

INSTITUTO TECNOLÓGICO DE AERONÁUTICA
MP-208: Optimal Filtering with Aerospace Applications
Computational Exercise 1

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Consider an MAV equipped with an ultrasonic sensor that provides a set $y_{1:N}$ of N measures of some physical quantity. Assume that $y_i \in \mathbb{R}$ is modeled by

$$y_i = \mathbf{h}_i \boldsymbol{\theta} + v_i, \quad i = 1, \dots, N,$$

where $\mathbf{h}_i \triangleq [i \ 1]$, $\boldsymbol{\theta} \triangleq [\theta_1 \ \theta_2]^T \in \mathbb{R}^2$ is a realization of $\boldsymbol{\Theta} \sim \mathcal{N}(\mathbf{m}_{\boldsymbol{\Theta}}, \mathbf{P}_{\boldsymbol{\Theta}})$, $v_i \in \mathbb{R}$ is a realization of $V_i \sim \mathcal{N}(0, R)$, $\{V_i\}$ is a white sequence, and V_i and $\boldsymbol{\Theta}$ are uncorrelated. Consider that $\mathbf{m}_{\boldsymbol{\Theta}} = [1 \ 2]^T$,

$$\mathbf{P}_{\boldsymbol{\Theta}} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.04 \end{bmatrix},$$

and $R = 0.01$.

- a.* Implement in a MATLAB script an LS estimator for $\boldsymbol{\theta}$ from $y_{1:N}$. Adopt a constant weight $W_i = 1$, $\forall i$. Conduct a Monte Carlo simulation with 100 realizations of $\hat{\boldsymbol{\Theta}}_N$. Obtain the sample mean and RMSE (square root of MSE) of the realizations. Present a histogram of the estimate realizations.
- b.* Implement in a MATLAB script an MAP estimator for $\boldsymbol{\theta}$ from $y_{1:N}$. Conduct a Monte Carlo simulation with 100 realizations of $\hat{\boldsymbol{\Theta}}_N$. Obtain the sample mean and RMSE of the realizations. Compute the theoretical mean and RMSE. Present an histogram of the estimate realizations.
- c.* Compare the sample means and RMSEs obtained in *a* and *b*.
- d.* Write a succinct report to present the results of the above items.