Contents

Defining ARA Coordinate Systems

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1 Introduction

In this document, we define the ARA coordinate system, and describe the station-centric coordinate systems to be used in ARA. Right now I still have all positions in ft. and am still keeping a large number of digits. In the interest of getting this out now, these will be adjusted later. Also a bunch of references need to be added.

2 Background: Coordinate System used by South Pole Surveyers

2.1 Eastings and Northings

The South Pole surveyors quotes locations in Eastings and Northings, expressed in feet. These are the x- and y-coordinates of a Cartesian system with its origin at a spot in the ice near the former location of the Dome. The origin moves with respect to the Earth's surface because the ice moves at a rate of about 10m/year along a line at approximately 40° W longitude.

"Northing" is defined to be in the direction of Grid North, which is parallel to the Prime Meridian. Note that this is not the same thing as pointing "North," which is always pointing away from the South Pole. "Easting" is defined to be always in the direction of Grid East, which points parallel to the 90° longitude line. Again, this is not the same thing as "East," which points in a different direction depending which longitude you are sitting at.

At the origin of the SP surveyor's coordinate system, Eastings and Northings are not (0,0), but instead N 50,000' E50,000'. These offsets are there to avoid negative values for eastings and northings.

2.2 Ice Flow, and Where is the South Pole?

Across the Antarcitic continent, the ice is moving relative to the earth's surface. At the South Pole, the ice flows along a line near 40° W longitude by about 10 meters per year. In the 2011-2012 season, according to the surveyors, the ice flow was in the direction of N36° 46'23"W in the northings-easting coordinate system.

The fact that the ice moves relative to the earth's surface means that the position of the South Pole changed each year with respect to the control point and benchmarks in the ice. On January 1, 2011, the South Pole was at 49722.88'E, 50530.11'N. On January 1, 2012, the South Pole was at N 50504.86 E 49744.05.

2.3 Geoid and Ice Thicknesses

The Earth's surface to a close approximation follows a "geiod" shape. The model used for most GPS systems is WGS84. It describes the earth as a spheroid with equatorial radius $a = 6378137$ and flattening $f = 1/298.257223560$. Then the derived polar radius is given by $c = a(1-f) = 6356752.3$.

If θ is geocentric latitude (angle of a line from the center of the earth to a point on the spheroid, relative to the equatorial plane), then:

$$r(\theta) = \left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{c^2} \right)^{-1/2}. \tag{1}$$

The geographic latitude θ_g (the one used for maps) is actually the angle of a line perpendicular to the spheroid surface, relative to the equatorial plane. Note that the line perpendicular to the spheroid surface does not in general pass through the center of the earth. It is given by the relation:

$$\tan\theta = \frac{c^2}{a^2}\tan\theta_g = (1-f)^2\tan\theta_g \tag{2}$$

According to Bedmap, the ice thickness at the SP is $I_{\mathrm{SP}} = 2646.28$ m.

2.4 Elevation

The South Pole surveyors report "elevation" in feet relative to a benchmark in the ice which is different from the "control point" used for easting and northing. The benchmark was moved in the 2010-2011 season when the dome was removed. Previous to the 2010-2011 season it was called the NCEL benchmark, established by the US Navy. The new benchmark is called the SPBM (South Pole BenchMark).

The SPBM is at N 51491.62 E 50592.62. The Lat./Long. for SPBM on Jan 1, 2012 was Lat. 89-59-47.22 S, Long. 40-41-38 E. Expressed in decimal notation, these are 89.99645°S, 40.69389°E. Latitude is geographical. Being so close to the SP, the geocentric latitude is nearly identical at 89.99643°. Bedmap shows this location to have a distance to the center of the earth of 6359420 m and the ice thickness at 2695.28 m.

The benchmark sinks by about 0.6 ft. each year as the snow packs. Therefore, the conversion from SPBM to MSL is different each year.

$$\mathrm{Elevation}_{\mathrm{MSL}} = \mathrm{Elevation}_{\mathrm{NCEL/SPBM}} + \mathrm{offset} \tag{3}$$

In the 2010/2011 season, the offset was 9264.37 ft. In the 2011/2012 season, the offset was 9263.98 ft. Notice that in the 2011-2012 season, the benchmark sank less than usual. Note also that this offset is not time dependent anymore once you get into deep ice, it is only while you are in the firn.

Kurt Skoog says, "The SPBM benchmark is a 70 foot long piece of pipe. The top of the pipe, which is where we take our measurements from is near the surface; but because the bottom of the pipe is 70 feet below the surface, the vertical movement of it is controlled by that level. At the time the benchmark was established, four years ago, the top was about two feet above the surface."

