Sorting and the Skill Premium: The Role of Nonhomothetic Housing Demand *

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Abstract

Since 1980 skilled and unskilled workers have increasingly sorted into different cities. In this paper we propose and quantify a novel driver of increased spatial sorting — diverging preferences over location attributes caused by diverging incomes across skill groups. The root cause of these diverging preferences is income inelastic housing demand, which we show to be a robust feature of consumption microdata. As the incomes of skilled workers rise relative to those of unskilled workers, their housing expenditure shares fall and they become more willing to cluster together in expensive, skill-intensive cities. By adding nonhomothetic preferences to an otherwise standard quantitative spatial model we show that this mechanism explains 22% of the increase in sorting since 1980.

1 Introduction

The gap in wages between workers with and without a college degree — the skill premium — has grown since 1980. The spatial literature has also documented a widening gap in the location choices of skilled and unskilled workers at the city level (Berry and Glaeser 2005), and pointed out the potential negative welfare consequences of this increase in sorting through productivity and amenity spillovers (Diamond 2016; Fajgelbaum and Gaubert 2020). In this paper we show that income inelastic housing demand

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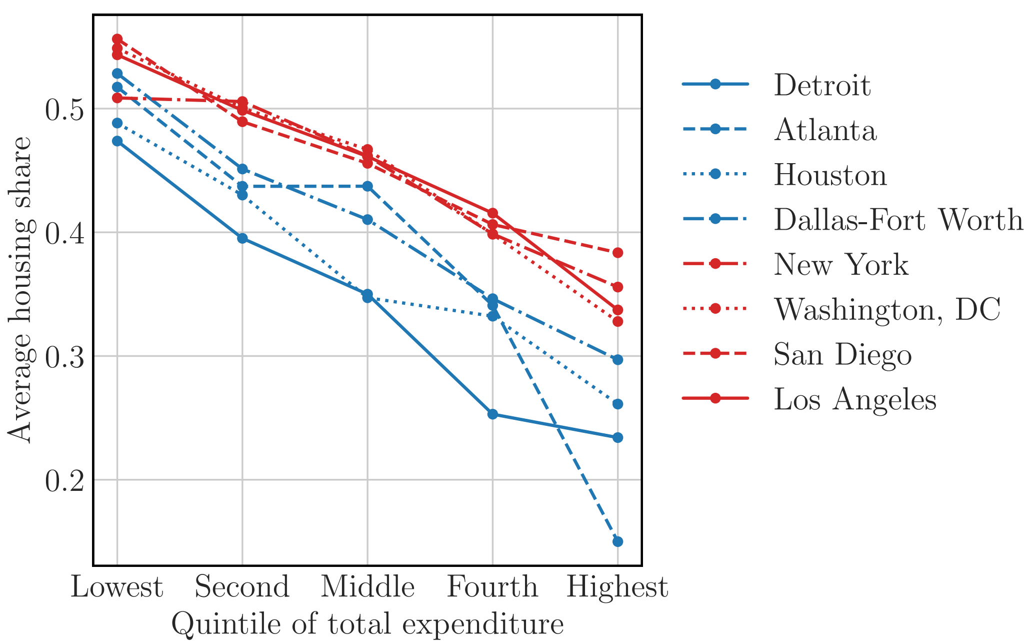
transmits shocks to the skill premium into changes in spatial sorting. The mechanism is simple. Less skilled, hence poorer, households spend more on housing relative to total expenditure and are therefore more sensitive to housing costs in choosing where to live. Skilled households, by contrast, have smaller housing expenditure shares and so put less weight on housing costs. As incomes across skill groups have diverged since 1980, so have these sensitivities to housing costs, and thus so have location choices. Under preferences consistent with consumption microdata, the growth of the skill premium accounts for just over a fifth of the total increase in sorting observed since 1980. We contribute to an emerging literature which connects nonhomothetic preferences measured at the individual level to aggregate outcomes — see for example Straub (2019) on savings and the wealth distribution, Borusyak and Jaravel (2018) on the gains from trade, and Comin, Danieli, and Mestieri (2020) on labor market polarization.

We begin by showing that housing demand is income inelastic. Figure 1 previews expenditure data from the Consumer Expenditure Survey (CEX). Each line plots housing expenditure shares by total expenditure for a set of cheap and expensive cities. Two facts are apparent. First, within each city, housing shares decline with total expenditure. Sec-

Figure 1: Housing share by MSA and expenditure



Source: CEX, 2006-2017, renters only. Housing share is defined as housing expenditure divided by total expenditure. The sample is cut into five groups ordered by total expenditure. We rank all cities identified in the CEX according to the BEA's regional price index for housing services. The plot features the four lowest price (blue) and four highest price (red) cities.

ond, at each given level of total expenditure, housing shares are higher in more expensive cities. This plot suggests that housing demand is income inelastic, but also points to the importance of controlling for local prices.

In Section 2 we formalize these insights by estimating nonhomothetic constant elasticity of substitution (NHCES) preferences over housing and nonhousing consumption. Our choice of NHCES preferences is motivated by three factors: they flexibly accommodate both income and price effects; they nest special cases usually employed in the spatial literature; and they are tractable enough to incorporate into a quantitative spatial model, as we show in Section 3. A crucial feature of our estimation strategy is controlling for local variation in housing prices. We show that failing to control for prices tends to bias estimates of the income elasticity towards homotheticity. Our estimated preferences display significant nonhomotheticity. For a household in the middle of the expenditure distribution, a 10% increase in total expenditure results in a 2.4% decrease in the housing expenditure share.

Several recent papers share our interest in estimating nonhomothetic preferences in a spatial setting. Particularly relevant is Albouy, Ehrlich, and Liu (2016), who also estimate NHCES preferences for housing, but use only aggregate variation in incomes and expenditure shares. Reassuringly our approach based on household level variation yields similar results. Also related is Handbury (2019), who estimates nonhomothetic price indices for groceries in different cities. Both of these papers focus on the implications of nonhomothetic preferences for inequality. Our focus is instead on how nonhomothetic preferences drive sorting across locations.

In Section 3 we embed NHCES preferences in a spatial model with skilled and unskilled workers to draw out their implications for sorting. In choosing where to live, households trade off wages, amenities and housing costs. Our empirical results establish that less skilled — and therefore poorer — households have higher housing expenditure shares. In our model this directly implies that they are more sensitive to housing costs relative to wages when choosing where to live. The difference in sensitivities by skill causes sorting, with skilled households disproportionately choosing to locate in expensive cities.

The sorting motive here is not new to the spatial literature — it emerges naturally in many papers which include some form of nonhomothetic housing demand (e.g Schmidheiny (2006), Gyourko, Mayer, and Sinai (2013)). Instead, our key theoretical contribution is to study how *location-neutral* changes in the wage distribution change the spatial distribution of skilled versus unskilled workers. In our model, an increase in the aggregate skill premium pushes the preferences of skilled and unskilled workers further apart and encourages ever more divergent location choices.

The mechanism described above differs significantly from alternative explanations for the 'Great Divergence' (Moretti 2012). Ganong and Shoag (2017) also consider the role of nonhomothetic housing demand, but focus on location-specific changes in housing supply regulations as the ultimate cause of divergence. Similarly, Diamond (2016) explains divergence as a result of local productivity shocks amplified by endogenous amenities. In contrast we focus on a shock (the rising national skill premium) which does not vary across locations, but which nevertheless generates very different outcomes across different locations. Another strand of the literature (Giannone (2019); Eckert, Ganapati, and Walsh (2020)) has explored the consequences of aggregate shocks to skilled productivity which alter the spatial distribution of economic activity by altering the spatial distribution of wages. Our mechanism operates through quite a different channel. Although in our quantitative model we endogenize wage and price responses, at its core our mechanism does not rely on them. Rather, rising inequality changes how households of different skill levels evaluate a fixed set of location characteristics by changing the weight they place on housing costs relative to wages.

Our paper is also related to a literature which studies the link between the income distribution and within-city sorting. Fogli and Guerrieri (2019) explore how a shock to the skill premium triggers an intensification of within-city segregation which ultimately amplifies the initial shock. Couture et al. (2019) explain gentrification since 1990 as a result of increasing income inequality coupled with endogenous neighborhood amenities. Both of these papers exploit an extreme form of nonhomotheticity — a unit housing requirement — which is standard in the within-city literature but which we will show is not consistent with the data at a between-city level. Therefore, in addition to the obvious differences in geography, our contribution relative to these papers lies in our estimation of nonhomothetic housing demand using consumption microdata.

Finally Sections 4 and 5 quantify the importance of the rising national skill premium for the increase in spatial sorting observed since 1980. We enrich our theoretical model with imperfect substitution between skilled and unskilled labor in production; heterogeneous housing supply elasticities across locations; and progressive income taxation. We use the quantitative model to infer how sorting would have evolved if the skill premium had remained constant at its 1980 level. We find that sorting would have risen by 22% less. Our model attributes the remaining 78% to location-specific productivity, amenity and housing supply changes such as those suggested by, e.g Diamond (2016) or Ganong and

Shoag (2017). The quantitative model highlights the importance of carefully estimating housing demand. We show that a Cobb-Douglas specification implies that the rising skill premium makes exactly zero contribution to the change in sorting; while a model with a unit housing requirement significantly overstates the role of the skill premium. In embedding nonhomothetic preferences in a quantitative spatial model, we follow Eckert and Peters (2018), who study the spatial implications of structural change. A crucial difference between their model and ours is that they shut down sorting across locations by income, whereas this sorting is at the heart of our mechanism.

2 Estimating the Income Elasticity of Housing Demand

In this section we estimate household preferences. In subsection 2.1 we describe the data. In subsection 2.2 we specify preferences for housing. Subsection 2.3 estimates the key parameters of our model. Before proceeding we briefly overview the main challenges involved. First, although it is standard to refer to the 'income elasticity' of housing demand, the conceptually appropriate elasticity is actually with respect to total expenditure. Therefore we require data on total expenditure. Second, expenditure is likely to be measured with error, which introduces substantial bias in our context. Therefore we require an instrument for expenditure. Finally, and most importantly, the price of housing varies widely across space, and does so in a way that is correlated with household income. Therefore we need to control for variation in housing prices. As we show below, failing to do so would strongly bias our results towards homotheticity.

2.1 Data

Consumption microdata

We measure the income elasticity of housing demand using the Panel Study of Income Dynamics (PSID). The PSID is a longitudinal survey which collects information on income, wealth, expenditure and other socioeconomic variables. Since 1999 the PSID has collected information on a number of expenditure categories, and since 2005 has included essentially all consumption covered by the Consumer Expenditure Survey (CEX) (Andreski et al. 2014). Because price data (discussed below) are not available until 2008, we use the 2009-2017 surveys. We restrict our sample to renting households because they have a clear measure of housing consumption.[1] We work with the restricted-access ver-

[1]In subsection 2.3.1 we present similar results using homeowners.

sion of the PSID, which includes county identifiers for each household. We map counties to the metropolitan statistical area (MSA) level and use this as our geography.

We prefer the PSID over the CEX for two reasons. First, the CEX has geographic identifiers only for households in large cities, covering less than half of the sample. This means we have to drop most CEX respondents in the analysis if we control for local prices. By contrast, the restricted-access PSID has geographic identifiers for the *full* sample, and we are able to link price data to about ninety percent of households (the remaining ten percent are rural areas for which no good housing price indices exist). The second advantage of the PSID is that we can follow the same households over time, allowing us to study how housing shares respond to changes in total expenditure within the same household.

Housing prices

Our primary data on housing prices comes from the Metropolitan Regional Price Parities produced by the Bureau of Economics Analysis (BEA 2020). For every MSA in the US, the BEA estimates price indices for goods, housing, and all other services besides housing. The housing index comes from American Community Survey (ACS) data on rents. The index explicitly controls for differences in housing quality and quantity across regions, making it a good measure of the price per unit of housing services.

2.2 Preferences

A household i in year t and location n has total expenditure e_{int}. There are two goods — housing and a numéraire, freely traded consumption good. The price of housing p_{nt} is location-year specific. Households have nonhomothetic constant elasticity of substitution (NHCES) preferences[2]. The utility U of household i consuming h units of housing and c units of the consumption good is implicitly defined by

$$1 = (\Omega_{int})^{\frac{1}{\sigma}} \left(hU^{-(1+\epsilon)}\right)^{\frac{\sigma-1}{\sigma}} + \left(cU^{-1}\right)^{\frac{\sigma-1}{\sigma}} \tag{1}$$

where $\epsilon \geq -1$ and $\sigma \in (0, 1)$ are parameters and Ω_{int} is a preference shifter which we parameterize below.[3] Maximizing (1) subject to a standard budget constraint yields household i's expenditure share on housing, denoted by $\eta\left(e, p_{nt}, \Omega_{int}\right)$ and implicitly defined

[2]For a detailed discussion of NHCES preferences, see Comin, Lashkari, and Mestieri (2015).

