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- What assumptions are we making when fitting linear mixed effects models?
- How can we select an appropriate model?



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When you want to generalize to a larger population of sample units

- Use fixed effects to model particular sites
- Use random effects (assumed to have a distribution) to model a population of sites

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- Allow for two types of predictions: population-averaged (average, across many realized "subjects") and subject-specific

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In practice, you may have only a few clusters (e.g., < 10):

- Makes it difficult to estimate variance parameters (describing how intercepts/slopes vary and covary among clusters).
- Is it realistic to "generalize" to a larger population of clusters?
- May be more appropriate to use fixed effects models to account for cluster-level differences.

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Level-1 predictors vary within clusters

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Implications:

- Degrees of freedom (amount of information) for level-1 predictors depends on the overall size of the data set (number of individuals and number of obs. per individual)
- Degrees of freedom for level-2 predictors depends on the number of clusters

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Can generalize to models that include more than 2 levels

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Distinctions between "types" of predictors (level-1 verus level-2) become even more important when we consider modeling non-normal data

- Implications for parameter interpretation
- Implications for analysis approach (generalized linear mixed effects models verus generalized estimating equations)

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$$R_{ij} = \beta_0 + b_{0i} + \beta_1 NAP_{ij} + \beta_2 Exposure_i + \epsilon_{ij}$$
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- Correlation among observations taken from different beaches = 0 (do not share either random term).

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The within-beach errors are independent and normally distributed, with constant variance: $\epsilon_{ij} \sim N(0, \sigma^2)$