Computational Physics Problem Set 9

John Ferrante

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GitHub: https://github.com/jferrante25/physga-2000

1 Problem 1 (Newman 8.6)

Part A: See Figure 1
Part B: See Figure 2; period of oscillation remains the same (2π).
Part C: See Figures 3 and 4; the rate of oscillation is faster in the second plot with an amplitude of 2 (about 13 complete oscillations in the same 50 second interval compared with 6 in the original)
Part D: See Figure 5
Part E: See Figures 6, 7, 8

2 Problem 2 (Newman 8.7)

Part A: The air resistance $F = -\frac{1}{2}\pi R^2 \rho C v^2$ along the direction of motion. The component of the air resistance in the x-direction will be $\dot{x}/v$ times this, and that in the y-direction $\dot{y}/v$ times it, with $v = \sqrt{\dot{x}^2 + \dot{y}^2}$. So,

$$F_x = m\ddot{x} = -\tfrac{1}{2}\pi R^2 \rho C \tfrac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}}(\dot{x}^2 + \dot{y}^2) = -\tfrac{1}{2}\pi R^2 \rho C \dot{x}\sqrt{\dot{x}^2 + \dot{y}^2}$$

and

$$\ddot{x} = -\tfrac{\pi R^2 \rho C}{2m}\dot{x}\sqrt{\dot{x}^2 + \dot{y}^2}$$

Taking the corresponding air resistance term for the y-component and adding -g for for the gravitational force gives:

$$\ddot{y} = -g - \tfrac{\pi R^2 \rho C}{2m}\dot{y}\sqrt{\dot{x}^2 + \dot{y}^2}$$

Rescaling Discussion:
If the original units are rescaled so $t = t'/T$, $x' = x\frac{\pi R^2 \rho C}{2m}$ (the distance scale for both x and y is rescaled this way), then

$$\frac{dy'}{dt'} = \frac{\pi R^2 \rho C T}{2m}\frac{dy}{dt}, \frac{d^2y'}{dt'^2} = \frac{\pi R^2 \rho C T^2}{2m}\frac{d^2y}{dt^2}$$

and

$$\frac{d^2y}{dt^2} = -g - \frac{\pi R^2 \rho C}{2m}\dot{y}\sqrt{\dot{x}^2 + \dot{y}^2}$$

becomes

$$\frac{2m}{\pi R^2 \rho C T^2}\frac{d^2y'}{dt'^2} = -g - \frac{\pi R^2 \rho C}{2m}(\frac{2m}{\pi R^2 \rho C T})^2 \dot{y}'\sqrt{\dot{x}'^2 + \dot{y}'^2}$$

$$\frac{d^2y'}{dt'^2} == -g\frac{\pi R^2 \rho C T^2}{2m} - \dot{y}'\sqrt{\dot{x}'^2 + \dot{y}'^2}$$

Part B: See Figure 9

Part C: See Figure 10. Estimating roughly from the plots, the 1 kg cannonball travels $\sim$ 260 meters in the x direction before returning to the ground (y=0), while the 5 kg ball travels $\sim$ 550 m and the kg $\sim$ 670 m. Based on these observations, it seems that the greater the mass of the cannonball, the further it travels, which makes sense given the 1/m factor in the air resistance expressions.