[3]The restriction that $\sigma < 1$ implies that housing demand is price inelastic, which will turn out to be the empirically relevant case. We impose this purely for ease of exposition. NHCES preferences in general do allow $\sigma > 1$.

as the solution to

$$\eta = \Omega_{int} e^{\epsilon(1-\sigma)} p_{nt}^{1-\sigma} (1-\eta)^{1+\epsilon}. \tag{2}$$

The parameters ϵ and σ jointly govern income and price effects. To clarify the role of each parameter, note that the elasticity of the relative housing share, $\eta/(1-\eta)$, with respect to utility, U, is

$$\frac{\partial \log\left(\frac{\eta}{1-\eta}\right)}{\partial \log U} = (1-\sigma)\epsilon$$

and the elasticity with respect to prices, p, is

$$\frac{\partial \log\left(\frac{\eta}{1-\eta}\right)}{\partial \log p} = (1-\sigma).$$

ϵ governs the relationship between housing demand and real consumption, cardinalized by U. A negative value of ϵ corresponds to housing as a necessity while positive ϵ implies that housing is a luxury.

NHCES preferences nest two specifications commonly used in the spatial literature. Taking $\epsilon = 0$ and $\sigma = 1$ gives Cobb-Douglas preferences.[4] In this case the expenditure share is given by

$$\eta\left(e, p_{nt}, \Omega_{int}\right) = \frac{\Omega_{int}}{1+\Omega_{int}} \tag{3}$$

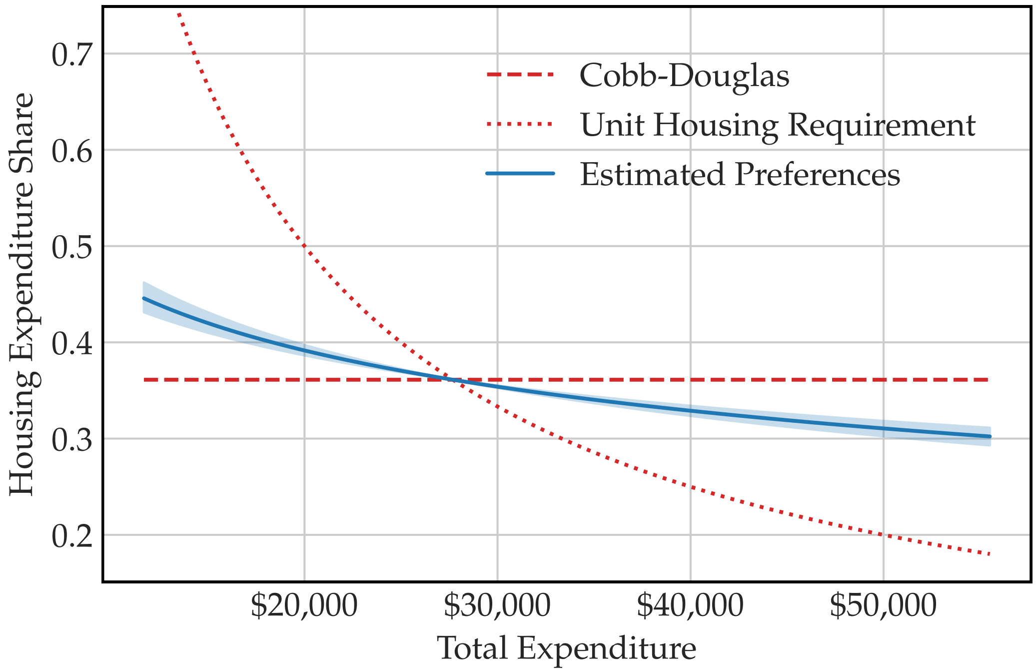
showing that under these preferences expenditure shares are unrelated to price or total expenditure. The opposite case can be obtained by setting $\epsilon = -1$ and $\sigma = 0$, yielding a unit housing requirement. In this case the expenditure share is

$$\eta\left(e, p_{nt}, \Omega_{int}\right) = \Omega_{int} p_{nt} e_{int}^{-1}. \tag{4}$$

The dashed and dotted lines in Figure 2 depict these two cases. For values of ϵ and σ between these two extremes housing demand is income and price inelastic, but not perfectly so.

[4] Strictly speaking, this is the limit as $\sigma \to 1$, and likewise the unit housing requirement is the limit as $\sigma \to 0$.

Figure 2: Housing Expenditure Shares



Notes: 'Cobb-Douglas' and 'Unit Housing Requirement' plot the preferences described by (3) and (4), respectively. 'Estimated Preferences' plots (2) at the parameter values in Table 1, Column (4). The shaded area represents a 99% confidence interval. In each case the scale parameter Ω is chosen to match an expenditure share of 0.37 at median total expenditure — this value is taken from our baseline sample.

2.3 Estimation

To build intuition for our estimation strategy, we log-linearize (2) to obtain

$$\hat{\eta}_{int} = \left(\frac{1-\bar{\eta}}{1-\bar{\eta}+(\epsilon+1)\bar{\eta}}\right)\left(\hat{\Omega}_{int} + \epsilon(1-\sigma)\hat{e}_{int} + (1-\sigma)\hat{p}_{nt}\right). \tag{5}$$

Here $\bar{\eta}$ denotes the median housing share and $\hat{x}$ denotes the log deviation of a variable from its median. Equation 5 can be more simply written as

$$\hat{\eta}_{int} = \omega_{int} + \beta\hat{e}_{int} + \psi\hat{p}_{nt} \tag{6}$$

Under the null of homothetic preferences, $\epsilon = \beta = 0$. We bring (6) to the data by modeling ω_{int} as a function of observables, year fixed effects and an additive error. Formally,

$$\hat{\eta}_{int} = \omega_t + \omega' X_{int} + \beta\hat{e}_{int} + \psi\hat{p}_{nt} + \zeta_{int} \tag{7}$$

where X_{int} is a vector of demographic characteristics, including the age, gender, and race of the household head; household size; and the number of earners in the household. Total expenditure e and the housing expenditure share η are observed in the data. The error term ζ_{int} represents potential measurement error in expenditure plus random shocks to household demand which are assumed to be uncorrelated with expenditure and prices conditional on the controls.

Table 1, columns (1) - (4), show the results of estimating equation 7. Column (1) estimates equation 7 by OLS without controlling for price $\hat{p}_{nt}$. The point estimate indicates significant nonhomotheticity, but two sources of bias are evident. First, measurement error in

Table 1: Preference Estimates

Dependent variable: Log housing share

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) GMM	(5) 2SLS
Log expenditure	-0.248***	-0.133***	-0.238***		-0.259***
	(0.018)	(0.028)	(0.031)		(0.076)
Log price			0.365***		0.364***
			(0.031)		(0.067)
Implied ϵ			-0.642***	-0.679***	-0.713***
			(0.079)	(0.074)	(0.229)
Implied σ			0.571***	0.563***	0.583***
			(0.043)	(0.044)	(0.102)
Demographic controls	✓	✓	✓	✓	
Household FE					✓
Year FE	✓	✓	✓	✓	✓
R^2	0.09	0.08	0.15	-	0.13
First-stage F-statistic	-	1,196.4	883.0	-	98.1
N	10,129	10,129	8,923	8,923	7,017
No. of clusters	4,816	4,816	4,229	4,229	2,377

Source: PSID and BEA

Note: Renters only. Instrument is log family income. Demographic controls are bins for family size, number of earners, and sex, race, and age of household head. Standard errors clustered at household level. See Appendix A for further details of sample construction. Null hypotheses are $\epsilon = 0$ and $\sigma = 1$, respectively.

* p < 0.1, ** p < 0.05, *** p < 0.01. Standard errors in parentheses.

expenditure is likely to bias $\hat{\beta}$ downwards.[5] Second, a positive correlation between prices and expenditure (reflecting the sorting of high income households into high price MSAs) will bias $\hat{\beta}$ upwards.

Column (2) addresses the first source of bias by instrumenting for log expenditure using log income. The exclusion restriction is that once a household has chosen its overall level of expenditure, income is irrelevant in determining housing expenditure. Consistent with the argument above, $\hat{\beta}$ rises and suggests only a small amount of nonhomotheticity. This is similar to the results in Aguiar and Bils (2015) or Davis and Ortalo-Magné (2011). Those papers use similar specifications and do not control for local prices.

Column (3) additionally controls for local prices with the constant-quality price indices from the BEA. $\hat{\beta}$ falls relative to column (2), consistent with the sorting of high income households into expensive locations. Unconditionally, high income households dedicate nearly as high a share of their expenditures to rent as low income households; but a substantial fraction of that reflects the higher prices they pay for each unit of housing services consumed because they choose to live in more expensive cities. In column (3) we also use (5) to back out estimates for the structural parameters ϵ and σ.

Column (4) shows our preferred specification. Here we estimate the nonlinear equation 2 directly by GMM. Similarly to column (3) we instrument for expenditure using income, use BEA price data and allow Ω_{int} to vary with demographic characteristics and year. The implied $\hat{\epsilon}$ and $\hat{\sigma}$ are close to their values in column (4). Moreover, $\hat{\epsilon}$ is significantly different from 0 at the 1% level. We therefore reject the null hypothesis of homothetic preferences. Since $\hat{\epsilon}$ and $\hat{\sigma}$ are far from -1, we can also reject a unit housing requirement. The solid line in Figure 2 plots our estimated housing demand function.

Column (5) explores an alternative specification for the shifter ω_{int}. We suppose that it can be written as a household fixed effect and an additive error term. Formally

$$\hat{\eta}_{int} = \omega_i + \omega_t + \beta \hat{e}_{int} + \gamma \hat{p}_{nt} + \zeta_{int}. \tag{8}$$

This specification implies that β is identified using only within-household variation in expenditure. It allows us to distinguish between our explicitly nonhomothetic preferences and alternative models in which expenditure shares may differ across different 'types' of household, but are fixed for a given type over time — as in Diamond (2016), for example. The reduced form coefficient is very similar to our baseline. The standard error of

[5] Because expenditure appears in the denominator of $\hat{\eta}$ this is not the usual classical measurement error result. See Appendix B for a short proof.

the structural parameter is large but the point estimate is also similar to our preferred specification. These results indicate that changes in total expenditure over time are associated with changes in the housing expenditure share, consistent with nonhomothetic preferences.

2.3.1 *Alternative Specifications*

We explore a number of alternative specifications and data sources in the Appendix.

In Table B.1, we present alternative specifications in the PSID. The results do not change when we control for local non-housing prices, when we use an alternative price index from Zillow, or when we use alternative instruments for expenditure. Estimates based on more granular price variation (at the county level) indicate if anything more nonhomotheticity than our baseline results. We replicate our main results using the Consumer Expenditure Survey (CEX), an alternative source for consumption microdata. We find similar parameter estimates in Table B.2. We then extend our results to homeowners using the CEX, which has superior coverage of homeowners relative to the PSID. The estimated parameters look very similar when we include homeowners. In Appendix B we relate our results to Davis and Ortalo-Magné (2011), who show that median housing shares are constant across space and offer this as evidence for Cobb-Douglas preferences. Briefly, we argue that the housing share mixes income and price effects, and that constant housing shares across space are necessary but not sufficient to conclude preferences are Cobb-Douglas. Finally, Appendix B.5 summarizes estimates of the income elasticity from the literature.

3 Model

Section 2 established that housing demand is income inelastic. In this section we study the implications of this fact for spatial sorting by skill, with a focus on how income inelastic housing demand connects the skill premium to sorting. We start with a simple model in which it is possible to analytically characterize the relationship between the skill premium and sorting. Then we construct a more realistic quantitative model which we use in counterfactuals in Section 5.

3.1 Simple Model

Production and Wages

There are two types of household, skilled and unskilled, with types denoted by $i = s, u$. Households supply labor to tradable goods producers in the location in which they live, denoted by n. These firms are perfectly competitive and produce using skilled and unskilled labor according to the function

$$F_n(l_{sn}, l_{un}) = z_n(A \cdot l_{sn} + l_{un}). \tag{9}$$

Note that skilled and unskilled labor are perfect substitutes, and that their relative productivities do not vary across locations. This implies that the skill premium is exogenous and equal to A in every location. Households do not save, so wages w_{in} are exactly equal to expenditure e_{in}. Expenditures and wages therefore satisfy

$$e_{sn} = w_{sn} = z_n A \tag{10}$$

$$e_{un} = w_{un} = z_n. \tag{11}$$

Location Choice and Preferences

We first describe the problem of a household in a given location, then turn to the household's choice of location. Households of type i in each location n have NHCES preferences (as in (1)) and solve the following problem

$$\begin{aligned} \max_{c,h} \quad & U \\ \text{s.t} \quad & 1 = (\Omega)^{\frac{1}{\sigma}} \left(hU^{-(1+\epsilon)}\right)^{\frac{\sigma-1}{\sigma}} + \left(cU^{-1}\right)^{\frac{\sigma-1}{\sigma}}, \\ & c + p_n h = e_{in} \end{aligned} \tag{12}$$

Below we denote the level of utility attained by a household of type i in location n by v_{in} and their housing expenditure share by η_{in}.

