Correlation, Causation, and Causal networks

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology



Learning objectives

- 1. Gain a deeper appreciation for why correlation (or association) is not the same as causation
- 2. Discover basic rules that allow one to determine dependencies (correlations) among variables from an assumed causal network
- 3. Understand how causal networks can be used to inform the choice of variables to include in a regression model

Causation versus correlation

Knowing what causes what makes a big difference in how we act. If the rooster's crow causes the sun to rise we could make the night shorter by waking up our rooster earlier and make him crow - say by telling him the latest rooster joke. - Judea Pearl (1936-), computer scientist

Regression

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Regression coefficients change depending on what other variables are included.

Regression and Causal Mechanisms

Often, we want to interpret models as capturing causal mechanisms so we can say what will happen if we intervene in the system:

- Will taking a daily vitamin improve long-term health?
- Will increasing the pay of teachers or reducing class size boost student performance?
- Will increasing taxes on the rich cause companies to relocate?
- Will we decrease deer population size if we institute an Earn-a-buck regulation?

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- The National Park Service commissioned a study to ask if fewer birds would have died from collisions had a bridge in Hastings MN been built differently.

We may also be interested in asking hypothetical questions. What would have happened if...

- A judge may have to decide if a worker's claim of sex discrimination is legitimate: would the worker have gotten the job if she was a male?
- The National Park Service commissioned a study to ask if fewer birds would have died from collisions had a bridge in Hastings MN been built differently.

These questions involve counterfactuals = something that did not happen, but would have happened if something had been different.

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Will taking a daily vitamin improve our long-term health?

People that take vitamins may have better health outcomes, but they may also...

- Exercise more
- Eat healthier
- Drive more cautiously

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- direct effect (negative) due to increased taxes
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Predicting the effect of an intervention requires something more complex...

Campaign Spending: Correlation vs. Causation

Campaign spending data from US Congressional elections

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Campaign Spending: Correlation vs. Causation

Campaign spending data from US Congressional elections

- Increased spending by those running for re-election (incumbents) is associated with lower vote percentages
- Does increased spending cause the incumbent to lose votes?
- Does the incumbent spend more when elections are tight?

Causal Networks

Causal network = Hypothetical model of how the system works

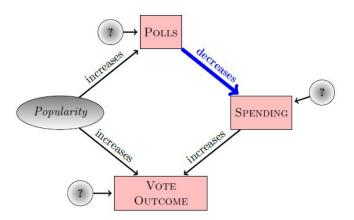


Figure 17.4: A hypothetical causal network describing how campaign spending by an incumbent candidate for political office is related to the vote outcome.

Links and Nodes

Nodes: represent variables or components in a system

- Boxes = observed
- Ellipses = not observed (latent)
- Circles = random noise, suggests *something* outside of the system influences the variable

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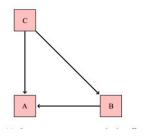
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Circles (things outside of the system) are said to be exogenous. Shapes with arrows pointing to them are influenced by things inside the system and are said to endogenous.

Direct, indirect, and total effects



- A direct effect is one that is represented by a path between two nodes (without consideration of any intermediate nodes): C → A
- An indirect effect is one that connects two nodes, but where we also consider an intermediate node: $C \rightarrow B \rightarrow A$. B is a called a mediator variable.
- The total effect is the sum of direct and indirect effects

Using Simulation to Explore Campaign Spending

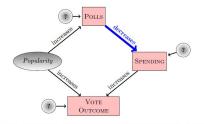


Figure 17.4: A hypothetical causal network describing how campaign spending by an incumbent candidate for political office is related to the vote outcome.

Linear Regression

```
fit.1<-lm(vote~spending, data=votedat)
summary(fit.1)
Call:
lm(formula = vote ~ spending, data = votedat)
Residuals:
    Min 10 Median 30 Max
-26.5315 -6.1569 -0.2512 6.4639 25.5007
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 67.51034 0.98980 68.21 <2e-16 ***
spending -0.34392 0.01812 -18.98 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.698 on 433 degrees of freedom
```

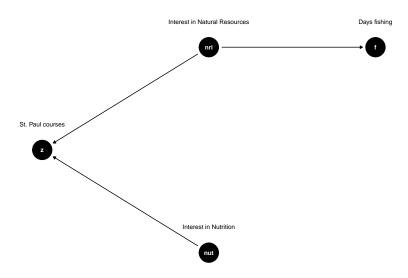
Multiple R-squared: 0.4541, Adjusted R-squared: 0.4528 F-statistic: 360.1 on 1 and 433 DF, p-value: < 2.2e-16