2.5 Effect of the Earth's Curvature

It may be important to keep in mind the size of the deviation of this Cartesian coordinate system from the Earth's curved surface as the size of our detector grows. If your origin sits at the edge of a detector of size d, the maximum radial distance between the Easting-Northing plane and a spherical Earth surface (assumed for this purpose only) becomes $x = d^2/2R$. For an IceCube size, 1 km scale detector, $x = 8$ cm. For an ARA 37 of 10 km scale, $x = 8$ m. For ARA 1000, it would be $x = 800$ m. This isn't necessarily a problem, just something to keep in mind.

2.6 The IceCube Coordinate System

The following statements summarize the coordinate system used by the IceCube experiment.

`The origin of the IceCube coordinate system shall be located at 46500'E, 52200'N, at an elevation of 2900 ft.`

This origin was chosen because it is near the geometric center of the IceCube detector, rounded to the nearest hundreds of feet. IceCube elevations (including the one in the preceding statement) are relative to MSL. IceCube defines $\hat{x}$ along Grid East, $\hat{y}$ along Grid North, and $\hat{z}$ as "up," normal to the Earth's surface. Over the IceCube distance scales, I guess any variations in "up" across the detector are not important to them, whereas it will be important for ARA.

3 Philosophy

We define an ARA "global" coordinate system as well as station-centric coordinate system for each station.

Like IceCube, ARA should use an ARA global coordinate system set in the ice that moves with the ice flow. We will, however, use a different origin from IceCube, and near the center of the planned ARA 37, rounded to the nearest hundreds of feet. This choice of origin is "success-oriented." The origin will be near the surface at that point, according to ice surfaces in Bedmap. We define $\hat{e}$ and $\hat{n}$ along the Grid East and Grid North directions respectively, and $\hat{u} = \hat{e} \times \hat{n}$. Note that in general $\hat{u}$ is going to be different from both "up" and "perpendicular to the surface."

For our station-centric coordinate systems, we call the axes $\hat{x}$, $\hat{y}$ and $\hat{z}$, with the $x - y$ plane tangent to the geoid at the time the station was deployed, with the $+\hat{x}$ approximately along the direction of the ice flow. Then $\hat{z} = \hat{x} \times \hat{y}$ is perpendicular to the geoid surface. This coordinate system is designed to be useful for ray tracing. Note, however, that the orientation of the geoid surface will change slightly with time, since the longitude and latitude of the station will be moving with the ice. The direction of the ice flow may change with time as well. However, the orientation of our station-centric coordinate system will remain fixed with respect to the ARA global coordinate system. For now, the origin of the station-centric coordinate system will be the southeast corner of the DAQ box on the as-built surveyor drawings. In the future, we can use the location of the station GPS once this is surveyed.

4 ARA Global Coordinate System

The southeast corner of the ARA1 DAQ box at deployment was at 51064.63' N, 38801.71' E (keeping two decimal places). The center of ARA37 is planned for -5 km easting and $\sqrt{3}/2$ km northing from ARA1. This means that the center of ARA37 should be at approximately 53907.39'N, 22399.60'E.

We wish to find the location near the ice surface at this point Using the Jan 1, 2012 position of the SP, the ARA origin is 53907.39-50504.86=3402.53' along the northing direction and 22399.60-49744.05= -27344.45' along the easting direction with respect to the SP. This is a distance of 8377.884 m from the SP in the Cartesian easting-northing plane. So the origin sits at a longitude of

$$\tan^{-1}\left(-27344.45/3402.53\right) = -82.92216^{\circ}, \tag{4}$$

which is 82.92216° W

SP Positions in ARA Global Coordinates			
All distances in ft.			
Season	$\hat{e}$	$\hat{n}$	$\hat{u}$
2010/2011	27344.05	-3395.14	-35.63
2011/2012	27322.88	-3369.89	-36.02

Table 1: Location of the ARA1 holes at the surface as surveyed during deployment.

We use Bedmap to find the latitude at which the surface of the ice sits at this distance (8377.884 m) from the SP in the easting-northing plane for this longitude[1] This occurs at a geographic latitude of $\mathrm{Lat_g} = -89.92525°$, which we translate to a geocentric latitude of $\mathrm{Lat} = -89.92474°$.

