Assessing the role of TIF and LIHTC in an equilibrium model of affordable housing development[☆]

Jaime Luque

ESCP Europe Business School, Madrid, Spain

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ABSTRACT

The goal of this paper is to understand the impact of the Tax Incremental Financing (TIF) and the Low Income Housing Tax Credit (LIHTC) programs in an equilibrium model of affordable housing development. A TIF policy can implement an equilibrium where the construction of affordable housing becomes feasible (it passes the "*but for*" test). TIF is also effective in ameliorating a housing affordability crisis resulting from supply frictions in the housing market (e.g., zoning constraints and NIMBYism). However, TIF has the pervasive effect of increasing the construction costs. TIF also has implications for global corporations that buy LIHTCs: it induces them to rebalance their portfolios of LIHTC equity away from municipalities that rely on TIF.

1. Introduction

Los Angeles, New York, and other cities in America are struggling to cope with the problem of homelessness and the lack of affordable housing.[1] On a single night in January 2015, more than 560,000 people nationwide were homeless, meaning they slept outside, in an emergency shelter, or in a transitional housing program. Almost a quarter were children.[2] Meanwhile, homeownership is hovering at 20-year lows, while about half of renters struggle to pay their landlords. Housing affordability problems were rapidly transmitted into the vacancy rates, which fell sharply in almost all major metropolitan areas in the United States between 2010 and 2015 (see Gabriel and Painter, 2017).[3] There is simply not enough housing available to provide affordable units to all of the community members who are struggling to pay their rents or are homeless.[4]

Policymakers are aware of these problems and are revising/expanding existing financial programs to increase the supply of affordable housing. Among these programs, the most important in the

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[1] Other medium size US cities face similar problems. For example, in Madison, Wisconsin, the homeless population has dramatically increased in the last years, with a staggering 40% since 2010. Roughly one third of the homeless population is composed of children.

[2] For example, last fall, Los Angeles Mayor Eric Carcetti asked the City Council to declare "a state of emergency" on homelessness and committed US$100 million to solving the problem, suggesting that subsidies would play a role.

[3] Segregation and housing inequality is also an important issue in other developed countries. For example, Ben-Shahar and Warszawski (2015) find that in Israel, housing affordability inequality has considerably increased in the past decade.

[4] See Luque and Pope, 2017 for a policy report on homelessness in Madison, Wisconsin.

United States (U.S.) are the Tax Incremental Financing (TIF, hereafter) and the Low Income Housing Tax Credit (LIHTC, hereafter - often pronounced "lie-tech"). TIF is a municipal development tool that allows developers to use tax dollars to spur private development in a predefined Tax Incremental District, or TID. In essence, a developer receives a loan that is paid back in subsequent periods using part (or all) of the property taxes that the new real estate asset generates. Under the LIHTC program, each state receives an annual allocation of tax credits, and then distributes theses tax credits among developers according to a well-defined Qualified Allocation Plan (QAP). Developers raise income by selling their LIHTCs to national investors (e.g., Prudential Financial and AEGON USA Realty Advisors). Typical LIHTC transactions allocate a 99.99% share of the ownership entity to the investor that buys the tax credits, while the developers keeps the remaining 0.01%. Importantly, the valuation of equity in this type of housing development projects is driven by the market prices of LIHTCs at the jurisdiction level. Hence, the phrase "LIHTC equity" is commonly used in the affordable housing development industry.

Our goal in this paper is to examine the role of TIF and LIHTC on affordable housing development. To this aim, we build a two period general equilibrium model with two jurisdictions where local developers provide affordable housing to local poor households, and finance these investments with a mixture of debt, TIF, and LIHTC equity. A corporation is the main buyer of LIHTCs in both jurisdictions, although developers can also keep some of their own LIHTCs. The other agents in our model are local rich households, who lend money in the debt market to consume in the second period, and a company that owns and sells construction materials in the first period. The capital structures of the new real estate assets, as well as the market prices of debt, LIHTC equity, affordable housing, and construction materials are endogenous in our model. For simplicity, we assume that each developer is endowed with a given amount of LIHTCs, which are then sold to a corporation at competitive prices. Because the development projects in different jurisdictions can differ in terms of productivity, size, availability of construction materials, or even in their capital structures, the competitive prices of LIHTCs can differ between jurisdictions. This feature of the model is consistent with the observed heterogeneity of LIHTC prices across different regions in the United States.

Another key element of our model is the property taxes that real estate owners pay in their respective jurisdictions. The revenue from these taxes reverts to the agents of the jurisdiction according to some exogenous sharing rule. When the jurisdiction's profits revert to local households, taxes can be seen as a standard redistributive device from owners of real estate to households. When the jurisdiction profits revert to developers, transfers can be seen as TIF. Implementing a TIF policy makes all agents in the jurisdiction other than developers worse off in terms of transfers of the jurisdiction's profits because the jurisdiction profit weights must add up to 1 across all agents of the jurisdiction. This critical aspect of the model captures how TIF limits the tax revenue received by all tax recipients within a TID (tax recipients include schools, fire stations, and many other essential programs to a local government).

To our knowledge, our paper is the first to analyze the TIF and LIHTC programs from an equilibrium perspective. Our approach allows us to address important questions related to these programs. First, our model captures the impact of TIF on affordable housing development. We show that TIF ameliorates the developer's need for additional equity and helps improve affordability in the jurisdiction that adopts a TIF policy. In essence, TIF liberates resources that make the developer's budget constraint less binding, in turn ameliorating the developer's pressure to sell LIHTCs.

TIF is a rare animal in the world of local politics, and can credit much of its success to how easily it can be customized to each locality. However, because capital markets are global, it is unclear how a TIF policy implemented at the local level affects global investors' decisions of where to allocate their capital (in the form of LIHTCs for afford-

able housing investments). The second insight of our model is that a generous TIF policy in one jurisdiction may reduce or even eliminate the necessity for LIHTC equity in the capital structure of an affordable housing development project, but in turn may channel more equity into affordable housing development projects in the other jurisdiction. This result illustrates the role of municipal public policy on global capital markets. Roughly speaking, TIF crowds out global investors' holdings of real estate equity in the jurisdiction with a TIF policy.

Third, the additional TIF resources that the developer receives may end up increasing the price of construction materials in the jurisdiction that implemented the TIF policy. In the no TIF jurisdiction, the price of construction materials may also increase due to the additional corporation's demand for LIHTC equity in that jurisdiction. However, our equilibrium simulations suggest that the latter increment in construction costs is smaller than the increment in construction costs in the jurisdiction that adopts a TIF policy.

Fourth, we show that TIF may have pervasive effects that prevent a development to occur. In particular, TIF may increase the ratio of debt to corporation's LIHTC equity. An equilibrium may fail to exist if this ratio goes above the threshold that the corporation tolerates when investing in affordable housing.

Fifth, another important question in municipal public policy is how to evaluate whether a proposed development project needs TIF to become feasible; in theory, a developer should only receive TIF if the project passes the "*but for*" test – the development project will not materialize if the developer feels the expected profit without TIF is not large enough given the risk. Our model sheds light on this question by showing how an affordable housing development project would not occur *but for* the use of TIF.

Finally, we use the "production uncertainty" feature of our model to understand how TIF can ameliorate the effects of housing market frictions on housing affordability. In our model, the production of affordable housing in the second period is subject to shocks in different states of nature.[5] We interpret these shocks as affordable housing supply frictions and think of them as zoning constraints and NIMBYism (an acronym for the phrase "Not In My Back Yard" used in the United States to capture the residents' opposition to a new controversial development that is close to them). Our simulations illustrate how TIF is effective in offsetting the negative impact of such frictions on housing affordability.

1.1. Relationship with the literature

The literature on affordable housing development is scarce. However, several academic[6] and government[7] initiatives suggest that this will rapidly change given the importance that low-income housing markets have had on people's welfare and public policy in recent years.

The theoretical literature has approached the economic modelling of real estate development with either a "real options" pricing partial equilibrium model (see e.g. Titman, 1985 and Childs et al., 1996) or in an equilibrium setting with entrepreneurial developers and a housing/land market (see, e.g., Helsley and Strange, 1997).

Recently, Faias and Luque (2017) have provided a third approach by considering a general equilibrium model that focuses on the financing

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[5] We follow Glaeser and Gyourko (2003) and measure affordability as the ratio of house prices to housing construction costs.

[6] See for instance the 2017 AEI-BoI-BGFRS-TAU-UCLA Conference on Housing Affordability, a three-year academic partnership exploring policy approaches to the housing affordability challenge. The partnership consists of the American Enterprise Institute, Bank of Israel, Board of Governors of the Federal Reserve System, Tel Aviv University Alrov Institute for Real Estate Research, and UCLA Ziman Center for Real Estate. See also "The Big Event on Housing Affordability and Homelessness" hosted by the Wisconsin School of Business in December 2017.

[7] For example, we think of New York City mayor Bill de Blasio's unprecedented plan to create 200,000 units of affordable housing over ten years.

aspects of commercial real estate development when jurisdictions compete to attract commercial real estate equity investments. Our equilibrium model departs from Faias and Luque in that their goal is to prove existence of an equilibrium in a model of commercial real estate development with segmented commercial equity markets. Our goal in the current paper is first to incorporate LIHTC and TIF into an equilibrium model of affordable housing development, and then to study the impact of TIF and LIHTC on the rents, construction costs, property taxes, and capital structure of two housing development projects in different jurisdictions. To our knowledge, our paper is the first to provide this type of analysis.

Our paper also departs from the "voting with feet" literature in which households choose where to live – see Tiebout (1956) and subsequent studies (e.g., Konishi, 2008; Luque, 2013). We assume that households have already made the decision of the jurisdiction where to live, and focus our analysis on how developers finance their affordable housing development projects when they compete for LIHTCs and rely on TIF money provided by the jurisdiction.

There are other papers in the literature, most of them empirical, that study the topic of affordable housing development, but which focus however on the impact of affordable housing on local neighborhoods; see, for instance, Baum-Snow and Marion (2009) for the impact of LIHTC financed developments in low income neighborhoods, Freedman and Owens (2011) for the impact of LIHTC developments on crime, and more recently, Diamond and McQuade (2017) for the impact of multi-family housing developments funded through the LIHTC program on surrounding property values; see also Green et al. (2002) for an early study on this issue. We refer to Luque (2016) and Luque et al. (2018) for careful reviews of this literature and the different proposed financing tools for creating affordable cities that serve the interests of the disadvantaged.