What happens when we include polls?

```
fit2<-lm(vote~spending+polls, data=votedat)
summary(fit2)
Call:
lm(formula = vote ~ spending + polls, data = votedat)
Residuals:
   Min 10 Median 30 Max
-13.910 -3.234 -0.065 3.812 13.843
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.90165 2.53144 -0.356 0.722
spending 0.27162 0.02466 11.013 <2e-16 ***
polls 0.74782 0.02690 27.799 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 5.214 on 432 degrees of freedom
```

Multiple R-squared: 0.8042, Adjusted R-squared: 0.8033 F-statistic: 887.4 on 2 and 432 DF, p-value: < 2.2e-16

Consider the following DAG:



I've simulated data using these assumptions (note interest in nutrition and fishing are not causally connected):

```
# Set seed of random number generator
set.seed (1040)
# number of students
n < -5000
# Interest in nutrition sciences
nut <- runif(n, 0, 10)
# Interest in natural resources
nri <- runif(n, 0, 10)
# Number of days fishing
f <- rpois(n, lambda=nri)</pre>
# Indicator variable (taking classes on St. Paul campus?)
p \leftarrow exp(-5 + 2*nut + 2*nri)/(1+exp(-5 + 2*nut + 2*nri))
z <- rbinom(n, 1, prob=p)</pre>
# Create data set
```

dagdata < -data.frame (nutrition.interest=nut, natresource.interest=nri,

Fishing is unrelated to interest in nutrition

```
mod1 <- lm(fishing ~ nutrition.interest, data=dagdata)</pre>
summarv (mod1)
##
## Call:
## lm(formula = fishing ~ nutrition.interest, data = dagdata)
##
## Residuals:
## Min 10 Median 30 Max
## -5.0049 -2.9686 -0.0049 2.0640 15.9993
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.926949 0.103496 47.60 <2e-16 ***
## nutrition.interest 0.007842 0.017838 0.44 0.66
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.646 on 4998 degrees of freedom
## Multiple R-squared: 3.867e-05, Adjusted R-squared: -0.0001614
## F-statistic: 0.1933 on 1 and 4998 DF, p-value: 0.6602
```

What if we "adjust" for whether the student is taking classes on

the St. Paul campus?

What if we "adjust" for whether the student is taking classes on the St. Paul campus?

```
mod2 <- lm(fishing ~ nutrition.interest + stpaulcampus, data=dagdata)</pre>
summary (mod2)
##
## Call:
## lm(formula = fishing ~ nutrition.interest + stpaulcampus, data = da
##
## Residuals:
## Min 1Q Median 3Q Max
## -5.4351 -2.9100 -0.3057 2.1591 16.1726
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.08727 0.26916 4.040 5.44e-05 ***
## stpaulcampus 4.37013 0.28389 15.394 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.563 on 4997 degrees of freedom
## Multiple R-squared: 0.04531, Adjusted R-squared: 0.04493
## F-statistic: 118.6 on 2 and 4997 DF, p-value: < 2.2e-16
```

Confused?

In the first example, we got the 'right answer' when we adjusted for polls.

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In the second example, adjusting for whether the student was taking courses on the St. Paul campus created a spurious (negative) correlation between interest in nurtition and fishing.

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In the second example, adjusting for whether the student was taking courses on the St. Paul campus created a spurious (negative) correlation between interest in nurtition and fishing.

Should we adjust or not? It depends on one's hypothetical model of the system (i.e., the causal network)!

A pathway between two nodes is a route between them (may pass through other nodes along the way)

- Correlating pathway follows in the direction of causal links
- Non-correlating pathway one that does not follow in the direction of causal links.

- A = a response variable
- B =an explanatory variable
- C = a covariate

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Three Linear pathways:

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- Causal mediator: $A \Leftarrow C \Leftarrow B$
- Common cause: $A \Leftarrow C \Rightarrow B$
- Witness: $A \Rightarrow C \Leftarrow B$

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Including *C* blocks the pathway, which is otherwise open

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Including *C* blocks the pathway, which is otherwise open

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• Witness: $A \Rightarrow C \Leftarrow B$

Including ${\cal C}$ opens the pathway, which is otherwise blocked.

A pathway between two variables (*A* and *B*) is correlating if there is a node on the pathway from which you can get to both variables.

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What about $A \Leftarrow C \Leftarrow D \Leftarrow B$?

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- Witness: $A \Rightarrow C \Leftarrow B$ (non-correlating)

What about $A \Leftarrow C \Leftarrow D \Leftarrow B$?

Correlating!

Recurrent (closed loop): $A \Rightarrow B \Rightarrow C \Rightarrow A$

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Raise questions of "when"... we won't deal with these

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 $A \Leftrightarrow B$ means that there is a non-causal connection between A and B.

Recurrent (closed loop): $A \Rightarrow B \Rightarrow C \Rightarrow A$

Raise questions of "when"... we won't deal with these

 $A \Leftrightarrow B$ means that there is a non-causal connection between A and B.

