Oblig 6 - FYS2140

Joakim Flatby

1.

a) $$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx} + \frac{1}{2}m\omega^2x^2\psi$$

$$a_+ = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$$

Momentum- og posisjon-operatoren i én dimensjon:

$$\hat{x} = x \qquad \hat{p} = \frac{\hbar}{i}\frac{d}{dx}$$

$$\Downarrow$$

$$a_+ = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{\hbar}{m\omega}\frac{d}{dx}\right)$$

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\psi_1 = A_1 a_+ \psi_0 \qquad \Big|\ A_n = \frac{1}{\sqrt{n!}} \Rightarrow A_1 = 1$$

$$\psi_1 = a_+ \psi_0$$

$$\psi_1 = \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{\hbar}{m\omega} \frac{d}{dx} \right) \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \sqrt{\frac{m\omega}{2\hbar}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(x\, e^{-\frac{m\omega}{2\hbar}x^2} - \frac{\hbar}{m\omega} \frac{d\left(e^{-\frac{m\omega}{2\hbar}x^2}\right)}{dx} \right)$$

$$= \sqrt{\frac{m\omega}{2\hbar}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(x - \frac{\cancel{\hbar}}{\cancel{m\omega}} \left(-\cancel{2} \frac{\cancel{m\omega}}{\cancel{2}\cancel{\hbar}} x \right) \right) e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \sqrt{\frac{m\omega}{2\hbar}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} 2x\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \sqrt{\frac{2m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} x\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\psi_2 = \frac{1}{\sqrt{2!}} a_+ \psi_1$$

$$= \frac{1}{\sqrt{2}} \sqrt{\frac{m\omega}{\cancel{2}\hbar}} \left(x - \frac{\hbar}{m\omega} \frac{d}{dx} \right) \sqrt{\frac{\cancel{2}m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} x\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \frac{m\omega}{\sqrt{2}\hbar} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(x - \frac{\hbar}{m\omega} \frac{d}{dx} \right) x\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \frac{m\omega}{\sqrt{2}\hbar} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(x^2 - \frac{\hbar}{m\omega} \left(1 - \frac{\cancel{2}m\omega}{\cancel{2}\hbar} x^2 \right) \right) e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \frac{m\omega}{\sqrt{2}\hbar} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(x^2 - \frac{\hbar}{m\omega} + x^2 \right) e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \frac{m\omega}{\sqrt{2}\hbar} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(2x^2 - \frac{\hbar}{m\omega} \right) e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(\frac{2m\omega}{\hbar} x^2 - 1 \right) e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\underline{\psi_2 = \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \left(\frac{2m\omega}{\hbar} x^2 - 1 \right) e^{-\frac{m\omega}{2\hbar}x^2}}$$

b)

