Models for Data with Zero Inflation

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology

Learning Objectives

- Be able to fit models to response data with lots of zeros (hurdle and zero-inflated models)
- Be able to describe these models and their assumptions using equations and text and match parameters in these equations to estimates in computer output.

Zero-Inflation

Zero-inflation deals with response data, Y_i, not predictors, X_i.

Zero inflation has received the most attention for count data:

- Covered in Zuur et al. Ch 11
- Kery Ch. 14

Also relevant to:

- Binary data (occupancy models, Kery Ch 20)
- Continuous data (e.g., Friederichs et al. 2011. Oikos 120:756-765)

Abundance Data and Zero Inflation

{From of Matt Russell, UMN}

Top 4 reasons why you might get a 0 when counting critters?

- Sites are not suitable for the species
- Density effects: a site is suitable, but unoccupied
- Design errors: sampling for too short of a time period, or during the wrong times
- Observer error: some species are difficult to identify/detect

Sampling and modeling macroinvertebrates

- Mayflies sampled using stratified random sampling along the Upper Mississippi River
- Characterized by a low-flow environment
- Samples collected with a 23 cm x 23cm sampler
- 43% of sample locations yielded zero mayflies

Gray 2005. *Ecol Model* 185:1-12

Univ. of Michigan

Center for Coastal Resources Management

Some examples: ingrowth of trees in a forest inventory

US Forest Service

- We don't measure all trees when sampling
- Typically establish a minimum diameter to sample (say 5.0 inches DBH)

PLOTID	ForestType	Year1	Year2	Number of ingrowth trees ha^{-1}
1	Aspen	2010	2015	0
2	Red pine	2010	2015	0
3	Aspen	2010	2015	20
4	Red Pine	2010	2015	0
5	Red Pine	2010	2015	15

Zeros and common statistical distributions

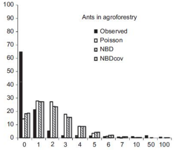
Count data:

- Poisson and Negative Binomial distributions allow for zeros, i.e., $P(Y = 0) \neq 0$.
- Need to ask, are there more zeros than expected for a Poisson($\hat{\lambda}$) or NegBin($\hat{\lambda}, \hat{\theta}$) distribution?

For continuous data:

- We do not expect a "piling" up of zeros
- We can apply "mixture models" (similar to the models you will here see for count data)
- For an example, see: Friederichs et al. 2011. Oikos 120:756-765. (on Moodle)

How can we determine if we have excess zeros?



Sileshi 2008 (on Moodle)

- Compare predicted and observed number of 0's (could use for a Goodness-of-fit test)
- Can also test for overdispersion (variation $>$ mean?)

Modeling Zero-Inflated Data

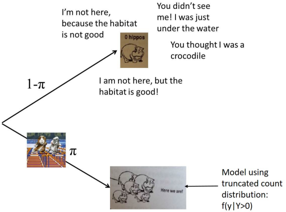
What do we do if we have zero-inflation?

- Hurdle models: model presence-absence (0 non-zero) and counts given presence
- Mixture models: allow for multiple ways to get a 0

For the in-class exercise, we will focus on the latter approach.

Hurdle Models

Group all 0's into a single category:



Hurdle: positive counts arise if you exceed some threshold (with probability π)

Hurdle Models

1. Presence-absence subcomponent:

$$Z_i = \left\{ \begin{array}{ll} 0 \text{ when } y = 0 & \text{occurs with probability } (1-\pi) \\ 1 \text{ when } y > 0 & \text{occurs with probability } \pi \end{array} \right\}$$

Can model Z_i using using logistic regression to allow presence-absence to depend on covariates

2. Count model subcomponent:

Model the non-zero data (using truncated distribution models)

- Poisson or negative binomial, modified to exclude the possibility of a 0

Can do this in two steps or use a single modeling framework (see Hurdle models Ch 11.5 in Zuur et al).

The non-zeros

Truncated distributions for non-zero count data:

$P(Y = y|Y > 0) = \frac{P(Y=y)}{P(Y>0)} = \frac{f(y)}{(1-f(0))}$

remember, $P(A|B)$=P(A and B)/P(B)

A truncated Poisson would look like...

$$P(Y = y|y > 0) = \frac{\frac{e^{-\lambda}\lambda^y}{y!}}{1-e^{-\lambda}}$$

We can incorporate covariates, using: $\log(\lambda) = \beta_0 + \beta_1 x + \ldots$

Note, however:

- We are modeling $E[Y|X, Y > 0] = \lambda_i$ and not $E[Y|X]$
- Need to be careful when plotting fitted model or constructing Bayesian p-values
- See Zuur et al. p. 288 for expressions for $E[Y|X]$ and $Var[Y|X]$

The non-zeros

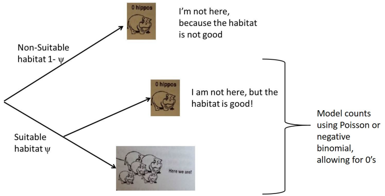
For continuous data:

- Log-normal, gamma distributions live on $(0, \infty)$ (so no need to truncate these)
- Or, can use truncated distributions (e.g., Normal) = $\frac{f(y)}{1-F(0)}$ where $F(y) = P(Y \leq y)$

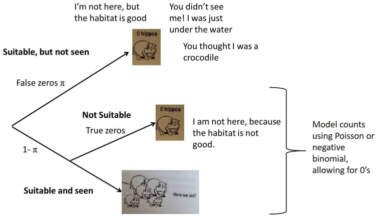
Which function in R is used to determine $F(Y)$? `pnorm`!