The remainder of this paper is structured as follows. In Section 2, we introduce our economy with two jurisdictions, several types of agents (including local affordable housing developers and households), and property taxes at the jurisdiction level. Section 3 reviews the main financing tools available for developers to use to construct affordable housing (including LIHTC and TIF) and explains how we incorporate these tools into our model. Sections 4 and 5 present the definition of equilibrium and its characterization, respectively. Section 6 presents thought-provoking examples of equilibria with and without a TIF policy, and assesses the role of TIF on LIHTC equity pricing, housing rents, construction costs, and other equilibrium values in view of these examples. Section 7 provides a conclusion and directions for future research.

2. Model set-up

Let us consider an economy with two periods, denoted by $t = 1, 2$, and two states of nature in the second period, which we denote by s_1 and s_2. In the first period and at each state of the second period, there is a numeraire good that facilitates trade, e.g., cash. The numeraire good, whose price we normalize to 1, is traded in the global market at each node of the event tree. We label the consumption of the numeraire good as "x_{10}" if we are in period 1 and "$x_0(s)$" if we are at state $s = s_1, s_2$ of the second period.

2.1. Agents

In our economy there are two jurisdictions, denoted by k_1 and k_2. Each jurisdiction $k = k_1, k_2$ has a representative local developer d_k, which finances the construction of affordable housing with debt, TIF financing, and the equity raised by selling ownership interests in the property to investors in exchange for LIHTCs.

In jurisdiction $k = k_1, k_2$, there are also two representative consumers: a high-income consumer H_k who only cares about the consumption of the numeraire good in the second period and a low-income household h_k whose consumption only consists of affordable housing in

the second period. We also refer to these two agents as rich and poor households, respectively. A low-income household is defined as one led by a person with a precarious job whose income mostly depends on government aid, such as housing vouchers.

We also consider two types of companies: a corporation c that buys LIHTCs from affordable housing developers, and a company m that owns and sells materials to the developers for the construction of housing. We assume that these two companies belong to both jurisdictions k_1 and k_2. Thus, the sets of agents in jurisdictions k_1 and k_2 are $\mathbf{k}_1 = \{h_1, H_1, d_1, c, m\}$ and $\mathbf{k}_2 = \{h_2, H_2, d_2, c, m\}$, respectively. We denote by $\mathbf{K}^i$ the set of jurisdiction(s) to which agent i belongs. For example, if $i = h_1$, then $\mathbf{K}^{h_1} = \{k_1\}$. If instead $i = c$, then $\mathbf{K}^c = \{k_1, k_2\}$. Finally, let $\mathbf{I} = \{h_1, h_2, H_1, H_2, d_1, d_2, c, m\}$ denote the set of all agents in the economy.

Agent i's endowments of the numeraire good in the first period and at state $s = s_1, s_2$ are denoted by ω_{10}^i and $\omega_0^i(s)$, respectively.

2.2. Construction and consumption of affordable housing

For simplicity, we assume that company m owns two types of composite construction materials in the first period, here labeled as "$1l$" with $l = 1, 2$ (thus, the first and second elements denote the time period and the type of composite material, respectively). Company m's endowment of the composite material $1l$ and its market price are denoted by ω_{1l}^m and p_{1l}, respectively. This endowments are finite.[8] We assume that the other agents of the economy have no endowments of construction materials. Thus, $\omega_{1l}^i = 0$, for all $i \neq m$ and $l = 1, 2$.

To construct affordable housing, developers need construction materials. We denote the choice variables for developer d_k's purchase of materials "11" and "12" by $b_{11}^{d_k}$ and $b_{12}^{d_k}$, respectively. The production function of affordable housing in jurisdiction k is Cobb-Douglas and has the following specific form:

$$y_k^{d_k} = TFP_k(b_{11}^{d_k})^{\alpha_k}(b_{12}^{d_k})^{1-\alpha_k},$$

where parameters $\alpha_k \leq 1$ and $TFP_k \geq 0$ stand for the exponent of the Cobb-Douglas function and the "Total Factor Productivity" of the housing construction project in jurisdiction k, respectively. Notice that parameter α_k captures the complementarity between the two construction materials. Also notice that when $\alpha_k = 1$ ($\alpha_k = 0$), developer d_k only needs construction material "11" ("12", respectively) to build affordable housing in his respective jurisdiction. We denote the housing development project in jurisdiction k by j_k.

Development of affordable housing occurs as follows. If developer d_k buys a positive amount of construction materials weighted by Cobb-Douglas exponents α_k and $1 - \alpha_k$, the construction of housing is initiated (instantaneously) in the first period. At the beginning of the second period, the construction is finalized, but the size of the project will depend on the state of nature. In particular, the final amount of affordable housing available in jurisdiction k at state s is given by the following function

$$f_k(y_k^{d_k})(s) = \varepsilon_k(s)y_k^{d_k},$$

where $\varepsilon_k(s) > 0$ is a state-dependent shock. The ε-shock can be interpreted as an underlying friction of the housing market. For example, we can think of a decrease in $\varepsilon_1(1)$ as more zoning regulations at state s_1 in the first jurisdiction. Lower affordable housing supply is expected as a result (see Glaeser and Gyourko, 2003 for empirical evidence). A

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[8] This is a standard assumption in general equilibrium models (we need to fix endowments and preferences to compute an equilibrium). Also notice that if we were to model the intermediate production sector of construction materials, we would still need to fix the inputs of the corresponding intermediate production function (e.g., trees, crushed rocks, sand, etc.), which are finite by nature.

decrease in $\varepsilon_1(1)$ can also be interpreted as NIMBYism,[9] which is similar to a constraint that limits the supply of affordable housing.

We denote the (endogenous) rent per unit of affordable housing occupied in jurisdiction k at state s by $p_k(s)$. At state s, the low-income household h_k rents an amount $a_k^{h_k}(s)$ of affordable housing space in jurisdiction k.[10]

For simplicity, we ignore the housing market segment for high-income households, and assume that affordable housing units are all filled with homogeneous low-income households and are located in a segregated low-income neighborhood. Thus, all construction in this model consists of affordable housing and relies on the LIHTC and TIF programs. We leave the modeling of the construction and sale of mixed-income housing projects for future research. See Dokow and Luque (2018) for the implications of considering mixed-income communities in terms of local public good provision and jurisdiction formation.

2.3. Preferences

Each agent $i \in \mathbf{I}$ assigns utility to the consumption bundle

$$x^i \equiv (x_{10}^i, b_{11}^i, b_{12}^i, (x_0^i(s), a_1^i(s), a_2^i(s))_{s=s_1,s_2})$$

composed of the agent's purchase of: (i) the numeraire good "10" and construction materials "11" and "12" in the first period, (ii) the numeraire good at states s_1 and s_2, and (iii) affordable housing in jurisdictions $k = 1$ and $k = 2$ at states s_1 and s_2. We consider the following log linear functional form for an agent i's utility function:

$$u^i(x^i) = \theta_{10}^i \ln x_{10}^i + \sum_{l=1,2} \theta_{1l}^i \ln b_{1l}^i$$

$$+ \sum_{s=s_1,s_2} \left(\theta_0^i(s) \ln x_0^i(s) + \sum_{k=1,2} \theta_k^i(s) \ln a_k^i(s) \right)$$

where the θ-parameters represent the agent i's utility weights corresponding to the different consumption goods. Poor households are the only agents that enjoy the consumption of affordable housing in their respective jurisdictions. Poor households also enjoy consumption of the numeraire good in the first period. Company m also prefers to consume the numeraire good in the first period (for this, company m seeks to sell its construction materials to local developers in equilibrium). The two rich households, H_1 and H_2, the two local developers, d_1 and d_2, and the corporation c only get positive utility from the consumption of the numeraire good in the second period. Formally.

(A1) Assumptions on preferences. *We assume that all θ -parameters are zero, except for $\theta_{10}^m > 0$, $\theta_{10}^{h_1} > 0$, $\theta_{10}^{h_2} > 0$, $\theta_0^{H_1}(s) > 0$, $\theta_0^{H_2}(s) > 0$, $\theta_0^{d_1}(s) > 0$, $\theta_0^{d_2}(s) > 0$, $\theta_0^c(s) > 0$, $\theta_{k_1}^{h_1}(s) > 0$, and $\theta_{k_2}^{h_2}(s) > 0$, for $s = s_1, s_2$.*

2.4. Jurisdiction property taxes and profits

We define a jurisdiction by a triplet $(\mathbf{k}, \alpha_k, \gamma_k)$ that specifies the set of players $\mathbf{k}$ in jurisdiction k, the technology α_k that developers use for the construction of affordable housing projects, and the property tax γ_k, respectively.

Each jurisdiction $k = k_1, k_2$ incurs some costs (e.g., public goods such as roads, sewerage, fire protection, police, etc.) in terms of the numeraire good. To finance these costs, jurisdictions charge property taxes to the owners of real estate properties. In our simple model, this

amounts only to the owners of affordable housing. We consider the following linear functional form for property taxes:

$$g_k(E_k^i) = \gamma_k E_k^i$$

where $\gamma_k > 0$ is a parameter that we refer to as the property tax rate and E_k^i is the agent i's ownership interest in property j_k. Equity owners pay a property tax that is proportional to their affordable housing equity holdings.[11] For simplicity, we assume that this tax is charged in the first period and is a function of the housing development size (this tax can be seen as the taxes charged to equity holders in period 2 that are discounted with the corresponding equilibrium shadow price deflators).[12]

We split the jurisdiction's costs to provide public goods between a fixed component and a variable component. Fixed costs are denoted by $\lambda_k > 0$. The variable cost for jurisdiction k is linear with functional form $\eta_k \sum_{i \in \mathbf{k}} E_k^i$, $\eta_k > 0$. The profit of jurisdiction k in the first period is given by

$$\pi_k \equiv \sum_{i \in \mathbf{k}} \gamma_k E_k^i - \left(\lambda_k + \eta_k \sum_{i \in \mathbf{k}} E_k^i \right)$$

In our model, property taxes have a redistributive effect because jurisdiction profits revert to the agents that live and do business in the jurisdiction according to some weights. To see this, consider an agent $i \in \mathbf{k}$ and let its share of jurisdiction k's profit be $\delta_k^i \in [0, 1]$. By choosing a vector $(\delta_k^i)_{i \in \mathbf{k}}$, such that $\sum_{i \in \mathbf{k}} \delta_k^i = 1$, the jurisdiction manager is effectively redistributing resources among agents in the jurisdiction.

3. Financing tools for affordable housing development

In this section we review the main financing tools available to developers in the construction of affordable housing and explain how we incorporate them into our model. The first tool is uncollateralized debt. The modeling of debt issuance is standard, and thus for the sake of brevity we just introduce the notation and the short sale constraint that a developer is subject to. The other two financing tools, LIHTC and TIF, require more discussion because, to our knowledge, it is the first time they are being incorporated into an equilibrium model. Thus, for each of them, we first provide a brief description of how the corresponding program works in the United States, which is informative for our modeling purposes, and then explain how we incorporate them into our framework.

