

1.

a. Lists are vertical

0	1	2	3	4	5	6	7	8	9	10
	20			16	44	94	12		13	
				5	88	39	23			
					11					

b.

0	1	2	3	4	5	6	7	8	9	10
11	39	20	5	16	44	88	12	23	13	94

c.

0	1	2	3	4	5	6	7	8	9	10
20		16	11	39	44	88	12	23	13	94

d. Assuming the mod11 is applied last

0	1	2	3	4	5	6	7	8	9	10
11	23	20	16	44	5	94	12	88	13	39

2.

put(k,v):

i <- h(k)

j <- 0

while A[i] != NULL or A[i] = SpecialMarker do

if A[i].key = k then

A[i] <- (k,v) //replace the old value with same key

j <- j+1

i <- h(k) + j\*j

A[i] <- (k,v) //found a nice empty index

get(k):

i <- h(k)

j <- 0

while A[i] != NULL do

if (A[i].key = k) then

return A[i]

j <- j+1

i <- h(k) + j\*j

return NULL //not found

remove(k):

i <- h(k)

```

j <- 0
while A[i] != NULL do
  if (A[i].key = k) then
    temp <- A[i]
    A[i] <- SpecialMarker //a non-NULL key that will never be used by regular values to
    represent a removed element
    return temp
  j <- j+1
  i <- h(k) + j*j
return NULL //key not found

```

3.

**Algorithm** matrixDFS( $G, v, P$ )

**Input:** A graph  $G$ , with  $n$  vertices labeled  $0, \dots, n-1$ , represented as an adjacency matrix, a starting vertex  $v$ , and an array  $C$  of size  $n$  that is initially 0s

**Output:** An integer array of size  $n$ , containing the preorder labelling of the vertices.

1.  $P[0] \leftarrow v$
2.  $C[v] \leftarrow 1$
3. for  $i$  from 0 to  $n$  do
4.   if  $G[v][i]$  is not 0 then
5.     if  $C[i] = 1$  then skip
6.     Else append DFS( $G, i, C$ ) to  $P$
7. return  $P$

4.

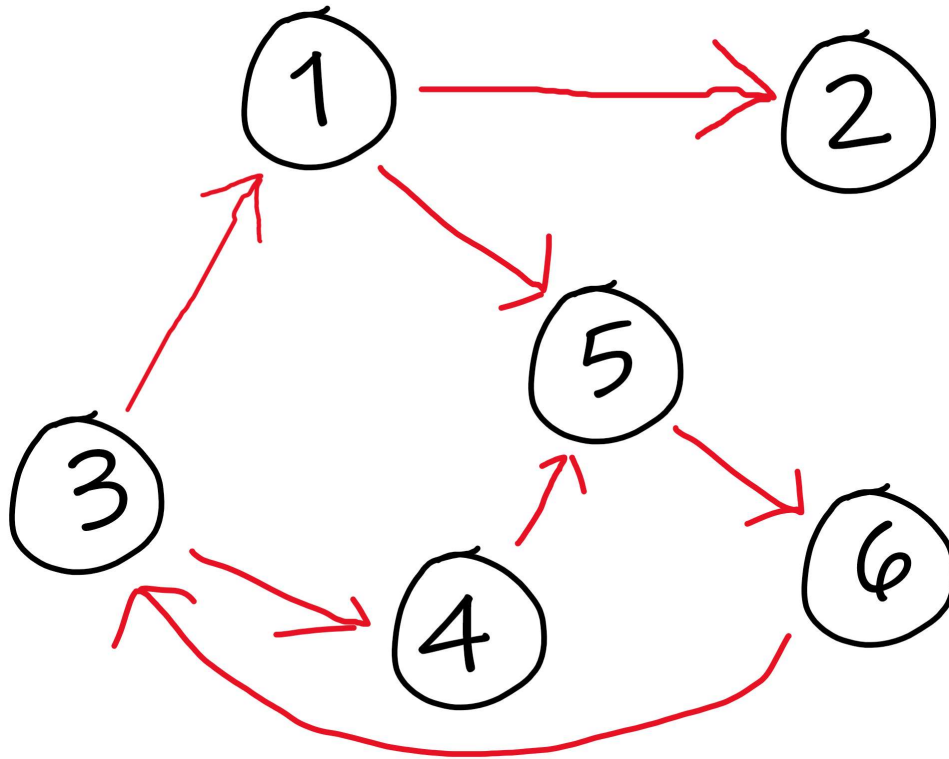
1. Takes 1 unit of time to append
2. Takes 1 unit of time
3. Loops  $n$  times per iteration
4. Takes 1 time per loop
5. Takes 1 unit of time to check
6. Recursive call
7. Takes 1 unit of time

This algorithm is  $O(n^2)$  in the best and worst case, because it will always conduct  $n$  loops over  $n$  iterations.

The adjacency list representation has a best/worst case of  $O(|V| + |E|)$ , where  $|V|$  is the number of vertices in the graph and  $|E|$  is the number of edges in the graph.

5.

a.



$G_0 =$

	1	2	3	4	5	6
1	0	1	0	0	1	0
2	0	0	0	0	0	0
3	1	0	0	1	0	0
4	0	0	0	0	1	0
5	0	0	0	0	0	1
6	0	0	1	0	0	0

Rows are from, columns are to.

b.

$G_1 =$

	1	2	3	4	5	6
1	0	1	0	0	1	0
2	0	0	0	0	0	0
3	1	1	0	1	1	0
4	0	0	0	0	1	0
5	0	0	0	0	0	1
6	0	0	1	0	0	0

$G_2$  has no changes since 2 has no outgoing edges

$G_3 =$

	1	2	3	4	5	6
1	0	1	0	0	1	0

<b>2</b>	0	0	0	0	0	0
<b>3</b>	1	1	0	1	1	0
<b>4</b>	0	0	0	0	1	0
<b>5</b>	0	0	0	0	0	1
<b>6</b>	1	1	1	1	1	0

$G_4$  has no changes since its only incoming edges, (3,4) and now (6,4), already have outgoing edges towards 5.

$G_5 =$

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>1</b>	0	1	0	0	1	1
<b>2</b>	0	0	0	0	0	0
<b>3</b>	1	1	0	1	1	1
<b>4</b>	0	0	0	0	1	1
<b>5</b>	0	0	0	0	0	1
<b>6</b>	1	1	1	1	1	0

$G_6 =$

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>1</b>	0	1	1	1	1	1
<b>2</b>	0	0	0	0	0	0
<b>3</b>	1	1	0	1	1	1
<b>4</b>	1	1	1	0	1	1
<b>5</b>	1	1	1	1	0	1
<b>6</b>	1	1	1	1	1	0