Computational Physics Problem Set 5

John Ferrante

October 8, 2024

GitHub: https://github.com/jferrante25/physga-2000

1 Problem 1 (5.15)

Code in GitHub. The analytic form of the derivative of $1 + \frac{1}{2}tanh(2x)$ is $1 - tanh^2(2x)$
See Figure 1 for plots.

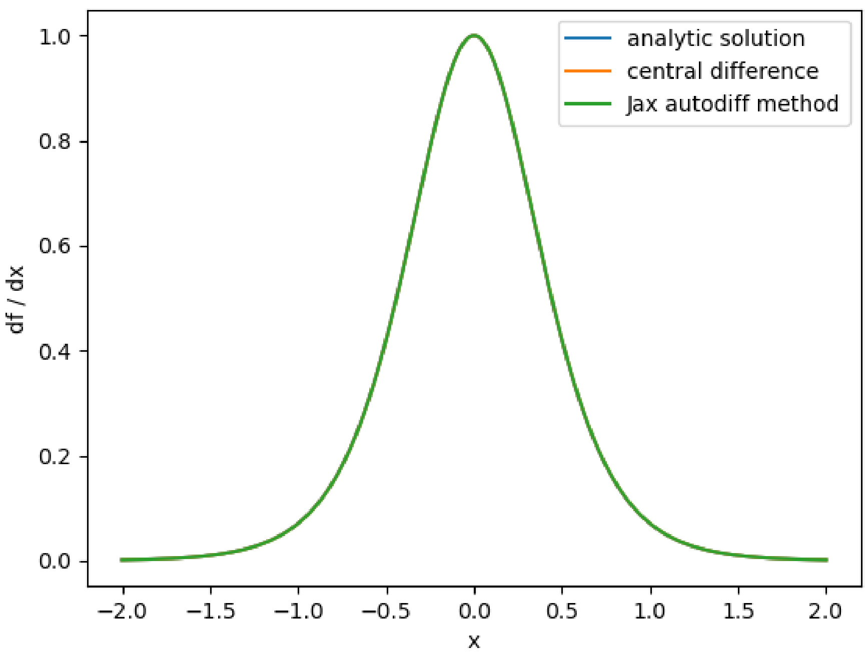

Figure 1: Derivative of 1 + 0.5 tanh(2x) on range -2 to 2, computed analytically, through central difference method, and autodiff with Jax

2 Problem 2 (5.17)

Part A: See code, Figure 2

Part B: $\dfrac{\mathrm{d}(x^{\text{a-1}}e^{\text{-x}})}{\mathrm{d}dx} = -x^{\text{a-1}}e^{\text{-x}} + (a-1)x^{\text{a-2}}e^{\text{-x}} = 0$

=> $x^{\text{a-1}}e^{\text{-x}} = (a-1)x^{\text{a-2}}e^{\text{-x}}$

=> $x^{\text{a-1}} = (a-1)x^{\text{a-2}}$

=> $x = (a-1)$

Part C: z=1/2 when x=c, so the since the peak falls at x=a-1, c should be set to a-1 for the peak to fall at z=1/2.

Part D: The integrand can be rewritten as: $e^{\text{(a-1)ln(x)-x}}$ This expression is preferable to the original, since we can avoid having to store the potentially very large value of $x^{\text{a-1}}$ and the very small value of $e^{\text{-x}}$ at the same time. Instead, a single value can be input as the argument of the exponent, and the values of (a-1) lnx and x that have to be stored to compute this value do not change as rapidly as the previous two do, and thus do not risk overflow/underflow as much.

Part E: gamma(3/2)= 0.886227208154826

Part F: gamma(3)= 2.0000000000000657 gamma(6) = 120.0 gamma(10) = 362879.9999999999