This must be because there is some unobserved variable, ${\cal U}$ producing the correlation:

• $A \Leftarrow U \Rightarrow B$

Blocking Back Door Pathways

To determine whether to adjust or not, consider these rules and follow the pathways between variables.

- Block correlating pathways: include at least one of the interior nodes on the pathway as a covariate
- DO not unblock non-correlating pathways: exclude all
 the interior nodes on the pathway; do not include any of
 them as covariates.

Back to Polls

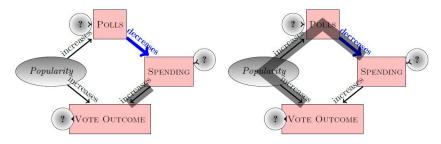
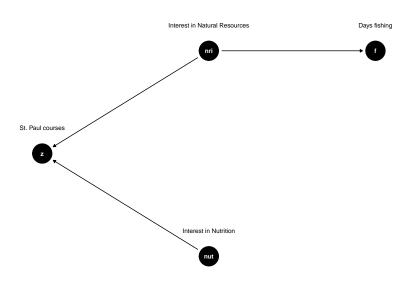


Figure 17.8: Pathways connecting spending to vote outcome. Both of these are correlating pathways.

And fishing/nutrition example



Selection Bias

What happens if we only survey students on the St. Paul campus?

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Selecting only individuals taking courses on the St. Paul campus has the same effect as adjusting for the variable in the regression.

Selection Bias

```
summary(lm(fishing~nutrition.interest,
          data=subset(dagdata, stpaulcampus==1)))
Call:
lm(formula = fishing ~ nutrition.interest, data = subset(dagdata,
    stpaulcampus == 1))
Residuals:
   Min 10 Median 30
                                 Max
-5.4340 -2.9801 -0.3135 2.2062 16.1717
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept) 5.45626 0.10864 50.224 < 2e-16 ***
nutrition.interest -0.06680 0.01841 -3.629 0.000287 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 3.619 on 4822 degrees of freedom
Multiple R-squared: 0.002724, Adjusted R-squared: 0.002518
F-statistic: 13.17 on 1 and 4822 DF, p-value: 0.000287
```

Some (Summary) Comments on Regression Modeling

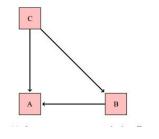
 Coefficients may change sign when we include or exclude other explanatory variables

Some (Summary) Comments on Regression Modeling

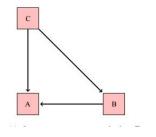
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Some (Summary) Comments on Regression Modeling

- Coefficients may change sign when we include or exclude other explanatory variables
- Whether we should include or exclude a particular variable depends on our hypothetical causal network
- Often useful to explore a few different hypothetical causal models, rather than fit (and average) over many models

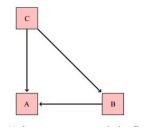


What if we want to estimate the direct effect of C on A?



What if we want to estimate the <u>direct effect</u> of C on A? Pathways:

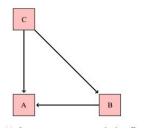
• $C \Rightarrow A$ (correlating), and the effect of interest.



What if we want to estimate the direct effect of C on A?

Pathways:

- $C \Rightarrow A$ (correlating), and the effect of interest.
- $C \Rightarrow B \Rightarrow A$ (correlating), an indirect effect of C on A that is mediated by B



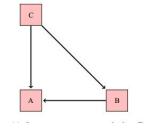
What if we want to estimate the direct effect of C on A?

Pathways:

- $C \Rightarrow A$ (correlating), and the effect of interest.
- $C \Rightarrow B \Rightarrow A$ (correlating), an indirect effect of C on A that is mediated by B Include B to block!

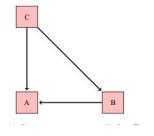
$$lm(A \sim C + B)$$

Estimates of Direct and Indirect Effects



What if we want to estimate the total effect of C on A?

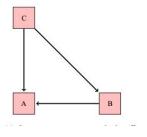
Estimates of Direct and Indirect Effects



What if we want to estimate the total effect of C on A?

- $C \Rightarrow A$ (correlating)
- $C \Rightarrow B \Rightarrow A$ (correlating)

Estimates of Direct and Indirect Effects



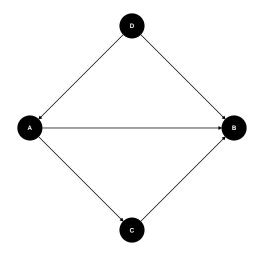
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Pathways:

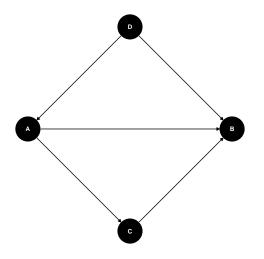
- $C \Rightarrow A$ (correlating)
- $C \Rightarrow B \Rightarrow A$ (correlating)

In this case, we would **not** want to include B as it would block the second pathway representing an indirect effect of *C* on *A*.