3.1. Debt

The first financing tool for developers is uncollateralized debt issuance. For simplicity, we ignore any issues regarding debt collateralization.[13] An agent i buying (selling) a face value of debt equal to D^i pays (receives) τD^i in the first period, where τ is the (endogenous) discount price of debt in the first period. We denote the short and long debt positions by $D^i < 0$ (borrower) and $D^i > 0$ (lender), respectively. Short sales are subject to the following constraint,

$$D^i \geq -\overline{D}^i, \tag{1}$$

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[9] NIMBY is an acronym for the phrase "Not In My Back Yard", which captures the opposition by residents to a proposal for a new development.

[10] Because housing is only consumed in one period ($t = 2$), we use the words rents and prices for housing interchangeably.

[11] We assume a "pass-through taxation" model, where the owners of the property are responsible for the property taxes and other expenses. This form of taxation includes Limited Liability Companies, one of most prevalent business forms in the United States.

[12] Optimality requires that equity taxes paid in the first period be equivalent to the discounted property taxes that the developer would pay in the second period using the developer's shadow prices of its budget constraints and additional sign constraints.

[13] We leave for future research an extension of this model in which debt is risky and collateralized by the housing asset. Also notice that assuming that debt is nominal is useful to guarantee the existence of an interior point in an agent's budget constraint. See Faias and Luque (2017).

where $\overline{D}^i > 0$ (notice that we allow this short sale constraint to be agent-type specific). At state s of the second period, the borrower (lender) pays (receives) $r(s)D^i$ nominal units.

3.2. Equity

An equity stake on an affordable housing project represents a claim to the future cash flows generated by this asset. The choice of how much equity the developer and the corporation agree to trade is endogenous in our model (see below). We denote these variables by $E_k^{d_k}$ and E_k^c, respectively. We assume that both poor and rich households and the company m that owns construction materials do not trade in the equity market of affordable housing development projects. Thus, we set $E_k^{H_k} = E_k^{h_k} = 0$ and $E_k^m = 0$ for $k = k_1, k_2$.

We denote by E_k the total amount of equity available in project j_k. If the corporation c buys $E_k^c < E_k$, then developer d_k holds the difference $E_k - E_k^c$. Formally, the market feasibility condition for equity shares corresponding to an affordable housing development project j_k is

$$E_k^{d_k} + E_k^c = E_k. \tag{2}$$

When $E_k^{d_k} = 0$, the developer sells the whole property to corporation c and does not keep any equity for itself. If instead $E_k^{d_k} \in (0, E_k)$, the developer keeps part but not all of the ownership on the property. When $E_k^{d_k} = E_k$, the developer owns all equity of the project.

In our model of affordable housing development, the developer d_k and corporation c's equity stakes on property j_k are determined by the amount of LIHTCs traded between the two agents. We explain this in the next section.

3.3. LIHTC

With more than 2.4 million affordable homes constructed or rehabilitated since 1986, the Low-Income Housing Tax Credit is seen as one of the most successful housing programs in U.S. history.[14] These credits are also commonly called Section 42 credits in reference to the applicable section of the Internal Revenue Code. Under this program, each state receives an annual allocation of tax credits, and then distributes theses tax credits among developers according to a well-defined Qualified Allocation Plan (QAP). In the Appendix, we provide a brief summary of the history of this program and the common criteria for developers to obtain LIHTCs under the QAP. We refer to Luque et al. (2018) for additional details of the LIHTC program.

The LIHTC program is a dollar-for-dollar tax credit, meaning that for each dollar of tax credits that an investor purchases, it can deduct a dollar from its federal income tax. Tax credits are sold at market prices and can vary across locations (depending on factors such as the size of the deals and the developers' expertise). The price per credit dollar is usually smaller than $1, but it is possible to see a price above $1 per credit dollar.[15]

Developers that obtain a state's tax credits raise equity by selling these credits to the investors. Typical transactions allocate a 99.99% share of the ownership entity to the investor that buys the tax credits, while the developers keep the remaining 1%. Thus, the equity raised by the developer to help finance the development is 0.9999 times the total equity value of the project. Fig. 1 illustrates a deal where a developer
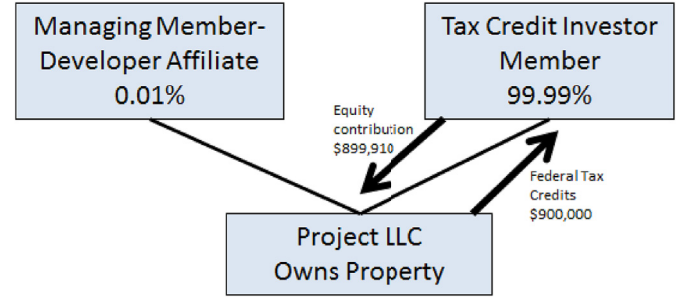

Fig. 1. This figure illustrates a typical LIHTC deal, in which a developer is allocated $1 million in tax credits and the market price of these credits is $0.90.

is allocated with $1 million in tax credits and the market price of tax credits is $0.90 for a dollar of credit.[16]

To incorporate LIHTC into our model, let us first assume that company c sets aside an amount ω_{10}^c (in terms of the numeraire good) to buy LIHTCs (for simplicity, we ignore corporation c's decision of how much to allocate for purchasing LIHTCs). For the sake of exposition, we denote jurisdiction k's housing authority allocation of LIHTCs to agent i by $\widehat{T}_k^i$, and assume that only the developer d_k receives a positive allocation of LIHTCs, i.e., $\widehat{T}_k^{d_k} > 0$ and $\widehat{T}_k^i = 0$ if $i \neq d_k$. This trick will help us simplify the budget constraints introduced below.

Corporation c chooses how many LIHTCs to buy. We denote this choice variable by $T_k^c \geq 0$. Because it is possible that the developer will keep some tax credits for itself ($T_k^{d_k} \geq 0$), market feasibility requires

$$T_k^c + T_k^{d_k} = \widehat{T}_k^{d_k} \tag{3}$$

The following facts motivate our modelling of the relationship between the investor's tax credits purchased and its equity stakes on the property. First, recall that if a corporation buys all the tax credits ($\widehat{T}_k$ in our terminology), then it becomes the owner of 99.99% of the property, and the developer keeps the remaining 1% of the ownership interest in the property. Second, although it is unusual, a corporation can buy fewer tax credits than the developer's total amount of LIHTCs (i.e., $T_k^c < \widehat{T}_k^{d_k}$). In that case, the developer keeps the remaining tax credits ($T_k^{d_k} = \widehat{T}_k^{d_k} - T_k^c$). Given these two facts, we can express the equity stakes that corporation c and developer d_k decide to hold in property j_k as

$$E_k^c = 0.9999 T_k^c \tag{4}$$

and

$$E_k^{d_k} = (\widehat{T}_k^{d_k} - T_k^c) + 0.0001 T_k^c, \tag{5}$$

respectively.

We denote the price that clears the market of LIHTCs corresponding to the affordable housing development project j_k by q_k. In our equilibrium notion below, we let T_k^c and $T_k^{d_k}$ be choice variables and infer E_k^c and $E_k^{d_k}$ from variables T_k^c and $T_k^{d_k}$ and equations (3)–(5). Notice that

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[14] For a detailed description of the LIHTC program, see Hobart and Schwarz (1995). We also refer the interested reader to the U.S. Department of Housing and Urban Development's website for further details.

[15] In 2017, LIHTC pricing has often risen above $1, and even in midwest states such as Wisconsin, pricing over $1 was not unheard of prior to the fall election of 2017.

[16] An allocation of $1 million in tax credits reflects a $100,000 annual allocation multiplied by 10 to reflect the 10-year period over which the credits can be taken. If the market price of tax credits is $0.90 for a dollar of credit, we have to multiply the $1 million by 0.90, indicating that the investor is willing to pay $900,000 for these credits. In actuality, the investor is buying a 99.99% share of the ownership entity, so the $900,00 would further be reduced to $899,910 when we multiply by 0.9999. So in this example, the equity raised by the developer to help finance the development is $899,910.

market clearing of LIHTCs in jurisdiction k (condition (3)) implies that the total amount of equity ($E_k = E_k^c + E_k^{d_k}$) must be such that

$$E_k = \widehat{T}_k^{d_k} \tag{6}$$

In other words, the total amount of tax credits available to developer d_k is equal to the nominal amount of project j_k's total equity.

For our two period economy, when corporation c purchases an amount $T_k^c > 0$ of LIHTCs, it gives $q_k 0.9999 T_k^c$ units of the numeraire good to developer d_k in the first period. At state s of the second period, corporation c is then entitled to an amount of tax credits T_k^c and the property cash flows $p_k(s) f_k(E_k^c)(s)$ corresponding to an amount of equity $E_k^c = 0.9999 T_k^c$. The developer's revenue in the second period consists of the remaining tax credits ($T_k^{d_k} = \widehat{T}_k^{d_k} - T_k^c$) and the cash flow $p_k(s) f_k(E_k^{d_k})(s)$ corresponding to an amount of equity $E_k^{d_k} = (\widehat{T}_k^{d_k} - T_k^c) + 0.0001 T_k^c$.[17] We can see these terms in the following budget constraints of an agent i:

$$(x_{10}^i - \widetilde{\omega}_{10}^i) + \sum_{l=1,2} p_{1k}(b_{1l}^i - \omega_{1l}^i) + \tau D^i + \sum_{k \in \mathbf{K}^i} q_k 0.9999(T_k^i - \widehat{T}_k^i)$$
$$+ \gamma_k E_k^i \leq 0 \tag{7}$$

$$(x_0^i(s) - \omega_0^i(s)) + \sum_{k \in \mathbf{K}^i} p_k(s) a_k^i(s) \leq \overline{r} D^i + \sum_{k \in \mathbf{K}^i} (T_k^i + p_k(s) f_k(E_k^i)(s)) \tag{8}$$

Constraints (7) and (8) correspond to an agent i's budget constraints in period 1 and state s of period 2, respectively. The term $\widetilde{\omega}_{10}^i$ in constraint (7) is defined as follows:

$$\widetilde{\omega}_{10}^i \equiv \omega_{10}^i + \sum_{k \in \mathbf{K}^i} \delta_k^i \pi_k$$

That is, in addition to the endowment of the numeraire good in period 1, an agent i that belongs to jurisdiction k receives a transfer $\delta_k^i \pi_k$ from its jurisdiction, where δ_k^i is the redistribution weight of profits in jurisdiction k distributed to agent i, and is such that $\sum_{i \in \mathbf{k}} \delta_k^i = 1$. If an agent belongs to more than one jurisdiction – as it is in the case of the corporation c and the company m – this agent may receive transfers from the two jurisdictions. For the case of a developer d_k, the set of jurisdiction memberships is single – it only belongs to jurisdiction k – and therefore its after-transfer endowment is $\widetilde{\omega}_{10}^{d_k} = \omega_{10}^{d_k} + \delta_k^{d_k} \pi_k$. Similarly, we can write $\widetilde{\omega}_{10}^{h_k}$ for household h_k as $\widetilde{\omega}_{10}^{h_k} = \omega_{10}^{h_k} + \delta_k^{h_k} \pi_k$. The interpretation of weight $\delta_k^{d_k}$ is key for our modeling of TIF, as we shall see below.

