Linear Regression Review

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology

Objectives

Review important statistical concepts within the context of linear regression using simulated data:

- Sampling Distributions
- T-tests for regression coefficients
- Confidence intervals
- P-values
- How to check assumptions

Sustainable trophy hunting of African Lions

Whitman et al. 2014 Nature 428:175-178

Important to know the age of male lions to help manage trophy hunting

- Removing males over 6 has little effect on social structure
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How, from afar, can we tell the age of a lion?

Its in the Nose!

After about 3 years of age, the fleshy part of the nose begins to freckle or become liver spotted. As the lion ages, more pigmentation appears until the nose is entirely black by about 8 years. A general rule of thumb is that by 6 years noses are >50% black.

Data are contained in `abd` library of Program R:

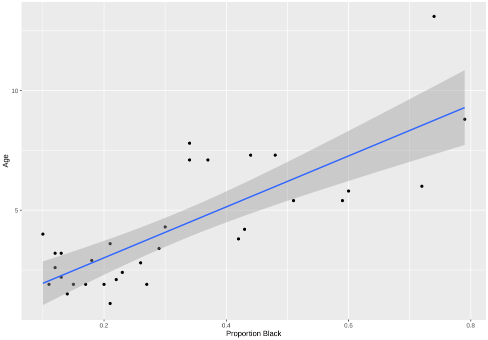
```r
install.packages("abd")  # only if not installed (do once)
```

```r
library(abd) # Each time you want to access the data
data(LionNoses)
head(LionNoses)
```

```
  age proportion.black
1 1.1             0.21
2 1.5             0.14
3 1.9             0.11
4 2.2             0.13
5 2.6             0.12
6 3.2             0.13
```

Lion’s Noses

```
library(ggplot2)
ggplot(LionNoses, aes(proportion.black, age)) +
  geom_point() +
  geom_smooth(method = "lm", formula = y ~ x)+ xlab("Proportion Black"
  ylab("Age")
```



Linear Regression Assumptions

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$
$$\epsilon_i \sim N(0, \sigma^2)$$

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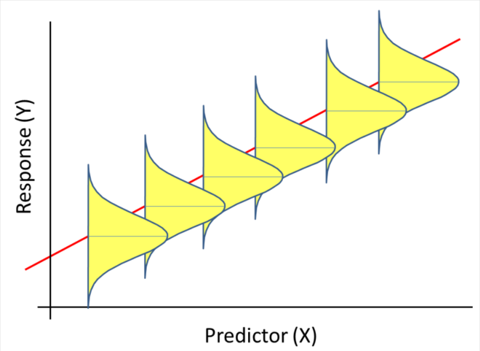
- **E**xistence (we observe random variables that have finite variance; we won't worry about this one)
- **Gauss**: ϵ_i come from a Normal (Gaussian) distribution

Regression Assumptions

- We specify a probability distribution for $Y_i|X_i \sim N(\mu_i, \sigma^2)$
- We have a model for the mean $\mu_i = \beta_0 + \beta_1 X_i$



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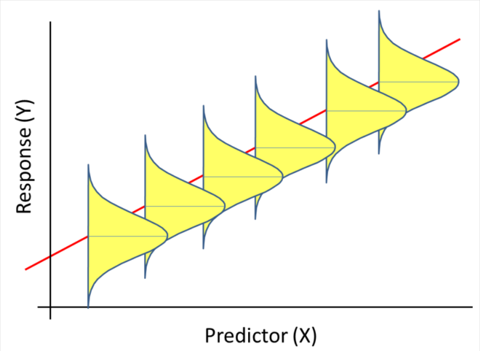


- How are these assumptions reflected in the figure? How can we evaluate the assumptions with our data?

