Math 4610 Tasksheet 3

Jacob Fitzgerald (A02261889)

Links

Code

https://jfitzusu.github.com/math4610/

Docs

https://jfitzusu.github.io/math4610/

Task 1

Bisection Method

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/**
* Returns the Root Approximation of a Function Using the Bisection Method
* *f(): Function to Find Root Of
* a: Lower Bound to Search
* b: Upper Bound to Search
* tol: Maximum Permissable Error
* Returns: Approximation of Root
*/
double bisect(double (*f)(), double a, double b, double tol) {
        double fa = f(a);
        double fb = f(b);

        // Invalid Boundaries
        if (fa * fb >= 0.0) {
            printf("There may not be a root in [a,b]: f(a) * f (b) = %e",
fa*fb);
            exit(-1);
        }

        double c;
        double fc;

        // Iterations Needed to Reach Tolerance
        int k = ((int) (log(tol) - log(b - a)) / log(0.5) + 1);

        for (int i=0; i<k;i++) {
            c = 0.5 * (a + b);
            fc = f(c);
```

```
            // Root in First Half
            if (fa * fc < 0.0) {
                b = c;
                fb = fc;

            }

            // Root in Second Half
            else if (fb * fc < 0.0) {
                a = c;
                fa = fc;
            }

            else if (fc == 0) {
                return c;
            }
        }

        return c;
    }
```

Fixed Point Iteration

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/**
* Approximates the Root of a Function Using Fixed Point Iteration
* *g(): The Modified, Fixed Point Form of the Function
* x0: Initial Approximation
* tol: Maximum Permissable Error
* maxIterations: Maximum Times to Try for Convergence
* Returns: Approximation of Root
*/
double fixedPoint(double (*g)(), double x0, double tol, int maxIterations) {
        double x1;
        for (int i=0; i<maxIterations; i++) {
            x1 = g(x0);

            // Root is Within Permissable Error
            if (fabs(x1 - x0) <= tol) {
                break;
            }
            x0 = x1;
        }

        return x1;
    }
```

Newton's Method

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/**
* Approximates Root of Function Using the Newton Method
* *f(): Function to Find Root In
* *g(): Derivitive of f()
* x0: Initial Guess
* tol: Maximum Permissible Error
* maxIterations: Maximum Times to Try for Convergence
* Returns: Approximation of Root
*/
double newton(double (*f)(), double(*g)(), double x0, double tol, int
maxIterations) {
        double x1;
        double gx;

        for (int i=0; i<maxIterations; i++) {
            gx = g(x0);

            // Newton Method Fails, Slope of Function is Flat
            if (gx == 0) {
                printf("Encountered Invalid Derivitaive Value at X = %e", x0);
                exit(-1);
            }

            x1 = x0 - f(x0) / gx;

            // Root Within Permissible Error
            if (fabs(x1 - x0) <= tol) {
                break;
            }

            x0 = x1;
        }

        return x1;
    }
```

Secant Method

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/**
* Approximates the Root of a Function Using the Secant Method
* *f(): Function to Approximate Root For
* x0: Initial Guess
* x1: Second Initial Guess
```

```
* tol: Maximum Permissable Error
* maxIterations: Maximum Times to Try for Convergence
* Reutnrs: Approximation of Root
*/
double secant(double (*f)(), double x0, double x1, double tol, int maxIterations)
{
        double f0 = f(x0);
        double f1 = f(x1);
        double x2 = 0;

        for (int i=0; i<maxIterations; i++) {
            x2 = x1 - f1 * (x1 - x0) / (f1 - f0);

            // Root Within Permissible Error
            if (fabs(x2 - x1) <= tol) {
                break;
            }

            x0 = x1;
            x1 = x2;
            f0 = f1;
            f1 = f(x1);
        }

        return x2;
    }
```

