

# Why Become a Bayesian

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology



# Why Bayes?

Some areas where Bayesian methods are a natural fit:

- Mixed effects models
- Generalized additive models (smoothing splines)
- Mixture distributions
- Models with conditionally independent (sometimes latent) random variables

# Random effects Models

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Frequentist methods (formally think of parameters as fixed, not random):

- Use Empirical Bayes to “predict” random parameters (BLUPs)
- Not clear how to account for uncertainty in these predictions

# Smoothing splines

$$PSS = \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \lambda \int f''(x) dx$$

- Penalties can be justified as priors (for amount of smoothness)
- Frequentist methods appeal to Bayesian philosophy when constructing uncertainty intervals

# Conditionally Independent Variables

Models with conditionally independent (sometimes latent) random variables

- Population time series with measurement error
- Mark-recapture
- Occupancy and N-mixture models
- Integrated population models

# State Space Models

State-space models (e.g., population time series data):<sup>1</sup>

$$N_{t+1} = N_t \exp(a + bN_t + \epsilon_t) \text{ (equation for changes in state)}$$

$$\epsilon_t \sim N(0, \sigma_p^2) \text{ captures "process variation"}$$

We count animals, leading to  $C_t \neq N_t$ , but our interest is in the latent variable,  $N_t$  (and  $a, b$ ).

---

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Observation model connects  $C_t$  to  $N_t$ :

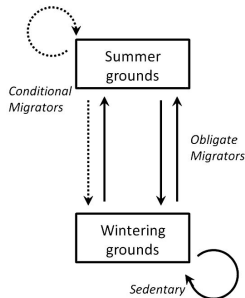
- $\log(C_t) \sim N(\log(N_t), \sigma_o^2)$  (observation process)
- or  $C_t \sim \text{Binomial}(N_t, p)$  (observation process)

---

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How does winter severity influence the probability that white-tailed deer will migrate to their winter range?<sup>2</sup>

- Obligate migrators migrate every year
- Facultative migrators migrate when winters are severe

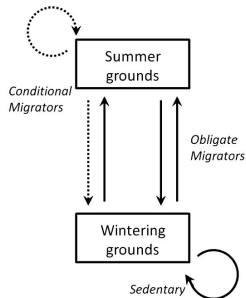


<sup>2</sup>Fieberg, J. and Conn, P. 2014. A hidden Markov model to identify and adjust for selection bias: An example involving mixed migration strategies. Ecology and Evolution 4(10):1903-1912.

$$y_{ij} = \begin{cases} 1 & \text{if deer } i \text{ migrates in year } j \\ 0 & \text{otherwise} \end{cases}$$

- $\Pr(\text{migrate} \mid \text{obligate migrator}) = 1$
- $\Pr(\text{migrate in year } j \mid \text{facultative migrator}) = \theta$

$$\theta_j = \text{plogis}(\beta_0 + \beta_1 WSI_j)$$

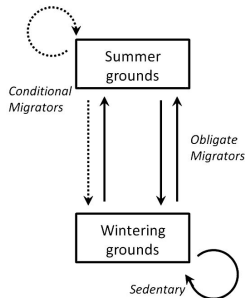


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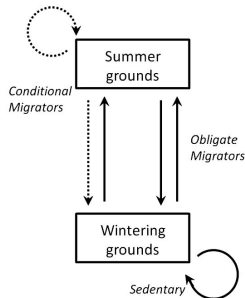
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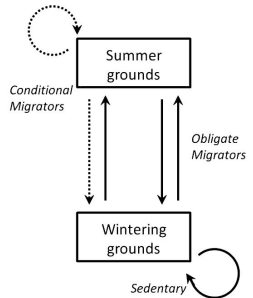
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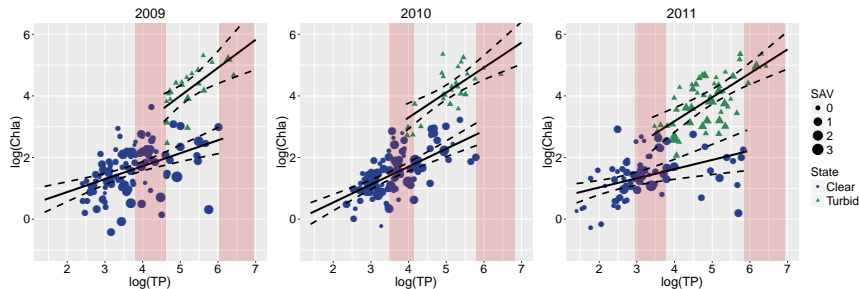
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$$Y_{ij} \sim \text{Bernoulli}(\Theta_{ij}); \Theta_{ij} = z_i + (1 - z_i)\theta_{ij}$$