 $lm(A \sim C)$

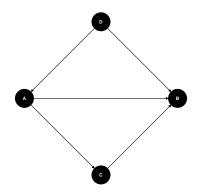


Goal: study the direct effect of A on B.



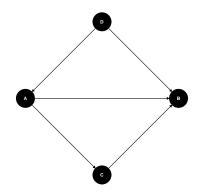
Goal: study the direct effect of A on B.

Need to block all other pathways between A and B.

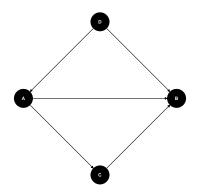


Pathways:

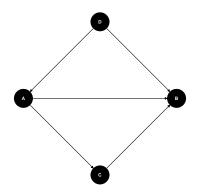
• $A \Rightarrow B$ (correlating), effect of interest.



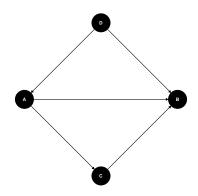
- ullet $A\Rightarrow B$ (correlating), effect of interest.
- $\bullet \ B \Leftarrow D \Rightarrow A$



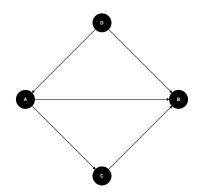
- ullet $A\Rightarrow B$ (correlating), effect of interest.
- $B \leftarrow D \Rightarrow A$ (correlating)



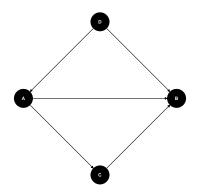
- $A \Rightarrow B$ (correlating), effect of interest.
- $B \Leftarrow D \Rightarrow A$ (correlating) Include D to block!



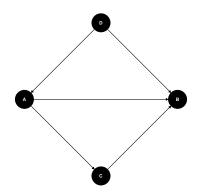
- $A \Rightarrow B$ (correlating), effect of interest.
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- $\bullet \ A \Rightarrow C \Rightarrow B$



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- $B \Leftarrow D \Rightarrow A$ (correlating) Include D to block!
- $A \Rightarrow C \Rightarrow B$ (correlating)



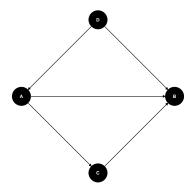
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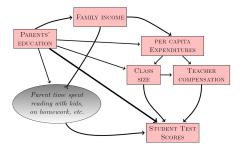
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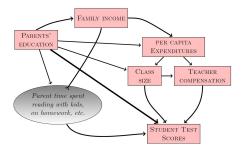
 $lm(B \sim A + C + D)$



To study the **total effect** of A on B, we would use $lm(B \sim A + D)$.

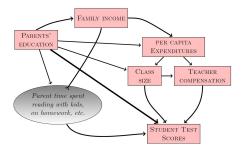


Effect of per-captita expenditures on Student Test Scores:



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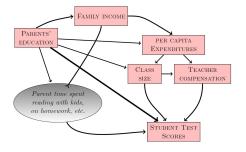
- Include Class Size?
- Include Teacher Compensation?



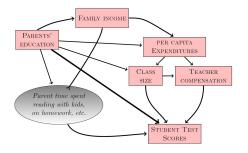
Effect of per-captita expenditures on Student Test Scores:

- Include Class Size?
- Include Teacher Compensation? (No and No)

Per-captia expenditures ⇒ (Class Size, Teacher Compensation) ⇒ Test Scores



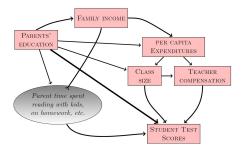
Include Parents' Education?



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Consider the path:

 $Per\text{-}capita\ expenditures \Leftarrow Parents'\ education \Rightarrow Test\ scores$

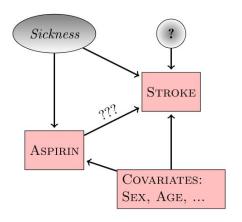


Include Parents' Education?

Consider the path:

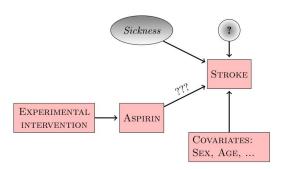
Per-capita expenditures \Leftarrow Parents' education \Rightarrow Test scores Include Parents' education to block the path!

Experiments



How do we measure "sickness' to block the backdoor pathway?

Experiments revisited



Randomly assigning aspirin (treatment) eliminates the connection between Sickness and Aspirin!