3.4. TIF

TIF enables municipalities to finance developments that, in theory, bring better infrastructure, more jobs, increased tax revenue, and other benefits to a city or district. TIF is also an important tool for affordable housing development. To implement a TIF policy, the municipality first has to create a Tax Incremental District (or TID), which is a catchment or neighborhood that is designated with a specific development need, such as affordable housing. Within a TID, all tax recipients have to agree before approving and offering TIF funds to a developer. Importantly, there needs to be a budget shortfall for a project to be eligible for TIF; otherwise the developer can move forward without needing the city's financial assistance.

Once an analyst has established the value of the project, the difference in tax revenue from the project now versus the increased tax revenue from the new project determines the increment, hence the name

Tax Increment Financing.[18] If the project is approved, then the city can offer a financing package for the amount of the increment to the developer, and take the increment as payment back on that financing.[19] Roughly speaking, the developer makes a payment back on these funds by simply paying its property taxes. Typically, a city would not lend out the entire increment to the developer; a general benchmark is 50–60% of the increment. The remaining is redistributed to the tax recipients, either as a lump sum payment or in the form of indirect municipal improvements, e.g., increasing sewer and storm water capacity, building parks and libraries, etc.[20]

To incorporate TIF into our model, let us go back to the term $\widetilde{\omega}_{10}^i \equiv \omega_{10}^i + \sum_{k \in \mathbf{K}^i} \delta_k^i \pi_k$ in budget constraint (7). When agent i is a developer, say d_k, transfer $\delta_k^{d_k} \pi_k$ can be seen as TIF. This lump sum payment makes the developer's budget constraint in the first period less binding, (partially) offsetting its property taxes. This in turn allows the developer to spend more resources on the development project if deemed necessary (optimality is a necessary condition for this argument to hold in equilibrium).

When $\delta_k^{h_k} > 0$, the interpretation is in terms of a lump sum pure redistribution transfer to the poor household. Because poor households do not own equity, and thus do not pay property taxes, this lump sum transfer (subsidy) is not offset by any other fiscal element in the poor household's budget constraint. The same argument applies to the case in which the agent is company m, rich household H_k, or corporation c. The intuition for $\delta_k^c \pi_k > 0$ is that the higher $\delta_k^c \pi_k$ is, the fewer "net property taxes" the corporation must pay to the jurisdiction.

The jurisdiction can adopt different policies by choosing the value of parameters $\delta_k^{d_k}$, δ_k^c, δ_k^m, $\delta_k^{h_k}$, and $\delta_k^{H_k}$. For instance, the jurisdiction may choose to increase $\delta_k^{d_k}$. This in turn makes the other agents in the jurisdiction worse off relative to the developer after the new redistribution policy is implemented, since the jurisdiction profit weights must add to 1 across the agents of the jurisdiction.

Notice that in reality the jurisdiction may collect taxes during later periods (period 2 in our model), after the real estate asset is developed. For simplicity, we opted to model property taxes as being collected in the first period but, as discussed above, we could have incorporated this mechanism into our model by discounting future property taxes with the agent's Lagrangian multipliers of the budget constraints. Because in that case the jurisdiction does not collect taxes in the first period, the jurisdiction may need to issue a municipal bond to extend a TIF loan to the developer, and then in the second period collect the property taxes and make the corresponding payments to the municipal bond holders. Our approach of having the taxes paid in the first period greatly simplifies the exposition of our model and the computation of equilibrium.

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[17] Notice that from condition (6) and the linearity of function f, we have that $f_k(E_k^c)(s) + f_k(E_k^{d_k})(s) = f_k(\widehat{T}_k^{d_k})(s)$.

[18] For instance, the city may currently receive $100,000 annually in property taxes from the existing structure, and the new development will increase tax revenue to $500,000 annually after construction is completed, a difference of $400,000. This is the most critical part of a TIF project analysis, because it determines the amount of TIF funds available for the city to distribute to the developer. Typically, these values are estimated for the next ten years. Simplistically, ten years in this case would mean there is a difference of $4,000,000. This difference is called the increment.

[19] A municipality can often receive lower interest rates on mortgages or even use bonds to get the increment monies up front, and it hands those funds over to the developer.

[20] For example, if the increment is $4,000,000 and the loan to the developer is $2,000,000, the developer would still pay back $400,000 every year for the next 10 years. However, the city is not using the full $400,000 for debt servicing; it is only using $200,000 for debt service. The remaining $200,000 can be distributed to the tax recipients, or it can be used for infrastructure development.

4. Equilibrium

In this section, we formalize our notion of equilibrium. Let $x^I = (x^{h_1}, x^{h_2}, x^{H_1}, x^{H_2}, x^{d_1}, x^{d_2}, x^m, x^c)$ denote the vector of all agents' consumption vectors. Similarly, we denote by D^I and T^I the vectors of all agents' debt and LIHTC choices, respectively.

Given commodity prices $p \in \mathbb{R}_+^3 \times \mathbb{R}_+^{2\cdot3}$, LIHTC prices $q \in \mathbb{R}_+^2$, and the price of debt $\tau \in \mathbb{R}_+$, an agent i's *optimization problem* consists of choosing a vector $(x^i, D^i, T^i) \in \mathbb{R}_+^9 \times \mathbb{R} \times \mathbb{R}_+^2$ that maximizes its utility function $u^i(x^i)$, subject to budget constraints (7) and (8), the debt short sale constraint (1), and the sign constraint on $T_k^i \geq 0$.

Definition 1. *A competitive equilibrium for this economy with two jurisdictions consists of a system* $(x^I, D^I, T^I, p, q, \tau)$, *such that:*

(i) *each agent "i" solves its optimization problem.*
(ii) *the following market clearing conditions hold:*
   - *(numeraire consumption good in period 1):* $\sum_{i \in \mathbf{I}} (x_{10}^i - \omega_{10}^i + \sum_{k=k_1,k_2} (\lambda_k + \eta_k E_k)) = 0$
   - *(construction input l = 1, 2):* $\sum_{i \in \mathbf{I}} (b_{1l}^i - \omega_{1l}^i) = 0$
   - *(numeraire consumption good at state $s = s_1, s_2$ of period 2):* $\sum_{i \in \mathbf{I}} \left(x_0^i(s) - \omega_0^i(s) \right) = 0$
   - *(affordable housing at state $s = s_1, s_2$):* $a_k^h(s) = f_k(y_k^{d_k})(s)$, $k = k_1, k_2$
   - *(debt):* $\sum_k (D^{d_k} + D^{H_k}) = 0$
   - *(LIHTC):* $T_k^{d_k} + T_k^c = \hat{T}_k^{d_k}$, $k = k_1, k_2$

The Proof of existence of equilibrium for this economy follows Faias and Luque (2017) and is thus omitted. We move to the characterization of equilibrium for our specific economy with LIHTC and TIF.

5. Characterization of equilibrium

In this section, we introduce mild assumptions to simply our economy and derive intuitive equilibrium properties, which will be used later in Section 6 to construct numerical examples of an equilibrium. We find useful to impose the following set of assumptions into the model.

(A2) Assumptions on agents' shares of jurisdiction profits. *For simplicity we assume that company m, corporation c and rich households H_1 and H_2 do not receive any jurisdiction's transfer of profits, thus $\delta_1^m = \delta_2^m = \delta_1^c = \delta_2^c = \delta_1^{H_1} = \delta_2^{H_2} = 0$.*

(A3) Assumptions on endowments. *We assume that only rich households H_1 and H_2 and corporation c are endowed with the numeraire good in the first period. Thus, $\omega_{10}^{H_1} > 0$, $\omega_{10}^{H_2} > 0$, $\omega_{10}^c > 0$, and $\omega_{10}^m = \omega_{10}^{h_1} = \omega_{10}^{h_2} = \omega_{10}^{d_1} = \omega_{10}^{d_2} = 0$. In the second period, only company m and poor households h_1 and h_2 have positive endowments of the numeraire good: $\omega_0^m(s) > 0$ and $\omega_0^{h_k}(s) > 0$, $k = 1, 2$, and $s = s_1, s_2$. For simplicity, we assume that $\omega_0^m(s_1) = \omega_0^m(s_2)$ and $\omega_0^{h_k}(s_1) = \omega_0^{h_k}(s_2)$, $k = 1, 2$. In addition, we assume that $\omega_0^c(s_1) = \omega_0^c(s_2) = \omega_0^{d_k}(s_1) = \omega_0^{d_k}(s_2) = \omega_0^{H_k}(s_1) = \omega_0^{H_k}(s_2) = 0$, $k = 1, 2$. Thus, agents c, d_1, d_2, H_1 and H_2 rely on their investment decisions to consume in the second period.*

Developers, who prefer consumption of the numeraire good tomorrow, have no commodity endowments whatsoever (since $\omega_0^{d_k} = \omega_{11}^{d_k} = \omega_{12}^{d_k} = \omega_0^{d_k}(s_1) = \omega_0^{d_k}(s_2) = 0$), so issuing debt, raising LIHTC equity, and obtaining TIF are the only means for them to construct affordable housing and receive cash flows in the second period. Next, we present our assumptions on debt and LIHTC equity.

(A4) Assumptions on debt. *We assume that debt is risk-free, in the sense that $r(s) = \bar{r}$ for $s = s_1, s_2$. In addition to developers, only rich households*

have access to the debt market. Thus, we impose $D^m = D^c = 0$. [21] Lastly, we assume that low-income households cannot lend their first period lump sum transfer (if $\delta_k^{h_k} > 0$) in the debt market and therefore $D^{h_1} = D^{h_2} = 0$.