# Mixture Distributions

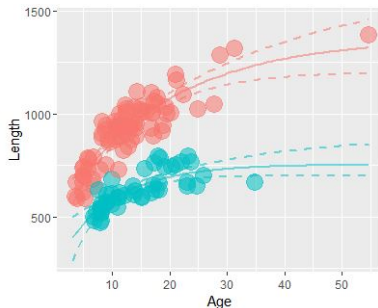


Relationship between Chla and Phosphorus depends on lake state (a latent variable)

$$\text{chla}_i = \alpha_0 + \tau S_i + b_0(1 - S_i)\log(P_i) + b_1 S_i \log(P_i) + \epsilon_i$$

$$S_i \sim \text{Bern}(p_i) = \begin{cases} 1 & \text{if the lake is turbid} \\ 0 & \text{if the lake is clear} \end{cases}$$

# Mixture Distributions: Cubera Snapper



$z_i = 1$  if morph type I and 0 if morph type II;  $z_i \sim dbern(p)$

# Kery and Schaub's book: Bayesian Population Analysis using WinBugs

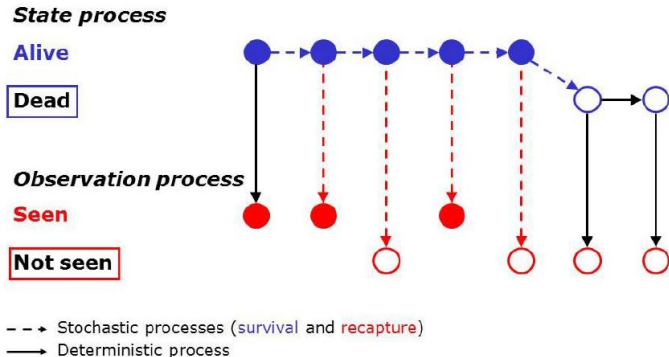


Fig. 7-1: Example of the state and observation process of a marked individual over time for the CJS model. The sequence of true states in this individual is  $z = [1, 1, 1, 1, 1, 0, 0]$ , and the observed capture-history is  $y = [1, 1, 0, 1, 0, 0, 0]$ .

# Maximum Likelihood versus Bayesian Methods

Likelihood formulation requires thinking of all the ways to get a capture-history (“alive, but not seen” or “dead so can’t be detected”)

- Think back to probability trees on your earlier homework

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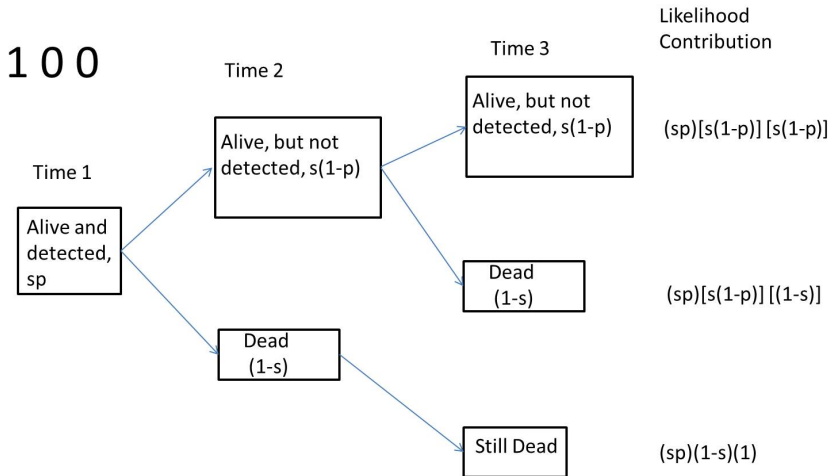
Bayesian methods are easier (I think), because we can break the problem down into conditionally independent events, modeled as Bernoulli random variables:

