Oblig 2 - MAT1120

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1.

$$A = \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \\ \end{array}$$
$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix} \qquad W = \mathrm{Col}\, A$$

a) Finn en basis for W:

Radreduserer A, og får

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\uparrow \quad \uparrow \quad \uparrow$$

De tre første kolonnene er pivotsøyler, og dermed er underrommet som utspennes av v_1, v_2 og v_3 i A en basis for W:

$$\text{Span}\left\{\begin{pmatrix}1\\0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right\}$$

Bruk Gram-Schmidts ortogonaliseringsprosess til å finne en ortogonal basis $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ for W:

$$\vec{u}_1 = \vec{v}_1 = \underline{\begin{pmatrix}1\\0\\1\\1\end{pmatrix}}$$

$$\vec{u}_2 = \vec{v}_2 - \text{proj}_{u_1}(\vec{v}_2)$$

$$= \begin{pmatrix}1\\1\\0\\0\end{pmatrix} - \frac{\begin{pmatrix}1\\0\\1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\1\\0\\0\end{pmatrix}}{\begin{pmatrix}1\\0\\1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\0\\1\\1\end{pmatrix}} \cdot \begin{pmatrix}1\\0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix} - \frac{1}{3}\begin{pmatrix}1\\0\\1\\1\end{pmatrix} = \underline{\begin{pmatrix}\frac{2}{3}\\1\\-\frac{1}{3}\\-\frac{1}{3}\end{pmatrix}}$$

$$\vec{u}_3 = \vec{v}_3 - proj_{u_1}(\vec{v}_3) - proj_{u_2}(\vec{v}_3)$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{3}\vec{u}_1 + \frac{1}{5}\vec{u}_2$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{1}{3} \\ 0 \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} + \begin{pmatrix} \frac{2}{15} \\ \frac{1}{5} \\ -\frac{1}{15} \\ -\frac{1}{15} \end{pmatrix} = \underline{\begin{pmatrix} -\frac{1}{5} \\ \frac{1}{5} \\ -\frac{2}{5} \\ \frac{3}{5} \end{pmatrix}}$$

$$\vec{u}_4 = \vec{v}_4 - proj_{u_1}(\vec{v}_4) - proj_{u_2}(\vec{v}_4) - proj_{u_3}(\vec{v}_4)$$

$$= \begin{pmatrix} 2 \\ 1 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{5}{3} \\ 0 \\ \frac{5}{3} \\ \frac{5}{3} \end{pmatrix} - \frac{4}{5}\begin{pmatrix} \frac{2}{3} \\ 1 \\ -\frac{1}{3} \\ -\frac{1}{3} \end{pmatrix} - \begin{pmatrix} -\frac{1}{5} \\ \frac{1}{5} \\ -\frac{2}{5} \\ \frac{3}{5} \end{pmatrix}$$

$$= \underline{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}$$

(Ettersom jeg har prestert å kalle kolonene i A for $\vec{v}_i$ så kaller jeg vektorene i den ortogonale basisen $\vec{e}_i$)

$$\vec{e}_1 = \operatorname{norm}(\vec{u}_1) = \frac{\vec{u}_1}{|\vec{u}_1|} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{1^2+1^2+1^2}} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

$$\vec{e}_2 = \operatorname{norm}(\vec{u}_2) = \begin{pmatrix} \frac{2}{\sqrt{15}} \\ \sqrt{\frac{3}{5}} \\ -\frac{1}{\sqrt{15}} \\ -\frac{1}{\sqrt{15}} \end{pmatrix}$$

$$\vec{e}_3 = \operatorname{norm}(\vec{u}_3) = \begin{pmatrix} -\frac{1}{\sqrt{15}} \\ \frac{1}{\sqrt{15}} \\ -\frac{2}{\sqrt{15}} \\ \sqrt{\frac{3}{5}} \end{pmatrix}$$