For a sanity check, we compare this to a geometrical estimate. If the distance from the SP in easting and northing is given by D, then the geocentric latitude of our origin on Jan 1, 2012 was approximately

$$\theta = \tan^{-1}\left[(c + I_{\mathrm{SP}})/D\right] = 89.92452°\mathrm{S}. \tag{5}$$

The geographic latitude is then:

$$\theta_g = \tan^{-1}\left[\frac{c + I_{\mathrm{SP}}}{D}\frac{a^2}{c^2}\right] = 89.925°\mathrm{S}. \tag{6}$$

At the geographic longitude and latitude obtained from Bedmap, it shows a distance from the earth's center to the ice surface of 6359440 m, with an ice thickness of 2689 m. We estimate the elevation of the surface relative to SPBM to be:

$$\mathrm{elev_{origin}} = \mathrm{r_{origin}}\sin\left(\mathrm{Lat_{origin}}\right) - \mathrm{r_{SPBM}}\sin\left(\mathrm{Lat_{SPBM}}\right) = 14.563981\ \mathrm{m} = 47.78\ \mathrm{ft.} \tag{7}$$

where remember that Lat is geocentric latitude for both, and r_{origin} and r_{SPBM} and the distance from the center of the earth to the ice surface at the ARA origin and the SPBM, according to Bedmap.

This is 47.78 ft.+9263.98 ft.=9312 ft. MSL for the 2011-2012 season.

Rounding to the nearest hundred feet, we find:

`The ARA global coordinate system will be at E 22400' N 53900' and 9300' MSL.`

4.1 Notable Landmarks in ARA Global Coordinates

Using the annual South Pole locations in Surveyor's Coordinates as in Section 2.2, we find the same locations in ARA global coordinates in Table 1.

`The location of the South Pole Telescope in ARA Global Coordinates in the 2011/2012 season was:` 24774.18$\hat{e}$ -871.35$\hat{n}$.

The location of the corners of the ICL building are listed in Table 2.

[1] A previous version wrongly had the longitude of the ARA origin at -84.17254, so the origin of the ARA location may have been derived from that instead.

ICL 2011/2012 Position in ARA Global Coordinates All distances in ft.		
Season	$\hat{e}$	$\hat{n}$
NW corner	24011.58	-1702.53
NE corner	24060.51	-1723.09
SE corner	24049.7	-1749.02
SW corner	24000.78	-1728.47

Table 2: Location of the ARA1 holes at the surface as surveyed during deployment.

4.2 Conversion from the Surveyer's Coordinate System to the ARA Global Coordinate System

To convert from a position $\vec{r}_{\text{surv}}$ quoted by the South Pole surveyor's in eastings, nothings and elevation to a position in the ARA global coordinate system, you use the following conversion:

$$\vec{r}_{\text{ARA}} = \vec{r}_{\text{Surv}} - \vec{t}_{\text{Surv}} \tag{8}$$

where

$$\vec{t}_{\text{Surv}} = 22400'\hat{e} + 53900'\hat{n} - Z(t)\hat{u} \tag{9}$$

where $Z(t)$ is an elevation offset which is time dependent due to the sinking of the SPBM. For the 2010-2011 season, $Z = 9300 - 9264.37 = 35.63$ ft. and for the 2011-2012 season, $Z = 9300 - 9263.98 = 36.02$ ft.

4.3 Conversion from the IceCube Coordinate system to the ARA Global Coordinate System

To convert from the IceCube Coordinate system to the ARA Global Coordinate System you use the following simple transformation:

$$\vec{r}_{\text{ARA}} = \vec{r}_{\text{IceCube}} - \vec{t}_{\text{IceCube}} \tag{10}$$

where

$$\vec{t}_{\text{IceCube}} = -24100'\hat{e} + 1700'\hat{n} + 6400'\hat{u} \tag{11}$$

Both coordinate systems use the same axis directions and both move with ice flow.

5 ARA1 Station-Centric Coordinate System

As noted above, the southeast corner of the ARA1 DAQ box at deployment was at 51064.63' N, 38801.71' E and 10.35' elevation relative to the SPBM, rounding to 2 decimal places.

We need to find the latitude of the station origin in order to know the orientation of the geoid. The distance in northing and easting from the SP is: N 51064.63-50504.86=559.77' =170.618 m, E 38801.71-49744.05=-10942.3'=-3335.21 m. The longitude is then $\phi = \tan^{-1} -10942.3/559.77 = -87.07151°$E. Then we geometrically estimate the geocentric latitude to be:

$$\theta = \tan^{-1}\left[(c + I_{\text{SP}})/D\right] = 89.99440°\text{S}. \tag{12}$$

The geographic latitude is then:

$$\ell_g = \tan^{-1}\left[\frac{c + I_{\text{SP}}}{D}\frac{a^2}{c^2}\right] = 89.97011°\text{S}. \tag{13}$$

So, the $\hat{x} - \hat{y}$ plane of the ARA1 coordinate system will make an angle of 89.97011° with the SP.

