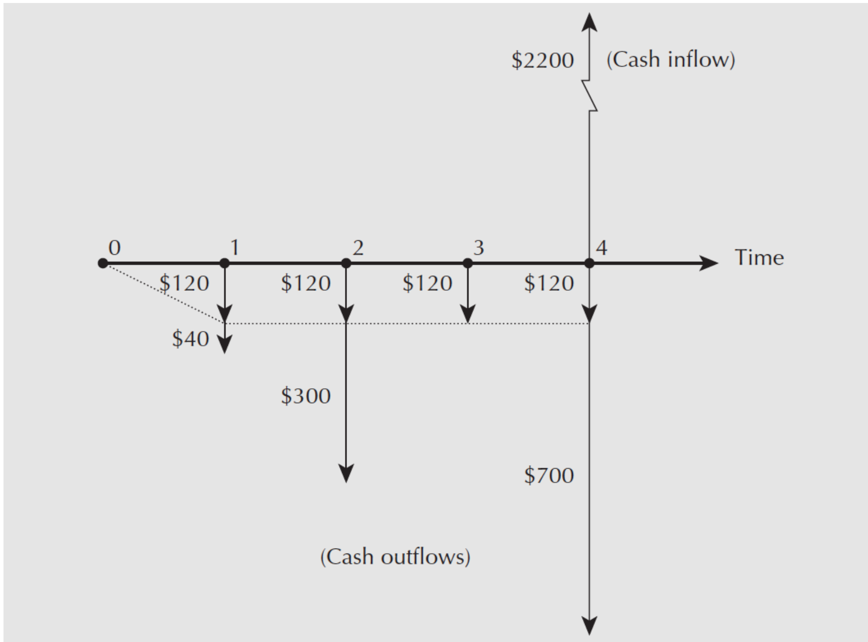
MSCI 261 Midterm Review (Chpt. 2-5)

Cash Flow Diagrams



- Cash inflows and outflows are represented by arrows
- Each "year" point represents the *beginning* of that year

Interest

- **Compound interest**: $F = P(1+i)^N$
  - F = future value (value at the end of year N)
  - P = present value (value at the beginning of year 0)
  - i = interest rate (per period)
  - N = number of compounding periods
- **Simple interest**: $F = PN(1+i)$
- **Nominal interest rate**: i_s
  - "Normal" way of stating interest rate
  - If annual nominal rate = 12%/year, then monthly nominal rate = 1%/month
- **Effective interest rate**: i_e
  - "Actual" interest rate
- **Converting from smaller period to large period**:
  - Suppose i_s is stated over a small period

- Then i_e over a large period, which consists of m small periods, is

  $$i_e = (1 + i_s)^m - 1$$

- i.e. effective interest is the rate such that $P(1 + i_s)^m = P(1 + i_e)$

- **Converting from large period to small period:**
  - If i_s is given over a large period = m small periods, then interest for small period is simply

    $$i = i_s/m$$

- **Converting nominal annual to effective annual rate**:
  - i.e. converting i_s to i_e for the same large period, which consists of m small compounding periods

    $$i_e = (1 + \frac{i_s}{m})^m - 1 \qquad \text{where } m = \# \text{ compounding periods in a year}$$

- Continuous compounding – compounding period is infinitesimally small

  $$\begin{aligned} i_e &= \lim_{m \to \infty} \left(1 + \frac{i_s}{m}\right)^m - 1 \\ &= e^{i_s} - 1 \end{aligned}$$

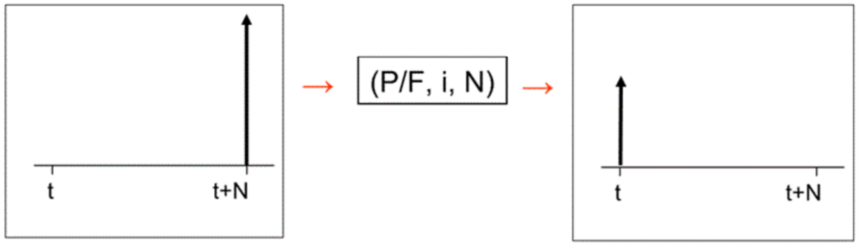
Compound Interest Factors

- Compound interest factors are just notations to represent formulas used to calculate F (future value), P (present value), or A (annuity).
- e.g. $(F/P, i, N) \rightarrow$ returns F, given P, i, and N
- **Compound amount factor** $= (F/P, i, N) = (1 + i)^N$
  - Given how much a payment is worth now, how much is it worth in N years?