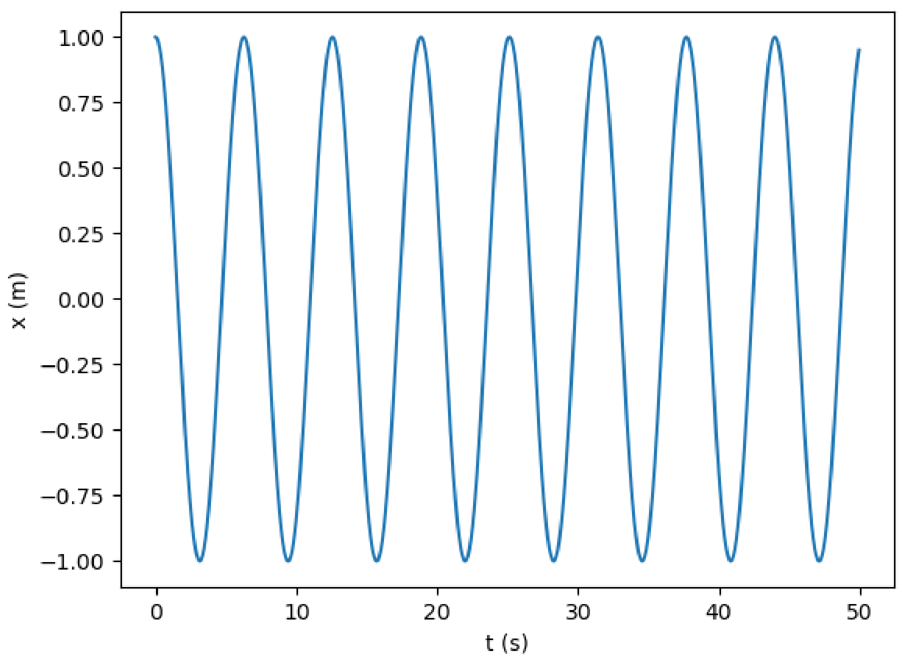

Figure 1: x as a function of time for harmonic oscillator, $\omega = 1$, $x_0 = 1$

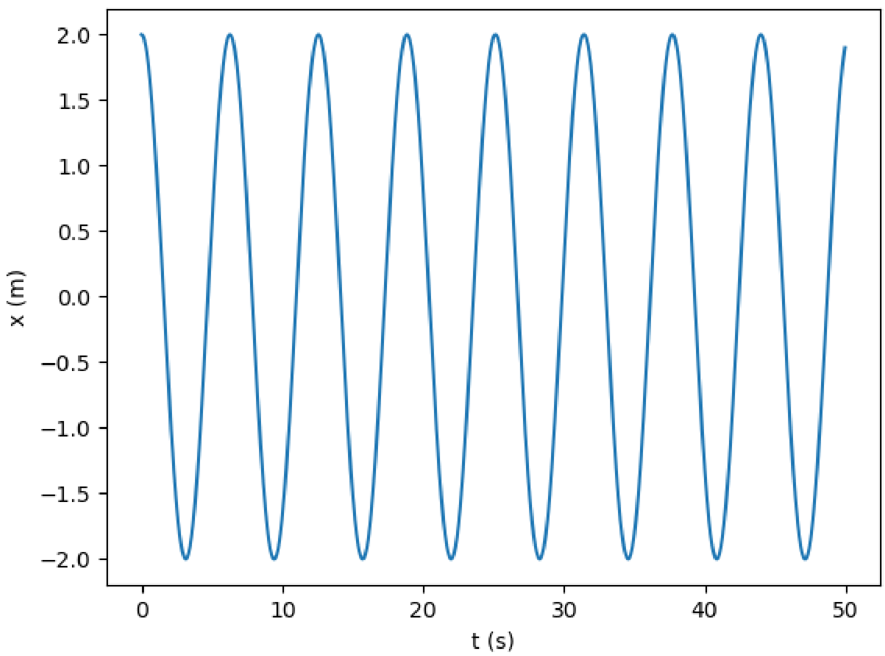

Figure 2: x as a function of time for harmonic oscillator, $\omega = 1$, $x_0 = 2$