Ortogonal basis for W:

$$B = \left\{ \begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}, \begin{pmatrix} \frac{2}{\sqrt{15}} \\ \sqrt{\frac{3}{5}} \\ -\frac{1}{\sqrt{15}} \\ -\frac{1}{\sqrt{15}} \end{pmatrix}, \begin{pmatrix} -\frac{1}{\sqrt{15}} \\ \frac{1}{\sqrt{15}} \\ -\frac{2}{\sqrt{15}} \\ \sqrt{\frac{3}{5}} \end{pmatrix} \right\}$$

b)

$$\text{proj}_W(\vec{y}) = \langle \vec{y}, \vec{u}_1 \rangle \vec{u}_1 + \langle \vec{y}, \vec{u}_2 \rangle \vec{u}_2 + \langle \vec{y}, \vec{v}_3 \rangle \vec{v}_3$$

$$= 3 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3} \\ 1 \\ -\frac{1}{3} \\ -\frac{1}{3} \end{pmatrix} + \frac{1}{5} \begin{pmatrix} -\frac{1}{5} \\ \frac{1}{5} \\ -\frac{2}{5} \\ \frac{3}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 0 \\ 3 \\ 3 \end{pmatrix} + \begin{pmatrix} \frac{2}{3} \\ 1 \\ -\frac{1}{3} \\ -\frac{1}{3} \end{pmatrix} + \begin{pmatrix} -\frac{1}{25} \\ \frac{1}{25} \\ -\frac{2}{25} \\ \frac{3}{25} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{272}{75} \\ \frac{26}{25} \\ \frac{194}{75} \\ \frac{209}{78} \end{pmatrix}$$

$$\vec{e}_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{272}{75} \\ \frac{26}{25} \\ \frac{194}{75} \\ \frac{209}{78} \end{pmatrix} = \begin{pmatrix} -\frac{197}{75} \\ -\frac{1}{25} \\ -\frac{119}{75} \\ -\frac{134}{75} \end{pmatrix}$$

$$C = \left\{ \begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}, \begin{pmatrix} \frac{2}{\sqrt{15}} \\ \sqrt{\frac{3}{5}} \\ -\frac{1}{\sqrt{15}} \\ -\frac{1}{\sqrt{15}} \end{pmatrix}, \begin{pmatrix} -\frac{1}{\sqrt{15}} \\ \frac{1}{\sqrt{15}} \\ -\frac{2}{\sqrt{15}} \\ \sqrt{\frac{3}{5}} \end{pmatrix}, \begin{pmatrix} -\frac{197}{75} \\ -\frac{1}{25} \\ -\frac{119}{75} \\ -\frac{134}{75} \end{pmatrix} \right\}$$

Kan bekreftes ved at den radreduserte versonen av matrisen:

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{15}} & -\frac{1}{\sqrt{15}} & -\frac{197}{75} \\ 0 & \sqrt{\frac{3}{5}} & \frac{1}{\sqrt{15}} & -\frac{1}{25} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{15}} & -\frac{2}{\sqrt{15}} & -\frac{119}{75} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{15}} & \sqrt{\frac{3}{5}} & -\frac{134}{75} \end{bmatrix}$$

blir: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c)

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

A transponert blir da:

$$A^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix}$$

$$A^T A \vec{x^*} = A^T \vec{y}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix} \cdot \begin{pmatrix} x^* \\ y^* \\ z^* \\ w^* \end{pmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 6 & 3 & 3 & 5 \\ 3 & 2 & 1 & 2 \\ 3 & 1 & 2 & 3 \\ 5 & 2 & 3 & 6 \end{bmatrix} \cdot \begin{pmatrix} x^* \\ y^* \\ z^* \\ w^* \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 6 \end{pmatrix}$$