Hybrid Newton's Method

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <stdbool.h>

/**
* Approximates the Root of a Function Using the  and Bisection Methods
* *f(): Function to Approximate Root For
* a: Miminum Value of Range to Search
* b: Maximum Value of Range to Serach
* tol: Maximum Permissable Error
* maxIterations: Maximum Times to Try for Convergence
* maxTries: Maximum Times to Reduce Interval
* strictInterval: If the Function Should Only Reutn rValues Wihtin the Interval
* Returns: Approximation of Root
*/
double hybridNewton(double (*f)(), double (*g)(), double a, double b, double tol,
int maxIterations, int maxTries, bool strictInterval) {

        double x2 = 0;
        double x0 = b;
        double x1 = 0;
        double f0 = f(x0);
        double f1 = f(x1);
        double gx;
```

```
        double error;

        for (int i=0; i<maxTries; i++) {

            // Initial Conditions for Newton Appromxiation
            x0 = b;
            x1 = 0;




            // Newton Approxmiation
            for (int j=0; j<maxIterations; j++) {
                gx = g(x0);
                if (gx == 0) {
                    printf("Encountered Invalid Derivitaive Value at X = %e",
x0);

                    exit(-1);
                }

                x1 = x0 - f(x0) / gx;
                error = fabs(x1 - x0);

                if (error <= tol) {
                    break;
                }

                x0 = x1;
            }

            // Only Returns Results Outisde the Interval if StrictInterval is
Off

            if (strictInterval) {
                if (error < tol && x1 < b && a < x1) {
                    return x1;
                }
            }
            else if (error < tol) {
                    return x1;
            }

            // Setup for Bisection
            double fa = f(a);
            double fb = f(b);
            double c;
            double fc;

            // Uses Bisection to Reduce Interval Size if Newton Fails
            for (int j = 0; j < 4; j++) {
                c = 0.5f * (a + b);
                fc = f(c);

                if (fa * fc < 0) {
                    fb = fc;
                    b = c;
                }
```

```
                else if (fb * fc < 0) {
                    fa = fc;
                    a = c;
                }

                else if (fc == 0) {
                    return c;
                }

                else {
                    printf("There may not be a root in [a,b]: f(a) * f (b) = %e",
fa*fb);
                    exit(-1);
                }
            }

            if (fabs(b - a) <= tol) {
                return b;
            }
        }

        return b;
    }
```

Hybrid Secant Method

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <stdbool.h>

/**
* Approximates the Root of a Function Using the Secant and Bisection Methods
* *f(): Function to Approximate Root For
* a: Miminum Value of Range to Search
* b: Maximum Value of Range to Serach
* tol: Maximum Permissable Error
* maxIterations: Maximum Times to Try for Convergence
* maxTries: Maximum Times to Reduce Interval
* strictInterval: If the Function Should Only Reutn rValues Wihtin the Interval
* Returns: Approximation of Root
*/
double hybridSecant(double (*f)(), double a, double b, double tol, int
maxIterations, int maxTries, bool strictInterval) {
        double x0 = b;
        double x1 = 0;
        double f0 = f(x0);
        double f1 = f(x1);
        double x2;
        double error;

        for (int i=0; i<maxTries; i++) {
```

```
        // Initial Conditions for Secant Appromxiation
        x0 = b;
        x1 = 10 * tol + x0;
        f0 = f(x0);
        f1 = f(x1);

        // Secant Approxmiation
        for (int j=0; j<maxIterations; j++) {
            x2 = x1 - f1 * (x1 - x0) / (f1 - f0);

            error = fabs(x2 - x1);
            if (error <= tol) {
                break;
            }

                        x0 = x1;
            x1 = x2;
            f0 = f1;
            f1 = f(x1);
        }

        // Only Returns Results Outisde the Interval if StrictInterval is
Off
        if (strictInterval) {
            if (error < tol && x1 < b && a < x1) {
                return x1;
            }
        }
        else if (error < tol) {
                return x1;
        }

        // Setup for Bisection
        double fa = f(a);
        double fb = f(b);
        double c;
        double fc;

        // Uses Bisection to Reduce Interval Size if Secant Fails
        for (int j = 0; j < 4; j++) {
            c = 0.5f * (a + b);
            fc = f(c);

            if (fa * fc < 0) {
                fb = fc;
                b = c;
            }

            else if (fb * fc < 0) {
                fa = fc;
                a = c;
            }

            else if (fc == 0) {
                return c;
```

```
                }

                else {
                    printf("There may not be a root in [a,b]: f(a) * f (b) = %e",
fa*fb);
                    exit(-1);
                }
            }

            if (fabs(b - a) <= tol) {
                return b;
            }
        }

        return b;
    }
```

Descriptions of these functions and their usages has already been covered in previous task sheets. These are simply their c-based implementations.