(A5) Assumptions on LIHTCs. *Because the LIHTC market of an affordable housing development project j_k is restricted to developer d_k and corporation c, we set $T_k^{h_k} = T_k^{h_{k'}} = T_k^{H_k} = T_k^{H_{k'}} = T_k^m = T_k^{d_{k'}} = 0$ with $k' \neq k$. Accordingly, we set $E_k^{h_k} = E_k^{h_{k'}} = E_k^{H_k} = E_k^{H_{k'}} = E_k^m = E_k^{d_{k'}} = 0$.*

Next, for the shake of clarity, we rewrite all agents' budget constraints in view of assumptions A1 to A5. First, recall that the poor household h_k may receive a lump sum transfer in the first period equal to $\delta_k^{h_k} \pi_k$. In addition, in the second period this household receives an endowment in terms of the numeraire good equal to $\omega_0^{h_k}(s) > 0$, which can be seen as a housing voucher. Thus, we can write his budget constraints in the first period and state s of the second period as follows, respectively:

$$x_{10}^{h_k} \leq \delta_k^{h_k} \pi_k \tag{9}$$

$$a_k^{h_k}(s) \leq \omega_0^{h_k}(s)/p_k(s) \tag{10}$$

Because in the first period the rich household H_k has access to the debt market and has endowments of the numeraire good, we can write his budget constraints in the first period and state s of the second period as follows, respectively:

$$\tau D^{H_k} \leq \omega_{10}^{H_k} \tag{11}$$

$$x_0^{H_k}(s) \leq \bar{r} D^{H_k} \tag{12}$$

Recall that company m only enjoys consuming the numeraire good in the first period and does not have access to the debt and LIHTC equity markets. Thus, the only relevant budget constraint for this agent is its first period budget constraint, which we can write in reduced form as follows:

$$x_{10}^m \leq p_{11}\omega_{11}^m + p_{12}\omega_{12}^m \tag{13}$$

We will see that in equilibrium company m seeks to sell its construction materials to local developers to consume as much of the numeraire good in the first period.

The corporation's budget constraints in the first period and at state s of the second period are, respectively:

$$\sum_k (q_k + \gamma_k)0.9999 T_k^c \leq \omega_{10}^c \tag{14}$$

$$x_0^c(s) \leq \sum_k \left(T_k^c + p_k(s)\epsilon_k(s)TFP_k(b_{11}^{d_k})^{\alpha_k}(b_{12}^{d_k})^{1-\alpha_k}(0.9999 T_k^c/\hat{T}_k^{d_k}) \right) \tag{15}$$

We will see that in equilibrium the corporation c uses its first period endowment to buy LIHTCs and then, given the proceeds from its holdings of LIHTC equity, it consumes the numeraire good in the second period. The term $0.9999 T_k^c/\hat{T}_k^{d_k}$ in constraint (15) captures the percentage of asset j_k's cash flows that corporation c is entitled to.

Finally, we present the developer d_k's budget constraints in period 1 and state s of the second period. These are, respectively:

$$p_{11}b_{11}^{d_k} + p_{12}b_{12}^{d_k} + \gamma_k 0.9999 T_k^{d_k} \leq \delta_k^{d_k}\pi - \tau D^{d_k} + q_k 0.9999(\hat{T}_k^{d_k} - T_k^{d_k}) \tag{16}$$

$$x_0^{d_k}(s) \leq rD^{d_k} + T_k^{d_k} + p_k(s)\epsilon_k(s)TFP_k(b_{11}^{d_k})^{\alpha_k}(b_{12}^{d_k})^{1-\alpha_k}$$

$$\times ((\widehat{T}_k^{d_k} - 0.9999T_k^c)/\widehat{T}_k^{d_k}) \qquad (17)$$

Notice that local developers can trade in the LIHTC equity market, debt market, and commodity markets. Local developers may also be entitled to jurisdiction lump sum transfers. The term $q_k 0.9999(\widehat{T}_k^{d_k} - T_k^{d_k})$ in budget constraint (16) is the revenue that developer d_k raises by selling equity to the corporation. In constraint (17), the term $(\widehat{T}_k^{d_k} - 0.9999T_k^c)/\widehat{T}_k^{d_k}$ captures the percentage of asset j_k's cash flows that developer d_k is entitled to.

We can rewrite the developer d_k's budget constraint (16) as follows:

$$ConstructionCost_k + PropertyTax_k^{d_k} = TIF_k + LIHTC_k^c + Debt^{d_k} \qquad (18)$$

where $TIF_k \equiv \delta_k^{d_k}\left(\left((\gamma_k - \eta_k)\,\widehat{T}_k^{d_k} - \lambda_1\right)\right)$, $ConstructionCost_k \equiv p_{11}b_{11}^{d_k} + p_{12}b_{12}^{d_k}$, $LIHTC_k^c \equiv q_k 0.9999T_k^c$, $PropertyTax_k^{d_k} \equiv \gamma_k 0.9999T_k^{d_k}$, and $Debt^{d_k} \equiv -\tau D_k^{d_k}$. Expression (18) says that the sum of the amount of TIF awarded to the developer, the income from the sale of LIHTCs to the corporation and the debt issued by the developer must cover the total construction costs and the developer's property taxes.

When analyzing the capital structure of a development project, analysts look at financial ratios. For example, a total debt to corporation's equity ratio higher than 2 indicates that the market value of a project's total debt is more than twice the market value of the corporation's equity in the project. Given the relevance of the total debt to corporation's equity ratio in our discussion of Section 6, we find useful to rewrite equation (18) in terms of financial ratios by dividing each term of the equation by the amount of equity held by the corporation in project:

$$\frac{TIF_k}{Equity_k^c} + \frac{LIHTC_k^c}{Equity_k^c} + \frac{Debt^{d_k}}{Equity_k^c} = \frac{ConstructionCost_k}{Equity_k^c} + \frac{PropertyTax_k^{d_k}}{Equity_k^c} \qquad (19)$$

where j_k $(Equity_k^c \equiv q_k E_k^c)$.

In equilibrium, equations (18) and (19) must hold with equality (otherwise market clearing conditions or the non-satiation property of agents' preferences condition may fail, a contradiction with equilibrium). Notice also that it may happen that an equilibrium fails to exist when there are additional restrictions on the equilibrium variables. For example, if the corporation does not tolerate a $Debt^{d_k}/Equity_k^c$ ratio above a given threshold in jurisdiction k, the amount of TIF and LIHTC granted by the local jurisdiction k may not be enough to cover the construction cost and property taxes. We will review this case in the next section.

Next, we present some interesting properties of an interior equilibrium (in the sense that $T_k^{d_k} > 0$, $T_k^c > 0$, $D^{d_k} > -\overline{D}^{d_k}$, $k = 1, 2$) satisfying assumptions A1-A5. We leave the proofs of these results for the Appendix.

Proposition 1. *In each jurisdiction, the ϵ-shock to affordable housing development drives the house price differential between states of nature, i.e.,*

$$p_1(1)/p_1(2) = \epsilon_1(2)/\epsilon_1(1) \qquad (20)$$

$$p_2(1)/p_2(2) = \epsilon_2(2)/\epsilon_2(1) \qquad (21)$$

For example, when housing supply frictions in jurisdiction k_1 are more intense at state s_1 than at state s_2 (i.e., $\epsilon_1(1) < \epsilon_1(2)$), we expect housing to be more expensive at state s_1 than at state s_2 (i.e., $p_1(1) > p_1(2)$).

Proposition 2. *The corporation's LIHTCs T_k^c decreases in the developer d_k's loan amount and the amount of TIF awarded to developer d_k by juris-*

diction k.

$$T_k^c = \frac{\widehat{T}_k^{d_k}r}{2 \cdot 0.9999 \cdot \omega_0^{h_k}(1)\cdot\tau}\left(\underbrace{\frac{\tau}{r}\omega_0^{h_k}(1)}_{} + \underbrace{\gamma_k\widehat{T}_k^{d_k}}_{j_k\text{'s property taxes}} - \underbrace{\omega_{10}^{H_k}}_{\text{developer }d_k\text{'s loan}} \right.$$

$$\left. - \underbrace{\delta^{d_k}((\gamma_k - \eta_k)\widehat{T}_k^{d_k} - \lambda_k)}_{\text{TIF}_k} \right) \qquad (22)$$

Proposition 2 provides a first insight into the role of municipal public policy on global capital markets. Roughly speaking, TIF crowds out global investors' holdings of real estate equity in the jurisdiction with a Tax Incremental District. In the next section, we shall elaborate more on this.

Our last characterization result is the following:

Proposition 3. *TIF increases the construction costs in the TIF jurisdiction.*

Proposition 3 does not depend on the degree of substitutability between construction materials.

In the next section, we provide numerical examples of an equilibrium that illustrate the above characterization result.

6. Assessing the role of TIF in the presence of LIHTCs

Our main goal in this section is to numerically evaluate the effect of TIF on equilibrium variables, as well as discuss the factors that might make an affordable housing development project unfeasible. We start with an example in which all jurisdiction profits go to the households, i.e., neither jurisdiction k_1 nor jurisdiction k_2 adopt a TIF policy.

Example 1. (No TIF): Let us consider an economy that satisfies the assumptions considered in Section 5, and consider the following parameter values for materials and the numeraire good endowments: $\omega_{11}^m = \omega_{12}^m = \omega_0^m(1) = \omega_0^m(1) = \omega_{10}^c = 1$, $\omega_{10}^{H_1} = \omega_{10}^{H_2} = 1$, and $\omega_0^{h_1}(s) = \omega_0^{h_2}(s) = 0.5$, $s = s_1, s_2$. We set the risk-free rate equal to $\overline{r} = 1$. The lower bound on the developer d_k's debt short sale constraint is $\overline{D}^{d_k} = 1$. Uncertainty about the size of the affordable housing development project is captured by the following parameter values: $\epsilon_1(1) = \epsilon_2(1) = 1$, $\epsilon_1(2) = \epsilon_2(2) = 0.5$. The Total Factor Productivity parameters are normalized to 1 (i.e., $TFP_1 = TFP_2 = 1$) and the α-parameters of the production functions are $\alpha_1 = 1$ and $\alpha_2 = 0$.[22]

Property taxes are the same in both jurisdictions and equal to $\gamma_1 = \gamma_2 = 0.5$. Variable and fixed costs are $\eta_1 = \eta_2 = 0.1$ and $\lambda_1 = \lambda_2 = 0.01$, respectively. We assume that all jurisdiction profits are redistributed to the households and thus there is no TIF money in this benchmark example, i.e., $\delta_1^m = 0$, $\delta_2^m = 0$, $\delta_1^{h_1} = \delta_2^{h_2} = 1$, $\delta_1^{H_1} = \delta_2^{H_2} = 0$, $\delta_1^{d_1} = \delta_2^{d_2} = 0$, $\delta_1^c = \delta_2^c = 0$. Finally, developers' LIHTCs endowments are $\widehat{T}_1^{d_1} = \widehat{T}_2^{d_2} = 0.4995$.