Having determined v_{in} for every location and type of household, we add a simple model of location choice based on Fréchet distributed preference shocks (Allen and Arkolakis 2014; Redding 2016). Each household ω draws an n-vector of idiosyncratic location preferences $\boldsymbol{\zeta}(\omega)$ from independent Fréchet distributions. The distribution of draws in n has scale B_n and shape θ. The preference shocks are multiplicative, so that the utility of household ω is

$$V_i(\omega) = \max_n \zeta_n(\omega) v_{in}.$$

Standard properties of the Fréchet distribution (e.g. Redding (2016)) imply that the number of households of type i who choose n is given by

$$l_{in} = \frac{v_{in}^{\theta} B_n}{\sum_m v_{im}^{\theta} B_m} L_i \tag{13}$$

where L_i is the exogenously given national population of households of type i. We refer to θ as the migration elasticity. Note that we impose that amenities do not differ by type i, an assumption we relax later.

Housing Supply

In the simple model, housing is supplied perfectly elastically at an exogenous price p_n in each location.

Equilibrium

Given parameters $\epsilon, \sigma, \Omega, \theta, A$ and fundamentals $(z_n, B_n, p_n)_n, (L_i)_i$, an equilibrium is a vector of populations l_{in}, wages w_{in}, and total expenditures e_{in} satisfying (10), (11), (12) and (13).

3.2 Analytical Results

In the simple model described above we can characterize changes in sorting analytically.[6] Our interest is in the log skill ratio in each location n, which we denote by s_n and define as the log ratio of skilled to unskilled households in location n

$$s_n = \log\left(\frac{l_{sn}}{l_{un}}\right).$$

Our main measure of sorting will be the variance of the log skill ratio, S, defined by

$$S = \mathrm{Var}\left(s_n\right).$$

[6]As in Section 2, we assume $\sigma \in (0,1)$. Proofs of all the statements in this section can be found in Appendix C.

This measure is zero when skilled workers are distributed in proportion to unskilled workers across space, and rises as they become more clustered. We emphasize this measure because it is invariant to proportional increases in the number of skilled workers in all locations, and therefore better reflects true changes in sorting. This is an especially desirable property given that between 1980 and 2010 the number of skilled workers in the US grew relative to the number of unskilled workers. However in Section 5 we will also report results for other measures of sorting.

We begin by characterizing the relationship between prices p_n and the skill ratio s_n.

Lemma 1 *Define the productivity-adjusted price of housing, $\tilde{p}_n$, by*

$$\tilde{p}_n = p_n z_n^{\epsilon}.$$

Then the log skill ratio s_n is a strictly increasing function of $\tilde{p}_n$, as long as $\epsilon < 0$. If $\epsilon = 0$ then s_n does not depend on $\tilde{p}_n$.

The intuition for this result is that unskilled households have higher housing expenditure shares, and are therefore more sensitive to housing prices. The result is that high price locations are disproportionately populated by skilled households. In this simple model, we have deliberately shut down sorting on wages and amenities, so differences in $\tilde{p}_n$ are the only motive for sorting. If $\epsilon = 0$ so that preferences are homothetic, housing costs do not drive sorting. In a richer environment the level of sorting would still be positively correlated with housing costs, conditional on wages and amenities, as long as $\epsilon < 0$.

We consider a small increase in the skill premium, $d \log A > 0$. Our main result characterizes how changes in the log skill ratio, ds_n, vary with the level of the log skill ratio, s_n.

Proposition 1 *Suppose $\epsilon < 0$. Then ds_n is a strictly increasing function of s_n. This implies that sorting rises, i.e $dS > 0$. If instead $\epsilon = 0$, then $ds_n = 0$ for all n and $dS = 0$.*

Proposition 1 tells us that increases in the skill premium amplify pre-existing patterns of sorting. To gain intuition for this result, it is helpful to think about how households trade off wages and housing costs using Roy's identity

$$\frac{\partial \log v_{in}}{\partial \log p_n} = -\eta_{in} \frac{\partial \log v_{in}}{\partial \log w_{in}}.$$

This equation states that the sensitivity of utility to housing costs relative to wages is an increasing function of the housing expenditure share. In Section 2 we showed that η_{in} is decreasing in total expenditure; in the model this corresponds to the assumption $\epsilon < 0$. As the total expenditure of skilled workers rises because of the rising skill premium, η_{sn} falls. Therefore the sensitivity of skilled workers to housing costs falls relative to that of unskilled workers. Equation 13 translates this into diverging location choices, with skilled workers moving towards more expensive cities. Because expensive cities are initially more skill intensive by Lemma 1, sorting rises.

Finally, the homothetic case clarifies the mechanics of the second part of Proposition 1. Rewriting the log skill ratio in terms of employment shares given by (13) yields

$$s_n = \varsigma + \theta \log\left(\frac{v_{sn}}{v_{un}}\right)$$

where ς is an endogenous constant. The Cobb-Douglas preferences used in most quantitative spatial models are log-linear in expenditure and prices: $v(e, p) = e/p^{\eta}$ for a constant η. This implies that $s_n = \varsigma + A$ is constant. By contrast, when preferences are nonhomothetic, the ratio $\frac{v_{sn}}{v_{un}}$ varies with the cost of living.

We close this subsection by briefly discussing the interpretation of Proposition 1. It can of course be thought of as a statement about changes over time in the skill premium while other fundamentals remain constant. But we can also think of Proposition 1 as saying — for any distribution of fundamentals, sorting will always be lower in a counterfactual economy with the same fundamentals but a lower skill premium. This interpretation will be the relevant one when we conduct our counterfactual experiment in Section 5.

3.3 Quantitative Model

To take the simple model above to the data, we enrich it on several dimensions: imperfect substitution between skilled and unskilled labor; inelastic housing supply; progressive taxation; and type-specific amenities.

We replace the production function (9) with a CES production function

$$F_n(l_s, l_u) = \left((A_{sn} l_s)^{\frac{\rho-1}{\rho}} + (A_{un} l_u)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

where ρ is the elasticity of substitution between skilled and unskilled labor. Note we allow the productivities A_{in} to vary freely. This implies wages

$$w_{in} = A_{in} l_{in}^{\frac{-1}{\rho}} \left((A_{sn} l_{sn})^{\frac{\rho-1}{\rho}} + (A_{un} l_{un})^{\frac{\rho-1}{\rho}} \right)^{\frac{1}{\rho-1}}. \tag{14}$$

We endogenize housing prices with a reduced form model of housing supply similar to Hsieh and Moretti (2019). The price of housing in location n is given by

$$p_n = \Pi_n (HD_n)^{\gamma_n} \tag{15}$$

where HD_n is (physical) housing demand in n and Π_n is an exogenous price shifter. γ_n is the (inverse) elasticity of housing supply, which is allowed to vary by location. Housing demand is the sum of housing consumption by both types of household

$$HD_n = \sum_i \eta_{in} e_{in} l_{in} p_n^{-1}. \tag{16}$$

We allow for progressive taxation following Heathcote, Storesletten, and Violante (2017). Expenditure equals after tax income, defined for a household of type i in location n as

$$e_{in} = \lambda w_{in}^{1-\tau}. \tag{17}$$

τ determines the progressivity of the tax system and λ is chosen so that the government budget balances.

Finally, we leave households' preferences within a location unchanged relative to the simple model, but allow amenities B_{in} to vary with household type. Equation 13 then becomes

$$l_{in} = \frac{v_{in}^{\theta} B_{in}}{\sum_m v_{im}^{\theta} B_{im}} L_i. \tag{18}$$

Equilibrium

Given parameters and fundamentals $(A_{in}, B_{in})_{i,n}, (\Pi_n)_n, (L_i)_i$, an equilibrium is a vector of populations l_{in}, wages w_{in}, total expenditures e_{in}, expenditure shares η_{in}, housing demands HD_n, and prices p_n satisfying equations (14), (17), (18), (12), (15) and (16).

A Neutrality Result

Finally, we show that part of Proposition 1 remains true in the quantitative model.

Proposition 2 *Suppose $\epsilon = 0$ so that preferences are homothetic. Then a proportional increase in A_{sn} in all locations has no effect on sorting and $dS = 0$.*

Please see Appendix C.3 for a proof. Inflating the productivity of skilled households by a constant factor in every location — a location-neutral increase in the aggregate skill premium — does not affect sorting when preferences are homothetic. Prices do not determine the relative attractiveness of a location to skilled versus unskilled households and so do not cause sorting. Proposition 2 is useful because it implies that any increases in sorting observed in our quantitative model are ultimately the result of nonhomothetic housing demand.

4 Calibration

We set ϵ and σ at the values obtained in Column (4) of Table 1. The remaining parameters to be calibrated are the elasticity of substitution between skilled and unskilled labor in production, ρ; the tax progressivity parameter, τ; the location specific inverse supply elasticities, γ_n; and the migration elasticity, θ. We choose units so that that the scaling factor Ω is normalized to one. We provide more detail on the calibration of each parameter in Appendix D.

4.1 Data

Location-level information on wages, rents and employment are from IPUMS (Ruggles et al. 2020). We use the 5% population samples of the 1980, 1990, and 2000 decennial Censuses and the 3% population sample from the 2009-2011 ACS. We work at the Metropolitan Statistical Area (MSA) level of geography. An MSA is defined as a set of counties with strong economic ties and a dense urban core, which corresponds to the notion of a region in our model. After ensuring locations are consistently defined across years we are left with a panel of 219 MSAs plus 50 residual state locations. Our Census sample constitutes prime-age adults who report strong labor force attachment. See Appendix A for more details, variable definitions, and sample construction.

4.2 Parameters

Elasticity of substitution The production side of the model is standard and we externally calibrate ρ. We set $\rho = 3.85$ to match the estimate from Card (2009).[7] That paper

[7] See Table 5, column 7 in Card (2009) for the negative inverse elasticity of -0.26.

estimates the elasticity of substitution between workers of different skill groups at the MSA level using immigration as an instrument for labor supply changes.

The elasticity is larger than canonical estimates from Katz and Murphy (1992) and Acemoglu and Autor (2011), who report a value close to 1.6. The discrepancy likely arises from the different sources of variation these papers exploit: Katz and Murphy (1992) model an *aggregate* production function fitted to time-series data, while Card (2009) models a *city-level* production function fitted to cross-sectional data. Empirically, studies that estimate the elasticity of substitution across cities tend to find values consistent with Card (2009). Bound et al. (2004) report a central estimate of 5; Beaudry, Doms, and Lewis (2010) report estimates between 3 and 4; Baum-Snow, Freedman, and Pavan (2018) find a central estimate of 4.6 in their full model; and Eckert, Ganapati, and Walsh (2020) estimate an elasticity of 3.6.[8]

Tax system To calibrate the progressivity parameter τ we follow the strategy of Heathcote, Storesletten, and Violante (2017). From (17), log post-tax income for individual i in year t is equal to

$$\log y_{it} = \log \lambda_t + (1-\tau) \log w_{it} \tag{19}$$

The tax system is not location specific, so we drop the location subscript. Allowing for an idiosyncratic error, we regress log post-tax income on log pre-tax income and a year fixed effect to obtain a consistent estimate of $1-\tau$. Pooled OLS in the PSID for 1980, 1990, 2000, and 2010 yields a point estimate of 0.174, close to the value of 0.181 reported by Heathcote, Storesletten, and Violante (2017) for 1978-2006.

