Risk Management for Financial Institutions

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Program

- December 5 and 6 - Market Risk, Theory and Risk Management Implementation
- December 12 and 13 - Credit Risk, Theory and Estimation

Market Risk and Credit Risk

- Market risk refers to risk arising from movements of interest rate, foreign exchange rates and prices of instruments in the money and capital markets which negatively affect the earning and capital of the financial institution. Market risks can be classified into 3 types which are interest rate risk, foreign exchange risk, and price risk.
- Credit risk arises from the inability of the counterparty to fulfil the stipulated conditions of the agreement resulting in losses to the financial institution

Introduction

- There is a vast and heterogeneous array of investment strategies across Alternative Investment Funds (AIFs).
- Most of them can be classified into Hedge Fund, Private Equity, Real Estate, and Fund of Fund strategies depending on the liquidity and nature of assets.
- During this short course we will focus on the implementation of risk management strategies from an *investment* perspective. Focusing on implementation rather than estimation.
- The regulation of the risk management best practices and requirements are ruled under the Alternative Investment Fund Managers (AIFM) regulatory framework.

The risk management process of an AIFM

- A governing body (the Board) responsible for the supervision of the activities of the AIFM and must be composed of at least three directors.
- At least two Conducting Officers (Senior Management) responsible for the day-to-day management. They must take all actions required within the scope of their responsibilities and tasks should be split to avoid conflicts of interest.
- Portfolio and risk management are the two core functions. Only one of them may be fully delegated to a third party ("no letter box").
- The AIFM must ensure the independent valuation of the assets of the AIFs. Delegation of the valuation function is possible to an external valuer. He/she must accept to be held liable for negligent valuation mistakes.
- These functions are usually delegated to third parties, some even must be delegated to third parties. One of the Conducting Officers must take care of the oversight of the delegated functions.

The Risk Management Process

- Identification
- Measurement
- **Management**
- Monitoring

The Risk Management Process

- The AIFM, in accordance with the AIFs rules or instruments of incorporation, periodically disclose to investors
  - The percentage of the AIF's assets which are subject to special arrangements arising from their illiquid nature
  - Any new arrangements for managing the liquidity of the AIF
  - The current risk profile of the AIF and the risk management systems employed by the AIFM to manage those risks
- In the case of levered strategies
  - Any change to the maximum level of leverage which the AIFM may employ on behalf of the AIF as well as any right of the reuse of collateral or any guarantee granted under the leveraging arrangement.
  - The amount of leverage employed.

Risk Management from a Portfolio Management Perspective

Two different areas

The AIFM shall functionally and hierarchically separate the functions of risk management from the operating units, including from the functions of portfolio management. But this is a difficult task.

- Persons engaged in the performance of the risk management function are not supervised by those responsible for the performance of the operating units, including the portfolio management function of the AIFM.
- Persons engaged in the performance of the risk management function are not engaged in the performance of activities within the operating units, including the portfolio management function.

Portfolio Management - Basics

Consider the following static portfolio allocation problem

- There is a Universe of $i \in \{1, ..., N\}$ tradeable assets that yield a random return r_i in the future.
- Finite first and second moments $\mathbb{E}[r_i] < \infty$, $\mathbb{E}[r_i^2] < \infty$
- No transaction costs, nor short-sell restrictions.
- Investors maximize a function $U : \mathbf{R}^N \to \mathbf{R}$ that depends on the moments of r_i, and potentially other "interesting" characteristics.

Example

$$\max_{\theta} \theta' \mu - \frac{\gamma}{2} \theta' \Sigma \theta$$
$$\theta' \mathbf{u} = 1$$

where

$$\theta = \underbrace{\begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \end{bmatrix}}_{\text{\% invested in asset } i}, \mathbf{u} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, \mu = \begin{bmatrix} \mathbb{E}[r_1] \\ \vdots \\ \mathbb{E}[r_N] \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & \dots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \dots & \sigma_N^2 \end{bmatrix}$$

Parameter γ captures the risk aversion of the investor.