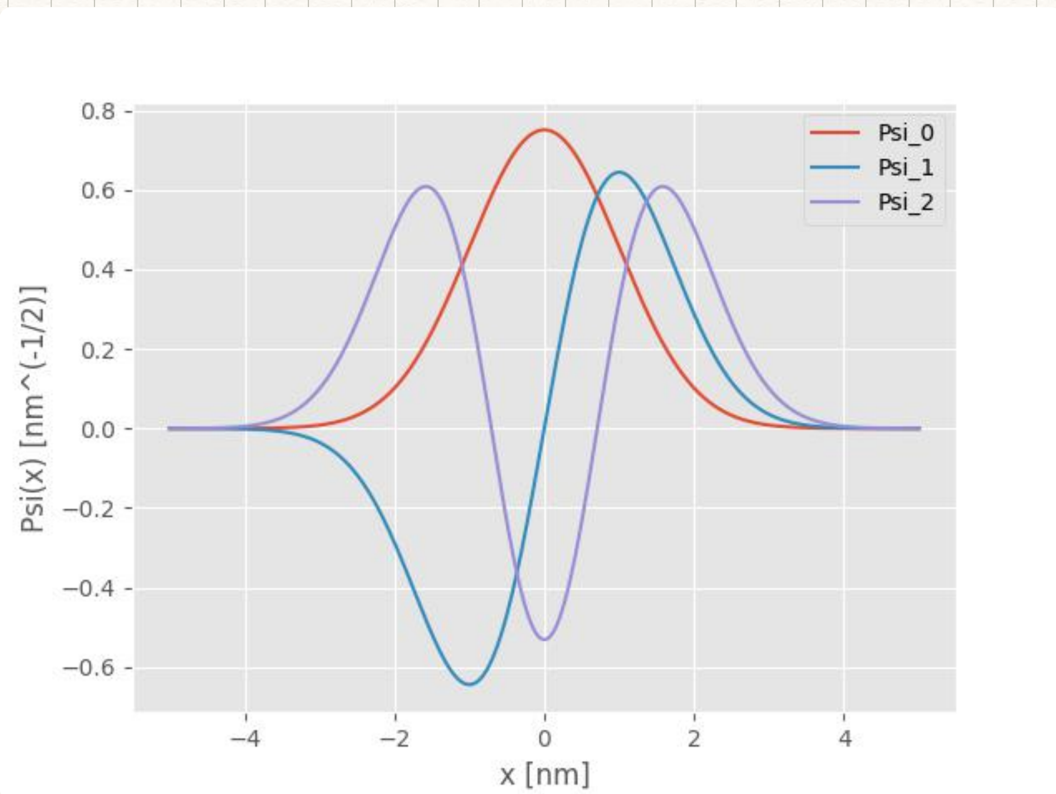
$$\frac{m\omega}{\hbar} = 1$$

$$\frac{m\sqrt{\frac{k}{m}}}{\hbar} = 1$$

$$\sqrt{mk} = \hbar$$

$$mk = \hbar^2$$

$$m = \frac{\hbar^2}{k}$$




```python
from __future__ import division
import numpy as np
import matplotlib.pyplot as plt
plt.style.use("ggplot")


h = 6.626e-34
hbar = h/(2*np.pi)
k = 1
m = (hbar)**2/k
omega = np.sqrt(k/m)
gamma = m*omega/hbar

x = np.linspace(-5, 5, 10000)

def psi_0(x):
    return (gamma/np.pi)**(1/4)*np.exp(-(gamma/2)*x**2)

def psi_1(x):
    return (gamma/np.pi)**(1/4) * np.sqrt(2*gamma)* x *np.exp(-(gamma/2)*x**2)

def psi_2(x):
    return 1/np.sqrt(2) * (gamma/np.pi)**(1/4)*(2*gamma*x**2-1)*np.exp(-(gamma/2)*x**2)

plt.plot(x, psi_0(x), x, psi_1(x), x, psi_2(x))
plt.xlabel("x [nm]")
plt.ylabel("Psi(x) [nm^(-1/2)]")
plt.legend(["Psi_0", "Psi_1", "Psi_2"])
plt.savefig("oblig6.png")
plt.show()
```

c) $\int_{-\infty}^{\infty} \psi_0^* \cdot \psi_1 \, dx$ vil uansett bli 0 ettersom

ψ_1 er antisymmetrisk rundt $x = 0$

og ψ_0 som er symmetrisk vil altså

bare "scale" ψ_1 og beholde antisymmetrien.

Det samme gjelder $\int_{-\infty}^{\infty} \psi_1^* \cdot \psi_2^* \, dx$.

Dermed trenger vi bare å regne ut

$$\int_{-\infty}^{\infty} \psi_0^* \cdot \psi_2 \, dx$$

$$\psi_0^* \cdot \psi_2 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2} \cdot \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \left(\frac{2m\omega}{\hbar}x^2 - 1\right) e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}} \left(\frac{2m\omega}{\hbar}x^2 - 1\right) e^{-\frac{m\omega}{\hbar}x^2}$$

$$= \sqrt{\frac{m\omega}{2\pi\hbar}} \left(\frac{2m\omega}{\hbar}x^2 e^{-\frac{m\omega}{\hbar}x^2} - e^{-\frac{m\omega}{\hbar}x^2}\right)$$

$$\sqrt{\frac{m\omega}{2\pi\hbar}} \int_{-\infty}^{\infty} \left(\frac{2m\omega}{\hbar}x^2 e^{-\frac{m\omega}{\hbar}x^2} - e^{-\frac{m\omega}{\hbar}x^2}\right) dx$$

$$\int \left(2Ax^2 e^{-Ax^2} - e^{-Ax^2}\right) dx$$

$$\boxed{A = \frac{m\omega}{\hbar}}$$

$$\int \left(-e^{-Ax^2}\right) dx$$

$$\boxed{\begin{array}{l|l} u = -e^{-Ax^2} & v' = dx \\ u' = 2Axe^{-Ax^2} dx & v = x \end{array}}$$