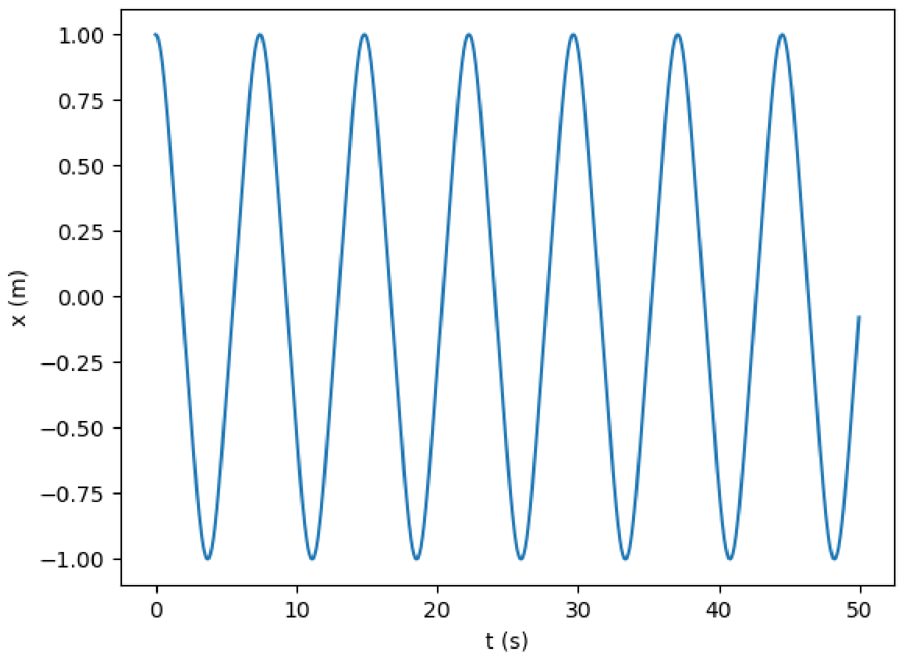

Figure 3: x as a function of time for anharmonic oscillator (equation $\frac{d^2x}{dt^2} = -\omega^2 x^3$, $\omega = 1$), $x_0 = 1$

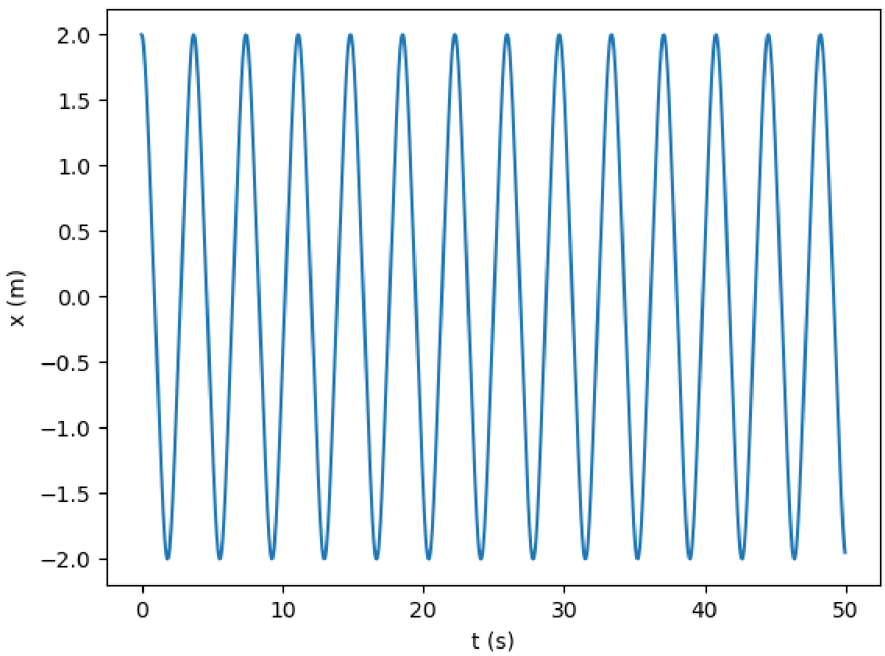

Figure 4: x as a function of time for anharmonic oscillator (equation $\frac{d^2x}{dt^2} = -\omega^2 x^3$, $\omega = 1$), $x_0 = 2$