For the above parameter values, we obtain a unique equilibrium interior solution where $q_1 = q_2 = 4.50$, $\tau = 2.50$, $p_{11} = p_{12} = 1.00$, $p_1(1) = p_2(1) = 0.50$, $p_1(2) = p_2(2) = 1.00$, $D^{d_1} = D^{d_2} = -0.40$, $T_1^c = T_2^c = 0.10$, and $T_1^{d_1} = T_2^{d_2} = 0.40$. Thus, $TIF_k = \delta_k^{d_k}\pi_k = 0.00$, $LIHTC_k^c = q_k T_k^c = 0.45$, $CC_k = p_{11}b_{11}^{d_k} + p_{12}b_{12}^{d_k} = 1.00$, $PropertyTax_k^{d_k} = \gamma_k E_k^{d_k} = 0.20$, and $Debt^{d_k} = \tau D^{d_k} = 1.00$, for both $k = k_1, k_2$.[23]

In terms of financial ratios, our equilibrium is characterized as follows. For both $k = k_1, k_2$, $\frac{TIF_k}{Equity_k^c} = 0.00$, $\frac{LIHTC_k^c}{Equity_k^c} = 1.00$, $\frac{Debt^{d_k}}{Equity_k^c} = 2.22$,

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[22] These parameter values on α_1 and α_2 imply that $x_{1k}^{d_k} = \omega_{1k}^m$, for $k = 1, 2$.

[23] Unless otherwise indicated, we write the equilibrium solution up to two decimals, and consider the rounding error as negligible.

Table 1

This table reports the values of the equilibrium variables and ratios given the parameters considered in Examples 1 and 2. We consider two decimals, and thus assume that the rounding error is negligible.

	Example 1		Example 2	
	k_1	k_2	k_1	k_2
q_k	4.50	4.50	4.69	4.69
τ	2.50	2.50	2.60	2.60
p_{1k}	1.00	1.00	1.07	1.02
D^d_k	−0.40	−0.40	−0.38	−0.38
T^c_k	0.10	0.10	0.09	0.11
$T^{d_k}_k$	0.40	0.40	0.41	0.39
TIF_k	0.00	0.00	0.11	0.00
$LIHTC^c_k$	0.45	0.45	0.41	0.49
$Debt^{d_k}_k$	1.00	1.00	1.00	1.00
$ConstructionCost_k$	1.00	1.00	1.07	1.02
$PropertyTax^{d_k}_k$	0.20	0.20	0.21	0.20
$Equity^c_k$	0.45	0.45	0.41	0.49
$TIF_k/Equity^c_k$	0.00	0.00	0.26	0.00
$LIHTC^c_k/Equity^c_k$	1.00	1.00	1.00	1.00
$Debt^{d_k}_k/Equity^c_k$	2.22	2.22	2.45	2.02
$ConstructionCost_k/Equity^c_k$	2.22	2.22	2.62	2.07
$PropertyTax^{d_k}_k/Equity^c_k$	0.44	0.44	0.51	0.40

$\frac{ConstructionCost_k}{Equity^c_k} = 2.22$, $\frac{PropertyTax^{d_k}_k}{Equity^c_k} = 0.44$. Thus, financial ratios are the same in both jurisdictions.

"But for" test: It is well known that a proper use of TIF requires the project to meet a standard called the "*but for*" test (see The Wisconsin Tax Payer, 2009). The idea is that the development project will not materialize if the developer feels the expected profit without TIF is not large enough given the risk. In terms of our previous example, we can capture this situation by taking preference parameters $\theta^{d_k}_0(s_1) = \theta^{d_k}_0(s_2) = 1$ for both developers d_1 and d_2, and then computing the local developer's expected utility, which in case of Example 1 is equal to ln0.8999. This utility can be seen as a proxy of the developer's profitability if constructing affordable housing. It stands to reason that affordable housing will not be developed if local developers require a expected indirect utility greater than ln0.9000. In the next example, jurisdiction k_1 will adopt an active TIF policy that ends up increasing the developer's expected profitability. We shall conclude that the affordable housing development would not occur *but for* the use of TIF.

Example 2. (The impact of TIF): Let us consider again the parameter values considered in our leading Example 1, but now suppose that jurisdiction k_1 introduces a TIF policy, which in terms of our parameters consists of choosing a positive $\delta^{d_1}_1$ while leaving parameter $\delta^{h_2}_2$ equal to 0. We set $\delta^{d_1}_1$ equal to 0.5. Thus, now developer d_1 receives half of the jurisdiction profits that before went to the poor household h_1. For these new parameter values, we obtain a unique equilibrium solution. We report the equilibrium solution and the corresponding financial ratios in Table 1. For the sake of comparison, we also report in this table the equilibrium solution corresponding to parameter values in Example 1.

Compared to the equilibrium values of our benchmark Example 1, we find that the new TIF policy for jurisdiction k_1 is now $TIF_1 = 0.11$ and has the following effects:

- TIF_1 induces the corporation to rebalance its LIHTC equity portfolio toward jurisdiction k_2. In particular, the corporation decreases (increases) its LIHTC equity exposure in the real estate asset of jurisdiction k_1 (k_2, respectively) by 8.9 percent.
- TIF_1 increases the total debt to corporation's equity ratio in jurisdiction k_1 by 10.36 percent, form 2.22 to 2.45. In jurisdiction k_2, this ratio decreases by 9.90 percent, from 2.22 to 2.02.

- TIF_1 increases construction costs in jurisdictions k_1 and k_2 by 7 percent and 2 percent, respectively.
- TIF_1 increases the developers d_1 and d_2's expected utility (a proxy of profitability) from 0.8999 to 0.9423 for developer d_1 and from 0.8999 to 0.9036 for developer d_2. Thus, affordable housing development passes the "but for" test.

We finish this section with five remarks that elaborate on the economic intuition.

Remark 1. TIF relaxes the developer d_1 's budget constraint and as a result less LIHTC equity from outside investors (here, the corporation c) is needed to finance the development project. In general equilibrium, this effect results in the corporation choosing to rebalance its portfolio of LIHTC equity toward the no TIF jurisdiction. But then, for a given amount of debt issuance ($\tau D^{d_1} = \tau D^{d_2} = -1$ in both Examples 1 and 2), the total debt to corporation's equity ratio in jurisdiction k_1 increases relative to the project in the jurisdiction without a TIF policy. If the corporation would only tolerate a ratio below 2.23, then the project with TIF would fail, since this ratio increases to 2.45 in Example 2.

Remark 2. If each jurisdiction had its own local corporation that could not invest across locations, then the equilibrium total debt to corporation's equity ratio would barely change after jurisdiction k_1 decides to implement a TIF policy. In particular, in such an economy, this ratio would only change 0.4 percent (from at 1.0769 to 1.0726) in jurisdiction k_1 after a TIF policy of $TIF_1 = 0.11$.[24] This result suggests that allowing a corporation to buy LIHTC in both jurisdictions is key to obtain an equilibrium where the corporation's LIHTC equity portfolio significantly rebalances away from the TIF jurisdiction.[25]

Remark 3. Construction cost in jurisdiction k_1 increases by 7 percent due to the additional developer d_1 's resources coming from TIF_1.[26] Construction cost in the no TIF jurisdiction k_2 also increases due to the corporation's higher LIHTC equity investment, but the increment is only 2 percent.

Remark 4. The affordable housing development project in jurisdiction k_1 passes the "but for" test, even when construction costs increase due to the TIF policy. As a result, jurisdiction k_1 approves the new TIF policy and the development project in jurisdiction k_1 occurs. Interestingly, because the local developer d_2 benefits from the corporation's additional demand for its LIHTC equity, the expected utility of this developer also increases above the threshold ln0.9000. As a result, affordable housing is also developed in jurisdiction k_2.

Remark 5. As discussed before, the ε-shock can be interpreted as underlying supply frictions of the housing market. Typical examples are zoning constraints and NIMBYism. Here we elaborate on this discussion by looking at the effect of a TIF policy in jurisdiction k_1, here denoted by TIF_1, on housing affordability in jurisdiction k_1 for different values

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[24] This small (insignificant) change is due to a slightly higher equity valuation in jurisdiction k_1 given the additional resources that TIF provides to the developer of that jurisdiction.

[25] We would obtain a similar result if we had assumed instead a situation where the corporation can access both jurisdictions but the amount of LIHTC for sale in jurisdiction k_2 is negligible. For example, if $\hat{T}^{d_1}_1 = 0.4995$ and $\hat{T}^{d_2}_2 = 0.0010$, then we get an equilibrium where the total debt to corporation's equity ratio in jurisdiction k_1 would only increase 4.19 percent (from 1.45 to 1.51) instead than 10.36 percent as in the benchmark case. This rebalancing effect is again driven by a higher equity valuation in jurisdiction k_1 following the TIF policy.

[26] Notice that in a more sophisticated economy with additional investment opportunities, such as additional development projects, stocks, and savings, TIF money could be channeled to those assets instead, and that would not necessarily be in the interest of the jurisdiction.
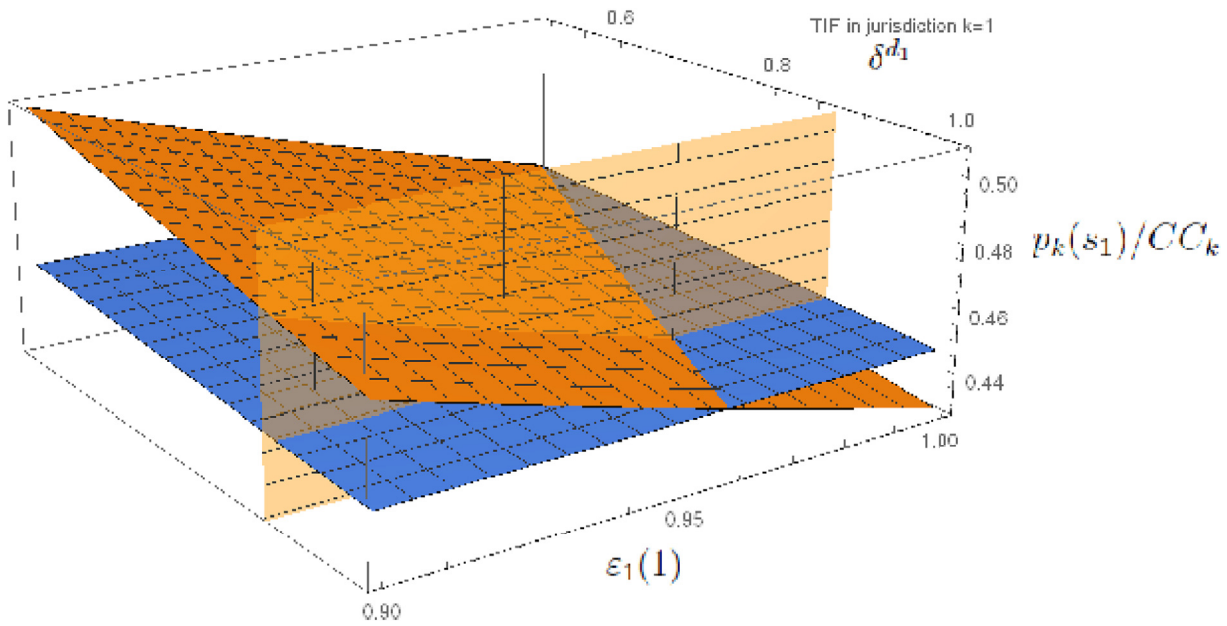
Fig. 2. This figure illustrates the jurisdiction k_1 and k_2's affordability indexes as a function of TIF_1 (in terms of δ^{d_1}) and shock $\varepsilon_1(1)$.

