An extension of Kyle's model

Juan F. Imbet

January 27, 2023

1 The Model

I extend Kyle [1985] in order to model Noise Traders (NT) as optimally choosing their trade volume based on their (probably incorrect) beliefs compared to Inside Traders (IT). This adds to the set-up the realism included by DeLong et al. [1990] in which noise traders trade on incorrect beliefs, which includes persistence in their trades and therefore risk to informed traders.

There are two periods, and an asset that pays $\tilde{v}$ at time one. Ex-ante, $\tilde{v} \sim N(p_0, \sigma_0^2)$. Informed traders know the realization of $\tilde{v}$ before it occurs, and place an order $\tilde{x}$. Noise traders on the other hand believe they are also informed traders, but their incorrect information makes them trade on $\tilde{n}$ instead of $\tilde{v}$, where ex-ante $\tilde{n} \sim N(p_0^*, \sigma_u^2)$ and $cov(\tilde{v}, \tilde{n}) = \sigma_{uv}$. Call the amount traded by NT as $\tilde{u}$. The covariance between both random variables capture the fact that noise traders could still have beliefs in the same direction as inside traders as well as contrarian beliefs which could increase trading risk. Finally, the market maker does not observe the individual actions $\tilde{x}, \tilde{u}$ but instead the total amount demanded $\tilde{y} = \tilde{x} + \tilde{u}$. Market makers supply competitively units to the market such that $\mathbb{E}[\tilde{v}|\tilde{y}] = p_0$. In order to solve for the rational equilibrium model each player conjectures a linear demand function of the other players and maximize their profit.

1.1 Inside Traders

Informed traders conjecture

$$\tilde{p} = \lambda \tilde{y} + \mu \text{ Price setting}$$
$$\tilde{u} = \gamma \tilde{n} + \theta \text{ NT demand}$$

They maximize ex-ante profits

$$\begin{aligned}\tilde{\pi} &= (\tilde{v} - \tilde{p})\tilde{x} - \frac{\tau\tilde{x}^2}{2} \\ &= (\tilde{v} - \lambda(\tilde{u} + \tilde{x}) - \mu)x - \frac{\tau\tilde{x}^2}{2} \\ &= (\tilde{v} - \lambda(\gamma\tilde{n} + \theta + \tilde{x}) - \mu)x - \frac{\tau\tilde{x}^2}{2} \\ &= [\tilde{v} - (\mu + \lambda\tilde{x} + \lambda\theta)]\tilde{x} - \lambda\gamma\tilde{n}\tilde{x} - \frac{\tau\tilde{x}^2}{2}\end{aligned}$$

$$\mathbb{E}[\tilde{\pi}|\tilde{v}] = [\tilde{v} - (\mu + \lambda\tilde{x} + \lambda\theta)]\tilde{x} - \lambda\gamma p_0^*\tilde{x} - \frac{\tau\tilde{x}^2}{2}$$

f.o.c

$$\begin{aligned}0 &= \tilde{v} - \mu - 2\lambda\tilde{x} - \lambda\theta - \lambda\gamma p_0^* - \tau\tilde{x} \\ \tilde{x} &= \frac{\tilde{v} - \mu - \lambda\theta - \lambda\gamma p_0^*}{2\lambda + \tau} \\ \tilde{x} &= \underbrace{(\frac{1}{2\lambda + \tau})}_{\beta}\tilde{v} + \underbrace{\frac{-\mu - \lambda\theta - \lambda\gamma p_0^*}{2\lambda + \tau}}_{\alpha}\end{aligned}$$

Market Makers and noise traders will conjecture similarly that $\tilde{x} = \alpha + \beta\tilde{v}$.

1.2 Noise Traders

Recall that noise traders will try to maximize their profit but do not internalize their price impact, plus they face transaction costs $\frac{\tau}{2}\tilde{u}^2$

$$\tilde{\pi} = (\tilde{n} - p_0)\tilde{u} - \frac{\tau\tilde{u}^2}{2}$$

where the f.p.c. implies

$$\tilde{u} = \underbrace{\frac{1}{\tau}}_{\gamma}\tilde{n} + \underbrace{-p_0}_{\theta}$$

1.3 Market Makers

Market Makers assume that IT and NT have linear demands on $\tilde{v}$ and $\tilde{u}$ and in perfect competition adjust prices such that

$$\begin{aligned}
\tilde{p} &= \mathbb{E}[\tilde{v}|\tilde{x}+\tilde{u}] \\
\tilde{p} &= p_0 + \frac{cov(\tilde{v}, \tilde{x}+\tilde{u})(\tilde{y} - \mathbb{E}[\tilde{x}+\tilde{u}])}{var(\tilde{x}+\tilde{u})} \\
\tilde{p} &= p_0 + \frac{cov(\tilde{v}, \alpha + \beta\tilde{v} + \gamma\tilde{n} + \theta)(\tilde{y} - \mathbb{E}[\alpha + \beta\tilde{v} + \gamma\tilde{n} + \theta])}{var(\alpha + \beta\tilde{v} + \gamma\tilde{n} + \theta)} \\
\tilde{p} &= p_0 + \frac{(\beta\sigma_0^2 + \gamma\sigma_{vn})(\tilde{y} - \alpha - \beta p_0 - \gamma p_0^* - \theta)}{\beta^2\sigma_0^2 + \gamma^2\sigma_u^2 + 2\gamma\beta\sigma_{vn}} \\
\tilde{p} &= \frac{\beta\sigma_0^2 + \gamma\sigma_{vn}}{\beta^2\sigma_0^2 + \gamma^2\sigma_u^2 + 2\gamma\beta\sigma_{vn}}\tilde{y} + p_0 + \frac{(-\alpha - \beta p_0 - \gamma p_0^* - \theta)(\beta\sigma_0^2 + \gamma\sigma_{vn})}{\beta^2\sigma_0^2 + \gamma^2\sigma_u^2 + 2\gamma\beta\sigma_{vn}} \\
\tilde{p} &= \underbrace{\frac{\beta\sigma_0^2 + \gamma\sigma_{vn}}{\beta^2\sigma_0^2 + \gamma^2\sigma_u^2 + 2\gamma\beta\sigma_{vn}}}_{\lambda}\tilde{y} + \underbrace{p_0 + \frac{(-\alpha - \beta p_0 - \gamma p_0^* - \theta)(\beta\sigma_0^2 + \gamma\sigma_{vn})}{\beta^2\sigma_0^2 + \gamma^2\sigma_u^2 + 2\gamma\beta\sigma_{vn}}}_{\mu}
\end{aligned}$$

1.4 Equilibrium

I focus first on deriving the equilibrium for λ, β and γ and solve for μ, α, γ later.

$$\frac{\beta\sigma_0^2 + \gamma\sigma_{vn}}{\beta^2\sigma_0^2 + \gamma^2\sigma_u^2 + 2\gamma\beta\sigma_{vn}} = \frac{1}{2}\Big(\frac{1}{\beta} - \frac{1}{\gamma}\Big)$$

Which can be solved numerically for β since $\gamma = \tau^{-1}$

References

J Bradford DeLong, Andrei Shleifer, Lawrence H Summers, and Robert J Waldmann. Noise trader risk in financial markets. *Journal of Political Economy*, 98(4):703–738, 1990.

Albert S Kyle. Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society*, pages 1315–1335, 1985.