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A new three-dimensional chaotic system, its dynamical analysis and electronic circuit applications

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ABSTRACT

In this paper, a new three-dimensional chaotic system is introduced, which contains the quadratic, cubic and quartic nonlinearities. Basic dynamical characteristics of this new system are studied such as equilibria, eigenvalue structures, Lyapunov exponent spectrum, fractal dimension and chaotic behaviors. Moreover, this paper investigates bifurcation analysis of the proposed chaotic system by means of a selected parameter. The chaotic system has been investigated by detailed numerical as well as theoretical analysis. Amplitude values are important in chaotic systems for real environment applications because of electronic components and materials limitations. Thus, the new chaotic system is rescaled and executed an electronic circuit implementation for real environment application. The obtained results show that this system has complex dynamics with some interesting characteristics and deserves a further detailed investigation. The new chaotic system can be useful in many engineering and scientific applications such as physics, control, cryptology and random number generator (RNG).

1. Introduction

The chaotic systems have been extensively studied and analyzed by many researchers due to its various applications in the field of population dynamics, electric circuits, cryptology, fluid dynamics, engineering, stock exchanges, etc. Most of the complex dynamical phenomenons are characterized by the chaotic and hyperchaotic system of nonlinear ODEs. For instance, variations in the population dynamics can be discussed and the predictions are made for the future scenarios. Moreover, Lorenz introduced a model in 1963 for weather forecasting [1], demonstrating the variations in temperature and wind speed. He observed that a small perturbations in the initial conditions of the proposed dynamical model lead to the surprising results. This effect is known as the “butterfly effect” as the system has sensitive dependence on the initial conditions.

The world “chaos” was used by Li and Yorke in 1975 for the first time in their research [2]. In 1976, Rossler considered a prototype equation to the Lorenz model that behaves similarly to the Lorenz model, but in low dimensional dissipative dynamical systems [3]. He also proposed an even simpler (algebraic) model in 1979 [4]. Matsumoto et al. developed an

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autonomous chaotic system in 1986 concerning the basic circuit layout. This circuit known in the literature as the "Chua circuit" [5]. Chen and Ueta discovered a chaotic system in 1999, known as the Chen system [6] based on the Lorenz model. Another chaotic system has been investigated in [7] by Lu and Chen, representing the transition between the Lorenz system and the Chen system. Sprott investigated generic three dimensional systems with quadratic nonlinearities [8–10]. He found nineteen different chaotic systems by an exhaustive computer research such that there is no obvious transformation of one to another. Many other chaotic systems have been introduced and analyzed in the literature, including the Liu and T system [11,12].

Thus the chaos is extremely complex nonlinear dynamical phenomenon in the nature. A lot of research has been reported in the literature investigating various kinds of characteristics associated with dynamical systems. Edward Lorenz discovered a "strange attractor" while finding the numerical solutions to the proposed weather model [13].

In the recent years, motivated by different applications, great works have been reported in constructing the chaotic and hyperchaotic dynamical models [14–20]. In this paper, a new chaotic system containing quadratic, cubic and quartic nonlinearities is proposed. Several dynamical features of the proposed system are discussed by means of both theoretical and numerical analysis. Moreover, a bifurcation analysis corresponding to the maximum Lyapunov exponent spectrum is also performed for the new chaotic system.

On the other hand; chaotic systems, have attracted lots of attention in recent years. Chaotic systems have been used in many different areas for different purposes. In this study, we executed the application of the chaotic system in real environment as electronic circuit implementation. The dynamic properties and simulations demonstrate that the proposed system is chaotic with interesting properties.

This paper is structured as follows: in Section 2, the new chaotic system is introduced. The basic dynamical properties of the proposed model have been discussed in the subsections. In Section 2.6, we discuss the bifurcation analysis and Lyapunov spectrum. The electronic circuit implementations are given in Section 3. And finally, conclusion based on the theoretical and numerical investigation is given in Section 4.

2. The new chaotic system and its dynamical properties

The new chaotic system investigated in the present paper is described as the following autonomous differential equations:

$$\begin{cases} x^{\cdot} = a(x-y), \\ y^{\cdot} = -4ay + xz + mx^3, \\ z^{\cdot} = -adz + x^3y + bz^2, \end{cases} \tag{1}$$

where x, y and z are state variables and a, b, d and m are parameters. The parameters values are set to $a = 1.8$, $b = -0.07$, $d = 1.5$ and $m = 0.12$. The system exhibits a chaotic behavior for the chosen values.

