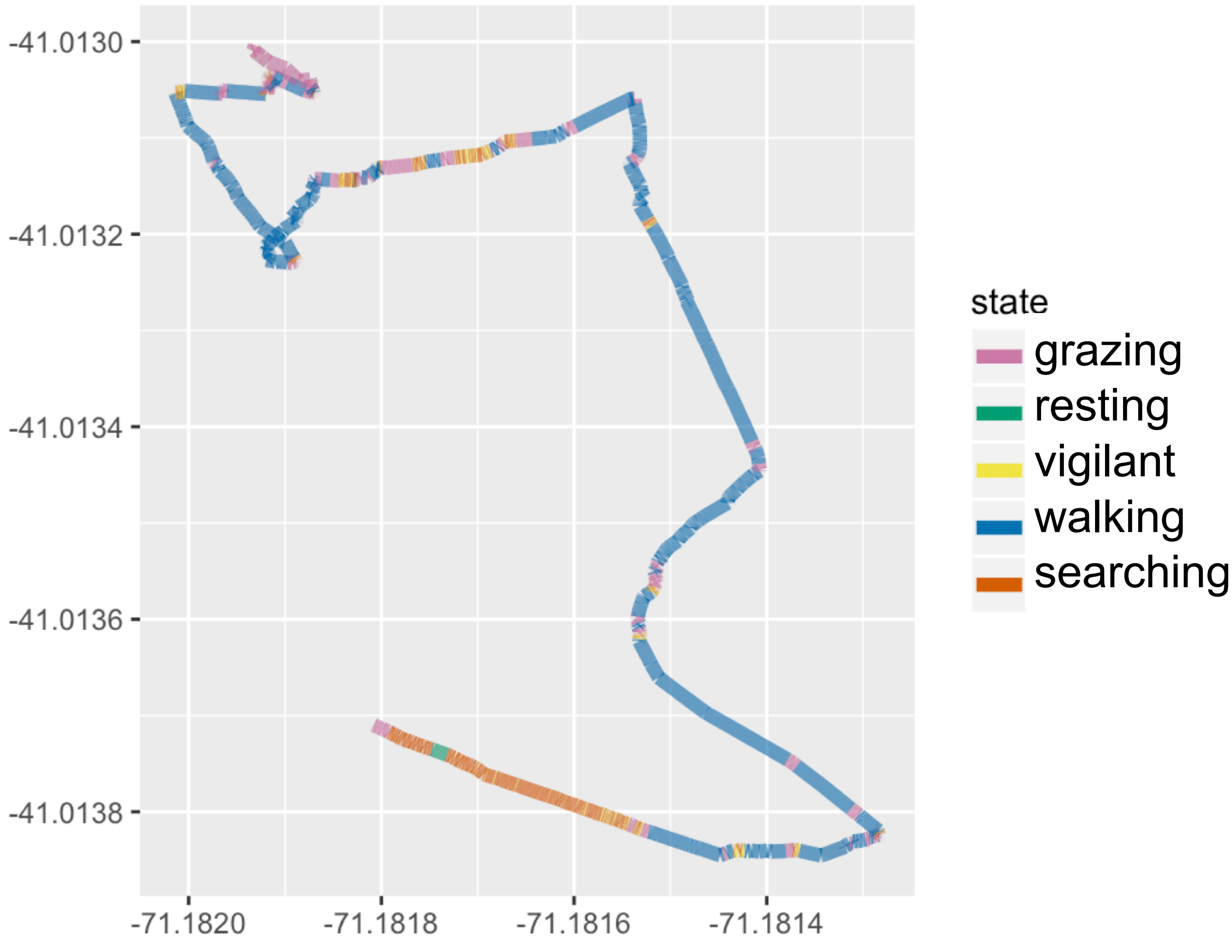
Discrete-time models

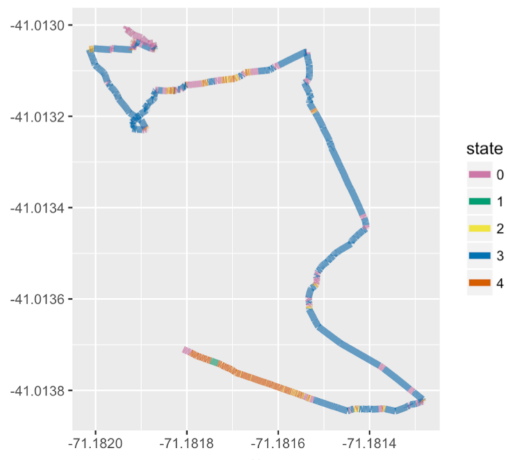
Movement discretized as Steps and Turns



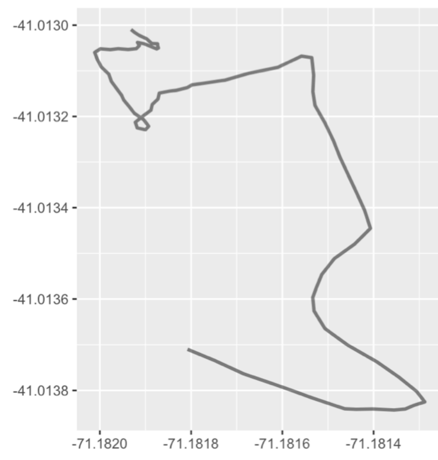
di Virgilio et al. 2018
PeerJ 6:e4867

Time scales

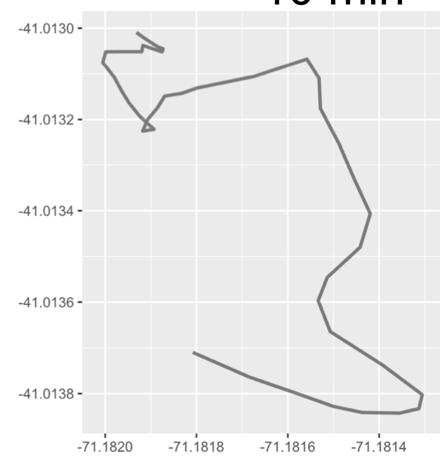
original



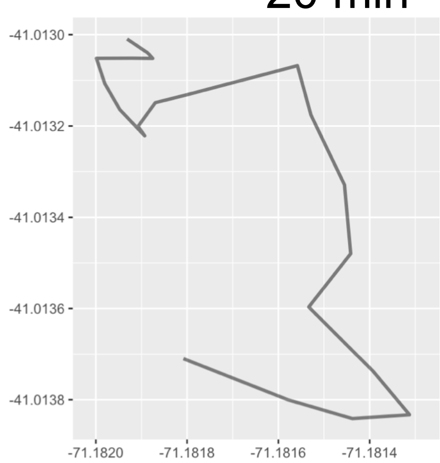
5 min



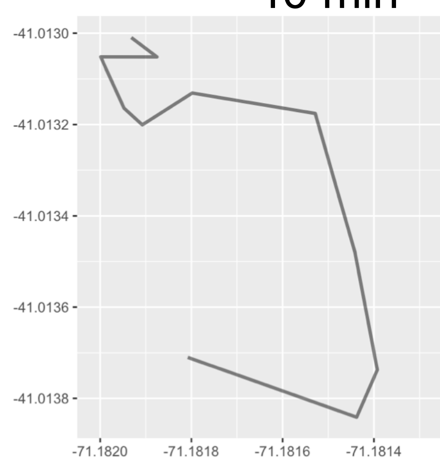
10 min



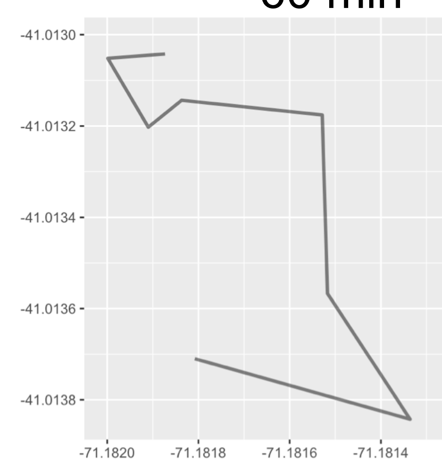
20 min



40 min



60 min



A time series

Assuming:

- Very little or no error on location

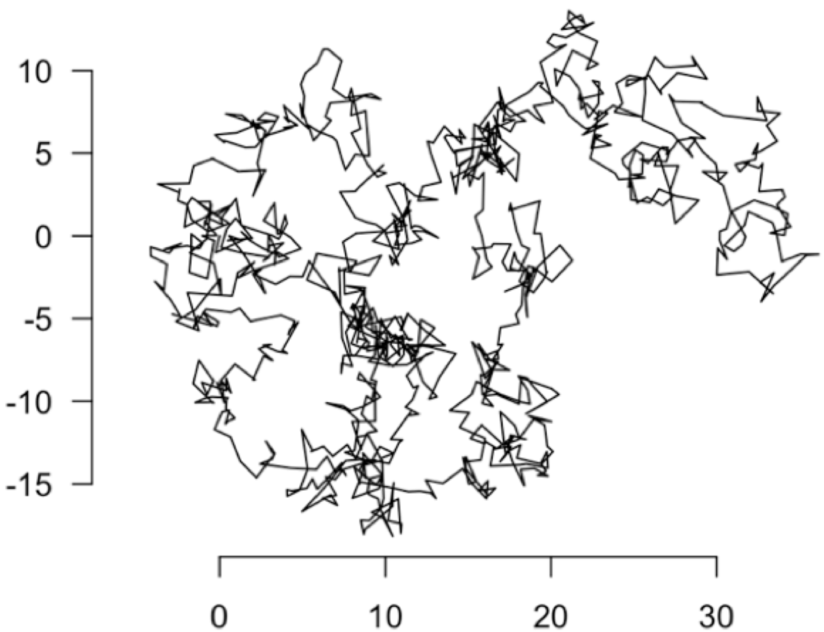
- Data collected at the **appropriate** and **fixed** temporal scale

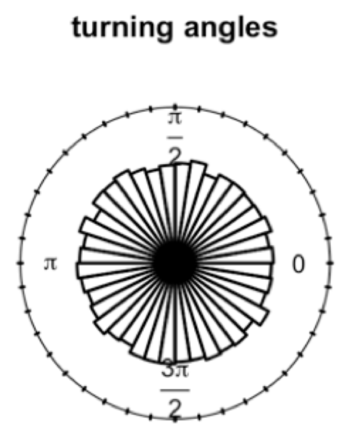
- Example:

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\varepsilon}_t,$$
$$\boldsymbol{\varepsilon}_t \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma})$$
$$\boldsymbol{\Sigma} \equiv \sigma^2\mathbf{I},$$
$$\boldsymbol{\mu}_t \sim \mathrm{N}(\boldsymbol{\mu}_{t-1}, \sigma^2\mathbf{I})$$

Steps and Turns

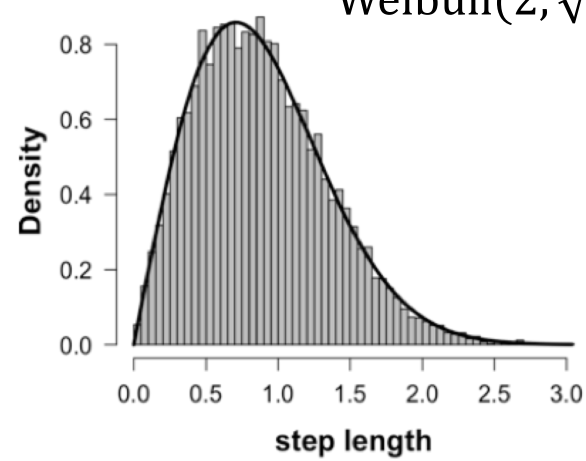
$$\boldsymbol{\mu}_t \sim \mathrm{N}(\boldsymbol{\mu}_{t-1}, \sigma^2\mathbf{I})$$





$\mathrm{Uniform}(-\pi, \pi)$

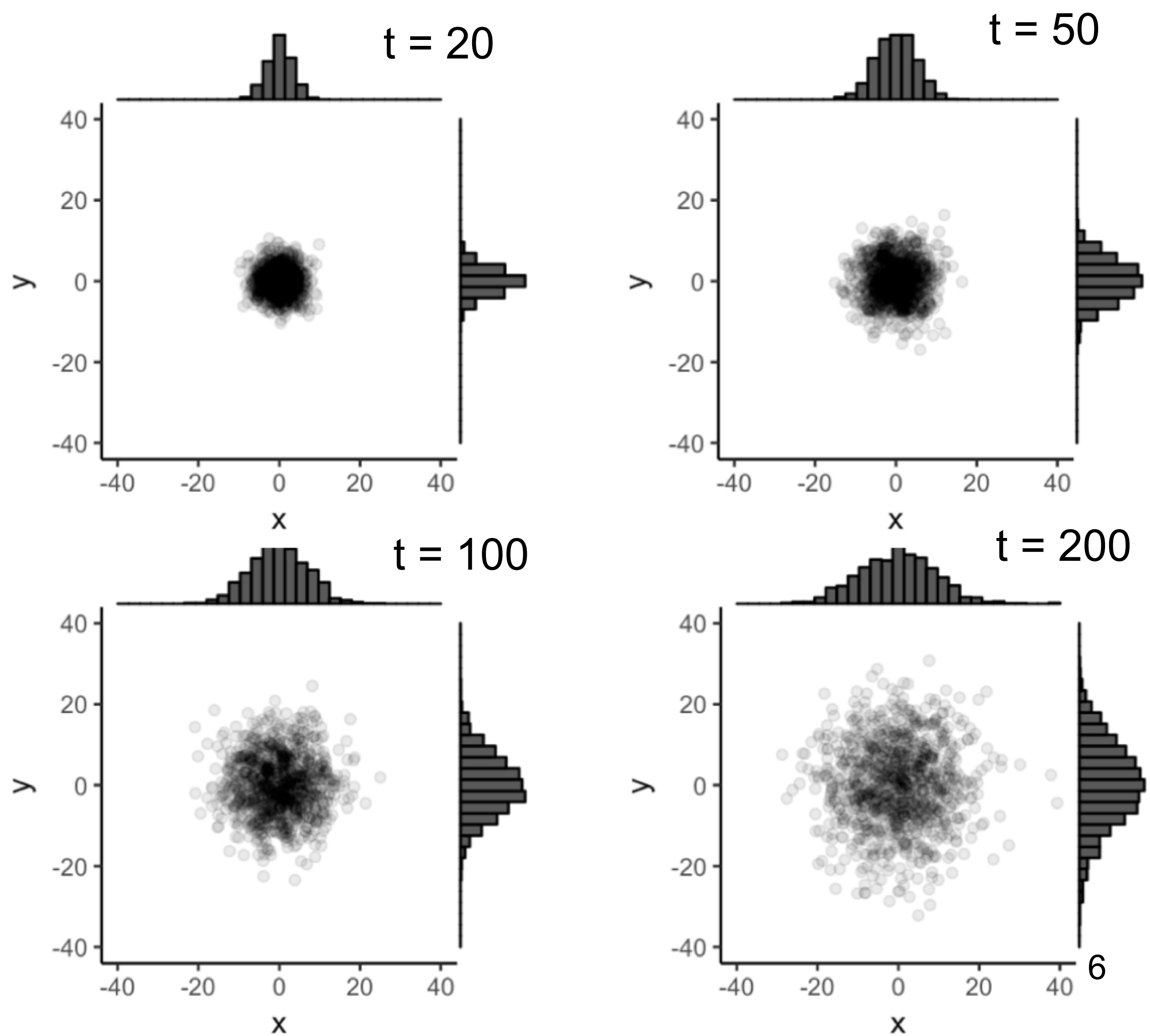
$\mathrm{Weibull}(2, \sqrt{2\sigma^2})$



Changes with time

$$\boldsymbol{\mu}_t \sim \mathrm{N}(\boldsymbol{\mu}_{t-1}, \sigma^2\mathbf{I})$$