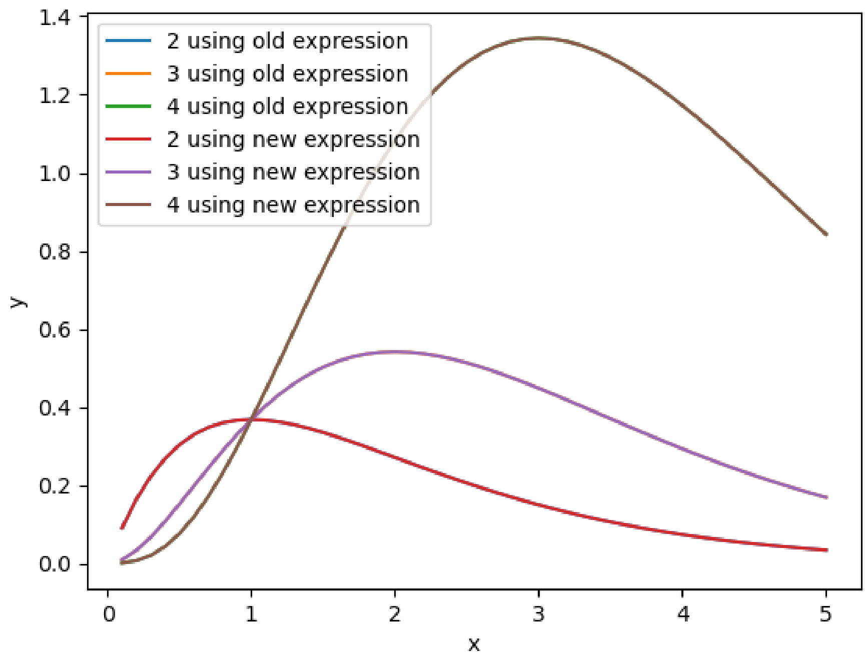

Figure 2: Graphs of the integrand for a=2,3,4, computed with the expressions from parts A and D

3 Problem 3

Part A: See code, Figure 3
Part B: See code, Figure 4
Part C: See code, Figure 5
The residuals are large, and fluctuate according to the original data. If we assumed this model gave a good account of the data, the fluctuating pattern of the original data that is reflected in the original would have to be accounted for by error. The error would be large in many places, and seemingly following a periodic, non-random pattern, which suggests that this model simply is not a good fit for the data.
Part D: See code, Figure 6
Using a 30th order polynomial, we obtain a fit that seems to match the trend of the data fairly well, and the condition number is on the order of 10^{12}, which is less then the inverse of machine precision (10^{15}), so it seems that there can be a reasonable polynomial model that provides a good explanation for the data.
Part E: See code, Figure 7
This model does a fairly good job explaining the data, capturing the oscillating trend correctly for the most part, but could improved likely using a longer sequence of sinusoidal functions. The data seems to have a periodicity of 1/8 of the full time span, about 0.42860 for the rescaled time units.