We would like the $+\hat{x}$ axis to point approximately in the direction of the ice flow and tangent to the geoid. Its direction will be the same as the line defined by the intersection of two planes: the plane the runs through the longitude line that points in the direction of the ice flow, and the plane tangent to the geoid at the ARA1 origin. If we find vectors perpendicular to each of the planes, which we will call $\hat{x}_1$ and $\hat{z}$, then the $+\hat{x}$ direction is the cross product $\hat{x}_1 \times \hat{z}$.

Let's call the ice flow longitude α. We want the find a vector perpendicular to the plane that contains the α longitude line. At the SP, α longitude is in the direction:

$$\hat{a} = (\sin\alpha)\,\hat{e} + (\cos\alpha)\,\hat{n} \tag{14}$$

Then the vector $\hat{x_1} = \hat{u} \times \hat{a}$ is perpendicular to the α longitude plane:

$$\hat{x_1} = -(\cos\alpha)\,\hat{e} + (\sin\alpha)\,\hat{n} \tag{15}$$

If an ARA station is at ℓ_g geographic latitude and ϕ longitude, then the vector pointing perpendicular to the geoid surface (and the station $\hat{z}$ axis) is:

$$\hat{z} = \sin\ell_g\hat{u} + \cos\ell_g\left(\sin\phi\hat{e} + \cos\phi\hat{n}\right) \tag{16}$$

Then the vector lying in the α longitude plane and tangent to the geoid surface (and thus the station $\hat{x}$ axis) is

$$\hat{x} = \hat{x_1} \times \hat{x_2} = (\sin\alpha\sin\ell_g)\,\hat{e} + (\cos\alpha\sin\ell_g)\,\hat{n} - \cos\ell_g\cos(\alpha - \phi)\,\hat{u} \tag{17}$$

Then, taking $\hat{z} \times \hat{x}$, we find our $\hat{y}$ axis:

$$\begin{aligned}
\hat{y} \;=\; & \left[-\cos^2\ell_g\cos\phi\cos(\alpha-\phi) - \sin^2\ell_g\cos\alpha\right]\hat{x} & (18)\\
& + \left[-\cos^2\ell_g\sin\phi\cos(\alpha-\phi) - \sin^2\ell_g\sin\alpha\right]\hat{y} & (19)\\
& + \left[\sin\ell_g\cos\ell_g\sin\phi - \alpha\right]\hat{z} & (20)
\end{aligned}$$

So, for the geographic latitude of 89.96971° S, longitude -80.4725°E and ice flow direction N36°46'23"W=36.77306°W, find the direction of the $\hat{x}$ axis of the ARA1 coordinate system, in the ARA global coordinate system to be:

$$\begin{aligned}
\hat{x} &= -0.5986468915\hat{e}0.8010128758\hat{n} - 0.000333208\hat{u} & (21)\\
\hat{y} &= -0.801013\hat{e} - 0.598647\hat{n} - 0.000401329\hat{u} & (22)\\
\hat{z} &= -0.000520943\hat{e} + 0.000026495\hat{n} + 1.\hat{u} & (23)
\end{aligned}$$

To take the position of the ARA1 origin in our global coordinate system to be the southeast corner of the DAQ box which is at E 38801.71', N 51064.63', and 10.35' elevation rel. to SPBM. Using the 2011/2012 conversion from SPBM to MSL, we find this elevation to be at 9274.33 ft. MSL. Subtracting the location of the ARA origin from these numbers, we find the position of the ARA1 coordinate system in the ARA global coordinate system:

`The position of the ARA1 coordinate system, in the ARA global coordinate system, is E 16401.71', N -2835.37', -25.67' elevation.`

To convert from the ARA global coordinates to ARA1 station coordinates, you need both a translation and a rotation. First, you translate by doing:

$$\vec{r}_{\mathrm{ARA1}} = \vec{r}_{\mathrm{global}} - \vec{t}_{\mathrm{ARA1}} \tag{24}$$

where

$$\vec{t}_{ARA1} = 16401.71'\hat{e} - 2835.37'\hat{n} - 25.67'\hat{u}. \tag{25}$$

Next you need to perform a rotation, given by:

$$R = \begin{pmatrix} -0.598647 & 0.801013 & -0.000332979 \\ -0.801013 & -0.598647 & -0.000401329 \\ -0.000520806 & 0.0000264661 & 1. \end{pmatrix}$$

To calculate R, first we found the three rotations, r_0, r_1, r_2, that take the $\hat{e}$ axis to the $\hat{x}$ axis. The first rotation, r_0, rotates the $\hat{e}$ axis about the $\hat{u}$ axis so that is aligned along the $\cos(\ell_g + 90°)\hat{e} - \sin(\ell_g + 90°)\hat{n}$ direction. Then, r_1 rotates it by $90° - \ell_g$ about the $\sin(\ell_g + 90°)\hat{e} + \cos(\ell_g + 90°)\hat{n}$ direction so that now at the longitude and latitude of ARA1, the vector sits tangent to the geoid. Now we need to rotate it so that it is aligned with the $\hat{x}$ direction as in Equation 17. First, we find the angle β between the vector $r_1 r_0 \hat{e}$ and $\hat{x}$ and then the final rotation r_2 is about the $\hat{z}$ direction (perpendicular to the geoid) in Equation 16 by the angle β. I have checked that $\hat{x} = r_2 r_1 r_0 \hat{e}$, $\hat{y} = r_2 r_1 r_0 \hat{n}$ and $\hat{z} = r_2 r_1 r_0 \hat{u}$. Then, R is the inverse of the product $r_2 r_1 r_0$.

