Partial regression and partial residual plots

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology

Learning Objective

Understand approaches for visualizing fitted multiple regression models

Visualizing Multiple Regression

$$Y \sim \beta_0 + X_1\beta_1 + X_2\beta_2 + \epsilon$$

β_1 reflects the “effect” of X_1 after accounting for X_2.

How can we visualize this “effect”?

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How can we visualize this "effect"?

- Added variable or partial *regression* plots
- Component + residual or partial *residual* plots

See the paper by Larano and Corcobado (2008) and Section 3.14 in the Book.

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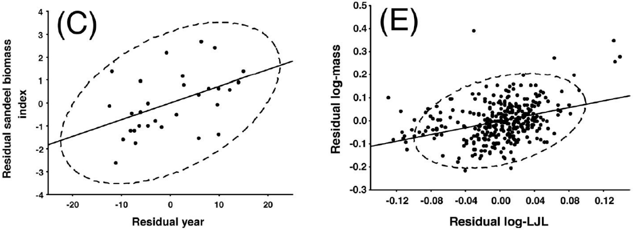
Plots the part of Y not explained by other predictors (i.e., X_{-i}) against the part of X_i not explained by the other predictors (X_{-i}).

Lets us visualize the effect of X_i after accounting for all other predictors.

Partial regression plots



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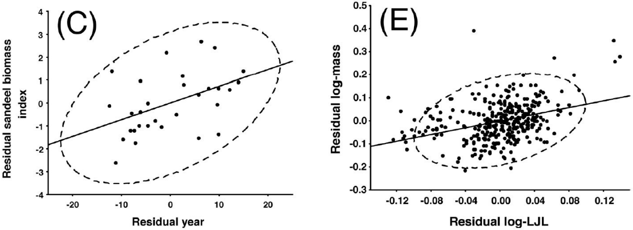

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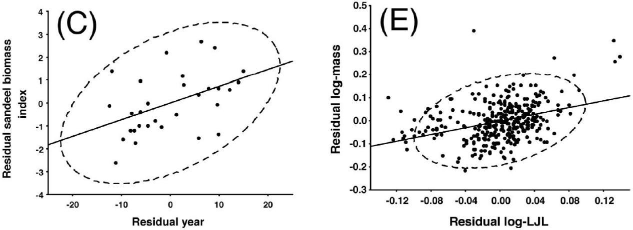

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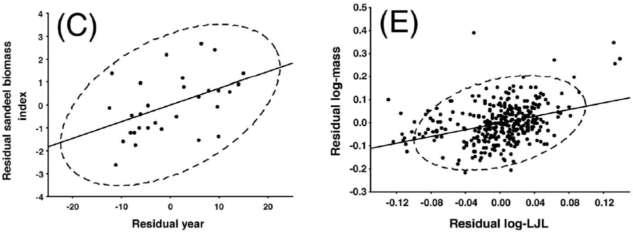

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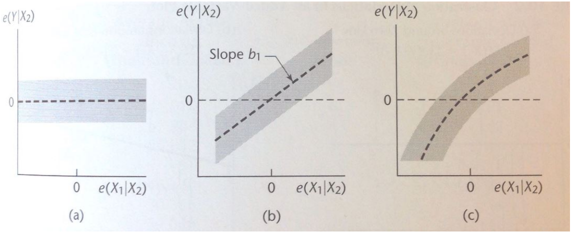
- Tells us about the importance of X_2 (given everything else already in the model)
- Can help with diagnosing non-linearities
- Helps visualize influential points and outliers

Added variable plot for X_1 (with one other predictor, X_2)



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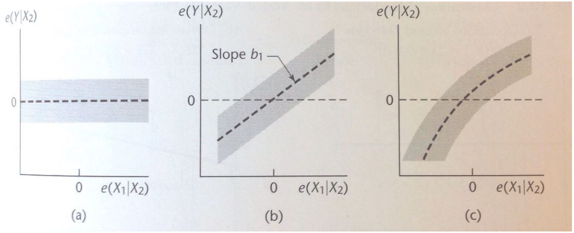

- Panel (a) suggests X_1 provides no additional information useful for predicting Y beyond that contained in X_2
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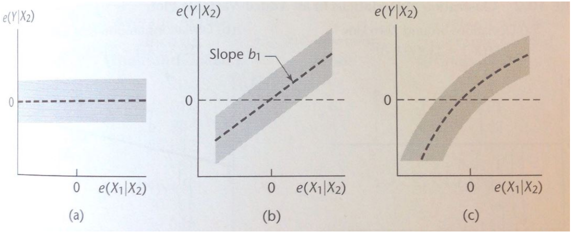