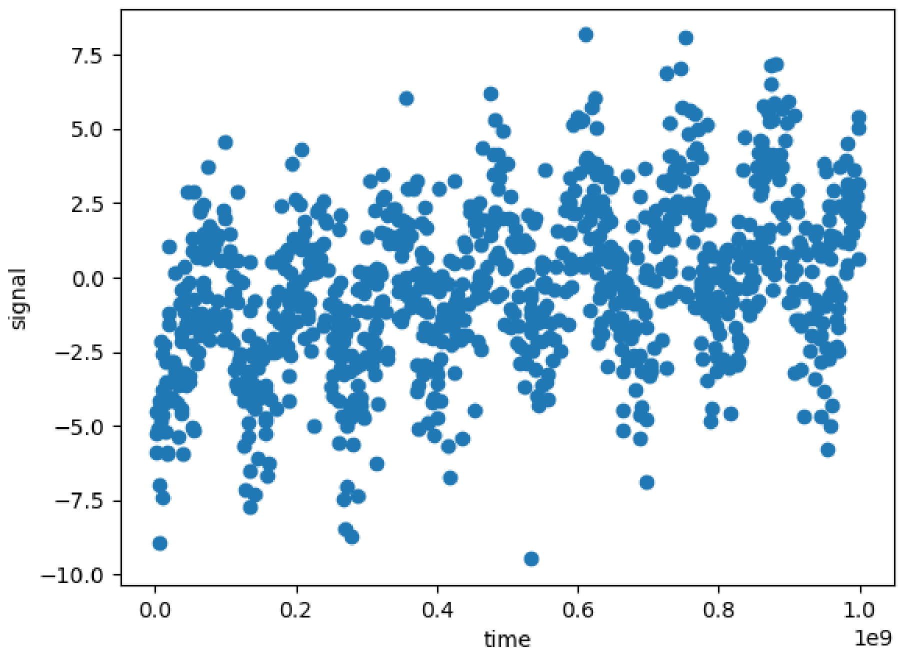

Figure 3: Plot of data before rescaling of t

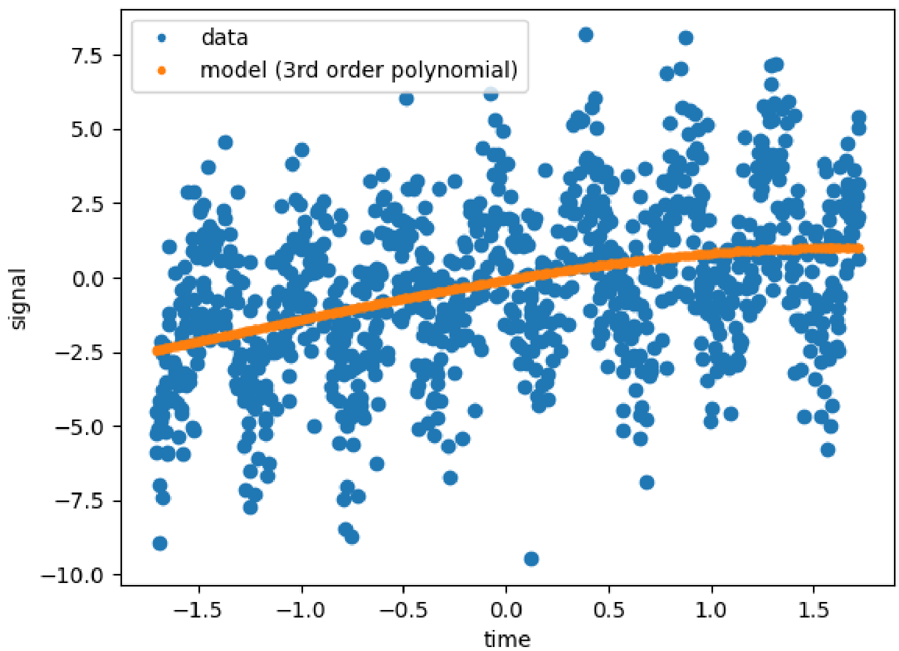

Figure 4: Plot of data with rescaling of t and SVD fit, 3rd order polynomial

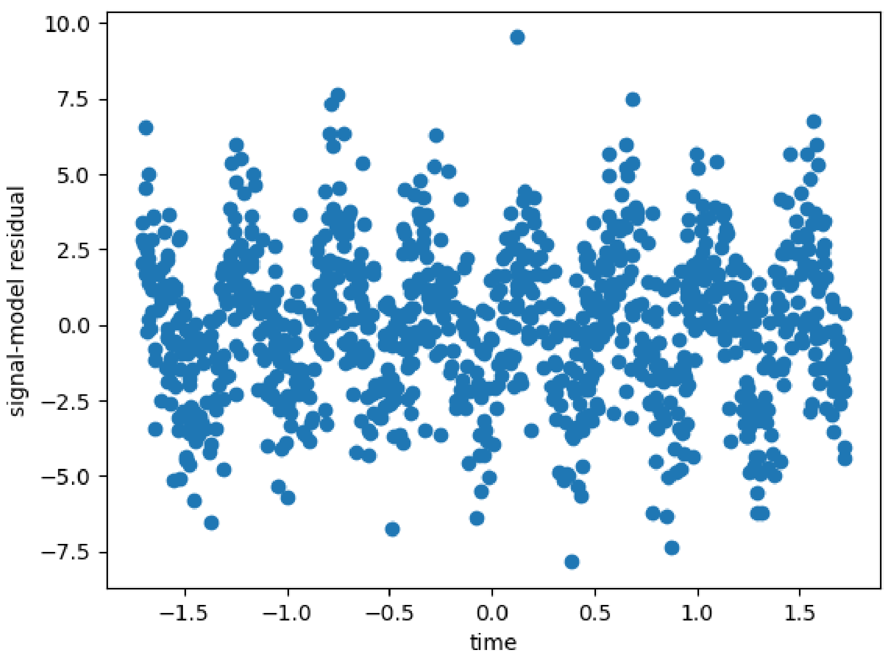

