Mixed Models Discussion Questions

- When/why would you consider using mixed models?
- What differentiates mixed models from traditional "fixed effects" regression models?
- What is the difference between a "level-1" and "level-2" predictor?
- How do mixed models "account for correlation"?
- What assumptions are we making when fitting linear mixed effects models?
- How can we select an appropriate model?

When/why would you consider using mixed models?

When you have more than one measurement on the same observational unit

- Multiple observations per lake, animal, study site, etc.

Experiments or surveys with different sizes of experimental units

- Split-plot designs (treatments applied to whole plots and subplots)
- Cluster samples (samples of households, individuals within households)

When you want to generalize to a larger population of sample units

- Use fixed effects to model particular sites
- Use random effects (assumed to have a distribution) to model a population of sites

What differentiates mixed models from traditional "fixed effects" regression models?

- Regression parameters (intercepts, slopes) are allowed to vary by cluster

Actually, we could include "fixed effects" to allow intercepts to vary by cluster; we could include interactions with these fixed effects to allow slopes to vary by cluster

- Random effects parameters (intercepts, slopes) are assumed to come from a distribution (usually normal).
- Allow for two types of predictions: population-averaged (average, across many realized "subjects") and subject-specific

Fixed versus random effects

In the Bayesian world, all parameters have a distribution (so, not a big leap to move from "fixed" to "random"; we will look at an in-class exercise that will hopefully help you think about this distinction)

In practice, you may have only a few clusters (e.g., < 10):

- Makes it difficult to estimate variance parameters (describing how intercepts/slopes vary and covary among clusters).
- Is it realistic to "generalize" to a larger population of clusters?
- May be more appropriate to use fixed effects models to account for cluster-level differences.

What is the difference between a “level-1” and “level-2” predictor?

Level-1 predictors vary within clusters

- NAP within beach
- Date or time in longitudinal studies where individuals are followed over time

Level-2 predictors are constant within a cluster

- Exposure within beach

Implications:

- Degrees of freedom (amount of information) for level-1 predictors depends on the overall size of the data set (number of individuals and number of obs. per individual)
- Degrees of freedom for level-2 predictors depends on the number of clusters

Multi-level models

Can generalize to models that include more than 2 levels

- Individuals within packs, within populations
- Eggs within nests from the same individual

Distinctions between “types” of predictors (level-1 verus level-2) become even more important when we consider modeling non-normal data

- Implications for parameter interpretation
- Implications for analysis approach (generalized linear mixed effects models verus generalized estimating equations)

How do mixed models “account for correlation”?

Observations that share common parameters (intercepts, slopes) will no longer be modeled as independent.

Consider the random intercept model:

$$R_{ij} = \beta_0 + b_{0i} + \beta_1 NAP_{ij} + \beta_2 Exposure_i + \epsilon_{ij}$$
$$b_{0i} \sim N(0, \sigma_b^2)$$
$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$$

- Correlation among observations taken at the same beach $= \frac{\sigma_b^2}{\sigma_b^2+\sigma_\epsilon^2}$ since they share b_{0i} but not ϵ_{ij}.
- Correlation among observations taken from different beaches = 0 (do not share either random term).

What assumptions are we making when fitting linear mixed effects models?

Our response, Y, can be described as a *linear function of covariates*.

$$R_{ij} = \beta_0 + b_{0i} + \beta_1 NAP_{ij} + \beta_2 Exposure_i + \epsilon_{ij}$$

The *intercepts* for each beach are *independent* and *normally distributed*:

$b_{0i} \sim N(0, \sigma_b^2)$ or, equivalently, $\alpha_i = \beta_0 + b_{0i} \sim N(\beta_0, \sigma_b^2)$

The within-beach errors are *independent* and *normally distributed*, with *constant variance*: $\epsilon_{ij} \sim N(0, \sigma^2)$