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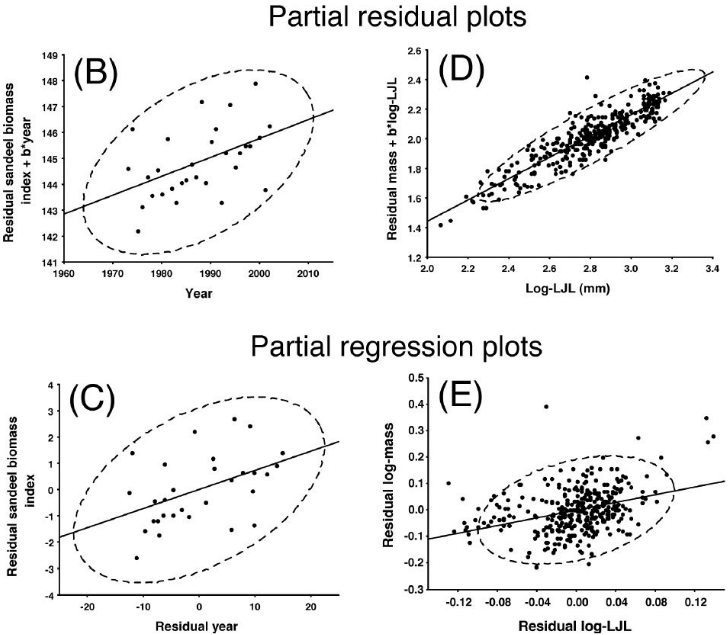
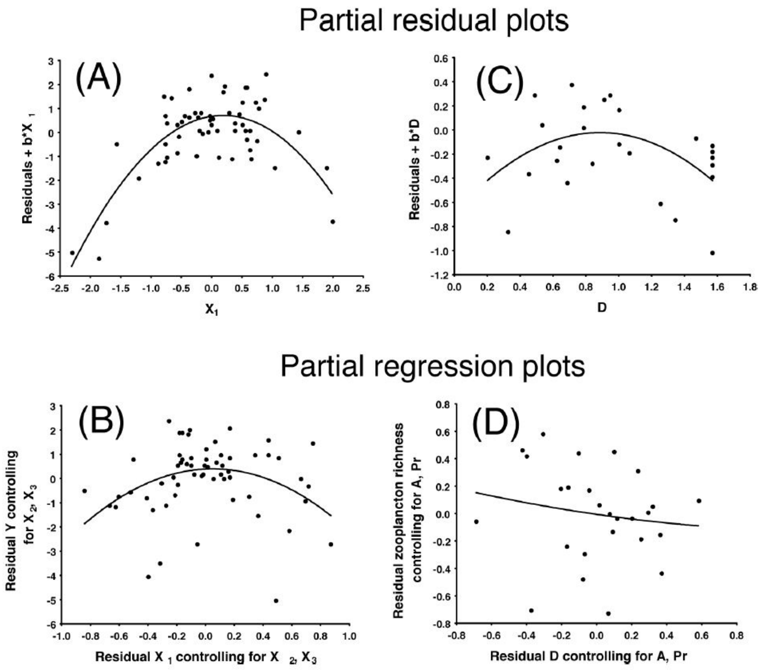
- Panel (b) suggest a linear relationship is appropriate (after accounting for X_2); the slope here is the same as that in the multiple regression model containing both X_1 and X_2
- Panel (c) suggests we may need to allow for a non-linear relationship between X_1 and Y

Component + residual plots or partial residual plot

Plots $X_i\beta_i + \hat{\epsilon}_i$ versus X_i.

- Better for diagnosing non-linearities
- X-axis depicts the scale of the focal variable (rather than the scale residuals)
- Not as good at depicting the amount of variability explained by the predictor (given everything else in the model).
- Easy to generalize to other regression models (see visreg package on Canvas)


Partial residual plots
(B)
Residual sandeel biomass index + b*year
Year
(D)
Residual mass + b*log-LJL
Log-LJL (mm)
Partial regression plots
(C)
Residual sandeel biomass index
Residual year
(E)
Residual log-mass
Residual log-LJL


Partial residual plots
(A)
Residuals + b*X 1
X1
(C)
Residuals + b*D
D
Partial regression plots
(B)
Residual Y controlling for X2, X3
Residual X 1 controlling for X 2, X3
(D)
Residual zooplancton richness controlling for A, Pr
Residual D controlling for A, Pr

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These two types of means are equivalent if there are no categorical predictors in the model.