

0.1 Calculating canonical ensemble averages

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 import os
4 import h5py, hickle

1 import sys
2 if 'src' not in sys.path: sys.path.append('src')
3 import wanglandau as wl

1 class CanonicalEnsemble:
2     def __init__(self, Es, lngs, name,  $\lambda$  = None):
3         self.Es = Es
4         self.lngs = lngs
5         self.name = name
6         # Choose to scale the exponent of Z to be a convenient size.
7         # This can be improved if needed by taking into account typical  $\beta$ *Es.
8         self. $\lambda$  = max(lngs) if  $\lambda$  is None else  $\lambda$ 
9     def Z( $\lambda$ , self,  $\beta$ ):
10         return np.sum(np.exp(-( $\beta$  * self.Es - self.lngs + self. $\lambda$ )))
11     def Z(self,  $\beta$ ):
12         return np.exp(self. $\lambda$ ) * self.Z( $\lambda$ , self,  $\beta$ )
13     def average(self, f,  $\beta$ ):
14         # return np.sum(f(self) * self.gs * np.exp(- $\beta$  * self.Es)) / self.Z( $\beta$ )
15         return np.sum(f(self) * np.exp(-( $\beta$  * self.Es - self.lngs + self. $\lambda$ ))) / self.Z( $\lambda$ , self,  $\beta$ )
16     def energy(self,  $\beta$ ):
17         return self.average(lambda ens: ens.Es,  $\beta$ )
18     def energy2(self,  $\beta$ ):
19         return self.average(lambda ens: ens.Es**2,  $\beta$ )
20     def heat_capacity(self,  $\beta$ ):
21         return self.energy2( $\beta$ ) - self.energy( $\beta$ )**2
22     def free_energy(self,  $\beta$ ):
23         return -np.log(self.Z( $\beta$ )) /  $\beta$ 
24     def entropy(self,  $\beta$ ):
25         return  $\beta$  * self.energy( $\beta$ ) + np.log(self.Z( $\beta$ ))

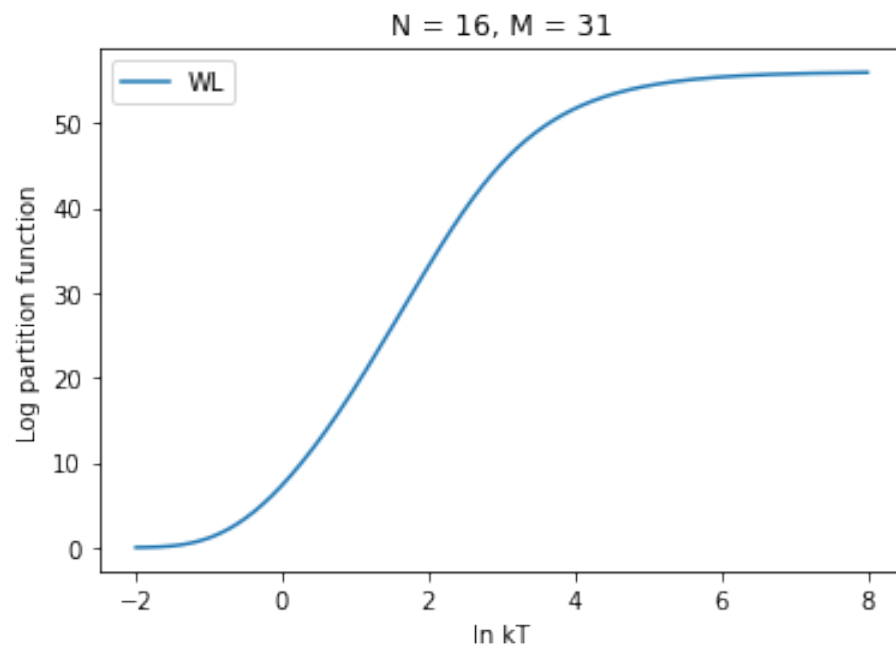
1 with h5py.File('data/simulation-l3t1t-sn.h5', 'r') as f:
2     h = hickle.load(f)
3     results = h['results']
4     params = h['parameters']

1 N, M = len(params['system']['StatisticalImage']['I0']), params['system']['StatisticalImage']['M']
2 wEs, S,  $\Delta$ S = wl.join_results(results)

1  $\beta$ s = [np.exp(k) for k in np.linspace(-8, 2, 500)]
2 wlens = CanonicalEnsemble(wEs, S - min(S), 'WL') # Wang-Landau results
3 # xens = CanonicalEnsemble(Es, gs, 'Exact') # Exact
4 # ensembles = [wlens, xens]
5 ensembles = [wlens]
```

Partition function