Residuals Versus Fitted

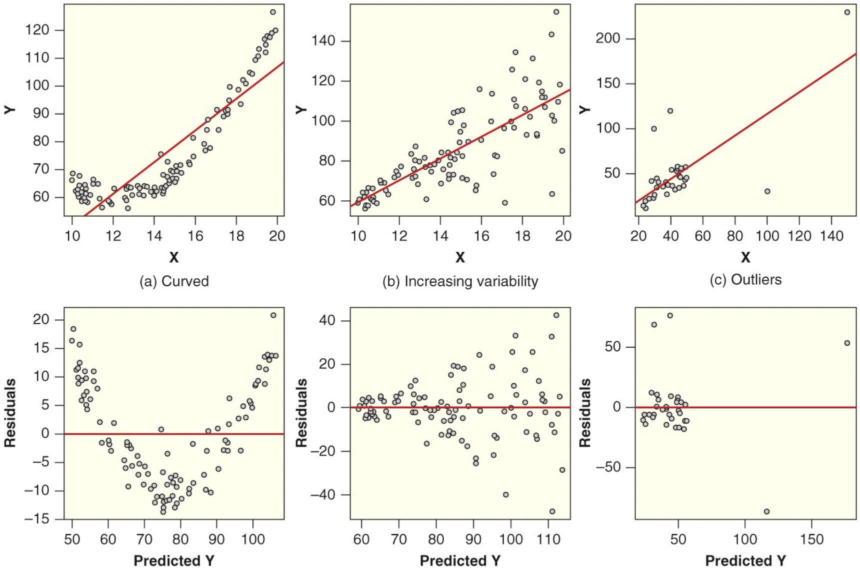
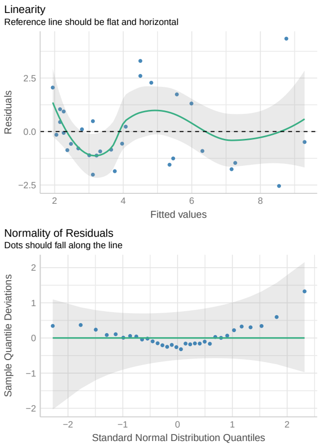


Figure 10.2

Graphical Check

```
lm.nose <- lm(age ~ proportion.black, data = LionNoses)
performance::check_model(lm.nose,
  check = c("linearity", "homogeneity", "qq", "normality"))
```



Interpretation: Intercept, Slope, t-tests and p-values, Residual Standard Error ($\hat{\sigma}$), R^2

```r
summary(lm.nose)
```

```
Call:
lm(formula = age ~ proportion.black, data = LionNoses)

Residuals:
    Min      1Q  Median      3Q     Max
-2.5449 -1.1117 -0.5285  0.9635  4.3421

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.8790     0.5688   1.545    0.133
proportion.black  10.6471     1.5095   7.053 7.68e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.669 on 30 degrees of freedom
Multiple R-squared:  0.6238,    Adjusted R-squared:  0.6113
F-statistic: 49.75 on 1 and 30 DF,  p-value: 7.677e-08
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Interpretation: Intercept and Slope

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But, proportion black < 1 for all lions.

? = $\frac{\triangle age}{\triangle 0.1 Proportion.Black}$ = 1.065.

$H_0 : \beta_1 = 0$ vs. $H_A : \beta_1 \neq 0$?

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See: AP stats guy videos in Ch. 1 of the textbook

SE $\hat{\beta}$

Think of many repetitions of:

- Collecting a new data set (of the same size)

SE $\hat{\beta}$

Think of many repetitions of:

- Collecting a new data set (of the same size)
- Fitting the same regression model

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SE = standard deviation of the sampling distribution!

Lets explore through simulation!

Simulation

Lets first generate a single data set consistent with our fitted model using the following code.

```
# Sample size of simulated observations
  n<-32

# Use the observed proportion.black to simulate obs.
  p.black<-LionNoses$proportion.black

# Use the estimated parameters to simulate data.
#  - can get these from the regression output
#  sigma<-summary(lm.nose)$sigma # residual variation about the line
#  betas<-coef(lm.nose) # Regression coefficients
  sigma<-1.67 # residual variation
  betas<-c(0.88, 10.65) #betas

# Create random errors (epsilons) and random responses
 epsilon<-rnorm(n,0, sigma) # Errors
 y<-betas[1] + p.black*betas[2] + epsilon # Response
```

Linear regression using `lm` function

```r
lmfit<-lm(y~p.black)
summary(lmfit)
```

```
Call:
lm(formula = y ~ p.black)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2401 -0.8812 -0.3871  0.9053  3.2192