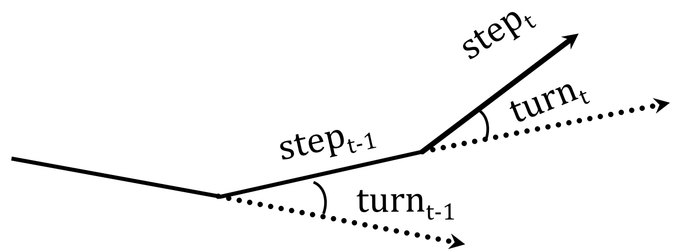
$$\boldsymbol{\mu}_0 = 0{,}0$$
$$\sigma^2 = 0.5$$
$$n = 1000$$



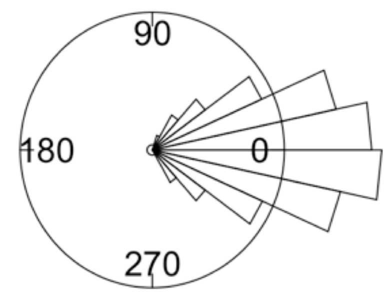
Some Types of Random Walks

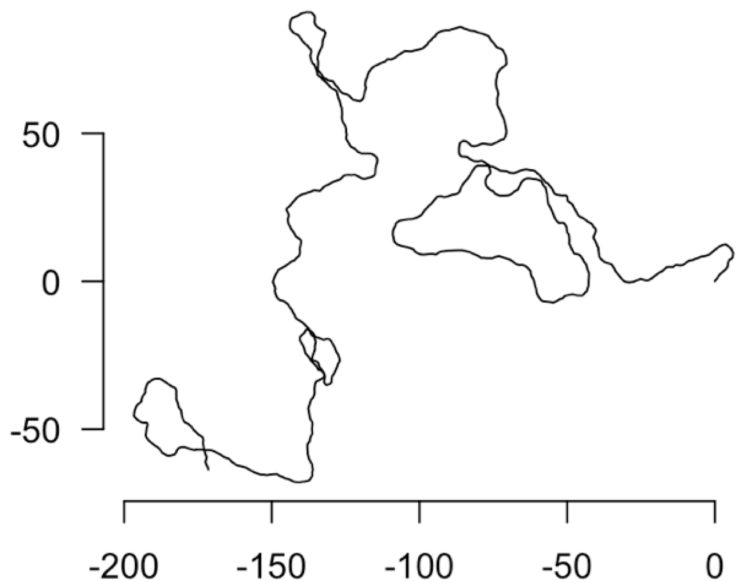
- By Type of turns:
  - Correlated Random Walk
  - Biased Random Walk
  - Biased Correlated Random Walk
- By distribution of steps
  - Levy Walk
  - Truncated Levy Walk

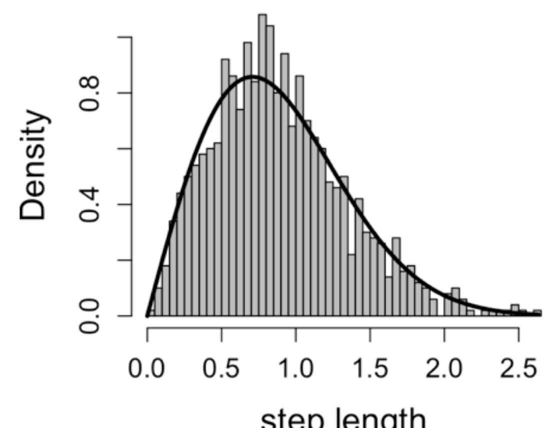
Correlated Random Walk



turning angles



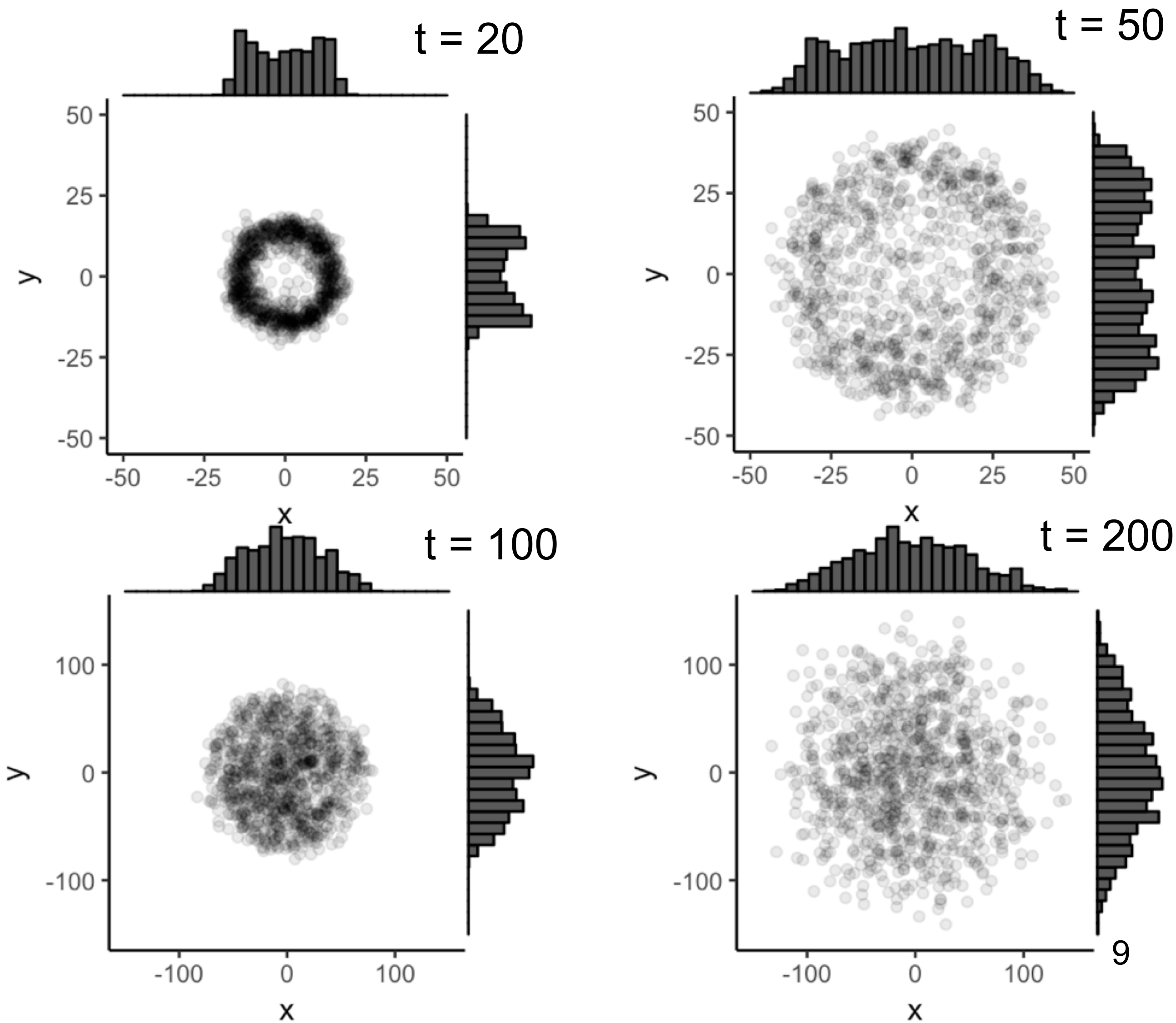




Changes with time (CRW)

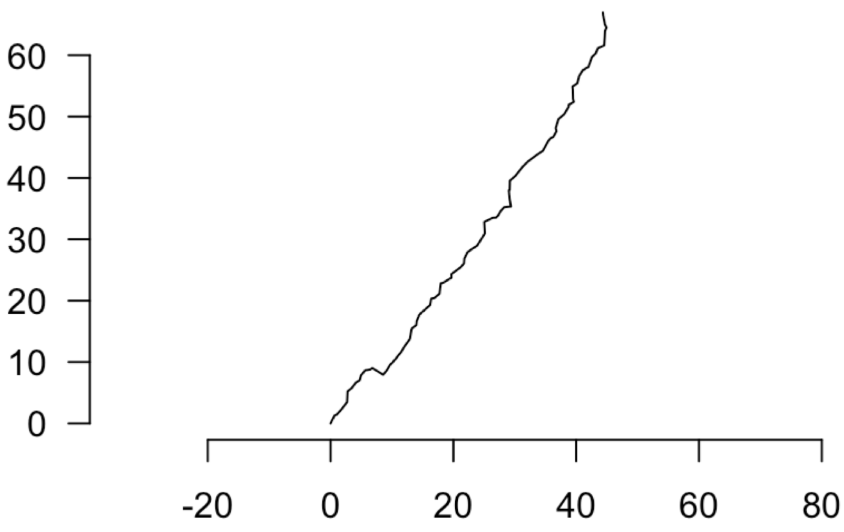
step ~ Weibull(a, b)
turn ~ von Mises(m, k)

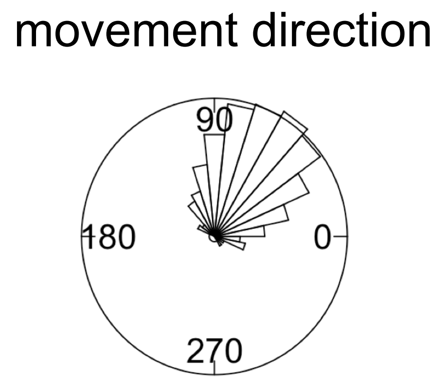
a = 1
b = 2
m = 0
k = 10
n = 1000

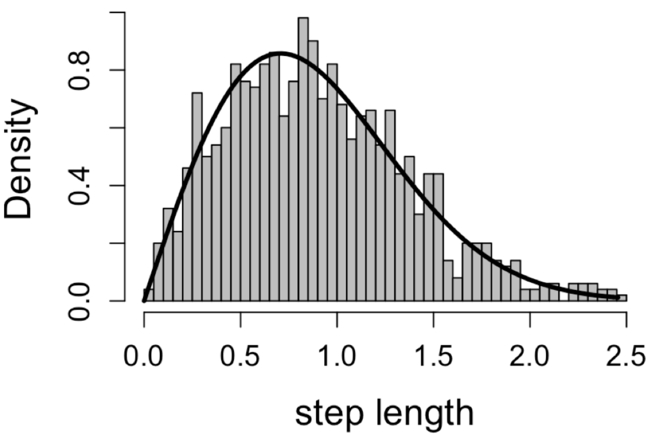


Biased Random Walk

step ~ Weibull(2, 1)
direction ~ von Mises(π/3 , 5)



