Asymptotic Analysis Cheatsheet

- Order Notations:
  - $f(n) \in O(g(n))$ if $\exists\ c > 0$ and $n_0 > 0$ such that $0 \leq f(n) \leq cg(n)\ \forall\ n \geq n_0$
    - f "grows no faster than" g
  - $f(n) \in \Omega(g(n))$ if $\exists\ c > 0$ and $n_0 > 0$ such that $0 \leq cg(n) \leq f(n)\ \forall\ n \geq n_0$
    - f "grows no slower than" g
  - $f(n) \in \Theta(g(n))$ if $\exists\ c_1, c_2 > 0$ and $n_0 > 0$ such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\ \forall\ n \geq n_0$
    - f ang g have the *same complexity*
  - $f(n) \in o(g(n))$ if $\forall\ c > 0, \exists\ n_0 > 0$ such that $0 \leq f(n) < cg(n)\ \forall\ n \geq n_0$
    - f has *lower complexity* than g
  - $f(n) \in \omega(g(n))$ if $\forall\ c > 0, \exists\ n_0 > 0$ such that $0 \leq cg(n) < f(n)\ \forall\ n \geq n_0$
    - f has *higher complexity* than g
  - $f \in O(g)$ and $f \in \Omega(g) \iff f \in \Theta(g)$
- Limit method: suppose $L = \lim_{n\to\infty} \dfrac{f(n)}{g(n)}$
  - $f \in o(g)$ if $L = 0$
  - $f \in \Theta(g)$ if $0 < L < \infty$
  - $f \in \omega(g)$ if $L = \infty$
- Useful facts for first-principles proofs:
  - $\log n \geq 1\ \forall\ n \geq 2$
  - $\log n \geq n\ \forall\ n \geq 0$
- Some math rules:
  - $\sum_{i=1}^{n} i^k \in \Theta(n^{k+1})$
  - $\sum_{i=1}^{n} c^i$
    - $\in \Theta(c^n)$ if $c > 1$
    - $\in \Theta(n)$ if $c = 1$
    - $\in \Theta(1)$ if $c < 1$
  - $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \lim_{x\to c} \dfrac{f'(x)}{g'(x)}$ if it exists (L'Hopital's Rule)
  - $\log(n!) \in \Theta(n \log n)$ (Stirling's Approximation)
  - $\sum_{i=1}^{n} \dfrac{1}{i} \in \Theta(\log n)$ (Harmonic series)