- Survival, given status (alive/dead) at previous time point
- Detection given current status (alive/dead)

Capture history 1 0 0



# Capture history 1 0 0



$$(sp)[s(1-p)][s(1-p)] + (sp)[s(1-p)][(1-s)] + (sp)(1-s)(1)$$

# Bayesian formulation: 1 0 0

Survival:

- $z_{i,t} = 1$  if individual  $i$  is alive at time  $t$  (0 if dead).
- $z_{i,t} | z_{i,t-1} \sim \text{Bernoulli}(sz_{i,t-1})$  (probability = 0 if already dead)

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## Detection:

- $y_{i,t} = 1$  if individual  $i$  is seen at time  $t$  (0 otherwise)
- $y_{i,t} \sim \text{Bernoulli}(pz_{i,t})$  (probability = 0 if dead)

# JAGS: 1 0 0

```
for (i in 1:nindividuals){  
  # Survive first time period to be detected?  
  z[i,1] ~ dbern(s)  
  
  # Detected given alive?  
  y[i,1] ~ dbern(z[i,1]*p)  
  
  for(t in 2:3){  
    # Survive (if alive previous time point z[i-1]=1)  
    z[i,t] ~ dbern(s*z[t-1])  
  
    # Detected given alive  
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  }  
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  }  
}
```

$z_{i,t}$  are only partially observed:

- If seen at time  $t$ , know  $z_{i,t} = 1$  (for all time points prior to  $t$ )
- If not seen at  $t$ ,  $z_{i,t} = \text{NA}$ .

JAGS will estimate/impute the missing  $z_{i,t}$ !

# Occupancy Models

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The processes affecting presence-absence may differ from those that influence detection | presence.

- $\text{logit}(\psi_i) = \alpha_0 + \alpha_1 x$  where  $\psi_i$  = probability species is present
- $\text{logit}(p_i) = \beta_0 + \beta_1 z$  where  $p_i$  = probability of detecting a species given it is present



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Two options for model fitting:

- Use a probability tree to determine all the ways to observe particular **capture histories**, then use maximum likelihood (e.g., `optim`)
- Consider partially observed variables and conditional independencies within a Bayesian formulation.

# Occupancy Models: Maximum Likelihood

Consider a single site and survey:

Define a random variable,  $Y_i$  as:

$$Y_i = \begin{cases} 1 & \text{if the species is observed at site } i \\ 0 & \text{if the species is not observed at site } i \end{cases}$$

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$$P(Y = 0) = P(\text{Not observed} \mid \text{present})P(\text{present}) + P(\text{not observed} \mid \text{not present})P(\text{not present}) = (1 - p)\psi + 1 - \psi = 1 - \psi p$$

## Likelihood: single visit

$$L = \prod_i (\psi p)^{Y_i} (1 - \psi p)^{(1-Y_i)}$$

- Likelihood contribution of a 1 is  $\psi p$
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Note: with single visits (and, with constant detection and occupancy probabilities), we cannot estimate both  $\psi$  and  $p$



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With multiple visits we can...

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Again,  $z_i$  are only partially observed:

- If see a species, know the site is occupied ( $z_i = 1$ )
- If do not see a species,  $z_i = \text{NA}$  (JAGS will impute/estimate)

# JAGS

```
for(i in 1:nsites){  
  # Occupied Y/N  
  z[i] ~ dbern(psi)  
  
  # Detected at a site (depends on psi and p)  
  y[i] ~ dbern(z[i]*p)  
}
```

But, we still need multiple visits to identify  $\psi$  and  $p$ .

