

# Linear Regression Review

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology



# Objectives

Review important statistical concepts within the context of linear regression using simulated data:

- Sampling Distributions
- T-tests for regression coefficients
- Confidence intervals
- P-values
- How to check assumptions

## Why Consider Simulated Data (Kery p. 7):

- Truth is known (can compare estimates to truth).
- Provides a check on coding errors.
- Facilitates understanding of **sampling distributions**.
- Provides a means to study properties of an estimator (mean, variance)
- ...and, an assessment of the effect of assumption violations.
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## Why Real Data

- Real data almost never meet all assumptions of common statistical models.

# Sustainable trophy hunting of African Lions

Whitman et al. 2014 Nature 428:175-178

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How, from afar, can we tell the age of a lion?

## Its in the Nose!



After about 3 years of age, the fleshy part of the nose begins to freckle or become liver spotted. As the lion ages, more pigmentation appears until the nose is entirely black by about 8 years. A general rule of thumb is that by 6 years noses are >50% black.

## Data are contained in abd library of Program R:

```
install.packages("abd") # only if not installed (do once)
```

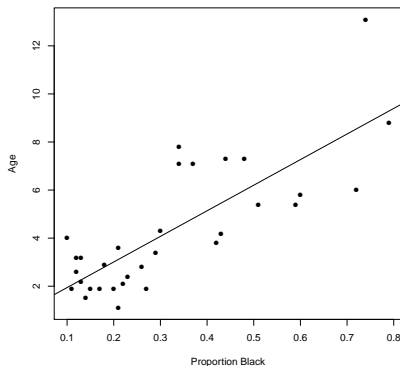
```
library(abd) # Each time you want to accesss the data  
data(LionNoses)  
head(LionNoses)
```

	age	proportion.black
1	1.1	0.21
2	1.5	0.14
3	1.9	0.11
4	2.2	0.13
5	2.6	0.12
6	3.2	0.13



# Lion's Noses

```
with(LionNoses,plot (proportion.black,age,   xlab="Proportion Black",  
                    ylab="Age",   pch=16))  
lm.nose<-lm(age~proportion.black,   data=LionNoses)  
abline(lm.nose)
```



# Linear Regression Assumptions

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\epsilon_i \sim N(0, \sigma^2)$$

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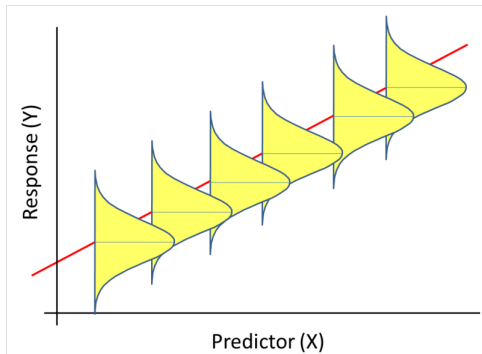
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- **Existence** (we observe random variables that have finite variance; we won't worry about this one)
- **Gauss**:  $\epsilon_i$  come from a Normal (Gaussian) distribution

# Regression Assumptions

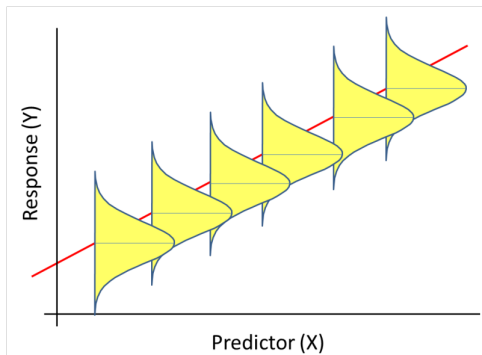
- We have a model for the mean  $\mu_i = \beta_0 + \beta_1 X_i$
- We specify a probability distribution for  $Y_i|X_i \sim N(\mu_i, \sigma^2)$





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- How are these assumptions reflected in the figure? How can we evaluate the assumptions with our data?

