Production Based Asset Pricing (Intro)

Juan F. Imbet
Université Paris Dauphine

Empirical Asset Pricing
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The q theory of investment

Consider a firm in partial equilibrium in discrete time with an infinite horizon. Capital is the only fixed factor of production and risk neutral managers acting on behalf of shareholders choose the capital stock each period to maximize the value of the firm. Firms total factor productivity z_t evolves stochastically. Managers will solve the following program

$$\max \mathbb{E}_t\Big[\sum_{j=t}^{\infty}\Big(\frac{1}{1+r}\Big)^j(\underbrace{z_j k_j}_{\text{Output}} - I_j - \underbrace{\frac{a}{2}\Big(\frac{I_j}{k_j}\Big)^2 k_j}_{\text{Adjustment costs}})\Big]$$

s.t.

$$k_{j+1} = (1-\delta)k_j + I_j$$

First Order Conditions

This model has no analytical solution. However, one can characterize the solution via an examination of the first-order conditions, and doing so helps lend intuition to the solution.

$$\mathcal{L} = \mathbb{E}_t\Big[\sum_{j=t}^{\infty}\Big(\frac{1}{1+r}\Big)^j\Big(z_j k_j - I_j - \frac{a}{2}\Big(\frac{I_j}{k_j}\Big)^2 k_j - \lambda_j(k_{j+1} - k_j(1-\delta) - I_j)\Big)\Big]$$

Derivating with respect to I_j

$$1 + a\frac{I_t}{k_t} = \lambda_t$$

Because λ_t is a Lagrange multiplier, it represents the shadow value of capital, so this first-order condition says that at an optimum the shadow value of capital equals its marginal cost, which has two components

First Order Conditions

Derivating with respect to k_{j+1}

$$\mathbb{E}_t\Big[\frac{1}{1+r}\Big(z_{t+1} + \frac{a}{2}\Big(\frac{I_{t+1}}{k_{t+1}}\Big)^2 + (1-\delta)\lambda_{t+1}\Big)\Big] = \lambda_t$$

Recursively and applying law of iterated expectations

$$\lambda_t = \mathbb{E}_t\Big[\sum_{j=0}^{\infty}\Big(\frac{1}{1+r}\Big)^j (1-\delta)^{j-1}(z_{t+1} + \frac{a}{2}\Big(\frac{I_{j+1}}{k_{j+1}}\Big)^2)\Big]$$

This equation shows that the shadow value of capital is the expected stream of future marginal benefits from using the capital. These benefits include both the marginal addition to profit and the reduction in installation costs. This quantity is usually called marginal q (Hayashi 1982)

Investment - q regressions

Putting everything together

$$1 + a\frac{I_t}{k_t} = q_t \rightarrow \frac{I_t}{k_t} = -\frac{1}{a} + \frac{1}{a}q_t = b_0 + b_1 q_t$$

$$\mathbb{E}_t\left[\frac{1}{1+r}\left(z_{t+1} + \frac{a}{2}\left(\frac{I_{t+1}}{k_{t+1}}\right)^2 + (1-\delta)q_{t+1}\right)\right] = 1 + a\frac{I_t}{k_t}$$

$$1 + a\frac{I_t}{k_t} = \frac{1}{1+r}\mathbb{E}_t[z_{t+1}] + \frac{a}{2(1+r)}\mathbb{E}_t\left[\left(\frac{I_{t+1}}{k_{t+1}}\right)^2\right] + \frac{1-\delta}{1+r}\mathbb{E}_t[q_{t+1}]$$

$$\frac{I_t}{k_t} = \underbrace{-\frac{1}{a}}_{b_0} + \underbrace{\frac{1}{a(1+r)}}_{b_1}\mathbb{E}_t[z_{t+1}] + \underbrace{\frac{1}{2(1+r)}}_{b_2}\mathbb{E}_t\left[\left(\frac{I_{t+1}}{k_{t+1}}\right)^2\right] + \underbrace{\frac{1-\delta}{a(1+r)}}_{b_3}\mathbb{E}_t[q_{t+1}]$$

$$\frac{I_t}{k_t} = b_0 + b_1 \times \underbrace{\mathbb{E}_t[z_{t+1}]}_{\text{Expected Profitability}} + b_2 \times \underbrace{\mathbb{E}_t\left[\left(\frac{I_{t+1}}{k_{t+1}}\right)^2\right]}_{\text{Expected Benefit Capital Installed}} + b_3 \times \underbrace{\mathbb{E}_t\left[q_{t+1}\right]}_{\text{Expected Marginal } q}$$

q theory and asset pricing

Consider risk averse managers facing the same investment decisions. For simplicity we consider a two period model were all capital is depreciated at $t+1$

$$\max z_t k_t - k_{t+1} - \frac{a}{2}\left(\frac{k_{t+1}}{k_t}\right)^2 k_t + \mathbb{E}_t\Big[M_{t+1}(z_{t+1}k_{t+1})\Big]$$

First order condition

$$1 + a\frac{k_{t+1}}{k_t} = \mathbb{E}_t\Big[M_{t+1}z_{t+1}\Big]$$

The price of the company (without the dividend)

$$P_t = \mathbb{E}_t\Big[M_{t+1}z_{t+1}k_{t+1}\Big]$$

Returns

The stock return cab be rewritten as

$$R_{t+1} = \frac{z_{t+1}k_{t+1}}{\mathbb{E}_t\left[M_{t+1}z_{t+1}k_{t+1}\right]} = \frac{z_{t+1}}{\mathbb{E}_t\left[M_{t+1}z_{t+1}\right]}$$

Replacing the first order condition

$$R_{t+1} = \frac{z_{t+1}}{1 + a\frac{k_{t+1}}{k_t}}$$

$$\mathbb{E}_t\left[R_{t+1}\right] = \frac{\mathbb{E}_t\left[z_{t+1}\right]}{1 + a\frac{k_{t+1}}{k_t}}$$

Firms with larger expected profitability and lower investment earn higher expected returns.