Housing supply elasticities Next we calibrate the set of housing supply elasticities, γ_n. The housing supply equation (15) is specified in terms of physical quantity of housing, HD_n, which is not observed. To obtain an estimating equation, rewrite (15) with HD_n expressed in terms of price and expenditure

$$p_n = \tilde{\Pi}_n \left(\sum_i R_{in} l_{in} \right)^{\chi_n}$$

[8] A notable exception is Diamond (2016), who estimates an elasticity close to 1.6 in line with the time-series results. From our review of the literature, hers is the only model featuring cities and yielding a low elasticity of substitution.

where $\chi_n = \gamma_n/(1+\gamma_n)$ and $\tilde{\Pi}_n = \Pi_n^{1/(1+\gamma_n)}$. Taking logs and differencing over time yields an equation which is linear in parameters

$$\Delta \log p_n = \Delta \log \tilde{\Pi}_n + \chi_n \Delta \log \left(\sum_i R_{in} l_{in} \right).$$

Similar to Saiz (2010), we parameterize χ_n as a function of geographical and regulatory constraints

$$\chi_n = \chi + \chi_L \Lambda_n + \chi_R WRLURI_n$$

where Λ_n is Saiz (2010)'s measure of the fraction of land unavailable for development and $WRLURI_n$ is the Wharton Residential Land Use Regulation Index developed by Gyourko, Saiz, and Summers (2008). This yields an estimating equation for χ, χ_L and χ_R

$$\Delta \log p_n = \Delta \log \tilde{\Pi}_n + (\chi + \chi_L \Lambda_n + \chi_R WRLURI_n)\, \Delta \log \left(\sum_i R_{in} l_{in} \right).$$

We use data on rents and wages from the 1980 and 2010 Censuses to construct $\Delta \log p_n$ using (2). To circumvent the endogeneity of rents and employment we follow Diamond (2016) and instrument using Bartik instruments for labor demand. These IVs predict regional wage changes using a region's sectoral composition in 1980 interacted with national wage trends. Denoting skill levels by i, regions by n, years by t, and industries by k, we define the Bartik IV as

$$Z_{int} = \sum_k w_{i,k,-n,t} \cdot \frac{l_{i,k,n,1980}}{l_{i,n,1980}}$$

where $w_{i,k,-n,t}$ is the log average wage of workers in group i and industry k and year t, in all regions *except* n. $l_{i,k,n,1980}$ is employment of group i in industry k and region n in 1980, and $l_{i,n,1980} = \sum_k l_{i,k,n,1980}$ is total employment of group i in region n in 1980. The identifying assumption is that a location's sectoral employment shares in 1980 are not correlated with future shocks to the housing supply shifter $\Delta \log \Pi_n$, but do impact employment and rental expenditures. To identify χ_L and χ_R we use the interaction of these Bartik shocks with Λ_n and $WRLURI_n$. We then set $\gamma_n = \chi_n/(1-\chi_n)$. The employment-weighted average of the γ_n obtained using this procedure is 0.78, very close to the value of 0.77 reported by Saiz (2010).

Migration elasticity We calibrate the migration elasticity, θ, by matching the long-run employment elasticity estimated by Hornbeck and Moretti (2019).[9] They run regressions of the form

$$\log Y_{n,2010} - \log Y_{n,1980} = \pi^Y (\log Z_{n,1990} - \log Z_{n,1980}) + \Gamma' X_n + v_n$$

where Y is an outcome, Z is a measure of TFP recovered from plant-level data, and X is a vector of controls. We consider the outcomes for earnings and employment.[10] The baseline results in Hornbeck and Moretti (2019) imply an employment elasticity of

$$\frac{\hat{\pi}^{\text{employment}}}{\hat{\pi}^{\text{earnings}}} = \frac{4.03}{1.46} = 2.76$$

We solve for the value of θ that produces the same response in our model.[11] This procedure implies $\theta = 3.89$. Although not strictly comparable because of the difference in utility functions and the presence of progressive taxation, this is similar to the values usually reported in the literature. For example, Hsieh and Moretti (2019) calibrate θ to be 3.33.

Table 2 summarizes the model parameters.

Table 2: Parameters

Parameter	Value	Source
ϵ	–0.68	PSID
σ	0.56	PSID
ρ	3.85	Card (2009)
τ	0.17	PSID
$\{\gamma_n\}$	0.78[a]	Census
θ	3.89	Indirect inference

[a] Employment-weighted mean

[9] This strategy is borrowed from Greaney (2020).

[10] In principle, this employment elasticity conflates migration with labor supply. Households in our model supply labor inelastically, so that we are interested in the migration margin only. Hornbeck and Moretti (2019) find that most of their estimated employment response is from migration rather than increased labor supply of incumbent workers.

[11] In the implementation, we hold amenities and housing supply shifters constant at their 1980 level, and feed into the model only the productivity shocks which occurred 1980-1990. We then calculate TFP shocks by averaging productivity shocks across skill groups.

5 The Skill Premium and Sorting

In this section we quantify the role of the rising national skill premium in the increase in sorting since 1980. The specific question we address is: how would sorting have evolved, had the skill premium remained at its 1980 level? Formally we proceed as follows:

(i) For each year $t \in \{1980, 1990, 2000, 2010\}$ we take wages, employment and rental expenditures by skill group i and MSA n from the Census. We solve for productivities A_{in}^t, amenities B_{in}^t, housing supply shifters Π_n^t and labor supplies L_i^t which rationalize the observed allocation as an equilibrium of our model.

(ii) For each year t we scale productivities so that (a) the skill premium in the model matches its value in the data in 1980, and (b) average unskilled wages are the same in the model as in the data in year t.

(iii) We solve the model for counterfactual wages, prices and employment levels in each year, using our rescaled productivities and keeping other fundamentals at the values obtained in (i).

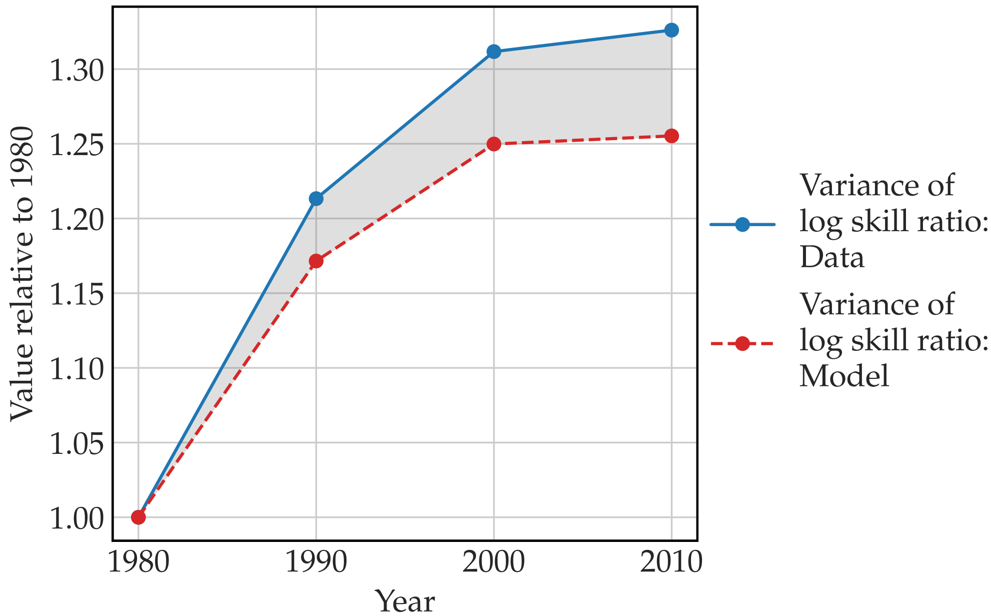
This counterfactual experiment eliminates the increase in the skill premium which occurred in the data between 1980 and 2010 while allowing unskilled wages to grow as they did in the data. Since the re-scaling of productivity is the same in all locations, this is a *location-neutral* shock. The difference between model and data identifies the causal effect of the rising national skill premium.

5.1 Main Results

Figure 3 shows our main result. In the absence of a rising national skill premium, sorting only rises by 25.5%, while in the data it rose by 32.6%. We conclude that without the increase in the skill premium, sorting would have risen by 7.1 percentage points less — 22% of the overall increase between 1980 and 2010. The shaded area in Figure 3 shows this difference. Our model explains the remaining 78% as the result of idiosyncratic amenity, productivity and housing supply shocks such as those highlighted by Diamond (2016) or Ganong and Shoag (2017). We obtain qualitatively similar results using other measures of sorting. For example, the difference in the log skill ratio between 90th percentile versus the 10th percentile rises by 19% less in the counterfactual, while the dissimilarity index, recently used by Fogli and Guerrieri (2019) to measure within-city segregation by income, rises by 15% less.

Figure 4 unpacks the mechanism behind the decrease in sorting observed in the model.

Figure 3: Sorting since 1980



Source: 'Data' calculated using Census data on employment by education level, 1980-2010. 'Model' is the output of the counterfactual experiment described in (i) - (iii) above, which removes the rise in the national skill premium but otherwise hold fundamentals at the level inferred from the data.

The left panel plots the difference in skill ratios between model and data in 2010 against log rents in the data in 2010. The negative slope shows that expensive MSAs generally lose skilled workers when we decrease the skill premium. Put another way, the rising skill premium made skilled households willing to tolerate high rents in cities like New York and Los Angeles (the two largest cities whose skill ratios fall relative to the median in our model). With a constant skill premium, they would have chosen to locate in cheaper cities. The right panel translates this into a statement about sorting. The low rent locations which gain skilled households in the model are typically less skill-intensive, so sorting falls.

The Role of Preferences

Next, we examine how our results depend on the specification of preferences over housing and nonhousing consumption. We consider two special cases which have been extensively used in the spatial and urban literatures: Cobb-Douglas preferences, which are perfectly homothetic and correspond to the parameterization $\epsilon = 0$ and $\sigma = 1$; and a unit housing requirement, which implies preferences are strongly nonhomothetic and corresponds to $\epsilon = -1$ and $\sigma = 0$.

Figure 4: City level changes in skill ratio

Source: y-axis plots log difference between skill ratio in 'Model' versus 'Data' in 2010. 'Data' calculated using Census data on employment by education level. 'Model' is the output of the counterfactual experiment described in (i) - (iii) above. Each dot represents an MSA, with size proportional to 1980 population. Solid lines represent regressions of difference in log skill ratio on x-axis variable.

The Cobb-Douglas case can be analyzed analytically thanks to Proposition 2. In this case we know that the difference between data and model will be exactly zero. Because preferences are homothetic, sensitivities to housing costs do not diverge as a result of the rising skill premium and so this force makes no contribution to the diverging location choices of skilled versus unskilled workers. To analyze the unit housing requirement, we repeat the calibration described in Section 4 and the counterfactual described above. We find that a unit housing requirement implies that the rising skill premium was responsible for 29% of the increase in sorting observed in the data, overstating the true effect (as measured by our estimated preferences) by about one third. These results show the importance of carefully estimating housing demand for evaluating the effects of the rising skill premium on sorting. Assuming homothetic preferences, as is standard in quantitative spatial models, shuts down the mechanism entirely; while a unit housing requirement substantially overstates its importance.

5.2 Taxation

In the preceding counterfactual exercise, we studied a location neutral shock to the incomes of skilled workers caused by productivity. We now present an alternative shock to the income distribution: a change in tax policy. The progressivity of the tax system is determined by τ as in equation 17, with a higher τ corresponding to more progressive tax system. As we explain below, a change in taxes is not location neutral in that it varies with the initial skill premium. Nevertheless, nonhomotheticity amplifies the effect of the policy.

We assess what would happen to sorting if the US were to adopt a Western-European style tax system. Chang, Chang, and Kim (2018) estimate (17) for a number of OECD countries. Germany has the most progressive tax system in their data, with $\tau_{Germany}$ a full 0.261 points higher than τ_{US} We recompute the model equilibrium at 2010 fundamentals and a higher level of τ, which we increase from 0.174 to 0.435 (i.e., imposing $\Delta\tau = \tau_{Germany} - \tau_{US}$). The results are in Table 3. The new tax regime is highly redistributive: the log skill premium net of taxes falls from 0.54 in the data to 0.35 in the counterfactual. The lower dispersion of incomes completely reverses the increase in sorting observed since 1980.

We compare the effect of the tax policy in our model to the same policy in a Cobb-Douglas environment. Unlike a proportional increase in skill productivities in all locations, a change in the tax parameter is non-neutral even under homothetic preferences. To see this, observe that in the Cobb-Douglas case, sorting depends on the log earnings premium, given by $(1-\tau)\log w_{sn}/w_{un}$ (as well as on amenity differences). A given change in τ therefore has a larger effect in regions with high skill premia. The third row of Table 3 shows that German-level taxation reverses about 75% of the increase in sorting since 1980 when preferences are Cobb-Douglas. In other words, nonhomothetic preferences amplify

Table 3: Taxation and sorting

Scenario	τ	Δ Sorting (% of change 1980-2010)	Log earnings premium
Data	0.174	-	0.536
German taxes, NHCES	0.435	-101	0.353
German taxes, C-D	0.435	-73	0.356

All rows describe sorting in 2010 under various scenarios. The first row is the data. The second and third increase τ to its level in Germany, under nonhomothetic CES (NHCES) and homothetic Cobb-Douglas (C-D) preferences respectively. The third column shows the percentage change in sorting under each scenario in 2010, as a fraction of the actual increase from 1980 to 2010. The fourth column is the log earnings premium, i.e. the log skill premium net of taxes.

the effect of the tax system on sorting by about one-third. Even though the aggregate earnings premium is similar in both cases, households in the nonhomothetic economy have divergent assessments of housing costs, hence divergent location choices.

Taken together, our two counterfactuals highlight the tight link between the income distribution and sorting. Compressing or dilating the income distribution—whether through productivity growth or policy changes—shifts households' relative preferences for cheap and expensive housing markets.