Closed Form Solution

The above problem allows for a closed form solution ($\gamma > 0$)

$$\mathcal{L} = \theta'\mu - \frac{\gamma}{2}\theta'\Sigma\theta - \lambda(\theta'\mathbf{u} - 1)$$

$$\mathcal{L}_\theta = \mu - \gamma\Sigma\theta - \lambda\mathbf{u} = 0 \to \theta = \frac{1}{\gamma}\Sigma^{-1}(\mu - \lambda\mathbf{u})$$

$$\mathbf{u}'\frac{1}{\gamma}\Sigma^{-1}(\mu - \lambda\mathbf{u}) = 1$$

$$\gamma = \mathbf{u}'\Sigma^{-1}(\mu - \lambda\mathbf{u}) = \mathbf{u}'\Sigma^{-1}\mu - \lambda\mathbf{u}'\Sigma^{-1}\mathbf{u}$$

$$\lambda = \frac{\mathbf{u}'\Sigma^{-1}\mu - \gamma}{\mathbf{u}'\Sigma^{-1}\mathbf{u}}$$

$$\theta = \frac{1}{\gamma}\Sigma^{-1}(\mu - \frac{\mathbf{u}'\Sigma^{-1}\mu - \gamma}{\mathbf{u}'\Sigma^{-1}\mathbf{u}}\mathbf{u})$$

Note that

$$\mathbf{u}'\theta = \frac{1}{\gamma}\mathbf{u}'\Sigma^{-1}(\mu - \frac{\mathbf{u}'\Sigma^{-1}\mu - \gamma}{\mathbf{u}'\Sigma^{-1}\mathbf{u}}\mathbf{u}) = \frac{1}{\gamma}\mathbf{u}'\Sigma^{-1}(\frac{\mathbf{u}'\Sigma^{-1}\mathbf{u}\mu - [\mathbf{u}'\Sigma^{-1}\mu - \gamma]\mathbf{u}}{\mathbf{u}'\Sigma^{-1}\mathbf{u}}) = \frac{1}{\gamma}\mathbf{u}'\Sigma^{-1}\frac{\gamma\mathbf{u}}{\mathbf{u}'\Sigma^{-1}\mathbf{u}} = 1$$

Is it realistic?

In general this simple portfolio allocation problem fails to capture the complexity of financial markets. More complex situations can be modelled but at the expense of losing a closed for solution.

- Irrelevant for asset managers with access to numerical methods.
- Realistic models can be solved using state of the art methods and commercial optimization solvers.
- Bottlenecks when $N >> 0$ as more complex problems are generally NP-Hard.
- Modelling *realistic* problems require a combination of convex and mixed integer formulations.
- Risk Management decisions will enter as *parameters* into the portfolio decision of portfolio managers.

Risk management decision: Market Exposure

How can we control the market risk of our portfolio? Market β

$$\max_{\theta} \underbrace{\theta'\mu}_{\text{PM}} - \underbrace{\frac{\gamma}{2}\theta'\Sigma\theta}_{\text{RM}}$$

$$\theta'\mathbf{u} = 1$$

$$\underbrace{\theta'\beta = \bar{\beta}}_{\text{RM}}$$

where $\bar{\beta} \in \mathbb{R}$ and $\beta = [\beta_1 \ldots \beta_N]'$ is the market beta of each asset.

Factor Exposure and concentration exposure

Exposure to common factors, consider a multifactor model with K factors that explains returns

$$r_t = \mathbf{B} f_t + \epsilon_t$$

where $\mathbf{B} \in \mathbb{R}^{N \times K}$ and $\bar{\beta} \in \mathbb{R}^K$

$$\begin{aligned} \max_{\theta} \theta' \mu - \frac{\gamma}{2} \theta' \Sigma \theta \\ \theta' \mathbf{u} = 1 \\ \mathbf{B}' \theta = \bar{\beta} \\ \bar{b} \geq \theta \geq \underline{b} \end{aligned}$$

where $\bar{\beta} \in \mathbb{R}$ and $\beta = [\beta_1 \dots \beta_N]'$ is the market beta of each asset.