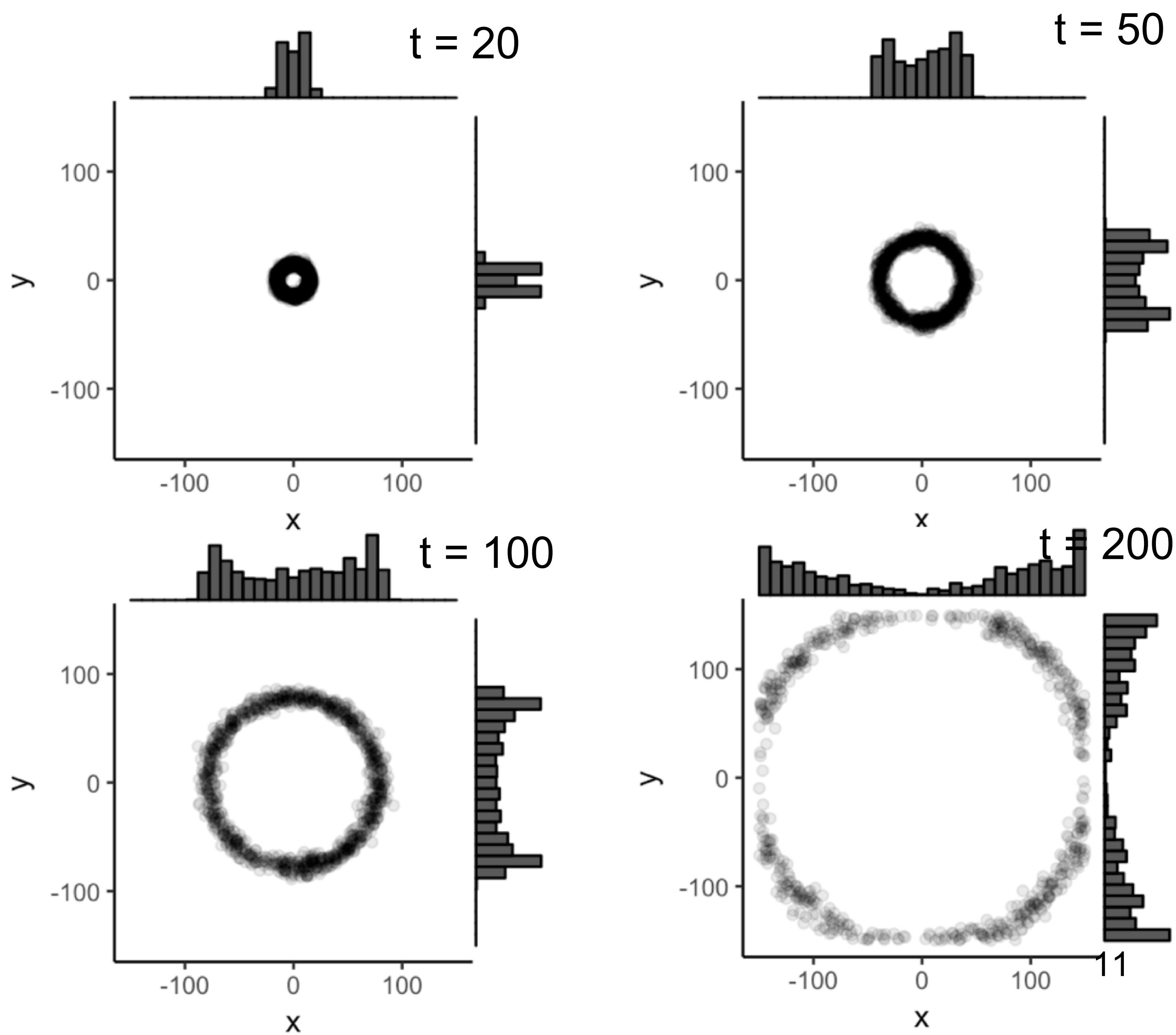




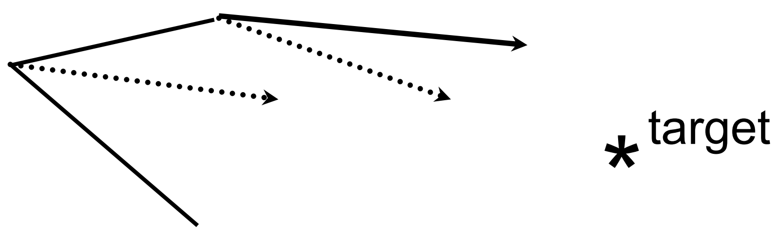
BRW, changes with time

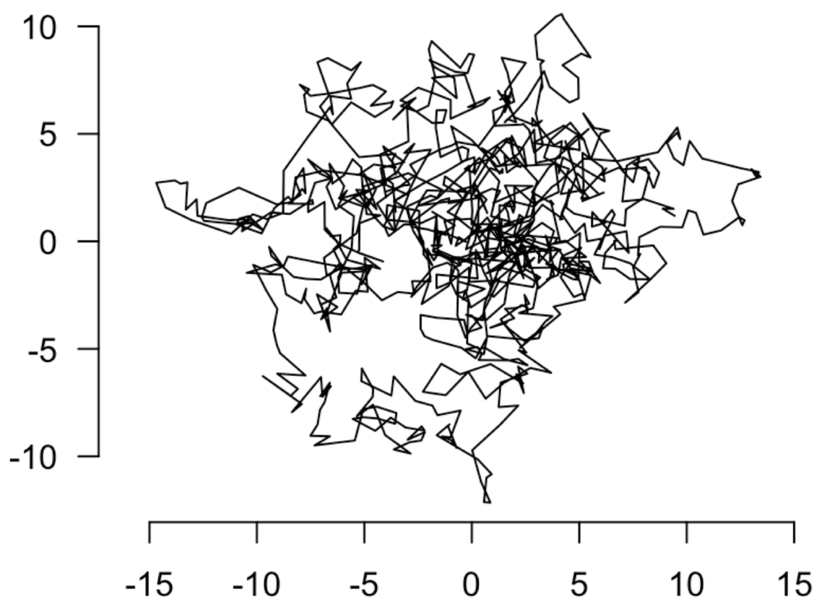
$\text{step} \sim \text{Weibull}(a, b)$
$\text{dir} \sim \text{von Mises}(m, k)$

$a = 1$
$b = 2$
$m \sim \text{Unif}(0, 2)$
$k = 5$
$n = 1000$

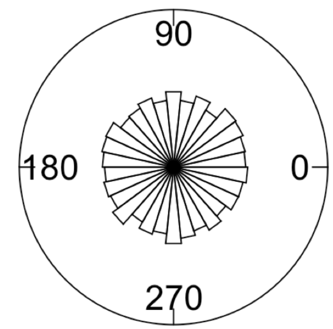


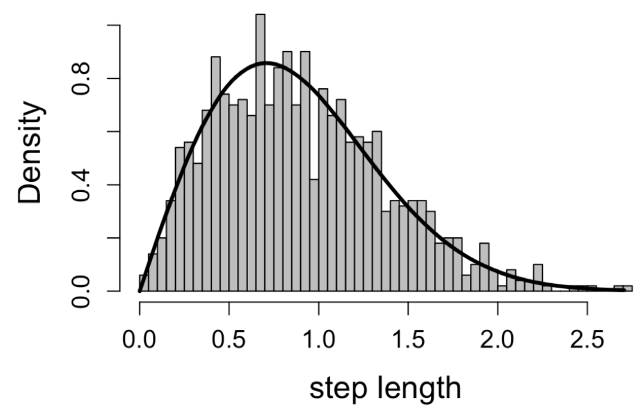
Centrally Biased RW





turning angles

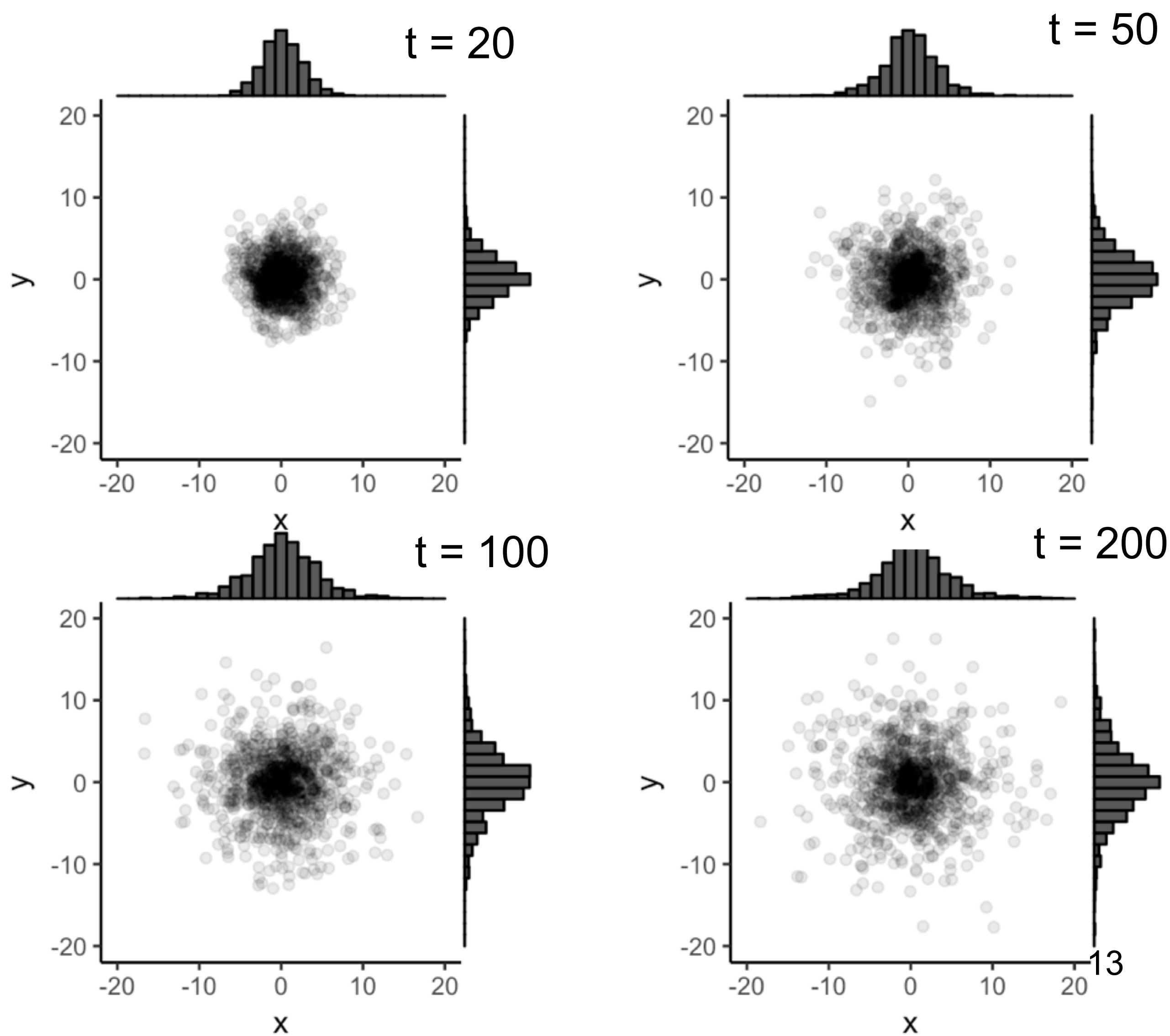




Centrally Biased RW

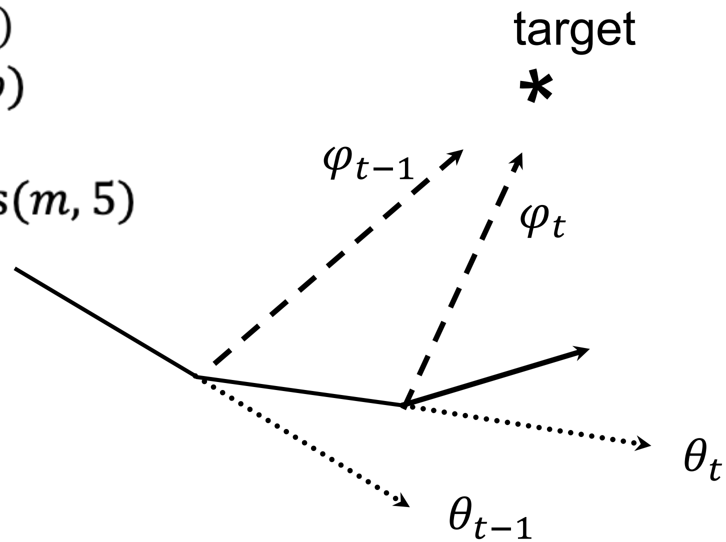
step ~ Weibull(a, b)
dir ~ von Mises(m, k)

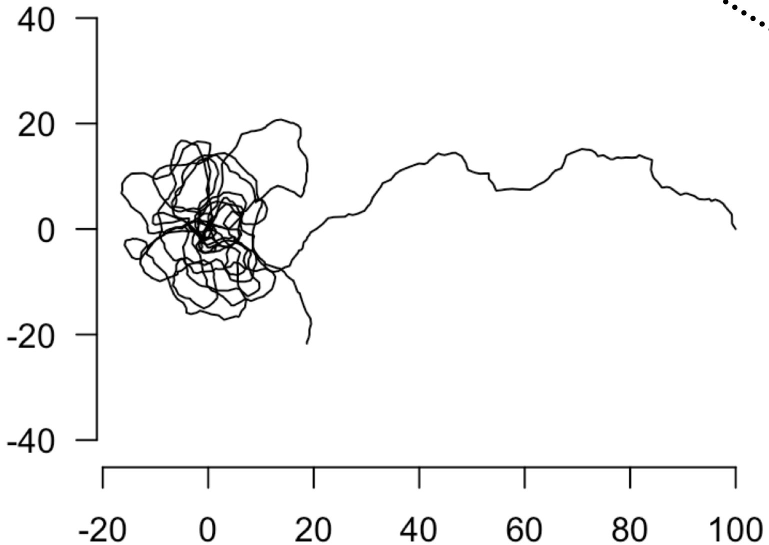
$\mathbf{w} = \mathbf{H} - \boldsymbol{\mu}_{t-1}$
$m = \text{atan2}(w_2, w_1)$



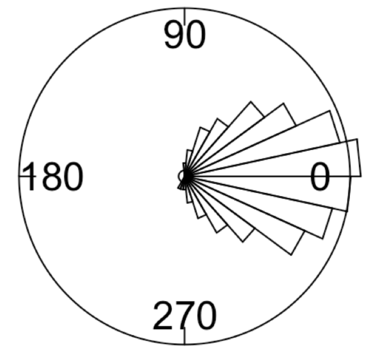
Biased Correlated RW

step ~ Weibull(2, 1)
$v = \sin(\theta) + \beta \sin(\varphi)$
$w = \cos(\theta) + \beta\cos(\varphi)$
$m = atan2(v, w)$
direction ~ von Mises$(m, 5)$

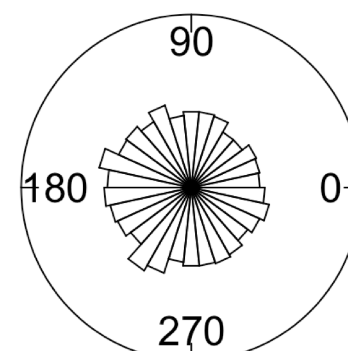




turning angles



direction



Velocity models

$$\mathbf{v}_t \equiv \boldsymbol{\mu}_t - \boldsymbol{\mu}_{t-1}$$

$$\mathbf{v}_t \sim N(\mathbf{0}, \sigma_\epsilon^2 \mathbf{I}) \longrightarrow \boldsymbol{\mu}_t \sim N(\boldsymbol{\mu}_{t-1}, \sigma_\epsilon^2 \mathbf{I})$$ no directional preference

$$\mathbf{v}_\mathrm{t} = \mathbf{M}\mathbf{v}_{\mathrm{t}-1} + \boldsymbol{\epsilon}_t$$ VAR(1) on velocity

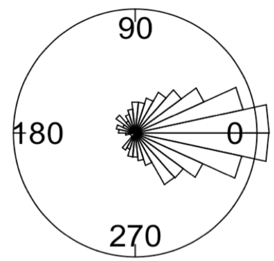
$$\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \sigma_\varepsilon^2 \mathbf{I})$$

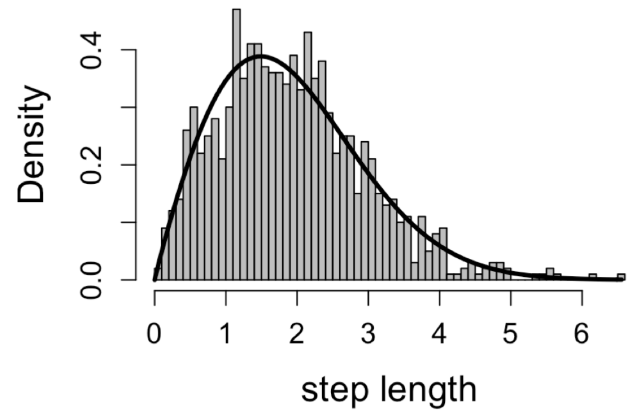
$$\mathbf{M} \equiv \gamma \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

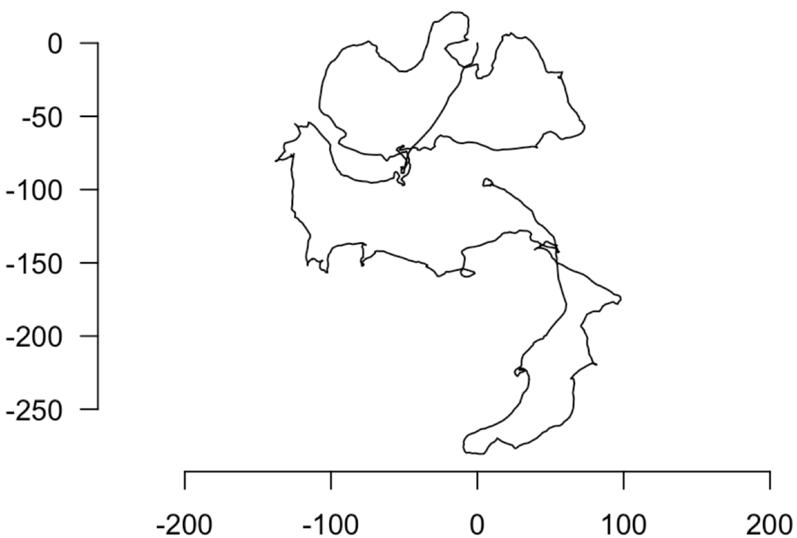
Jonsen, I. D., Flemming, J. M. and Myers, R. A. (2005)
Robust state–space modeling of animal movement data. Ecology, 86: 2874-2880