5.3 Extensions and Robustness

Below we discuss several extensions and robustness checks, focusing on how the results of our main counterfactual change.[12]

We begin by considering how our results change as the parameters of our NHCES preferences change. In particular we focus on two alternative sets of estimates: the estimates based on the household fixed effects specification in Table 1, Column (5), which imply $\epsilon = -0.713$ and $\sigma = 0.583$; and estimates based on the CEX which include homeowners (see Table B.2, Column (5)) and imply $\epsilon = -0.500$ and $\sigma = 0.549$. We choose these estimates as all our other estimates of ϵ (using an instrument and controlling for MSA-level prices) lie between them. We find that the share of the increase in sorting explained by the rising skill premium ranges from 23% for the smaller ϵ, to 15% for the larger ϵ. The direction of change is sensible — when preferences are more nonhomothetic, changes in income cause larger changes in location choices — and the similar magnitudes give us confidence that our results are robust to reasonable changes in our estimation strategy.

We then examine how endogenous amenities might change our results. Diamond (2016) models amenities as a function of the skill ratio, and finds that skilled households value these amenities more. In Appendix E.1 we incorporate these insights into the simple model of Section 3. We show that endogenous amenities amplify the effect of the skill premium on spatial sorting in the presence of nonhomothetic housing demand. The intuition for this result is straightforward. An increase in the skill premium makes skilled households less sensitive to housing costs and encourages movement towards more expensive cities, just as in our baseline model. Then amenities endogenously rise in expensive cities, encouraging further skilled in-migration. As a result, sorting rises more than in our baseline model. Next, we extend the neutrality result in Proposition 2 to an envi-

[12] Whenever we alter a parameter or add a new feature to the model, we re-calibrate the model following the same steps as in Section 4.

ronment with endogenous amenities. When housing demand is homothetic, changes in the skill premium have no effect on sorting, even in the presence of endogenous amenities. In summary, endogenous amenities are likely to amplify our mechanism, but they do not create an independent link between the skill premium and spatial sorting.

We next turn to the production side of our model. We consider a lower value of ρ equal to 1.6, taken from Acemoglu and Autor (2011). Using this value of ρ, the share of the increase in sorting explained by the rising skill premium falls to 13%. The fact that the share explained falls is intuitive — when skilled and unskilled labor are not close substitutes, the influx of skilled workers into expensive cities is dampened by falling skilled wages. This implies that the impact of the rising skill premium is limited. Our quantitative results are therefore fairly sensitive to changes in this parameter. Nevertheless, as we argued in Section 4, city-level estimates of the elasticity of substitution are consistently higher than aggregate estimates like the one used here, and our baseline value of $\rho = 3.85$ is around the middle of such estimates.

Finally, we consider the role of agglomeration externalities. We focus on simple, skill-neutral externalities and in Appendix E.2 show that the neutrality result of Proposition 2 continues to apply: agglomeration externalities do not link the skill premium to spatial sorting when housing demand is homothetic. To investigate how agglomeration might interact with nonhomothetic housing demand, we enrich our quantitative mode with a constant elasticity productivity spillover. We parameterize the model to match the elasticity of 0.06 reported in Ciccone and Hall (1996) and repeat our main counterfactual. The share explained remains very similar to our baseline model at 22%.

6 Conclusion

Since 1980, skilled and unskilled households have increasingly sorted into different locations. Between-group earnings inequality has grown at the same time. We link growth in sorting to growth in the skill premium through the channel of income inelastic housing demand. As skilled households become richer, they reduce their expenditure shares on local housing and become less sensitive to housing costs. Skilled households flock to expensive cities, unskilled households flee to cheap cities, and sorting rises.

We present a theory of sorting based on changes in aggregate fundamentals. Our model features heterogeneous households with NHCES preferences over housing and nonhousing consumption, and can be characterized analytically. We show that shocks which are location neutral—for example, a proportional increase in skilled wages everywhere—

nevertheless changes sorting when preferences are nonhomothetic. Even in a richer environment featuring taxation, endogenous amenities, and agglomeration, the standard alternative of homothetic preferences imposes a tight restriction on how aggregate shocks transmit to changes in sorting.

To quantify the role of this mechanism, we estimate nonhomothetic preferences using consumption microdata and embed them in a quantitative spatial model. We find that the rising skill premium explains just over a fifth of the increase in sorting observed since 1980. In creating a parsimonious model to highlight the role of nonhomothetic housing demand, we have abstracted from channels which directly link sorting to aggregate efficiency or welfare. Doubtless such channels exist (Diamond 2016; Fajgelbaum and Gaubert 2020), presenting unexplored links between inequality and aggregate efficiency. Studying such links is an exciting avenue for future research.

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A Data

A.1 PSID

The primary consumption microdata comes from the Panel Study of Income Dynamics. The PSID is administered biannually, with about 9,000 households responding in each wave. It included a consumption module starting in 1999 and added several categories in 2005. The survey now covers about 70% of spending in the national accounts (Blundell, Pistaferri, and Saporta-Eksten 2016). Total expenditure is computed as the sum of all reported consumption categories: rent, food, utilities, telephone and internet, automobile expenses (including car loans, down payments, lease payments, insurance, repairs, gas, and parking), other transportation expenses, education, childcare, healthcare, home repairs, furniture, computers (2017 only), clothing, travel, and recreation. The PSID imputes a small number of observations to handle invalid responses. To match the definition in IPUMS, housing expenditure is equal to rent plus utilities. Homeowners were not asked to estimate the rental value of their home until 2017, so we restrict attention to renters and analyze homeowners with the CEX.

We select our sample according to the following criteria. We cut respondents in the top and bottom 1% of the pre-tax income distribution in each year to guard against serious misreporting errors. We also select households in which the head is prime-age (25-55, inclusive) and attached to the labor force (head or spouse reports usually working at least 35 hours per week). The controls included in the regressions are dummies for family size bins, number of earners, age bins, sex of household head, race of household head, and year. We use PSID sample weights in all regressions.

Using the restricted access county identifiers, we are able to assign local prices to 88% of households in the PSID sample. The missing 12% is by construction: the BEA estimates price indices only for MSAs, not for micropolitan statistical areas or rural areas. MSAs, in turn, cover only about 86% of the US population. Because the price data is available only starting in 2008, we use the 2009-2017 waves of the PSID (note that each survey asks about consumption in the previous year).

A.2 Regional Price Parities

The BEA has published metropolitan price indices since 2008 (Aten and Figueroa 2019). In this section we discuss the construction of the rental index; for more information about the price parities for other consumption components, we refer the interested reader to

Real Personal Income and Regional Price Parities (2020).

Using ACS data, the BEA estimates a standard hedonic regression model of the form

$$\log r_{in} = \alpha_n + X'_{in}\beta + \varepsilon_{in} \tag{20}$$

where r is rent, α_n is an MSA dummy and X is the set of observable dwelling characteristics available in the ACS: the type of structure interacted with the number of bedrooms, the total number of rooms, building age, a rural dummy, and a dummy for whether utilities are included in the contract rent. The log price indices are given by $\{\alpha_n\}$.

A.3 CEX

We append the 2006-2017 Consumer Expenditure Surveys (CEX) together and annualize at the household level. We define rental expenditure as actual rent paid for renters (`rendwe`) and self reported rental-equivalent (`renteqvx`) for owners. As in PSID, we add utilities `util` to be consistent with the data available in the Census. To solicit rental equivalent, homeowners are asked "If someone were to rent your home today, how much do you think it would rent for monthly, unfurnished and without utilities?" We define total consumption expenditure as equal to total reported expenditure `totexp` *less* retirement and pension savings `retpen`, cash contributions `cashco`, miscellaneous outlays `misc` (which includes mortgage principal), and life and personal insurance `lifins`. We apply the exact same sample selection criteria and controls in the CEX as in the PSID (see Section A.1). We use CEX sample weights in all regressions.

In 2006, the CEX added more detailed geographic identifiers in the variable `psu`. The primary sampling unit, i.e. the MSA of residence, is available for a subset of households. The CEX identifies twenty-four large MSAs, which cover about 45% of households in the survey. In regressions with price data, we include data starting in 2008.

A.4 Census

We use the 5% public use samples from the 1980, 1990, and 2000 Censuses. For the final period of data, we use the 2009-2011 American Community Survey, a 3% sample. For convenience we refer to this as the "2010 data." IPUMS attempts to concord geographic units across years, although complete concordance is not possible because of data availability and disclosure rules. We classify MSAs according to the variable `metarea`. We produce a balanced panel using the following rule: if an MSA appears in all four years, then it is kept. If an MSA does not appear in all four years, then we assign all individuals in that MSA across all years to a residual state category. For example, Charlottesville, VA appears in 1980, 2000, and 2010, but not in 1990. Therefore we assign all individuals in Charlottesville in every year to "Virginia." This procedure gives us 219 MSAs (including Washington, D.C.) and 50 residual state categories, for a total of 269 regions. The share of national employment which can be assigned to an MSA, rather than a state residual, is 70% in 1980, 72% in 1990, and 75% in 2000 and 2010.

A worker is considered skilled if she or he has completed at least a four year college degree according to the variable `educ`. By this metric, the national fraction of workers who are skilled is 22.5% in 1980, 26.5% in 1990, 30.2% in 2000, and 35.7% in 2010.

We compute wages and employment for each region, skill level, and year. Wages are from the IPUMS variable `incwage`. To be included in the wage and employment sample, workers must be between 25 and 55 years old, inclusive; not have any business or farm income; work at least 40 weeks per year and 35 hours per week; and earn at least one-half the federal minimum wage. For each region, skill level, and year, we average rents paid by renting households whose head is included in the wage and employment sample. Rents are from the IPUMS variable `rentgrs`. Wages and rents are adjusted to 2000 real values using BLS' Non-Shelter CPI.

A.4.1 Reconciling income and expenditure in the data

The necessary model inputs are (1) average total expenditures by skill, location, and year, denoted by e_{int}; and (2) average housing expenditures by skill, location, and year, denoted by r_{int}. Rents and wages in the data, which we denote $\tilde{r}_{int}$ and $\tilde{w}_{int}$ do not exactly correspond to their model counterparts because of differences in savings and household composition. In addition, we only observe rents paid by renting households, who tend to have lower incomes and total expenditures than non-renting households. We now describe the steps to address these discrepancies in the Census data.

Income Compute post-tax income as $\tilde{y}_{int} = \lambda_t \tilde{w}_{int}^{1-\tau}$, where λ_t is chosen to balance the budget. We assume that the elasticity of expenditure to permanent post-tax income is unity, so from the household's budget constraint it is immediate that expenditure is equal to permanent post-tax income, $e_{int} = \tilde{y}_{int}$. Households save in the data, but savings wash out in the aggregate since we focus on permanent income.

We could relax this assumption following Straub (2019). Suppose that expenditure were given by $e_{int} = \bar{c}_{int} \tilde{y}_{int}^{\phi}$. If $\phi < 1$, expenditure would be nonhomothetic in permanent income. Qualitatively, this feature would increase the strength of our sorting mechanism. If consumption were less important for high-income households (relative to income), then they would be less sensitive to the price of local housing consumption (again, relative to income). We do not pursue a quantitative treatment of nonhomothetic total expenditure, which would require a dynamic quantitative spatial model beyond the scope of our paper.

Rent In the model, households are unitary decisionmakers. In the data, rent may be apportioned among multiple earners or dependents. To reconcile the two, we multiply rents in the data by a scalar, κ, such that the average rental share in the Census data is equal to the average rental share in the CEX, in each year. Specifically, κ_t is chosen to fit

$$\frac{\sum_i \sum_n \frac{\kappa_t \tilde{r}_{int}}{e_{int}} l_{int}}{\sum_i \sum_n l_{int}} = \overline{\eta}_t$$

where e, r, and l are for renters only. The final step is to impute rents by homeowners in the Census. This can be backed out given the data on renters and our estimated preferences. From (2), the expenditures of any two groups a and b in the same MSA satisfy

$$\frac{r_a}{r_b} = \left(\frac{e_a}{e_b}\right)^{-\epsilon\sigma} \left(\frac{e_a - r_a}{e_b - r_b}\right)^{1+\epsilon}$$

We compute r_{int}, rental expenditure by all households, as the solution to this equation given data on rental expenditure by renters, total expenditure by renters, and total expenditure by all households.

A.4.2 Bartik instruments

In order to obtain instrumental variables for labor demand, we construct Bartik shift-share variables. The share is a region's industrial composition in 1980, and the shift is change in average wages nationwide (excluding the region itself).

We use the industry categories in the Census variable `ind1990`. Harmonizing the industries with our own crosswalk yields 208 industries which are consistently defined over all four periods. We drop individuals who cannot be classified into any industry ($\approx 0.3\%$ of workers) or who are in the military ($\approx 0.9\%$ of workers).

B Estimation

We first describe how measurement error biases OLS estimates of the log-linearized estimating equation (7). We then describe alternative specifications to estimate the preferences in Section 2.

