1 Toy Systems and Quantum Master Equations
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How do quantum systems behave when interacting with their environments? This question is usually addressed by considering Markovian quantum master equations like the Lindblad equation. However, the assumptions of memoryless dynamics or being able to average over some environment behavior are often invalid. To assess the validity of these assumptions and to understand the more general dynamics, I propose to find exact (numerical) solutions for at least one toy system and to evaluate the effect of the environment.

In particular, I propose to start with a transverse-field Ising model with the Hamiltonian

$$\mathsf{H} = -\sum_{i\in\mathsf{Z}_n} \left(J_i\sigma_{iz}\sigma_{(i+1)z} + \mu_0 h_i\sigma_{ix}\right),$$

where $\sigma_{1z} = \sigma_z \otimes \mathsf{I} \otimes \cdots \otimes \mathsf{I}$, etc. and $\sigma_{i0} = \mathsf{I}$ that is coupled to a Markovian bath so that

$$\dot{\rho} = \frac{[\mathsf{H}, \rho]}{i\hbar} + \sum_{i\in\mathsf{Z}_n,\, a\in\{x,z\}} \gamma_{ia}\left(\mathsf{L}_{ia}\rho\mathsf{L}_{ia}^\dagger - \frac{1}{2}\left\{\mathsf{L}_{ia}\mathsf{L}_{ia}^\dagger, \rho\right\}\right),$$

where $\mathsf{L}_{iz} = \left(\sigma_{ix} - i\sigma_{iy}\right)/2$ and $\mathsf{L}_{iz} = \left(\sigma_{iy} - i\sigma_{iz}\right)/2$, similar to in [3, 2].

I would consider one or a few spins as a subsystem, and the rest as the (inner) environment. First, I would exactly diagonalize the closed system with $\Gamma_{ia} = 0$ and study the behavior of the subsystem as the size n of the full system grows, which I expect will reach to about 16. What are the stationary states and what is the time evolution of the expectation values of quantities like the subsystem Hamiltonian, net magnetization, and von Neumann entropy? Does the closed system thermalize any of these quantities as it grows?

I would then study how the quantities mentioned behave differently as the interaction with a Markovian environment is turned on in three regimes where

- the isolated subsystem of a few spins interacts with the bath,
- the full Ising chain including subsystem interacts with the bath, and
- the environment Ising chain but not subsystem interacts with the bath.

That is, how do different kinds of composite environments effect the behavior of the few-spin subsystem in question?

While I have proposed this investigation for the Ising model, the target system and desired quantities may change as I research the Ising model, various generalizations like the Heisenberg or Potts models, and other possible target systems like coupled harmonic oscillators or discrete-level systems, like in [1].

References

[1] M. Bhattacharya et al. "Understanding the Damping of a Quantum Harmonic Oscillator Coupled to a Two-Level System Using Analogies to Classical Friction". In: *American Journal of Physics* 80.9 (Aug. 2012), pp. 810–815.

[2] Jiasen Jin et al. "Phase Diagram of the Dissipative Quantum Ising Model on a Square Lattice". In: *Physical Review B* 98.24 (Dec. 2018), p. 241108.

[3] Nobuyuki Yoshioka and Ryusuke Hamazaki. "Constructing Neural Stationary States for Open Quantum Many-Body Systems". In: *Physical Review B* 99.21 (June 2019), p. 214306.

2 Dynamics of an electronic oscillator with step nonlinearity and bandpassed delayed feedback

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This thesis would extend and verify the work of Kees and myself from prior summers. While Colleen Werkheiser's thesis has shown that fast-oscillating periodic solutions in a time-delayed system with lowpass filtering are unstable, it is possible that these fast solutions may be stabilized with the use of bandpass feedback.

The use of a programmable time-delay device enables the experiments with an electronic circuit to potentially be automated. Thus, a detailed examination of the parameter space would be possible. This experimental approach would also be checked against theoretical predictions and similarly detailed numerical computations.

By forming a full understanding of fast-oscillating solutions in this system, we inform the study of more complex systems and the conditions which give rise to so-called virtual chimera states [1].

References

[1] Laurent Larger, Bogdan Penkovsky, and Yuri Maistrenko. "Virtual Chimera States for Delayed-Feedback Systems". In: *Physical Review Letters* 111.5 (Aug. 2013), p. 054103.