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.7028     0.5627   3.026  0.00505 **
p.black       8.9392     1.4934   5.986 1.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.651 on 30 degrees of freedom
Multiple R-squared:  0.5443,    Adjusted R-squared:  0.5291
F-statistic: 35.83 on 1 and 30 DF,  p-value: 1.45e-06
```

Sampling Distribution

Use a for loop to:

- Generate 5000 data sets using the same code
- Fit a linear regression model to each data set
- For each fit, store $\hat{\beta}$

In-class exercise

Sampling distribution

When conducting hypothesis tests or constructing confidence intervals, we will work with the distribution of:

$$\frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$$

Sampling distribution of the t-statistic

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Think of many repetitions of:

- Collecting a new data set (of the same size)
- Fitting the regression model
- Calculating: $t = \frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)}$

A histogram of the different t values should be well described by a Student's t-distribution with $n - 2$ degrees of freedom.

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Helpful hints:

- β_1 = true value used to simulate the data, `coef(lm.nose)[2]` = 10.6471
- $\hat{\beta}_1$ is extracted using: `coef(lmfit)[2]`
- $\widehat{SE}(\hat{\beta}_1)$ is extracted using `sqrt(vcov(lm.temp)[2,2])`

In-class exercise

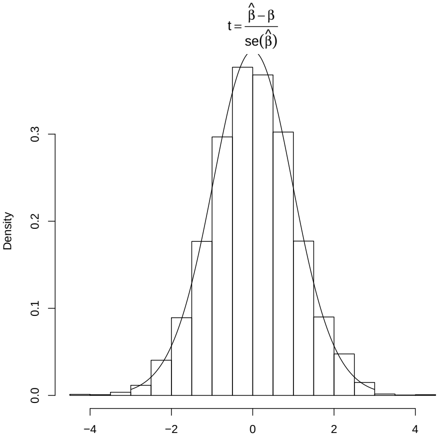
My code

My code

```
nsims<-5000 # number of simulations
tsamp.dist<-matrix(NA, nsims,1) # matrix to hold results
for(i in 1:5000){
  epsilon<-rnorm(n,0,sigma) # Errors
  y<-betas[1] + betas[2]*p.black + epsilon # Response
  lm.temp<-lm(y~p.black) # lm

# Here is our statistic, calculated for each sample
  tsamp.dist[i]<-(coef(lm.temp)[2]-betas[2])/sqrt(vcov(lm.temp)[2,2])
}
# Plot results
hist(tsamp.dist, xlab="",
     main=expression(t==frac(hat(beta)-beta, se(hat(beta)))), freq=FAL
tvalues<-seq(-3,3, length=1000) # xvalues to evaluate t-distribution
lines(tvalues,dt(tvalues, df=30)) # overlay t-distribution
```

Sampling distribution of t-statistic



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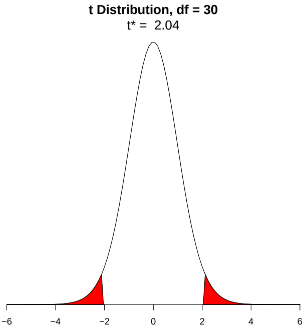
- The parameter is *fixed*, but the endpoints of the interval are *random*

Confidence Interval

$$\frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$$



Note:

$t_{0.025,n-2}$ = `qt(p=0.025, df=30)` = -2.04

$t_{0.975,n-2}$ = `qt(p=0.975, df=30)` = 2.04

Confidence Interval

$$P(t_{0.025,n-2} < \frac{\hat{\beta} - \beta}{\widehat{SE}(\hat{\beta})} < t_{0.975,n-2}) = 0.95$$

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$$P(\hat{\beta} + 2.04\widehat{SE}(\hat{\beta}) > \beta > \hat{\beta} - 2.04\widehat{SE}(\hat{\beta})) = 0.95$$

So, take $(\hat{\beta} - 2.04\widehat{SE}(\hat{\beta}), \hat{\beta} + 2.04\widehat{SE}(\hat{\beta}))$ as the the 95% confidence interval.

`confint` function