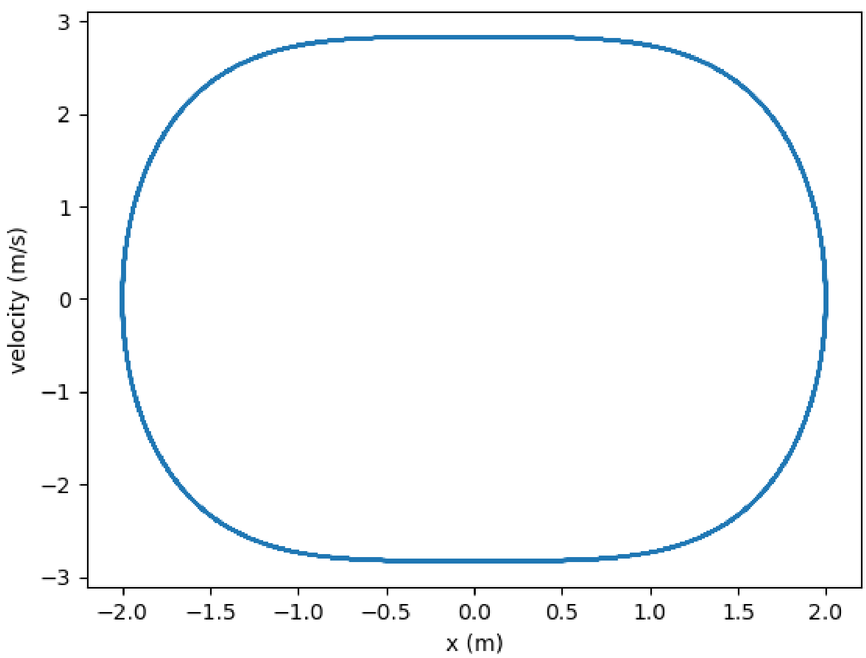

Figure 5: Phase-space plot for anharmonic oscillator

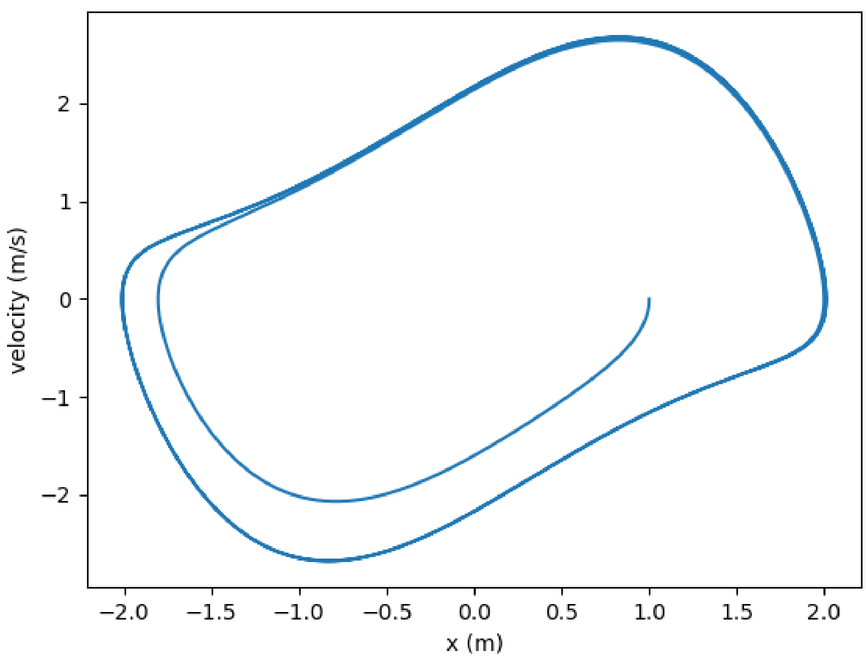

Figure 6: Phase portrait for van der Pol oscillator, $\mu = 1$