Utvidet matrise blir da:

$$\begin{bmatrix} 6 & 3 & 3 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ 3 & 1 & 2 & 3 & 1 \\ 5 & 2 & 3 & 6 & 6 \end{bmatrix} \underset{\sim}{\overset{\text{som radreduseres til}}{}} \begin{bmatrix} 1 & 0 & 1 & 0 & -4 \\ 0 & 1 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

dermed har vi

$$\begin{aligned} x + z &= -4 \\ y - z &= 4 \\ w &= 3 \\ z &= \text{fri} \end{aligned} \quad \Rightarrow \quad \begin{aligned} x &= -4 - z \\ y &= 4 + z \\ w &= 3 \\ z &= z \end{aligned}$$

$$x^* = \begin{pmatrix} -4 - z \\ 4 + z \\ z \\ 3 \end{pmatrix} \Rightarrow \underline{\underline{\begin{pmatrix} -4 \\ 4 \\ 0 \\ 3 \end{pmatrix} + z \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}}}$$

2.

$$A = \begin{bmatrix} 0 & -3 & -3 \\ -1 & 2 & -1 \\ -2 & -2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 3 & -6 \\ -1 & 0 & 0 \\ -2 & -6 & 3 \end{bmatrix}$$

$$\det(A - \lambda I_3) \qquad \det(B - \lambda I_3)$$

$$\begin{vmatrix} -\lambda & -3 & -3 \\ -1 & 2-\lambda & -1 \\ -2 & -2 & 1-\lambda \end{vmatrix} \qquad \begin{vmatrix} -\lambda & 3 & -6 \\ -1 & -\lambda & 0 \\ -2 & -6 & 3-\lambda \end{vmatrix}$$

$$\Downarrow \qquad \Downarrow$$

$$\underline{-\lambda^3 + 3\lambda^2 + 9\lambda - 27} \qquad \underline{-\lambda^3 + 3\lambda^2 + 9\lambda - 27}$$

b)

$$\Downarrow$$

$$\lambda_1 = -3 \qquad \lambda_2 = 3$$

$$\vec{v} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

Dersom $\vec{v}$ er en egenvektor til A, skal:

$$A\vec{v} = \lambda \vec{v}$$

$$\begin{bmatrix} 0 & -3 & -3 \\ -1 & 2 & -1 \\ -2 & -2 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 + 0 + 0 \\ -3 + 2 - 2 \\ 3 + 1 - 1 \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} = \lambda_1 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$= (-3) \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\underline{\underline{\begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix}}}$$

Finn alle egenverdiene og basiser for egenrommene til både A og B.

Egenverdier funnet i a) ved å finne nullpkt. til de karakteristiske polynomene:

$\lambda_1 = -3 \qquad \lambda_2 = 3$

Basis for egenrom:

A:

$$A - \lambda I_3 = 0$$

$$\begin{bmatrix} -\lambda & -3 & -3 & 0 \\ -1 & 2-\lambda & -1 & 0 \\ -2 & -2 & 1-\lambda & 0 \end{bmatrix}$$

$\lambda_1 = -3$: $\begin{bmatrix} 3 & -3 & -3 & 0 \\ -1 & 5 & -1 & 0 \\ -2 & -2 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$\Downarrow$$

$$x - \tfrac{3}{2}z = 0 \qquad\qquad x = \tfrac{3}{2}z$$

$$y - \tfrac{1}{2}z = 0 \quad \Rightarrow \quad y = \tfrac{1}{2}z$$

$$z = \text{fri}$$

velger $\underline{\underline{z=2}}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{3}{2}z \\ \frac{1}{2}z \\ z \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} z = \underline{\underline{\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}}}$$

basis for egenrommet

tilhørende $\lambda = -3$

for både A og B

$$\lambda_2 = 3: \quad \left[\begin{array}{ccc|c} -\lambda & -3 & -3 & 0 \\ -1 & 2-\lambda & -1 & 0 \\ -2 & -2 & 1-\lambda & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -3 & -3 & -3 & 0 \\ -1 & -1 & -1 & 0 \\ -2 & -2 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Downarrow$$