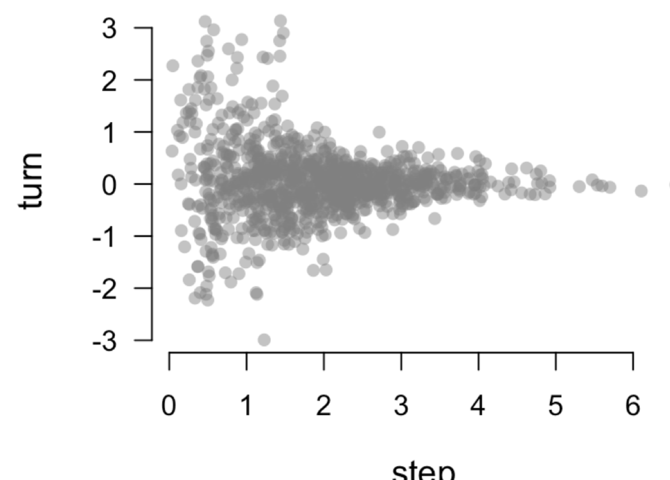
Velocity Models

$$\theta = 0$$
$$\gamma = 0.9$$



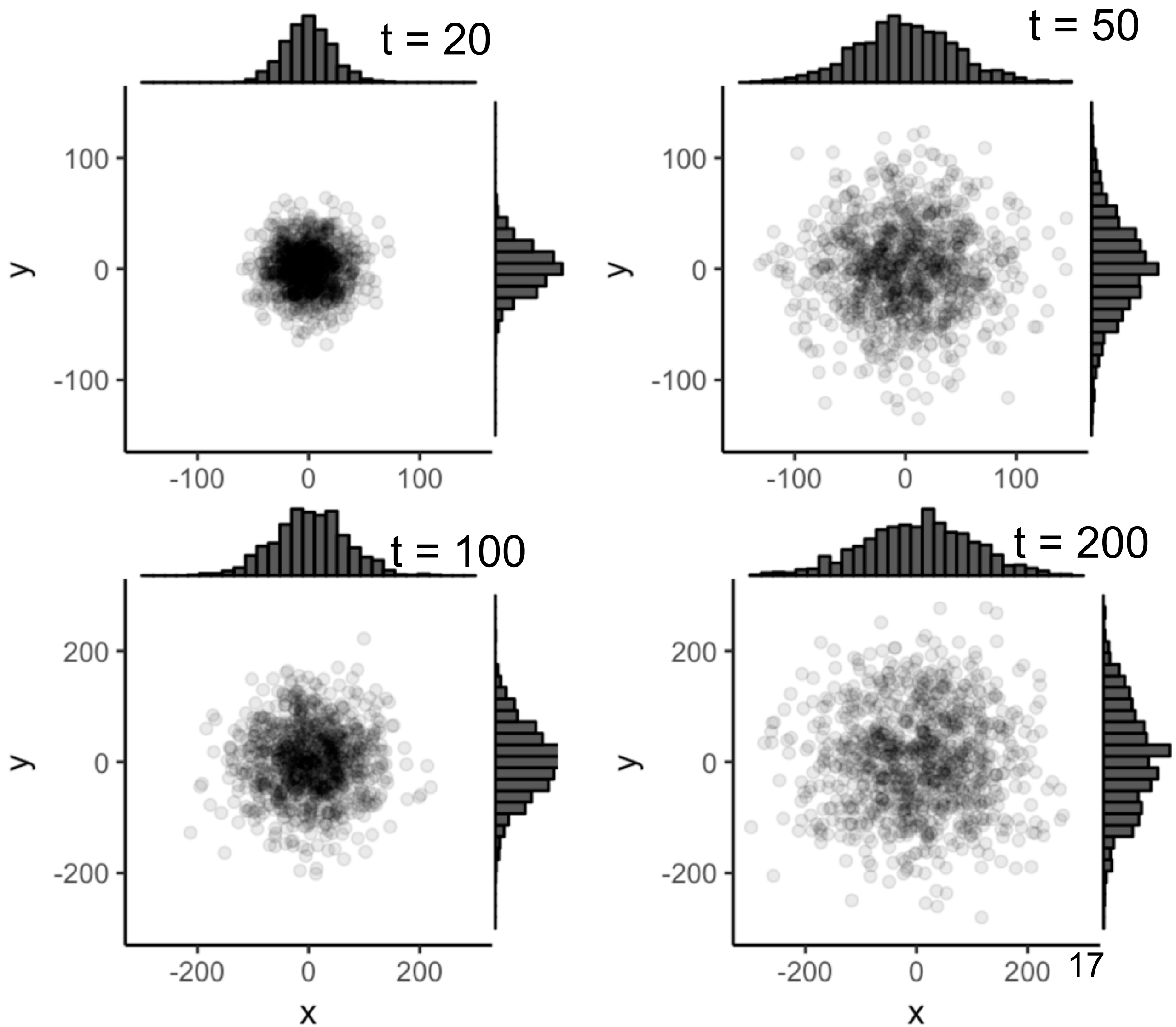




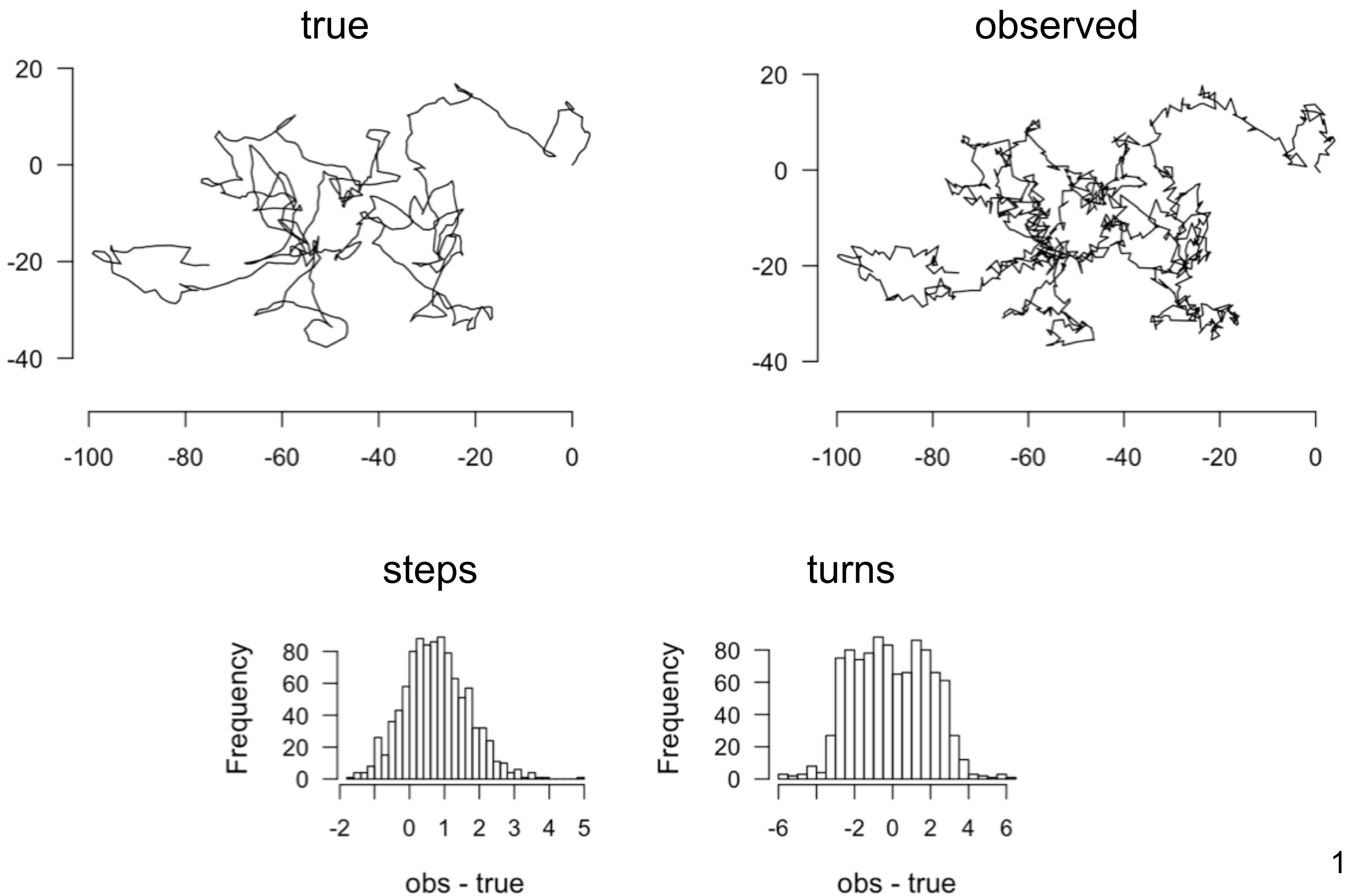


Changes with time

$\theta = 0$
$\gamma = 0.9$

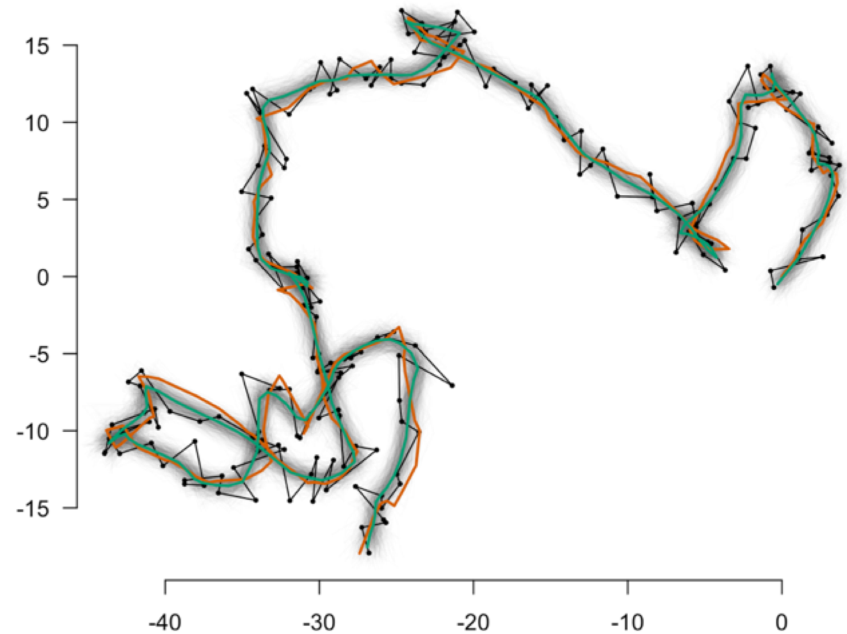


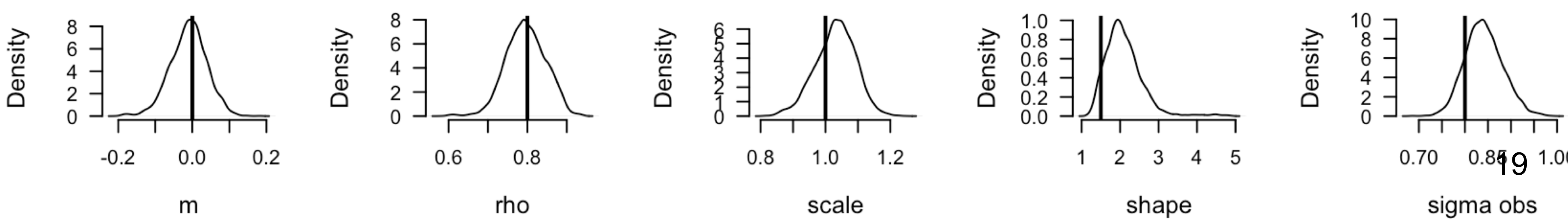
CRW with measurement error



Measurement Error

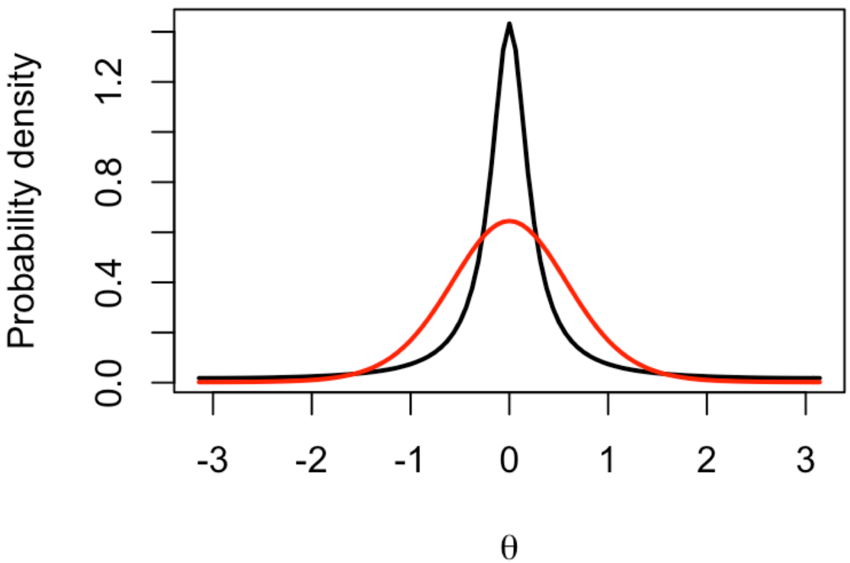
```
transformed parameters{
  vector[T] x;
  vector[T] y;
  x[1] = x1;
  y[1] = y1;
  for(i in 2:T){
    x[i] = x[i-1]+cos(h[i])*steps[i-1];
    y[i] = y[i-1]+sin(h[i])*steps[i-1];
    }
}
model {
  for(i in 2:T){
    steps[i-1] ~ weibull(shape, scale);
    h[i] ~ wrappedCauchy(rho, h[i-1] + m);
    }

    for (t in 2:T) {
      xo[t] ~ normal(x[t], sigma);
      yo[t] ~ normal(y[t], sigma);
      }
}
```

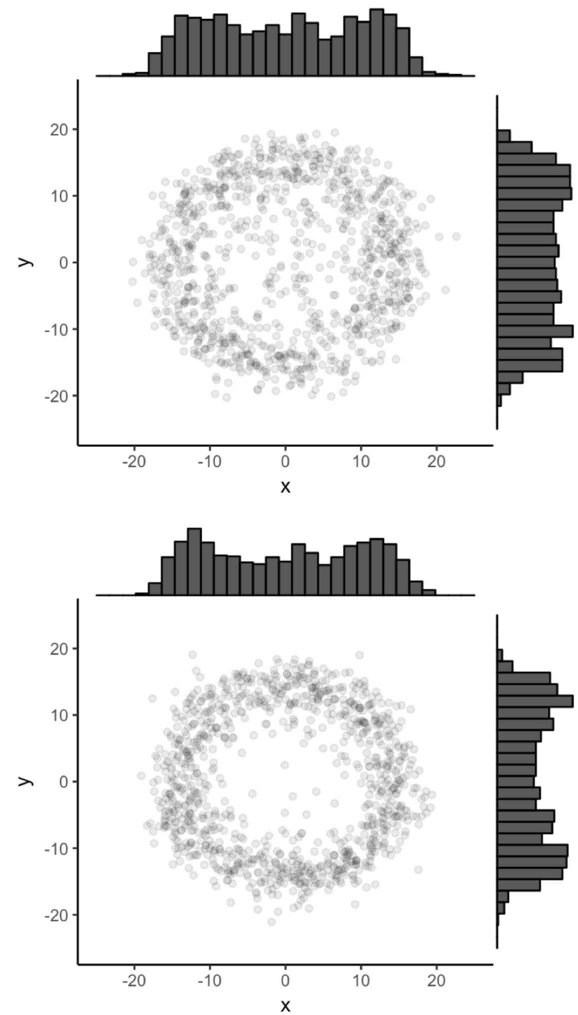




What distribution to use?