Code for testing these methods can be found in tasks 2 and 3.

Task 2

Code for Testing Newton:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <stdbool.h>
#include "../src/bisect.c"
#include "../src/newton.c"
#include "../src/secant.c"
#include "../src/fixedpoint.c"
#include "../src/hybridnewton.c"
#include "../src/hybridsecant.c"


double fval(double);
double gval(double);
double fprimeval(double);

int main() {
    printf("Newton Root: %f \n", newton(fval, fprimeval, 0, 0.0001, 100));
}

double fval(double xval) {
    return xval * exp(-xval);
}

double fprimeval(double xval) {
    return exp(-xval) - xval * exp(-xval);
}
```

This code sets up our mathematical function: f(x) = x * e ^ -x as a c function, and passes it to our newton method as a pointer, printing the result to the console. Additionally, the derivative of our function (fprimeval) is also passed to our Newton method in the same way, so that it can actually work.

Results of Running the Code:

```
Newton Root: 0.000000
```

As 0 is one of the roots of our funciton, we can be somewhat certain that our Newton method actually works now.

Task 3

Code for Testing All Methods:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <stdbool.h>
#include "../src/bisect.c"
#include "../src/newton.c"
#include "../src/secant.c"
#include "../src/fixedpoint.c"
#include "../src/hybridnewton.c"
#include "../src/hybridsecant.c"


double fval(double);
double gval(double);
double fprimeval(double);

int main() {
    printf("Bisection Root: %f \n", bisect(fval, -3.9, 4, 0.0001));
    printf("Fixed Point Root: %f \n", fixedPoint(gval, -0.1, 0.0001, 100));
    printf("Newton Root: %f \n", newton(fval, fprimeval, 0, 0.0001, 100));
    printf("Secant Root: %f \n", secant(fval, 0, 1, 0.0001, 100));
    printf("Hybrid Newton Root: %f \n", hybridNewton(fval, fprimeval, -3.9, 4,
0.0001, 100, 10, true));
    printf("Hybrid Secant Root: %f \n", hybridSecant(fval, -3.9, 4, 0.0001, 100,
10, true));

}

double fval(double xval) {
    return xval * exp(-xval);
}

double gval(double xval) {
    return xval - fval(xval);
}

double fprimeval(double xval) {
```

```
    return exp(-xval) - xval * exp(-xval);
}
```

This code uses each of our methods descripted in Task 1 to approximate the root of f(x) = x * e ^ -x. It's the same as the code for testing the Newton method, just applied to every method. It uses a c function to represent our mathematical function, and passes it to each of our methods by pointer. A g(x) = x - f(x) is used for fixed point methods, and the derivative of the function (fprimeval) is passed to the newton/hybrid newton methods in order to allow them to complete their approximations.

Testing Output:

```
Bisection Root: -0.000026
Fixed Point Root: 0.000000
Newton Root: 0.000000
Secant Root: 0.000000
Hybrid Newton Root: -0.000000
Hybrid Secant Root: 0.000000
```

