Quantifying Mental States in Work Environment: Mathematical Perspectives

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Abstract

This article presents a study involving 87 participants exposed to a stressful scenario in a virtual reality (VR) environment. An algorithm was developed to assign a positive or negative valence based on questionnaire responses. EEG signals were recorded, and a k-nearest neighbors (KNN) algorithm was trained to infer emotional valence from these signals. Our objective is to further develop mathematical models capable of describing the dynamic evolution of emotional and mental states.

Keywords: keyword1, Keyword2, Keyword3, Keyword4

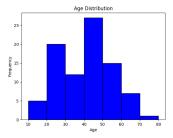
1 Introduction

Emotional states continuously emerge during our daily activities. Emotions and their implication in behavior have been a topic of research forever and have been analyzed since ancient civilizations through many prisms over time. This goes from philosophy to politics, from moral and religion to law [1]. Modern paradigms have initially been shaped by psychology and psychiatry. With advances in technology and aggregation of large data sets, biochemistry and neuroscience, supported by data analysis, provide now a data-driven scientific framework [2–7]. In a previous contribution, wishing to propose a mathematical framework for a descriptive approach of how feelings arise in many aspects of human daily lives, we represented the human mental state as a dynamical system: the mental state is akin to a physical state in which emotional fluxes arise. This mental state evolves as a function of time and results from external stimulus and internal reactions, see [8]. In several aspects, our model encompasses and goes beyond certain psychological theories such as the Discrete Theory of Emotions [5, 9, 10], the Dimensional theory [11-15] and the Affective Events Theory [16-18]. The affective events theory particularly focuses on the affects triggered by events at works. A fundamental question is how to measure these affects. In the present article, we communicate about the results of a study that involved 87 participants who went through a work-type event on a virtual reality (VR) headset. These subjects were asked to visualize an interactive scenario in which they played the role of a bank employee confronted by an angry client. From time to time, they had to select between few possible responses to the client's behavior, which then determined how the scenario would unfold. At the end, they were given a grade within the VR headset. While participants engaged in the experience using the headset, we recorded EEG signals using 4 channels (openBCI electrodes, [19–21]). We also recorded electrodermal activity, Photoplethysmography (known as PPG which is used to derive heart rate) and temperature thanks to the emotibit framework [21, 22]. Finally, subjects were asked to answer a questionnaire at the end of the experiment. The scenario was initially developed at the Crédit Agricole Paris for internal training purpose. The aim of the study is to establish a realistic framework for recording data on psychological states, to support the development of dynamical systems models that describe their evolution over time.

Participants were recruited in France, in the regions of Paris-Ile de France and Normandie. The age (mean=40.6, std=13.7) and sex (M=45, F=42) distributions are represented in Figure 1.

The analysis of the experiments includes the following different parts:

- a qualitative and statistical examination of the participants' responses,
- An automatic analysis of the questionnaire responses was conducted to generate a valence score. The questionnaire included several items designed to evaluate participants' internal states during the experiment, including an open-ended question inviting them to freely describe the thoughts, reasoning processes, and emotions they



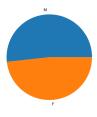


Fig. 1 Age and Sex Distributions of the sample of participants

experienced. Based on these responses, we developed an algorithm that assigns each participant a score reflecting the overall positivity or negativity of their reported feelings. This score is represented as a binary variable in 0,1, which can be interpreted as a discrete operationalization of the concept of valence commonly discussed in the psychological literature, see [11–15].

- An analysis of the EEG signals: EEG data were preprocessed and analyzed using localized spectral frequencies techniques,
- a machine learning algorithm aimed at predicting the valence number from the EEG,
- a reflection on mathematical models to be used to produce relevant signals.

2 Results

2.1 Analysis of the questionnaire and Generation of the valence number

Analysis of the questionnaire responses revealed that 3 of 87 participants (3.4 %) reported experiencing anger during the experiment, while 18 participants (20.7%) reported feelings of annoyance and 22 participants (25.3%) reported a sense of judgment toward the client (see figure 2.1). We note that although the client's behavior in VR is deliberately designed to provoke emotions such as anger—if it was real life - only three participants reported actually feeling anger during the VR experience. These findings suggest that, although the scenario was designed to provoke strong negative emotions if it was in real life, the majority of participants experienced only mild to moderate affective reactions, which indicates a degree of emotional distancing afforded by the VR setting. Indeed, participants are aware they are not in a real-life situation while immersed in VR during this experiment. This distinction makes the VR experiment particularly suitable for training and educational purposes, for which it was designed: individuals can learn to recognize and reflect on potential scenarios without being overwhelmed by the critical emotional states they might experience in reality. In this way, VR allows participants to reason through situations in a controlled environment, better preparing them for real-world encounters where emotions are more likely to arise. When asked "On a scale from 0 to 10, to what extent do you think the interaction with the client would have affected your internal state if it had occurred in real life?", participants reported a mean score of 4.0 (SD = 2.5). This distribution, illustrated in Figure 2.1, indicates a moderate perceived impact on

