

Frequentist versus Bayesian statistics

FW8051 Statistics for Ecologists

Department of Fisheries, Wildlife and Conservation Biology



Learning Objectives

- Understand differences in how probability is defined in Frequentist and Bayesian statistics
- Understand how to estimate parameters and their uncertainty using Bayesian methods
- Compare Bayesian and Frequentist inference, starting with a simple problem that we can solve analytically.

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Goal: make 'good' decisions with high probability (across potential repeated experiments)

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Still want to make ‘good’ decisions with high probability (across potential repeated experiments)...*calibrated Bayes!*

Key Difference: Probability

Frequentist: relative frequency of events

Bayesian: belief about the system

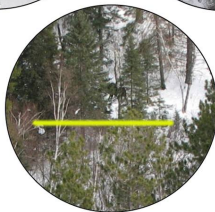
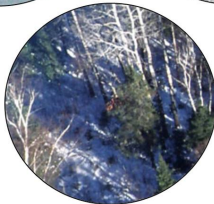
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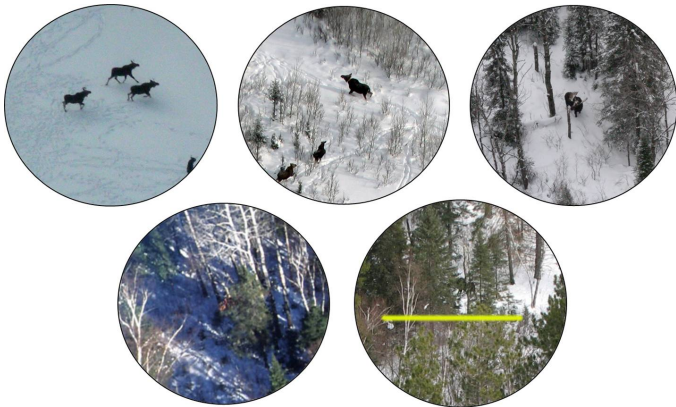
Bayesian: belief about the system

Lets compare inference from the two methods with a simple example

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Goal: estimate p and characterize uncertainty wrt p .

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3. Estimate p using Maximum Likelihood

Maximum Likelihood

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$$\hat{p} = y/n$$

$$\text{var}(\hat{p}) = \text{var}(y/n) = \text{var}(y)/n^2 = p(1-p)/n$$

$$= I^{-1}(p), \text{ where } I(p) = E\left(-\frac{\partial^2 \log L(p)}{\partial p^2}\right)$$

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For large n , a 95% CI = $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

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$P(-z_{1-\alpha/2} \leq \frac{\hat{p}-p}{\sqrt{\text{var}(\hat{p})}} \leq z_{\alpha/2}) = \alpha$ where z is a standard normal random variable.

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$$P(\hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} > p > \hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}) \approx 0.95$$

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```
# Estimate and SE  
(theta.hat<-59/124)
```

```
## [1] 0.4758065
```

```
(se.theta.hat<-sqrt(theta.hat*(1-theta.hat)/124))
```

```
## [1] 0.04484873
```

```
# Confidence Interval  
round(rep(theta.hat,2)+ c(-1.96,1.96)*se.theta.hat,2)
```

```
## [1] 0.39 0.56
```

How well do these work?

- Simulate 10,000 binomial random variables using:
`x<-rbinom(10,000, size = n, p)` with $n = 15, 30, 100$; $p=0.1, 0.5, 0.9$
- Estimate 10,000 \hat{p} values (x/n) and 10,000 95% CIs
- Determine how many of these CIs include p .

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```
# 10,000 repeated samples of size 124 and theta = theta.hat  
ys<-rbinom(10000,size=124,prob=59/124)
```

```
# Calculate 10,000 theta^'s, SE(theta^)'s, CI's  
theta.hats<-ys/124  
se.theta.hats<-sqrt(theta.hats*(1-theta.hats)/124)  
up.CIs<-theta.hats+1.96*se.theta.hats  
low.CIs<-theta.hats-1.96*se.theta.hats
```

```
# Determine coverage  
inCI<-I(low.CIs < 59/124 & up.CIs > 59/124) # true theta is in the in  
sum(inCI)/10000
```

```
[1] 0.9421
```

Frequentist Interpretation

In reality, we get 1 data set. We ended up with $\hat{p} = 0.48$, with 95% CI = (0.39, 0.56)

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The procedure we used should result in an interval that contains the true parameter 95% of the time.

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2. Specify a **prior distribution** for the parameters, $\pi(p)$, reflecting our *a priori* belief about p
3. Use Bayes rule to determine the **posterior distribution** of p given the data, $p(p|y)$:

$$p(p|y) = \frac{L(y|p)\pi(p)}{p(y)} = \frac{L(y|p)\pi(p)}{\int L(y|p)\pi(p)dp}$$

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The **posterior distribution** captures our belief about the parameters after having collected data!

Bayes Theorem

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$p(y) = \int L(y|p)\pi(p)dp$ is the *marginal* distribution of y which requires integrating over p .