So, the transformation from ARA global coordinates to ARA1 coordinates is given by:

$$\vec{r}_{\mathrm{ARA1}} = R\left(\vec{r}_{\mathrm{global}} - \vec{t}_{\mathrm{ARA1}}\right) \tag{26}$$

5.1 ARA1 Antenna Positions

Table 3 shows the location of the ARA1 holes at the surface as surveyed at deployment. Table 4 shows the depth of each antenna in inches below the surface deduced from measurements made at deployment. This numbers came from Christian Miki. Table 5 lists the positions of the ARA1 antenna feeds in the ARA global coordinate system. Table 9 lists the positions of the ARA1 borehole antenna feeds in the ARA1 local coordinate system, using the transformation described in Equation 38. For the ARA1 surface antennas, the positions are that of the end of the top-most tube that had the smallest easting value. The positions in both coordinate systems will change with time due to settling in the firn, and any ice stretching and straining. However, since the coordinate systems move with the ice drift (just as the IceCube coordinate system does), the positions will not change due to ice flow, as long the local ice drift is parallel to the ice drift at the origin of the surveyor's coordinate system.

For the surface antennas, Table 9 gives the positions in ARA1 coordinates of the end of the top-most tube that had the smallest easting value. Sebastian said, "Each antenna consists of 6 tubes that are arranged in a hexagon (and joined together at the feedpoint). If this were deployed perfectly flat, two tubes would be at the same height, but things are rarely "perfect" at the pole. The surveyors judged by eye which of the tubes is the highest, and held their stick on the end of that one." The magnitude of the vectors in Table 12 give the length of the top-most tube and the direction gives the orientation of the tube and thus the antenna.

Table 12 gives the distance vector from one end to the other of the top-most tube on each of the surface antennas, both in surveyor's coordinates or ARA global coordinates (they have the same axes) and then also in ARA1 coordinates.

	Surveyer Coordinates			
	# on Survey Map	Eastings (ft.)	Northings (ft.)	elev. rel. to SPBM (ft.)
Hole 1:	121	38789.83154	51039.22553	13.14117
Hole 2:	114	38781.73567	51084.75208	12.92174
Hole 3:	124	38834.86521	51049.48963	12.70895
Hole 4:	108	38825.69905	51091.61495	12.98194
Hole Pulser 1:	118	38677.94350	51039.36389	12.77118
Hole Pulser 2:	111	38777.92027	51196.69942	13.05694

Table 3: Location of the ARA1 holes at the surface as surveyed during deployment.

	Antenna Type	Depth from surface (inches)
Hole 1:	TH	2464
	TV	2545
	BH	3154
	BV	3288
Hole 2:	TH	2143
	TV	2224
	BH	2792
	BV	2928
Hole 3:	TH	2104
	TV	2185
	BH	2792
	BV	2928
Hole 4:	TH	2383
	TV	2464
	BH	3032
	BV	3156
Pulsers:	P1	1452
	P2	1692

Table 4: Depth of the ARA1 antenna feeds from the surface above them, from Christian Miki.

		ARA Global Coordinates of ARA1 Station		
		$\hat{e}$ (ft.)	$\hat{n}$ (ft.)	$\hat{u}$ (ft.)
Surface	1	16219.7	-2734.23	-21.9785
	2	16581.2	-2751.59	-22.9535
	3	16394.6	-3029.28	-23.5252
	4	16407.9	-2861.97	-22.5693
Hole 1:	TH	16389.93793	-2860.792206	-228.212135
	TV	16389.94143	-2860.792789	-234.9621341
	BH	16389.96772	-2860.797173	-285.7121271
	BV	16389.97351	-2860.798137	-296.8787922
Hole 2:	TH	16381.82820	-2815.263346	-201.6815687
	TV	16381.83170	-2815.263929	-208.4315678
	BH	16381.85622	-2815.268017	-255.7648946
	BV	16381.86209	-2815.268996	-267.0982263
Hole 3:	TH	16434.95605	-2850.525515	-198.6443591
	TV	16434.95955	-2850.526098	-205.3943582
	BH	16434.98576	-2850.530467	-255.9776846
	BV	16434.99163	-2850.531446	-267.3110163
Hole 4:	TH	16425.80194	-2808.402203	-221.6213659
	TV	16425.80544	-2808.402786	-228.371365
	BH	16425.82996	-2808.406875	-275.7046918
	BV	16425.83532	-2808.407767	-286.0380237
Pulsers:	P1	16278.00619	-2860.646562	-144.2488033
	P2	16377.99333	-2703.312759	-163.9630405

Table 5: Location of the ARA1 antennas in the ARA global coordinate system.