$$= u \cdot v - \int u' \cdot v$$

$$= -xe^{-Ax^2} - \int 2Ax^2 e^{-Ax^2} dx$$

Dermed har vi:

$$-xe^{-Ax^2} - \cancel{\int 2Ax^2 e^{-Ax^2} dx} + \cancel{\int 2Ax^2 e^{-Ax^2} dx}$$

$$= -xe^{-Ax^2} = -xe^{-\frac{m\omega}{\hbar}x^2}$$

$$\sqrt{\frac{m\omega}{2\pi\hbar}} \int_{-\infty}^{\infty} \left(\frac{2m\omega}{\hbar} x^2 e^{-\frac{m\omega}{\hbar}x^2} - e^{-\frac{m\omega}{\hbar}x^2}\right) dx$$

$$= \sqrt{\frac{m\omega}{2\pi\hbar}} \left[-xe^{-\frac{m\omega}{\hbar}x^2}\right]_{-\infty}^{\infty}$$

$$= \sqrt{\frac{m\omega}{2\pi\hbar}} (0 - 0) \quad \left(e^{-\infty} = 0 \text{ for begge grenser}\right)$$

$$\underline{\underline{= 0}}$$ Dermed er disse også ortogonale.

2

a)

$$\alpha = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}$$

$$u = \sqrt{\frac{m\omega}{\hbar}}\, x$$

$$\alpha^2 x = \frac{1}{\sqrt{\pi}} u$$

$$\underline{u = \alpha^2 x \sqrt{\pi}}$$

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2} = \alpha\, e^{-\frac{1}{2}u^2}$$

$$\psi_1 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \sqrt{\frac{2m\omega}{\hbar}}\, x\, e^{-\frac{m\omega}{\hbar}x^2} = \alpha\sqrt{2}\, u\, e^{-\frac{1}{2}u^2}$$

Vi vet at ψ_0 er symmetrisk og ψ_1 er antisymmetrisk rundt 0. Derfor er både $|\psi_0|^2$ og $|\psi_1|^2$ symmetriske.

Dermed vil $x|\psi_0|^2$ og $x|\psi_1|^2$ være antisymmetriske, og integralet fra $-\infty$ til ∞ blir 0 for begge:

$$\langle x \rangle = \int_{-\infty}^{\infty} x\,|\psi_n|^2\, dx = 0, \quad \text{for} \quad n = 1, 2.$$

Ettersom $\langle p \rangle$ defineres som:

$\langle p \rangle = m\dfrac{d\langle x \rangle}{dt}$ vil begge disse også bli 0.

ψ_0:

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2 \, |\psi_0|^2 \, dx$$

$$= \int_{-\infty}^{\infty} \alpha^2 x^2 e^{-u^2} \, dx$$

$u = \alpha^2 x \sqrt{\pi} \Rightarrow x = \frac{u}{\alpha^2\sqrt{\pi}}$

$\frac{du}{dx} = \alpha^2\sqrt{\pi} \Rightarrow dx = \frac{1}{\alpha^2\sqrt{\pi}} du$

$$= \int_{-\infty}^{\infty} \alpha^2 \cdot \frac{u^2}{\alpha^4 \cdot \pi} \cdot e^{-u^2} \cdot \frac{1}{\alpha^2\sqrt{\pi}} \, du$$

$$= \frac{1}{\alpha^4 \pi^{\frac{3}{2}}} \int_{-\infty}^{\infty} u^2 \cdot e^{-u^2} \, du$$

$$= \frac{1}{\alpha^4 \pi^{\frac{3}{2}}} \cdot \frac{\sqrt{\pi}}{2}$$

$$= \frac{1}{2\alpha^4 \pi}$$

$$= \underline{\underline{\frac{\hbar}{2m\omega}}}$$

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \psi_0^* \left(\frac{\hbar}{i} \frac{d}{dx} \right)^2 \psi_0 \, dx$$

$$u = \sqrt{\frac{m\omega}{\hbar}}\, x$$

$$\alpha = \left(\frac{m\omega}{\pi \hbar} \right)^{\frac{1}{4}}$$

$$\Downarrow$$

$$u = \alpha^2 \sqrt{\pi}\, x$$

$$\frac{du}{dx} = \alpha^2 \sqrt{\pi}$$

$$\boxed{\begin{aligned} \frac{\hbar}{i} \frac{d}{dx} &= \frac{\hbar}{i} \frac{du}{dx} \frac{d}{du} \\ &= \frac{\hbar}{i} \alpha \sqrt{\pi} \frac{d}{du} \end{aligned}}$$

