

INSTITUTO TECNOLÓGICO DE AERONÁUTICA
MP-208: Optimal Filtering with Aerospace Applications
Computational Exercise 2

Prof. Davi Antônio dos Santos

9 de novembro de 2020

Consider a discrete-time second-order dynamic system described, with a sampling time of $T = 0.1$ s, by:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k + \mathbf{w}_k$$

$$y_{k+1} = \mathbf{C}\mathbf{x}_{k+1} + v_k$$

where $\mathbf{x}_k \in \mathbb{R}^2$ is the state vector, $y_k \in \mathbb{R}$ is the measured output, $\{\mathbf{w}_k \in \mathbb{R}^2\}$ is a realization of the SP $\{\mathbf{W}_k\}$, with $\mathbf{W}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ and $\mathbf{Q} = \text{diag}(1 \times 10^{-2}, 4 \times 10^{-2})$, $\{v_k \in \mathbb{R}\}$ is a realization of the SP $\{V_k\}$, with $V_k \sim \mathcal{N}(0, R)$ and $R = 1 \times 10^{-2}$, $\mathbf{x}_1 \in \mathbb{R}^2$ is a realization of the RV $\mathbf{X}_1 \sim \mathcal{N}(\bar{\mathbf{x}}, \bar{\mathbf{P}})$, with $\bar{\mathbf{x}} = [1 \ 0]^T$ and $\bar{\mathbf{P}} = \text{diag}(1 \times 10^{-4}, 1 \times 10^{-8})$, the sequence $\{\{V_k\}, \{\mathbf{w}_k\}, \mathbf{X}_1\}$ is uncorrelated, $u_k \in \mathbb{R}$ is a control input¹

$$u_k = 10 \left(\bar{y}_k - \mathbf{e}_1^T \mathbf{x}_k \right) - 2\mathbf{e}_2^T \mathbf{x}_k,$$

in which $\bar{y}_k \in \mathbb{R}$ is the command input, consider $\bar{y}_k = 5$, $\forall k$, and finally,

$$\mathbf{A} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

- a. Simulate the above system in the period from 0 to 20 s. Obtain a unique plot containing 10 realizations $\{y_k\}$ of the measured output together with the input command $\{\bar{y}_k\}$.
- b. Using the true parameters (of the given model), implement a conventional Kalman filter for this system. Simulate the filter on all the ten measurement realizations $\{y_k\}$ obtained in *a*. Obtain two separate plots, one for $i = 1$ and the other for $i = 2$, with the ten realizations of the (true) estimation error $\mathbf{e}_i^T \tilde{\mathbf{x}}_{k|k}$ and the respective sample means and RMS values, as well as the theoretical standard deviations (calculated from $\mathbf{P}_{k|k}$).
- c. Repeat item *b* using the information filter presented in Chapter 5.
- d. Write a report to objectively present the obtained results, together with the respective analysis.

¹ $\mathbf{e}_1 \triangleq [1 \ 0]^T$, $\mathbf{e}_2 \triangleq [0 \ 1]^T$.