    $$F = P(F/P, i, N)$$

- **Present worth factor** $= (P/F, i, N) = \dfrac{1}{(1 + i)^N}$
  - Given how much a payment will be worth in N years, how much is it worth now?

    $$P = F(P/F, i, N)$$

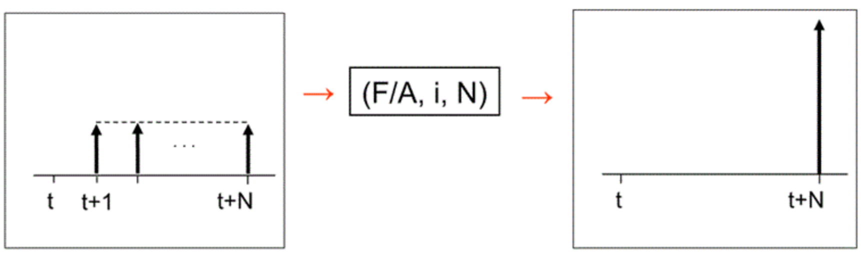


- **Sinking fund factor** $= (A/F, i, N) = \dfrac{i}{(1+i)^N - 1}$
  - Given how much an amount should be worth in N years, how much should I deposit/pay each year (i.e. annuity) ?

$$A = F(A/F, i, N)$$

- **Uniform series compound amount factor** $= (F/A, i, N) = \dfrac{(1+i)^N - 1}{i}$
  - If I deposit/pay A each year, how much will it be worth in N years?

$$F = A(F/A, i, N)$$

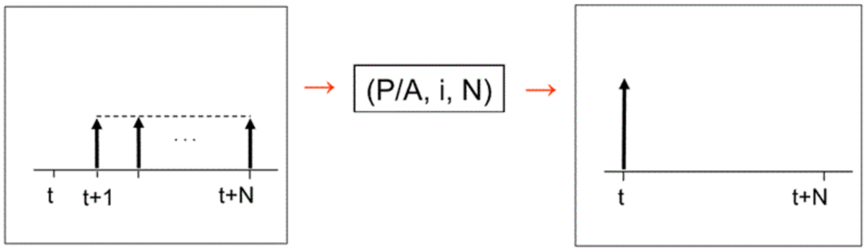


- **Capital recovery factor** $= (A/P, i, N) = \dfrac{i(1+i)^N}{(1+i)^N - 1}$
  - Given how much a payment is worth now, how much should I deposit/pay each year in order to recover this payment in N years?

$$A = P(A/P, i, N)$$

- **Series present worth factor** $= (P/A, i, N) = \dfrac{(1+i)^N - 1}{i(1+i)^N}$
  - If I despoit/pay A each year for N years, how much is it all worth today?

$$P = A(P/A, i, N)$$



Compound Interest Factor	Excel Function
$P = A(P/A, i, N)$	$P = PV(i, N, -A)$
$P = F(P/F, i, N)$	$P = PV(i, N, 0, -F)$
$F = A(F/A, i, N)$	$F = FV(i, N, -A)$
$F = P(F/P, i, N)$	$F = FV(i, N, 0, -P)$
$A = P(A/P, i, N)$	$A = PMT(i, N, -P)$
$A = F(A/F, i, N)$	$A = PMT(i, N, 0, -F)$