```
1 for ens in ensembles:
2     plt.plot(-np.log( $\beta$ s), np.log(np.vectorize(ens.Z)( $\beta$ s)), label=ens.name)
3     plt.xlabel("ln kT")
4     plt.ylabel("Log partition function")
5     plt.title('N = {}, M = {}'.format(N, M))
6     plt.legend();
```

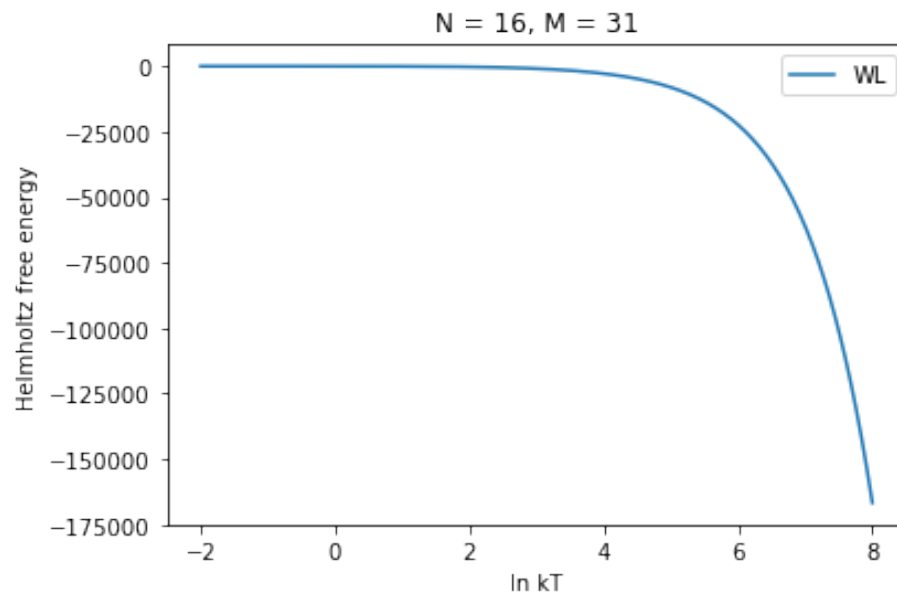


Helmholtz free energy

```

1  for ens in ensembles:
2      plt.plot(-np.log( $\beta$ s), np.vectorize(ens.free_energy)( $\beta$ s), label=ens.name)
3  plt.xlabel("ln kT")
4  plt.ylabel("Helmholtz free energy")
5  plt.title('N = {}, M = {}'.format(N, M))
6  plt.legend();

```

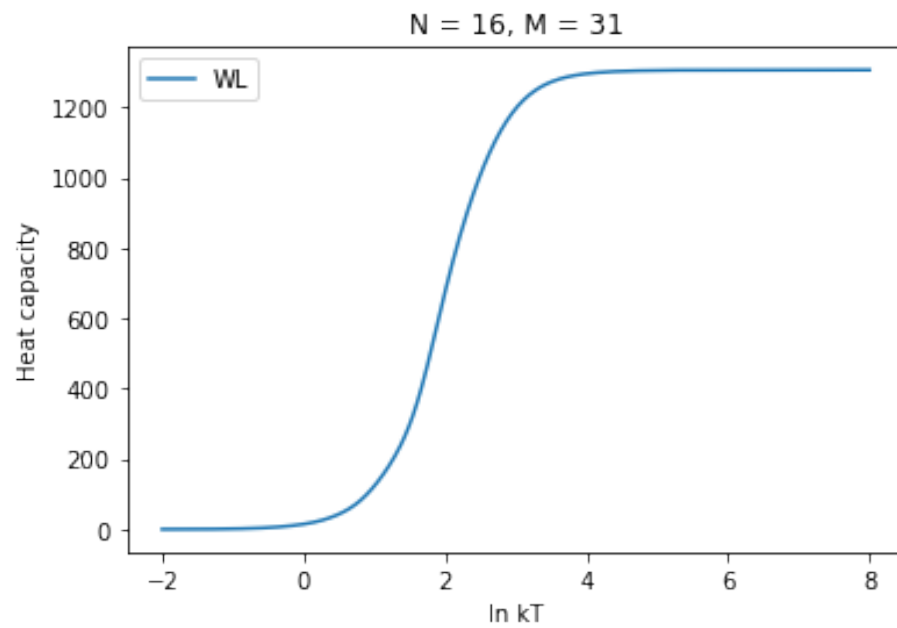


Heat capacity

```

1  for ens in ensembles:
2      plt.plot(-np.log( $\beta$ s), np.vectorize(ens.heat_capacity)( $\beta$ s), label=ens.name)
3  plt.xlabel("ln  $kT$ ")
4  plt.ylabel("Heat capacity")
5  plt.title('N = {}, M = {}'.format(N, M))
6  plt.legend();

```



Entropy

```

1  for ens in ensembles:
2      plt.plot(-np.log( $\beta$ s), np.vectorize(ens.entropy)( $\beta$ s), label=ens.name)
3  plt.xlabel("ln kT")
4  plt.ylabel("Canonical entropy")
5  plt.title('N = {}, M = {}'.format(N, M))
6  plt.legend();

```