2.1. Symmetry and invariance

It is simple to see the invariance of system (1) under the coordinate transformation

$$(x, y, z) \to (-x, -y, z).$$

That is, the system (1) has symmetry around the z−axis [20]. Hence, from Fig. 1 (phase portrait for x–z and y–z), we can observe that the variables x and y are symmetrical with respect to the origin $O(0, 0, 0)$.

2.2. Dissipativity

The proposed dynamical system (1) can be written in the vector form as

$$X^{\cdot} = F(X) = [f_1(X), f_2(X), f_3(X)]^T. \tag{2}$$

The divergence of the vector field $F(X)$ on $\mathbb{R}^3$ is given by

$$\nabla \cdot F = \frac{\partial f_1(X)}{\partial x} + \frac{\partial f_2(X)}{\partial y} + \frac{\partial f_3(X)}{\partial z}. \tag{3}$$

For the system (1), the divergence is computed as $\nabla \cdot F = -3a - ad + 2bz < 0$, which is negative if $z \geq 0$, so that volume (V) decreases at rate

$$\frac{dV(t)}{dt} = (-3a - ad + 2bz)V. \tag{4}$$

Solving the above differential equation, we have

$$dV(t) = V(0)e^{(-3a-ad+2bz)}t = V(0)e^{-3a-ad+2bz}t. \tag{5}$$

So the volume of the system reduces exponentially fast to 0, and hence the dynamical model (1) is a dissipative system for $z \geq 0$. That is, every volume consisting of the system trajectory shrinks to zero as $t \to \infty$ at an exponential rate of $(-3a - ad + 2bz)$. Thus, all the system orbits are confined to a specific subset containing zero volume and the asymptotic motion converges onto an attractor [21,22].

2.3. System equilibria

The equilibria of the chaotic system (1) are found by setting $\dot{x} = \dot{y} = \dot{z} = 0$, i.e.,

$$\begin{cases} a(x-y) = 0, \\ -4ay + xz + mx^3 = 0, \\ -adz + x^3 y + bz^2 = 0. \end{cases} \tag{6}$$

The system (1) has four equilibrium points:

$$O(0,0,0),$$

$$E_1\left(0,0,a\,d/b\right);$$

$$E_2\left(\sqrt{\frac{4\,a - z^\star}{m}}, \sqrt{\frac{4\,a - z^\star}{m}}, z^\star\right),$$

$$E_3\left(-\sqrt{\frac{4\,a - z^\star}{m}}, -\sqrt{\frac{4\,a - z^\star}{m}}, z^\star\right),$$

where $z^\star = \frac{(adm^2+8a)+am\sqrt{d^2m^2+16d-64b}}{2(1+bm^2)}$.

The local behavior of the system around these equilibrium points can be investigated by using the following Jacobian matrix:

$$J(x,y,z) = \begin{bmatrix} a & -a & 0 \\ 3mx^2 + z & -4a & x \\ 3x^2 y & x^3 & 2bz - ad \end{bmatrix} \tag{7}$$

For the equilibrium point O, the above Jacobian matrix becomes the following

$$J(O) = \begin{bmatrix} 1.8000 & -1.8000 & 0 \\ 0 & -7.2000 & 0 \\ 0 & 0 & -2.7000 \end{bmatrix} \tag{8}$$

To obtain its eigenvalues, let $\det(\lambda I - J(O)) = 0$. Then, the characteristic equation has the following form:

$$\lambda^3 + 8.1\lambda^2 + 1.62\lambda - 34.992 = 0$$

Solving the above characteristic equation, the eigenvalues are found as

$$\lambda_1 = 1.8, \quad \lambda_2 = -7.2, \quad \lambda_3 = -2.7.$$

Here λ_1 is a positive real number, λ_2 and λ_3 are two negative real numbers. That means the equilibrium $O(0, 0, 0)$ is a saddle point. So, this equilibrium point $O(0, 0, 0)$ is unstable. Next, linearizing the system (1) about the other equilibria, we obtain the following Jacobian matrix

$$J(E_1) = \begin{bmatrix} 1.8000 & -1.8000 & 0 \\ -38.5714 & -7.2000 & 0 \\ 0 & 0 & 2.7000 \end{bmatrix} \tag{9}$$