$$x + y + z = 0 \quad \Rightarrow x = -y - z$$

$$y = \text{fri}$$

$$z = \text{fri}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y - z \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ y \\ 0 \end{pmatrix} + \begin{pmatrix} -z \\ 0 \\ z \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} y + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} z$$

velger $z = 1$ og $y = 1$ og basisen blir:

$$\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

tilhørende $\lambda_2 = 3$ for A

For B:

$$\begin{bmatrix} -\lambda & 3 & -6 \\ -1 & -\lambda & 0 \\ -2 & -6 & 3-\lambda \end{bmatrix} = \begin{bmatrix} -3 & 3 & -6 \\ -1 & -3 & 0 \\ -2 & -6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x + \tfrac{3}{2} z &= 0 \\ y - \tfrac{1}{2} z &= 0 \end{aligned} \implies \begin{aligned} x &= -\tfrac{3}{2} z \\ y &= \tfrac{1}{2} z \\ z &= \text{fri} \end{aligned}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} z \\ \frac{1}{2} z \\ z \end{pmatrix} \overset{z=2}{=} \underline{\underline{\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}}}$$

basis for egenrommet tilhørende $\lambda = 3$ for B

c) Ettersom egenrommene til A er:

$$\left\{\begin{pmatrix}3\\1\\2\end{pmatrix}\right\} \quad \text{og} \quad \left\{\begin{pmatrix}-1\\1\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\1\end{pmatrix}\right\}$$

ser vi at summen av dimensjonene til egenrommene er 3, som er lik antall dimensjoner i A. Dermed er A diagonaliserbar.

$$P = \begin{bmatrix}\vec{v}_1 & \vec{v}_2 & \vec{v}_3\end{bmatrix} \qquad D = \begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_2\end{bmatrix}$$

$$\underline{P = \begin{bmatrix}3 & -1 & -1\\ 1 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}} \qquad \underline{D = \begin{bmatrix}-3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}}$$

Summen av dimensjonene til egenrommene til B er $2 \neq 3$, og dermed er B <u>ikke</u> diagonaliserbar.

3.

Ettersom A er symmetrisk, dvs., $A = A^T$, så vet vi at A må være ortogonalt diagonaliserbar

$$\det(A - \lambda I_3)$$

$$\begin{vmatrix} 1-\lambda & 0 & \sqrt{2} \\ 0 & 2-\lambda & 0 \\ \sqrt{2} & 0 & -\lambda \end{vmatrix} = (2-\lambda)(\lambda^2 - \lambda - 2)$$

$$(2-\lambda)(\lambda^2 - \lambda - 2) = 0$$

$$\Downarrow$$

$$\lambda_1 = -1$$

$$\lambda_2 = 2$$

$\lambda_1 = -1$:

$$\begin{bmatrix} 2 & 0 & \sqrt{2} \\ 0 & 3 & 0 \\ \sqrt{2} & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Downarrow$$

$$x = -\frac{1}{\sqrt{2}} z$$

$$y = 0$$

$$z = \text{fri}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} z \\ 0 \\ z \end{pmatrix} \overset{z=1}{=} \underline{\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ 1 \end{pmatrix}}$$

$\lambda_2 = 2$:

$$\begin{bmatrix} -1 & 0 & \sqrt{2} \\ 0 & 0 & 0 \\ \sqrt{2} & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\sqrt{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x = \sqrt{2}\, z$$

$$y = \text{fri}$$

$$z = \text{fri}$$

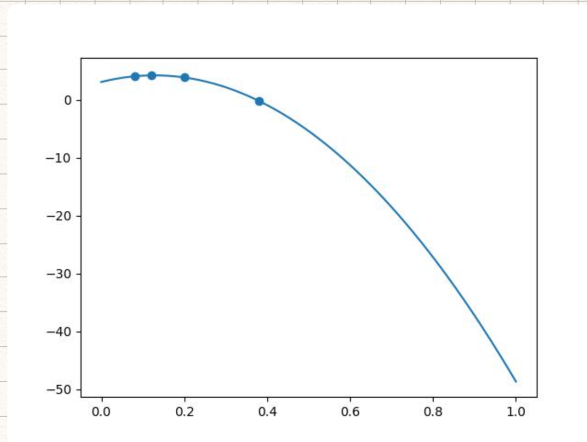
$$\begin{pmatrix} \sqrt{2}z \\ y \\ z \end{pmatrix} \overset{z=1}{=} \begin{pmatrix} \sqrt{2} \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$P = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

4.

