

# Logistic regression models for binary data

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology



# Outline

- Logistic regression
  - Model formulation
  - Interpretation of regression parameters
- Comparing Models and Assessing Fit
  - Likelihood Ratio tests, AIC
  - Goodness-of-fit testing
  - Residual plots
- Plotting/visualizing model fit
  - Logit scale versus probability scale
  - Confidence intervals on probability scale
- Bayesian formulation (practice!)

**Model formulation**

Model Comparisons and Assessing fit

Bayesian fitting

Other topics not covered here

# Logistic regression

Model for binary (0/1) data or binomial data (number of 1's out of  $n$  trials).

$$Y_i | X_i \sim \text{Binomial}(n_i, p_i)$$

$$\text{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_{1,i} + \dots \beta_p X_{p,i}$$

- Random component = Bernoulli or binomial distribution
- Systematic component:  $\text{logit}(p_i)$  or  $\log(\text{odds})$  = linear combination of predictors

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Remember, for binary data,  $E[Y_i|X_i] = p_i$

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$$\Rightarrow p_i = \frac{\exp(\beta_0 + \beta_1 X_{1,i} + \dots \beta_p X_{p,i})}{1 + \exp(\beta_0 + \beta_1 X_{1,i} + \dots \beta_p X_{p,i})}$$

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$\frac{p}{1-p}$  is referred to as the **odds**.

The link function,  $\log\left(\frac{p}{1-p}\right)$ , is referred to as **logit**.

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Thus, we can describe our model in the following ways:

- We are modeling  $\log\left(\frac{p}{1-p}\right)$  as a linear function of  $X_1, \dots, X_p$ .
- We are modeling the logit of  $p$  as a linear function of  $X_1, \dots, X_p$ .
- We are modeling the log odds of  $p$  as a linear function of  $X_1, \dots, X_p$ .

$$\text{Odds} = \frac{p}{1-p}$$

If the probability of winning a bet is  $= 2/3$ , what are the odds of winning?

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Table 6.1. Various probabilities, odds and log odds. The table shows how log odds are calculated from probabilities.

$P_i$	0.001	0.1	0.3	0.4	0.5	0.6	0.7	0.9	0.999
$1 - P_i$	0.999	0.9	0.7	0.6	0.5	0.4	0.3	0.1	0.001
$O_i$	0.001	0.11	0.43	0.67	1	1.5	2.33	9	999
$\text{Ln}(O_i)$	-6.91	-2.20	-0.85	-0.41	0	0.41	0.85	2.20	6.91

{From Zuur et al. 2007. Analyzing Ecological data}

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Odds can vary between 0 and  $\infty$ , so  $\log(\text{odds})$  can live on  $-\infty$  to  $\infty$ .

## Odds Ratios: $\exp(\beta)$

Consider a regression coefficient for a categorical variable:

$$\text{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 I(\text{group} == B)_i$$

$I(\text{group} == B)_i = 1$  if observation  $i$  is from Group B and 0 if Group A

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Consider the ratio of these odds:

$$\frac{\frac{p_B}{1-p_B}}{\frac{p_A}{1-p_A}} = \frac{\exp(\beta_0 + \beta_1)}{\exp(\beta_0)} = \frac{e^{\beta_0} e^{\beta_1}}{e^{\beta_0}} = \exp(\beta_1)$$

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So,  $\exp(\beta_1)$  gives an **odds ratio** (or ratio of odds) for Group B relative to group A.

## Odds Ratios: $\exp(\beta)$

Consider a continuous predictor,  $X$ :

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$\beta_1$  gives the change in log odds per unit change in  $X$ .

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- Odds when  $X_i = a$  is given by  $\frac{p_i}{1-p_i} = \exp(\beta_0 + \beta_1 a)$
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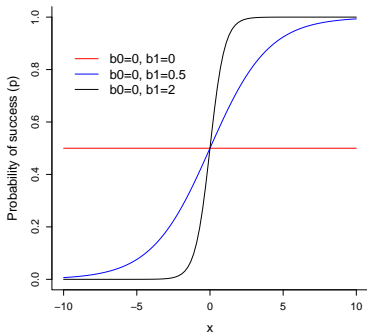
So,  $\exp(\beta_1)$ , gives the odds ratio for two observation that differ by 1 unit of  $X$ .

# Multiple predictors

For multiple predictor models,

$\exp(\beta_i)$  gives the odds ratio for observations where  $X_i$  differs by 1 unit, while holding everything else constant!