The corresponding eigenvalues are

$$\lambda_1 = 6.7699, \quad \lambda_2 = -12.1699, \quad \lambda_3 = 2.7000.$$

Thus, the equilibrium point E_1 is unstable. Similarly, for the equilibrium points E_2 and E_3, we have the corresponding eigenvalues, respectively;

$$\lambda_1 = 0.826 - 3.828i, \quad \lambda_2 = 0.826 + 3.828i, \quad \lambda_3 = -10.747.$$

and

$$\lambda_1 = 0.826 - 3.828i, \quad \lambda_2 = 0.826 + 3.828i, \quad \lambda_3 = -10.747.$$

For both the equilibrium points E_2, and E_3, we have λ_3 is a negative real number, while λ_1 and λ_2 a conjugate pair of complex eigenvalues having positive real parts. This implies that both the equilibrium points E_2, and E_3 are saddle-focus points; so, these equilibria are unstable points.

2.4. Lyapunov exponent and fractional dimension

It is well known that the Lyapunov exponent is a way for analyzing the nonlinear behavior of the dynamical systems. In fact, it measures the exponential rates of either convergence or divergence of the trajectories in the phase space [23]. If at least one Lyapunov exponent is positive, then the system can be considered as the chaotic. The Lyapunov exponents of the dynamical system (1) are calculated numerically for the parameter values $a = 1.8$, $b = -0.07$, $d = 1.5$ and $m = 0.02$ with the initial state as $x(0) = (0.5, 0, 0)$. The corresponding values of the Lyapunov exponents are

$$L_1 = 0.23108, \quad L_2 = 0.0038593, \quad L_3 = -9.3662$$

The Lyapunov dimension of the new chaotic system (1) is fractional given by

$$D_L = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0243. \tag{10}$$

Eq. (10) means system (1) is really a dissipative system, and the Lyapunov dimensions of the system are fractional. Thus, having a positive Lyapunov exponent and the strange attractor, it is obvious that the proposed system is chaotic.

2.5. Phase portraits

The initial values of the system are chosen as (0.5, 0, 0) and simulations have been completed. This nonlinear system exhibits the chaotic dynamic behavior. In Fig. 1, we produce the $x-y$, $x-z$, $y-z$ and $x-y-z$ phase portraits of the chaotic system (1) using the MATLAB ode45 function. Fig. 1 demonstrate the phase portraits for $m = 0.12$.

2.6. Lyapunov spectrum and bifurcation analysis

Figs. 2 and 3 show the Lyapunov spectrum of the new system for a varying parameter m, and constant parameters $a = 1.8$, $b = -0.07$, $d = 1.5$. One can observe from the Lyapunov exponents spectrum, when m is in the range (0.062, 0.185), the new system is chaotic with a positive Lyapunov exponent.

Figs. 4–6 show the bifurcation diagrams which corresponds directly to the maximum Lyapunov exponent spectrum, shown in Figs. 2 and 3. As can be seen in Figs. 4 and 6, the new system has chaotic behavior in the interval $0.062 \le m \le 0.185$, except one interval 0.101–0.105. Furthermore, in Fig. 5, system has chaotic behavior in the interval $0.215 \le m \le 0.225$. On the other hand, system has limit cycle behavior in the intervals $0.186 \le m \le 0.214$ and $0.226 \le m \le 0.4$.

2.7. Sensitivity to initial conditions

Sensitivity to initial conditions means that any arbitrarily small perturbations in the initial state of the system can lead to the significantly distant behavior of the future state of the chaotic system. This dependence of initial conditions in the system makes the long-term prediction impossible. Obtained time series using ode45 function for $y = 0$ and $y = 0.001$ have given in Fig. 7. It shows that the evolution of the chaos trajectories is very sensitive to initial conditions.