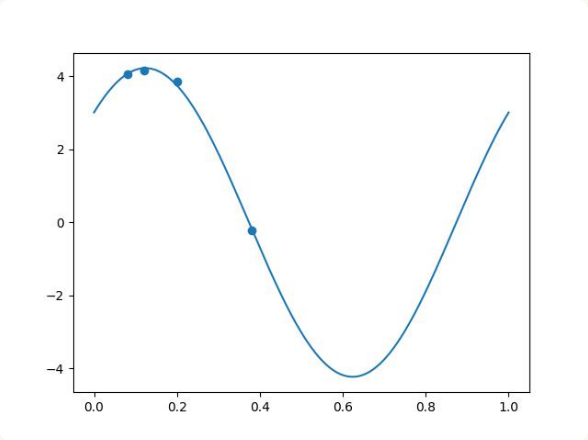
a)

```python
from __future__ import division
import numpy as np
import matplotlib.pyplot as plt

data = np.array([
        [0.08, 4.05],
        [0.12, 4.15],
        [0.20, 3.85],
        [0.38, -0.22]
    ])

def least_squares_fit_1():
    #Design matrix
    X = np.insert(data, 0, 1, axis=1)[:, :-1]
    X = np.insert(X, 2, X[:, 1:].flatten()**2, axis=1)

    #y-values
    y = data[:, 1]

    X_T = np.transpose(X)

    left_side = np.dot(X_T, X)
    right_side = np.dot(X_T, y)

    #inverse left side and dot with right side

    B = np.dot(np.linalg.inv(left_side), right_side)

    x = np.linspace(0, 1, 1000)
    plt.plot(x, B[0] + B[1]*x + B[2]*x**2)
    plt.scatter(data[:, 0], data[:, 1])
    #plt.show()

    return B
```



b)

```python
def least_squares_fit_2():
    #Design matrix
    x_values = data[:, :-1]

    X = np.sin(2*np.pi*x_values)
    X = np.insert(X, 1, np.cos(2*np.pi*x_values).flatten(), axis=1)

    y_values = data[:, 1:]

    X_T = np.transpose(X)

    left_side = np.dot(X_T, X)
    right_side = np.dot(X_T, y_values)

    a_b = np.dot(np.linalg.inv(left_side), right_side)

    x = np.linspace(0, 1, 1000)
    plt.plot(x, a_b[0]*np.sin(2*np.pi*x) + a_b[1]*np.cos(2*np.pi*x))
    plt.scatter(data[:, 0], data[:, 1])
    #plt.show()

    return a_b

```



c)

```python
def calculate_error(beta, a_b):
    #Beta-values from A:
    x_values = data[:, :-1]
    y_values = data[:, 1:]

    epsilon1 = y_values - (beta[0] + beta[1]*x_values + beta[2]*x_values**2)
    norm1 = np.linalg.norm(epsilon1)

    epsilon2 = y_values - (a_b[0]*np.sin(2*np.pi*x_values) + a_b[1]*np.cos(2*np.pi*x_values))
    norm2 = np.linalg.norm(epsilon2)

    print norm1, norm2


calculate_error(least_squares_fit_1(), least_squares_fit_2())
```

Terminal

```
1x-193-157-250-219:MAT1120 joakimflatby$ python oppg4.py
0.0452961133406 0.137044526055
```

Ut ifra dette mener jeg tilpasningen i oppgave a) er best. Dette passer også med at man kan se på plottet i b) at noen punkter ikke passer så bra.