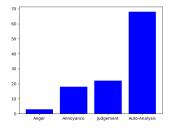


Fig. 2 This figure summarizes some of the answers to the questionnaire following the VR experiment: 3 of 87 participants (3.4 %) reported experiencing anger during the experiment, while 18 participants (20.7%) reported feelings of annoyance and 22 participants (25.3%) reported a sense of judgment toward the client; 68 declared to be used to analyze their emotions and internal psychological states.

internal state, with considerable variability across participants. It was not uniform but displayed a bimodal tendency, with clusters around low values (1-2) and higher values (6–8). This pattern suggests heterogeneity in participants' anticipated emotional reactivity, with some perceiving minimal potential impact and others anticipating a more substantial effect. However, it appears to be much more higher that the declared 3.4% of anger in the VR setting. The questionnaire further examined whether participants engaged in regular self-monitoring of their emotional states, a factor that may systematically influence response patterns; 68 participants (78 %) affirmed that they habitually reflect on their emotions. Finally, participants were asked to freely describe the thoughts, reasoning processes, and emotions that arose during the experiment. A selection of responses is presented below: "I kept my temper in front of the customer"; "The woman spoke badly to m"; "I felt empathy. I was worried about the customer's case. At the same time, I was annoyed by the customer's behavior because she was not kind, even though I suggested a solution"; "I have been in this situation many times. I am able to control my emotions in such contexts because I have worked in this field for more than 20 years"; "I felt stress at the beginning, but as soon as I understood her needs, I was able to manage the situation".

Our algorithm, when applied to the questionnaire responses, assigned each participant a valence score—0 indicating predominantly negative feelings and 1 indicating predominantly positive feelings regarding the experiment. Of the 87 participants, 60 (69.0%) were classified with a positive valence (1), while the remaining 27 (31.0%) were classified with a negative valence (0). See

2.2 EEG Analysis and Prediction of Valence from EEG

Next, we analyzed the EEG data from each participant and extracted spectral features within sub-time windows. These features were then used to train a K-Nearest Neighbors (KNN) algorithm to predict valence using 59 of the participants. The KNN algorithm predicted valence in the test set (25 participants) with an accuracy of 81.2% for valence = 1 and 66.7% for valence = 0. Figure 2.2 summarizes the results.

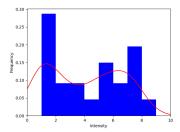


Fig. 3 This figure summarizes some of the answers to the questionnaire following the VR experiment: 3 of 87 participants (3.4 %) reported experiencing anger during the experiment, while 18 participants (20.7%) reported feelings of annoyance and 22 participants (25.3%) reported a sense of judgment toward the client; 68 declared to be used to analyze their emotions and internal psychological states.

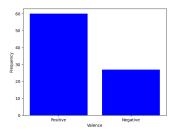


Fig. 4 Frequencies of positive and negative valences assigned by our algorithm to each participant's feelings from the responses to the questionnaire. Of the 87 participants, 60 (69.0%) were classified with a positive valence (1), while the remaining 27 (31.0%) were classified with a negative valence (0)

2.3 Mathematical Models

Figure 2.3 displays the preprocessed EEG time series of one participant (No. 85) alongside a spectral analysis of the same data. The extracted spectral features served as inputs to the prediction model. As a natural next step, we sought to reproduce relevant signals using mathematical models. For this purpose, we explored several approaches, including neural networks driven by Hodgkin–Huxley equations, a reduced stochastic model, reaction–diffusion models, and a forced Poisson equation. Figure 2.3 shows the simulation results of the reduced stochastic model, which is a stochastic differential equation derivated from a type FitzHugh-Nagumo model. The relevance of this model is that it models a switch between two attracting states which is observed in the raw data (not shown here). A more comprehensive study—covering parameter estimation and comparison with experimental data—will be presented in a forthcoming article. Additional details are provided in the Results and Discussion section.

3 Methods

Experiments were conducted using a Vive HTC virtual reality headset. Questionnaire responses were processed using a custom MATLAB implementation based on thresholding. For each response series, a scalar score was derived and subsequently classified

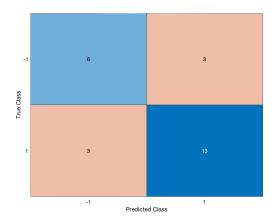


Fig. 5 The KNN algorithm predicted valence in the test set with an accuracy of 81.2% for valence = 1 and 66.7% for valence = 0.

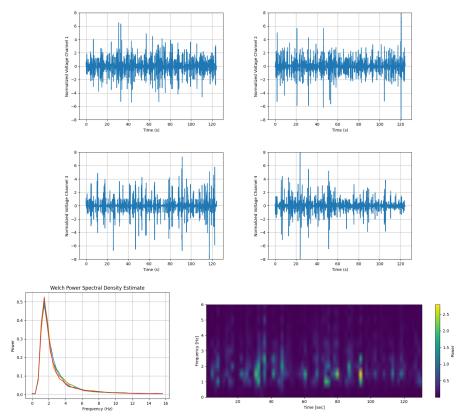


Fig. 6 EEG data recorded from participant No. 85. The top two rows display the preprocessed signals from four recording channels. The lower-left panel shows the corresponding power spectral densities (PSDs), and the lower-right panel presents the spectrogram of the first channel.