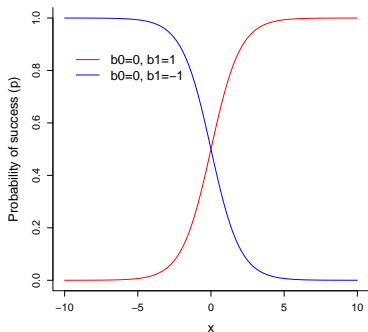
$$\text{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_i$$



The slope coefficient  $\beta_1$  controls how quickly we transition from 0 to 1.

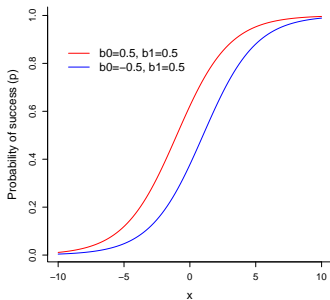


$$\text{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 x_i$$



The sign of  $\beta_1$  determines if  $p$  increases or decreases as we increase  $X$ .

$$\text{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_i$$

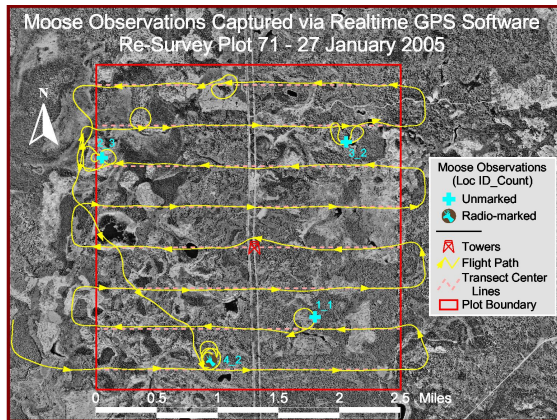


$\beta_0$ :

- Controls the height of the curve when  $X = 0$ .
- Gives the log odds of detection when all predictor variables = 0
- $E[Y_i | X_i = 0] = \frac{\exp(\beta_0)}{1 + \exp(\beta_0)}$  (equals 1/2 if  $\beta_0 = 0$ ).

# Sightability Surveys: Minnesota Moose

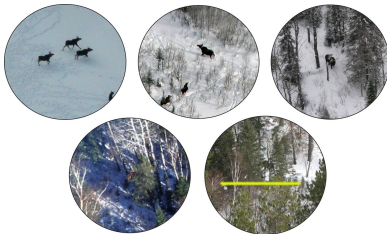
$\frac{124 \text{ 'trials', 2005-2007}}{n_0 = 65 \text{ missed groups}}$   
 $n_1 = 59 \text{ observed groups}$



- Binary observations,  $Y_i = 0$  (missed) or 1 (seen).
- Covariates thought to influence detection.

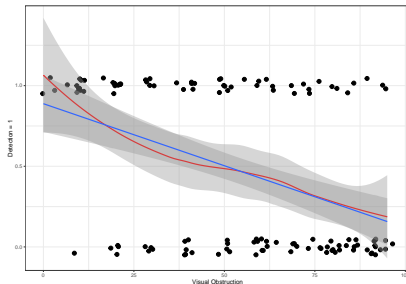
# Covariates

- Visual obstruction
- Survey year (may be due to different observers)



# Visual Obstruction

```
ggplot(exp.m, aes(voc, observed)) + theme_bw() +  
  geom_point(position = position_jitter(w = 2, h = 0.05), size = 3) +  
  geom_smooth(colour = "red") + geom_smooth(method = "lm") +  
  xlab("Visual Obstruction") +  
  ylab("Detection = 1")
```



- 1m would eventually predict  $p_i \geq 1$  and  $p_i \leq 0$
- would assume constant variance when  $\text{var}(p_i) = p_i(1 - p_i)$

$$Y_i|X_i \sim \textit{Bernouli}(p_i)$$

$$\textit{logit}(p_i) = \log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 \textit{voc}_i$$

Assumptions:

- observations are independent
- log odds is a linear function of *voc*
- mean and variance depend on *voc*

$E[Y_i|X_i] = p_i; \textit{var}[Y_i|X_i] = p_i(1-p_i)$  with:

$$p_i = \frac{\exp(\beta_0 + \beta_1 \textit{voc}_i)}{1 + \exp(\beta_0 + \beta_1 \textit{voc}_i)}$$

```
mod1<-glm(observed~voc, data=exp.m, family=binomial())
summary(mod1)
```

