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- What assumptions are we making when fitting linear mixed effects models?
- How can we select an appropriate model?

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When you want to generalize to a larger population of sample units

- Use fixed effects to model particular sites
- Use random effects (assumed to have a distribution) to model a population of sites

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- Random effects parameters (intercepts, slopes) are assumed to come from a distribution (usually normal).
- Allow for two types of predictions: population-averaged (average, across many realized "subjects") and subject-specific

# Fixed versus random effects

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- Makes it difficult to estimate variance parameters (describing how intercepts/slopes vary and covary among clusters).
- Is it realistic to "generalize" to a larger population of clusters?
- May be more appropriate to use fixed effects models to account for cluster-level differences.

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Level-1 predictors vary within clusters

- NAP within beach
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Implications:

- Degrees of freedom (amount of information) for level-1 predictors depends on the overall size of the data set (number of individuals and number of obs. per individual)
- Degrees of freedom for level-2 predictors depends on the number of clusters

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Can generalize to models that include more than 2 levels

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Distinctions between "types" of predictors (level-1 verus level-2) become even more important when we consider modeling non-normal data

- Implications for parameter interpretation
- Implications for analysis approach (generalized linear mixed effects models verus generalized estimating equations)

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Consider the random intercept model:

$$R_{ij} = \beta_0 + b_{0i} + \beta_1 NAP_{ij} + \beta_2 Exposure_i + \epsilon_{ij}$$

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- Correlation among observations taken from different beaches = 0 (do not share either random term).

# What assumptions are we making when fitting linear mixed effects models?

Our response, $Y$, can be described as a *linear function of covariates*.

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The *intercepts* for each beach are *independent* and *normally distributed*:

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The within-beach errors are *independent* and *normally distributed*, with *constant variance*: $\epsilon_{ij} \sim N(0, \sigma^2)$