of parameters $\varepsilon_1(1)$ and TIF_1. We focus on state s_1 and assume that the ε-shock in jurisdiction k_2 is 1. We then vary the ε-shock in jurisdiction k_1 from 0.90 to 1.00. For example, we interpret $\varepsilon_1(1) = 0.95$ as a shock that diminishes affordable housing supply by 5 per cent. We follow Glaeser and Gyourko (2003) and measure affordability as the ratio of house prices to housing construction costs, i.e., $p_k(s_1)/CC_k$ for jurisdiction $k = k_1, k_2$.[27] The lower this ratio, the more affordable is housing in jurisdiction k at state s.

Fig. 2 illustrates the value of ratio $p_k(s_1)/CC_k$ for jurisdictions $k = k_1, k_2$ and different parameter values of $\varepsilon_1(1)$ and δ^{d_1} (our proxy of TIF_1). For this exercise, we assume that local developer d_1 requires an indirect utility of 0.97 to build affordable housing. For this utility threshold, we find that a TIF of $\delta^{d_1} \geq 0.88$ is able to implement an equilibrium with construction of affordable housing. In Fig. 2, we can also see a trade-off between $\varepsilon_1(1)$ and δ^{d_1}. Roughly speaking, a higher TIF_1 can offset the reduction in jurisdiction k_1 's affordability index relative to jurisdiction k_2 driven by additional housing supply frictions.

7. Conclusions and directions for future research

This paper provides a general equilibrium model of affordable housing development with two jurisdictions to assess the role of the Tax Incremental Financing (TIF) and the Low Income Housing Tax Credit (LIHTC) programs. To this aim, we incorporate and endogenize in a consistent way several important financial variables, such as the rents that low income households pay for living in the newly constructed affordable housing units, as well as the developer's capital structure, composed of debt, TIF, and LIHTC equity.

Our characterization of equilibrium and numerical examples illustrate thought-provoking ways in which TIF may affect the equilibrium outcome. Our main results are that TIF induces the corporation to rebalance its portfolio of LIHTC equity toward the no TIF jurisdiction. Construction costs increase in the jurisdiction with a TIF policy

due to the additional TIF resources that the developer receives. Construction costs also increase in the no TIF jurisdiction, although significantly less than in the TIF jurisdiction, due to the corporation's higher demand for LIHTC equity – and thus higher equity prices – in that jurisdiction. A TIF policy can implement an equilibrium where the construction of affordable housing becomes feasible. TIF is also effective in ameliorating a housing affordability crisis resulting from supply frictions in the housing market, such as new zoning constraints or NIMBYism. However, TIF also has the pervasive effect of increasing the ratio of debt to corporation's equity. This in turn may preclude an affordable housing project to occur if the corporation tolerates a lower ratio.

The model proposed in this paper can be extended in many interesting directions. For example, a common concern in affordable housing development is financial efficiency. Often, developers are known to understate the equity available to them, as using more leverage increases their return. In addition, developers can often overstate the construction expenses and understate the final value of the project. These incorrect assumptions can be made in an effort to be cautious about setting expectations for the project, but they can also stem from intentionally trying to mislead the TIF board in order to receive more funds, as TIF is seen as "free" money for developers. These considerations suggest that extending our theory to allow for information asymmetries between the developer and municipal authorities would be a fruitful line of research with implications for public policy.[28]

Another important issue with TIF is that sometimes the costs of increased public services are paid by the residents and businesses outside the TID (see The Wisconsin Tax Payer, 2009). A political economy

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[27] Glaeser and Gyourko argue that "a housing affordability crisis means that housing is expensive relative to its fundamental costs of production—not that people are poor."

[28] For example, public overinvestment in an area may require more private development to repay the infrastructure costs. If this additional development does not materialize, the municipality may choose to increase property taxes or incur in a larger deficit. A less common scenario occurs when a developer underestimates the construction expenses or complications of a project. This public underinvestment causes significant issues when TIF is involved, as there would be little or no private invesment, in turn leaving the municipality responsible for unpaid costs and stalled development projects.

theory should be developed to understand the opposition to TIF of certain groups at a higher hierarchy level (e.g., at the state level).

Other extensions of our model are the following. First, incorporate collateralized non-recourse mortgage debt, where in case of default the affordable housing asset would be seized by the lender. With this approach we would be able to understand the impact of default risk on affordable housing development. Second, allow for multiple housing projects and heterogeneous households within a jurisdiction. With this extension we would be able to understand the impact of TIF and LIHTC on house prices across the different residential real estate assets in a jurisdiction, connecting in this way with the empirical literature of housing externalities. Third, allow for the issuance of municipal bonds to finance the deficit of a jurisdiction with an ambitious housing pro-

gram. In this setting, we would be able to examine the impact of liquidity in the municipal bond market on affordable housing development. These extensions may also provide us with a better understanding of the market mechanisms behind the different housing development programs.

Empirical research to study our model predictions as well as the above open questions is also welcome. However, to our knowledge, this is challenging because the availability of TIF data is limited (to this end, it would help if municipalities show TIF amounts separately on property tax bills). To test the predicitions of this paper, researchers may also considering constructing a database with capital structure information for a sufficiently large number of development projects across different geographical regions.

A. Appendix

A.1. History and institutional aspects of the LIHTC program

The creation of the LIHTC program goes back to 1986, when the administration implemented a federal income tax reform that provided a ten-year tax credit for investors in affordable housing.[29] This initiative was made permanent by Congress in 1993. The most innovative aspect of this program is that it makes federal subsidies a tax expenditure administered by the Internal Revenue Service (IRS) rather than a federal expense administered by the U.S. Department of Housing and Urban Development (HUD).

The National Association of Home Builders (NAHB) estimates that the one-year local impact of constructing 100 units for a typical family LIHTC development includes $7.9 million in local income, $827,000 in taxes and other revenue for local governments, and 122 local jobs. The annual recurring impact of those 100 family units includes $2.4 million in local income, $441,000 in taxes and other revenue for local governments, and 30 local jobs.[30]

Another important innovative aspect is that it allows for some local control because credit allocators usually are the state housing financial authorities. Each state receives an annual allocation of tax credits, and then distributes theses tax credits among local developers according to a well-defined Qualified Allocation Plan (QAP). The QAP is updated every year or two to reflect current priorities.

In their review of tax credit applications, the state agency in charge of allocating the tax credits uses a point scoring system to evaluate projects. These points are distributed among different categories. For example, the Wisconsin Housing and Economic Development Authority (WHEDA) 2017 QAP assigns a total of 284 points to the following categories (points in parentheses): lower-income areas (5), energy efficiency and sustainability (32), mixed-income incentive (12), serves large families (5), serves lowest-income residents (60), supportive housing (20), rehab/neighborhood stabilization (25), universal design (18), financial participation (25), eventual tenant ownership (3), development team (12), readiness to proceed (12), credit usage (30), and opportunity zones (25). Developers prepare their affordable housing proposals trying to score the maximum points for as many categories as possible, but sometimes financial and physical barriers prevent developers from getting high scores in some of these categories. The maximum annual credit allocation is usually either 4% or 9% of the eligible basis of a project,[31] depending on factors such as the type of affordable housing development project (e.g., rehabilitation versus new construction) and the use of tax exempt bonds.[32]

A.2. Institutional aspects of TIF

Once a Tax Incremental District (TID) is created, a developer can come forward with a proposal for his project. Before asking for Tax Increment Financing (TIF), the developer will need the correct entitlements in place. These requirements may vary across TIDs. As we can see in Table 2 below, the developer's proposal needs to include the value of the project annually, typically for the next ten years (for simplicity, we restrict our model to two dates). The proposal should also include building renderings, budgets, and the project's current capital structure and shortfall.

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[29] The compliance period is 15 years, reflecting that investors are not out at year 10.

[30] See report by National Association of Home Builders (2010).

[31] The eligible basis of a project is the cost of acquiring an existing building if there is one (but not the cost of the land), plus construction and other construction-related costs to complete the project.

[32] The choice of the annual allocation rate comes down to whether the developer is using tax exempt bond financing or not. Tax exempt bonds are often combined with the 4% credits and are more often used for acquisition rehab projects. The bond financing can qualify the developer for 4% credits as long as thresholds are met, and is not as competitive as the 9% credit in many states. In a 4% deal, the amount of credit is so much less than the developer can generate with 9% credits that it is difficult to make new construction or adaptive re-use deals work without additional soft financing sources. Tax exempt bonds and 9% credits cannot be combined.

Table 2

In this example we consider an existing structure that generates $100,000 annually in property taxes, and a new development that increase tax revenue to $500,000 annually after construction is completed. Thus, there is a difference of $400,000 per year. After 10 years, the increment becomes $4,000,000. The tax recipients only take the original tax basis, the $100,000 annually in this example, and the remaining $400,000 is given back to the original lender, either the bond owners or the bank. We assume that the developer only gets a TIF loan equal to 50% of the increment. In this case, the developer still pays back $400,000 every year for the next 10 years. However, the city is not using the full $400,000 for debt servicing; it is only using $200,000 for debt service. The remaining $200,000 can be distributed to the tax recipients, or it can be used for infrastructure development.

Original Tax Basis	$100,000
New Tax Basis	$500,000
Increment	$400,000
Ten Year Increment	$4,000,000
Loan Amount	$2,000,000
Debt Service (Annual)	$200,000
Excess Funds (Annual)	$200,000

Once the developer submits his proposal, the city proceeds to value the project. The value might decrease initially if an existing, functioning structure needs to be demolished before construction of the project can begin, but should increase significantly upon completion of the improvements. The correct valuation of the proposed development is critical, because the new tax basis upon completion is the foundation for the TID to calculate available TIF monies, and typically the city will employ a consultant, either full-time or ad hoc, to independently appraise the project.