B.1 Measurement error

Recall that the log-linearized estimating equation is

$$\hat{\eta}_{int} = \omega_t + \omega' X_{int} + \beta \hat{e}_{int} + \gamma \hat{p}_{nt} + \zeta_{int}$$

We address measurement error in expenditure in the following way. First, partialling out observable demographics and prices, write the reduced-form relationship between expenditure shares and total expenditure as

$$\eta = \beta^0 e + \zeta \tag{21}$$

where we have suppressed subscripts and hats for notational convenience. Expenditure and rental expenditure are measured with error: $\tilde{e} \equiv e + v$, $\tilde{r} \equiv r + \xi$, and $\tilde{\eta} \equiv \tilde{r} - \tilde{e}$. ξ and v are assumed to be uncorrelated with e, r, and ζ.

The OLS estimate of β^0 is asymptotically

$$\begin{aligned} \widehat{\beta^0}_{OLS} &= \frac{\text{cov}(\tilde{\eta}, \tilde{e})}{\text{var}(\tilde{e})} \\ &= \frac{\beta^0 \sigma_e^2 + \sigma_{\xi,v} - \sigma_v^2}{\sigma_e^2 + \sigma_v^2} \end{aligned}$$

The attenuation bias $\sigma_e^2 / (\sigma_e^2 + \sigma_v^2)$ is familiar from classical measurement error. There are two additional sources of bias: (1) measurement error in expenditure appears on both the left- and right-hand sides of (21); and (2) measurement errors in expenditure and rent are mechanically correlated. The direction of the bias is ambiguous, although in practice the OLS estimate is lower than the 2SLS estimate.

B.2 Alternative specifications in PSID

We present several alternative specifications in Table B.1, still using our baseline sample of renters in the PSID.

Table B.1: Preferences, alternative specifications (PSID)

Dependent variable: Log housing share

	(1) GMM Local prices	(2) GMM Zillow	(3) GMM Zillow	(4) 2SLS Fixed effects	(5) 2SLS Fixed effects	(6) GMM Alt. IV	(7) GMM Alt. IV
Log expenditure				-0.222***	-0.183***		
				(0.030)	(0.029)		
Implied ϵ	-0.650***	-0.705***	-0.890***			-0.594***	-0.626***
	(0.077)	(0.096)	(0.101)			(0.096)	(0.114)
Implied σ	0.534***	0.623***	0.707***			0.578***	0.587***
	(0.051)	(0.047)	(0.033)			(0.053)	(0.056)
MSA FE				✓			
State + Urban FE					✓		
Demographic controls	✓	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓	✓
Excluded IV	log expenditure	log expenditure	log expenditure	log expenditure	log expenditure	lagged log expenditure	education
R^2	-	-	-	0.12	0.10	-	-
First-stage F-statistic	-	-	-	802.1	984.2	-	-
N	8,923	7,052	7,133	8,879	8,915	5,392	8,772
No. of clusters	4,229	3,640	3,674	4,201	4,224	2,743	4,148

Source: PSID, BEA, and Zillow.

Note: Column (1) extends the model to incorporate variation in local non-housing prices. Column (2) uses Zillow metropolitan price data while column (3) uses Zillow county price data. Column (4) uses MSA fixed effects. Column (5) uses state fixed effects and a dummy for living in an urban area. Columns (6) and (7) instrument log expenditure with lagged log expenditure and education, respectively. Standard errors clustered at the household level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses.

B.2.1 *Non-housing prices*

In the baseline model, we assume that the price of non-housing consumption, C, is equalized across space. Here, we relax that assumption. Let non-housing consumption be a Cobb-Douglas aggregate of goods, with share ϱ, and non-housing services, with share $1-\varrho$. This modification implies that equation 2 becomes

$$\eta_{int} = \Omega_{int}\left(\frac{e_{int}}{P_{nt}}\right)^{\epsilon(1-\sigma)}\left(\frac{p_{nt}}{P_{nt}}\right)^{1-\sigma}(1-\eta_{int})^{1+\epsilon}. \tag{22}$$

where $P_{nt} = p_{Gnt}^{\varrho} p_{Snt}^{1-\varrho}$ is the local price index of goods and non-housing services.

We measure each index using the BEA's Metropolitan Regional Price Parities and set $\varrho = 0.51$ to replicate the BEA expenditure weights. We estimate (22) by GMM in column (1) of Table B.1. The estimated coefficients are not significantly different from the baseline values.

B.2.2 *Alternative measures of price*

In column (2) we show that the estimates are not sensitive to our choice of price index. Instead of the BEA Regional Price Parities, we use a Rent Index from Zillow, a real estate analytics firm. Zillow estimates the market price per square foot of rented units in most metropolitan areas starting in 2011. Despite the difference in datasets, the estimated coefficients are reassuringly close to the baseline values.

To study sorting *across* cities in a tractable way, we have assumed that prices are equalized within each MSA. However, urban models typically feature within-city price heterogeneity.[13] We relax the assumption of common prices by assigning households to a *county-level* rather than a *metropolitan-level* price.

We use the county price index from Zillow. The estimated parameters in column (3) imply more nonhomotheticity than our MSA-level results. Incorporating more granular price data, for example at the neighborhood or Census tract level, is a worthwhile avenue for future research.

Columns (4) and (5) dispense with price data altogether. Instead, we re-estimate the linearized equation 7, replacing p with location fixed effects. The advantage of this strategy is that we can be agnostic on the correct measure of housing prices; the disadvantage is that we can no longer identify the price elasticity. Column (5) uses MSA fixed effects. The

[13] Differentiated neighborhood quality or commuting costs will generate price dispersion within a city.

estimated coefficient on log expenditure is nearly identical to the baseline estimate. Alternatively, column (5) uses state fixed effects plus a dummy equal to one if the household lives in an urban area. We include this robustness check because it can be done using the public-access PSID.

B.2.3 Alternative instruments

In columns (6) and (7) we consider alternative instruments for log expenditure. Column (6) uses the log of lagged expenditure, which is valid as long as measurement error in expenditure is not serially correlated. Column (7) instead uses education as an instrument. We assign households into four bins based on the household head's level of education: less than high school graduate, high school graduate, some college, and college graduate or higher.[14] The exclusion restriction is that, conditional on the true level of expenditure and observable demographics like age and family size, education does not determine the housing share. The coefficient estimates are stable with these alternative instruments.

B.3 Consumer Expenditure Survey (CEX)

In this section we present additional results from the Consumer Expenditure Survey (CEX). Reassuringly, all findings are close to our main results.

In the first column of Table B.2, we re-estimate our baseline specification in the CEX. The estimated expenditure elasticity is slightly higher, but the difference is not statistically significant.

B.3.1 Homeowners

Thus far we have focused on renting households because we do not observe expenditure on owner-occupied housing. In this section we explore whether our results extend to homeowners too. An appropriate measure of housing expenditure by homeowners is *rent equivalent*. Respondents are asked: "If someone were to rent your home today, how much do you think it would rent for monthly, unfurnished and without utilities?" The PSID consumption module did not elicit rent equivalent until 2017, but rent equivalent is available in all recent waves of the CEX. Therefore we use the CEX to study homeowners.

Column (2) of Table B.2 pools renting and owning households together. The estimate is consistent with significant nonhomotheticity. Restricting attention only to owners (col-

[14]Results are similar if we use years of education.

Table B.2: Preferences (CEX)

Dependent variable: Log housing share

	(1)	(2)	(3)	(4)	(5)
	GMM	GMM	GMM	GMM	GMM
	Baseline	Pooled	Owners	Pooled, out-of-pocket	Owners, out-of-pocket
ϵ	-0.626***	-0.537***	-0.681***	-0.623***	-0.500***
	(0.068)	(0.037)	(0.036)	(0.042)	(0.063)
σ	0.559***	0.552***	0.438***	0.506***	0.549***
	(0.045)	(0.026)	(0.029)	(0.031)	(0.041)
Demographic controls	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
N	2,503	7,009	4,506	7,009	4,506

Source: CEX and BEA

Note: Column (1) replicates our baseline specification of Table 1 column (5), using the CEX. Column (2) adds homeowners, measuring housing expenditure by self-reported rental equivalent. Column (3) restricts to homeowners only. Columns (4) and (5) instead measures housing expenditure as out-of-pocket expenses, defined as mortgage interest (but not principal), property taxes, insurance, maintenance, and repairs. Instrument is log household income. Robust standard errors in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses.

umn (3)) yields even stronger nonhomotheticity than the baseline estimate for renters.

In columns (4) and (5), we use an alternative measure of housing expenditure for homeowners, *out-of-pocket expenses*. We define out-of-pocket expenses as the sum of mortgage interest, property tax, insurance, maintenance, and repairs. We omit payments on mortgage principal since these payments are savings, not consumption. Out-of-pocket expenses reflect the user cost of housing, which is equal to the rental value of the house in equilibrium. The estimates are close to our baseline results.

B.3.2 Imputing rents from home values

Because data on rent equivalent is (until very recently) unavailable in the PSID, the standard approach has been to impute rents as a constant fraction of self-reported home value, generally six percent (Attanasio and Pistaferri 2016; Straub 2019). We argue that this is not an appropriate strategy. The six percent figure is from Poterba and Sinai (2008), who compute the user cost of housing with data from the Survey of Consumer Finances. Poterba and Sinai (2008) document considerable variation in the user cost across different types of homeowners, with a mean of six percent. We provide further evidence on heterogeneity in rent-to-value ratios from the CEX in Figure B.1, panel (a). The median is 7.8% with a mean of 8.5%. Crucially, there is a clear negative relationship: more valuable properties have systematically lower rent-to-value ratios. The same pattern appears in the Residential Financial Survey, used by the BEA to impute rents in the national accounts (Katz 2017). Imputing rent as a constant fraction of home value tends to deflate the housing shares of households with low home values, decreasing the degree of nonhomotheticity in the data.

B.4 Comparison to Davis and Ortalo-Magné (2011)

We reconcile our paper with Davis and Ortalo-Magné (2011), who argue in favor of Cobb-Douglas preferences over housing and non-housing consumption.

First, Davis and Ortalo-Magné (2011) show that median rent-to-income ratios were roughly constant across time and space between 1980 and 2000. We replicate and extend this result in Table B.3. The mean is constant from 1980-2000 but increases after 2000, suggesting that the main findings in Davis and Ortalo-Magné (2011) may be sensitive to the period studied.

Second, constant housing shares over space are necessary but not sufficient to conclude that preferences are Cobb-Douglas. Cross-city comparisons of housing shares reflect both

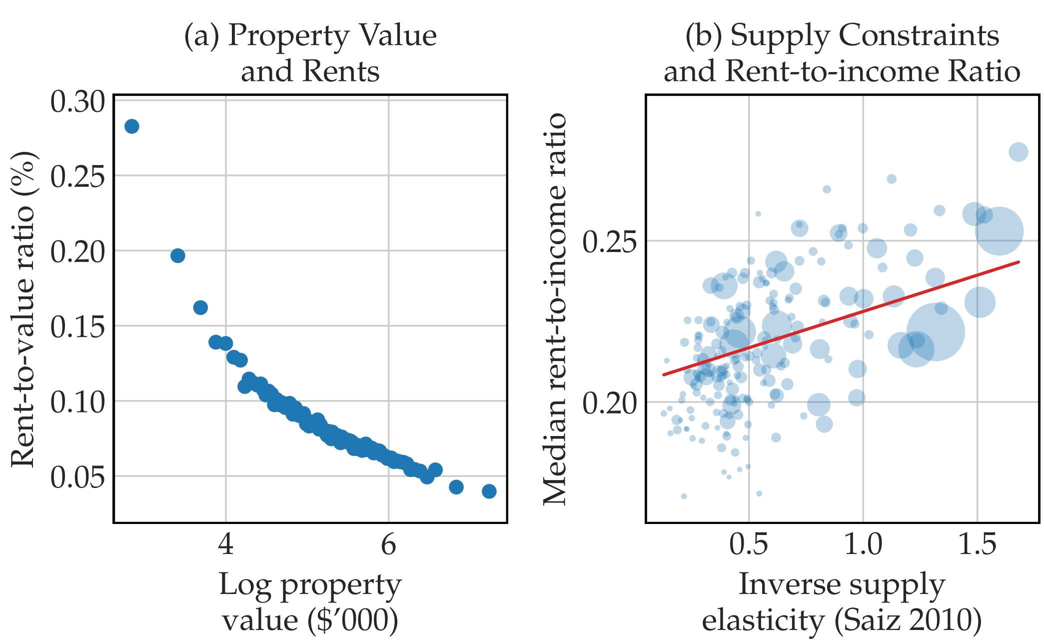
Table B.3: Median housing share across cities

	DOM sample		Full sample	
	Mean	Std. dev.	Mean	Std. dev.
1980	0.22	0.02	0.22	0.02
1990	0.23	0.02	0.22	0.02
2000	0.22	0.02	0.22	0.02
2010	0.27	0.03	0.26	0.03

Source: Census. These are unweighted statistics of the median rent-to-income ratio across 50 US metropolitan areas ("DOM sample") and 219 US metropolitan areas ("Full sample")

income and price effects. Households in expensive cities have a higher housing share than households in inexpensive cities at every level of income; but the composition of expensive cities is tilted toward high-income households, who tend to have lower housing shares. In Figure B.1, panel (b), we plot the median rent-to-income ratio and the inverse housing supply elasticity from Saiz (2010), a reduced-form price shifter. Expenditure is positively correlated with prices at the city level, with a correlation of 0.59. Cobb-Douglas preferences imply zero correlation.