```r
confint(lm.nose)
```

```
                    2.5 %    97.5 %
(Intercept)      -0.2826733  2.040686
proportion.black  7.5643082 13.729931
```

What is wrong with the following interpretation?

$$P(7.56 \leq \beta \leq 13.72) = 0.95$$

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We are 95% sure that the true slope (relating proportion of nose that is black and age) falls between 7.56 and 13.73.

Explore CIs through simulation

Simulate another 5000 data sets in R:

Determine 95% confidence limits for each data set and examine whether or not the CI contains the true β.

My Code

My Code

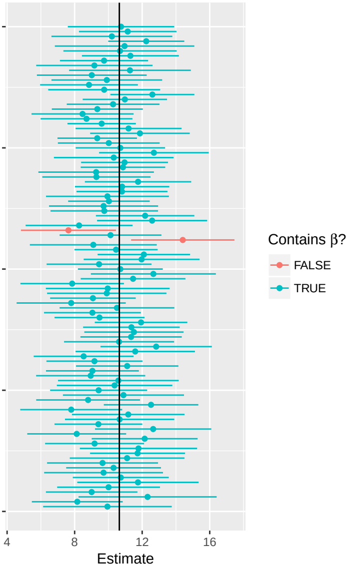
```
nsims<-5000 # number of simulations
Limits<-matrix(NA,nsims,2) # to hold results
beta.hats<-matrix(NA,nsims,1) # to hold estimates
for(i in 1:nsims){
  epsilon<-rnorm(n,0, sigma) # Errors
  y<-betas[1] + betas[2]*p.black + epsilon # Response
  lm.temp<-lm(y~p.black)

# Beta.hat & Confidence limits
  beta.hats[i]<-coef(lm.temp)[2]
  Limits[i,]<-confint(lm.temp)[2,]
}
# True parameter in interval?
I.in<-betas[2]>=Limits[,1] & betas[2] <= Limits[,2]

# Coverage
sum(I.in)/nsims
```

```
[1] 0.9444
```

First 100 simulations



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P-values

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Is the value we get for $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)}$ = 7.053 consistent with $H_0 : \beta_1 = 0$?

P-values

If the null hypothesis, $H_0 : \beta_1 = 0$, is true, then:

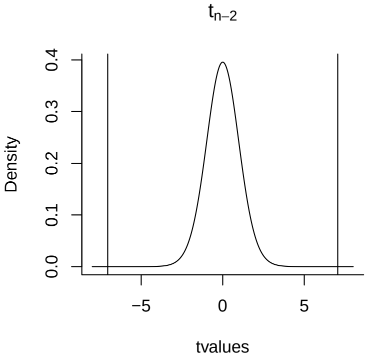
$$t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$$

Is the value we get for $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)}$ = 7.053 consistent with $H_0 : \beta_1 = 0$?

- Overlay $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)}$ = 7.053 on a t_{n-2} distribution
- Determine the probability of getting a t-statistic as or more extreme as the one we observed.

Hypothesis test

- t-distribution tells about the sampling distribution of $t = \frac{\hat{\beta}_1 - 0}{\widehat{SE}(\hat{\beta}_1)} \sim t_{n-2}$ *when the null hypothesis is true*
- our t-statistic falls in the tail of this distribution (so, the Null hypothesis is unlikely to be true!)

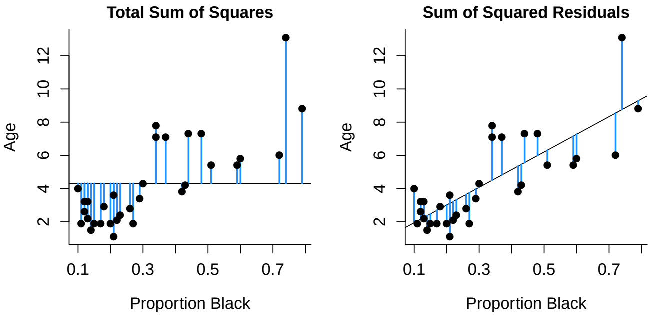


R^2