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Keep focus on:

Posterior distribution \propto Likelihood \times prior distribution

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- $\pi(p)$: Beta distribution(α, β) = $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha-1}(1-p)^{\beta-1}$

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[Plot this using `curve (dbeta (x, 1, 1), from=0, to=1)]`

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Use $\pi(p)$ and $p(y|p)$ and Bayes Theorem to calculate $p(p|y)$, the **posterior distribution**.

Bayes Theorem

$$p(p|y) = \frac{p(y|p)\pi(p)}{p(y)} = \frac{p(y|p)\pi(p)}{\int_{-\infty}^{\infty} p(y|p)\pi(p)d(p)}$$

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This is a beta distribution with parameters (60, 66).

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The **posterior distribution** gives us the probability distribution of the parameter, given the data and our prior beliefs.

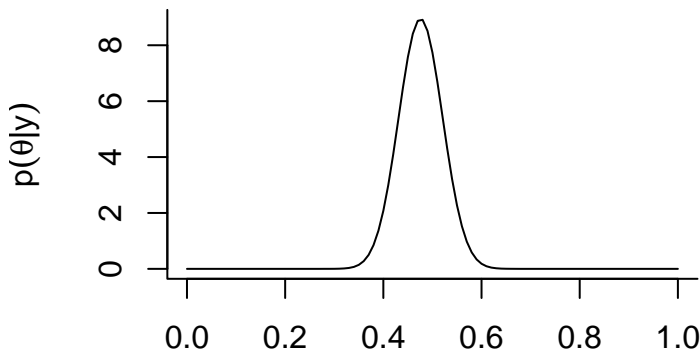
Bayesian Inference

Use `curve` to plot the posterior distribution = $\text{Beta}(60,66)$.

Bayesian Inference

Use curve to plot the posterior distribution = Beta(60,66).

```
par(bty="l", mar=c(2,4.1,1,2.1))  
# Plot the Posterior Distribution of theta  
plot(curve(dbeta(x, 60, 66), from=0, to=1),  
      type="l", xlab=expression(theta), ylab=c(expression(p(group("", theta, "|") * y))))
```



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```
# 95% credible interval  
round(qbeta(c(0.025, 0.975), 60, 66), 2)
```

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Same endpoints as Frequentist confidence interval, different interpretation!

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Interpretation: p has a 95% chance of being in the interval.

- Represents our *belief* based on *data* and our *prior* assumptions

Frequentist vs. Bayesian

Typical Conclusions

Frequentist

Bayesian

Estimation

I have 95% confidence that the population mean is between 12.7 and 14.5 mcg/liter.

There is a 95% probability that the population mean is in the interval 136.2 g to 139.6 g.

Hypothesis Testing

If H_0 is true, we would get a result as extreme as the data we saw only 3.2% of the time. Since that is smaller than 5%, we would reject H_0 at the 5% level. These data provide significant evidence for the alternative hypothesis.

The odds in favor of H_0 against H_A are 1 to 3.

{Mary Parker, <http://www.austinncc.edu/mparker/stat/nov04/>}

Advantages of Bayesian statistics

• Bayesian statistics is a more coherent and consistent approach to statistics

• Bayesian statistics is a more flexible approach to statistics

• Bayesian statistics is a more powerful approach to statistics

• Bayesian statistics is a more intuitive approach to statistics

• Bayesian statistics is a more practical approach to statistics

• Bayesian statistics is a more robust approach to statistics

• Bayesian statistics is a more accurate approach to statistics

• Bayesian statistics is a more efficient approach to statistics

• Bayesian statistics is a more effective approach to statistics

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- Easy to characterize uncertainty for functions of the parameters
- Intuitive appeal of credibility intervals (vs. confidence intervals)
- Coherent philosophy of statistics
 - All inferences come from the posterior distribution
 - No separate theories for estimation, hypothesis testing, multiple comparisons, etc.

Disadvantages of Bayesian statistics

• Subjective prior distributions

• Subjective choice of loss function

• Subjective choice of utility function

• Subjective choice of prior distribution

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- Perceived subjectivity
- Computationally demanding when using MCMC

“Ecologists should be aware that Bayesian methods constitute a radically different way of doing science. Bayesian statistics is not just another tool to be added into ecologists’ repertoire of statistical methods. Instead, Bayesians categorically reject various tenets of statistics and the scientific method that are currently widely accepted in ecology and other sciences. The Bayesian approach has split the statistics world into warring factions (ecologists’ “density independence” vs “density dependence” debates of the 1950s pale by comparison), and it is fair to say that the Bayesian approach is growing rapidly in influence” - Brian Dennis (1996, Ecological Applications, p.1095-1103).

Pragmatic Statistician

Many do consider Bayesian methods another tool in the toolbox...

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We will often fit models using both frequentist and Bayesian statistics (often, with similar answers)!

When is Bayesian Inference “Easier” or Preferred?

Dorazio 2016. Population Ecology 58:31-44

- “**Hierarchical models** of data that link a submodel of sampling processes with a submodel of ecological processes.”
- Inference for latent (i.e., unobserved) state variables
- Missing data problems
- Intractable likelihood functions
- Complex models of different sources and types of data (with shared parameters)