Return Predictability

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Introduction

How do we test if returns are predictable?

$$R_{t \to t+k} = a + bx_t + \epsilon_{t \to t+k}$$

this is equivalent to say that expected returns are time-varying

$$\mathbf{E}_t[R_{t \to t+k}] = a + bx_t$$

The Classic Efficient Market view

- In the classic efficient-market view, stock prices are not predictable, (loosely, the random walk view) so we should see $b = R^2 = 0$.
- Modern asset pricing acknowledges that expected returns can vary over time and this does not necessarily contradicts market efficiency.

In fact you can predict returns on T bills almost with a $R^2 = 1$.

$$P_t = e^{-r^f_{t \to t+1}} C \to r^f_{t \to t+1} = -\ln(P_t) - \ln(C)$$

New Facts

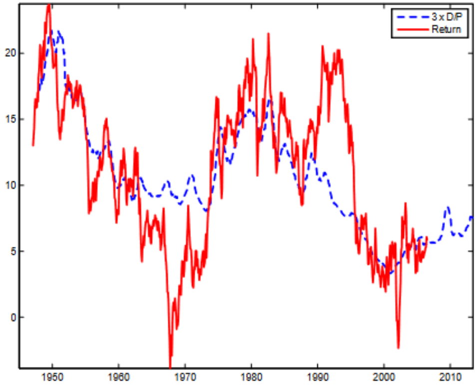
Dividends and prices should forecast returns, and this is the result of ”simple” accounting relations

$$R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t} = \frac{D_t}{P_t}\frac{D_{t+1}}{D_t} + \frac{P_{t+1}}{P_t}$$

here the predictive power of the dividend yield depends on the relative dividend growth and price growth.

Can dividend yields predict returns?

Yes, over long horizons.



Dividend yield and following seven-year return. The dividend yield is multiplied by four. Both series use the CRSP value-weighted market index.

Longer Horizons

Return predictability should growth with time horizon. Define $D_t/P_t = x_t$ and introduce notation (treat all variables as deviations from their means to avoid constants)

$$r_{t+1} = bx_t + \epsilon_{t+1}$$
$$x_{t+1} = \phi x_t + \delta_{t+1}$$

Compounding

$$r_{t+1} + r_{t+2} = bx_t + \epsilon_{t+1} + b(\phi x_t + \delta_{t+1})$$
$$r_{t+1} + r_{t+2} = b(1+\phi)x_t + b\delta_{t+1} + \epsilon_{t+1}$$
$$\sum_{s=1}^{T} r_{t+s} = b\sum_{s=1}^{T}(\phi^{s-1})x_t + \text{error}$$

If dividend yields are very persistent, predictability also occurs for longer horizons $\phi \approx 1$

Predicting single returns in the future

The fact that x_t is highly persistent, also implies that it can predict single returns and not only cumulative returns in the future

$$
\begin{aligned}
r_{t+2} &= bx_{t+1} + \epsilon_{t+2} \\
r_{t+2} &= b(\phi x_t + \delta_{t+1}) + \epsilon_{t+2} \\
r_{t+2} &= b\phi x_t + \text{error} \\
r_{t+T} &= b\phi^{T-1} x_t + \text{error}
\end{aligned}
$$

Present Value Identities

Notation, consider a one period model

$$R_{t+1} = \frac{D_{t+1}}{P_t}$$
$$\frac{R_{t+1}}{D_t} = \frac{\frac{D_{t+1}}{D_t}}{P_t}$$
$$\frac{P_t}{D_t} = \frac{\frac{D_{t+1}}{D_t}}{R_{t+1}}$$

or in logs

$$p_t - d_t = \Delta d_{t+1} - r_{t+1}$$

which holds both ex post (at time $t+1$) and ex-ante (in expectations)

$$p_t - d_t = \mathbf{E}_t \Delta d_{t+1} - \mathbf{E}_t r_{t+1}$$

Understanding present value relations

$$p_t - d_t = \mathbf{E}_t \Delta d_{t+1} - \mathbf{E}_t r_{t+1}$$

Implies that changes in $p_t - d_t$ observed today (mostly p since dividends are very sticky), must come from changes in expectations of dividend growth (cash flow channel) or changes in expectations from discount rates (discount rate channel.) This idea implies that naturally we should study the following predictability regressions

$$r_{t+1} = b_r(d_t - p_t) + \epsilon^r_{t+1}$$
$$\Delta d_{t+1} = b_d(d_t - p_t) + \epsilon^d_{t+1}$$

since $p_t - d_t = r_{t+1} - \Delta d_{t+1}$

$$d_t - p_t = (b_r - b_d)(d_t - p_t) + \epsilon^r_{t+1} - \epsilon^d_{t+1}$$

which ex-post implies $\epsilon^r_{t+1} = \epsilon^d_{t+1}$ and ex-ante $b_r - b_d = 1$