# Residuals Versus Fitted

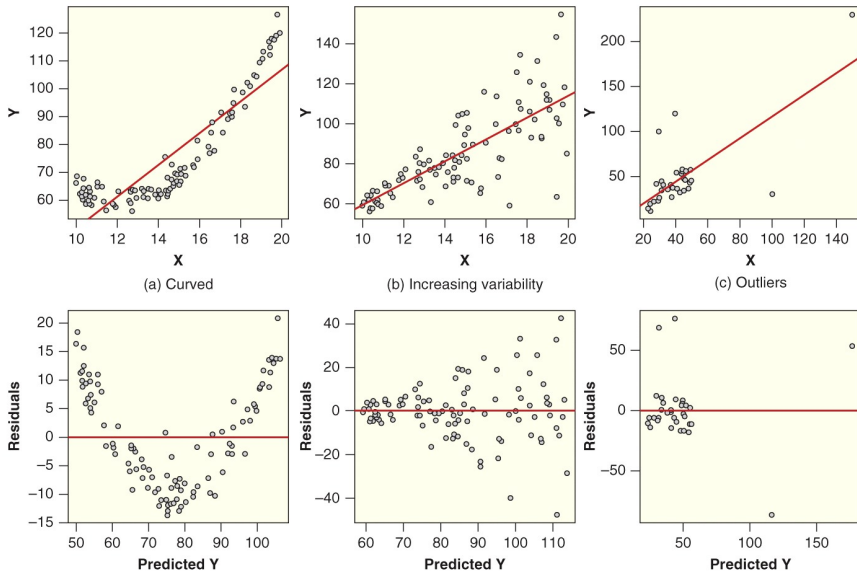


Figure 10.2

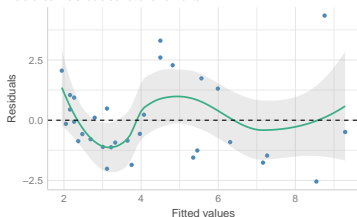
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# Graphical Check

```
performance::check_model(lm.nose,  
  check = c("linearity", "homogeneity", "qq", "normality"))
```

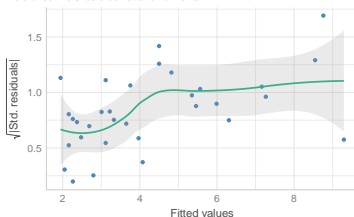
## Linearity

Reference line should be flat and horizontal



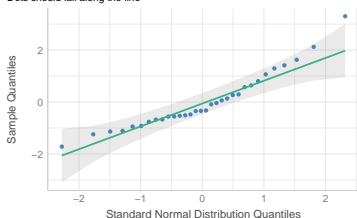
## Homogeneity of Variance

Reference line should be flat and horizontal



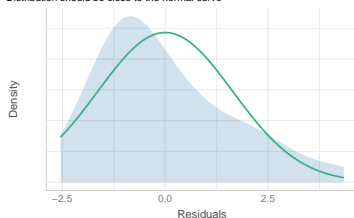
## Normality of Residuals

Dots should fall along the line



## Normality of Residuals

Distribution should be close to the normal curve



## Interpretation: Intercept, Slope, t-tests and p-values, Residual Standard Error ( $\hat{\sigma}$ ), $R^2$

```
summary(lm.nose)
```

Call:

```
lm(formula = age ~ proportion.black, data = LionNoses)
```

Residuals:

Min	1Q	Median	3Q	Max
-2.5449	-1.1117	-0.5285	0.9635	4.3421

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.8790	0.5688	1.545	0.133
proportion.black	10.6471	1.5095	7.053	7.68e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.669 on 30 degrees of freedom

Multiple R-squared: 0.6238, Adjusted R-squared: 0.6113

F-statistic: 49.75 on 1 and 30 DF, p-value: 7.677e-08

## Interpretation: Intercept and Slope

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But, proportion black  $< 1$  for all lions.

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But, proportion black < 1 for all lions.

$$? = \frac{\Delta \text{age}}{\Delta 0.1 \text{Proportion.Black}} = 1.065.$$

$H_0 : \beta_1 = 0$  vs.  $H_A : \beta_1 \neq 0$ ?

```
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```

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See: AP stats guy videos in Ch. 1 of the textbook



SE  $\hat{\beta}$

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- Collecting a new data set (of the same size)

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Lets explore through simulation!

# Simulation

Lets first generate a single data set consistent with our fitted model using the following code.

