u>annuity</u>) ?

$$A = F(A/F, i, N)$$

- **Uniform series compound amount factor** $= (F/A, i, N) = \dfrac{(1+i)^N - 1}{i}$
  - If I deposit/pay A each year, how much will it be worth in N years?

    $F = A(F/A, i, N)$

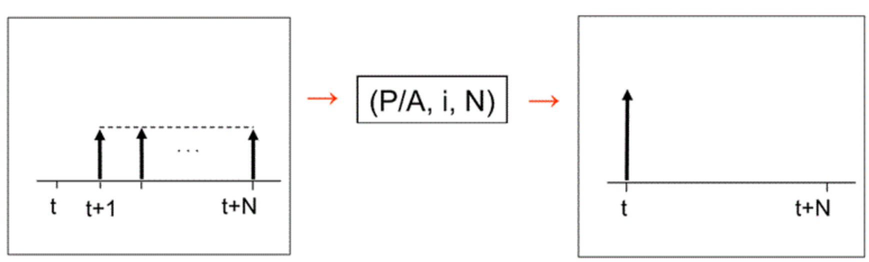


- **Capital recovery factor** $= (A/P, i, N) = \dfrac{i(1+i)^N}{(1+i)^N - 1}$
  - Given how much a payment is worth now, how much should I deposit/pay each year in order to recover this payment in N years?

    $A = P(A/P, i, N)$

- **Series present worth factor** $= (P/A, i, N) = \dfrac{(1+i)^N - 1}{i(1+i)^N}$
  - If I despoit/pay A each year for N years, how much is it all worth today?

    $P = A(P/A, i, N)$

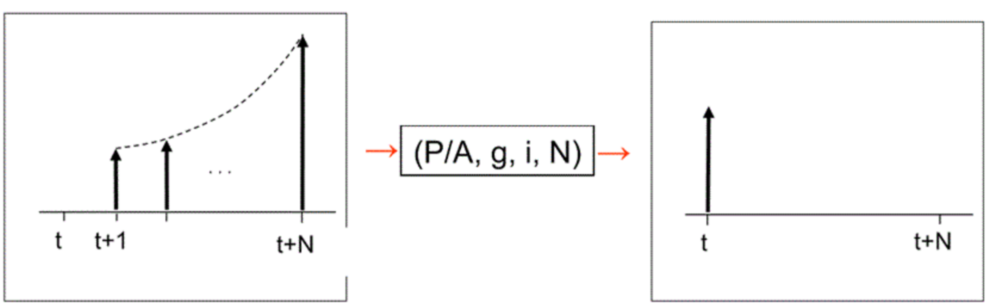


Conversion Factors

- **Arithmetic gradient to annuity conversion factor** $= (A/G, i, N) = \dfrac{1}{i} - \dfrac{N}{(1+i)^N - 1}$

    - Returns an annuity value, **not** the present worth
    - Annuity increases/decreases by an amount G each year
        - Year 1: $A = A'$
        - Year 2: $A = A' + G$
        - Year 3: $A = A' + 2G$
        - Year N: $A = A' + (N-1)G$
    - First find $A_{total} = A' + G(A/G, i, N)$, then $P = A_{total}(P/A, i, N)$

- **Geometric gradient to present worth conversion factor** $= (P/A, g, i, N) = \dfrac{(P/A, i^o, N)}{1+g}$
    - Annuity grows by a rate g each year
        - Year 1: $A = A'$
        - Year 2: $A = A'(1+g)$
        - Year 3: $A = A'(1+g)^2$
        - Year N: $A = A'(1+g)^{N-1}$
    - **Growth-adjusted interest rate** $= i^o = \dfrac{1+i}{1+g} - 1$
    - If $g = i > 0$, the growth rate cancels the interest rate so $i^o = 0$, and $P = \dfrac{NA}{1+g}$



Compound Interest Factor	Excel Function
$P = A(P/A, i, N)$	$P = PV(i, N, -A)$
$P = F(P/F, i, N)$	$P = PV(i, N, 0, -F)$
$F = A(F/A, i, N)$	$F = FV(i, N, -A)$
$F = P(F/P, i, N)$	$F = FV(i, N, 0, -P)$
$A = P(A/P, i, N)$	$A = PMT(i, N, -P)$
$A = F(A/F, i, N)$	$A = PMT(i, N, 0, -F)$

Calculating Present and Future Worth

- Present worth can also be calculated as = sum of revenue − cost in each year, divided by the discount in that year $(1+i)^k$
  - $PW = -C_{initial} + \frac{R_1 - C_1}{(1+i)^1} + \frac{R_2 - C_2}{(1+i)^2} + \ldots + \frac{R_N - C_N}{(1+i)^N}$
- **Capital recovery formula**:
  - Given initial purchase cost P (year 0) and final salvage value S (year N), what's the annual saving A required to justify this purchase?

$$A = P(A/P, i, N) - S(A/F, i, N) = (P - S)(A/P, i, N) + S \cdot i$$

- If payment period $\neq$ compound period for annuities:
  - Method 1: calculate PV or FV of each annuity individually and sum
    - PV of each year $= A(P/F, i, N)$
    - FV of each year $= A(F/P, i, N - \text{current year})$
  - Method 2: convert compounding period → payment period (i.e. find effective interest)
    - $i_e = (1+i)^m - 1$, where m = the # of compounding periods in a payment period
  - Method 3: convert annuity → equivalent annual annuity (can't use this for annuities with gradients)
    - i.e. an annuity payment at the end of m years is the FV of m years of equivalent annual annuities
    - $A_{annual} = A(A/F, i, m)$, then find PV or FV over total # of compounding years
- If $N \to \infty$:
  - Present worth of a project that continues indefinitely, with <u>infinite series of uniform cash flows</u> is called the **capitalized value**

$$P = \lim_{N \to \infty} A(P/A, i, N) = \frac{A}{i}$$

Comparison Methods

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