Mixture Model: Suitable and Non-Suitable Habitat (Kery)

Two ways to get a 0:



Mixture Models: true and false zeros (Zuur et al)



Reality

Zero-inflation:

- Kery suggests we think of the extra zeros as arising from non-suitable habitat
- Zuur et al. suggests we view the extra zeros as suitable habitat where species are not detected

Assigning meaning to the zero-inflation process can in some cases be useful, but it also requires a leap of faith!

See comments on this blog: https://statisticalhorizons.com/zero-inflated-models

ZIP model: Zero-inflated Poisson

Probability Mass Function: $f(y) = \frac{e^{-\lambda}\lambda^y}{y!}$

Let: π be the probability of a zero-inflated response

ZIP model (Zuur):

$$P(Y = y) = f(y) = \begin{cases} \pi + (1-\pi)e^{-\lambda} & \text{if } y = 0 \\ (1-\pi)\frac{e^{-\lambda}\lambda^y}{y!} & \text{if } y = 1, 2, 3, \ldots \end{cases}$$

Get a 0 two ways:

- Zero-inflated process leads to a 0, occurs with probability π
- Non-zero inflated 0, occurs with probability $(1-\pi)f(0)$

Non-zero responses: $(1-\pi)f(y)$

ZIP model: Zero-inflated Poisson

Zuur and `zeroinfl` function in `pscl` R package:

- Parameterizes in terms of π = the probability of a zero-inflated response

Kery:

- Parameterizes in terms of $\psi = 1 - \pi$ = the probability of a NON zero-inflated response

ZIP model (Zuur and `zeroinfl`):

$$P(Y = y) = f(y) = \begin{cases} \pi + (1-\pi)e^{-\lambda} & \text{if } y = 0 \\ (1-\pi)\frac{e^{-\lambda}\lambda^y}{y!} & \text{if } y = 1, 2, 3, \ldots \end{cases}$$

ZIP model (Kery):

$$P(Y = y) = f(y) = \begin{cases} 1 - \psi + \psi e^{-\lambda} & \text{if } y = 0 \\ \psi\frac{e^{-\lambda}\lambda^y}{y!} & \text{if } y = 1, 2, 3, \ldots \end{cases}$$

ZINB model: Zero-inflated Negative Binomial

Probability Mass Function: $f(y) = \binom{y+\theta-1}{y}\left(\frac{\theta}{\mu+\theta}\right)^\theta\left(\frac{\mu}{\mu+\theta}\right)^y$

ZINB model (Zuur et al):

$$f(y) = \begin{cases} \pi + (1-\pi)\left(\frac{\theta}{\mu+\theta}\right)^\theta & \text{if } y = 0 \\ (1-\pi)\binom{y+\theta-1}{y}\left(\frac{\theta}{\mu+\theta}\right)^\theta\left(\frac{\mu}{\mu+\theta}\right)^y & \text{if } y = 1, 2, 3, \ldots \end{cases}$$

ZINB model (Kery):

$$f(y) = \begin{cases} 1 - \pi + \pi\left(\frac{\theta}{\mu+\theta}\right)^\theta & \text{if } y = 0 \\ \pi\binom{y+\theta-1}{y}\left(\frac{\theta}{\mu+\theta}\right)^\theta\left(\frac{\mu}{\mu+\theta}\right)^y & \text{if } y = 1, 2, 3, \ldots \end{cases}$$

Fitting Models in R

We can use the `zeroinfl` function in the `pscl` package in R to fit:

- Both types of models (Hurdle model, mixture)
- With both the Poisson and Negative Binomial distributions (see in class exercise)

Can also code models in JAGS (see Kery Ch 14) and fit using other packages (e.g. `glmmTMB`)

zeroinfl versus Kery

Remember:

- `zeroinf`: models probability of a zero-inflated response (i.e., "false" zero) = π_i
- Kery: models the probability of a NON zero-inflated response (i.e., probability of a "true" zero or a count > 0) = ψ_i

As a result, the sign of the coefficients will differ between the two approaches.

Model Comparisons

Can compare Poisson, Negative Binomial, Zero-inflation models

- Using AIC
- Graphs of observed vs expected proportion of zeros in a dataset
- Graphs of the sample mean–variance relationship.

My experience, and that of others, is that a Negative Binomial model (without zero-inflation) often "wins" (but not always)

- See Warton (2005) on Canvas, as well as Gray (2005), Sileshi (2008)

Also, zero-inflated negative binomial models can sometimes be difficult to fit (past homework problem)