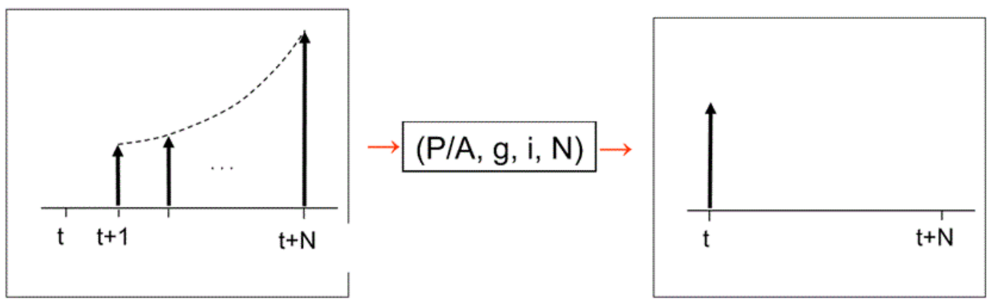
Conversion Factors

- **Arithmetic gradient to annuity conversion factor** $= (A/G, i, N) = \dfrac{1}{i} - \dfrac{N}{(1+i)^N - 1}$
  - Returns an annuity value, **not** the present worth
  - Annuity increases/decreases by an amount G each year
    - Year 1: $A = A'$
    - Year 2: $A = A' + G$
    - Year 3: $A = A' + 2G$
    - Year N: $A = A' + (N-1)G$
  - First find $A_{total} = A' + G(A/G, i, N)$, then $P = A_{total}(P/A, i, N)$

- **Geometric gradient to present worth conversion factor** $= (P/A, g, i, N) = \dfrac{(P/A, i^o, N)}{1+g}$
  - Annuity grows by a rate g each year

- Year 1: $A = A'$
- Year 2: $A = A'(1+g)$
- Year 3: $A = A'(1+g)^2$
- Year N: $A = A'(1+g)^{N-1}$

- **Growth-adjusted interest rate** $= i^o = \dfrac{1+i}{1+g} - 1$
- If $g = i > 0$, the growth rate cancels the interest rate so $i^o = 0$, and $P = \dfrac{NA}{1+g}$



Calculating Present and Future Worth

- Present worth can also be calculated as = sum of revenue − cost in each year, divided by the discount in that year $(1+i)^k$
  - $PW = -C_{initial} + \dfrac{R_1 - C_1}{(1+i)^1} + \dfrac{R_2 - C_2}{(1+i)^2} + \ldots + \dfrac{R_N - C_N}{(1+i)^N}$
- **Capital recovery formula**:
  - Given initial purchase cost P (year 0) and final salvage value S (year N), what's the annual saving A required to justify this purchase?

$$A = P(A/P, i, N) - S(A/F, i, N) = (P - S)(A/P, i, N) + S \cdot i$$

- If payment period $\neq$ compound period for annuities:
  - Method 1: calculate PV or FV of each annuity individually and sum
    - PV of each year $= A(P/F, i, N)$
    - FV of each year $= A(F/P, i, N - \text{current year})$
  - Method 2: convert compounding period → payment period (i.e. find effective interest)
    - $i_e = (1+i)^m - 1$, where m = the # of compounding periods in a payment period
  - Method 3: convert annuity → equivalent annual annuity (can't use this for annuities with gradients)
    - i.e. an annuity payment at the end of m years is the FV of m years of equivalent annual annuities
    - $A_{annual} = A(A/F, i, m)$, then find PV or FV over total # of compounding years

- If $N \to \infty$:
  - Present worth of a project that continues indefinitely, with infinite series of uniform cash flows is called the **capitalized value**

$$P = \lim_{N \to \infty} A(P/A, i, N) = \frac{A}{i}$$

Relations Between Projects

- **Independent**: costs and benefits of each project is not affected by whether any other project is chosen; any projects (or none) can be chosen
- **Mutually exclusive**: choosing one project excludes all other projects from being chosen; only one of many projects can be chosen
- **Related but not mutually exclusive**: costs and benefits of a project depends on another project, but both can also be chosen
  - Put projects into mutually exclusive sets
  - n projects can be put into 2^n sets
  - Example: need to pick a number of projects with various costs, given a total budget
    - Evaluate all combinations of projects, and those combinations' present worths and costs
    - Find the combination of projects that has the highest PW, while satisfying the budget