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \psi_0^* \left(\frac{\hbar}{i} \alpha^2 \sqrt{\pi} \frac{d}{du} \right)^2 \psi_0 \, dx$$

$$= \int_{-\infty}^{\infty} \psi_0^* \left(-\hbar^2 \alpha^4 \pi \frac{d^2 \psi_0}{du^2} \right) dx$$

$$\frac{d^2 \psi_0}{du^2} = \frac{d}{du} \left(\frac{d\left(\alpha e^{-\frac{1}{2}u^2}\right)}{du} \right)$$

$$= \frac{d}{du} \left(-\alpha u e^{-\frac{1}{2}u^2} \right)$$

$$= \alpha e^{-\frac{1}{2}u^2} (u^2 - 1)$$

$$\int_{-\infty}^{\infty} \psi_0^* \left(-\hbar^2 \alpha^4 \pi \cdot \alpha e^{-\frac{1}{2}u^2}(u^2-1)\right) dx$$

$$\int_{-\infty}^{\infty} \alpha e^{-\frac{1}{2}u^2} \left(-\hbar^2 \alpha^5 \pi e^{-\frac{1}{2}u^2}(u^2-1)\right) dx$$

$$\int_{-\infty}^{\infty} -\alpha^6 \hbar^2 \pi e^{-u^2}(u^2-1)\, dx$$

$$-\alpha^6 \hbar^2 \pi \int_{-\infty}^{\infty} \left(u^2 e^{-u^2} - e^{-u^2}\right) dx$$

$$-\frac{\alpha^6 \hbar^2 \pi}{\alpha^2 \sqrt{\pi}} \int_{-\infty}^{\infty} \left(u^2 e^{-u^2} - e^{-u^2}\right) du$$

$$-\alpha^4 \hbar^2 \sqrt{\pi} \left(-\frac{\sqrt{\pi}}{2}\right)$$

$$\frac{\alpha^4 \hbar^2 \pi}{2}$$

$$= \frac{m\omega \hbar^2 \pi}{2\hbar\pi} = \frac{1}{2} m\omega\hbar$$

$$u = \alpha^2 \sqrt{\pi}\, x$$

$$\frac{du}{dx} = \alpha^2 \sqrt{\pi}$$

$$dx = \frac{1}{\alpha^2 \sqrt{\pi}}\, du$$

ψ_1 :

$$\psi_1 = \alpha\sqrt{2}\,u\,e^{-\frac{1}{2}u^2}$$

$\langle x^2 \rangle$:

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} |\psi_1|^2 x^2 \, dx$$

$$= \int_{-\infty}^{\infty} 2\alpha^2 u^2 e^{-u^2} x^2 \, dx$$

$$\left| \begin{array}{l} u = \alpha^2\sqrt{\pi}\,x \Rightarrow x = \frac{1}{\alpha^2\sqrt{\pi}} u \\ dx = \frac{1}{\alpha^2\sqrt{\pi}} du \end{array} \right.$$

$$= \int_{-\infty}^{\infty} 2\alpha^2 u^2 e^{-u^2} \frac{1}{\alpha^4\pi} u^2 \frac{1}{\alpha^2\sqrt{\pi}} du$$

$$= \int_{-\infty}^{\infty} \frac{2}{\alpha^4\pi\sqrt{\pi}} u^4 e^{-u^2} du$$

$$= \frac{2}{\alpha^4\pi\sqrt{\pi}} \int_{-\infty}^{\infty} u^4 e^{-u^2} du$$

(Dette integralet gjør jeg lenger ned når jeg regner ut $\langle p^2 \rangle$ for ψ_1)

$$= \frac{2}{\alpha^4\pi\sqrt{\pi}} \left(\frac{3\sqrt{\pi}}{4} \right)$$

$$= \frac{3}{2\alpha^4\pi}$$

$$= \frac{3}{2} \frac{\hbar\cancel{\pi}}{m\omega\cancel{\pi}} = \underline{\underline{\frac{3}{2} \frac{\hbar}{m\omega}}}$$