# Multiple Visits: Occupancy model

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There is also a cost to visiting sites multiple times

# Multiple visits: start with 2

4 Possibilities:

- $(0,0)$
- $(0,1)$
- $(1,0)$
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Write down the probability of each **capture history**.

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$$(0,0) = \psi(1 - p)(1 - p) + (1 - \psi)$$

$$(0,1) \text{ or } (1,0) = \psi p(1 - p)$$

$$(1,1) = \psi p^2$$

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Assume we have

- $n_{00}$  of these (0,0)
- $n_{01}$  of either (0,1) or (1,0)
- $n_{11}$  of (1,1)

$$L \propto [\psi(1-p)(1-p) + (1-\psi)]^{n_{00}} [\psi p(1-p)]^{n_{01}} [\psi p^2]^{n_{11}}$$

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Could use `optim` to solve for  $p$  and  $\psi$

(see also Excel spreadsheet Todd Arnold put together under **Handouts**, Occupancy tab)



# JAGS

```
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  z[i] ~ dbern(psi[i])  
}  
for(j in 1:nvisits){  
  # Detected at a site (depends on psi and p)  
  y[i,j] ~ dbern(z[i]*p)  
}
```

# Willow Tit Example

In-class Exercise using data from:

Royle, J.A. and R. M. Dorazio. 2008. Hierarchical Modeling and inference in Ecology. Academic Press, New York, NY.



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- Abundance:  $\text{logit}(\psi) = \beta_0 + \beta_1 * \text{elevation}$

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These models are considerably more complex than logistic regression models.

They also require repeated visits to sites.

When is the effort worth it?

# Logistic Regression

When fitting logistic regression models to data that are impacted by perfect detection, we are essentially modeling:

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Consequences (Kery 238):

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$$\text{logit}(\psi p) = \beta_0 + x\beta_1$$

Consequences (Kery 238):

- Estimators of occupancy probabilities will be biased low whenever  $p < 1$
- Estimators of  $\beta$ s, describing covariate relationships with occupancy rates, will be biased toward 0 whenever  $p < 1$

# Logistic Regression

When fitting logistic regression models to data that are impacted by perfect detection, we are essentially modeling:

$$\text{logit}(\psi p) = \beta_0 + x\beta_1$$

Consequences (Kery 238):

- Estimators of occupancy probabilities will be biased low whenever  $p < 1$
- Estimators of  $\beta$ s, describing covariate relationships with occupancy rates, will be biased toward 0 whenever  $p < 1$
- Variables that explain variability in detection probabilities may show up as important predictors of *apparent presence*

# The other side of the coin: Brian McGill's first blog<sup>3</sup>

Welsh AH, Lindenmayer DB, Donnelly CF (2013) Fitting and Interpreting Occupancy Models. PLoS ONE 8: e52015.

- Maximum likelihood can be *hard*
  - Many cases with multiple solutions; answer you get may depend on starting values you supply to the solver

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  - Solutions may converge to the boundary (where  $\psi$  and/or  $p$  were either 0 or 1; theoretically impossible)
  - Problems are worse when data were sparse
- Detection may depend on abundance, so correcting for detection may give you a worse index of abundance.

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## Follow-up<sup>4</sup>

Guillera-Arroita G, Lahoz-Monfort JJ, MacKenzie DI, Wintle BA, McCarthy MA (2014) Ignoring imperfect detection in biological surveys is dangerous: a response to 'Fitting and Interpreting Occupancy Models'. PLoS ONE 9: e99571.

- If  $\psi$  is low ( $< 50\%$ ) and  $p$  is also low, ignoring detection probabilities gives a **better** estimate of  $\psi$  (in terms of mean-squared error).

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<sup>4</sup><https://dynamicecology.wordpress.com/2014/09/15/detection-probabilities-statistical-machismo-and-estimator-theory/>

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- Accounting for detection works best when  $\psi$  is high ( $> 50\%$ ) and  $p$  is low.