Comparison Methods

- **Minimum acceptable rate of return (MARR)**:
  - Rate of return required for investing in a project to be acceptable
    - "The investment should return *at least* this much"
    - i.e. investing in a project that returns at less than MARR is deemed undesirable, since the money could be invested elsewhere with higher returns
  - When doing comparisons, MARR is used as the interest rate
- **Present/future worth comparison**
  - Calculate PW or FW for all cash flows for each project, and compare
  - For independent projects, choose all with $PW \geq 0$
  - For mutually exclusive projects, choose the one with highest PW
- **Annual worth comparison**
  - Convert all cash flows to annual worth
  - Useful when many cash flows are given as annuities
- Comparing projects with different timespans

- **Note**: no special techniques are required when comparing using the AW method
- **Repeated lives**: repeat the "service life" of each alternative so that the timespans under consideration are equal
  - i.e. find the least common multiple of the service lives
- **Study period**: choose a period over which the alternatives' PWs will be compared
  - If the period < the lifespan of an alternative, its salvage value at that time must be assumed

- **Payback period**
  - $\dfrac{\text{First cost}}{\text{Annual return}}$ = the # of years for an investment to be recovered, assuming $i = 0$
  - For non-constant annual returns, deduct each year's savings from the first cost one year at a time until the cost has been returned completely
    - e.g. First cost = \$100k; annual savings = \$10k, \$12k, \$14k . . .
    - Year 1: \$100k – \$10k = \$90k; Year 2: \$90k – \$12k = \$78k . . . continue until \$0
  - Shorter payback period is preferred

- **Internal rate of return**
  - IRR = interest rate at which the project breaks even
  - i.e. IRR is i^* such that

$$\sum_{t=0}^{N} \frac{R_t - D_t}{(1+i^*)^t} = 0 \qquad \text{where } R_t = \text{receipts in year } t \text{ and } D_t = \text{disbursements in year } t$$

  - i^* can be solved by setting $PW(receipts) = PW(disbursements)$, or similarly using FW or AW
    - Without spreadsheets, i^* can be estimated by trial and error, then determined through linear interpolation
  - For independent projects, choose all projects with IRR ≥ MARR
  - For mutually exclusive projects:
    - Having the highest IRR does not necessarily indicate being the best alternative
    - First find i^* for all alternatives; eliminate those that are < MARR
    - Begin with the alternative with the *lowest first cost* as the *current best*
    - Compare *current best* with the alternative with the *next lowest first cost*
      - Evaluate the IRR of their increment (receipts and disbursements of A – B)
      - If IRR < MARR, current best does not change
      - If IRR ≥ MARR, the new alternative becomes the current best
    - Go through all alternatives from lowest to highest first cost; then the *current best* returns the best alternative
  - Ex: Alternatives A, B, C are ordered from lowest to highest first cost

        - *Current best* begins as alternative A
        - Solve for i^* using
          $PW(disbursement(B) - disbursement(A)) = PW(receipts(B) - receipts(A))$
          i.e. the *increment* from A to B
        - If $i^* \geq$ MARR, B becomes *current best*
        - Solve for i^* for C – B, etc ...

- **Multiple IRRs**
    - **Descartes' Rule of Signs**: the # of positive solutions in $a_0+a_1x+a_2x^2+\ldots+a_nx^n = 0$ is *less than or equal to* the # of sign changes in the coefficients
    - This also corresponds to the sign changes in net cash flow of each year
- **External rate of return**
    - Use ERR (i_e^*) instead when there are multiple IRRs
    - i.e. return earned by cash flow that's not invested into the project
    - Assumed to be invested at MARR
    - Calculating **approximate ERR**:
        - Take all positive net cash flows (net receipts) forward to the final year, at MARR (i.e. take F/P or F/A)
        - Take all negative net cash flows (net disbursements) forward also to the final year, at an unkown i_e^*
        - This produces the *transformed series*
        - Find the IRR of the transformed series; i.e. set $FW(disbursements)_{i_e^*} = FW(receipts)_{MARR}$, solve for i_e^*
    - The project is deemed acceptable if ERR $\geq$ MARR