```
# Sample size of simulated observations
n<-32

# Use the observed proportion.black to simulate obs.
p.black<-LionNoses$proportion.black

# Use the estimated parameters to simulate data.
# - can get these from the regression output
# sigma<-summary(lm.nose)$sigma # residual variation about the line
# betas<-coef(lm.nose) # Regression coefficients
sigma<-1.67 # residual variation
betas<-c(0.88, 10.65) #betas

# Create random errors (epsilons) and random responses
epsilon<-rnorm(n,0, sigma) # Errors
y<-betas[1] + p.black*betas[2] + epsilon # Response
```

# Linear regression using lm function

```
lmfit<-lm(y~p.black)
summary(lmfit)
```

Call:

```
lm(formula = y ~ p.black)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.2401	-0.8812	-0.3871	0.9053	3.2192

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	1.7028	0.5627	3.026	0.00505	**
p.black	8.9392	1.4934	5.986	1.45e-06	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.651 on 30 degrees of freedom

Multiple R-squared: 0.5443, Adjusted R-squared: 0.5291

F-statistic: 35.83 on 1 and 30 DF, p-value: 1.45e-06



# Sampling Distribution

Use a for loop to:

- Generate 5000 data sets using the same code
- Fit a linear regression model to each data set
- For each fit, store  $\hat{\beta}$

In-class exercise

# Sampling distribution of $\hat{\beta}$

Because the sampling distribution of  $\hat{\beta}$  depends on  $\sigma$ , we usually work with the distribution of:

$$\frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$$

which does not!

## Sampling distribution of the t-statistic

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- Fitting the regression model
- Calculating:  $t = \frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)}$

A histogram of the different  $t$  values should be well described by a Student's  $t$ -distribution with  $n - 2$  degrees of freedom.

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Helpful hints:

- $\beta_1$  = true value used to simulate the data,  
`coef(lm.nose)[2] = 10.6471`
- $\hat{\beta}_1$  is extracted using: `coef(lmfit)[2]`
- $\widehat{SE}(\hat{\beta}_1)$  is extracted using `sqrt(vcov(lm.temp)[2,2])`

In-class exercise

My code

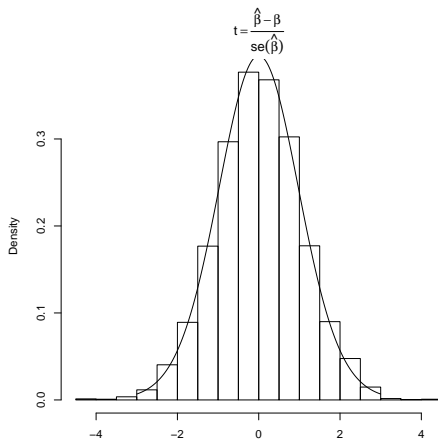
# My code

```
nsims<-5000 # number of simulations
tsamp.dist<-matrix(NA, nsims,1) # matrix to hold results
for(i in 1:5000){
  epsilon<-rnorm(n,0,sigma) # Errors
  y<-betas[1] + betas[2]*p.black + epsilon # Response
  lm.temp<-lm(y~p.black) # lm

  # Here is our statistic, calculated for each sample
  tsamp.dist[i]<-(coef(lm.temp)[2]-betas[2])/sqrt(vcov(lm.temp)[2,2])
}
# Plot results
hist(tsamp.dist, xlab="",
      main=expression(t==frac(hat(beta)-beta, se(hat(beta)))), freq=FALSE)
tvalues<-seq(-3,3, length=1000) # xvalues to evaluate t-distribution
lines(tvalues,dt(tvalues, df=30)) # overlay t-distribution
```



# Sampling distribution of t-statistic



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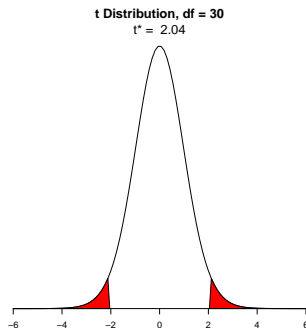
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- A 95% confidence interval should contain the true parameter for 95% of all samples
- The parameter is *fixed*, but the endpoints of the interval are *random*

# Confidence Interval

$$\frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$$



Note:

$$t_{0.025, n-2} = \text{qt}(p=0.025, \text{df}=30) = -2.04$$

$$t_{0.975, n-2} = \text{qt}(p=0.975, \text{df}=30) = 2.04$$

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$$P(\hat{\beta} + 2.04\widehat{SE}(\hat{\beta}) > \beta > \hat{\beta} - 2.04\widehat{SE}(\hat{\beta})) = 0.95$$

So, take  $(\hat{\beta} - 2.04\widehat{SE}(\hat{\beta}), \hat{\beta} + 2.04\widehat{SE}(\hat{\beta}))$  as the the 95% confidence interval.