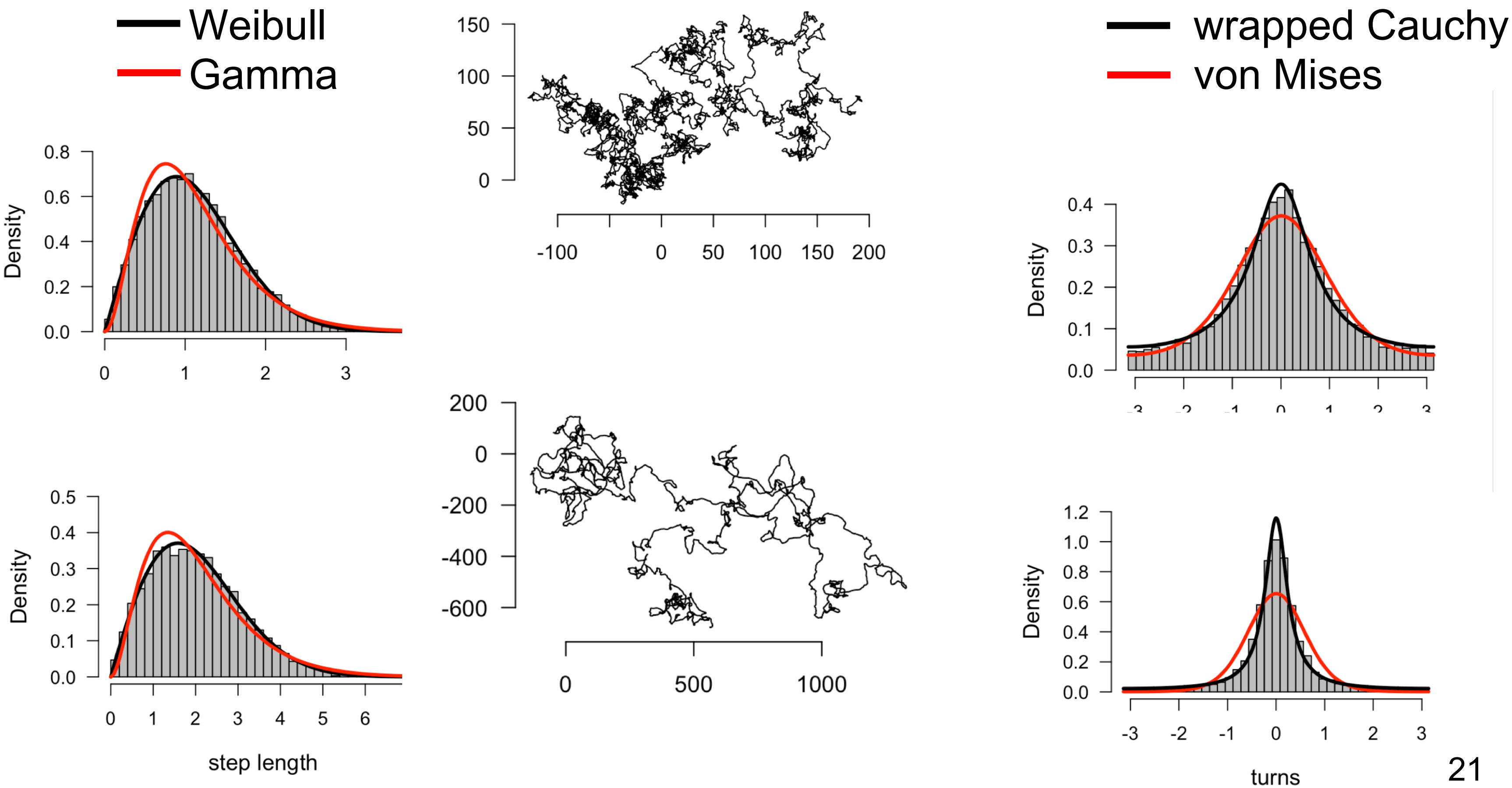
wrapped Cauchy has fatter tails than von Mises



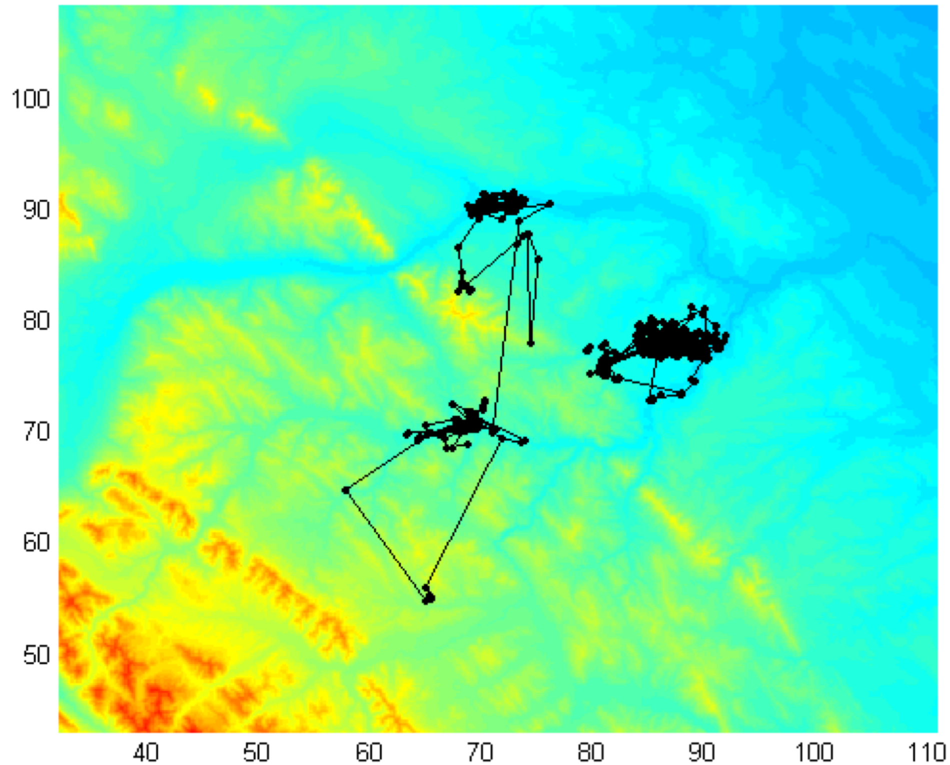


What distribution to use?

Simulations with a velocity model



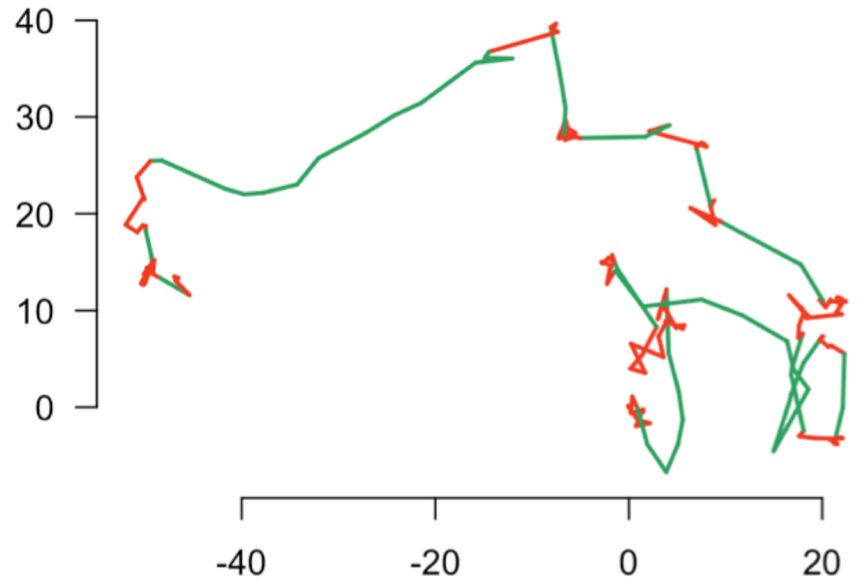
Animals Don't Always Move in the Same Way…



kilometers

State-dependent movement models

Different “states” have different movement characteristics and parameters



Blackwell (2003) *Biometrika*, Morales et al. (2004) *Ecology*, Jonsen et al (2005) *Ecology*, McClintock et al. (2012) *Ecological Monographs,* Langrock et al. (2012) *Ecology*

Two movement states

$$r_t \sim \text{Weibull}(a_t, b_t),$$

$$\theta_t \sim \text{wCauchy}(m_t, \rho_t)$$

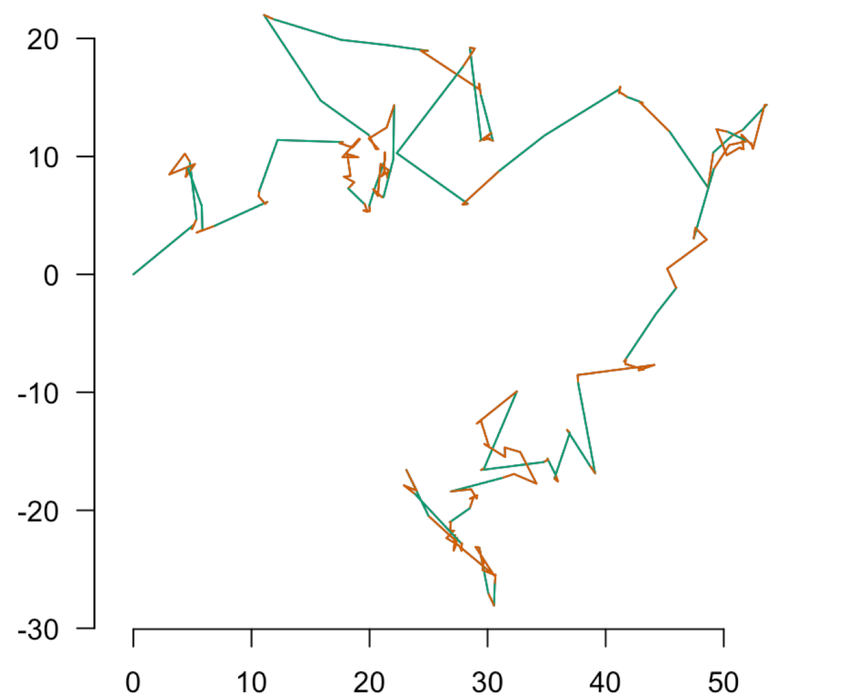
$$a_t = \begin{cases} a_1 & \text{if } z_t = 1 \\ a_2 & \text{if } z_t = 0 \end{cases},$$

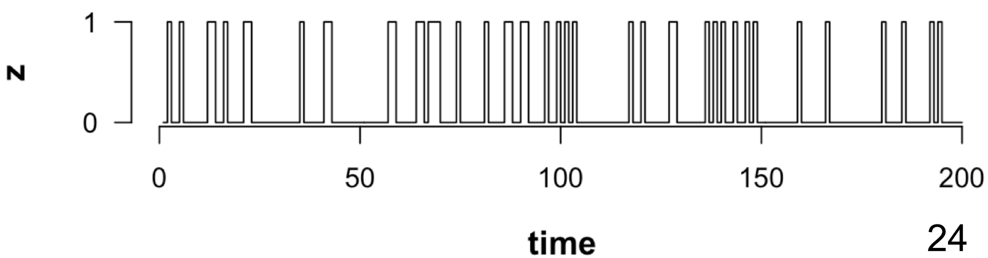
$$b_t = \begin{cases} b_1 & \text{if } z_t = 1 \\ b_2 & \text{if } z_t = 0 \end{cases},$$

$$m_t = \begin{cases} m_1 & \text{if } z_t = 1 \\ m_2 & \text{if } z_t = 0 \end{cases},$$

$$\rho_t = \begin{cases} \rho_1 & \text{if } z_t = 1 \\ \rho_2 & \text{if } z_t = 0 \end{cases},$$