A critical aspect of TIF is that it limits the tax revenue received by all tax recepients within a TID. Tax recipients include schools, fire stations, and many other essential programs to a local government (programs that often need significant financial support). Tax recipients, and occasionally local law makers as well, typically make up a TIF committee that approves or rejects projects requesting TIF assistance. The TIF committee is not required to take the developer's valuation into account when valuing the project or at any point during the process of determining whether or not to support the development with TIF monies.

As mentioned in Section 3.4, there needs to be a budget shortfall for a project to be eligible for TIF (otherwise the developer can move forward without needing the city's financial assistance). While this may at first appear to be a market inefficiency, one should keep in mind that we are dealing with TIF projects that include an affordable housing component, and also that many of these developments occur in blighted areas, where soil contamination and other factors can complicate construction. There are complicating factors that surround these deals, thus placing them in the position where the market cannot fully support their weight. Also, notice that taxes and regulations (such as soil requirements, building ordinances, and setbacks) create additional restrictions for developers that sometimes make their projects unfeasible. For instance, in downtown Madison, Wisconsin, there is significant demand for multi-family affordable housing, but because of the State Capitol Dome and beautiful lake views, there are strict height requirements which can prevent developers from being able to build to the needs of the downtown area, as they are only allowed to build so many stories. In a way, TIF remediates the burden of restrictions originally created by regulation through taxation policy.

TIF is relevant at the policy level in the U.S. because the strategies and policies of municipal governments often differ significantly, even between cities of similar size with similar demographics and similar industries supporting their existence. For instance, two cities can exist next to each other, in the same state, with roughly the same amount of people, with the majority of their workers employed in similar vocations, and yet they will be governed in entirely different manners. This individualized structure of local government in the U.S. makes standardization almost impossible, and very few policy tools exist that can be used across the country at the municipal level.

A.3. Proofs

Proof of Proposition 1. Conditions (20) and (21) follow from assumption $\omega_0^{h_k}(s_1) = \omega_0^{h_k}(s_2)$, the poor household h_k 's budget constraint (10) at state s, and the market clearing condition for affordable housing at state s. ∎

Proof of Proposition 2. Expression (22) follows from developer d_k's budget constraint in the first period and the Khun-Tucker condition $T_k^c v_k^c = 0$, where v_k^c is the shadow value of the non-negativity constraint $T_k^c \geq 0$. Notice also that $\tau D^{d_k} = -\omega_{10}^{H_k}$. ∎

Proof of Proposition 3. The proof follows from expression

$$\underbrace{CC_k \equiv p_{11}b_{11}^{d_k} + p_{12}b_{12}^{d_k}}_{\text{construction cost of project } j_k} = \frac{\tau}{r}\omega_0^{h_k}(1)\left(1 - 0.9999\frac{T_k^c}{\widehat{T}_k^{d_k}}\right) \tag{23}$$

and the fact that T_k^c decreases with τ. The latter follows from the developer's budget constraint (16), optimality conditions (27), (28), (29) and (33), and the Khun-Tucker condition $v_k^{d_k} = 0$, where $v_k^{d_k}$ is the developer d_k's Lagrange multiplier of constraint $T_k^{d_k} \geq 0$.

Next, we prove that condition (23) holds in equilibrium. First, using the developer d_k's optimality conditions with respect to the purchase of construction materials $l = 1$ and $l = 2$, we can write:

$$p_{11} = \frac{\sum_s \lambda^{d_k}(s)p_k(s)\varepsilon_k(s)}{\lambda_1^{d_k}} TFP_k\alpha_k\left(b_{11}^{d_k}\right)^{\alpha_k-1}\left(b_{12}^{d_k}\right)^{1-\alpha_k}\left(1 - 0.9999\frac{T_k^c}{\widehat{T}_k^{d_k}}\right) \tag{24}$$

$$p_{12} = \frac{\sum_s \lambda^{d_k}(s) p_k(s) \varepsilon_k(s)}{\lambda_1^{d_k}} TFP_k(1-\alpha_k)\left(b_{11}^{d_k}\right)^{\alpha_k}\left(b_{12}^{d_k}\right)^{-\alpha_k}\left(1 - 0.9999\frac{T_k^c}{\widehat{T}_k^{d_k}}\right) \tag{25}$$

where $\lambda_1^{d_k}$ and $\lambda^{d_k}(s)$ are the developer d_k's Lagrange multipliers associated with his budget constraints in period 1 and state s of period 2, respectively.

Now, the developer d_k's optimality condition with respect to D^{d_k} assuming a non-binding short sale constraint (interior solution) implies the following condition:

$$\frac{\sum_s \lambda^{d_k}(s)}{\lambda_1^{d_k}} = \frac{\tau}{r} \tag{26}$$

Using the poor household h_k's budget constraint at state s and the market clearing equation for affordable housing in jurisdiction k, we find the following equilibrium price of affordable housing at state s:

$$p_k(s) = \frac{\omega_0^{h_k}(s)}{\varepsilon_k(s) TFP_k\left(b_{11}^{d_k}\right)^{\alpha_k-1}\left(b_{12}^{d_k}\right)^{1-\alpha_k}} \tag{27}$$

Finally, we use equations (24)–(27) and assumption $\omega_0^{h_k}(1) = \omega_0^{h_k}(2)$ to obtain the following reduced form expressions for the price of construction materials 11 and 12:

$$p_{11} = \frac{\tau}{r}\left(\frac{\omega_0^{h_k}(1)}{b_{11}^{d_k}}\right)\alpha_k\left(1 - 0.9999\frac{T_k^c}{\widehat{T}_k^{d_k}}\right) \tag{28}$$

$$p_{12} = \frac{\tau}{r}\left(\frac{\omega_0^{h_k}(1)}{b_{12}^{d_k}}\right)(1-\alpha_k)\left(1 - 0.9999\frac{T_k^c}{\widehat{T}_k^{d_k}}\right) \tag{29}$$

The equilibrium equation (23) for the construction cost of a project j_k follows from equations (28) and (29), and definition $CC^k \equiv p_{11}b_{11}^{d_k} + p_{12}b_{12}^{d_k}$. Equation (23) is independent of the degree of sustitutability between construction materials. ∎

A.4. Numerical examples

To compute numerical examples 1 and 2, we first find the value of $b_{11}^{d_1}$, $b_{12}^{d_2}$, T_1^c, T_2^c and τ by solving a system of five equations together with the following impliciy assumption

$$\omega_0^m(s) = \widehat{T}_1^{d_1} + \widehat{T}_1^{d_1}, s = s_1, s_2 \tag{30}$$

The implicit assumption (30) follows from the market clearing condition of the numeraire good at state s and budget constraints (15) and (17).

The system of five equations that solve for variables $b_{11}^{d_1}$, $b_{12}^{d_2}$, T_1^c, T_2^c and τ are (22), market clearing equations $b_{11}^{d_1} = \omega_{11}^m$ and $b_{12}^{d_2} = \omega_{12}^m$, and the following two conditions:

$$\omega_{10}^c = \frac{1}{r}\left(1 + \frac{0.9999\omega_0^{h_1}(1)}{\widehat{T}_1^{d_1}}\right)\tau T_1^c + \frac{1}{r}\left(1 + \frac{0.9999\omega_0^{h_2}(1)}{\widehat{T}_2^{d_2}}\right)T_2^c \tag{31}$$

$$\frac{\tau}{r}\omega_0^{h_1}(1)\sum_k\left(\alpha_k\frac{\omega_{11}^m}{b_{11}^{d_k}} + (1-\alpha_k)\frac{\omega_{12}^m}{b_{12}^{d_k}}\right)\left(1 - \frac{0.9999T_k^c}{\widehat{T}_k^{d_k}}\right) = F \tag{32}$$

where $F = \omega_{10}^c + \sum_k\left(\omega_{10}^{H_k} + \left((1-\delta^{h_k})\left(\lambda_k + \eta_k\widehat{T}_k^{d_k}\right) + \delta^{h_k}\gamma_k\widehat{T}_k^{d_k}\right)\right)$.

Equation (31) follows from the corporation's first period budget constraint (14) and the following optimality pricing condition

$$q_k = \frac{\tau}{r}\left(\frac{1}{0.9999} + \frac{\omega_0^{h_k}(1)}{\widehat{T}_k^{d_k}}\right) - \gamma_k + \frac{v_k^c}{\lambda_0^c 0.9999} \tag{33}$$

which in turn follows from the corporation's first order optimality condition with respect to T_k^c, the household h_2's budget constraints (9) and (10), and the market clearing condition for affordable housing at state s. Here v_k^c and λ_0^c denote the corporation's Lagrange multipliers of constraints $T_k^c \geq 0$ and (14), respectively. The corresponding Khun-Tucker condition for the first Lagrange multiplier is $v_k^c T_k^c = 0$.

Equation (32) follows from the market clearing equation of the numeraire good in the first period and the following equations derived from agents m and h_k' first period budget constraints:

$$x_{10}^m = p_{11}\omega_{11}^m + p_{12}\omega_{12}^m \tag{34}$$

$$x_{10}^{h_k} = \delta^{h_k}\pi_k \tag{35}$$

where p_{11} and p_{12} take the value given by expressions (28) and (29), respectively.

Once we find the values for $b_{11}^{d_1}$, $b_{12}^{d_2}$, T_1^c, T_2^c and τ, we can find the values of the remaining equilibrium variables since these are written in terms of $b_{11}^{d_1}$, $b_{12}^{d_2}$, T_1^c, T_2^c, τ and the parameters of the economy. These follow from our pricing equations (27)–(29) and (33), and reduced form buget constraints (9)–(17).

We finish this section with two remarks. First, the developer's short sale constraints on debt are non-binding in equilibrium when $\overline{D}^{d_k} = 1$, so the corresponding Lagrange multiplier is zero (in fact, we find an equilibrium with $D^{d_k} = -D^{H_k} < \overline{D}^{d_k}$, for $k = 1, 2$, which in turn satisfies debt market clearing condition $D^{d_1} + D^{d_2} + D^{H_1} + D^{H_1} = 0$ given the equilibrium market price τ). Similarly, the developer and corporation's sign constraints on LIHTC are non-binding since we find an interior equilibrium where $T_1^c > 0$, $T_2^c > 0$, $T_1^{d_1} > 0$, $T_2^{d_2} > 0$. Thus, the corresponding shadow prices are zero.

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