Figure B.1: Additional Figures



Notes: (a) CEX, 2006-2017. We compute the average ratio of rent equivalent to property value for 100 property value bins. (b) Census data from 2000 and Saiz (2010). Size of marker indicates city size. Solid line is a regression of median rent-to-income ratio on inverse supply elasticity.

More generally, preferences are not identified from aggregate data on housing shares. Turning to the microdata, Davis and Ortalo-Magné (2011) report that the elasticity of rent (net of utilities) to total expenditure in the CEX is 0.99. Aguiar and Bils (2015) report an elasticity of 0.92 with the same data. This is the elasticity *unconditional on prices*, and hence not appropriate for a model of sorting across cities. We replicate and extend Davis and Ortalo-Magné (2011) in Table B.4. To facilitate a fair comparison, we estimate the elasticity of rent (net of utilities) to total expenditure in the CEX.[15] As in Davis and Ortalo-Magné (2011), the unconditional expenditure elasticity is close to unity. Crucially, controlling for local prices—whether by directly including prices as in column (2), or by using MSA fixed effects as in column (3)—confirms that preferences are significantly nonhomothetic. In columns (4) and (5) we repeat the exercise for utilities, which we model as part of housing.

Table B.4: Separate rent and utilities

	Log net rent			Log utilities	
	(1)	(2)	(3)	(4)	(5)
Log expenditure	0.982***	0.824***	0.844***	0.541***	0.639***
	(0.025)	(0.026)	(0.026)	(0.029)	(0.037)
Log price		0.576***			
		(0.033)			
MSA FE			✓		✓
Demographic controls	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
First stage F-stat.	2,404	1,137	1,305	2,382	1,295
R^2	0.38	0.49	0.50	0.37	0.44
N	5,803	2,501	3,029	5,795	3,022

Source: CEX and BEA

Note: Controls are bins for age, race, and gender of household head; household size; and number of earners in household. Instrument is log household income. Robust standard errors in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses.

[15] Our baseline specifications define housing costs as rent plus utilities.

B.5 Income elasticities from the literature

Table B.5 summarizes estimates of the income elasticity of housing demand from the literature. Controlling for local prices, using expenditure on the right hand side, and accounting for measurement error with an IV are all key in obtaining a consistent estimate of the elasticity.

C Theory

C.1 Proof of Lemma 1

We start by taking logs of (18)

$$\log l_{in} = \theta \log v_{in} + \log B_n - \log U_i$$

where U_i is just the denominator in (18). We difference this across types in the same location n and use the definition of the log skill ratio s_n

$$s_n = \theta \log\left(\frac{v_{sn}}{v_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Now we need a tractable expression for v_{in}. After some rearrangement, the utility maximization problem in (1) yields

$$v_{in} = e_{in}\left(1 - \eta_{in}\right)^{\frac{1}{1-\sigma}}.$$

Noting that in the simple model expenditure is just equal to wages, we obtain

$$s_n = \frac{\theta}{1-\sigma}\log\left(\frac{1-\eta_{sn}}{1-\eta_{un}}\right) + \theta \log A - \log\left(\frac{U_s}{U_u}\right).$$

The key object here is the ratio of expenditure shares. From (2), this can be written as

$$\frac{\eta_{sn}}{\eta_{un}} = \left(\frac{e_{sn}}{e_{un}}\right)^{\epsilon(1-\sigma)}\left(\frac{1-\eta_{sn}}{1-\eta_{un}}\right)^{1+\epsilon}.$$

Again noting that expenditures are just wages, this becomes

$$\frac{\eta_{sn}}{\eta_{un}} = A^{\epsilon(1-\sigma)}\left(\frac{1-\eta_{sn}}{1-\eta_{un}}\right)^{1+\epsilon}.$$

Table B.5: Income elasticities in the literature

Paper	Elasticity	Sample	Local prices?	Expenditure?	IV?
Rosenthal (2014)[a]	-0.88	Renters	✓		
Ioannides, Zabel, et al. (2008)[b]	-0.79	Owners	✓		
Hansen, Formby, Smith, et al. (1996)[c]	-0.73	Renters			
Larsen (2014)[d]	-0.67	Owners			
Zabel (2004)[e]	-0.52	Owners	✓		
Albouy, Ehrlich, and Liu (2016)[f]	-0.28	Renters	✓		
Lewbel and Pendakur (2009)[g]	-0.28	Renters		✓	✓
Attanasio et al. (2012)[h]	-0.22	Both		✓	
Aguiar and Bils (2015)[i]	-0.08	Both		✓	✓
Davis and Ortalo-Magné (2011)[j]	-0.01	Both		✓	✓
Paper benchmark[k]	-0.24	Renters	✓	✓	✓

[a] American Housing Survey, 1985-2011. Table 5, column 1.
[b] American Housing Survey, 1985-1993. Table 5, column 1.
[c] American Housing Survey, 1989. Table 5, column 2, last row.
[d] Norweigian Rental Survey and Consumer Expenditure Survey, 2007. Table 2, row 5.
[e] American Housing Survey, 2001. Table 3, row 3.
[f] US Census, 1970-2014. Table 1, column 3.
[g] Canadian Family Expenditure Surveys, 1969-1996. Median uncompensated elasticity computed using authors' replication file following their Appendix VII.1.
[h] British Household Panel Survey, 1991-2002. Table 4, panel B. Estimates for high- and low-education groups are averaged with weights one-third and two-thirds, respectively.
[i] US CEX, 1980-2010. Table 2, column 1.
[j] US CEX, 1982-2003. Text, page 253.
[k] PSID and BEA, 2009-2017.

We define $q_n = \eta_{sn}/\eta_{un}$. Taking the derivative of the expression above with respect to η_{un} shows that q_n is a strictly increasing function of η_{un}. Substituting, we obtain

$$s_n = \left(\frac{\theta}{1-\sigma}\frac{1}{1+\epsilon}\right)\log q_n + \left(\frac{\theta}{1+\epsilon}\right)\log A - \log\left(\frac{U_s}{U_u}\right). \tag{23}$$

Therefore s_n is a strictly increasing function of q_n. It is therefore also a strictly increasing function of η_{un}. The final step is to show that η_{un} is a strictly increasing function of $\tilde{p}_n = pz_n^{\epsilon}$. This directly follows from substituting the definition of $\tilde{p}_n$ into (2) for the unskilled type and differentiating with respect to $\tilde{p}_n$. Therefore we have the desired result: s_n is a strictly increasing function of $\tilde{p}_n$.

The case of $\epsilon = 0$ is straightforward. Preferences are homothetic and so $\eta_{sn} = \eta_{un}$. Therefore $q_n = 1$ for all n. Therefore from (23) s_n is constant across locations.

C.2 Proof of Proposition 1

We first assume $\epsilon < 0$. Equation 2 shows that in every location n η_{un} is unaffected by changes in A. Using the definition $q_n = \eta_{sn}/\eta_{un}$, we have

$$q_n = A^{\epsilon(1-\sigma)}\left(\frac{1-\eta_{un}q_n}{1-\eta_{un}}\right)^{1+\epsilon}.$$

Taking logs and differentiating with respect to $\log A$ yields

$$\frac{d\log q_n}{d\log A} = \epsilon(1-\sigma)\left(1+(1+\epsilon)\frac{\eta_{sn}}{1-\eta_{sn}}\right)^{-1}.$$

Since $\epsilon < 0$ and $\sigma < 1$ this is strictly negative for all n. But since the term in parentheses is increasing η_{sn}, $\frac{d\log q_n}{d\log A}$ is less negative for locations with a large η_{sn}. Inspection of (2) shows that η_{sn} is a strictly increasing function of $\tilde{p}_n$. Therefore $\frac{d\log q_n}{d\log A}$ is a strictly increasing function of $\tilde{p}_n$. Now return to (23) and difference between any two locations n and m

$$s_n - s_m = \left(\frac{\theta}{1-\sigma}\frac{1}{1+\epsilon}\right)(\log q_n - \log q_m).$$

Differentiate with respect to $\log A$

$$\frac{ds_n}{d\log A} - \frac{ds_m}{d\log A} = \left(\frac{\theta}{1-\sigma}\frac{1}{1+\epsilon}\right)\left(\frac{d\log q_n}{d\log A} - \frac{d\log q_m}{d\log A}\right).$$

Since the elasticity of q_n with respect to A is a strictly increasing function of $\tilde{p}_n$, this is positive if and only if $\tilde{p}_n \geq \tilde{p}_m$. This implies that ds_n is an increasing function of $\tilde{p}_n$. Since Lemma 1 established that s_n is a strictly increasing function of $\tilde{p}_n$, this implies that ds_n is a strictly increasing function of s_n.

Now we turn to sorting S, defined as the variance of s_n. We prove this statement for the weighted variance with positive (and fixed) weights ω_n which sum to 1 since we will weight by employment in our empirical application. By definition

$$S = \sum_n \omega_n \left(s_n - \bar{s}\right)^2$$

$$\bar{s} = \sum_n \omega_n s_n.$$

Differentiating

$$dS = 2\sum_n \omega_n \left(ds_n - d\bar{s}\right)\left(s_n - \bar{s}\right) = 2Cov(ds_n, s_n).$$

Since ds_n is a strictly increasing function of s_n, this covariance is positive and so $dS > 0$. This completes the proof for the case of $\epsilon < 0$.

For the sake of brevity we do not deal with the case of $\epsilon = 0$ here. It is a special case of Proposition 2, which is proved below.

C.3 Proof of Proposition 2

We start by taking logs of (18)

$$\log l_{in} = \theta \log v_{in} + \log B_{in} - \log U_i$$

where U_i is just the denominator in (18). We difference this across types in the same location n and use the definition of the log skill ratio s_n

$$s_n = \theta \log\left(\frac{v_{sn}}{v_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Now when $\epsilon = 0$ preferences are homothetic, and v_{in} is given by

$$v_{in} = y_{in}\left(1 + \Omega p_n^{1-\sigma}\right)^{\frac{-1}{1-\sigma}}.$$

This implies that the ratio v_{sn}/v_{un} is just the ratio of incomes. Therefore

$$s_n = \theta \log\left(\frac{y_{sn}}{y_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Now use (17) to replace incomes with wages

$$s_n = \theta(1-\tau)\log\left(\frac{w_{sn}}{w_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Next, we replace wages with productivities and labor supplies, using (14), and rearrange

$$(1+\theta(1-\tau)\rho^{-1})s_n = \theta(1-\tau)\log\left(\frac{A_{sn}}{A_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Now we turn to an increase in the skill premium. Suppose all A_{sn} increase by a factor of A. The above equation becomes

$$(1+\theta(1-\tau)\rho^{-1})s_n = \theta(1-\tau)\log\left(\frac{A_{sn}}{A_{un}}\right) + \theta(1-\tau)\log A + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Now difference this across any two locations n and m

$$\left(1+\theta(1-\tau)\rho^{-1}\right)(s_n - s_m) = \theta(1-\tau)\log\left(\frac{A_{sn}}{A_{un}}/\frac{A_{sm}}{A_{um}}\right) + \log\left(\frac{B_{sn}}{B_{un}}/\frac{B_{sm}}{B_{um}}\right).$$

We can see that the difference between the log skill ratio in any two locations does not depend on A. This directly implies that the variance of s_n, which we use as our measure of sorting, does not depend on the skill premium A.

D Calibration

Tax system

We use data from the 1981/91/2001/11 waves of the PSID (each containing summary information on the *prior* year's income). Using the same sample restrictions as in section 2, we run the PSID data through the NBER's TAXSIM program. For each household, pre-tax income is computed as adjusted gross income minus Social Security transfers. Post-tax income is computed as pre-tax income minus federal and state taxes (including payroll taxes) plus Social Security transfers. We estimate (19) by pooled OLS over the four periods. Our estimated $\hat{\tau}$ is 0.174 (robust s.e. 0.003). The R^2 of the regression is 0.98, suggesting that, despite its parsimony, a log-linear tax equation is a good approximation to the actual tax system in the United States. Our estimate is quite close to Heathcote, Storesletten, and Violante (2017), who estimate $\hat{\tau} = 0.181$ despite constructing a household sample with slightly different inclusion criteria.