$$z_t \sim \text{Bernoulli}(p)$$

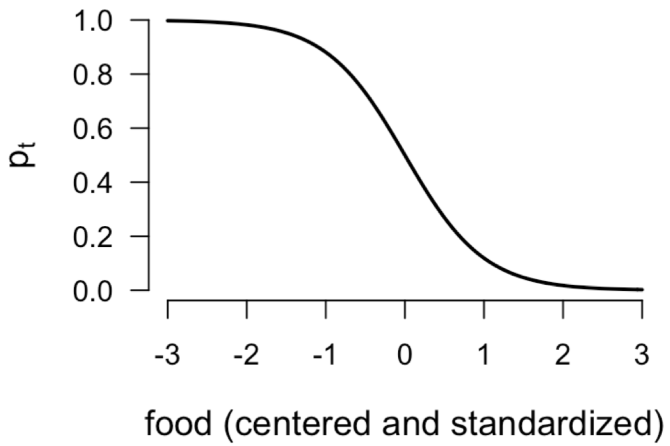


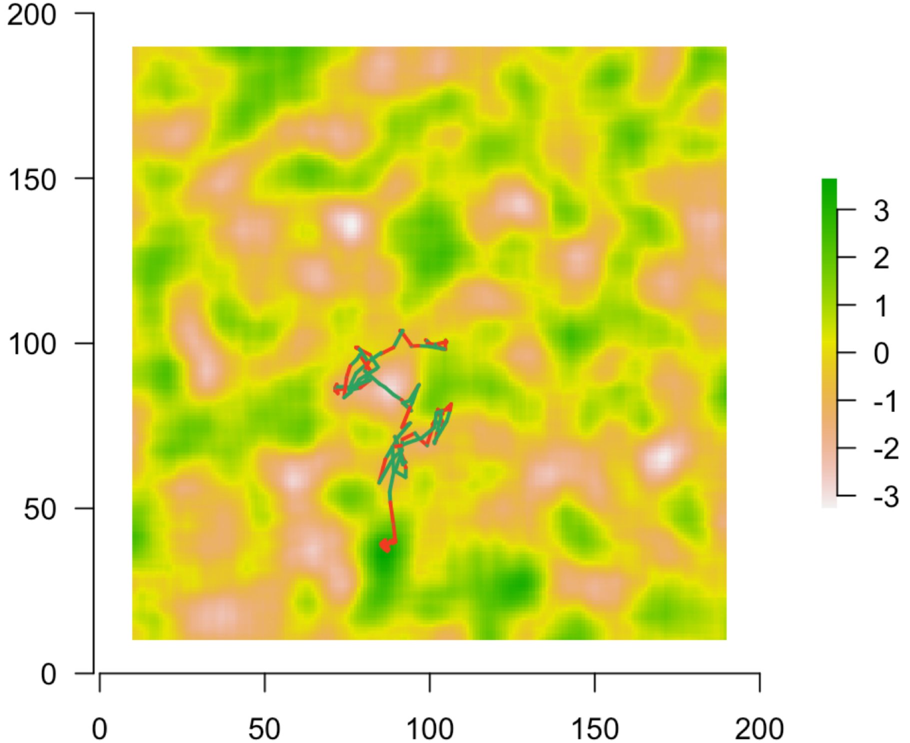


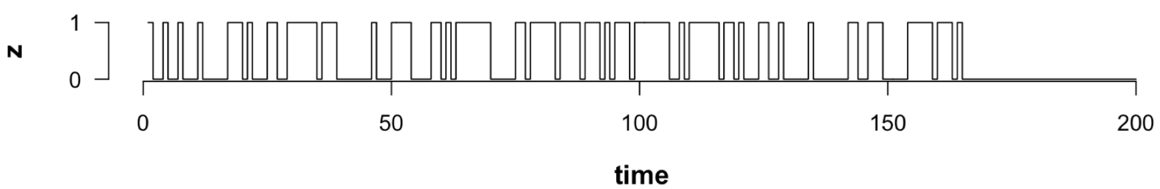
Covariates on state probability

$$z_t \sim \text{Bernoulli}(p_t)$$

$$\text{logit}(p_t) = \boldsymbol{x}_t'\boldsymbol{\beta}$$

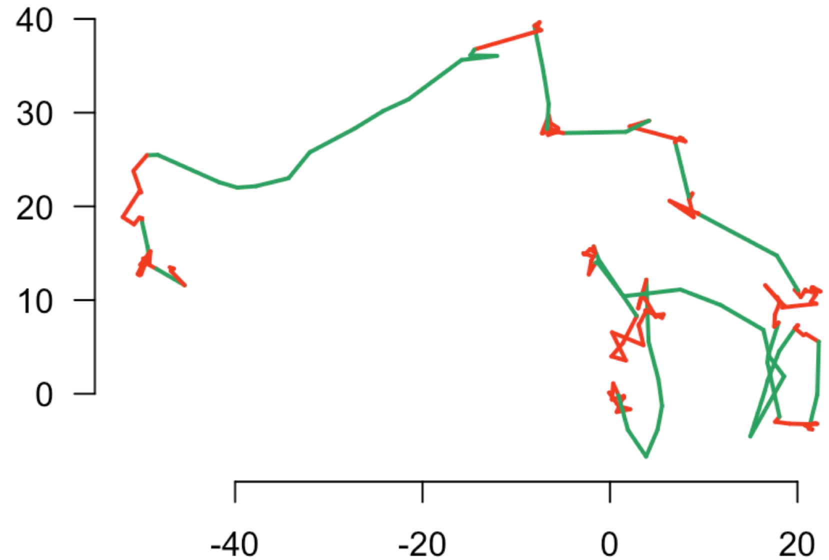






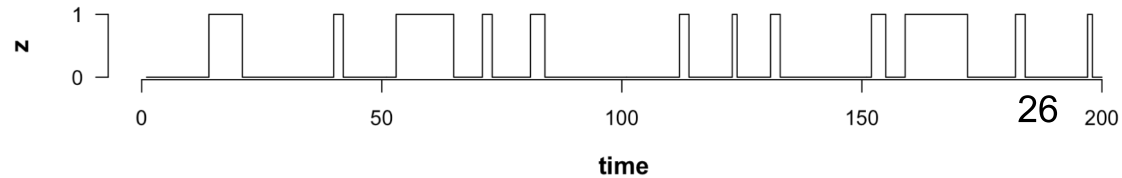
Modeling behavioral changes (HMM)

$$z_t \sim \begin{cases} \mathrm{Bern}(p_1) & \text{if } z_{t-1} = 1 \\ \mathrm{Bern}(p_2) & \text{if } z_{t-1} = 0 \end{cases}.$$



transition probability matrix

$$\begin{array}{cc} & \overbrace{\begin{array}{cc} 1 & \quad 0 \end{array}}^{z_t} \\ z_{t-1}\begin{cases} 1 \\ 0 \end{cases} & \begin{bmatrix} p_1 & 1 - p_1 \\ p_2 & 1 - p_2 \end{bmatrix} \end{array}$$

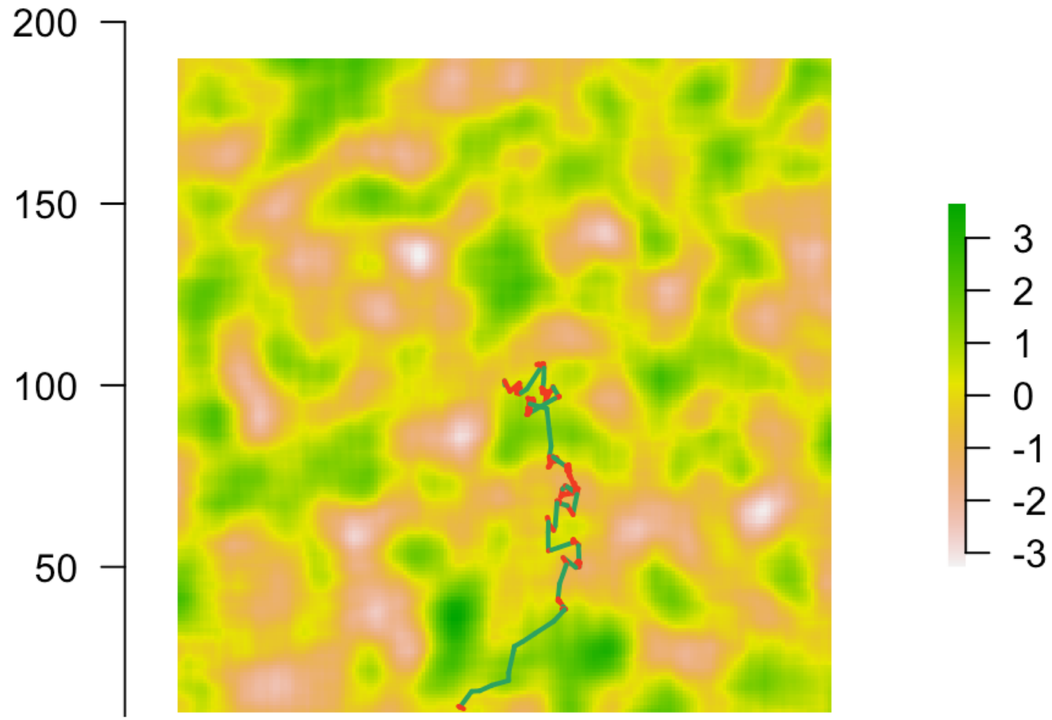


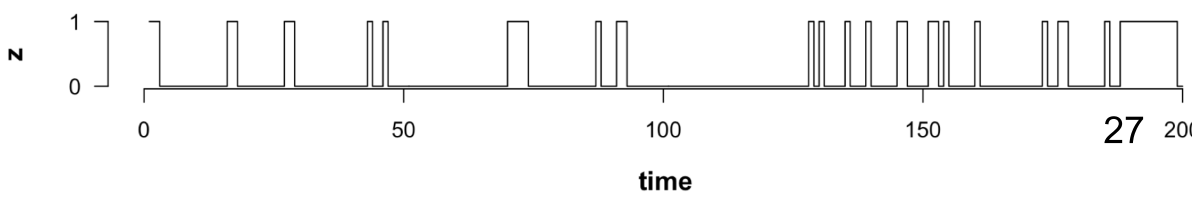
Covariates on transition probabilities

$$z_t \sim \begin{cases} \text{Bern}(p_1) & \text{if } z_{t-1} = 1 \\ \text{Bern}(p_2) & \text{if } z_{t-1} = 0 \end{cases}.$$

$$\text{logit}(p_{1,t}) = \boldsymbol{x}_t' \boldsymbol{\beta}_1$$

$$\text{logit}(p_{2,t}) = \boldsymbol{x}_t' \boldsymbol{\beta}_2$$





Changes based on internal state

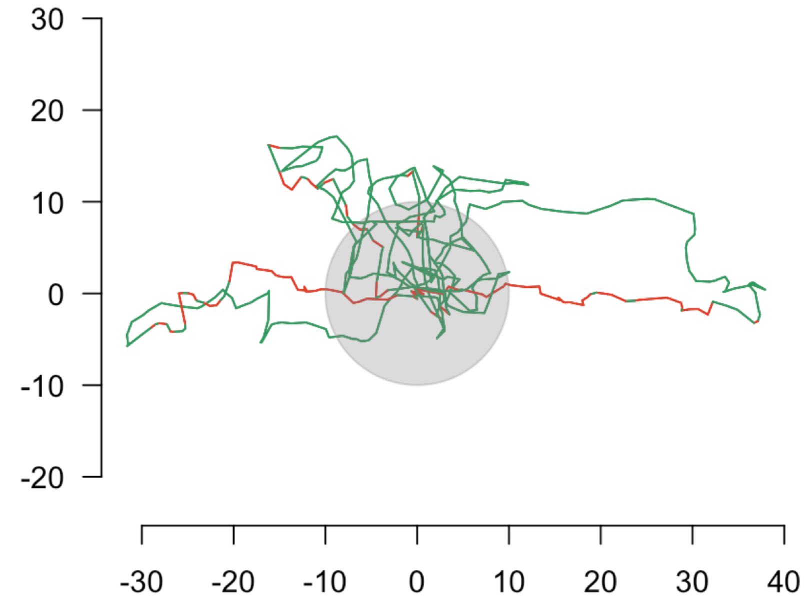
$$\text{step}_t \sim \text{Weibull}(a, b)$$

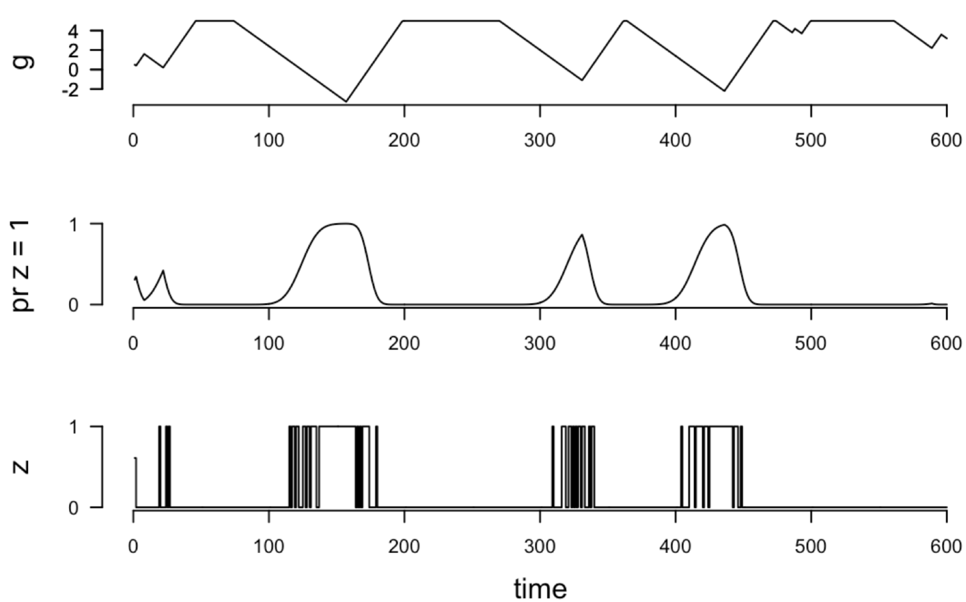
$$\text{direction}_t \sim \text{wCauchy}(\lambda_t, \rho)$$

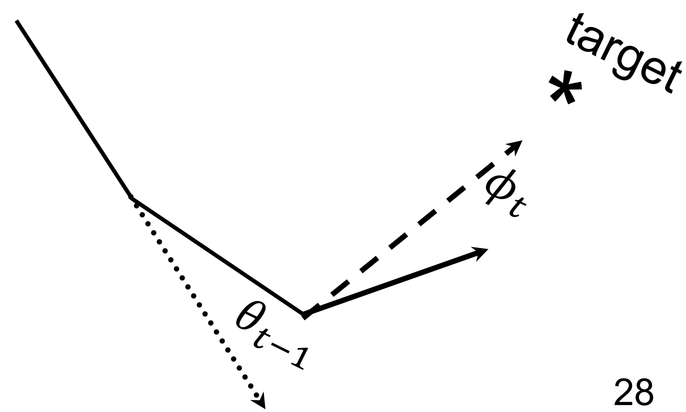
$$\lambda_t = \begin{cases} \theta_{t-1} & \text{if } z_t = 0 \\ \phi_t & \text{if } z_t = 1 \end{cases}$$