6 ARA2 Station-Centric Coordinate System

`The position of the ARA2 coordinate system, in the ARA global coordinate system, is E 13126.70', N -8519.62', -18.72' elevation.`

The axes of the ARA2-centric coordinate system are:

$$\hat{x} = -0.5986437007\hat{e} + 0.8010086064\hat{n} - 0.000970503\hat{u} \quad (27)$$
$$\hat{y} = -0.801013\hat{e} - 0.598647\hat{n} - 0.000401329\hat{u} \quad (28)$$
$$\hat{z} = -0.00311277\hat{e} - 0.00111477\hat{n} + 0.999994533955\hat{u} \quad (29)$$

To convert from the ARA global coordinates to ARA2 station coordinates, you need both a translation and a rotation. First, you translate by doing:

$$\vec{r}_{\mathrm{ARA2}} = \vec{r}_{\mathrm{global}} - \vec{t}_{\mathrm{ARA2}} \quad (30)$$

where

$$\vec{t}_{ARA2} = 13126.70'\hat{e} - 8519.62'\hat{n} - 18.72'\hat{u}. \quad (31)$$

Next you need to perform a rotation, given by:

$$R = \begin{pmatrix} -0.598647 & 0.801013 & -0.000970507 \\ -0.801007 & -0.598646 & -0.00316072 \\ -0.00311277 & -0.00111477 & 0.999995 \end{pmatrix}$$

So, the transformation from ARA global coordinates to ARA2 coordinates is given by:

$$\vec{r}_{\mathrm{ARA2}} = R\left(\vec{r}_{\mathrm{global}} - \vec{t}_{\mathrm{ARA2}}\right) \quad (32)$$

ARA2 antenna positions in ARA Global Coordinates are in Table 6.

7 ARA3 Station-Centric Coordinate System

`The position of the ARA3 coordinate system, in the ARA global coordinate system, is E 9848.35', N -2835.19', -12.70' elevation.`

The axes of the ARA3-centric coordinate system are:

$$\hat{x} = -0.59864439636\hat{e} + 0.8010089581\hat{n} - 0.00198193\hat{u} \quad (33)$$
$$\hat{y} = -0.801005\hat{e} - 0.598647\hat{n} - 0.00247503\hat{u} \quad (34)$$
$$\hat{z} = -0.00316902\hat{e} + 0.000105871\hat{n} + 0.999994973049\hat{u} \quad (35)$$

To convert from the ARA global coordinates to ARA2 station coordinates, you need both a translation and a rotation. First, you translate by doing:

$$\vec{r}_{\mathrm{ARA3}} = \vec{r}_{\mathrm{global}} - \vec{t}_{\mathrm{ARA3}} \quad (36)$$

where

$$\vec{t}_{ARA3} = 9848.35'\hat{e} - 2835.19'\hat{n} - 12.70'\hat{u}. \quad (37)$$

Next you need to perform a rotation, given by:

		ARA Global Coordinates of ARA2 Station in 2012-2013		
		$\hat{e}$ (ft.)	$\hat{n}$ (ft.)	$\hat{u}$ (ft.)
Hole 1:	BV	13101.7	-8495.71	-640.493
	BH	13101.7	-8495.72	-630.793
	TV	13101.5	-8495.77	-582.833
	TH	13101.5	-8495.79	-568.284
Hole 2:	BV	13146.5	-8485.86	-640.473
	BH	13146.4	-8485.87	-629.696
	TV	13146.3	-8485.93	-577.964
	TH	13146.2	-8485.94	-568.264
Hole 3:	BV	13111.1	-8543.03	-640.311
	BH	13111.	-8543.04	-630.072
	TV	13110.9	-8543.1	-577.801
	TH	13110.9	-8543.11	-567.562
Hole 4:	BV	13155.5	-8530.45	-640.555
	BH	13155.5	-8530.46	-629.778
	TV	13155.4	-8530.52	-578.585
	TH	13155.3	-8530.53	-568.885
Hole 5:	V	13101.7	-8383.93	-651.601
	H	13101.7	-8383.94	-640.823
Hole 6:	V	13000.7	-8541.77	-570.115
	H	13000.7	-8541.78	-559.337