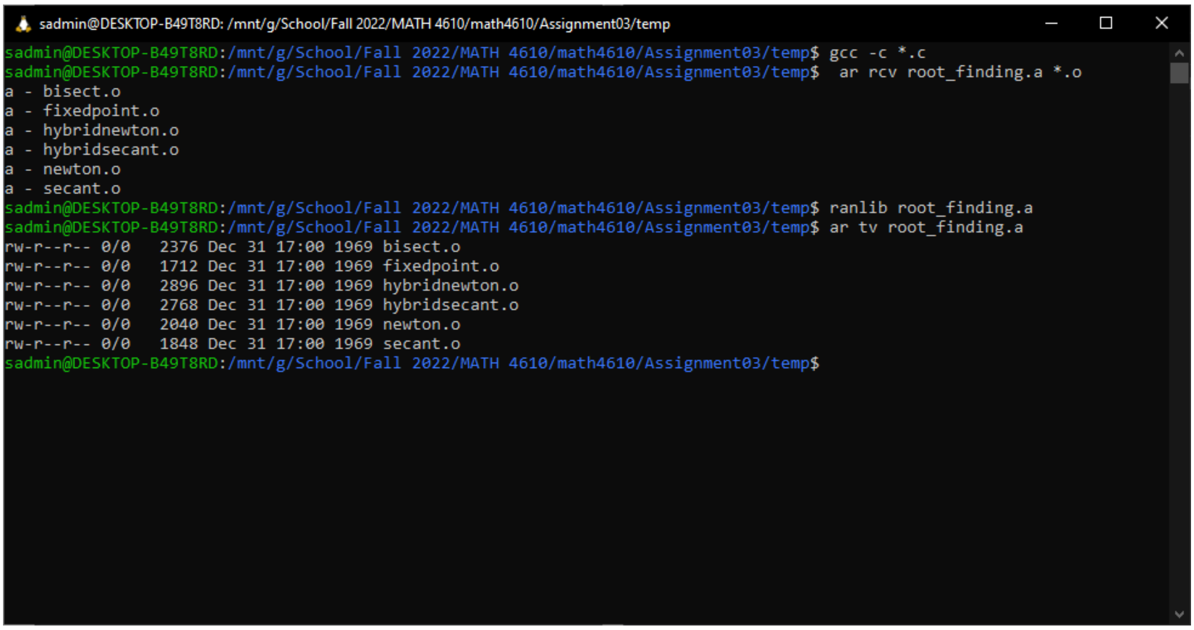
As you can see, our output for the Newton method remains the same as in task 2, which means we haven't screwed anything up. Additionally, all the other methods return the same result, which is a very good sign. Our Bisection is a little off, but it's well within the tolerance specified.

Task 4

Console Output:

```
sadmin@DESKTOP-B49T8RD:/mnt/g/School/Fall 2022/MATH
4610/math4610/Assignment03/temp$ gcc -c *.c
sadmin@DESKTOP-B49T8RD:/mnt/g/School/Fall 2022/MATH
4610/math4610/Assignment03/temp$  ar rcv root_finding.a *.o
a - bisect.o
a - fixedpoint.o
a - hybridnewton.o
a - hybridsecant.o
a - newton.o
a - secant.o
sadmin@DESKTOP-B49T8RD:/mnt/g/School/Fall 2022/MATH
4610/math4610/Assignment03/temp$ ranlib root_finding.a
sadmin@DESKTOP-B49T8RD:/mnt/g/School/Fall 2022/MATH
4610/math4610/Assignment03/temp$ ar tv root_finding.a
rw-r--r-- 0/0   2376 Dec 31 17:00 1969 bisect.o
rw-r--r-- 0/0   1712 Dec 31 17:00 1969 fixedpoint.o
rw-r--r-- 0/0   2896 Dec 31 17:00 1969 hybridnewton.o
rw-r--r-- 0/0   2768 Dec 31 17:00 1969 hybridsecant.o
rw-r--r-- 0/0   2040 Dec 31 17:00 1969 newton.o
rw-r--r-- 0/0   1848 Dec 31 17:00 1969 secant.o
sadmin@DESKTOP-B49T8RD:/mnt/g/School/Fall 2022/MATH
4610/math4610/Assignment03/temp$
```

Screenshot:



Task 5

Results of Running Newly Compiled Tests:

```
Bisection Root: -0.000026
Fixed Point Root: 0.000000
Newton Root: 0.000000
Secant Root: 0.000000
Hybrid Newton Root: -0.000000
Hybrid Secant Root: 0.000000
```

The results of our tests remain the same after library compilation, which is good to see. Usually, this wouldn't be an issue, but c can compile things in weird ways, with different builds and the like, which can actually have an effect on the accuracy of your results. It's good to see that nothing has changed.