Value at Risk

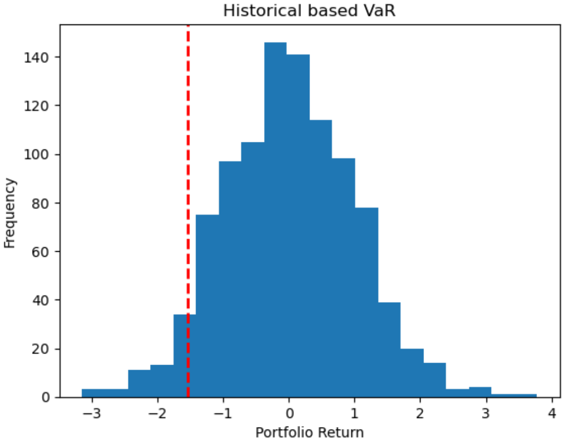
The Value-at-risk (VaR) of a portfolio is a measure of the risk of loss for an investment. Most practitioners know how to compute the VaR of a portfolio, not how to control it.

$$VaR_\alpha(X) = -\inf\{x \in \mathbb{R} : F_X > \alpha\}$$

For a given portfolio, time horizon, and probability α, the α VaR can be defined informally as the maximum possible loss during that time after excluding all worse outcomes whose combined probability is at most α. The most difficult task is to compute F_x which could be done either by

- Historical data
- Distribution Assumptions
- Non-parametric approaches

Visualization



VaR management - Normal Distribution

For simplicity, it is common to only focus on individual variances when computing the value at risk. This is the method originally proposed by the RiskMetrics department at JP Morgan in the 1980s. Given $\eta = \Phi^{-1}(\alpha)$ define

$$V = \eta\sqrt{\text{diag}(\Sigma)}$$

and the correlation matrix

$$C = \begin{bmatrix} 1 & \dots & \rho_{1N} \\ \vdots & \ddots & \vdots \\ \rho_{N1} & \dots & 1 \end{bmatrix} \tag{1}$$

Include the VaR in the portfolio allocation

$$\begin{aligned} \max_{\theta} \theta'\mu - \frac{\gamma}{2}\theta'\Sigma\theta \\ \theta'\mathbf{u} = 1 \\ \sqrt{\theta' VCV\theta} \leq \text{VaR} \end{aligned}$$

Leverage

Leverage allows investors to boost the performance of investment strategies at the expense of a larger exposure to risks. Leverage is required to open short positions and is bounded by specific margin requirements of the broker. Gross leverage is defined as

$$L = \frac{\text{Total Value Positions}}{\text{Capital}} \tag{2}$$

Consider a fund that receives X dollars, it takes a fraction $\theta^+ \geq 0$ and buys stocks with it, and it takes short positions for a fraction $\theta^- \geq 0$. The gross leverage of the fund is

$$L = \frac{(\theta^+ + \theta^-)X}{X} = \theta^+ + \theta^- \tag{3}$$

Recall that $\theta^+ + \theta^- \neq 1$ most of the times.

Introducing Leverage

We introduce a new asset, cash, that yields a constant return of 0 and cannot be shorted. Imagine this the cash the investor has in the broker account. The proceedings from short positions do not need to necessarily finance a long position in another asset.

$$\max_{\theta} \theta' \mu - \frac{\gamma}{2} \theta' \Sigma \theta$$
$$\theta' \mathbf{u} + \theta_c = 1$$

Leverage depends on the **absolute** value of the elements of θ. However, it is inelegant to include the function $|.|$ in optimization formulations, as most optimization solvers wont allow it.

$$\theta = \theta^+ - \theta^-$$
$$\theta^+ \geq 0$$
$$\theta^- \geq 0$$

Controlling Leverage

$$
\begin{aligned}
\max_{\theta} \theta' \mu - \frac{\gamma}{2} \theta' \Sigma \theta \\
\theta' \mathbf{u} + \theta_c = 1 \\
\theta = \theta^+ - \theta^- \\
\theta^+ \geq 0 \\
\theta^- \geq 0 \\
\mathbf{u}'(\theta^+ + \theta^-) \leq L
\end{aligned}
$$

While mutual funds tend to have a gross leverage close to 1, arbitrage funds can go as far as $\times 10$.

