Summary of log likelihood in statsmodel:
CDF
$\frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)}$

Log-likelihood of the multinomial logit model.
$\ln L = \sum_{i=1}^{n} \sum_{j=0}^{J} d_{ij} \ln \left(\frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)} \right)$

Log-likelihood of the multinomial logit model for each observation.
$\ln L_i = \sum_{j=0}^{J} d_{ij} \ln \left(\frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)} \right)$

Score matrix for multinomial logit model log-likelihood.
$\frac{\partial \ln L}{\partial \beta_j} = \sum_i \left(d_{ij} - \frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)} \right) x_i$

Jacobian matrix for multinomial logit model log-likelihood.
$\frac{\partial \ln L_i}{\partial \beta_j} = \left(d_{ij} - \frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)} \right) x_i$

Multinomial logit Hessian matrix of the log-likelihood.
$\frac{\partial^2 \ln L}{\partial \beta_j \partial \beta_l} = -\sum_{i=1}^{n} \frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)} \left[\mathbf{1}\left(j = l\right) - \frac{\exp\left(\beta_l' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)} \right] x_i x_l'$
where $\mathbf{1}\left(j = l\right)$ equals 1 if $j = l$ and 0 otherwise.

Summary of equivalent values for the bid-choice function from Hurtubia:
CDF (for each i HH)
$\frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{L} \exp\left(\beta_k' x_i\right)} \frac{\exp\left(\frac{-R_i - a - \gamma r_i}{2\sigma^2}\right)}{\sqrt{2\pi\sigma^2}}$

Log-likelihood of the BC model.
$\ln L = \sum_{i=1}^{H} \sum_{j=0}^{L} d_{ij} \ln \left(\frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{L} \exp\left(\beta_k' x_i\right)} \frac{\exp\left(\frac{-R_i - a - \gamma r_i}{2\sigma^2}\right)}{\sqrt{2\pi\sigma^2}} \right)$

Log-likelihood of the BC model for each HH observation.

$$\ln L_i = \sum_{j=0}^{L} d_{ij} \ln \left(\frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{L} \exp\left(\beta_k' x_i\right)} \frac{\exp\left(\frac{-R_i - a - \gamma r_i}{2\sigma^2}\right)}{\sqrt{2\pi\sigma^2}} \right)$$

Score matrix for BC model log-likelihood.

$$\frac{\partial \ln L}{\partial \beta_j} = \sum_{i=1}^{H} \left(d_{ij} - \left(\frac{d_{ij}\gamma}{2\sigma^2} + 1 \right) \frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{J} \exp\left(\beta_k' x_i\right)} \right) x_i$$

$$\frac{\partial \ln L}{\partial \sigma} = \sum_{i=1}^{H} \left(d_{ij} \left(R_i - a - \gamma \ln \left(\sum_j \exp\left(\beta_j' x_i\right) \right) \right) - \frac{2}{\sigma} \right)$$

$$\frac{\partial \ln L}{\partial a} = \sum_{i=1}^{H} \left(\frac{-d_{ij}}{2\sigma^2} \right)$$

$$\frac{\partial \ln L}{\partial \gamma} = \sum_{i=1}^{H} \left(\frac{-d_{ij} \ln\left(\sum_j \exp\left(\beta_j' x_i\right)\right)}{2\sigma^2} \right)$$

Jacobian matrix for BC model log-likelihood.

$$\frac{\partial \ln L_i}{\partial \beta_j} = \left(d_{ij} - \left(\frac{d_{ij}\gamma}{2\sigma^2} + 1 \right) \frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{L} \exp\left(\beta_k' x_i\right)} \right) x_i$$

$$\frac{\partial \ln L_i}{\partial \sigma} = \left(d_{ij} \left(R_i - a - \gamma \ln \left(\sum_j \exp\left(\beta_j x_i\right) \right) \right) - \frac{2}{\sigma} \right)$$

$$\frac{\partial \ln L_i}{\partial a} = \left(\frac{-d_{ij}}{2\sigma^2} \right)$$

$$\frac{\partial \ln L_i}{\partial \gamma} = \left(\frac{-d_{ij} \ln\left(\sum_j \exp\left(\beta_j' x_i\right)\right)}{2\sigma^2} \right)$$

BC Hessian matrix of the log-likelihood.

$$\frac{\partial^2 \ln L}{\partial \beta_j \partial \beta_l} = -\sum_{i=1}^{H} \left(\frac{d_{ij}\gamma}{2\sigma^2} + 1 \right) \frac{\exp\left(\beta_j' x_i\right)}{\sum_{k=0}^{L} \exp\left(\beta_k' x_i\right)} \left[\mathbf{1}\left(j = l\right) - \frac{\exp\left(\beta_l' x_i\right)}{\sum_{k=0}^{L} \exp\left(\beta_k' x_i\right)} \right] x_i x_l'$$

where $\mathbf{1}\left(j = l\right)$ equals 1 if $j = l$ and 0 otherwise.

$$\frac{\partial^2 \ln L}{\partial \beta_j \partial \sigma} = \sum_{i=1}^{H} \left(\frac{\gamma d_{ij} x_i}{\sigma^3} \right) \frac{\exp\left(\beta_j' x_i\right)}{\sum_j \exp\left(\beta_j' x_i\right)}$$

$$\frac{\partial^2 \ln L}{\partial \beta_j \partial a} = \textstyle\sum_{i=1}^{H} 0$$

$$\frac{\partial^2 \ln L}{\partial \beta_j \partial \gamma} = \textstyle\sum_{i=1}^{H} \left(\frac{-d_{ij}}{2\sigma^2}\right) \frac{\exp\left(\beta_j' x_i\right)}{\sum_j \exp\left(\beta_j' x_i\right)}$$

$$\frac{\partial^2 \ln L}{\partial \sigma \partial a} = \textstyle\sum_{i=1}^{H} \frac{d_{ij}}{\sigma^3}$$

$$\frac{\partial^2 \ln L}{\partial \sigma \partial \gamma} = \textstyle\sum_{i=1}^{H} \frac{d_{ij} \ln\left(\sum_j \beta_j' x_i\right)}{\sigma^3}$$

$$\frac{\partial^2 \ln L}{\partial a \partial \gamma} = \textstyle\sum_{i=1}^{H} 0$$