Table 6: Location of the ARA2 antennas in the ARA global coordinate system.

$$R = \begin{pmatrix} -0.598646 & 0.801011 & -0.00198193 \\ -0.801008 & -0.598649 & -0.00247504 \\ -0.00316902 & 0.000105871 & 0.999995 \end{pmatrix}$$

So, the transformation from ARA global coordinates to ARA3 coordinates is given by:

$$\vec{r}_{\text{ARA3}} = R\left(\vec{r}_{\text{global}} - \vec{t}_{\text{ARA3}}\right) \tag{38}$$

ARA3 antenna positions in ARA Global Coordinates are in Table 7.

8 Testbed Station-Centric Coordinate System

The position of the Testbed coordinate system, in the ARA global coordinate system, is E 17246.69’, N -540.82’, -21.86’ elevation.

8.1 Testbed Positions

Testbed positions in ARA Global Coordinates are in Table 8.

	ARA Global Coordinates of ARA3 Station in 2012-2013		
	$\hat{e}$ (ft.)	$\hat{n}$ (ft.)	$\hat{u}$ (ft.)
Hole 1: BV	9866.37	-2805.22	-644.029
BH	9866.34	-2805.21	-633.359
TV	9866.17	-2805.21	-581.52
TH	9866.14	-2805.21	-571.281
Hole 2: BV	9820.13	-2814.05	-644.998
BH	9820.1	-2814.05	-635.299
TV	9819.94	-2814.04	-583.567
TH	9819.91	-2814.04	-573.867
Hole 3: BV	9829.57	-2859.03	-644.858
BH	9829.54	-2859.02	-635.159
TV	9829.37	-2859.02	-583.965
TH	9829.36	-2859.02	-579.116
Hole 4: BV	9876.01	-2849.25	-644.499
BH	9875.98	-2849.25	-634.26
TV	9875.81	-2849.25	-581.989
TH	9875.78	-2849.25	-572.29
Hole 5: BV	9818.86	-2703.17	-663.01
BH	9818.83	-2703.17	-652.23
Hole 6: BH	9719.46	-2859.59	-629.94
BH	9719.42	-2859.58	-619.16

Table 7: Location of the ARA3 antennas in the ARA global coordinate system.

9 Conclusions

Enjoy!

10 Acknowledgements

Many thanks to Kurt Skoog and Kurt Woschnagg for supplying a wealth of information on South Pole surveying.

		ARA Global Coordinates of Testbed Station in 2011-2012		
		$\hat{e}$ (ft.)	$\hat{n}$ (ft.)	$\hat{u}$ (ft.)
Near- Surface	1 V	17256.16055	-543.9032991	-25.83655145
	1 H	17256.16225	-543.9035825	-29.11739089
	2 H	17244.39079	-524.5402321	-25.77714532
	3 V	17233.59802	-544.1282521	-25.76935976
Hole 1:	TH	17274.83263	-554.3001215	-89.09124119
	TV	17274.84113	-554.3015384	-105.4954384
Hole 2:	TH	17276.80643	-520.0848685	-112.0912754
	TV	17276.79793	-520.0834515	-95.68707814
Hole 3:	TH	17244.78163	-504.5062551	-96.43294865
	TV	17244.79013	-504.5076721	-112.8371459
Hole 5:	TH	17213.44901	-553.3273649	-122.1288519
	TV	17213.44051	-553.3259479	-105.7246547
Hole 6:	TH	17254.42644	-572.2516154	-121.6216164
	TV	17254.41964	-572.2504818	-108.4982586
Cal 1:	H	17245.22083	-636.7340319	-79.25777648
	V	17245.22012	-636.7340319	-95.66197369
Cal 2:	H	17245.21846	-636.7340318	-134.1106907
	V	17245.21917	-636.7340319	-117.7064935
Cal 3:	H	17245.22315	-636.7340319	-25.60717082
	V	17245.22315	-636.7340319	-25.60588471
Wind Turbine #1		46166.49	52159.43	
Wind Turbine #2		22546.95	-2594.03	
Wind Turbine #3		18662.31	-3552.64	

Table 8: Location of the Testbed antennas in the ARA global coordinate system.