## confint function

```
confint(lm.nose)
```

	2.5 %	97.5 %
(Intercept)	-0.2826733	2.040686
proportion.black	7.5643082	13.729931

What is wrong with the following interpretation?

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- $\beta$  is either in this particular interval ( $P = 1$ ) or it is not ( $P = 0$ )
- $P(L \leq \beta \leq U) = 0.95$ , where  $L$  and  $U$  are random variables that determine the upper and lower limits of the 95% confidence interval

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- $\beta$  is either in this particular interval ( $P = 1$ ) or it is not ( $P = 0$ )
- $P(L \leq \beta \leq U) = 0.95$ , where  $L$  and  $U$  are random variables that determine the upper and lower limits of the 95% confidence interval

We are 95% sure that the true slope (relating proportion of nose that is black and age) falls between 7.56 and 13.73.

# Explore CIs through simulation

Simulate another 5000 data sets in R:

Determine 95% confidence limits for each data set and examine whether or not the CI contains the true  $\beta$ .

# My Code

My Code

My Code

My Code

My Code

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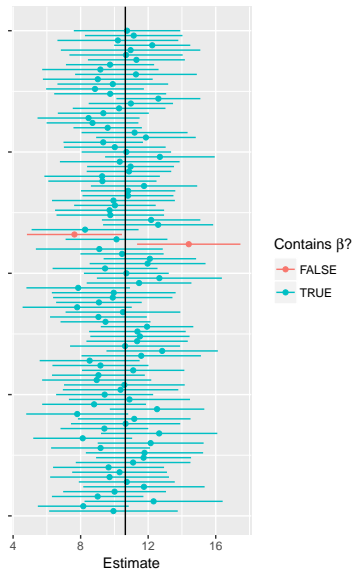
```
nsims<-5000 # number of simulations
Limits<-matrix(NA,nsims,2) # to hold results
beta.hats<-matrix(NA,nsims,1) # to hold estimates
for(i in 1:nsims){
  epsilon<-rnorm(n,0, sigma) # Errors
  y<-betas[1] + betas[2]*p.black + epsilon # Response
  lm.temp<-lm(y~p.black)

  # Beta.hat & Confidence limits
  beta.hats[i]<-coef(lm.temp)[2]
  Limits[i,]<-confint(lm.temp)[2,]
}
# True parameter in interval?
I.in<-betas[2]>Limits[,1] & betas[2] <= Limits[,2]

# Coverage
sum(I.in)/nsims
```

```
[1] 0.9444
```

# First 100 simulations



```
summary(lm.nose)
```

Call:

```
lm(formula = age ~ proportion.black, data = LionNoses)
```

Residuals:

Min	1Q	Median	3Q	Max
-2.5449	-1.1117	-0.5285	0.9635	4.3421

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.8790	0.5688	1.545	0.133
proportion.black	10.6471	1.5095	7.053	7.68e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.669 on 30 degrees of freedom

Multiple R-squared: 0.6238, Adjusted R-squared: 0.6113

F-statistic: 49.75 on 1 and 30 DF, p-value: 7.677e-08

## P-values

If the null hypothesis,  $H_0 : \beta_1 = 0$ , is true, then:

$$t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$$



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Is the value we get for  $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} = 7.053$  consistent with  $H_0 : \beta_1 = 0$ ?

# P-values

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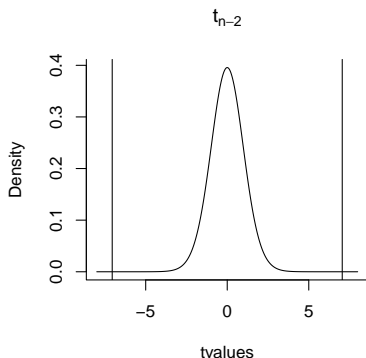
$$t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$$

Is the value we get for  $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} = 7.053$  consistent with  $H_0 : \beta_1 = 0$ ?

- Overlay  $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} = 7.053$  on a  $t_{n-2}$  distribution
- Determine the probability of getting a t-statistic as or more extreme as the one we observed.

# Hypothesis test

- t-distribution tells about the sampling distribution of  $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$  when the null hypothesis is true
- our t-statistic falls in the tail of this distribution (so, the Null hypothesis is unlikely to be true!)