Housing Supply Elasticities

Our estimating equation is

$$\Delta \log p_n = \Delta \log \tilde{\Pi}_n + (\chi + \chi_L \Lambda_n + \chi_R WRLURI_n)\, \Delta \log \left(\sum_i R_{in} l_{in} \right). \tag{24}$$

We bring this to the data using changes between 1980 and 2010. Saiz (2010) reports values of land unavailability Λ_n and regulatory constrains $WRLURI_n$ for a subset of MSAs. After dropping those for which these measures are missing, we are left with 193 MSAs. We use Census data on rents and incomes to construct model-consistent prices p_n using (2), setting $\Omega = 1$ throughout. We use Census data on rents and employment to construct housing expenditure $\sum_i R_{in} l_{in}$ for each MSA. Finally we use the Bartik shifter Z_{int} (and its interactions with Λ_n and $WRLURI_n$) described in the text as an instrument for housing expenditure. Table D.1 reports the result of estimating (24) by 2SLS. For the 193 locations with complete data, we then define

$$\gamma_n = \frac{\chi_n}{1 - \chi_n}$$

where

$$\chi_n = \chi + \chi_L \Lambda_n + \chi_R WRLURI_n.$$

Table D.1: Housing Supply Elasticity Estimates
Dependent variable: Log price change, 1980-2010

χ	-0.177
	(0.208)
χ_L	0.363
	(0.106)
χ_R	0.526
	(0.118)

Source: Census. Robust standard errors in parentheses.

This procedure results in a negative value for one MSA. We replace this with a zero. Of the remaining locations, 50 are the nonmetro portions of states and 26 are MSAs for which Λ_n and $WRLURI_n$ are not available. For the 26 MSAs, we define γ_n to be the median among the 193 MSAs with complete information. For the 50 state residuals, we set γ_n to the lowest value among the 193 MSAs with complete information, on the assumption that supply is likely to be more elastic in nonmetro areas.

Migration elasticity

We estimate θ by requiring our model to match the results of Hornbeck and Moretti (2019). That paper estimates the causal effect of TFP shocks between 1980 and 1990 on employment and wages. We mimic their setting by shutting down all shocks other than shocks to productivity and then repeating their regressions using the output of our model. Our target is the ratio of the effect on employment to the effect on wages by 2010 — the long run elasticity of employment to wages. This implies a target of $4.03/1.46 = 2.76$ (see Table 2, Column (3) of Hornbeck and Moretti (2019)). Formally we proceed as follows:

(i) Guess θ

(ii) Invert the model in 1980 and 1990 to obtain fundamentals $\left(A_{in}^t, B_{in}^t\right)_{i,n}, \left(\Pi_n^t\right)_n, \left(L_i^t\right)_i$ for $t = 1980, 1990$.

(iii) Solve the model with fundamentals $\left(A_{in}^{90}, B_{in}^{80}\right)_{i,n}, \left(\Pi_n^{80}\right)_n, \left(L_i^{80}\right)_i$ to obtain $\left(\hat{l}_{in}^{90}, \hat{w}_{in}^{90}\right)_{i,n}$.

(iv) Define $L_n^{80} = \sum_i l_{in}^{80}$, $W_n^{80} = \sum_i l_{in}^{80} w_{in}^{80} / \sum_i l_{in}^{80}$ and $\log Z_n^{80} = \sum_i l_{in}^{80} \log A_{in}^{80} / \sum_i l_{in}^{80}$ and likewise for $\hat{L}_n^{90}$, $\hat{W}_n^{90}$ and $\log \hat{Z}_n^{90}$

(v) Estimate the models below by OLS, weighting by 1980 employment:

$$\log \hat{L}_n^{90} - \log L_n^{80} = \pi^L \left(\hat{Z}_n^{90} - Z_n^{80}\right) + v_n^L$$
$$\log \hat{W}_n^{90} - \log W_n^{80} = \pi^W \left(\hat{Z}_n^{90} - Z_n^{80}\right) + v_n^W.$$

Note that the fact that we only study changes between 1980 and 1990 is innocuous, because our model has no transitional dynamics.

(vi) Calculate π^L/π^W.

(vii) Update θ until π^L/π^W converges to the target value.

This procedure yields $\theta = 3.89$.

E Counterfactual

E.1 Endogenous Amenities

Diamond (2016) shows the importance of endogenous amenities for understanding the location choices of skilled versus unskilled workers. In this subsection we consider how our results might change in the presence of endogenous amenities.

We start by incorporating them into the simple model described in Section 3. Following Diamond (2016) we model amenities as

$$B_{in} = b_{in} \left(\frac{l_{sn}}{l_{un}} \right)^{\beta_s}. \tag{25}$$

That is, for both types amenities depend on the skill ratio, but different types may value them differently — this is captured by β_s. In the context of our simple model, we do not allow exogenous differences in amenities across types, and so we impose $b_{sn} = b_{un} = b_n$. Following the same steps as in the proof of Lemma 1, we arrive at an expression for s_n

$$s_n = \left(\frac{\theta}{1-\sigma} \frac{1}{1+\epsilon} \right) \log q_n + \left(\frac{\theta}{1+\epsilon} \right) \log A + \log \left(B_{sn}/B_{un} \right) - \log \left(\frac{U_s}{U_u} \right).$$

Using the expression for B_{in} above, we obtain

$$s_n = \left(\frac{\theta}{1-\sigma} \frac{1}{1+\epsilon} \right) \log q_n + \left(\frac{\theta}{1+\epsilon} \right) \log A + \left(\beta_s - \beta_u \right) s_n - \log \left(\frac{U_s}{U_u} \right)$$

and thus

$$s_n = \left(1 - \left(\beta_s - \beta_u\right)\right)^{-1} \left(\left(\frac{\theta}{1-\sigma} \frac{1}{1+\epsilon} \right) \log q_n + \left(\frac{\theta}{1+\epsilon} \right) \log A - \log \left(\frac{U_s}{U_u} \right) \right). \tag{26}$$

To see the implications of endogenous amenities for sorting, it is helpful to consider two economies — one without endogenous amenities, whose variables we denote using $\bar{x}$, and one with endogenous amenities, which we denote using $\tilde{x}$. We fix a location n, and difference across these economies

$$\tilde{s}_n - \bar{s}_n = \left(\left(1 - \left(\beta_s - \beta_u\right)\right)^{-1} - 1 \right) \left(\frac{\theta}{1-\sigma} \frac{1}{1+\epsilon} \right) \log \bar{q}_n + C \tag{27}$$

where we have used the fact that $\tilde{q}_n = \bar{q}_n$, which follows from the definition of q_n, and we have collected constants which do not depend on location into C.

Diamond (2016) shows that skilled households value endogenous amenities more than unskilled households, implying $\beta_s > \beta_u$. This implies that $\tilde{s}_n - \bar{s}_n$ is an increasing function of q_n. If $\epsilon < 0$ then $\tilde{s}_n - \bar{s}_n$ is also an increasing function of $\tilde{p}_n$. This leads to the first result of this section:

Proposition 3 *Suppose $\beta_s > \beta_u$. If $\epsilon < 0$, sorting is higher in the presence of endogenous amenities, i.e $\tilde{S} > \bar{S}$. If instead $\epsilon = 0$ then $\tilde{S} = \bar{S} = 0$.*

Proof. If $\beta_s > \beta_u$ and $\epsilon < 0$ then (27) implies $\tilde{s}_n - \bar{s}_n$ is a strictly increasing function of $\tilde{p}_n$. Then

$$\begin{aligned}
\tilde{S} &= Var(\tilde{s}_n) \\
&= Var(\bar{s}_n + (\tilde{s}_n - \bar{s}_n)) \\
&= \bar{S} + 2Cov(\bar{s}_n, (\tilde{s}_n - \bar{s}_n)) + Var(\tilde{s}_n - \bar{s}_n) \\
&> \bar{S}
\end{aligned}$$

where the last line follows from the fact that $\bar{s}_n$ and $(\tilde{s}_n - \bar{s}_n)$ are both increasing in $\tilde{p}_n$ and therefore have a positive covariance. If $\epsilon = 0$ then (27) implies $\tilde{s}_n$ differs from s_n by a constant. Therefore $\tilde{S} = \bar{S}$. From Lemma 1 we know that $\bar{S} = 0$ when $\epsilon = 0$. ■

Proposition 3 tells us that endogenous amenities amplify the effects of nonhomothetic housing demand. Nonhomothetic housing demand ensures that high price locations have a higher skill ratio. Endogenous amenities then encourage even more skilled workers to locate there. But it is important to note that when $\epsilon = 0$, there is no sorting even with endogenous amenities, showing that they do not create an independent motive for sorting in our model, but rather amplify existing ones.

We now proceed to our next result, concerning the effect of an increase in the skill premium, $d \log A > 0$

Proposition 4 *Suppose $\beta_s > \beta_u$. If $\epsilon < 0$, sorting increases more when amenities are endogenous. Formally, $d\tilde{S} > d\bar{S}$. When $\epsilon = 0$ then $d\tilde{S} = d\bar{S} = 0$*

Proof. We return to (27) and difference between any two locations n and m. We obtain

$$(\tilde{s}_n - \bar{s}_n) - (\tilde{s}_m - \bar{s}_m) = \left((1 - (\beta_s - \beta_u))^{-1} - 1 \right) \left(\frac{\theta}{1-\sigma} \frac{1}{1+\epsilon} \right) \log \left(\bar{q}_n / \bar{q}_m \right).$$

Differentiating, we obtain

$$\left(\frac{d\tilde{s}_n}{d\log A}-\frac{d\bar{s}_n}{d\log A}\right)-\left(\frac{d\tilde{s}_m}{d\log A}-\frac{d\bar{s}_m}{d\log A}\right)=$$
$$\left((1-(\beta_s-\beta_u))^{-1}-1\right)\left(\frac{\theta}{1-\sigma}\frac{1}{1+\epsilon}\right)\left(\frac{d\log\bar{q}_n}{d\log A}-\frac{d\log\bar{q}_m}{d\log A}\right).$$

From Proposition 1 we know that the right hand side is positive iff $\tilde{p}_n > \tilde{p}_m$. Therefore $(d\tilde{s}_n - d\bar{s}_n)$ is an increasing function of $\tilde{p}_n$. Then

$$\begin{aligned}
d\tilde{S} &= 2Cov(d\tilde{s}_n, \tilde{s}_n)\\
&= 2Cov(d\bar{s}_n + (d\tilde{s}_n - d\bar{s}_n), \tilde{s}_n)\\
&= 2Cov(d\bar{s}_n, \tilde{s}_n) + 2Cov((d\tilde{s}_n - d\bar{s}_n), \tilde{s}_n)\\
&> 2Cov(d\bar{s}_n, \tilde{s}_n)\\
&= 2Cov(d\bar{s}_n, \bar{s}_n + (\tilde{s}_n - \bar{s}_n)) > d\bar{S}
\end{aligned}$$

which completes the proof for the case of $\epsilon < 0$. For $\epsilon = 0$ q_n is a constant, from which it follows $d\tilde{S} = d\bar{S} = 0$. ■

Proposition 4 shows that endogenous amenities amplify the mechanism we focus on in this paper — diverging incomes causing diverging sensitivities to housing costs and thus diverging location choices – but do not independently link the skill premium to spatial sorting.

Finally, we extend Proposition 2 to a richer environment with endogenous amenities. We drop the assumption that $b_{sn} = b_{un}$. Adding endogenous amenities does not change the derivation presented in the proof of Proposition 2, so we start from

$$(1+\theta(1-\tau)\rho^{-1})s_n = \theta(1-\tau)\log\left(\frac{A_{sn}}{A_{un}}\right)+\log\left(\frac{B_{sn}}{B_{un}}\right)-\log\left(\frac{U_s}{U_u}\right).$$

Inserting our definition of B_{in} and rearranging, we obtain

$$(1+\theta(1-\tau)\rho^{-1}-(\beta_s-\beta_u))s_n = \theta(1-\tau)\log\left(\frac{A_{sn}}{A_{un}}\right)+\log\left(\frac{b_{sn}}{b_{un}}\right)-\log\left(\frac{U_s}{U_u}\right).$$

Following exactly the same steps as in Proposition 2, we obtain our final result:

Proposition 5 *Suppose $\beta_i \neq 0$. Suppose also $\epsilon = 1$ so that preferences are homothetic. Then a proportional increase in A_{sn} in all locations has no effect on sorting and $dS = 0$.*

Proposition 5 tells us that even in the quantitative model, if preferences are homothetic then endogenous amenities do not independently link the skill premium to spatial sorting.

E.2 Agglomeration Externalities

We model agglomeration externalities as

$$A_{in} = a_{in} \left(l_{sn} + l_{un}\right)^{\delta}$$

In the context of the quantitative model introduced in Section 3, although agglomeration externalities affect wages (and therefore expenditures) differently in different locations, they are skill-neutral in the sense that they do not alter the ratio of skilled to unskilled wages in a given location. Therefore when $\epsilon = 0$, and sorting depends only on relative wages (and relative amenities), agglomeration does not link the skill premium to sorting and Proposition 2 continues to apply.