		ARA1 Coordinates		
		$\hat{x}$ (ft.)	$\hat{y}$ (ft.)	$\hat{z}$ (ft.)
Surface	1	189.866	85.134	3.79378
	2	-40.41	-193.99	2.63145
	3	-151.128	121.72	2.13181
	4	-25.1154	10.8557	3.09518
Hole 1:	TH	-13.2399	24.7215	-202.538
	TV	-13.2399	24.7215	-209.288
	BH	-13.24	24.7213	-260.038
	BV	-13.24	24.7213	-271.205
Hole 2:	TH	28.0742	3.95222	-176.
	TV	28.0742	3.9522	-182.75
	BH	28.0741	3.95206	-230.083
	BV	28.074	3.95203	-241.416
Hole 3:	TH	-31.9772	-17.4954	-172.993
	TV	-31.9772	-17.4954	-179.743
	BH	-31.9774	-17.4955	-230.326
	BV	-31.9774	-17.4956	-241.659
Hole 4:	TH	7.25284	-35.3715	-195.961
	TV	7.25282	-35.3716	-202.711
	BH	7.25271	-35.3717	-250.045
	BV	7.25268	-35.3717	-260.378
Pulsers:	P1	53.8527	114.263	-118.517
	P2	120.03	-60.0085	-138.26

Table 9: Location of the ARA1 antennas in the ARA1 coordinate system. For the surface antennas, the coordinates given are those of the end of the uppermost rod with the smallest easting value.

		ARA2 Coordinates		
		$\hat{x}$ (ft.)	$\hat{y}$ (ft.)	$\hat{z}$ (ft.)
Hole 1:	BV	34.7348	7.68624	-621.718
	BH	34.7349	7.68644	-612.019
	TV	34.7352	7.68746	-564.059
	TH	34.7353	7.68777	-549.509
Hole 2:	BV	15.8246	-34.0692	-621.849
	BH	15.8247	-34.069	-611.071
	TV	15.825	-34.0679	-559.339
	TH	15.8251	-34.0677	-549.639
Hole 3:	BV	-8.79942	28.4907	-621.512
	BH	-8.79936	28.4909	-611.274
	TV	-8.79901	28.4921	-559.003
	TH	-8.79895	28.4923	-548.764
	BV	-25.3388	-14.6568	-621.91
Hole 4:	BH	-25.3388	-14.6566	-611.132
	TV	-25.3384	-14.6555	-559.939
	TH	-25.3384	-14.6553	-550.239
Hole 5:	V	124.259	-59.2214	-632.951
	H	124.259	-59.2211	-622.173
Hole 6:	V	58.2267	115.93	-550.975
	H	58.2268	115.93	-540.197

Table 10: Location of the ARA2 antennas in the ARA2 coordinate system.

		ARA3 Coordinates		
		$\hat{x}$ (ft.)	$\hat{y}$ (ft.)	$\hat{z}$ (ft.)
Hole 1:	BV	14.4718	-30.8184	-631.38
	BH	14.4719	-30.8183	-620.71
	TV	14.4726	-30.8174	-568.87
	TH	14.4728	-30.8172	-558.631
Hole 2:	BV	35.0771	11.511	-632.204
	BH	35.0772	11.5112	-622.504
	TV	35.0779	11.512	-570.772
	TH	35.078	11.5122	-561.072
Hole 3:	BV	-6.59522	30.879	-632.098
	BH	-6.59509	30.8792	-622.398
	TV	-6.5944	30.8801	-571.205
	TH	-6.59434	30.8801	-566.355
Hole 4:	BV	-26.5719	-12.1721	-631.885
	BH	-26.5718	-12.1719	-621.646
	TV	-26.5711	-12.171	-569.375
	TH	-26.571	-12.1709	-559.675
Hole 5:	BV	124.689	-53.8016	-650.198
	BH	124.689	-53.8014	-639.421
Hole 6:	BV	58.8426	119.374	-616.827
	BH	58.8427	119.375	-606.05

Table 11: Location of the ARA3 antennas in the ARA3 coordinate system.

	ARA1 Surface Antenna Orientations		
	$\hat{e}$ (ft.)	$\hat{n}$ (ft.)	$\hat{u}$ (ft.)
Surveyor or ARA global coordinates	9.68975	7.41258	-0.33599
	11.9147	2.81338	0.60837
	11.9517	2.36813	0.18344
	12.094	2.98601	0.13911
	$\hat{x}$ (ft.)	$\hat{y}$ (ft.)	$\hat{z}$ (ft.)
ARA1 coordinates	0.13696	-12.199	-0.340335
	-4.87935	-11.2282	0.602485
	-5.25802	-10.9912	0.177497
	-4.84826	-11.4751	0.133147

Table 12: The table gives the vector that points from one end of the top-most tube to the other. The vector was chosen to begin at the end of the tube with the smallest easting and it has a length equal to that of the tube. Its direction gives the orientation of the tube and thus the antenna. The top half of the table gives this distance vector for the ARA1 surface antennas in the surveyor's or ARA global coordinate system (which have the same axes). The second half of the table gives the same vector for each antenna in the ARA1 coordinate system.