Recovering Expenditures and Prices in a Spatial Model

Notation

$\mathcal{C}$ - set of consumption goods. N_c.
C - vector of consumption expenditures. $N \times N_c$.
$\mathcal{S}$ - set of sectors. N_s.
Γ - IO matrix. $N_s \times N_s$.
Δ - mapping from $\mathcal{C}$ to $\mathcal{S}$. $N_c \times N_s$.
Π^s - trade flow matrix. $N \times N$.
R - revenue matrix. $N \times N_s$.
F - final demands. $N \times N_s$.

Equilibrium

(Definition) $F = C\Delta$
(Market clearing) $R_{is} = \sum_n \pi^s_{in} \left(\sum_{c \in \mathcal{C}} \delta_{sc} C_{nc} + \sum_{q \in \mathcal{S}} \gamma_{sq} R_{nq} \right)$

Solution

Assume R, Π and Γ are observed. Define

$$b_s = R^s - \Pi^s R \Gamma^s$$

which is $N \times 1$. Then

$$b_s = \Pi^s F^s.$$

Clearly to recover final demands we must have Π^s invertible. This is equivalent to assuming that knowing where production happens is informative about where consumption happens, which is true whenever we have some trade costs. For example this is trivial when the sector s good is nontradable. Π^s is *not* invertible under the assumption that the sector s good is perfectly tradable. This case is dealt with separately below.

If Π^s is invertible then we can recover a matrix of final demands F^s which satisfies

$$F = C\Delta.$$

Clear to recover C we need to invert Δ. A necessary condition is that $N_c \leq N_s$, i.e we cannot have more consumption goods than we have sectors. More generally we need no two consumption goods to have the same sectoral composition. It is not a problem to have some sectors which do not map to any consumption goods (e.g machinery is not part of any consumption good, just an input into other consumption goods).

Recovering Prices

Now assume that each C_i vector is the result of some demand system, i.e $C_i = D(p_i, f_i)$ where p_i is the vector of prices in location i and f_i is the income distribution in i.