Call:

```
glm(formula = observed ~ voc, family = binomial(), data = exp.m)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-1.8056	-0.9071	-0.6218	0.9745	1.8647

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	1.759933	0.460137	3.825	0.000131	***
voc	-0.034792	0.007753	-4.487	7.21e-06	***

---

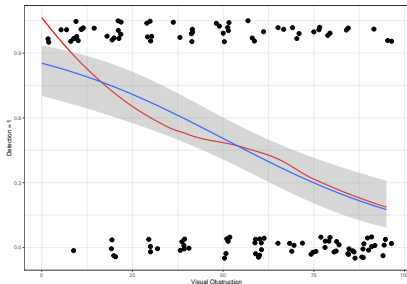
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 171.61 on 123 degrees of freedom  
Residual deviance: 147.38 on 122 degrees of freedom  
AIC: 151.38

Number of Fisher Scoring iterations: 4

```
ggplot(exp.m, aes(voc,observed))+ theme_bw() +
  geom_point(position = position_jitter(w = 2, h = 0.05), size=3) +
  xlab("Visual Obstruction") + geom_smooth(se=F, colour="red") +
  stat_smooth(method="glm", method.args = list(family = "binomial"))
ylab("Detection = 1")
```





```
mod1$coef
```

```
(Intercept)          voc  
1.75993309 -0.03479153
```

Regression coefficient for voc (visual obstruction) = -0.039.

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Regression coefficient for voc (visual obstruction) = -0.039.

- The log odds of being detected decreases by 0.039 per unit increase in visual obstruction
- The odds of being detected decreases by  $\exp(0.039) = 0.96$  per unit increase in visual obstruction

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- The odds of being detected decreases by  $\exp(-0.039) = 0.96$  per unit increase in visual obstruction

Intercept = 2.12 =  $\log(\text{odds})$  when VOC = 0.

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mod1$coef
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Intercept = 2.12 =  $\log(\text{odds})$  when VOC = 0.

```
# p(Y=1|voc=0) = exp(coef(mod1)[1]) / (1+exp(coef(mod1)[1]))  
plogis(coef(mod1)[1])
```

```
(Intercept)  
0.8532013
```

```
mod1$coef
```

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(Intercept)          voc  
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Regression coefficient for voc (visual obstruction) = -0.039.

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- The odds of being detected decreases by  $\exp(0.039) = 0.96$  per unit increase in visual obstruction

Intercept = 2.12 =  $\log(\text{odds})$  when VOC = 0.

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plogis(coef(mod1)[1])
```

```
(Intercept)  
0.8532013
```

We see roughly 90% of moose if there is no visual obstruction.

```
exp.m$year<-as.factor(exp.m$year)
mod2<-glm(observed~voc+year, data=exp.m, family=binomial())
summary(mod2)
```

Call:

```
glm(formula = observed ~ voc + year, family = binomial(), data = exp.m)
```

Deviance Residuals:

	Min	1Q	Median	3Q	Max
	-1.9351	-0.8411	-0.4561	0.9493	1.8680

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	2.453203	0.622248	3.942	8.06e-05 ***
voc	-0.037391	0.008199	-4.560	5.11e-06 ***
year2006	-0.453862	0.516567	-0.879	0.3796
year2007	-1.111884	0.508269	-2.188	0.0287 *

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 171.61 on 123 degrees of freedom  
 Residual deviance: 142.23 on 120 degrees of freedom  
 AIC: 150.23

Number of Fisher Scoring iterations: 4

```
coef(mod2)
```

```
(Intercept)          voc    year2006    year2007  
2.45320264 -0.03739118 -0.45386154 -1.11188432
```



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(Intercept)          voc    year2006    year2007  
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Year 2004:  $\log(p_i/(1 - p_i)) = 2.75 - 0.0422VOC$

Year 2005:  $\log(p_i/(1 - p_i)) = 2.75 - 0.0422VOC + 0.006879$

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So, 0.006879 gives the difference in log odds between years 2005 and 2004 (if we hold VOC constant).

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Year 2004:  $\log(p_i/(1 - p_i)) = 2.75 - 0.0422VOC$

Year 2005:  $\log(p_i/(1 - p_i)) = 2.75 - 0.0422VOC + 0.006879$

So, 0.006879 gives the difference in log odds between years 2005 and 2004 (if we hold VOC constant).