3. The electronic circuit implementation and real environment application

3.1. Rescaling of the new chaotic system

Amplitude values are important in chaotic systems for real environment applications because of electronic components and materials limitations. To avoid saturation property of electronic components like the analog multipliers and operational amplifiers, the amplitude values of chaotic systems are usually reduced by a linear rescaling of the variables.

In Fig. 1 are seen some of the signals, x, y, and z values in the interval of (0, 45). There is no need to rescale x value because of in the interval of (−5, 5). But, y and z need to be re-scaled for real environment applications.

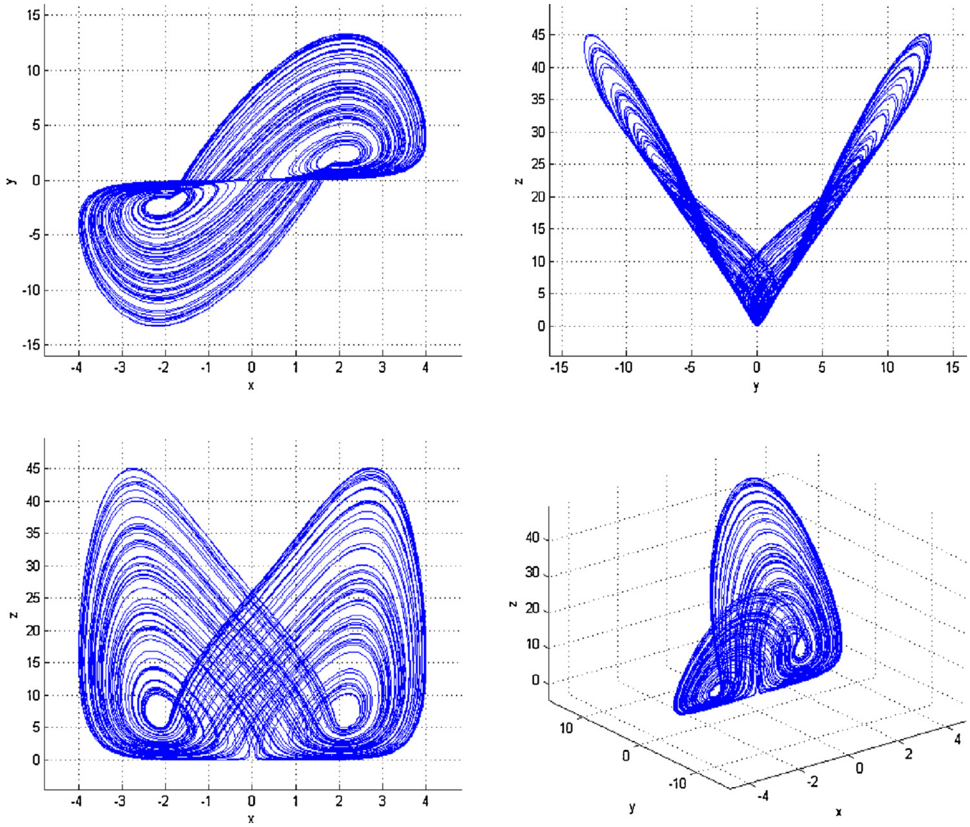

Fig. 1. Phase portraits of system (1) with $a = 1.8$, $b = -0.07$, $d = 1.5$ and $m = 0.12$.

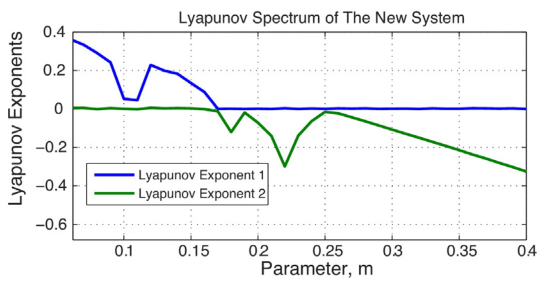

Fig. 2. Dynamics of the Lyapunov exponents with $a = 1.8$, $b = -0.07$, $d = 1.5$ and $m = 0.12$.