$R^2$ 

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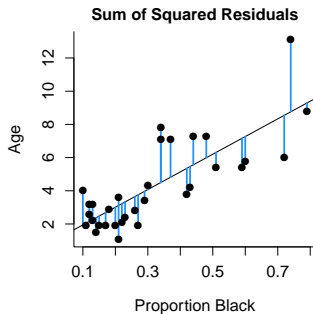
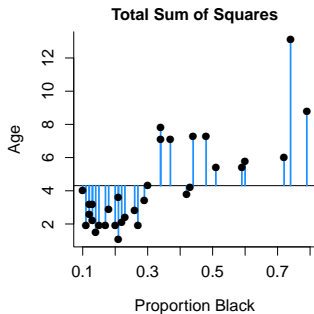
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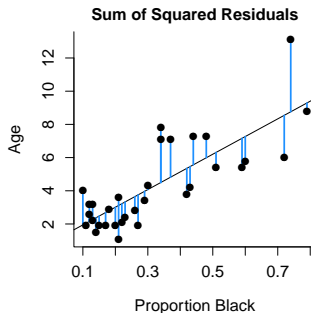
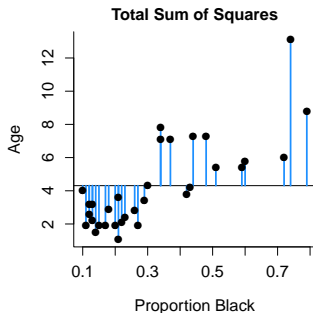
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# Sum of Squares

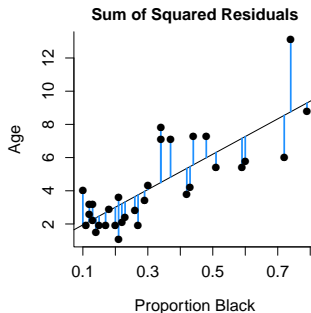
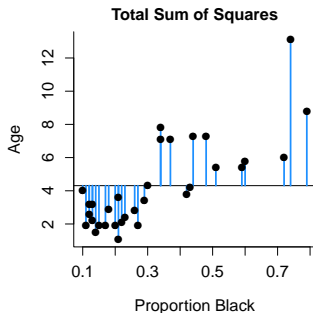


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$$\text{SST (Total sum of squares)} = \sum_i^n (Y_i - \bar{Y})^2$$

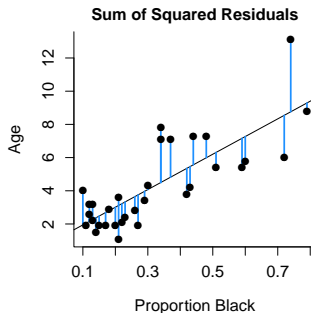
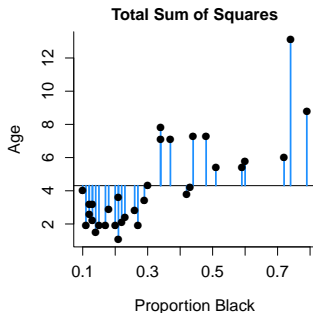
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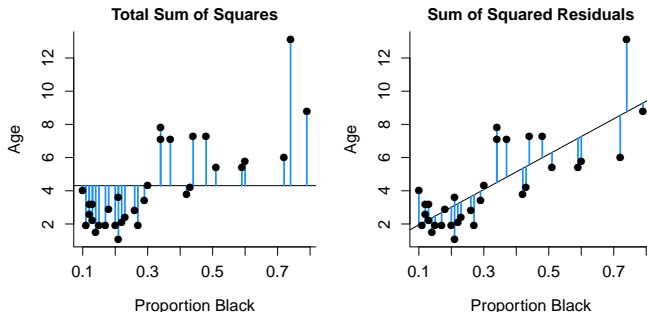
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$$\text{SSR (sum of squares regression)} = \text{SST} - \text{SSE} = \sum_i^n (\hat{Y}_i - \bar{Y})^2$$



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$$SSR \text{ (sum of squares regression)} = SST - SSE = \sum_i^n (\hat{Y}_i - \bar{Y})^2$$

$$R^2 = \frac{SST - SSE}{SST} = \frac{SSR}{SST} = \text{proportion of the variation explained by the linear model.}$$

Residual standard error =  $\hat{\sigma}$

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\epsilon_i \sim N(0, \sigma^2)$$

- $\sigma$  describes the amount of variability about the regression line

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- $n - p$  since we lose one degree of freedom for each parameter we estimate

# Residual Standard Error

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We expect 95% of the observations to fall within  $2 \times 1.669$  of the regression line.