Liquidity

The introduction of cash in the portfolio allocation dynamics allows us to also manage liquidity. This is particularly relevant for institutions that face withdrawals. E.g.

$$\theta_c \geq \bar{c} \geq 0$$

We will see asset specific restrictions on liquidity later.

Non-convex non-linear dynamics

Some real-world features require a more advanced modelling technique. These include:

- Sector concentration
- Portfolio size
- Higher Moments
- Transaction costs
- Asset liquidity
- Heterogeneous Preferences

Stock picking dynamics

- Given a potentially large set $N >> 0$ some investments strategies require to focus only on a small subset n of assets. Standard convex optimization formulations will not allow to "choose" a subset of assets.
- Penalization methodologies commonly used in Machine Learning (e.g. LASSO) require on measures of "distance" and are not suitable for this task.
- We can borrow from the *Operations Research* field, a Mixed Integer Programming formulation.
- Under this formulation, some variables can only take certain integer values.
- Most common optimization algorithms "branch" and "bound" solutions.
- Introduce binary variables that indicate whether or not an asset has been "chosen" in the portfolio, and if it is part of the long or short leg of the portfolio.

$$x_i, x_i^+, x_i^- \in \{0, 1\}$$

Stock picking dynamics

We introduce a mathematical formulation suitable for implementing numerical solutions. Some of these definitions rely on what is called a **Big** M formulation.

- If an asset is picked it is either long or short, but not both.

$$x_i = x_i^+ + x_i^-$$

- $\theta_i = 0 \iff x_i = 0$
- $\theta_i = 0 \rightarrow x_i = 0$, for a large $M >> 0$.

$$M\theta_i^+ \geq x_i^+$$
$$M\theta_i^- \geq x_i^-$$

- $x_i = 0 \rightarrow \theta_i = 0$

$$Mx_i^+ \geq \theta_i^+$$
$$Mx_i^- \geq \theta_i^-$$

Stock picking dynamics

- Minimum position if stock is picked, avoids computational rounding errors, if $x_i^+ = 1 \rightarrow \theta_i^+ \geq m$.

$$\theta_i^+ \geq x_i^+ m$$
$$\theta_i^- \geq x_i^- m$$

- Hold n_l stocks long and n_s stocks short

$$\mathbf{u}' x^+ = n_l$$
$$\mathbf{u}' x^- = n_s$$

Soft constraints

Constraints of the form $\mathbf{B}'\theta = \bar{\beta}$ can be too "strong" if $n_s + n_l$ is small. We can impose "soft-constraints"

- Soft constraint on factor exposure (recall that the bounds can be factor-specific)

$$\mathbf{B}'\theta \geq \bar{\beta} - \beta_l$$
$$\mathbf{B}'\theta \leq \bar{\beta} + \beta_u$$

Sector concentration

- Consider S sectors, asset classes, or any clustering, and an indicator matrix

$$\mathbf{S} = [\mathbf{S}_{is}] \text{ if asset } i \text{ belongs to sector } s$$

- Maximum concentration % per sector (long or short)

$$\mathbf{S}'(\theta^{+} + \theta^{-}) \leq \bar{S} \in \mathbb{R}^{S}$$

- Maximum number of stocks per sector (long or short)

$$\mathbf{S}'x \leq n_{\bar{S}} \in \mathbb{N}^{S}$$

Practical Example

$$
\begin{aligned}
\max_{\theta} \theta' \mu - \frac{\gamma}{2} \theta' \Sigma \theta \\
\theta' \mathbf{u} + \theta_c = 1 \\
\theta = \theta^+ - \theta^- \\
\theta^+ \geq 0 \\
\theta^- \geq 0 \\
\mathbf{u}'(\theta^+ + \theta^-) \leq L \\
\mathbf{B}' \theta = \bar{\beta} \\
\bar{b} \geq \theta \geq \underline{b} \\
\sqrt{\theta' VCV \theta} \leq \text{VaR}
\end{aligned}
$$