$\langle p^2 \rangle$:

$$\psi_1 = \alpha\sqrt{2}\, u\, e^{-\frac{1}{2}u^2}$$

$$u = \alpha^2\sqrt{\pi}\, x$$

$$\frac{du}{dx} = \alpha^2\sqrt{\pi}$$

$$dx = \frac{1}{\alpha^2\sqrt{\pi}}\, du$$

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \psi_1^* \left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 \psi_1\, dx$$

$$= \int_{-\infty}^{\infty} \psi_1^* \left(\frac{\hbar\alpha^2\sqrt{\pi}}{i}\frac{d}{du}\right)^2 \psi_1\, dx$$

$$= \int_{-\infty}^{\infty} \psi_1^* \left(-\hbar^2\alpha^4\pi\, \frac{d^2\psi_1}{du^2}\right) dx$$

$$= \int_{-\infty}^{\infty} \alpha\sqrt{2}\, u\, e^{-\frac{1}{2}u^2} \left(-\hbar^2\alpha^4\pi\, \frac{d^2\psi_1}{du^2}\right) \cdot \frac{1}{\alpha^2\sqrt{\pi}}\, du$$

$$= \int_{-\infty}^{\infty} -\alpha^3\hbar^2\sqrt{2\pi}\, u\, e^{-\frac{1}{2}u^2} \frac{d^2\psi_1}{du^2}\, du$$

$$\frac{d^2\psi_1}{du^2} = \frac{d}{du}\left(\frac{d}{du}\left(\alpha\sqrt{2}\, u\, e^{-\frac{1}{2}u^2}\right)\right)$$

$$= \frac{d}{du}\left(-\sqrt{2}\,\alpha\, e^{-\frac{1}{2}u^2}(u^2-1)\right)$$

$$= \sqrt{2}\,\alpha\, u\, e^{-\frac{1}{2}u^2}(u^2-3)$$

$$-\alpha^3 \hbar^2 \sqrt{2\pi} \int_{-\infty}^{\infty} u e^{-\frac{1}{2}u^2} \left(\sqrt{2}\, \alpha u e^{-\frac{1}{2}u^2} (u^2 - 3) \right) du$$

$$-2\alpha^4 \sqrt{\pi} \hbar^2 \int_{-\infty}^{\infty} u^2 e^{-u^2} (u^2 - 3)\, du$$

$$= -2\alpha^4 \sqrt{\pi} \hbar^2 \int_{-\infty}^{\infty} u^4 e^{-u^2} - 3u^2 e^{-u^2}\, du$$

$$= -2\alpha^4 \sqrt{\pi} \hbar^2 \left(\int_{-\infty}^{\infty} u^4 e^{-u^2}\, du - \int_{-\infty}^{\infty} 3u^2 e^{-u^2}\, du \right)$$

$$\int_{-\infty}^{\infty} u^4 e^{-u^2}\, du =$$

Rottmann s. 155

$$\int_0^{\infty} e^{-\lambda x^2} x^k\, dx = \frac{1}{2} \lambda^{-\frac{k+1}{2}} \Gamma\left(\frac{k+1}{2}\right)$$

Ettersom vi vet at u^4 og e^{-u^2} er symmetriske blir produktet også symmetrisk og vi kan bruke integralet fra 0 til ∞ gangert med 2.

$$2 \int_0^{\infty} e^{-1u^2} u^4\, du = 2 \cdot \frac{1}{2} (1)^{-\frac{5}{2}} \Gamma\left(\frac{5}{2}\right)$$

$$= \frac{3}{4} \sqrt{\pi}$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(x+1) = x\Gamma(x)$$

$$\Gamma\left(\tfrac{3}{2} + 1\right) = \tfrac{3}{2} \Gamma\left(\tfrac{3}{2}\right)$$

$$\tfrac{3}{2} \Gamma\left(\tfrac{1}{2} + 1\right) = \tfrac{3}{2} \cdot \tfrac{1}{2} \Gamma\left(\tfrac{1}{2}\right)$$

$$= \tfrac{3}{2} \cdot \tfrac{1}{2} \cdot \sqrt{\pi} = \tfrac{3}{4} \sqrt{\pi}$$

$$3\int_{-\infty}^{\infty} u^2 e^{-u^2}\,du = 2\int_{0}^{\infty} e^{-u^2} u^2\,du$$

$$= 2\cdot 3\cdot \frac{1}{2}\,(1)^{-\frac{3}{2}}\,\Gamma\left(\frac{3}{2}\right)$$