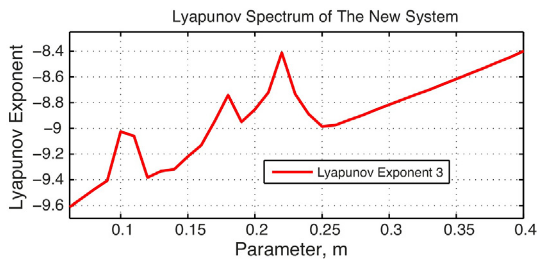

Fig. 3. Dynamics of the Lyapunov exponents with $a = 1.8$, $b = -0.07$, $d = 1.5$ and $m = 0.12$.

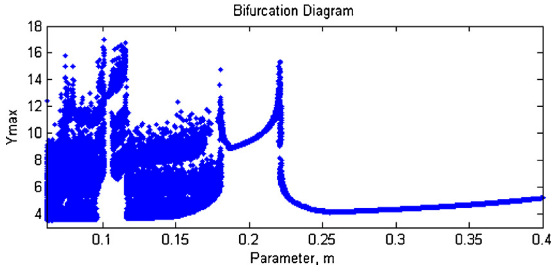

Fig. 4. Bifurcation diagram (between 0.062 and 0.4).

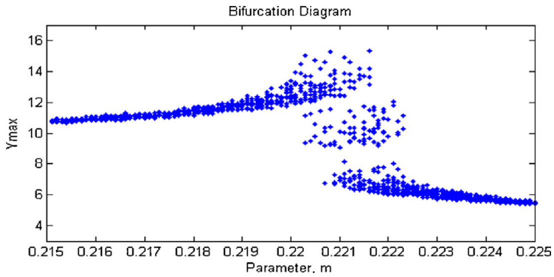

Fig. 5. Bifurcation diagram (between 0.215 and 0.225).

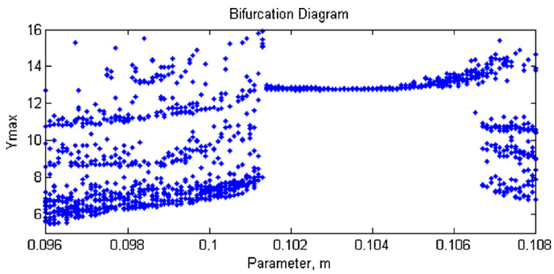

Fig. 6. Bifurcation diagram (between 0.062 and 0.108).

For example, let $u=x$, $v=y/5$, and $w=z/10$, and then setting the original state variables x, y, z instead of the variables u; v; w, the rescaled system firstly becomes the following.

$$\begin{cases} \dot{x} = a(x-y), \\ \dot{y} = -4ay + xz + mx^3, \\ \dot{z} = -adz + x^3y + bz^2, \end{cases} \tag{11}$$

$$\begin{cases} u = x, \\ v = y/5, \\ w = z/10, \end{cases} \tag{12}$$

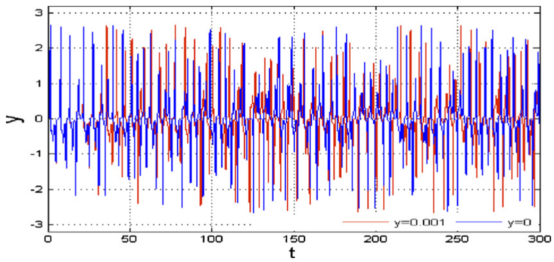

Fig. 7. Time series for $y=0$ and $y=0.001$.