$\exp(0.006879) = 1.0069 = \text{odds ratio (year 2005 to year 2004)}$

$\text{odds ratio} = \frac{p_{2005}/(1-p_{2005})}{p_{2004}/(1-p_{2004})} = 1.0069$

If confidence limits for  $\beta$  include 0 or confidence limits for  $\exp(\beta)$  include 1, then we do not have enough evidence to say that years differ in their detection probabilities.

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```
mod2$coef
```

```
(Intercept)          voc    year2006    year2007  
  2.45320264 -0.03739118 -0.45386154 -1.11188432
```

```
sqrt(diag(vcov(mod2)))
```

```
(Intercept)          voc    year2006    year2007  
0.622247867 0.008199483 0.516567443 0.508269279
```

```
exp(rep(0.006879, 2)+c(-1.96, 1.96)*0.53664) # exp(beta +/-1.96SE)
```

```
[1] 0.3517145 2.8826021
```

If confidence limits for  $\beta$  include 0 or confidence limits for  $\exp(\beta)$  include 1, then we do not have enough evidence to say that years differ in their detection probabilities.

```
mod2$coef
```

```
(Intercept)          voc    year2006    year2007  
  2.45320264 -0.03739118 -0.45386154 -1.11188432
```

```
sqrt(diag(vcov(mod2)))
```

```
(Intercept)          voc    year2006    year2007  
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```

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```
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```

95% Confidence interval for odds ratio = (0.35, 2.88) includes 1  
(not statistically significant)

# Confin

```
(ci.prof<-confint(mod2))
```

Waiting for profiling to be done...

	2.5 %	97.5 %
(Intercept)	1.30341777	3.7586692
voc	-0.05448153	-0.0221268
year2006	-1.48479529	0.5516852
year2007	-2.14380706	-0.1382692

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These are profile-likelihood based confidence intervals based on “inverting” the likelihood ratio test (see Maximum Likelihood notes).



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```
exp(ci.prof[3,])
```

	2.5 %	97.5 %
	0.2265487	1.7361764

Profile-likelihood based intervals should have better statistical properties with small data sets (better **coverage** rates).

Model formulation

**Model Comparisons and Assessing fit**

Bayesian fitting

Other topics not covered here

# Likelihood ratio tests

We can again use difference in deviances (equivalent to likelihood ratio tests) to compare full and reduced models.

```
drop1(mod2, test="Chisq")
```

Single term deletions

Model:

observed ~ voc + year

	Df	Deviance	AIC	LRT	Pr(>Chi)
<none>		142.23	150.23		
voc	1	168.20	174.20	25.9720	3.464e-07 ***
year	2	147.38	151.38	5.1558	0.07593 .

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

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voc is an important predictor, the importance of year is less clear.

# ANOVA function (car package)

Or use Anova in car package

```
library(car)
```

```
Anova(mod2)
```

Analysis of Deviance Table (Type II tests)

Response: observed

	LR	Chisq	Df	Pr(>Chisq)
voc	25.9720	1	3.464e-07	***
year	5.1558	2	0.07593	.

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

# AIC

We can compare nested or non-nested models using the AIC function

