Interest Rate and Credit Models

Homework Assignment #1

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January 27, 2020

Problems

1. (**Programming project**.) A very simple interest rate fitting framework, popular among economists (including the US Treasury Department) and some buy side firms is the *Nelson-Siegel model*. We consider the (currently observed) instantaneous forward rate $f_0(s)$, which in the following will be denoted by $y(s)$. In the NS model, $y(s)$ is assumed to have a parametric form given by:

$$y(s) = \beta_0 + \beta_1 e^{-\lambda s} + \beta_2 \lambda s e^{-\lambda s}, \tag{1}$$

where $\beta_0, \beta_1, \beta_2 \in \mathbb{R}$, and $\lambda > 0$ are the model parameters. The zero coupon rate $Y(T)$ for maturity T corresponding to the instantaneous forward rate $y(s)$ is given by

$$Y(T) = \frac{1}{T} \int_0^T y(s) ds. \tag{2}$$

(i) Show that

$$Y(T) = \beta_0 + \beta_1 \frac{1 - e^{-\lambda T}}{\lambda T} + \beta_2 \Big(\frac{1 - e^{-\lambda T}}{\lambda T} - e^{-\lambda T} \Big). \tag{3}$$

(ii) Get a snapshot of the benchmark *zero coupon* swap rates from Bloomberg for all available maturities $Y = 1$ year, $Y = 2$ years, through 30 years.

(iii) Formulate the curve fitting problem as a nonlinear least squares problem and use an optimization algorithm of your choice (Gauss-Newton, Levenberg-Marquardt, BFGS, ...) to calibrate the model parameters. Report the calibration error.

(iv) The decay rate parameter λ tends to calibrate poorly. For that reason, it is usually set to a positive value, and not calibrated through the optimizer. Come up with a reasonable choice of the value of λ and investigate how your choice impacts the calibration error.

(v) Investigate the stability of the parameters $\beta_0, \beta_1, \beta_2$ by comparing their values calibrated to the market snapshots on several different days.

You should notice that the calibration errors for the NS model are fairly large. For that reason, this model is never used on dealers' interest rate trading desks.

2. Loss given default (LGD) is a measure of the severity of a default, and is defined as the fraction of the face value of a bond that is not repaid to the investor. In other words,

$$LGD = (1 - R) \times F.$$

(i) Find the probability distribution of the LGD in Merton's model.

(ii) Implement the formula for the density function and plot it for $T = 10$ and $V_0 = 95, 99, 120, 140$ (as usual, $F = 100$).

3. Find a closed form expression for the recovery value $B^{\text{rec}}(t, T)$ in the Black and Cox model by evaluating the integral on page 39 of Lecture Notes #1. Note that all the integrals that you encounter during the computation are Gaussian, and you should be able to express the result in terms of the cumulative normal distribution function.

4. Consider the Cox model where the intensity follows the normal process

$$d\lambda(t) = \theta dt + \sigma dW(t),$$

with constant θ and $\lambda(0) = \lambda_0$.

(i) Calculate the survival probability $S(0, t)$ for this Cox process.

(ii) Design and implement in computer code an algorithm for simulating events from this Cox process.

(iii) Verify the correctness of your algorithm by comparing the estimated survival probabilities with the closed form formula. Use $\lambda_0 = 0.03$, $\theta = 0.005$, $\sigma = 0.008$, and a range of values of t, say $t = 1, 2, \ldots, 10$.

This assignment is due on February 17