

**2012 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part A****Time—30 minutes****Number of problems—2****A graphing calculator is required for these problems.**

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.
- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?
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