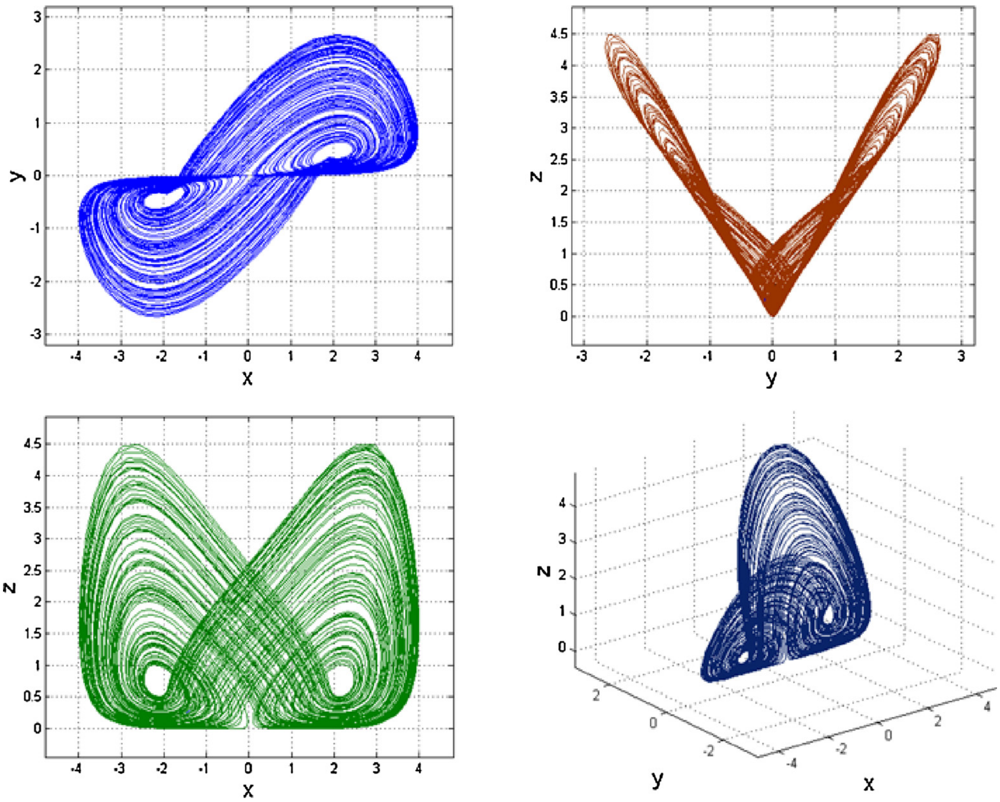

Fig. 8. The new phase portraits of rescaled chaotic system with $a = 1.8$, $b = -0.07$, $d = 1.5$ and $m = 0.12$.

$$\begin{cases} x = u, \\ y = 5v, \\ z = 10w, \end{cases} \tag{13}$$

$$\begin{cases} \dot{x} = a(u - 5v) = au - 5av, \\ \dot{y} = -4a(5v) + (u)(10w) + mu^3 = -20av + 10uw + mu^3, \\ \dot{z} = -ad(10w) + u^3(5v) + b(10w)^2 = -10adw + 5u^3v + 100bw^2. \end{cases} \tag{14}$$

According to new values (u, v, w), derivatives of all values are:

$$\begin{cases} \dot{u} = \dot{x}, \\ \dot{v} = \dot{y}/5, \\ \dot{w} = \dot{z}/5. \end{cases} \tag{15}$$

After the new values, rescaled system becomes the following.

$$\begin{cases} \dot{u} = \dot{x} = au - 5av, \\ \dot{v} = \dot{y}/5 = (-20av + 10uw + mu^3)/5 = -4av + 2uw + (m/5)u^3, \\ \dot{w} = \dot{z}/10 = (-10adw + 5u^3v + 100bw^2)/10 = -adw + 0.5u^3v + 10bw^2. \end{cases} \tag{16}$$

Finally, rescaled chaotic system are given by

$$\begin{cases} \dot{u} = au - 5av, \\ \dot{v} = -4av + 2uw + (m/5)u^3, \\ \dot{z} = -adw + 0.5u^3v + 10bw^2. \end{cases} \tag{17}$$