$$z_t \sim \text{Bernoulli}\big(1 - \Phi(g_t)\big)$$

$$g_t = \min(g_{t-1} + \beta_{in} I_{in} - \beta_{out}(1 - I_{in}), g_{max})$$







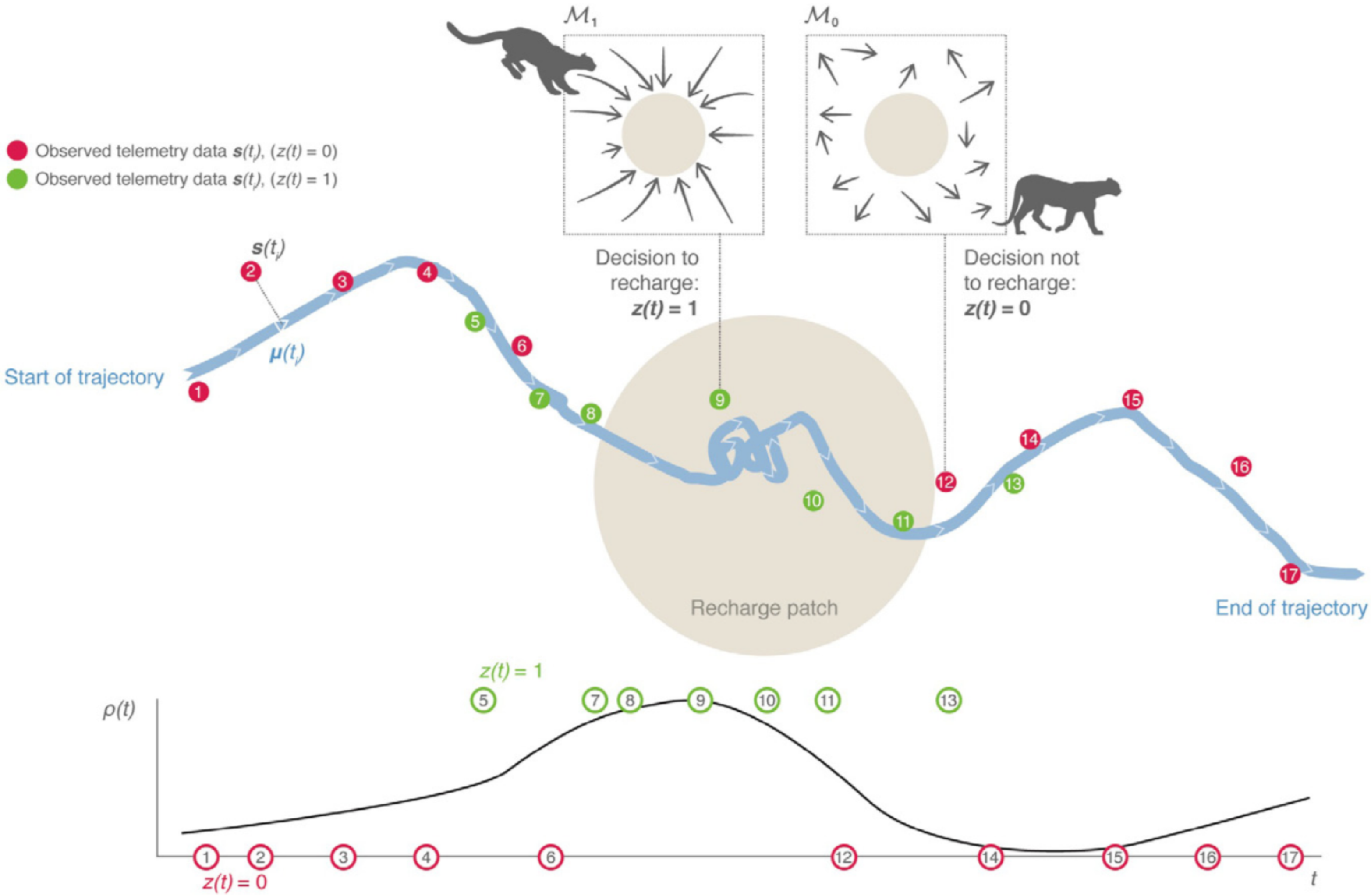
ECOLOGY LETTERS

Ecology Letters, (2018) doi: 10.1111/ele.13198

METHOD

Running on empty: recharge dynamics from animal movement data

Mevin B. Hooten,[1]* Henry R. Scharf[2] and Juan M. Morales[3]



How to fit all of these different (multistate) discrete-time models?

	Bayesian	BRW	CRW	Measurement Error	Temporal irregularity	Random effects	Recharge
BUGS	Yes	Yes	Yes	Yes	Yes	Yes	Yes
STAN	Yes	Yes	Yes	Yes	Yes	Yes	Yes
moveHMM	No	No	Yes	No	No	No	No
momentuHMM	No	Yes	Yes	Yes	Yes	Yes	Yes

fitting these models with BUGS and Stan

- BUGS (winBUGS, OpenBUGS or JAGS)
  - pros:
    - easy to set up
    - straightforward estimation of posteriors for hidden states
  - cons:
    - tricky to implement circular distributions
    - slow
- Stan
  - pros:
    - easy to implement circular distributions (or anything!)
    - faster (efficient)
  - cons:
    - need to marginalize states
    - need to decode states

HMM in BUGS

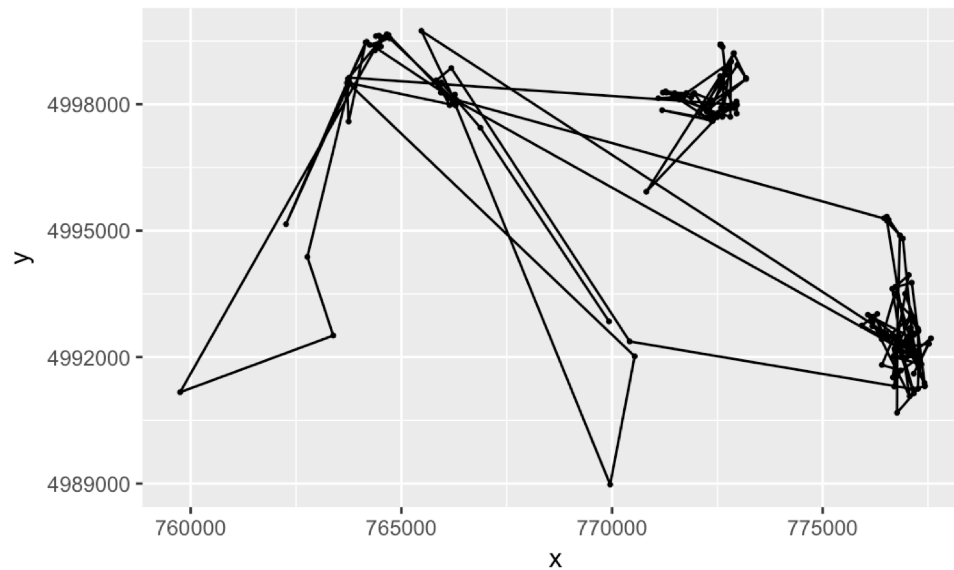
```
model{
   for (t in 2:T) {
       steps[t] ~ dweib(b[z[t]], a[z[t]]) #b=shape

       ones[t] ~ dbern( wc[t] )
       wc[t] <- (1/(2*Pi)*(1-pow(rho[z[t]],2))/
                (1+pow(rho[z[t]],2) -
                2*rho[z[t]] * cos(turns[t]- mu[z[t]])))/100

       turns[t] ~ dunif(-3.14159265359, 3.14159265359)

       z[t] ~ dcat(p[t,])

       p[t,1] <- q[z[t-1]]
       p[t,2] <- 1-q[z[t-1]]
   }
```

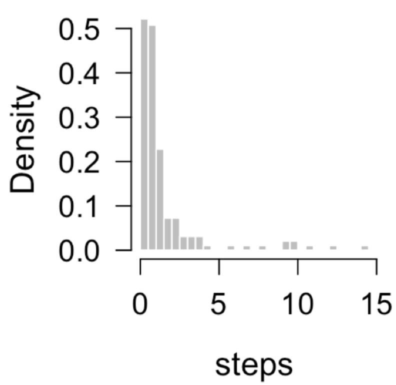
HMM in Stan

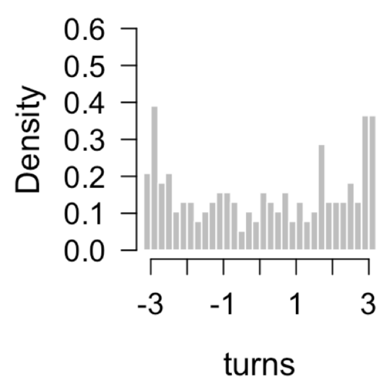
```
model {
  vector[N] lp;    // for forward variables
  vector[N] lp_p1; // for forward variables

  // likelihood
  for (t in 1:T) {
    for (n in 1:N) {
      lp_p1[n] = log_sum_exp(to_vector(log_theta_tr[t,n]) + lp);
        if(steps[t]>=0)
          lp_p1[n] = lp_p1[n] +
              weibull_lpdf(steps[t] | b[n], a[n]);
        if(turns[t]>=(-pi()))
           lp_p1[n] = lp_p1[n] +
               wrappedCauchy_lpdf(turns[t] | mu[n], rho[n]);
    }
    lp = lp_p1;
    target += log_sum_exp(lp);
  }
}
```

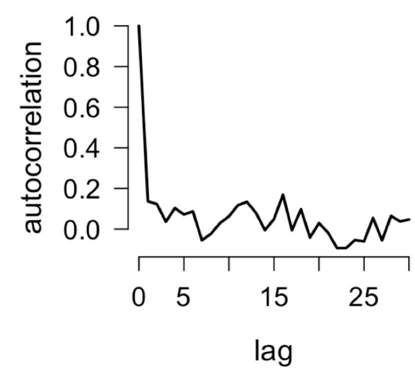

See Stan case study by Vianey Leos Barajas and Théo Michelot arxiv.org/abs/1806.10639

Some Bayesian examples with elk data

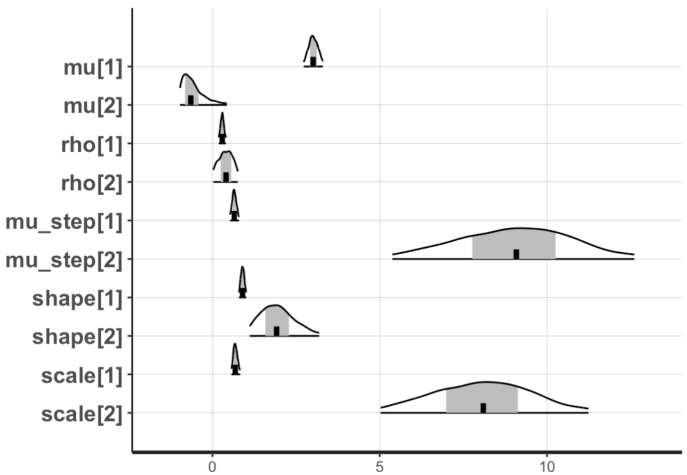


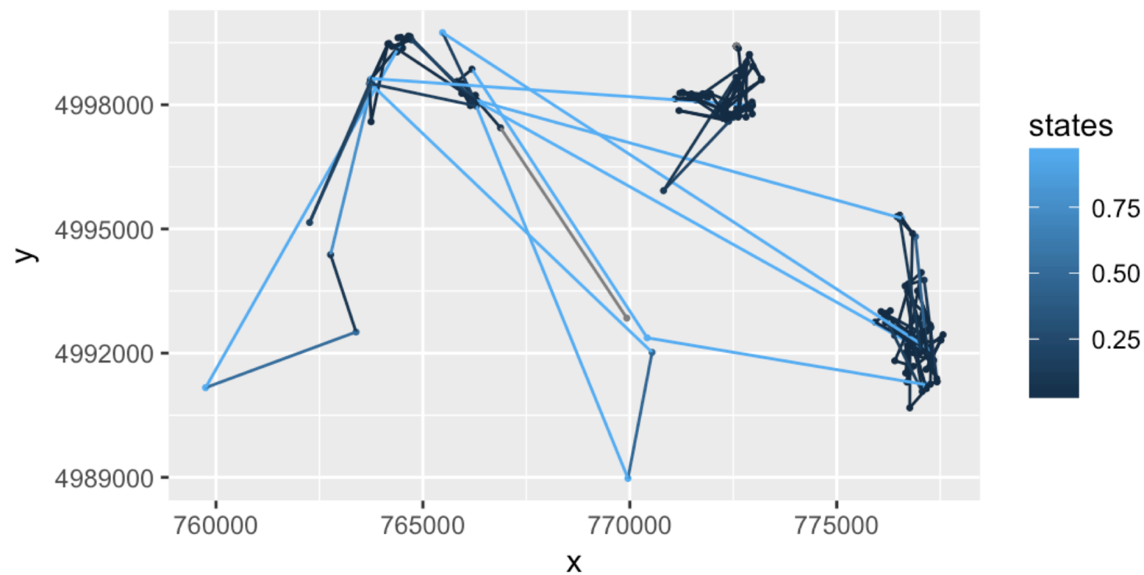


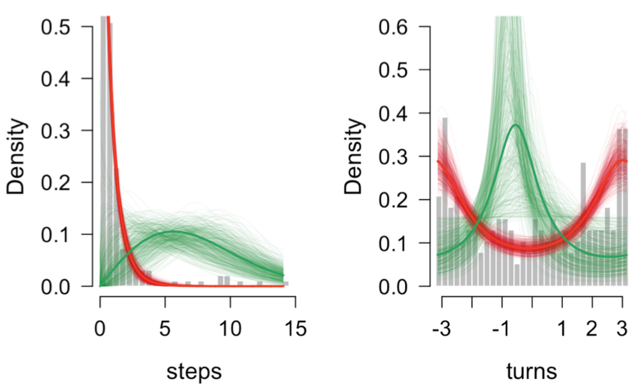


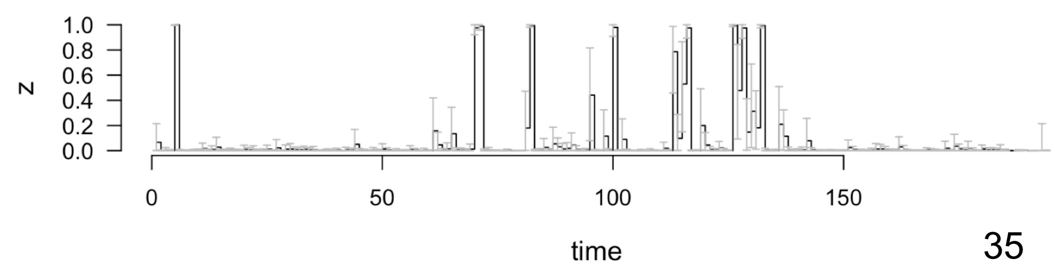


Mix of two states (non-Markov)

