Time Series Analysis

Homework assignment #1

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Problems

1. Is the following $AR(2)$ process :

$$X_t = 1.92 - 1.1X_{t-1} + 0.18X_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1), \tag{1}$$

covariance stationary? If so, calculate its mean and all autocovariances.

2. Carry out a detailed discussion of the conditions on β_1 and β_2 under which the $AR(2)$ time series

$$X_t = \alpha + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma^2), \tag{2}$$

is covariance stationary.

3. Consider the time series of daily returns on two ETFs tacking broad market indices: SPY (tracking S&P 500) and IWV (tracking Russel 3000) over the last 10 years, and let X_t denote the difference of these returns. Try to model X_t as an $AR(p)$ time series model, and discuss the results.

4. (**Bonus problem**) Show that the $AR(1)$ time series model

$$X_t = \alpha + \beta X_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma^2), \tag{3}$$

with $0 < \beta < 1$ can be viewed as a result of discretization of the continuous time Ornstein-Uhlenbeck process:

$$dX_t = \lambda(\mu - X_t)dt + \gamma dW_t, \tag{4}$$

where $\lambda, \gamma > 0$. Find the mapping between the parameters of these two models.

This assignment is due on September 5