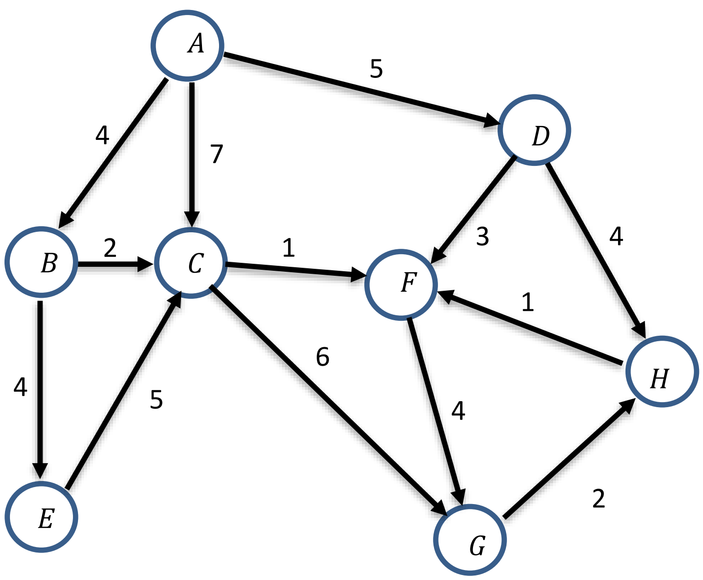
CSC 226 SUMMER 2020
ALGORITHMS AND DATA STRUCTURES II
ASSIGNMENT 4
UNIVERSITY OF VICTORIA

1. The table below purports to give the length of the shortest routes connecting the cities. It contains an error. Correct the table and draw the corresponding map. (Assume it's undirected).

	Providence	*Westerly*	*New London*	*Norwich*
Providence	0	53	54	48
Westerly	53	0	18	101
New London	54	18	0	12
Norwich	48	101	12	0

2. Consider the following weighted, directed graph G. There are 8 vertices and 13 edges. The edge list E is as follows:

$$\{(H,F),(G,H),(F,G),(C,G),(E,C),(C,F),(B,E),(B,C),(D,H),(D,F),(A,D),(A,C),(A,B)\}$$



The Bellman-Ford algorithm makes $|V| - 1 = 8 - 1 = 7$ passes through the edge list E. Each pass relaxes the edges in the order they appear in the edge list. As with Dijkstra's algorithm, we record the current best known cost $D[V]$ to reach each vertex V from the start vertex S. Initially $D[A] = 0$ and $D[V] = +\infty$ for all the other vertices $V \neq A$. Run Bellman-Ford on the given graph, starting at vertex A, and using the order of set E above, show me the contents of array $D[]$ after each iteration until there is no change in the array from one iteration to the next.

3. Consider a weighted, directed graph G with n vertices and its corresponding adjacency matrix M, where $M(i,j) = 0$ if $i = j$, $M(i,j) = weight((i,j))$ if edge (i,j) is in G and $M(i,j) = +\infty$ otherwise. Let

$$M^2 = \min\{M(i,1) + M(1,j), M(i,2) + M(2,j), \dots, M(i,n) + M(n,j)\}$$

where + is regular addition. If $M^2(i,j) = d$, what may we conclude about vertices i and j? What about M^k for any $1 \leq k \leq n$?

4. Consider the following baseball standings for one division. Use Network Flow to determine if Detroit and Toronto are eliminated from the playoffs at this time.

Team	wins	games left	vs. NY	vs. Bal	vs. Bos	vs. Tor	vs. Det
New York	75	28	-	4	10	9	5
Baltimore	71	28	4	-	5	11	8
Boston	69	27	10	5	-	2	10
Toronto	63	27	9	11	2	-	5
Detroit	49	28	5	8	10	5	-

5. Let N be a flow network, and let f be a flow for N. Prove that for any cut, χ, of N, the value of f is equal to the flow across cut χ, that is, $|f| = f(\chi)$. Use an induction proof on the number of vertices in set T of the cut. That is, the base case would be $|T| = 1$, where $T = \{t\}$.