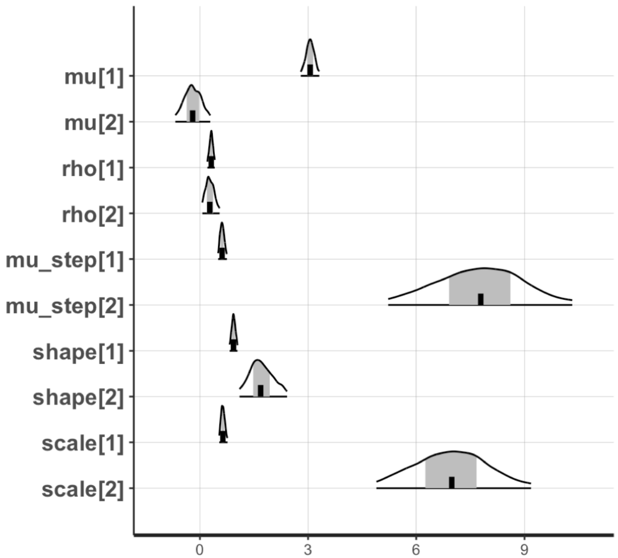


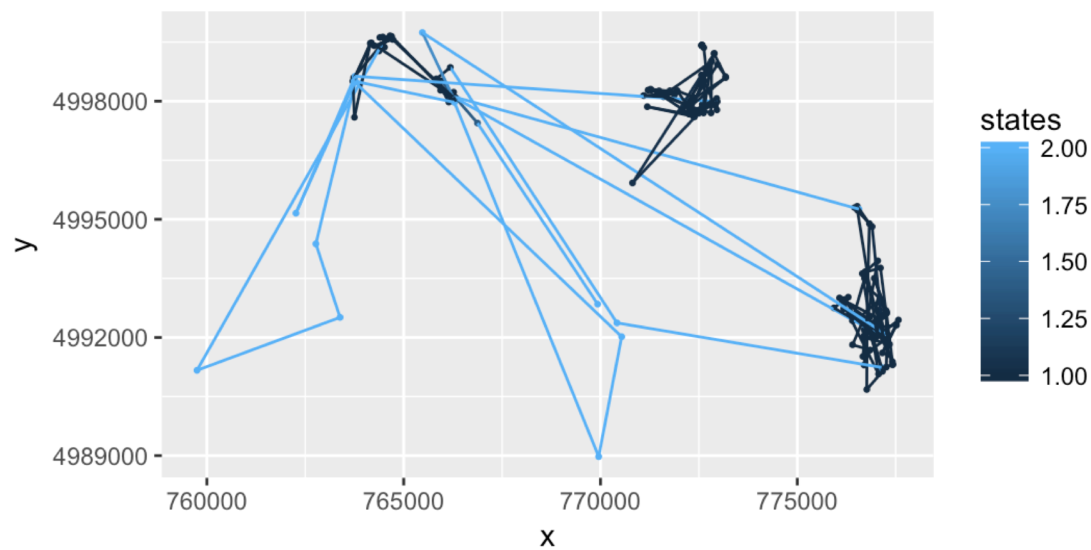


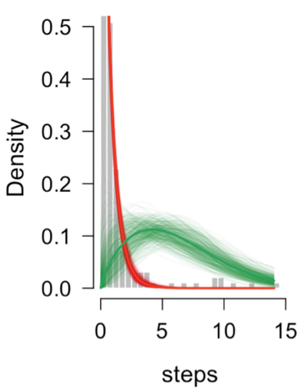


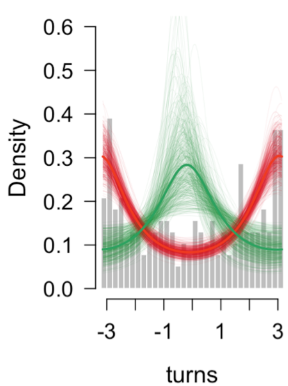


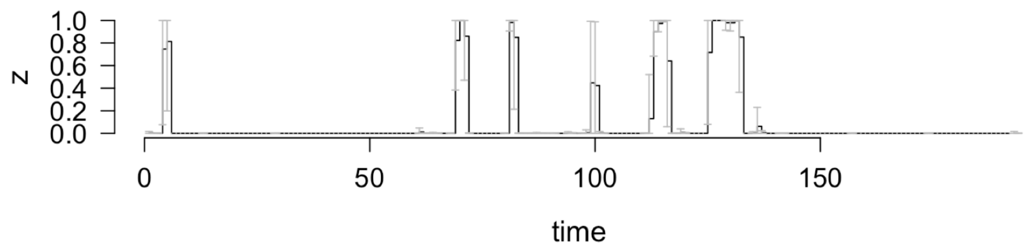
HMM


mu[1]
mu[2]
rho[1]
rho[2]
mu_step[1]
mu_step[2]
shape[1]
shape[2]
scale[1]
scale[2]
0
3
6
9

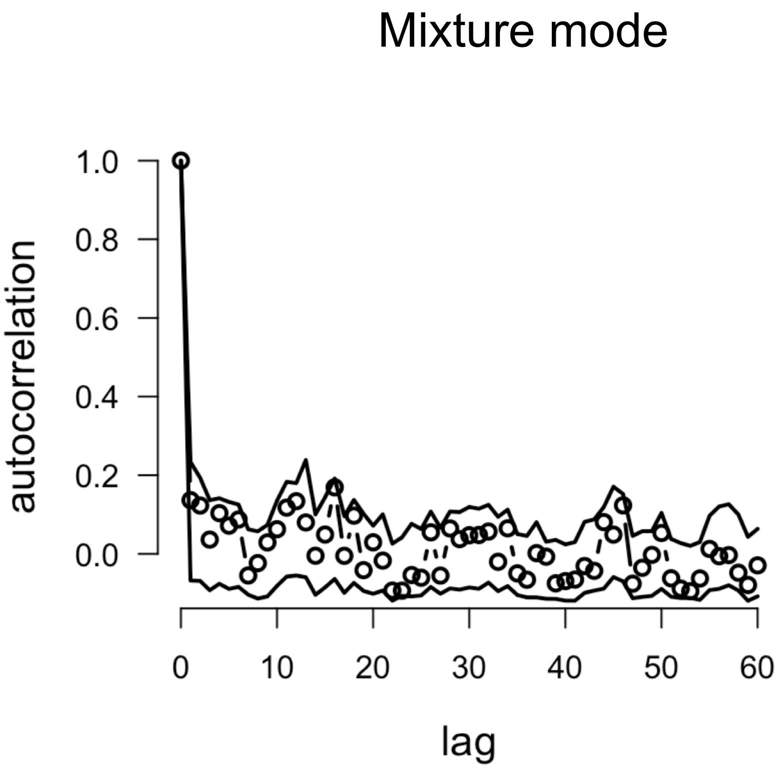

4998000
4995000
4992000
4989000
y
760000
765000
770000
775000
x
states
2.00
1.75
1.50
1.25
1.00

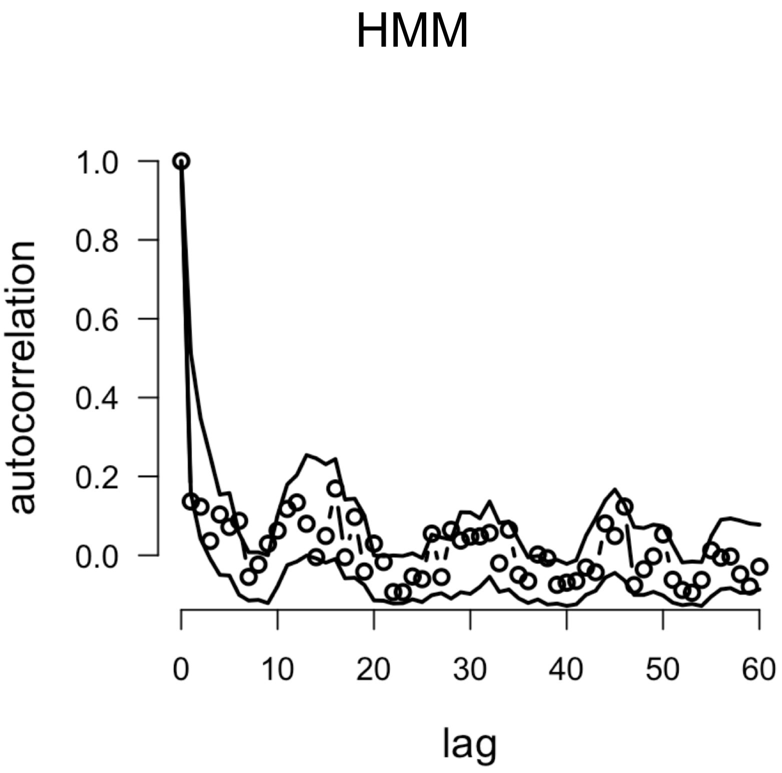

Density
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
steps


Density
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3
-1
1
3
turns

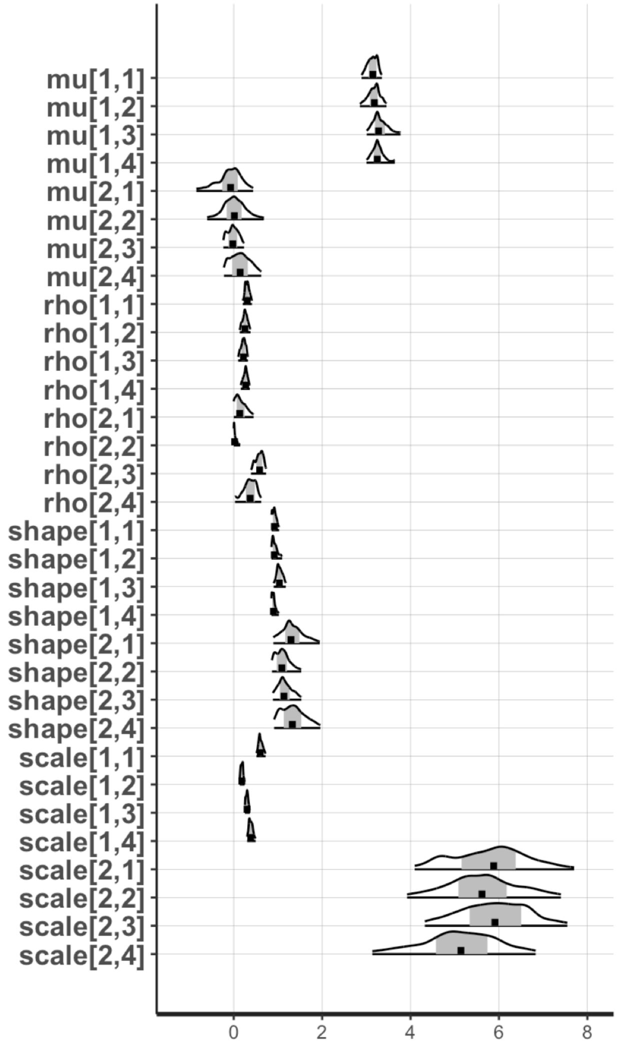

z
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150
time

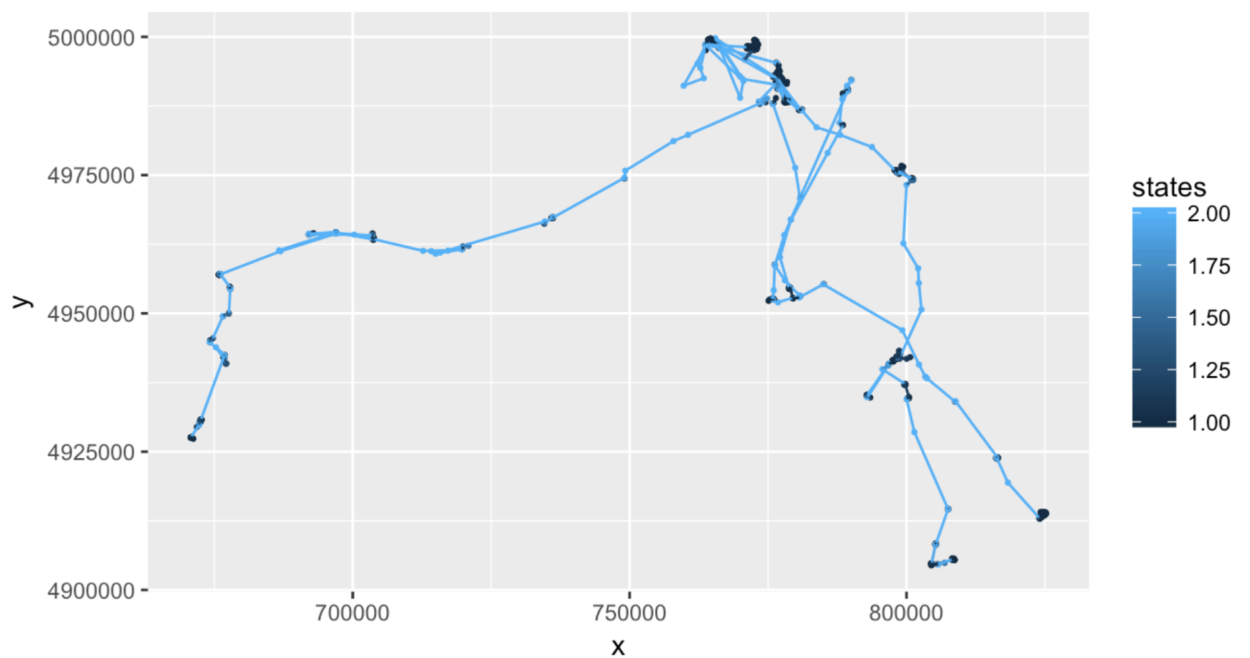
Posterior predictive check on ACF





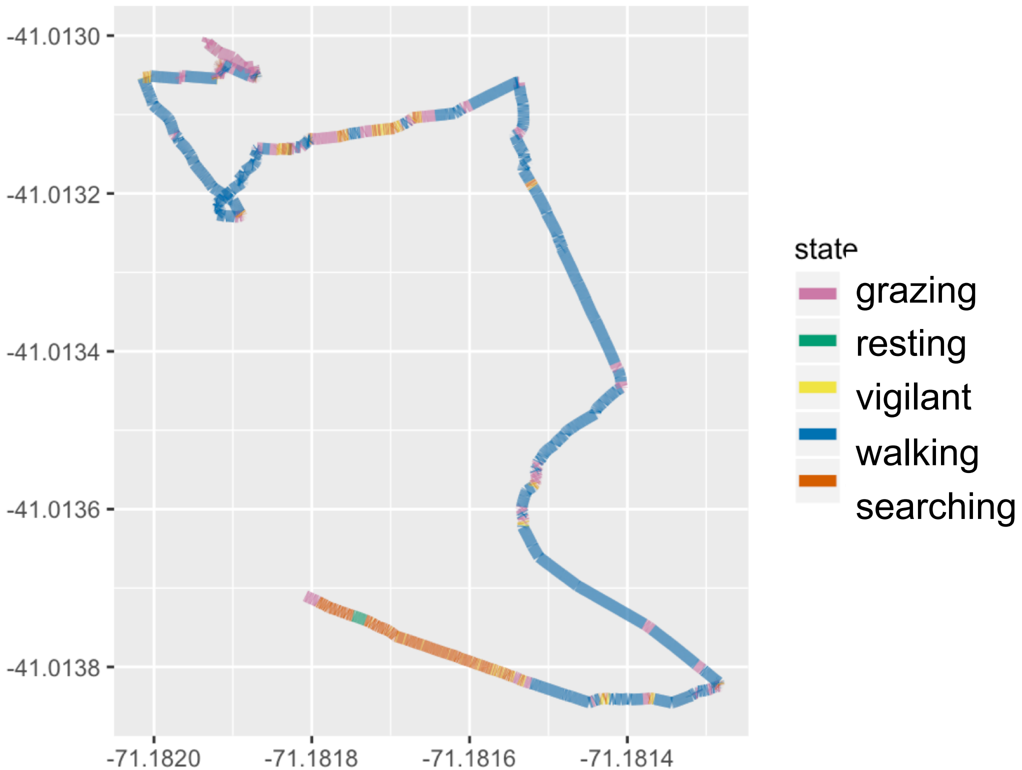
Individual “random effects”





Interpretation and caveats

- Do states really correspond to different behaviors?
- Importance of auxiliary information and bio-logging



di Virgilio et al. 2018
PeerJ 6:e4867

Movement, Space Use, and Population Dynamics