# INSTITUTO TECNOLÓGICO DE AERONÁUTICA
# MP-208: Optimal Filtering with Aerospace Applications
# Computational Exercise 3

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Consider a system described by:

$$\dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), u(t)\big) + \mathbf{w}(t),$$

$$y_{k+1} = h\left(\mathbf{x}_{k+1}\right) + v_{k+1},$$

where $\mathbf{x}(t) \triangleq [x_1(t)\ x_2(t)]^{\mathrm{T}} \in \mathbb{R}^2$ is the state vector at the continuous time $t$, $\mathbf{x}_k \triangleq \mathbf{x}(t_k)$ is the state vector at the discrete time $k$, $u(t) \in \mathbb{R}$ is the control input, $\{\mathbf{w}(t) \in \mathbb{R}^2\}$ is the state noise, $y_k \in \mathbb{R}$ is the measured output, $\{v_k \in \mathbb{R}\}$ is the measurement noise, and

$$\mathbf{f}(\mathbf{x}(t), u(t)) \triangleq \begin{bmatrix} -x_1(t) + x_2(t) \\ -0.1x_1(t)^2 - 1 + u(t) \end{bmatrix}, \tag{1}$$

$$h(\mathbf{x}) = x_1. \tag{2}$$

For the sake of convenience, consider the control input

$$u(t) = -10y(t) + 10, \tag{3}$$

where $y(t) \in \mathbb{R}$ is the continuous-time counterpart of $y_k$.

Let us adopt the parameters presented in Table 1.

Table 1: System parameters.

| **Description** | **Value** |
|---|---|
| Covariance of the state noise | $\mathbf{Q}(t) = 0.01\mathbf{I}_2$ |
| Covariance of the measurement noise | $R_k = 0.01$ |
| Statistics of the initial state | $\bar{\mathbf{x}} = \mathbf{0}_2$, $\bar{\mathbf{P}} = \mathbf{I}_2$ |
| Sensor sampling time | $T_s = 0.1$ s |

*a.* Simulate the system described above using a MATLAB script and the 4th-order Runge-Kutta method.

*b.* Design and implement (in a MATLAB script) a CDEKF for estimating $\{\mathbf{x}(t)\}$. This script has to contain an outer Monte Carlo loop allowing for an arbitrary number $N$ of runs (or realizations).

*c.* Using the MATLAB scripts implemented in the above questions, conduct a simulation study to evaluate the performance of the CDEKF over 100 realizations. For each of the two state components (say, the $i$th component), this study must include one graphic containing:

- all the realizations of $\tilde{X}_i \triangleq X_i - \hat{X}_i$ vs time; and
- the filtered standard deviation computed by the filter (*i.e.*, the $i$th diagonal element of $\sqrt{P_{k|k}}$.) vs time.

*d.* Write a report to objectively present the obtained results, together with the respective analysis.