Solutions Notebook for The Real Numbers and Real Analysis by Ethan D. Bloch

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4.2 The Derivative

Definition 4.2.1. Let $I \subseteq \mathbb{R}$ be an open interval, let $c \in I$, and let $f : I \to \mathbb{R}$ be a function.

1. The function f is **differentiable** at c if
$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c} \tag{4.1}$$
exists; if this limit exists, it is called the **derivative** of f at c, and it is denoted $f'(c)$.

2. The function f is **differentiable** if it is differentiable at every number in I. If f is differentiable, the **derivative** of f is the function $f' : I \to \mathbb{R}$ whose value at x is $f'(x)$ for all $x \in I$.

Lemma 4.2.2. Let $I \subseteq \mathbb{R}$ be an open interval, let $c \in I$, and let $f : I \to \mathbb{R}$ be a function. Then f is differentiable at c if and only if
$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \tag{4.2}$$
exists, and if this limit exists it equals $f'(c)$.

4.2.1 Exercises

Exercise 4.2.1. Using only the definition of derivatives and Lemma 4.2.2, find the derivative of each of the following functions.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x - 8$ for all $x \in \mathbb{R}$.
2. Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x^3$ for all $x \in \mathbb{R}$.

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