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## Follow-up<sup>4</sup>

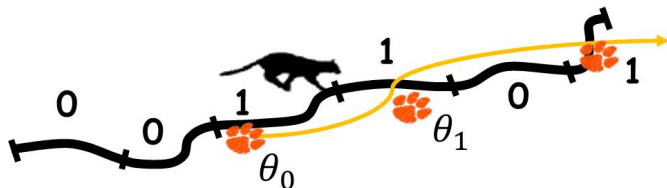
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- If  $\psi$  is low ( $< 50\%$ ) and  $p$  is also low, ignoring detection probabilities gives a **better** estimate of  $\psi$  (in terms of mean-squared error).
- Accounting for detection works best when  $\psi$  is high ( $> 50\%$ ) and  $p$  is low.
- When  $p$  is high ( $> 50\%$  and definitely when  $> 80\%$ ), both methods work equally well regardless of  $\psi$

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# Occupancy and Correlated Data



- $\pi$  = probability species is present to the left of the first segment
- $\theta_0$  = probability species is present given it was not present in the previous replicate
- $\theta_1$  = probability species is present given it was present in the previous replicate

Hines et al. (2010), Hines et al. (2014), Aing et al. (2011)

## Capture history: 0101

From Hines et al. 2014:

$$\Pr(h_i = 0101) = \psi\left([\pi\theta' + (1 - \pi)\theta](1 - p)\theta' + [\pi(1 - \theta') + (1 - \pi)(1 - \theta)]\theta\right)p \times [(1 - \theta')\theta + \theta'(1 - p)\theta']p$$

eqn 1

Previously non-sampled site was occupied or not, first sample occupied or not. . . .

## Capture history: 00

$$\begin{aligned}\Pr(h_i = 00) = & (1 - \psi) + \psi \{ ([\pi\theta' + (1 - \pi)\theta](1 - p)\theta' + [\pi(1 - \theta') \\ & + (1 - \pi)(1 - \theta)]\theta)(1 - p) + [\pi\theta' + (1 - \pi)\theta](1 - p)(1 - \theta') \\ & + [\pi(1 - \theta') + (1 - \pi)(1 - \theta)](1 - \theta) \}\end{aligned}$$

# Bayesian

Conditional Bernoulli random variables:

- $z_i \sim \text{dbern}(\psi_i)$  Site occupied ( $z_i = 1$ ) or not ( $z_i = 0$ )
- $f_i \sim \text{dbern}(z_i\pi)$  segment 0 (not sampled) within site  $i$  occupied “locally” or not.
- $y_{i1} \sim \text{dbern}(z_i f_i \theta' + z_i(1 - f_i)\theta)$  first sampled segment at site  $i$  occupied locally or not.
- $y_{ij} \sim \text{dbern}(z_i y_{ij-1} \theta' + z_i(1 - y_{ij-1})\theta)$  segment  $j$  at site  $i$  occupied locally or not
- $o_{ij} \sim \text{dbern}(y_{ij}p)$  observed on segment  $j$  at site  $i$ .

```

# Likelihood
for(i in 1:nsites){ # M = number of sites
  # Occupied
  z[i] ~ dbern(psi[i])      # Occupied Y/N
  logit(psi[i]) <- b0 + b1*AGR[i] + b2*SeaSTM[i]

  # Site "0" (non-surveyed site) local presence
  first.obs[i]~dbern(z[i]*pi)

  # For site 1
  # y = probability of local presence | site presence (z=1)
  loc.psi[i,1]<-z[i]*theta[1+first.obs[i]]
  y[i,1] ~ dbern(loc.psi[i,1]) # local presence site 1

  # Detection / local abundance
  logit(p[i])<-g0+g1*FNS3km[i]
  obs[i,1]~dbern(p[i]*y[i,1])

  for(j in 2:nsurveys){ # number of visits
    # y = probability of local presence | site presence (z=1)
    loc.psi[i,j]<-z[i]*theta[1+y[i,j-1]]
    y[i,j] ~ dbern(loc.psi[i,j]) # local presence site 1

    # Detection / local abundance
    obs[i,j]~dbern(p[i]*y[i,j])

  }
}

```