Understanding present value relations

$$1 = (b_r - b_d)$$

$$1 = \frac{cov(r_{t+1}, d_t - p_t)}{var(d_t - p_t)} - \frac{cov(\Delta d_{t+1}, d_t - p_t)}{var(d_t - p_t)}$$

$$var(d_t - p_t) = cov(r_{t+1}, d_t - p_t) - cov(\Delta d_{t+1}, d_t - p_t) > 0$$

It must be that the dividend price ratio predicts returns, dividend growth, or both.

Longer - horizons

Campbell and Shiller Decomposition

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\frac{P_{t+1}}{D_{t+1}} + 1}{\frac{P_t}{D_t}} \frac{D_{t+1}}{D_t}$$

$$r_{t+1} = \log(\frac{P_{t+1}}{D_{t+1}} + 1) + \Delta d_{t+1} - (p_t - d_t)$$

$$r_{t+1} = \log(e^{p_{t+1} - d_{t+1}} + 1) + \Delta d_{t+1} - (p_t - d_t)$$

expand $\log(e^{p_{t+1} - d_{t+1}} + 1)$ around the exponent of the unconditional mean e^{x_0} since $x_0 = \mathbf{E}(p_t - d_t)$

$$\log(e^{p_{t+1} - d_{t+1}} + 1) \approx \log(e^{x_0} + 1) + \frac{e^{x_0}}{e^{x_0} + 1}(p_t - d_t - x_0)$$

Campbell and Shiller Decomposition

Define $\rho = \frac{e^{x_0}}{e^{x_0}+1}$

$$\begin{aligned}
\log(e^{p_{t+1}-d_{t+1}}+1) &\approx [\log(e^{x_0}+1)-\rho x_0] + \rho(p_{t+1}-d_{t+1}) \\
r_{t+1} &\approx [\log(e^{x_0}+1)-\rho x_0] + \rho(p_{t+1}-d_{t+1}) + \Delta d_{t+1} - (p_t - d_t) \\
r_{t+1} &\approx k + \rho(p_{t+1}-d_{t+1}) + \Delta d_{t+1} - (p_t - d_t)
\end{aligned}$$

if you substract the unconditional expectation (and replace the notation)

$$\begin{aligned}
r_{t+1} - \mathbf{E}r_{t+1} &\approx (k-k) + \rho(p_{t+1}-d_{t+1}-x_0) + \Delta d_{t+1} - \mathbf{E}\Delta d_{t+1} - (p_t - d_t - x_0) \\
r_{t+1} &\approx \rho(p_{t+1}-d_{t+1}) + \Delta d_{t+1} - (p_t - d_t)
\end{aligned}$$

Campbell and Shiller Decomposition

$$p_t - d_t \approx \rho(p_{t+1} - d_{t+1}) + (\Delta d_{t+1} - r_{t+1})$$

Iterating forward

$$p_t - d_t \approx \rho(\rho(p_{t+2} - d_{t+2}) + (\Delta d_{t+2} - r_{t+2})) + (\Delta d_{t+1} - r_{t+1})$$
$$p_t - d_t \approx \rho(\rho(\rho(p_{t+3} - d_{t+3}) + (\Delta d_{t+3} - r_{t+3})) + (\Delta d_{t+2} - r_{t+2})) + (\Delta d_{t+1} - r_{t+1})$$
$$p_t - d_t \approx \sum_{j=1}^{k} \rho^{j-1}(\Delta d_{t+j} - r_{t+j}) + \rho^k(p_{t+k} - d_{t+k})$$

The last term is called the ”rational bubble term” or the transversality condition, and should approach zero as k increases. In other words

$$p_t - d_t \approx \mathbf{E}_t \sum_{j=1}^{\infty} \rho^{j-1}(\Delta d_{t+j} - r_{t+j})$$

Predictions

- Price-dividend ratios can move if and only if there is news about current dividends, future dividend growth or future returns.
- For $p_t - d_t$ to move there must be expected variation in dividend growth or in returns.

Long term regressions

Introduce the same notation (think of ∞ as a long sample period

$$\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} = b_d(d_t - p_t) + \epsilon_d$$

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = b_r(d_t - p_t) + \epsilon_r$$

replacing

$$b_r - b_d \approx 1$$

$$\epsilon_r - \epsilon_d \approx 0$$