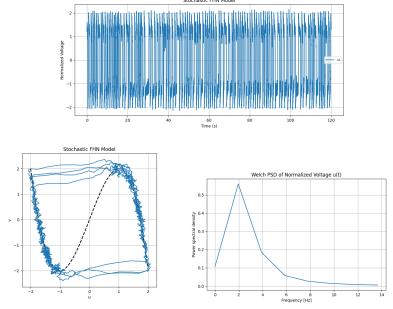


Fig. 7 Numerical simulation of a SDE of FHN type. Numerical simulation of the stochastic FitzHugh–Nagumo (FHN) system. The top panel shows the time series of the fast variable u. The lower-left panel depicts the corresponding phase-space trajectory over a five-second interval, while the lower-right panel presents the power spectral density (PSD) of the rescaled signal 1.6u.

as positive or negative according to a predefined cutoff criterion, yielding an operational measure of emotional state during the experiment. Electroencephalographic (EEG) signals were recorded with the OpenBCI Ganglion Board (4-channel, 200 Hz sampling rate), following the international 10–20 electrode placement system. Active electrodes were positioned at Fp1 and Fp2 (frontopolar sites) and T3 and T4 (temporal sites). Reference and ground electrodes were placed at A1 and A2 (left and right earlobes, respectively). The raw signals were digitized onboard and transmitted wirelessly to the acquisition computer. Preprocessing included band-pass filtering between 1–40 Hz (Butterworth, 4th order), notch filtering at 50 Hz to suppress line noise, and visual inspection to reject epochs contaminated by motion or muscle artifacts. Data were normalized prior to analysis. For spectral analysis, power spectral densities (PSDs) were estimated using Welch's method with 2-second Hamming windows, 75% overlap. For the classification of EEG signals, we employed the k-nearest neighbors (KNN) algorithm using the spectral features extracted from four EEG channels. The model was trained on data from 62 participants and subsequently evaluated on an independent set of 25 participants. Several mathematical models can be employed to reproduce signals with relevant spectral properties in EEG context. In this context, our own repertoire includes networks of excitatory and inhibitory Hodgkin-Huxley-type neurons [23], reaction-diffusion systems [24–31], and time-driven Poisson equations such as in [32]. The signals presented in the Results section, 2.3 were generated through simulations of the following equations:

$$\begin{cases}
\epsilon du_t = f(u_t) - v_t + \epsilon \sigma_1 dB_t^1 \\
dv_t = bu_t - v_t + \sigma_2 dB_t^2
\end{cases}$$
(1)

with $f(u) = -u(u - a^2)$, $a = \sqrt{3}$, b = 1.8, $\sigma_1 = \sigma_2 = 0.5$. With this parameters the underlying ordinary differential equation (ODE) of (1) has two stationary stable points $(\sqrt{a^2 - b}, b\sqrt{a^2 - b})$ and one unstable stationary point (0, 0). The two stationary stable points lie close to the local maxima points of the cubic $-u(u - a^2)$, on the stable manifold part. The Brownian terms make the trajectory jump from one part of the cubic to the other part. This phenomenon provides the observed frequency.

4 Discussion

In this study, we analyzed EEG data from a cohort of 87 participants exposed to a stressful work-related scenario presented through a virtual reality headset. Emotional valence during the task was assessed using both self-report questionnaires and a custom threshold-based algorithm that classified responses as positive or negative. Spectral features were extracted from the EEG recordings and subsequently used to train a k-nearest neighbors (KNN) classifier. The model was trained on data from 59 participants and evaluated on an independent test set of 25 participants, achieving an accuracy of 81.2% for positive valence (valence = 1) and 66.7% for negative valence (valence = 0). Beyond empirical analysis, we also explored mathematical models capable of reproducing signals with spectral characteristics comparable to those observed in the experimental data, thereby providing a framework for linking theoretical modeling with empirical findings. Two important remarks are necessary when interpreting these results. First, the VR-based scenarios, although effective in creating controlled experimental conditions, cannot fully replicate the affective dynamics of real-life situations. For example, the proportion of participants reporting anger was only 3.4%, which is considerably lower than would be expected in naturalistic contexts or even when viewing emotionally evocative film material [33]. This discrepancy likely reflects the original purpose of the VR scenarios, which were designed for employee training rather than affect induction, leading participants to adopt a more analytical than experiential stance during the task. Second, the EEG recordings were limited to four electrodes, which restricts the spatial resolution of the neural data. Despite these constraints, the classification algorithm performed robustly, achieving a good accuracy in distinguishing emotional valence, underscoring both the feasibility and the potential of combining EEG-based spectral features with machine learning for dynamic emotion recognition, see [34]. Our objective is to construct simplified mathematical models capable of reproducing signals indicative of neural electrical activity and corresponding emotional states. Future work will involve a systematic comparison between these simulated signals and empirical EEG recordings, as well as the integration and analysis of data acquired from EmotiBit devices.

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