## 0.1 Thermal calculations on images

```
import numpy as np
   from scipy import interpolate, special
   import sys
   if 'src' not in sys.path: sys.path.append('src')
   import wanglandau as wl
    0.1.1 Parallel Simulation
   N = 16
   M = 2**5 - 1
   I0 = np.zeros(N, dtype=int)
   system_params = {
      'StatisticalImage': {
          'I0': I0,
          'I': I0.copy(),
          'M': M
      }
   }
   # L = 16
   # system_params = {
   # 'IsingModel': {
           'spins': np.ones((L, L), dtype=int)
   #
   # }
6
   params = {
      'system': system_params,
      'simulation': {
          'max_sweeps': 500_000_000,
          'flat_sweeps': 10_000,
         'eps': 1e-8,
         'logf0': 1,
          'flatness': 0.1
      'parallel': {
         'bins': 8,
11
          'overlap': 0.25,
          'sweeps': 1_000_000
13
14
15
       'save': {
          'prefix': 'simulation-',
16
          'dir': 'data'
17
      }
18
   }
19
   params.pop('parallel', None) # Single run
   wlresults = wl.run(params, log=True)
   Run parameters
```

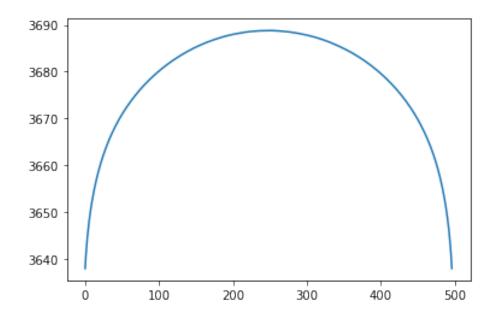
```
'M': 31}},
 'simulation': {'max_sweeps': 500000000,
                'flat_sweeps': 10000,
                'eps': 1e-08,
                'logf0': 1,
                'flatness': 0.1},
 'save': {'prefix': 'simulation-', 'dir': 'data'},
 'log': True}
Running ...
Wang-Landau START
fiter
         steps
                     max steps
     1120000
                 4852245278
1
2
     480000
                 6230406265
3
     800000
                 7059975221
4
     640000
                 7515304503
5
     960000
                 7753865876
     1600000
                 7875971497
6
7
     1760000
                 7937743507
8
     2240000
                 7968810956
9
     2080000
                 7984390249
     3520000
10
                 7992191314
11
     4800000
                 7996094704
12
     9600000
                 7998047114
13
     10880000
                 7999023498
14
     17760000
                 7999511734
15
     27200000
                 7999755864
     35040000
                 7999877931
16
     21280000
17
                 7999938966
18
     67200000
                 7999969483
19
     32480000
                 7999984742
20
     142400000
                 7999992371
21
     104800000
                 7999996186
     24000000
22
                 7999998093
23
     288480000
                 7999999047
24
     29280000
                 799999524
25
     38560000
                 7999999762
26
     41600000
                 7999999881
27
     97120000
                 799999941
     28960000
                 7999999971
Done: 1036640000 total MC iterations; converged.
... done in 116 seconds.
Writing results ... done: data/simulation-gozi5xqv.h5
[r['converged'] for r in wlresults['results']]
```

## [True]

```
wlEs, S, \Delta = wl.join_results(wlresults['results'])
```

## 0.1.2 Results

```
import matplotlib.pyplot as plt
for i, r in enumerate(wlresults['results']):
    plt.plot(r['Es'][:-1], r['S'] + \( \Delta S[i] \))
```



N, M = len(system\_params['StatisticalImage']['I0']), system\_params['StatisticalImage']['M']

Fit a spline to interpolate and optionally clean up noise, giving WL g's up to a normalization constant.

```
gspl = interpolate.splrep(wlEs, S, s=0*np.sqrt(2))
wlgs = np.exp(interpolate.splev(wlEs, gspl) - min(S))
```

## 0.1.3 Exact density of states

We only compute to halfway since g is symmetric and the other half's large numbers cause numerical instability.

```
def reflect(a, center=True):
    if center:
        return np.hstack([a[:-1], a[-1], a[-2::-1]])
    else:
        return np.hstack([a, a[::-1]])
```

The exact density of states for uniform values. This covers the all gray and all black/white cases. Everything else (normal images) are somewhere between. The gray is a slight approximation: the ground level is not degenerate, but we say it has degeneracy 2 like all the other sites. For the numbers of sites and values we are using, this is insignificant.

```
def bw_g(E, N, M, exact=True):
   return sum((-1)**k * special.comb(N, k, exact=exact) * special.comb(E + N - 1 - k*(M + 1), E - k*(M +

→ 1), exact=exact)

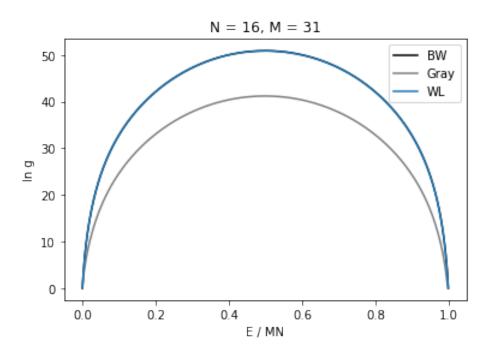
       for k in range(int(E / M) + 1))
def exact_bw_gs(N, M):
   Es = np.arange(N*M + 1)
   gs = np.vectorize(bw_g)(np.arange(1 + N*M // 2), N, M, exact=False)
   return Es, reflect(gs, len(Es) % 2 = 1)
def gray_g(E, N, M, exact=True):
   return 2 * bw_g(E, N, M, exact=exact)
def exact_gray_gs(N, M):
   Es = np.arange(N*M + 1)
   gs = np.vectorize(gray_g)(np.arange(1 + N*M // 2), N, M, exact=False)
   return Es, reflect(gs, len(Es) % 2 = 1)
    Expected results for black/white and gray.
bw_Es, bw_gs = exact_bw_gs(N=N, M=M)
gray_Es, gray_gs = exact_gray_gs(N=N, M=-1 + (M + 1) // 2)
```

Choose what to compare to.

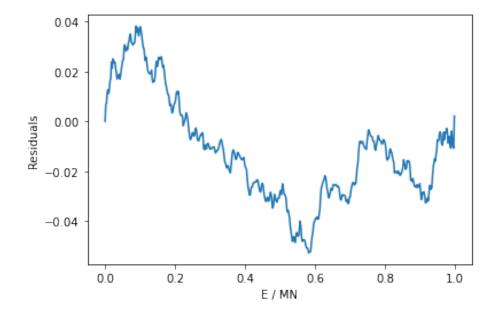
```
Es, gs = bw_Es, bw_gs
```

Presumably all of the densities of states for different images fall in the region between the all-gray and all-black/white curves.

```
plt.plot(bw_Es / len(bw_Es), np.log(bw_gs), 'black', label='BW')
plt.plot(gray_Es / len(gray_Es), np.log(gray_gs), 'gray', label='Gray')
plt.plot(wlEs / len(wlEs), S - min(S), label='WL')
plt.xlabel('E / MN')
plt.ylabel('ln g')
plt.title('N = {}, M = {}'.format(N, M))
plt.legend();
```



```
# plt.plot(wlEs / len(wlEs), np.abs(wlgs - bw_gs) / bw_gs)
# plt.ylabel('Relative error')
plt.plot(wlEs / len(wlEs), S - np.log(bw_gs) - min(S))
plt.ylabel('Residuals')
plt.xlabel('E / MN');
```



print('End of job.')

End of job.