```
AIC(mod1, mod2)
```

	df	AIC
mod1	2	151.3824
mod2	4	150.2266

# Goodness-of-fit

Can adapt our general approach for testing goodness-of-fit using Pearson residuals ( $r_i$ )

$$r_i = \frac{Y_i - E[Y_i|X_i]}{\sqrt{Var[Y_i|X_i]}}$$

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- $Var[Y_i|X_i] = p_i(1 - p_i)$

See textbook for an implementation of this test...

# Hosmer-Lemeshow test (similar test)

Group Observations by deciles of their predicted values to form groups, then calculate the expected and observed number of successes and failures for each group:

Model Results				
	$G_1 = [0, \hat{\pi}_1]$	$G_2 = (\hat{\pi}_1, \hat{\pi}_2]$	...	$G_{10} = (\hat{\pi}_9, 1]$
Successes	$\sum_{x_i \in G_1} \hat{\pi}_i$	$\sum_{x_i \in G_2} \hat{\pi}_i$	...	$\sum_{x_i \in G_{10}} \hat{\pi}_i$
Failures	$n_1 - \sum_{x_i \in G_1} \hat{\pi}_i$	$n_2 - \sum_{x_i \in G_2} \hat{\pi}_i$	...	$n_{10} - \sum_{x_i \in G_{10}} \hat{\pi}_i$

Observed Results				
	$G_1 = [0, \hat{\pi}_1]$	$G_2 = (\hat{\pi}_1, \hat{\pi}_2]$	...	$G_{10} = (\hat{\pi}_9, 1]$
Successes	$\sum_{x_i \in G_1} y_i$	$\sum_{x_i \in G_2} y_i$	...	$\sum_{x_i \in G_{10}} y_i$
Failures	$n_1 - \sum_{x_i \in G_1} y_i$	$n_2 - \sum_{x_i \in G_2} y_i$	...	$n_{10} - \sum_{x_i \in G_{10}} y_i$

See: **Goodness of fit with binary data** here:

<http://www.unc.edu/courses/2010fall/ecol/563/001/docs/lectures/lecture21.htm>

# Hosmer-Lemeshow Test

$$\chi^2 = \sum_{i=1}^{n_g} \frac{(O_i - E_i)^2}{E_i} \sim \chi_{g-2}^2$$

where  $g$  = number of groups.

# Hosmer-Lemeshow test

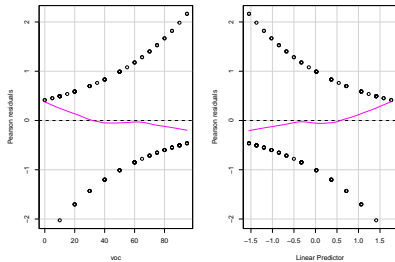
```
library(ResourceSelection)
hoslem.test(exp.m$observed, fitted(mod1), g=8)
```

Hosmer and Lemeshow goodness of fit (GOF) test

```
data:  exp.m$observed, fitted(mod1)
X-squared = 3.2505, df = 6, p-value = 0.7768
```

# Residual plots

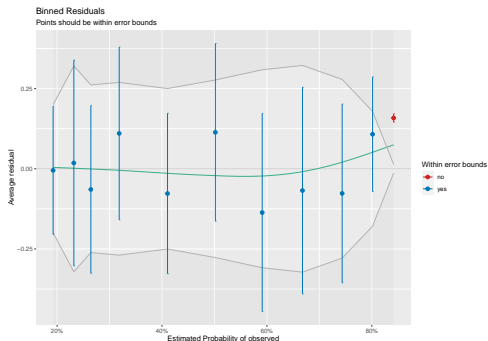
```
residualPlots(mod1)
```



	Test stat	$\Pr(> \text{Test stat} )$
voc	0.6873	0.4071

# Binned residual plot

```
binplot<-performance::binned_residuals(mod1)
plot(binplot)
```



# Visualizing models

# Probability Scale

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 $P(\text{detect animal}|\text{voc})$ :



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The estimates of  $\beta$  are maximum likelihood estimates, found by maximizing:

$$L(\beta; y, x) = \prod_{i=1}^n p_i^{y_i} (1 - p_i)^{1-y_i}, \text{ with}$$

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- Generate confidence intervals for  $\text{logit}(p)$ , then back-transform to get confidence intervals for  $p$
- Ensures the confidence intervals will live on the (0,1) scale
- Intervals will not be symmetric

# Confidence Intervals

Statistical Theory:

- $Var(ax + by) = a^2 var(x) + b^2 var(y) + 2abCov(x, y)$
- If  $x$  and  $y$  are normally distributed,  $ax + by$  will be normally distributed

Let  $X$  = matrix of data

- Here,  $X$  has 2 columns (column of 1's, column with  $x$ )
- $n$  rows (one for each observation).
- We can use `model.matrix` to see it

Let  $\beta = p \times 1$  vector of parameters

- Here,  $p = 2$ ,  $\beta = (\beta_0, \beta_1)$

We can generate  $logit(\hat{p}|X)$  and its SE using matrix multiplication:

- $logit(\hat{p}|X) = X\hat{\beta}$  (use `%*%` in R to perform matrix multiplication)
- Variance/covariance of  $logit(\hat{p}|X) = X\Sigma_{\beta}X^T$

# Confidence Intervals

In practice, we can use `predict(model, newdata=, type="link", se=TRUE)` to get predictions on logit scale.

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# A note on model visualization

Model 2:  $\text{observed} \sim \text{voc} + \text{year}$  is additive on the logit scale

- Differences in  $\text{logit}(p)$  among years will not depend on voc
- Differences in  $p$ , will however, depend on voc!

See: [LogisticModelsFrequentist.html](#)

## A note on model visualization

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- Differences in  $\text{logit}(p)$  among years will not depend on voc
- Differences in  $p$ , will however, depend on voc!

See: [LogisticModelsFrequentist.html](#)

- Can always create your own “effect” plots by calculating predicted values for different combinations of your predicted values (as in the in-class exercise)
- Can use the `effects` package to do something similar

# Effect plots on probability scale

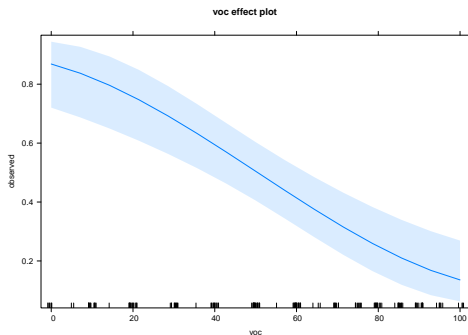
Use `effects` package:

- Fixes all continuous covariates (other than the one of interest) at their mean values
- Categorical predictors: averages predictions on link scale, weighted by proportion of data in each category

# Effect plots

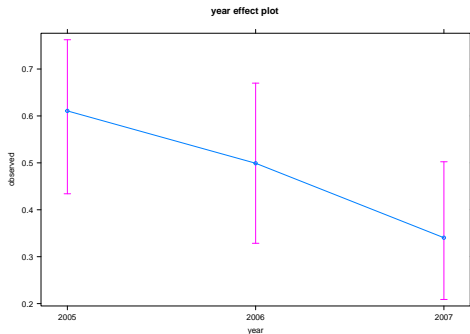
Use `type="response"` to plot on response scale