Figure 5: Plot of residual for 3rd order polynomial fit

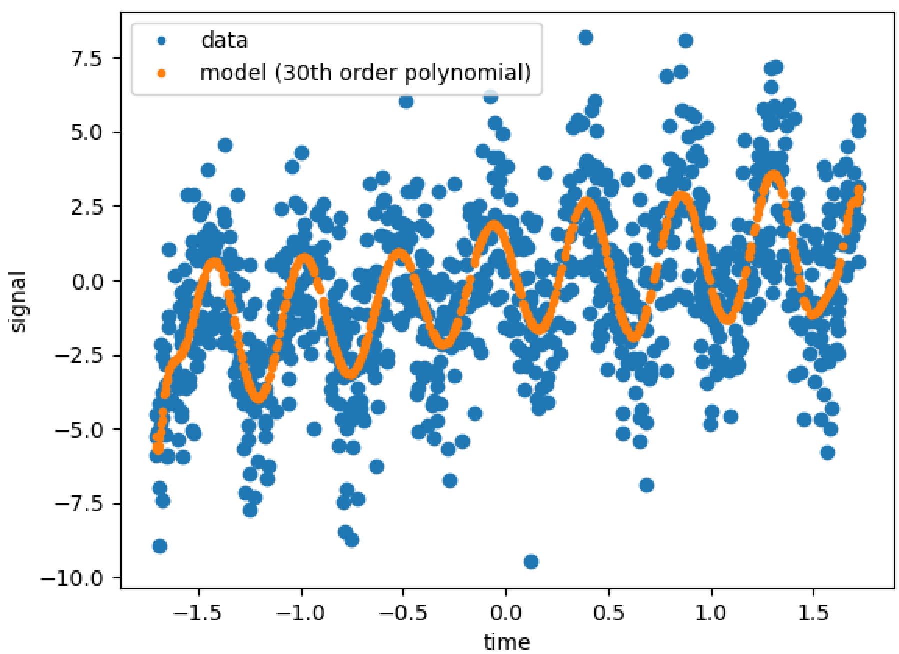

Figure 6: Plot of data with rescaling of t and SVD fit, 30th order polynomial fit

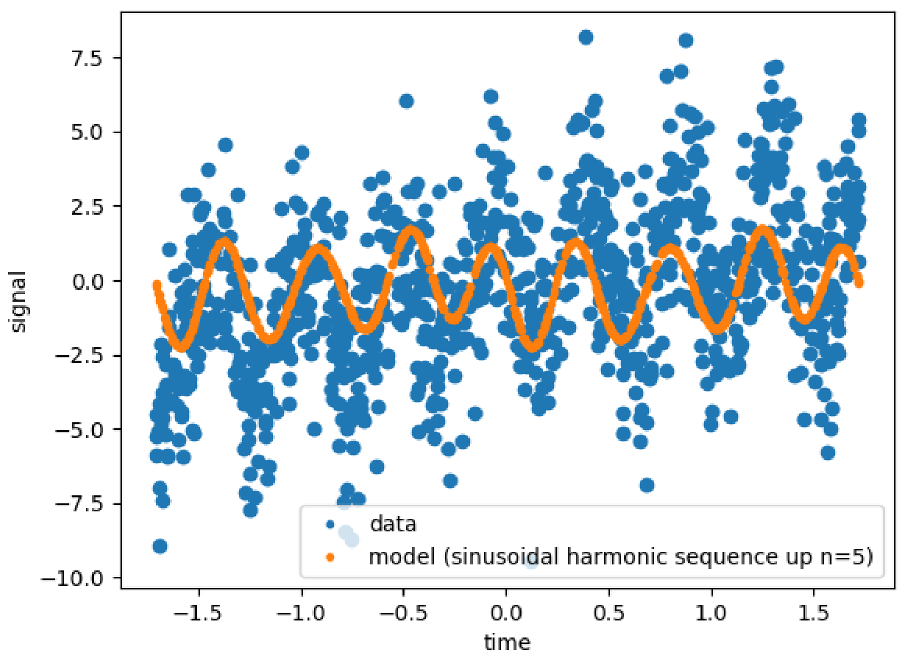

Figure 7: Plot of data with rescaling of t and SVD fit using sinusoidal functions