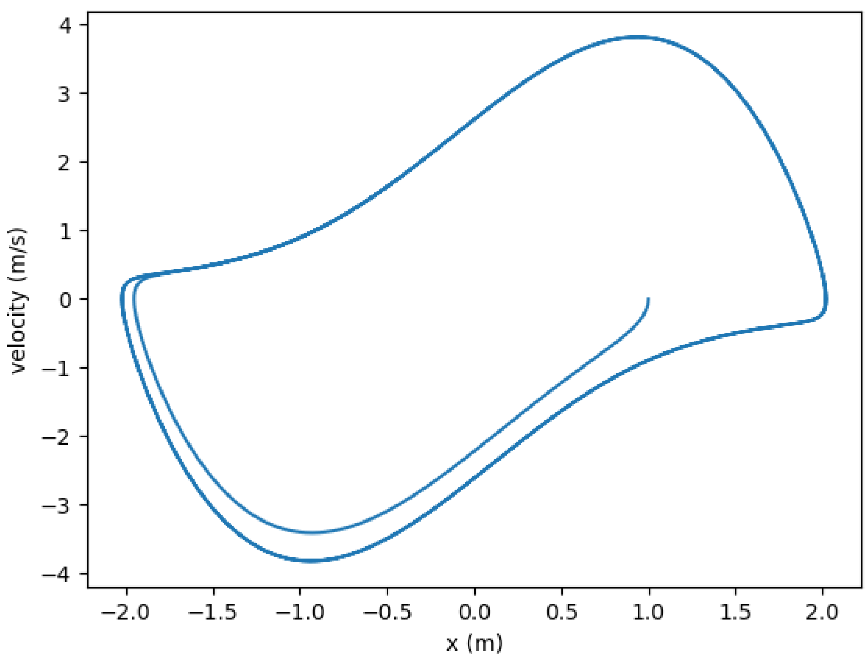

Figure 7: Phase portrait for van der Pol oscillator, $\mu = 2$

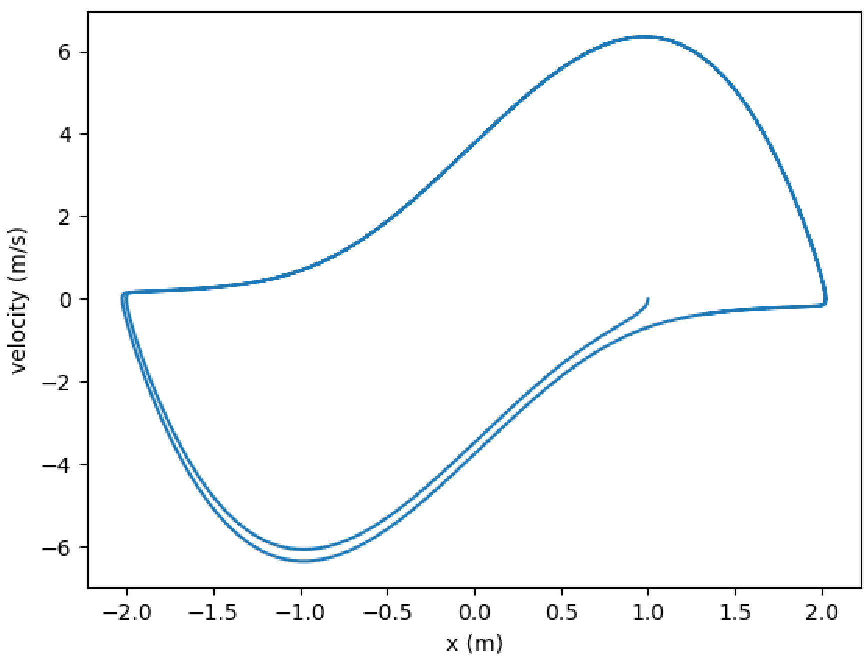

Figure 8: Phase portrait for van der Pol oscillator, $\mu = 4$