$$= 2\cdot 3\cdot \frac{1}{2}\cdot \frac{1}{2}\sqrt{\pi}$$

$$= 2\cdot 3\cdot \frac{1}{4}\sqrt{\pi}$$

$$= \frac{3}{2}\sqrt{\pi}$$

$$-2a^4\sqrt{\pi}\hbar^2\left(\int_{-\infty}^{\infty} u^4 e^{-u^2}\,du - \int_{-\infty}^{\infty} 3u^2 e^{-u^2}\,du\right)$$

$$= -2a^4\sqrt{\pi}\hbar^2\left(\frac{3}{4}\sqrt{\pi} - \frac{3}{2}\sqrt{\pi}\right)$$

$$= -2a^4\sqrt{\pi}\,\hbar^2\left(-\frac{3}{4}\sqrt{\pi}\right)$$

$$= \frac{3}{2}a^2\pi\hbar^2$$

$$= \frac{3}{2}\left(\frac{m\omega}{\hbar\pi}\right)\pi\hbar^2$$

$$= \frac{3}{2}m\omega\hbar$$

b) Ψ_0:

$$\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} \qquad \sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

$$= \sqrt{\langle x^2\rangle} \qquad = \sqrt{\langle p^2\rangle}$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \qquad = \sqrt{\frac{m\omega\hbar}{2}}$$

Uskarphetsprinsippet: $\sigma_x \sigma_p \geq \frac{\hbar}{2}$

$$\sqrt{\frac{\hbar}{2\cancel{m\omega}}} \cdot \sqrt{\frac{\cancel{m\omega}\hbar}{2}} = \frac{\hbar}{2}$$

$$\sqrt{\frac{\hbar^2}{4}} = \frac{\hbar}{2}$$

$$\underline{\frac{\hbar}{2} = \frac{\hbar}{2}}$$

Altså gjelder uskarphetsprinsippet for Ψ_0

Ψ_1:

$$\sigma_x = \sqrt{\frac{3\hbar}{2m\omega}} \qquad \sigma_p = \sqrt{\frac{3m\omega\hbar}{2}}$$

$$\sigma_x \sigma_p = \sqrt{\frac{3\hbar}{2\cancel{m\omega}}} \cdot \sqrt{\frac{3\cancel{m\omega}\hbar}{2}}$$

$$= \sqrt{\frac{9\hbar^2}{4}}$$

$$\underline{= \frac{3\hbar}{2} > \frac{\hbar}{2}}$$

c)

Forventningsverdien for den totale energien:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

Forv. verdi for kinetisk energi:

$$\langle K \rangle - \frac{\hbar^2}{2m}\int \psi^* \frac{d^2\psi}{dx^2}\,dx$$

og pot. energi:

$$\langle V \rangle = \langle \tfrac{1}{2} m\omega^2 x^2 \rangle = \tfrac{1}{2} m\omega^2 \langle x^2 \rangle$$

ψ_0:

<u>$\langle K \rangle$</u>:

$$\langle K \rangle = \frac{p^2}{2m} = \frac{\langle p^2 \rangle}{2m}$$

$$= \frac{\cancel{m}\omega\hbar}{4\cancel{m}}$$

$$= \frac{1}{4}\omega\hbar$$

$\langle V \rangle$:

$$\langle V \rangle = \frac{1}{2} m\omega^2 \langle x^2 \rangle$$

$$= \frac{1}{2} \cancel{m} \omega^{\cancel{2}} \cdot \frac{\hbar}{2 \cancel{m\omega}}$$

$$= \underline{\frac{1}{4} \omega \hbar}$$

$$E_n = \left(n + \frac{1}{2}\right) \hbar\omega$$

$$\underline{= \frac{1}{2} \hbar\omega \quad = \frac{1}{4} \omega\hbar + \frac{1}{4} \omega\hbar}$$

Altså er summen av forventningsverdiene for V og K lik den totale Energien E_0

ψ_1:

$\langle K \rangle$: $$\langle K \rangle = \frac{\langle p^2 \rangle}{2m}$$

$$= \frac{3 \cancel{m} \omega \hbar}{4 \cancel{m}}$$

$$= \underline{\frac{3}{4} \omega h}$$

$\langle V \rangle$: $\langle V \rangle = \frac{1}{2} m\omega^2 \langle x^2 \rangle$

$$= \frac{1}{2} m\omega^2 \cdot \frac{3\hbar}{2m\omega}$$

$$\underline{= \frac{3}{4} \omega\hbar}$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$= \frac{3}{2}\hbar\omega = \frac{3}{4}\hbar\omega + \frac{3}{4}\hbar\omega$$

Forventningsverdiene stemmer også her.