```
library(effects)  
plot(effect("voc", mod2), type="response")
```



# Effect plots

```
plot(effect("year", mod2), type="response")
```



# Year Effects

```
effect("year", mod2)
```

```
year effect
year
      2005      2006      2007
0.6107432 0.4991440 0.3404147
```

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Estimates of  $P(\text{Observed} = 1 | \text{year} = \text{year}_i, \text{voc} = \text{voc})$

# Year Effects

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year effect
year
      2005      2006      2007
0.6107432 0.4991440 0.3404147
```

Estimates of  $P(\text{Observed} = 1 | \text{year} = \text{year}_i, \text{voc} = \bar{\text{voc}})$

```
newdata<-data.frame(voc=rep(mean(exp.m$voc),3),
                    year=c("2005","2006","2007"))
predict(mod2, newdata=newdata, type="resp")
```

```
      1      2      3
0.6107432 0.4991440 0.3404147
```



# VOC Effect

```
effect("voc", mod2)
```

```
      voc effect  
      0      20      50      80     100  
0.8684553 0.7575962 0.5044524 0.2490051 0.1356677
```

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$P(\text{Observed} = 1 | VOC, year)?$

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```

$P(\text{Observed} = 1 | VOC, year)?$

Weighted mean (across categories, here = “year”), with weights given by the proportion of observations in each category.

# VOC Effect

```
effect("voc", mod2)
```

```
      voc effect  
voc  
      0      20      50      80     100  
0.8684553 0.7575962 0.5044524 0.2490051 0.1356677
```

```
p.years<-table(exp.m$year)/nrow(exp.m)  
newdata<-data.frame(expand.grid(year=c("2005", "2006", "2007"),  
                                voc=seq(0,100,20)))  
newdata$pred<-predict(mod2, newdata=newdata, type="link")  
plogis(sum(newdata$pred[1:4]*p.years))
```

```
[1] 0.8654255
```

# Confidence intervals

```
summary(effect("year", mod2))
```

```
year effect
year
      2005      2006      2007
0.6107432 0.4991440 0.3404147

Lower 95 Percent Confidence Limits
year
      2005      2006      2007
0.4340997 0.3285547 0.2088034

Upper 95 Percent Confidence Limits
year
      2005      2006      2007
0.7624246 0.6699327 0.5023149
```