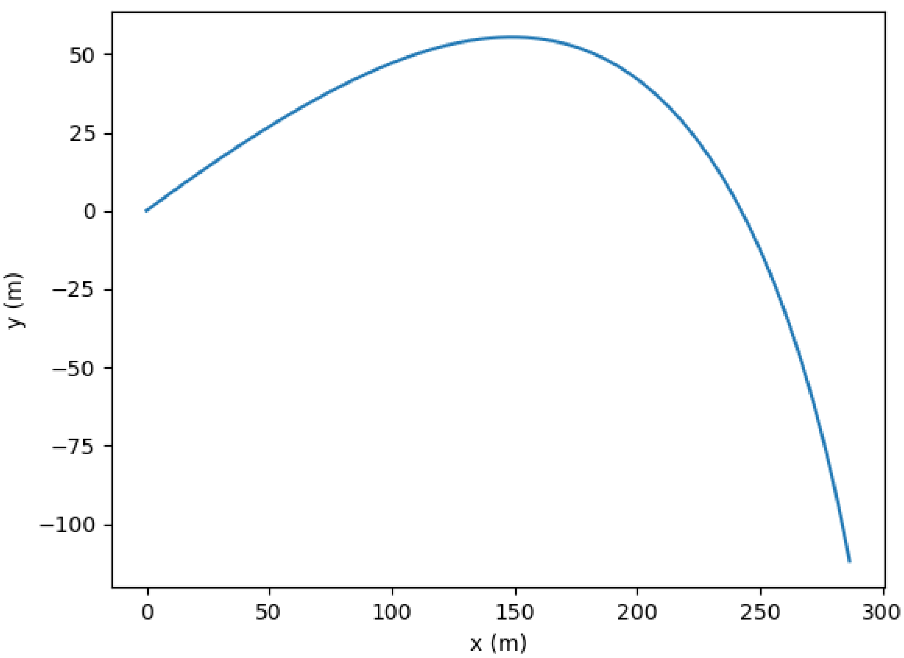

Figure 9: Trajectory of 1 kg cannonball

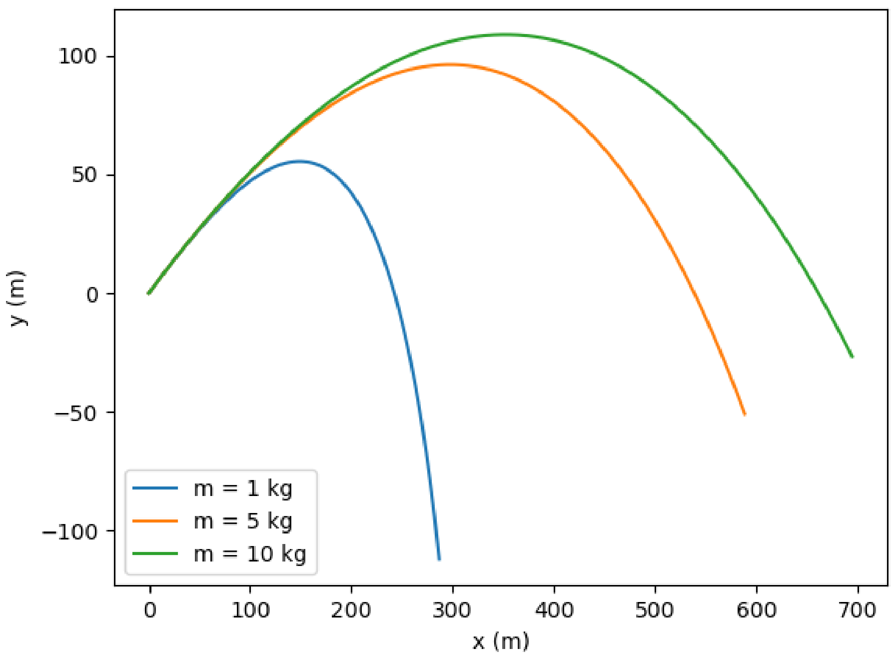

Figure 10: Trajectory of 1 kg, 5 kg, and 10 kg cannonballs