```r
summary(lm.nose)
```

```

Call:
lm(formula = age ~ proportion.black, data = LionNoses)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.5449 -1.1117 -0.5285  0.9635  4.3421 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)        0.8790     0.5688   1.545    0.133    
proportion.black  10.6471     1.5095   7.053 7.68e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.669 on 30 degrees of freedom
Multiple R-squared:  0.6238,    Adjusted R-squared:  0.6113 
F-statistic: 49.75 on 1 and 30 DF,  p-value: 7.677e-08
```

Sum of Squares



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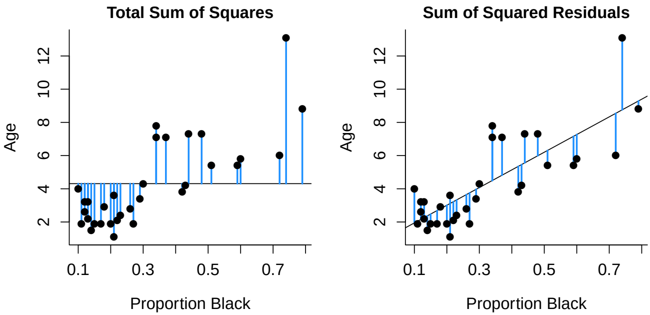

SST (Total sum of squares) = $\sum_i^n (Y_i - \bar{Y})^2$

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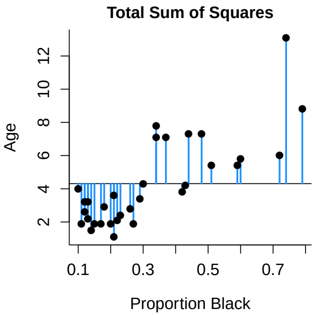

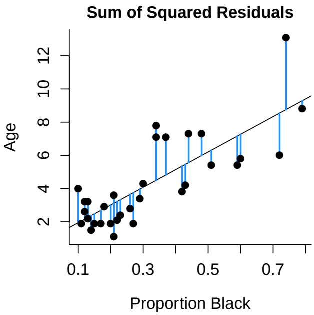

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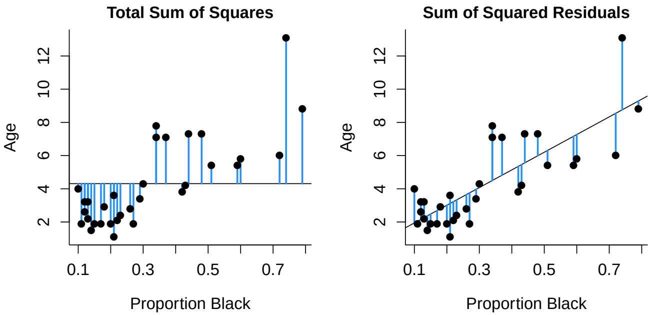

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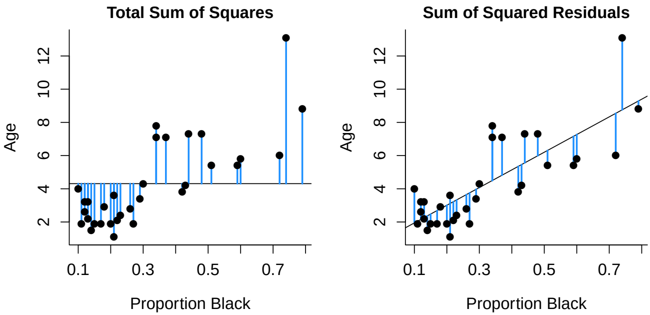

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SSR (sum of squares regression) = SST - SSE = $\sum_i^n (\hat{Y}_i - \bar{Y})^2$

$R^2 = \frac{SST-SSE}{SST} = \frac{SSR}{SST}$ = proportion of the variation explained by the linear model.

Residual standard error = $\hat{\sigma}$

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\epsilon_i \sim N(0, \sigma^2)$$

- σ describes the amount of variability about the regression line

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- Listed as `Residual Standard Error` in R output from `summary` function
- $n - p$ since we lose one degree of freedom for each parameter we estimate

Residual Standard Error

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We expect 95% of the observations to fall within 2*1.669 of the regression line.