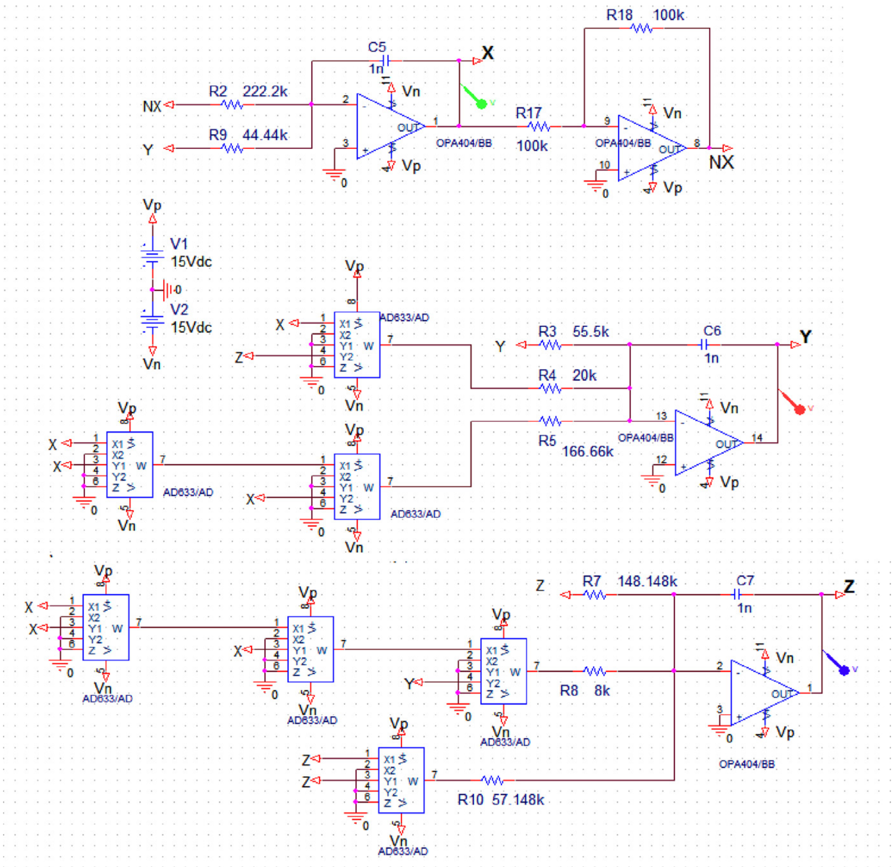

Fig. 9. The electronic circuit schematic of the rescaled new chaotic system.

In Fig. 8 are shown the new phase portraits of rescaled chaotic system with $a = 1.8$, $b = -0.07$, $d = 1.5$ and $m = 0.12$. As can be seen in Fig. 8 below, amplitude values of y and z decreased according to phase portraits of first system which is in Fig. 1. Now, we can execute electronic circuit implementation as the example of real environment application.

3.2. Circuit implementation of the rescaled new chaotic system

The electronic circuit of the new rescaled chaotic system was designed in OrCAD-PSpice program. The electronic circuit schematic of the new system is seen in Fig. 9. The circuit includes simple electronic elements such as resistors, operational amplifiers.

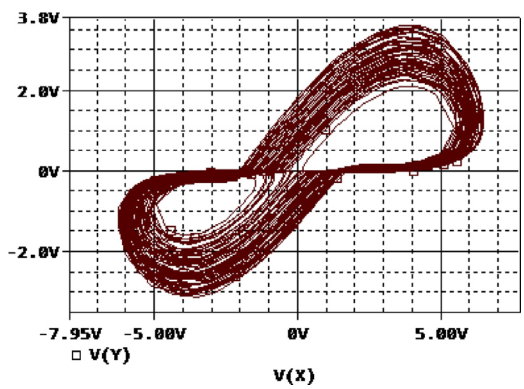

Fig. 10. Phase portrait obtained from OrCAD-PSpice simulation (x–y).

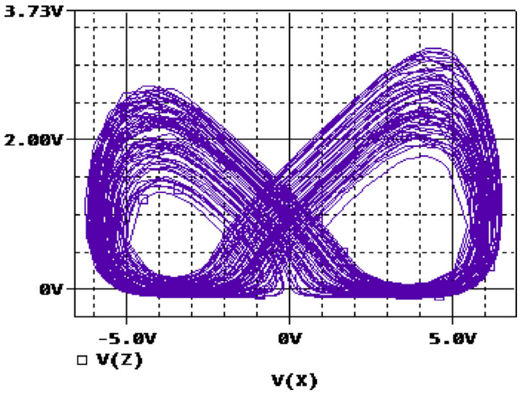

Fig. 11. Phase portrait obtained from OrCAD-PSpice simulation (*x*–*z*).