# Models with interactions

```
mod3<-glm(observed~voc+year+voc:year, data=exp.m, family=binomial())
summary(mod3)
```

Call:

```
glm(formula = observed ~ voc + year + voc:year, family = binomial(),
    data = exp.m)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-1.86221	-0.98652	-0.06984	0.86697	2.03839

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	4.23935	1.45979	2.904	0.00368	**
voc	-0.06661	0.02259	-2.949	0.00319	**
year2005	-2.70469	1.68093	-1.609	0.10761	
year2006	-2.34628	1.70158	-1.379	0.16793	
year2007	-2.15134	1.65108	-1.303	0.19258	
voc:year2005	0.04437	0.02600	1.707	0.08789	.
voc:year2006	0.03127	0.02730	1.145	0.25212	
voc:year2007	0.01286	0.02717	0.473	0.63604	

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

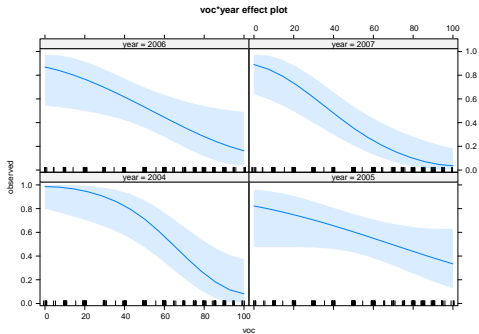
(Dispersion parameter for binomial family taken to be 1)

Null deviance: 221.81 on 159 degrees of freedom  
Residual deviance: 171.08 on 152 degrees of freedom  
AIC: 187.08

Number of Fisher Scoring iterations: 5

# Effect Plot

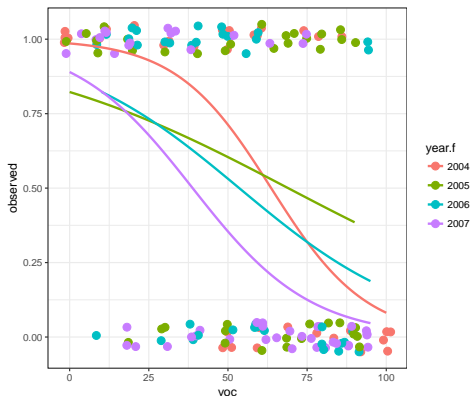
```
plot(effect("voc:year", mod3), type="response")
```



# ggplot2

```
ggplot(exp.m, aes(x=voc, y=observed, colour=year)) + theme_bw() +  
  geom_point(position = position_jitter(w = 2, h = 0.05), size=3) +  
  stat_smooth(method="glm", method.args = list(family = "binomial"),  
             se=FALSE)
```

`'geom_smooth()'` using formula `'y ~ x'`

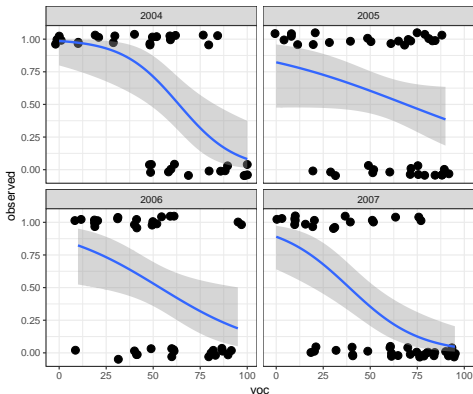




# ggplot2

```
ggplot(exp.m, aes(x=voc, y=observed)) + theme_bw()+  
  geom_point(position = position_jitter(w = 2, h = 0.05), size=3) +  
  stat_smooth(method="glm", method.args = list(family = "binomial")) +  
  facet_wrap(~year)
```

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Model formulation

Model Comparisons and Assessing fit

Bayesian fitting

Other topics not covered here

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Gelman's recommendations (see [arxiv.org/pdf/0901.4011.pdf](http://arxiv.org/pdf/0901.4011.pdf)):

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In class exercise: adapt the JAGS code for fitting mod1 (voc only) to allow fitting of mod2 (voc + year).



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# ROC curves

ROC = Receiver operating characteristic curve

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We could use a threshold,  $T$ , for  $\hat{p}_i$  and set  $\hat{Y}_i$  equal to 1 when  $\hat{p}_i \geq T$  and 0 otherwise. Then, compare  $\hat{Y}_i$  to  $Y_i$ .

- Results would depend on our chosen threshold,  $T$ .
- ROC curve: considers all possible thresholds and plots True Positive Rate,  $P(\hat{Y}_i = 1|Y_i = 1)$  versus False Positive Rate,  $P(\hat{Y}_i = 1|Y_i = 0)$
- AUC gives the area under the ROC curve (higher values suggest better predictive value)