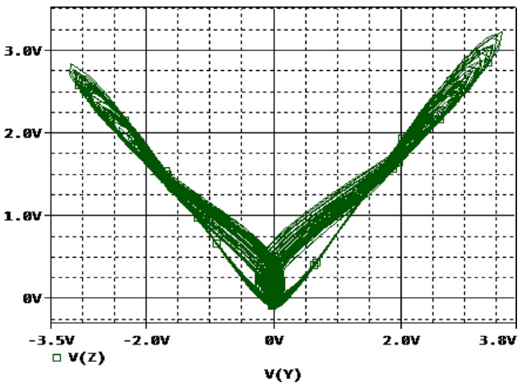

Fig. 12. Phase portrait obtained from OrCAD-PSpice simulation (*y*–*z*).

Experimental electronic circuit of the new chaotic system was designed for parameter $a = 1.8$, $b = -0.07$, $d = 1.5$, $m = 0.12$, and initial conditions $x(0) = 0.5$, $y(0) = 0$, $z(0) = 0$. Since only x re-quires the initial condition voltage for executing the circuit, implementation of the new circuit in real time is easier.

TL081 opamps, and the Analog Devices AD633 multipliers were used and R2 = 222.2 kΩ, R3 = 55.5 kΩ, R4= 20 kΩ, R5 = 166.6 kΩ, R7 = 148.148 kΩ, R8 = 8 kΩ, R9 = 44.4 kΩ, R10 = 57.148 kΩ, R17 = R12 = 100 kΩ, C1 = C2 = C3 = 1 nF, Vn = 15 V, Vp = 15 V were chosen. Acceptable inputs to the AD633 multiplier IC are −10 V to +10 V. The output voltage is the product of the inputs divided by 10 V.

The OrCAD-PSpice simulation oscilloscope outputs (phase portraits) of circuitry of the re-scaled new system, for parameters $a = 1.8$, $b = -0.07$, $d = 1.5$, $m = 0.12$, are seen in Figs. 10–12. As they can be seen from the Matlab ode45 function simulation outputs in Fig. 8 and the Or-CAD-PSpice simulation outputs in Figs. 10–12, the results are similar.

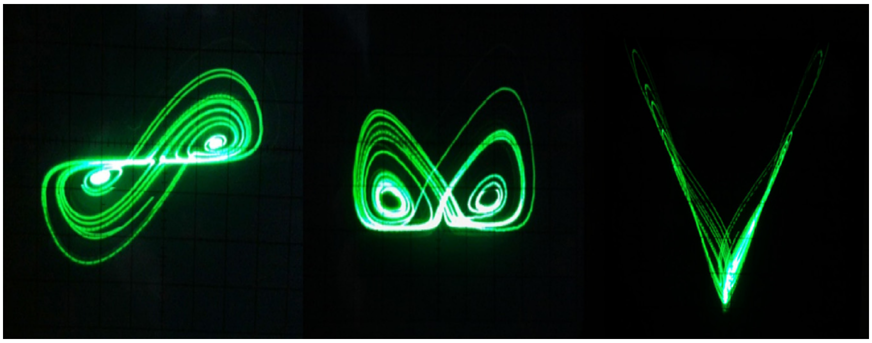

Fig. 13. *x*–*y*, *x*–*z* and *y*–*z* phase portraits on the oscilloscope.

3.3. Real environment application of the new chaotic circuit design

Real environment application of the new chaotic system was implemented with electronic components on bread-board. Fig. 13 exhibits the phase portrait outputs of the electronic circuit application on an oscilloscope. The outputs show that results of the chaotic system which was modeled on Matlab verify the real circuit results and ORCAD-PSpice simulation results.

4. Conclusion

This article has reported and analyzed a new three-dimensional autonomous chaotic system. The dynamical properties of the new system have been discussed by means of equilibria, Lyapunov dimension and chaotic behaviors. We have investigated the proposed chaotic system using theoretical analysis and numerical simulations. Bifurcation diagrams and Lyapunov spectrum are also performed. The obtained results confirm the complex dynamical behaviors. In this paper, in addition to basic dynamical analysis, the new chaotic have been executed for its simulations using the designed electronical circuit in OrCAD PSpice program. The new chaotic system is confirmed through an electronic circuit design. The simulation results of the Matlab and the OrCAD-PSpice programs were obtained for the similar shaped phase portraits. The proposed new chaotic system, they can be useful in many engineering applications such as chaos based cryptology, coding information, information compression, random number generator, chaos based music and image generator, etc. in the near future.

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